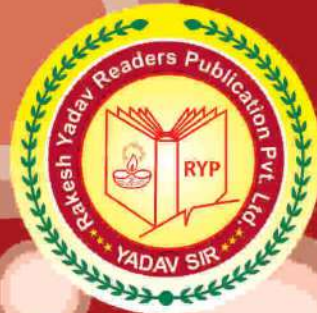


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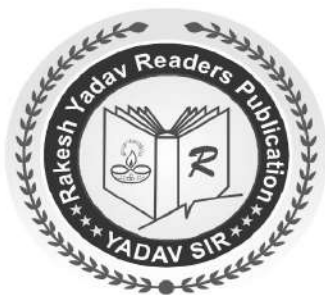
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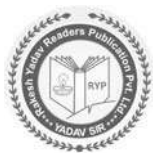
My Beloved Daughters
Unnati Yadav & Pragati Yadav

जीतने की इच्छा सभी में होती है, मगर जीतने के लिए तैयारी करने की इच्छा बहुत कम लोगों में होती है।

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Editor-in-chief
Karan Chaudhary

Preface

Nothing thrills a writer more than the success of his book. With this book, I hope to reach a much wider section of the student community and others, who relentlessly compete for various Government – jobs.

I am thankful to Almighty and my family (My parents, brother, wife, daughters and son), who extended their help in various invisible ways. I sincerely hope, the book **ADVANCE MATHS** will meet a good response. I would humbly appreciate suggestions, doubt, etc. concerned with this book at the following.

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Author
Rakesh Yadav

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UNIT DIGIT

Last Digit of number is called Unit Digit

1 2 3 4
↓
unit digit

In This no. 4 is unit digit.

The unit digit of the Resultant value depends upon The unit digits of all participating numbers.

Ex.1: $23 + 34 + 46 + 78 = 181$, unit digit of 181.

Sol. $\therefore$ unit digit = 1

It is clear that the unit digit of the Resultant value 181 depends upon the unit digits 3, 4, 6, 8

$$3 + 4 + 6 + 8 = 21$$

So, units digit = 1

Ex.2: What is the unit digit of ?

$$31 \times 37 \times 36 \times 46 \times 89$$

sol. $31 \times 37 \times 36 \times 46 \times 89$

Unit digit = 1, 7, 6, 6, 9

multiply the unit digits = $1 \times 7 \times 6 \times 6 \times 9$

$$\Rightarrow 1 \times 7 = 7$$

$$\Rightarrow 7 \times 6 = 42$$

$$\Rightarrow 2 \times 6 = 12$$

$$\Rightarrow 2 \times 9 = 18$$

unit digit = 8

Ex.3: What is the unit digit of?

$$31 \times 33 \times 37 \times 39 \times 43$$

Sol. $31 \times 33 \times 37 \times 39 \times 43$

multiply the unit digits

$$= 1 \times 3 \times 7 \times 9 \times 3$$

unit digit = 7

Ex.4: What is the unit digit of ?

$$91 \times 93 \times 95 \times 96 \times 97 \times 98$$

Sol. multiply the unit digit

$$1 \times 3 \times 5 \times 6 \times 7 \times 8 = 0$$

Ex.5: Find the unit digit of $135 \times 136 \times 170$

Sol. The unit digits = 5, 6, 0

multiply the units digit

$$= 5 \times 6 \times 0$$

= unit digit = 0

Ex.6: Find the unit digit at the product of all the odd prime numbers.

sol. The prime numbers are 3, 5, 7, 11, 13, 17, etc.

Now we know that if 5 is multiplied by any odd number it always gives the last digit 5. So the required unit digit will be '5',

Ex.7: Find the unit digit of $584 \times 328 \times 547 \times 613$

Sol. The unit digits = 4, 8, 7, 3
multiplying the unit digits
 $= 4 \times 8 \times 7 \times 3$
 $=$ unit digit = 2

Ex.8: Find the unit digit of the product of all the even numbers

Sol. The even number are 2, 4, 6, 8, 10, 12, etc.
Now we know that if '0' is multiplied by any number it always gives the last digit 0. so the required unit digit will be 0.

Ex.9: Find the unit digit 4!

$$\text{Sol. } 4! = 4 \times 3 \times 2 \times 1 = 24$$

unit digit = 4

Ex.10: Find the unit digit 5!

$$\text{Sol. } 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

unit digit = 0

* Factorial 5 and more than 5 express gives unit digit 0.

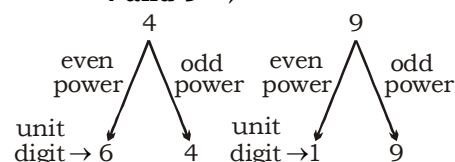
Unit digit when 'N' is Raised to a power
unit digit of 0, 1, 5 and 6 has any power (odd or even) no change

Ex.11: $(3765)^{437}$
unit digit = $(5)^{437} = 5$

Ex.12: $(6736)^{32567}$
unit digit = $(6)^{32567} = 6$

Ex.13: $(32541)^{325}$
unit digit = $(1)^{325} = 1$

* **4 and 9** $\rightarrow$



Ex.14: Find the unit place $(67354)^{1237}$

Sol. $(67354)^{1237}$
unit digit = $(4)^{1237} = (4)^{\text{odd power}}$
So, unit digit = 4

Ex.15: Find the unit place $(3259)^{1214}$

Sol. $(3259)^{1214}$
unit digit = $(9)^{1214} = (9)^{\text{even power}}$
unit digit = 1

Ex.16: Find the unit place $(6734)^{312}$

Sol. $(6734)^{312}$
unit digit = $(4)^{312} = (4)^{\text{even}}$
unit digit = 6

Rule of (2, 3, 7 and 8) $\rightarrow$

unit digit when 'N' is raised to a power

If the value of the power is

unit digit ↓	Power $\rightarrow$			
	1 or $4n+1$	2 or $4n+2$	3 or $4n+3$	4 or $4n+4$
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6

here $n \rightarrow$ Natural No.

If those number which unit digit 2, 3, 7 and 8. $\rightarrow$

all unit digit have cyclicity 4

Ex.18: Find the unit place 3^{35}

Sol. $3^{35} = 3^{32} \times 3^3$
Break the power form of $4n$
 $(3^4)^8 \times 3^3 = (\dots 1) \times (\dots 7)$
unit place = $1 \times 7 = 7$

Ex.19: Find the unit place $(127)^{39}$

Sol. $(127)^{39}$
unit place = $(7)^{39}$
 $= (7)^{36} \times (7)^3 = (7^4)^9 \times (7)^3$
 $= (\dots 1) \times (\dots 3)$
unit place = $1 \times 3 = 3$

Ex.20: Find the unit place $(678)^{562}$

Sol. $(678)^{562}$
unit digit = $(8)^{562}$
 $= (8)^{560} \times (8)^2$
 $= (8^4)^{140} \times (8)^2$
 $= (\dots 6) \times (\dots 4)$
unit digit = $6 \times 4 = 24 = 4$

Ex.21: Find the unit place $(327)^{640}$

Sol. $(327)^{640}$
unit digit = $(7)^{640}$
640 is multiple of 4
then = $(7^4)^{160}$
unit digit = $(1)^{160} = 1$

Ex.22: Find the unit digit of $(2137)^{753}$

Sol. $(2137)^{753}$
unit digit = $(7)^{753}$
= $(7)^{752} \times 7^1$
= $(7^4)^{188} \times 7^1$
= $(...1) \times 7$
unit digit = $1 \times 7 = 7$

Ex.23: Find the unit digit of $(13)^{2003}$

Sol. $(13)^{2003}$
unit digit = $(3)^{2003}$
= $3^{2000} \times 3^3$
= $(3^4)^{500} \times 3^3$
= $(...1)^{500} \times 27$
= $1 \times 27 = 27$
unit digit = 7

Ex.24: Find the unit digit of $(22)^{23}$

Sol. $(22)^{23}$
unit digit = $(2)^{23}$
= $(2)^{20} \times 2^3 = (2^4)^5 \times 8$
= $(...6)^5 \times 8$
unit digit = $6 \times 8 = 48 = 8$

Ex.25: Find the unit digit of $(37)^{105}$

Sol. $(37)^{105}$
unit digit = $(7)^{105}$
= $(7)^{104} \times 7^1$
= $(7^4)^{26} \times 7^1$
= $(...1)^{26} \times 7$
unit digit = $1 \times 7 = 7$

Ex.26: Find the unit place

$(23)^{21} \times (24)^{22} \times (26)^{23} \times (27)^{24} \times (25)^{25}$

Sol. $(23)^{21} \times (24)^{22} \times (26)^{23} \times (27)^{24} \times (25)^{25}$
unit digit = $(3)^{21} \times (4)^{22} \times (6)^{23} \times (7)^{24} \times (5)^{25}$

Break the power multiple of 4

$$\begin{array}{ccccccc} 3^{20} \times 3^1 & \times & 4^{22} & \times & 6^{23} & \times & (7^4)^6 \times 7^1 \times 5^{25} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{even} & & \text{same} & & \text{same} & & \text{same} \\ \text{power} & & \text{digit} & & \text{digit} & & \text{digit} \end{array}$$

= $3 \times 6 \times 6 \times 1 \times 5$
unit digit = 0

Note:- unit digit = even $\times 5 = '0'$

Ex.27: Find the unit place

$(235)^{215} + (314)^{326} + (6736)^{213} + (3167)^{112}$
unit digit

$(5)^{215} + (4)^{326} + (6)^{213} + (7)^{112}$
Same even power Same $(7^4)^{28}$

= $5 + 6 + 6 + 1$
unit digit = $18 = 8$

Ex.28: Find the unit place of

$\frac{12^{55}}{3^{11}} + \frac{8^{48}}{16^{18}}$

Sol. $\frac{12^{55}}{3^{11}} + \frac{8^{48}}{16^{18}}$

= $\frac{(3 \times 4)^{55}}{3^{11}} + \frac{(2^3)^{48}}{(2^4)^{18}}$

= $\frac{3^{55} \times 4^{55}}{3^{11}} + \frac{2^{144}}{2^{72}}$

= $3^{44} \times 4^{55} + 2^{72}$

unit digit = $(...1) \times (...4) + 6$

= $4 + 6 = 10$,

unit digit = 0

EXERCISE

- Find the unit digit of $584 \times 389 \times 476 \times 786$
(a) 7 (b) 3 (c) 4 (d) 6
- Find the unit digit of $641 \times 673 \times 677 \times 679 \times 681$
(a) 9 (b) 3 (c) 6 (d) 7
- Find the unit digit of $(5627)^{153} \times (671)^{230}$
(a) 7 (b) 9 (c) 3 (d) 1
- Find the unit digit of $(3625)^{333} \times (4268)^{645}$
(a) 6 (b) 3 (c) 4 (d) 0
- Find the unit digit of $(3694)^{1793} \times (615)^{317} \times (841)^{941}$
(a) 5 (b) 3 (c) 4 (d) 0
- Find the unit digit of $(7^{95} - 3^{58})$
(a) 7 (b) 3 (c) 4 (d) 0
- Find the unit place of $(17)^{1999} + (11)^{1999} - (7)^{1999}$
(a) 0 (b) 1 (c) 2 (d) 7
- Find the unit digit of $3^6 \times 4^7 \times 6^3 \times 7^4 \times 8^2 \times 9^5$
(a) 6 (b) 9 (c) 0 (d) 2
- Find the unit digit of $111!$ (factorial 111).
(a) 0 (b) 1 (c) 5 (d) 3
- The last digit of the number obtained by multiplying the numbers $81 \times 82 \times 83 \times 84 \times 85 \times 86 \times 87 \times 88 \times 89$ will be
(a) 0 (b) 9 (c) 7 (d) 2
- Find the units digit of the expression $25^{6251} + 36^{528} + 22^{853}$
(a) 4 (b) 3 (d) 6 (d) 5
- Find the units digit of the expression $55^{725} + 73^{5810} + 22^{853}$
(a) 4 (b) 0 (c) 6 (d) 5
- Find the units digit of the expression $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$.
(a) 1 (b) 9 (d) 7 (d) 0
- Find the last digit of the number $1^3 + 2^3 + 3^3 + 4^3 + \dots + 99^3$.
(a) 0 (b) 1 (d) 2 (d) 5
- Unit digit in $(264)^{102} + (264)^{103}$ is:
(a) 0 (b) 4 (c) 6 (d) 8
- Unit digit $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^4 + 259]$ is
(a) 1 (b) 4 (c) 5 (d) 6
- The unit digit in the expansion of $(2137)^{754}$ is
(a) 1 (b) 3 (c) 7 (d) 9
- The digit in unit's place of the product $(2153)^{167}$ is :
(a) 1 (b) 3 (c) 7 (d) 9
- The digit in unit's place of the product $(2464)^{1793} \times (615)^{317} \times (131)^{491}$ is
(a) 0 (b) 2 (c) 3 (d) 5
- What will be the unit digit in the product of 7^{105} ?
(a) 5 (b) 7 (c) 9 (d) 1
- what is the number of unit place in $(329)^{78}$
(a) 1 (b) 7 (c) 9 (d) 3
- unit digit of the number $(22)^{23}$ is:
(a) 4 (b) 6 (c) 6 (d) 8
- The unit digit in the product $(122)^{173}$ is:
(a) 2 (b) 4 (c) 6 (d) 8
- The unit digit in the sum of $(124)^{372} + (124)^{373}$ is :
(a) 5 (b) 4 (c) 2 (d) 0
- The last digit of $(1001)^{2008} + (1002)$ is:
(a) 0 (b) 3 (c) 4 (d) 6

26. Find the unit digit in the product: $(4387)^{245} \times (621)^{72}$.
(a) 1 (b) 2 (c) 5 (d) 7
27. The unit digit of the expression $25^{6251} + 36^{528} + 73^{54}$ is
(a) 6 (b) 5 (c) 4 (d) 0
28. The unit's digit in the product $7^{71} \times 6^{63} \times 3^{65}$ is
(a) 1 (b) 2 (c) 3 (d) 4
29. The last digit of 3^{40} is
(a) 1 (b) 3 (c) 7 (d) 9
30. The digit in unit's place of the number $(1570)^2 + (1571)^2 + (1572)^2 + (1573)^2$ is :
(a) 4 (b) 1 (c) 2 (d) 3
31. The unit digit in $3 \times 38 \times 537 \times 1256$ is
(a) 4 (b) 2 (c) 6 (d) 8
32. The unit digit in the product $(2467)^{153} \times (341)^{72}$ is
(a) 1 (b) 3 (c) 7 (d) 9
33. The unit digit in the product $(6732)^{170} \times (6733)^{172} \times (6734)^{174} \times (6736)^{176}$
(a) 1 (b) 3 (c) 4 (d) 5
34. Find the unit digit of the product of all the prime number between 1 and 99999
(a) 9 (b) 7 (c) 0 (d) N.O.T.
35. Find the unit digit of the product of all the elements of the set which consists all the prime numbers greater than 2 but less than 222.
(a) 4 (b) 5 (c) 0 (d) N.O.T.
36. Find the last digit of $222^{888} + 888^{222}$
(a) 2 (b) 6 (c) 0 (d) 8
37. Find the last digit of $32^{32^{32}}$
(a) 4 (b) 8 (c) 6 (d) 2
38. Find the last digit of the expression:
 $1^2 + 2^2 + 3^2 + 4^2 + \dots + 100^2$.
(a) 0 (b) 4 (c) 6 (d) 8
39. Find the unit digit of $1^1 + 2^2 + 3^3 + \dots + 10^{10}$.
(a) 9 (b) 7 (c) 0 (d) N.O.T.
40. Find the unit digit of $13^{24} \times 68^{57} + 24^{13} \times 57^{68} + 1234 + 5678$.
(a) 4 (b) 7 (c) 0 (d) 8
41. The unit digit of the expression
$$\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{100}$$

(a) 7 (b) 9 (c) 8 (d) N.O.T.
42. Find the unit digit of the expression $888^{92351} + 222^{92351} + 666^{23591} + 999^{99991}$.
(a) 5 (b) 9 (c) 3 (d) None of these
43. The last digit of the following expression is: $(1!)^1 + (2!)^2 + (3!)^3 + (4!)^4 + \dots + (10!)^{10}$
(a) 4 (b) 5 (c) 6 (d) 7
44. The last 5 digits of the following expression will be $(1!)^5 + (2!)^4 + (3!)^3 + (4!)^2 + (5!)^1 +$
 $(10!)^5 + (100!)^4 + (1000!)^3 + (10000!)^2 + (100000!)^1$
(a) 45939 (b) 00929
(c) 20929
(d) Can't determined
45. The unit digit of the following expression $(1!)^{99} + (2!)^{98} + (3!)^{97} + (4!)^{96} + \dots + (99!)^1$ is:
(a) 1 (b) 3 (c) 7 (d) 6
46. The unit digit of $(12345k)^{72}$ is 6. The value of k is:
(a) 8 (b) 6 (c) 2
(d) all of these
47. The unit digit of the expression $(1!)^{11} + (2!)^{21} + (3!)^{31} + \dots + (100!)^{1001}$
(a) 0 (b) 1 (c) 2 (d) 7
48. The last digit of the expression $4 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 \times \dots \times 4^{99} \times 9^{100}$ is :
(a) 4 (b) 6 (c) 9 (d) 1
49. The last digit of the expression $4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^6 + \dots + 4^{99} + 9^{100}$ is:
(a) 0 (b) 3 (c) 5
(d) None of these
50. The unit digit of $2^{3^4} \times 3^{4^5} \times 4^{5^6} \times 5^{6^7} \times 6^{7^8} \times 7^{8^9}$ is:
(a) 0 (b) 5
(c) Can't be determined
(d) None of these

ANSWER KEY

1. (d)	6. (c)	11. (b)	16. (b)	21. (a)	26. (d)	31. (d)	36. (c)	41. (c)	46. (d)
2. (a)	7. (b)	12. (c)	17. (d)	22. (d)	27. (d)	32. (c)	37. (c)	42. (b)	47. (d)
3. (a)	8. (a)	13. (b)	18. (c)	23. (a)	28. (d)	33. (c)	38. (a)	43. (d)	48. (b)
4. (d)	9. (a)	14. (a)	19. (a)	24. (d)	29. (a)	34. (c)	39. (b)	44. (b)	49. (a)
5. (d)	10. (a)	15. (a)	20. (b)	25. (b)	30. (a)	35. (b)	40. (a)	45. (c)	50. (a)

SOLUTION

1. (d) $584 \times 389 \times 476 \times 786$
unit digit 4, 9, 6, 6
Multiplying the unit digit
 $= 4 \times 9 \times 6 \times 6$
unit digit = 6
2. (a) $641 \times 673 \times 677 \times 679 \times 681$
unit digit = 1, 3, 7, 9, 1
Multiply the unit digit
 $= 1 \times 3 \times 7 \times 9 \times 1$
 $= 21 \times 9 = 189$

3. (a) $(5627)^{153} \times (671)^{230}$
unit digit $(7)^{153} \times (1)^{230}$
 $= (7)^{152} \times 7^1 \times 1$
 $= (7^4)^{38} \times 7 \times 1$
 $= (\dots 1)^{38} \times 7$
unit digit = $1 \times 7 = 7$
4. (d) $(3625)^{333} \times (4268)^{645}$
unit digit $(5)^{333} \times (8)^{645}$
 $= 5 \times (8)^{644} \times 8^1$

- $= 5 \times (8^4)^{161} \times 8^1$
 $= 5 \times (6)^{161} \times 8$
unit digit = $5 \times 6 \times 8 = 240 = 0$
5. (d) $(3694)^{1793} \times (615)^{317} \times (841)^{941}$
unit digit $(4)^{1793} \times (5)^{317} \times (1)^{941}$
4^{odd power} = 4
 $5^n = 5$
 $4 \times 5 \times 1 = 20$
Hence, unit digit = 0

6. (c) $7^{95} - 3^{58}$
 $= 7^{92} \times 7^3 - 3^{56} \times 3^2$
 $= (7^4)^{23} \times 343 - (3^4)^{14} \times 9$
 $= (\dots 1)^{23} \times 3 - (\dots 1)^{14} \times 9$
unit digit = $(\dots 3) - (\dots 9)$
 $= 13 - 9 = 4$
7. (b) $(17)^{1999} + (11)^{1999} - (7)^{1999}$
unit digit = $(7)^{1999} + (1)^{1999} - (7)^{1999}$
 $\therefore (7)^{1999} - (7)^{1999}$ gives = 0

Then, unit digit = 1

8. (a) Unit digit = $3^6 \times 4^7 \times 6^3 \times 7^4 \times 8^2 \times 9^5$

The unit digit of $3^6 = 3^4 \times 3^2 = 9$

The unit digit of $4^7 = 4$

The unit digit of $6^3 = 6$

The unit digit of $7^4 = 1$

The unit digit of $8^2 = 4$

The unit digit of $9^5 = 9^4 \times 9^1 = 9$

multiply the unit digits = $9 \times 4 \times 6 \times 1 \times 4 \times 9$

unit digit = 6

9. (a) $111! = 1 \times 2 \times 3 \times 4 \times 5 \times \dots \times 100 \times 111$

Since there is product of 5 and 2 hence it will give zero as the unit digit.

Hence the unit digit of $111!$ is 0 (zero).

10. (a) $81 \times 82 \times 83 \times 84 \times 85 \times 86 \times 87 \times 88 \times 89$

Unit digits = 1, 2, 3, 4, ..., 9

Product of unit digits

$= 1 \times 2 \times 3 \times \dots \times 9$

Because 5 multiply any even no. Then

we gets unit digit = 0

11. (b) $25^{6251} + 36^{528} + 22^{853}$
unit digit = $(5)^{6251} + (6)^{528} + (2)^{853}$

$$\begin{aligned} \text{unit digit} &= (\dots 5) + (\dots 6) + (2)^{853} \times 2 \\ &= (\dots 5) + (\dots 6) + (2^4)^{213} \times 2 \\ &= 5 + 6 + (6)^{213} \times 2 \end{aligned}$$

$$\begin{aligned} \text{Sum of unit digit} &= 5 + 6 + 6 \times 2 \\ &= 5 + 6 + 12 = 23 \end{aligned}$$

Hence, unit digit = 3

12. (c) $55^{725} + 73^{5810} + 22^{853}$
unit digit = $(5)^{725} + (3)^{5810} + (2)^{853}$
 $= (\dots 5) + (3^4)^{1452} \times 3^2 + (2^4)^{213} \times 2^1$
 $= 5 + (1)^{1452} \times 9 + (16)^{213} \times 2^1$
Sum of unit digit = $5 + 1 \times 9 + 6 \times 2 = 5 + 9 + 12 = 26$
unit digit = 6

13. (b) $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$
unit digit = $(1)^1 + (2)^2 + (3)^3 + (4)^4 + (5)^5 + (6)^6$

Sum of unit digit = $1 + 4 + 7 + 6 + 5 + 6 = 29$

unit digit = 9

14. (a) $1^3 + 2^3 + 3^3 + 4^3 \dots + 99^3$
Sum of cube of natural no.

$$= \left(\frac{n(n+1)}{2} \right)^2 = \left(\frac{99(99+1)}{2} \right)^2$$

$$= \left(\frac{99 \times 100}{2} \right)^2 = (99 \times 50)^2$$

$$= (4850)^2$$

Unit digit = 0

15. (a) $(264)^{102} + (264)^{103}$

unit digit

$$4^1 \rightarrow 4 \rightarrow 4$$

$$4^2 \rightarrow 16 \rightarrow 6$$

$$4^3 \rightarrow 64 \rightarrow 4$$

Rule: When 4 has odd power, then unit digit is: 4

When 4 has even power, then unit digit is 6

$$(264)^{102} + (264)^{103}$$

$$\begin{array}{c} \downarrow \quad \downarrow \\ (4)^{102} + (4)^{103} \end{array}$$

(even power) (odd power)

$$\text{unit digit } 6 + 4 = 10 \rightarrow 0$$

Alternate :

$$\Rightarrow (264)^{102} + (264)^{103}$$

$$\Rightarrow (264)^{102} (1 + 264)$$

$$\Rightarrow (264)^{102} \times 265$$

Multiplication of 5 & 2 = 0

Hence, unit digit is 0.

16. (b) $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^4 + (259)]$

unit place of 1, 5 and 6 will remain same

$$= [(1)^{98} + (1)^{29} - (6)^{100} + (5)^{35} - (6)^4 + 9]$$

$$= [1 + 1 - 6 + 5 - 6 + 9]$$

$$\Rightarrow 16 - 12 = 4$$

Hence, unit digit = 4

17. (d) $(2137)^{754}$
 $= (7)^{754}$ will give unit digit

$7^1 = 7$	unit digit $\rightarrow 7$	754 divide by 4 = $\frac{754}{4}$ So, remainder is 2 $7^2 = 9$
$7^2 = 49$	$\rightarrow 9$	
$7^3 = 343$	$\rightarrow 3$	
$7^4 = 2401$	$\rightarrow 1$	
$7^5 = 16807$	$\rightarrow 7$	

& will repeat

Unit Place = 9

18. (c) $(2153)^{167}$
unit digit = 3^{167}

unit digit

$$3^1 \rightarrow 3 \rightarrow 3$$

$$3^2 \rightarrow 9 \rightarrow 9$$

$$3^3 \rightarrow 27 \rightarrow 7$$

$$3^4 \rightarrow 81 \rightarrow 1$$

This cycle will continue

$\Rightarrow$ divide the power of 3 by 4

$$\frac{167}{4} \Rightarrow \text{remainder is 3}$$

$$3^3 \Rightarrow 7$$

Unit digit = 7

19. (a) $(2464)^{1793} \times (615)^{317} \times (131)^{491}$

$$4^1 \rightarrow 4 \rightarrow 4$$

$$4^2 \rightarrow 16 \rightarrow 6$$

$$4^3 \rightarrow 64 \rightarrow 4$$

So odd power of 4 will have 4 as unit digit and even power will have 6 as unit digit 5 and 1 have same unit digits respectively

$$\begin{array}{ccc} (2464)^{1793} & \times (615)^{317} & \times (131)^{491} \\ \downarrow & \downarrow & \downarrow \\ \text{odd power} & & \\ \text{unit digit} & \rightarrow 4 & \times 5 \quad \times 1 = 20 \end{array}$$

$$\Rightarrow 20 \Rightarrow 0 \text{ unit digit}$$

20. (b) 7^{105}

$$\Rightarrow 7^1 \rightarrow 7 \rightarrow 7$$

$$\Rightarrow 7^2 \rightarrow 49 \rightarrow 9$$

$$\Rightarrow 7^3 \rightarrow 343 \rightarrow 3$$

$$\Rightarrow 7^4 \rightarrow 2401 \rightarrow 1$$

Divide power of 7 by 4

$$\frac{105}{4} \rightarrow \text{remainder} = 1 \Rightarrow 7^1 \text{ is left}$$

unit digit = 7

21. (a) $(329)^{78}$

$\Rightarrow$ If power of 9 is odd, then unit digit number be 9. If power is even then unit digit number be 1.

Hence, unit digit = 1

22. (d) $(22)^{23}$
- | | | | |
|-------|--------|------------|-----------------|
| | Result | unit digit | |
| 2^1 | 2 | 2 | Cycle completes |
| 2^2 | 4 | 4 | |
| 2^3 | 8 | 8 | |
| 2^4 | 16 | 6 | |
| 2^5 | 32 | 2 | |

So divide power of 22 by 4

$$\frac{23}{4} = \text{remainder } 3$$

$$2^3 = 8$$

unit digit = 8

23. (a) $(122)^{173}$

	Unit digit	
$2^1 \rightarrow 2$	$2 \rightarrow 2$	Cycle
$2^2 \rightarrow 4$	$4 \rightarrow 4$	
$2^3 \rightarrow 8$	$8 \rightarrow 8$	
$2^4 \rightarrow 16$	$6 \rightarrow 6$	
$2^5 \rightarrow 32$	$2 \rightarrow 2$	

$$2^{173} = 2^{4 \times 43 + 1} = 2^{4 \times 43} \times 2 = 16^{43} \times 2$$

$$= 6^{43} \times 2 = 6 \times 2 = 12$$

unit digit = 2

24. (d) $(124)^{372}$ $(124)^{373}$

$$\downarrow \quad \downarrow$$

$$4^{372} \quad 4^{373}$$

When 4 has odd power then unit digit is 4 when 4 has even power then unit digit is 6

$$4^1 \rightarrow 4 \rightarrow 4$$

$$4^2 \rightarrow 16 \rightarrow 6$$

$$4^3 \rightarrow 64 \rightarrow 4$$

$$4^4 \rightarrow 256 \rightarrow 6$$

$$4^{372} \quad 4^{373}$$

$$\downarrow \quad \downarrow$$

$$6 \quad 4 = 10$$

last (unit) digit = 0

25. (b) $(1001)^{2008} + 1002$

$$\downarrow$$

$$\text{Unit digit} \rightarrow 1^{2008} + 1002$$

Unit digit will be 1 in case of 1

$$\Rightarrow 1 + 1002 = 1003$$

unit digit (last digit) = 3

26. (d) unit place

$7^1 \rightarrow 7$	$7 \rightarrow 7$	Cycle
$7^2 \rightarrow 49$	$9 \rightarrow 9$	
$7^3 \rightarrow 343$	$3 \rightarrow 3$	
$7^4 \rightarrow 2401$	$1 \rightarrow 1$	
$(4387)^{245} \times (621)^{72}$		

$$\downarrow$$

$$(7)^{245} \times (1)^{72}$$

$$\downarrow$$

$$(7)^{4 \times 61 + 1} \times 1$$

$$\downarrow$$

$$(7)^{4 \times 61} \times 7 \times 1$$

$$\downarrow$$

$$(1)^{61} \times 7 \times 1$$

unit digit = 7

27. (d) 5 always gives unit digit 5 and 6 always gives unit digit 6

	unit digit	
$3^1 \rightarrow 3$	$3 \rightarrow 3$	Cycle
$3^2 \rightarrow 9$	$9 \rightarrow 9$	
$3^3 \rightarrow 27$	$7 \rightarrow 7$	
$3^4 \rightarrow 81$	$1 \rightarrow 1$	

$$25^{6251} + 36^{528} + 72^{54}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$5^{6251} + 6^{528} + 3^{54}$$

$$\text{unit digit} \rightarrow 5 + 6 + 3^{54/4 = r=2}$$

$$= 5 + 6 + 9 = 20 = 0$$

Hence, unit digit = 0

28. (d) $7^{71} \times 6^{63} \times 3^{65}$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\text{unit place } 7^3 \quad 6^3 \quad 3^1$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\text{unit digit} \Rightarrow 3 \times 6 \times 3 = 54$$

$$\Rightarrow 4$$

29. (a) 3^{40} :

$$\text{Divide} = \frac{40}{4} \Rightarrow \text{remainder} = 0$$

	Unit digit	
$3^1 \rightarrow 3$	$3 \rightarrow 3$	Cycle
$3^2 \rightarrow 9$	$9 \rightarrow 9$	
$3^3 \rightarrow 27$	$7 \rightarrow 7$	
$3^4 \rightarrow 81$	$1 \rightarrow 1$	

Hence, unit digit of 3^{40} of completing all cycle = 1

30. (a)

$$(1570)^2 + (1571)^2 + (1572)^2 + (1573)^2$$

$$\text{unit digit} \rightarrow 0^2 + 1^2 + 2^2 + 3^2$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$0 + 1 + 4 + 9 = 14$$

$$\text{unit digit} = 4$$

31. (d) $3 \times 38 \times 537 \times 1256$

Note:- Always multiply only unit digit of first no. to second and product's unit digit no. with 3rd no. Again product of last's unit digit to fourth and so on.

Hence, unit digit = 8

32. (c) $(2467)^{153} \times (341)^{72}$

$$\downarrow \quad \downarrow$$

$$(7)^{153} \times (1)^{72}$$

$$\downarrow$$

$$[153/4 = \text{remainder} = 1]$$

$$\Rightarrow 7^1 \times 1 = 7$$

	Result	Unit digit
$7^1 =$	7	7
$7^2 =$	49	9
$7^3 =$	343	3
$7^4 =$	2401	1

Hence, unit digit = 7

33. (c) $(6732)^{170} \times (6733)^{172} \times (6734)^{174} \times (6736)^{176}$

$$\text{unit digit} = (2)^{170} \times (3)^{172} \times (4)^{174} \times (6)^{176}$$

$$= (2^4)^{42} \times 2^2 \times (3^4)^{43} \times (4)^{174} \times (6)^{176}$$

$$= (...6) \times 4 \times (...1) \times (...6) \times (...6)$$

Multiplication of unit digit

$$= 6 \times 4 \times 1 \times 6 \times 6 = 864$$

Hence, unit digit = 4

34. (c) The set of prime number S = {2, 3, 5, 7, 11, 13, ...}

Since there is one 5 and one 2 which gives 10 after multiplying mutually, it means the unit digit will be zero.

Hence, unit digit = 0

35. (b) The set of required prime number = The set of required prime number = {3, 5, 7, 11, ...}

Since there is no any even number in the set so when 5 will multiply with any odd number, it will always give 5 as the last digit.

Hence the unit digit will be 5.

36. (c) The last digit of the expression will be same as the last digit of $2^{888} + 8^{222}$.

Now the last digit of 2^{888} is 6 and the last digit of the 8^{222} is 4.

$$\therefore 6 + 4 = 10.$$

Hence, unit digit = 0

37. (c) Find the last digit of $2^{32^{32}}$

$$\text{But } 2^{32^{32}} = 2^{32 \times 32 \times 32 \dots \times 32 \text{ times}}$$

$$\Rightarrow 2^{32^{32}} = 2^{4 \times 8 \times (32 \times 32 \dots \times 32 \text{ times})}$$

$$\Rightarrow 2^{32^{32}} = 2^{4n}$$

where $n = 8 \times (32 \times 32 \dots \times 32 \text{ times})$
Again $2^{4n} = (16)^n \Rightarrow$ unit digit is 6, for every $n \in \mathbb{N}$

Hence, the required unit digit = 6

38. (a) Sum of square natural

$$\text{number} = \frac{n(n+1)(2n+1)}{6}$$

Here, $n = 100$

$$= \frac{100 \times 101 \times 201}{6} = 338350$$

Then, Unit digit = 0

39. (b) Find the unit digit of $1^1 + 2^2 + 3^3 + \dots + 10^{10}$.

The unit digit of $1^1 = 1$
 The unit digit of $2^2 = 4$
 The unit digit of $3^3 = 7$
 The unit digit of $4^4 = 6$
 The unit digit of $5^5 = 5$
 The unit digit of $6^6 = 6$
 The unit digit of $7^7 = 3$
 The unit digit of $8^8 = 6$
 The unit digit of $9^9 = 9$
 The unit digit of $10^{10} = 0$
 Thus the unit digit of the given expression will be 7.
 $(\therefore 1 + 4 + 7 + 6 + 5 + 6 + 3 + 6 + 9 = 47)$

40. (a) The unit of 3^{24} is 1
 The unit digit of 8^{57} is 8
 The unit digit of 4^{13} is 4
 The unit digit of 7^{68} is 1
 So the resultant value of the unit digits
 $= 1 \times 8 + 4 \times 1 + 4 + 8$
 $= 8 + 4 + 4 + 8 = 24$
 Thus the unit digit of the whole expression is 4.

41. (c) Since in the numerator of the product of the expression there will be 2 zeros at the and these two zeros will be cancelled by 2 zeros of the denominator. Hence finally we get a non-zero unit digit in the expression.

$$\text{Now, } \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{100}$$

$$= \frac{1 \times 2^1 \times 3^1 \times 2^2 \times 5^1 \times 2^1 \times 3^1 \times 7^1 \times 2^3 \times 3^2 \times 2^1 \times 5^1}{5^2 \times 2^2}$$

$$= \frac{1 \times 2^8 \times 3^4 \times 5^2 \times 7^1}{5^2 \times 2^2}$$

$$= 1 \times 2^6 \times 3^4 \times 7$$

Therefore, the unit digit of the given expression will be same as that of $1 \times 2^6 \times 3^4 \times 7$.

- Now, The unit digit of $1 \times 2^6 \times 3^4 \times 7$ is 8.

$(\therefore \text{the product of unit digits of } 1, 2^6, 3^4, 7 \text{ is } 1 \times 4 \times 1 \times 7 = 28)$

Hence, the unit digit of $\frac{10!}{100}$ is 8.

42. (b) First of all we find the unit digit individually of all the four terms,

So, the unit digit of $888^{9235!}$ is equal to the unit digit of $8^{9235!}$

Now, the unit digit of $8^{9235!}$ equal to the unit of 8^4 (since $9235!$ is divisible by 4), which is 6.

$$\text{unit digit of } (888)^{9235!} = (8)^{4n} = 6$$

$$\text{unit digit of } (222)^{9235!} = (2)^{4n} = 6$$

$$\text{unit digit of } (666)^{2359!} = (6)^{\text{any power}} = 6$$

$$\text{unit digit of } (999)^{9999!} = (9)^{\text{even power}} = 1$$

Thus the unit digit of the expression is 9. $(\therefore 6 + 6 + 6 + 1 = 19)$

43. (d) The unit digit of the given expression will be equal to the unit digit of the sum of the unit digits of every term of the expression.

Now, The unit digit of $(1!)^2 = 1$

$$\text{The unit digit of } (2!)^2 = 4$$

$$\text{The unit digit of } (3!)^3 = 6$$

$$\text{The unit digit of } (4!)^4 = 6$$

$$\text{The unit digit of } (5!)^5 = 0$$

$$\text{The unit digit of } (6!)^6 = 0$$

Thus the last digit of the $(7!)^7$, $(8!)^8$, $(9!)^9$, $(10!)^{10}$ will be zero.

So, the unit digit of the given expression = 7

$$(\therefore 1 + 4 + 6 + 6 + 0 + 0 + 0 + 0 + 0 = 17)$$

44. (b) The last digit of $(1!)^5 = 1$

$$\text{The last digit of } (2!)^4 = 16$$

$$\text{The last digit of } (3!)^3 = 216$$

$$\text{The last digit of } (4!)^2 = 576$$

$$\text{The last digit of } (10!)^5 = 00000$$

$$\text{The last digit of } (100!)^4 = 00000$$

$$(1000!)^3 = 00000$$

$$(10000!)^2 = 00000$$

$$(100000!)^1 = 00000$$

Thus the last 5 digits of the given expression = 00929

$$[\therefore 1 + 16 + 216 + 576 + 120 + 00000 + 00000 + 00000 + 00000 + 00000 = 00929]$$

45. (c) $(1!)^{99} + (2!)^{98} + (3!)^{97} + (4!)^{96} + \dots + (99!)^1$

$$\text{unit digit } (1!)^{99} = (1!)^{99} = 1$$

$$\text{unit digit } (2!)^{98} = 1 \times 2 = (2)^{98} = 4$$

$$\text{unit digit } (3!)^{97} = 1 \times 2 \times 3 = (6)^{97} = 6$$

$$\text{unit digit } (4!)^{96} = 1 \times 2 \times 3 \times 4 = (4)^{96} = 6$$

$$\text{unit digit } (5!)^{95} = 1 \times 2 \times 3 \times 4 \times 5 = (0)^{95} = 0$$

.....

 same unit digit $(99!)^1 = (1 \times 2 \times \dots \times 99)$

$$= (0)^1 = 0$$

$$\text{Then, Sum of unit digit} = 1 + 4 + 6 + 6 + 0 + 0 + \dots + 0 = 17$$

$$\text{unit digit} = 7$$

46. (d) unit digit $(12345k)^{72} = 6$
 if we put the value of $k = 2, 6, 8$
 Then we get unit digit = 6

47. (d) $(1!)^{11} + (2!)^{21} + (3!)^{31} + \dots + (100!)^{100!}$

$$\text{unit digit } (1!)^{11} = 1^1 = 1$$

$$\text{unit digit } (2!)^{21} = (2)^2 = 4$$

$$\text{unit digit } (3!)^{31} = (6)^6 = 6$$

$$\text{unit digit } (4!)^{41} = (4)^{24} = 6$$

$$\text{unit digit } (5!)^{51} = (0)^{120} = 0$$

.....

 unit digit $(100!)^{100!}$

$$\Rightarrow = (0)^{1 \times 2 \times 3 \times \dots \times 100} = 0$$

$$\text{Sum of unit digit} = 1 + 4 + 6 + 6 + 0 + 0 + 0 = 17$$

$$\text{unit digit} = 7$$

48. (b) $4 \times 9^2 \times 4^3 \times 9^4 \times 4^5 \times 9^6 \times \dots \times 4^{99} \times 9^{100}$

$$\text{unit digit } 4^1 = 4$$

$$\text{unit digit } 9^2 = 1$$

$$\text{unit digit } 4^3 = 4$$

$$\text{unit digit } 9^4 = 1$$

$$\text{unit digit } 4^5 = 4$$

.....

 unit digit $4^{99} = 4$

$$\text{unit digit } 9^{100} = 1$$

Then multiply the unit digit
 $\frac{4}{4} \times \frac{1}{4} \times \frac{4}{4} \times \frac{1}{4} \times \frac{4}{4} \times \frac{1}{4} \times \dots \times \frac{4}{4} \times \frac{1}{4}$
 Pair of 4×1 (4) is equal 50
 we can say this expression = 4^{50}
 Then, unit place = 6

49. (a) $4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^6 + \dots + 4^{99} + 9^{100}$

$$\text{unit digit } 4^1 = 4$$

$$\text{unit digit } 9^2 = 1$$

$$\text{unit digit } 4^3 = 4$$

$$\text{unit digit } 9^4 = 1$$

$$\text{unit digit } 4^5 = 4$$

.....

 unit digit $4^{99} = 4$

$$\text{unit digit } 9^{100} = 1$$

Then,
 Sum of unit place
 $4 + 1 + 4 + 1 + 4 + 1 + \dots + 4 + 1$
 Pair of $4 + 1$ (5) is equal to 50
 We can say this expression
 $= 50 \times 5 = 250$
 unit digit = 0

50. (a) $2^{3^4} \times 3^{4^5} \times 4^{5^6} \times 5^{6^7} \times 6^{7^8} \times 7^{8^9}$

We know unit digit of

$$2^{3^4} = 2^{81} = 2$$

$$\text{unit digit of } 5^{6^7} = 5$$

Then we know that when even number is multiplied by 5 then we get unit place = '0'
 So, last digit = 0

NUMBER OF ZEROES

Number of zeroes in an Expression

zero:- zero will be formed by 2 and 5

Ex. $10 = 2 \times 5$

$100 = 2^2 \times 5^2$

$1000 = 2^3 \times 5^3$

⇒ We can say that for 'n' number of zeroes at end of the product. We need exactly 'n' combinations of 5 and 2

Ex.1 Find the number of zeroes at the end of the product:-

$5 \times 7 \times 9 \times 2 \times 11$

Sol. $5 \times 7 \times 9 \times 2 \times 11$

In this product we see

Number of 2's = 1

Number of 5's = 1

Number of pair 2's and 5's = 1

∴ Number of zero = 1

Ex.2 Find the number of zeroes at the end of the product:- $12 \times 27 \times 63 \times 113 \times 1250 \times 24 \times 650$

Sol. $12 \times 27 \times 63 \times 113 \times 1250 \times 24 \times 650$

Break the numbers form of 2 and 5 multiple

In this series 27, 63 & 113 are not multiple of 2 & 5.

∴ The multiple of 2 & 5 are

12, 1250, 24 & 650

⇒ $12 = 2 \times 2 \times 3 = 2^2 \times 3$

⇒ $1250 = 2 \times 5 \times 5 \times 5 \times 5 = 2^1 \times 5^4$

⇒ $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

⇒ $650 = 2 \times 5 \times 5 \times 13 = 2^1 \times 5^2 \times 13$

$2^2 \times 3 \times 27 \times 63 \times 113 \times 2^1 \times 5^4 \times 2^3 \times 3 \times 2^1 \times 5^2 \times 13$

Number of 2's = 7

Number of 5's = 6

⇒ Number of pair 2's and 5's = 6
there are 7 two's and 6 fives.
Hence we shall be able to form only 6 pairs of 2 and 5,
Hence there will be 6 zeroes at the end of the product of numbers.

Ex.3 Find the number of zeroes at the end of the product:-

$1 \times 3 \times 5 \times 7 \times 9 \dots 97 \times 99$

Sol. $1 \times 3 \times 5 \times 7 \times 9 \dots 97 \times 99$

In this series the number of zero and the end of the product is "0". Because there is no even number present in this series while it is necessary to be 2 and 5 for the Zero

The highest power of k that can exactly divided n! we divide n by k, n by k², n by k³ and so on till we get

$\left[\frac{n}{k} \right]$ equal to 1 and then add up as.

$\left[\frac{n}{k} \right] + \left[\frac{n}{k^2} \right] + \left[\frac{n}{k^3} \right] + \left[\frac{n}{k^4} \right] + \dots + \left[\frac{n}{k^x} \right]$

Ex.4 Find the largest power of 5 contained in 124!

Sol. $\left[\frac{124}{5} \right] + \left[\frac{124}{5^2} \right] = 24 + 4 = 28$

[We cannot do it further since 124 is not divisible by 5³]

Hence, there are 28 times 5 alternate as a factor in 124!

Alternate:-

Divide successive quotients till we get 0 as the last quotient

$$\begin{array}{r} 5 \overline{)124} \\ 5 \overline{)24} \rightarrow 28 \text{ (add up all} \\ 4 \rightarrow \text{the quotients)} \end{array}$$

Ex.5 Find the largest power of 3 that can divide 270!

Sol. $\left[\frac{270}{3} \right] + \left[\frac{270}{3^2} \right] + \left[\frac{270}{3^3} \right] + \left[\frac{270}{3^4} \right]$

$+ \left[\frac{270}{3^5} \right] = 90 + 30 + 10 + 3 + 1 = 134$

Hence, there are 134 times 3 involved as a factor in 270!

Alternate:-

Divide successive quotients till we get 0 as the last quotient

$$\begin{array}{r} 3 \overline{)270} \\ 3 \overline{)90} \rightarrow \\ 3 \overline{)30} \rightarrow 134 \text{ (add up all the} \\ 3 \overline{)10} \rightarrow \text{quotients)} \\ 3 \overline{)3} \rightarrow \\ 3 \overline{)1} \rightarrow 0 \end{array}$$

* Alternate method is easier than first.

Ex.6 Find the largest power of 2 that can contained in:-

$1 \times 2 \times 3 \times 4 \dots 22$

Sol. $1 \times 2 \times 3 \times 4 \dots 22$

$$\begin{array}{r} 2 \overline{)22} \\ 2 \overline{)11} \rightarrow \\ 2 \overline{)5} \rightarrow \\ 2 \overline{)2} \rightarrow \\ 2 \overline{)1} \rightarrow 19 \end{array}$$

Number of 2's = 11 + 5 + 2 + 1 = 19

Hence, there are 19 times 2 involved as a factor in 22!

Ex.7 Find the largest power of 5 that can contained in

$1 \times 2 \times 3 \times 4 \dots 41 \times 42$

Sol.
$$\begin{array}{r} 5 \overline{)42} \\ 5 \overline{)8} \rightarrow 9 \\ 5 \overline{)1} \rightarrow 0 \end{array}$$

Hence, there are 9 times 5 involved as a factor in 42!

Ex.8 Find the largest power of 7 that can exactly divide 777!

Sol.
$$\begin{array}{r} 7 \overline{)777} \\ 7 \overline{)111} \rightarrow \\ 7 \overline{)15} \rightarrow 128 \text{ (add up all the} \\ 7 \overline{)2} \rightarrow \text{quotients)} \\ 7 \overline{)0} \end{array}$$

Thus the highest power of 7 is 128 by which 777! can be divided.

Ex.9 Find the number of zeroes at the end of the product 10!

Sol. $10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

$$\begin{array}{c} \swarrow \searrow \\ 2 \times 5 \end{array}$$

* In this type expression it is clear that in any factorial value, the number of 5's will always be lesser than the number of 2's. In this condition, all we need to do is to count the number of 5's
Number of 5's = 2
Number of 2's = 5
But pair of 2's and 5's are = 2
Then,
Number of zero's = 2

Alternate:-

Highest power of 5's in 10!

$$\frac{10}{5} = 2$$

Number of 5's = 2

Then number of zeroes = 2

Ex.10 Find the number of zeroes at the end of the product 100!

$$\text{Sol. } \begin{array}{r} 5 \overline{)100} \rightarrow \\ 5 \overline{)20} \rightarrow \\ 5 \overline{)4} \rightarrow \\ 5 \overline{)0} \rightarrow \end{array} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} 24 \text{ (add up all the quotients)}$$

Number of 5's = 20 + 4 = 24

then number of zeroes = 24

Ex.11 Find the number of zeroes at the end of the product 1000!**Sol.** $1 \times 2 \times 3 \times \dots \times 999 \times 1000$

$$\begin{array}{r} 5 \overline{)1000} \rightarrow \\ 5 \overline{)200} \rightarrow \\ 5 \overline{)40} \rightarrow \\ 5 \overline{)8} \rightarrow \\ 5 \overline{)1} \rightarrow \\ 5 \overline{)0} \rightarrow \end{array} \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} 249$$

The highest power of Number 5's = 200 + 40 + 8 + 1 = 249
then number of zeroes at the end of the product = 249

Ex.12 Find the number of zeroes at the end of the product $1 \times 3 \times 5 \times 7 \dots \dots \dots 73 \times 1024$ **Sol.** $1 \times 3 \times 5 \times 7 \dots \dots \dots 73 \times 1024$
Number of 5's from 1 to 73

$$\begin{array}{r} 5 \overline{)73} \rightarrow \\ 5 \overline{)14} \rightarrow \\ 5 \overline{)2} \rightarrow \\ 5 \overline{)0} \rightarrow \end{array} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} 16 \text{ (add up all the quotients)}$$

Total number of 5's = 14 + 2 = 16

we know that

$$1024 = 2^{10}$$

number of 2 = 10

number of pairs (2 and 5) = 10

then number of zeroes = 10

Ex.13 Find the number of zeroes at the end of the product $12 \times 13 \times 14 \dots \dots \dots 84$ **Sol.** In this types expression first for us complete the series
$$\frac{1}{1} \times \frac{2}{2} \times \frac{3}{3} \dots \dots \dots \frac{11}{11} \times \frac{12}{12} \times \frac{13}{13} \dots \dots \dots \frac{84}{84} \times \frac{1}{1} \times \frac{2}{2} \times \frac{3}{3} \dots \dots \dots \frac{11}{11}$$

Number of zero (1 to 84)

$$\begin{array}{r} 5 \overline{)84} \rightarrow \\ 5 \overline{)16} \rightarrow \\ 5 \overline{)3} \rightarrow \\ 5 \overline{)0} \rightarrow \end{array} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} 19$$

Number of zero (1 to 11)

$$\begin{array}{r} 5 \overline{)11} \rightarrow \\ 5 \overline{)2} \rightarrow \\ 5 \overline{)0} \rightarrow \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} 2$$

Number of zeroes = 19 - 2 = 17

Ex.14 Find the number of zeroes at the end of the product $512 \times 513 \dots \dots \dots 1120$ **Sol.** $\frac{1}{1} \times \frac{2}{2} \times \frac{3}{3} \dots \dots \dots \frac{511}{511} \times \frac{512}{512} \times \frac{513}{513} \dots \dots \dots \frac{1120}{1120} - \frac{1}{1} \times \frac{2}{2} \times \frac{3}{3} \dots \dots \dots \frac{511}{511}$

$$\begin{array}{r} 5 \overline{)1120} \rightarrow \\ 5 \overline{)224} \rightarrow \\ 5 \overline{)44} \rightarrow \\ 5 \overline{)8} \rightarrow \\ 5 \overline{)1} \rightarrow \\ 5 \overline{)0} \rightarrow \end{array} \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} 277$$

$$\begin{array}{r} 5 \overline{)511} \rightarrow \\ 5 \overline{)102} \rightarrow \\ 5 \overline{)20} \rightarrow \\ 5 \overline{)4} \rightarrow \\ 5 \overline{)0} \rightarrow \end{array} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} 126$$

Number of zeroes = 277 - 126 = 151

Ex.15 Find the number of zeroes at the end of the product $1^5 \times 2^5 \times 3^5 \dots \dots \dots 32^5$ **Sol.** In this type every second terms has power of 2's. It means power of 2's more than that of 5 So count the power of 5's

power of 5's = total power of 5's

$$5^5 = (1 \times 5)^5 = 1^5 \times 5^5 = 5$$

$$10^5 = (2 \times 5)^5 = 2^5 \times 5^5 = 5$$

$$15^5 = (3 \times 5)^5 = 3^5 \times 5^5 = 5$$

$$20^5 = (4 \times 5)^5 = 4^5 \times 5^5 = 5$$

$$25^5 = (5 \times 5)^5 = 5^5 \times 5^5 = 10$$

$$30^5 = (6 \times 5)^5 = 6^5 \times 5^5 = 5$$

Number of 5's power = 35

then number of zeroes at the end of the product = 35

Ex.16 Find the number of zeroes at the end of the product $1^1 \times 2^2 \times 3^3 \times 4^4 \dots \dots \dots 28^{28}$ **Sol.** count the number of 5's
power of 5's = total power of 5's

$$5^5 = (1 \times 5)^5 = 1^5 \times 5^5 = 5$$

$$10^{10} = (2 \times 5)^{10} = 2^{10} \times 5^{10} = 10$$

$$15^{15} = (3 \times 5)^{15} = 3^{15} \times 5^{15} = 15$$

$$20^{20} = (4 \times 5)^{20} = 4^{20} \times 5^{20} = 20$$

$$25^{25} = (5 \times 5)^{25} = 5^{25} \times 5^{25} = 50$$

Number of 5's power

$$= 5 + 10 + 15 + 20 + 50$$

$$= 100$$

Then number of zeroes at the end of product = 100

Ex.17 Find the number of zeroes at the end of the product $a = 1^3, b = 2^4, c = 3^5, \dots, z = 26^{28}$ $a \times b \times c \times d \dots \dots \times z$ **Sol.** Count the number of 5's
power of 5's = total power of 5's

$$5^7 = (1 \times 5)^7 = 1^7 \times 5^7 = 7$$

$$10^{12} = (2 \times 5)^{12} = 2^{12} \times 5^{12} = 12$$

$$15^{17} = (3 \times 5)^{17} = 3^{17} \times 5^{17} = 17$$

$$20^{22} = (4 \times 5)^{22} = 4^{22} \times 5^{22} = 22$$

$$25^{27} = (5 \times 5)^{27} = 5^{27} \times 5^{27} = 54$$

Number of 5's power

$$= 7 + 12 + 17 + 22 + 54 = 112$$

Then number of zeroes at the end of product = 112

Ex.18 Find the number of zeroes at the end of the product $1^1 \times 2^2 \times 3^3 \times 4^4 \dots \dots \dots 100^{100}$ **Sol.** Count the power of 5's

$$5^5 = 5$$

$$10^{10} = 10$$

$$15^{15} = 15$$

$$\dots$$

$$\dots$$

$$\dots$$

$$\dots$$

$$\dots$$

$$100^{100} = 100$$

$$5 + 10 + 15 \dots \dots + 100$$

it is an a.p.series

we use a.p. formula

$$\text{number of term} = \frac{l - a}{d} + 1$$

 l = last term of a.p. a = first term of a.p. d = common difference

$$\text{number of term} = \frac{100 - 5}{5} + 1 = 20$$

sum of n^{th} term of a.p.

$$= \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{20}{2} [2 \times 5 + (20-1) \times 5]$$

$$= 10 [10 + 19 \times 5]$$

$$= 10 [105] = 1050$$

but $25^{25} = (5 \times 5)^{25} = 5^{25} \times 5^{25}$

$$50^{50} = (2 \times 5 \times 5)^{50} = 2^{50} \times 5^{50} \times 5^{50}$$

$$75^{75} = (3 \times 5 \times 5)^{75} = 3^{75} \times 5^{75} \times 5^{75}$$

$$100^{100} = (4 \times 5 \times 5)^{100} = 4^{100} \times 5^{100} \times 5^{100}$$

then number of 5's power

$$= 25 + 50 + 75 + 100$$

$$= 250$$

then number of total zeroes at the end of product

$$= 1050 + 250 = 1300$$

Ex.19 Find the number of zeroes at the end of the product $10 \times 20 \times 30 \dots \dots \dots 80$ **Sol.** $10^1 \times 1 \times 10^1 \times 2 \times 10^1 \times 3 \dots \dots 10^1 \times 8 = 10^8 [1 \times 2 \times 3 \dots \dots 8]$

from 1 to 8, number of 0's = 1
 $\therefore$ Only one pair (2 & 5)
 then total number of 0's
 = 1 + 8 = 9

Ex.20 Find the number of zeroes at the end of the product
 $10 \times 20 \times 30 \dots\dots\dots 1000$

Sol. $10^1 \times 1 \times 10^1 \times 2 \times 10^1 \times 3 \dots\dots 10^1 \times 100$
 $= 10^{100} [1 \times 2 \times 3 \times \dots\dots 100]$
 from 1 to 100 number of 0's

$$\begin{array}{r} 5 \overline{) 100} \\ 5 \overline{) 20} \rightarrow \\ 5 \overline{) 4} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 100} \\ 5 \overline{) 20} \rightarrow \\ 5 \overline{) 4} \rightarrow \\ 0 \end{array}} \right\} 24 \text{ (add up all the quotients)}$$

number of 0's = 20 + 4 = 24
 and 10^{100} , here number of zero
 = 100

total number of 0's
 = 24 + 100 = 124

then number of zeroes = 124

EXERCISE

- Find the number of zeroes at the end of the product 47!
 (a) 8 (b) 9 (c) 10 (d) 11
- Find the number of zeroes at the end of the product 125!
 (a) 25 (b) 30 (c) 31 (d) 28
- Find the number of zeroes at the end of the product 378!
 (a) 93 (b) 90 (c) 75 (d) 81
- Find the number of zeroes at the end of the product 680!
 (a) 163 (b) 169 (c) 170 (d) 165
- Find the number of zeroes at the end of the product 1000!
 (a) 200 (b) 249 (c) 248 (d) 250
- Find the number of zeroes at the end of the product 500!
 (a) 100 (b) 124 (c) 120 (d) 125
- Find the number of zeroes at the end of the product 1132!
 (a) 280 (b) 271 (c) 281 (d) 272
- Find the number of zeroes at the end of the product 1098!
 (a) 280 (b) 270 (c) 271 (d) 262
- Find the number of zeroes at the end of the product 2346!
 (a) 580 (b) 583 (c) 575 (d) 580
- Find the number of zeroes at the end of the product 2700!
 (a) 673 (b) 670 (c) 669 (d) 675
- Find the number of zeroes at the end of the product $10 \times 15 \times 44 \times 28 \times 70$
 (a) 2 (b) 3 (c) 4 (d) 5
- Find the number of zeroes at the end of the product $12 \times 5 \times 15 \times 24 \times 13 \times 30 \times 75$
 (a) 4 (b) 5 (c) 2 (d) 3
- Find the number of zeroes at the end of the product $2 \times 4 \times 6 \times \dots\dots 48 \times 50$
 (a) 6 (b) 12 (c) 7 (d) 5
- Find the number of zeroes at the end of the product $1 \times 3 \times 5 \times 7 \times 9 \times 11 \dots\dots 99 \times 101$
 (a) 24 (b) 5 (c) 2 (d) 0
- Find the number of zeroes at the end of the product $21 \times 22 \times 23 \dots\dots 59 \times 60$
 (a) 14 (b) 4 (c) 10 (d) 12
- Find the number of zeroes at the end of the product $35 \times 36 \times 37 \times \dots\dots 89 \times 90$
 (a) 21 (b) 7 (c) 14 (d) 20
- Find the number of zeroes at the end of the product $41 \times 42 \dots\dots 109 \times 110$
 (a) 26 (b) 9 (c) 17 (d) 25
- Find the number of zeroes at the end of the product $140! \times 5 \times 15 \times 22 \times 11 \times 44 \times 135$
 (a) 34 (b) 35 (c) 36 (d) 37
- Find the number of zeroes at the end of the product $25! \times 32! \times 45!$
 (a) 10 (b) 23 (c) 22 (d) 7
- Find the number of zeroes at the end of the product $1140! \times 358! \times 171!$
 (a) 282 (b) 325 (c) 411 (d) 370
- Find the number of zeroes at the end of the product $1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \dots\dots 49^{49}$
 (a) 225 (b) 250 (c) 240 (d) 245
- Find the number of zeroes at the end of the product $100^1 \times 99^2 \times 98^3 \times 97^4 \dots\dots 1^{100}$
 (a) 970 (b) 1124 (c) 875 (d) 975
- Find the number of zeroes at the end of the product $1^{11} \times 2^{21} \times 3^{31} \times 4^{41} \dots\dots 10^{101}$
 (a) 51 (b) 10 (c) $5! + 10!$ (d) N.O.T
- Find the number of zeroes at the end of the product $2^2 \times 5^4 \times 4^2 \times 10^8 \times 6^{10} \times 15^{12} \times 8^{14} \times 20^{16} \times 10^{18} \times 25^{20}$
 (a) 98 (b) 90 (c) 94 (d) 100
- Find the number of zeroes at the end of the product $3200 + 1000 + 40000 + 32000 + 15000$
 (a) 15 (b) 13 (c) 2 (d) 3
- Find the number of zeroes at the end of the product $3200 \times 1000 \times 40000 \times 32000 \times 15000$
 (a) 15 (b) 2 (c) 14 (d) 16
- Find the number of zeroes at the end of the product $20 \times 40 \times 7600 \times 600 \times 300 \times 1000$
 (a) 11 (b) 10 (c) 2 (d) 3
- Find the number of zeroes at the end of the product $100! + 200!$
 (a) 24 (b) 25 (c) 49 (d) N.O.T
- Find the number of zeroes at the end of the product $1^1 \times 2^2 \times 3^3 \times 4^4 \dots\dots 10^{10}$
 (a) 10 (b) 15 (c) 5 (d) N.O.T
- Find the number of zeroes at the end of the product $100! \times 200!$
 (a) 49 (b) 24 (c) 73 (d) N.O.T
- Find the number of zeroes at the end of the product $5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 45 \times 50$
 (a) 8 (b) 12 (c) 10 (d) 14
- Find the number of zeroes at the end of the product $2 \times 4 \times 6 \times 8 \times 10 \dots\dots 200$
 (a) 49 (b) 24 (c) 25 (d) 50
- Find the number of zeroes at the end of the product $1 \times 3 \times 5 \times 7 \dots\dots 99 \times 64$
 (a) 23 (b) 6 (c) 0 (d) 5
- Find The No. zero at the end of the product of $2^{222} \times 5^{555}$
 (a) 222 (b) 555 (c) 777 (d) 333
- Find the number of zeroes at the end of the product $10 + 100 + 1000 + \dots\dots 1000000000$
 (a) 8 (b) 28 (c) 0 (d) 1
- Find the number of zeroes at the end of the product $10^1 \times 10^2 \times 10^3 \times 10^4 \dots\dots 10^{10}$
 (a) 10 (b) 55 (c) 50 (d) 45

37. Find the number of zeroes at the end of the product
 $2^1 \times 5^2 \times 2^3 \times 5^4 \times 2^5 \times 5^6 \times 2^7 \times 5^8 \times 2^9 \times 5^{10}$
 (a) 30 (b) 25
 (c) 55 (d) 50

38. Find the number of zeroes at the end of the product
 $(3^{123} - 3^{122} - 3^{121}) (2^{121} - 2^{120} - 2^{119})$
 (a) 1 (b) 0 (c) 119 (d) 120
39. Find the number of zeroes at the end of the product

$$(8^{123} - 8^{122} - 8^{121}) (3^{223} - 3^{222} - 3^{221})$$

- (a) 1 (b) 2 (c) 0 (d) 3
- 40 Find the number of zeroes at the end of the product
 $5 \times 10 \times 15 \dots 75$
 (a) 11 (b) 15 (c) 10 (d) 18

ANSWER KEY

1. (c)	5. (b)	9. (b)	13. (a)	17. (c)	21. (b)	25. (c)	29. (b)	33. (b)	37. (b)
2. (c)	6. (b)	10. (a)	14. (d)	18. (d)	22. (b)	26. (d)	30. (c)	34. (a)	38. (a)
3. (a)	7. (c)	11. (b)	15. (c)	19. (b)	23. (c)	27. (a)	31. (a)	35. (d)	39. (b)
4. (b)	8. (c)	12. (b)	16. (c)	20. (c)	24. (a)	28. (a)	32. (b)	36. (b)	40. (a)

SOLUTION

1. (c)
$$\begin{array}{r} 5 \overline{) 47} \\ 5 \overline{) 9} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array}$$

No. of zeroes = $9 + 1 = 10$

2. (c)
$$\begin{array}{r} 5 \overline{) 125} \\ 5 \overline{) 25} \rightarrow \\ 5 \overline{) 5} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 125} \\ 5 \overline{) 25} \rightarrow \\ 5 \overline{) 5} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array}} \right\} 31$$

No. of zeroes
 $= 25 + 5 + 1 = 31$

3. (a)
$$\begin{array}{r} 5 \overline{) 378} \\ 5 \overline{) 75} \rightarrow \\ 5 \overline{) 15} \rightarrow \\ 5 \overline{) 3} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 378} \\ 5 \overline{) 75} \rightarrow \\ 5 \overline{) 15} \rightarrow \\ 5 \overline{) 3} \rightarrow \\ 0 \end{array}} \right\} 93$$

No. of zeroes = $75 + 15 + 3 = 93$

4. (b)
$$\begin{array}{r} 5 \overline{) 680} \\ 5 \overline{) 136} \rightarrow \\ 5 \overline{) 27} \rightarrow \\ 5 \overline{) 5} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 680} \\ 5 \overline{) 136} \rightarrow \\ 5 \overline{) 27} \rightarrow \\ 5 \overline{) 5} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array}} \right\} 169$$

No. of zeroes = $136 + 27 + 5 + 1 = 169$

5. (b)
$$\begin{array}{r} 5 \overline{) 1000} \\ 5 \overline{) 200} \rightarrow \\ 5 \overline{) 40} \rightarrow \\ 5 \overline{) 8} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 1000} \\ 5 \overline{) 200} \rightarrow \\ 5 \overline{) 40} \rightarrow \\ 5 \overline{) 8} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array}} \right\} 249$$

No. of zeroes = $200 + 40 + 8 + 1 = 249$

6. (b)
$$\begin{array}{r} 5 \overline{) 500} \\ 5 \overline{) 100} \rightarrow \\ 5 \overline{) 20} \rightarrow \\ 5 \overline{) 4} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 500} \\ 5 \overline{) 100} \rightarrow \\ 5 \overline{) 20} \rightarrow \\ 5 \overline{) 4} \rightarrow \\ 0 \end{array}} \right\} 124$$

No. of zeroes = $100 + 20 + 4 = 124$

7. (c)
$$\begin{array}{r} 5 \overline{) 1132} \\ 5 \overline{) 226} \rightarrow \\ 5 \overline{) 45} \rightarrow \\ 5 \overline{) 9} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 1132} \\ 5 \overline{) 226} \rightarrow \\ 5 \overline{) 45} \rightarrow \\ 5 \overline{) 9} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array}} \right\} 281$$

No. of zeroes = $226 + 45 + 9 + 1 = 281$

8. (c)
$$\begin{array}{r} 5 \overline{) 1098} \\ 5 \overline{) 219} \rightarrow \\ 5 \overline{) 43} \rightarrow \\ 5 \overline{) 8} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 1098} \\ 5 \overline{) 219} \rightarrow \\ 5 \overline{) 43} \rightarrow \\ 5 \overline{) 8} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array}} \right\} 271$$

No. of zeroes = $219 + 43 + 8 + 1 = 271$

9. (b)
$$\begin{array}{r} 5 \overline{) 2346} \\ 5 \overline{) 469} \rightarrow \\ 5 \overline{) 93} \rightarrow \\ 5 \overline{) 18} \rightarrow \\ 5 \overline{) 3} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 2346} \\ 5 \overline{) 469} \rightarrow \\ 5 \overline{) 93} \rightarrow \\ 5 \overline{) 18} \rightarrow \\ 5 \overline{) 3} \rightarrow \\ 0 \end{array}} \right\} 583$$

No. of zeroes = $469 + 93 + 18 + 3 = 583$

10. (a)
$$\begin{array}{r} 5 \overline{) 2700} \\ 5 \overline{) 540} \rightarrow \\ 5 \overline{) 108} \rightarrow \\ 5 \overline{) 21} \rightarrow \\ 5 \overline{) 4} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 2700} \\ 5 \overline{) 540} \rightarrow \\ 5 \overline{) 108} \rightarrow \\ 5 \overline{) 21} \rightarrow \\ 5 \overline{) 4} \rightarrow \\ 0 \end{array}} \right\} 673$$

No. of zeroes = $540 + 108 + 21 + 4 = 673$

11. (b) $10 \times 15 \times 44 \times 28 \times 70$

$$\underline{2} \times \underline{5} \times \underline{3} \times \underline{5} \times \underline{2} \times \underline{2} \times \underline{11} \times \underline{2} \times \underline{2} \times \underline{7} \times \underline{2} \times \underline{5} \times \underline{7}$$

In this expression

No. of 2's = 6

No. of 5's = 3

Pair of 2's and 5's = 3

So, No. of zeroes = 3

12. (b) $12 \times 5 \times 15 \times 24 \times 13 \times 30 \times 75$

$$\underline{2} \times \underline{2} \times \underline{3} \times \underline{5} \times \underline{3} \times \underline{5} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{3} \times \underline{13} \times \underline{2} \times \underline{3} \times \underline{5} \times \underline{5} \times \underline{5} \times \underline{3}$$

No. of 2's $\rightarrow$ 6

No. of 5's $\rightarrow$ 5

Pair of 2's and 5's = 5

No. of zeroes = 5

13. (a) $2 \times 4 \times 6 \times \dots 48 \times 50$

$$\Rightarrow 2 \times 1 \times 2 \times 2 \times 2 \times 3 \times \dots 2 \times 24 \times 2 \times 25$$

$$\Rightarrow 2^{25} (1 \times 2 \times 3 \times 4 \times \dots 25)$$

There are many 2's In This series we count 5's

$$\begin{array}{r} 5 \overline{) 25} \\ 5 \overline{) 5} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 25} \\ 5 \overline{) 5} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array}} \right\} 6$$

No. of 5's = $5 + 1 = 6$

Then No. of zeroes = 6

14. (d) $1 \times 3 \times 5 \times 7 \times 9 \times 11 \times \dots 99 \times 101$

There is no 'zero' in this expression because there is no even present here.

15. (c) $21 \times 22 \times 23 \times \dots 59 \times 60$

$$1 \times 2 \times 3 \times \dots 19 \times 20 \times 21 \times 22 \times 23 \times \dots 59 \times 60$$

$$\underline{\hspace{1cm}} - 1 \times 2 \times 3 \times \dots 20 \underline{\hspace{1cm}}$$

$$\begin{array}{r} 5 \overline{)60} \\ 5 \overline{)12} \rightarrow 14 \\ 5 \overline{)2} \rightarrow 4 \\ 0 \end{array}$$

No. of zeroes 1 to 60 = 12 + 2 = 14

No. of zeroes 1 to 20 = 4

No. of zeroes 21 to 60 = 14 - 4 = 10

16. (c) $35 \times 36 \times 37 \times \dots \times 89 \times 90$
 $1 \times 2 \times 3 \times 4 \times \dots \times 34 \times 35 \times 36 \times \dots \times 89 \times 90$
 $\frac{1 \times 2 \times 3 \times \dots \times 34 \times 35 \times 36 \times \dots \times 89 \times 90}{1 \times 2 \times 3 \times \dots \times 34 \times 35 \times 36 \times \dots \times 89 \times 90}$

No. of zeroes 1 to 90 = 18 + 3 = 21

$$\begin{array}{r} 5 \overline{)90} \\ 5 \overline{)18} \rightarrow 21 \\ 5 \overline{)3} \rightarrow 0 \end{array}$$

No. of Zeroes 1 to 34

$$\begin{array}{r} 5 \overline{)34} \\ 5 \overline{)6} \rightarrow 7 \\ 5 \overline{)1} \rightarrow 0 \end{array}$$

No. of Zeroes = 6 + 1 = 7

No. of zeroes 35 to 90 = 21 - 7 = 14

17. (c) $41 \times 42 \times \dots \times 109 \times 110$
 $1 \times 2 \times 3 \times 4 \times \dots \times 40 \times 41 \times 42 \times \dots \times 109 \times 110$
 $\frac{1 \times 2 \times 3 \times 4 \times \dots \times 40 \times 41 \times 42 \times \dots \times 109 \times 110}{1 \times 2 \times 3 \times 4 \times \dots \times 40 \times 41 \times 42 \times \dots \times 109 \times 110}$

No. of zeroes 1 to 110 = 22 + 4 = 26

No. of zeroes 1 to 40 = 8 + 1 = 9

$$\begin{array}{r} 5 \overline{)110} \\ 5 \overline{)22} \rightarrow 26 \\ 5 \overline{)4} \rightarrow 0 \end{array} \quad \begin{array}{r} 5 \overline{)40} \\ 5 \overline{)8} \rightarrow 9 \\ 5 \overline{)1} \rightarrow 0 \end{array}$$

No. of zeroes 41 to 110 = 26 - 9 = 17

18. (d) 140! would have 28 + 5 + 1 = 34

$$\begin{array}{r} 5 \overline{)140} \\ 5 \overline{)28} \rightarrow 34 \\ 5 \overline{)5} \rightarrow 0 \\ 5 \overline{)1} \rightarrow 0 \end{array}$$

Now Remaining part
 $5 \times 15 \times 22 \times 11 \times 44 \times 135$

$5 \times 3 \times 5 \times 2 \times 11 \times 11 \times 2 \times 2 \times$

$11 \times 5 \times 27$

No. of 2's = 3

No. of 5's = 3

Pair of (2 & 5) = 3

Remaining part of the Expression would have 3 zeroes

Total No. of zeroes = 34 + 3 = 37

19. (b) $\begin{array}{r} 5 \overline{)25} \\ 5 \overline{)5} \rightarrow 6 \\ 5 \overline{)1} \rightarrow 0 \end{array} \quad \begin{array}{r} 5 \overline{)32} \\ 5 \overline{)6} \rightarrow 7 \\ 5 \overline{)1} \rightarrow 0 \end{array}$

$$\begin{array}{r} 5 \overline{)45} \\ 5 \overline{)9} \rightarrow 10 \\ 5 \overline{)1} \rightarrow 0 \end{array}$$

No. of zeroes In 25! = 5 + 1 = 6

No. of zeroes In 32! = 6 + 1 = 7

No. of zero In 45! 9 + 1 = 10

Total No. of zero = 6 + 7 + 10 = 23

20. (c)

$$\begin{array}{r} 5 \overline{)1140} \\ 5 \overline{)228} \rightarrow 283 \\ 5 \overline{)45} \rightarrow 87 \\ 5 \overline{)9} \rightarrow 0 \\ 5 \overline{)1} \rightarrow 0 \end{array} \quad \begin{array}{r} 5 \overline{)358} \\ 5 \overline{)71} \rightarrow 87 \\ 5 \overline{)14} \rightarrow 0 \\ 5 \overline{)2} \rightarrow 0 \end{array}$$

$$\begin{array}{r} 5 \overline{)171} \\ 5 \overline{)34} \rightarrow 41 \\ 5 \overline{)6} \rightarrow 0 \\ 5 \overline{)1} \rightarrow 0 \end{array}$$

No. of zeroes in 1140!

= 228 + 45 + 9 + 1 = 283

No. of zeroes in 358!

= 71 + 14 + 2 = 87

No. of zeroes in 171!

= 34 + 6 + 1 = 41

Total No. of zeroes = 283 + 87 + 41 = 411

21. (b) The Fives will be less than the two's Hence, we need to count only the Fives

Thus,

$$5^5 = (5 \times 1)^5 = 5$$

$$10^{10} = (5 \times 2)^{10} = 10$$

$$15^{15} = (3 \times 5)^{15} = 15$$

$$20^{20} = (4 \times 5)^{20} = 20$$

$$25^{25} = (5 \times 5)^{25} = 50$$

$$30^{30} = (5 \times 6)^{30} = 30$$

$$35^{35} = (5 \times 7)^{35} = 35$$

$$40^{40} = (5 \times 8)^{40} = 40$$

$$45^{45} = (5 \times 9)^{45} = 45$$

$$5 + 10 + 15 + 20 + 50 + 30 + 35 + 40 + 45$$

No. of Fives = 250

Then,

Number of zeroes = 250

22. (b) The Five will be less than the two's Then count the number of five

$$100^1 \times 95^6 \times 90^{11} \times \dots \times 10^{91} \times 5^{96}$$

(1 + 6 + 11 + \dots + 91 + 96) using sum of A.P.

a = 1,

d = 5

$$\text{No. of term} = \frac{96 - 1}{5} + 1 = 20$$

$$S_n = \frac{20}{2} [2 \times 1 + 19 \times 5]$$

$$= 10 [2 + 95] = 970$$

But

$$100^1 = (5 \times 5 \times 4)^1 = 5^1 \times 5^1$$

$$75^{26} = (5 \times 5 \times 3)^{26} = 5^{26} \times 5^{26}$$

$$50^{51} = (5 \times 5 \times 2)^{51} = 5^{51} \times 5^{51}$$

$$25^{76} = (5 \times 5)^{76} = 5^{76} \times 5^{76}$$

Then no of zeroes

$$= 1 + 26 + 51 + 76 = 154$$

Total number of zeroes

$$= 154 + 970 = 1124$$

23. (c) Count the No. of 5's

5^{51} and 10^{101}

No. of 5's = 5! + 10!

Then ,

No. of zeroes = 5! + 10!

24. (a) Count the No. of 5's

Then

$$5^4 \times 10^8 \times 15^{12} \times 20^{16} \times 25^{20}$$

$$= 4 + 8 + 12 + 16 + 40$$

$$= 80$$

So, No. of zero = 80

25. (c)

$$3200$$

$$1000$$

$$40000$$

$$32000$$

$$+ 15000$$

$$91200$$

No. or zero = 2

26. (d) $3200 \times 1000 \times 40000 \times 32000 \times 15000$

No. of zero's 2 + 3 + 4 + 3 + 3

$$= 15$$

But 1500 = 3 \times 5 \times 100

Here 5 is present

When 5 is multiply by even number, then unit digit '0' is get.

Then,

No. of Total zero = 15 + 1 = 16

27. (a) $20 \times 40 \times 7600 \times 600 \times 300 \times 1000$

No. of zeroes = 1 + 1 + 2 + 2 + 2 + 3 = 11

28. (a) 100! + 200!

No. of zeroes In 100! = 20 + 4 = 24

No. of zeroes In 200! = 40 + 8 + 1 = 49

When you add the two Number (One with 24 zeroes and the other with 49 zeroes at It's end)

The Total No. of zeroes = 24

29. (b) $1^1 \times 2^2 \times 3^3 \times 4^4 \times \dots \times 10^{10}$

Count the Number of 5's

5^5 no of fives = 5

10^{10} No. of Fives = 10

No. of zeroes = 5 + 10 = 15

30.(c) 100!

$$\begin{array}{r} 5 \overline{) 100} \\ 5 \overline{) 20} \rightarrow \\ 5 \overline{) 4} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 100} \\ 5 \overline{) 20} \rightarrow \\ 5 \overline{) 4} \rightarrow \\ 0 \end{array}} \right\} 24$$

No. of zeroes In 100! = 24

No. of zeroes In 200! = 49

When you multiply two numbers (One with 24 zeroes and the other with 49 zeroes at It's end). The Resultant Total No. of zeroes = $24 + 49 = 73$

31.(a) $5 \times 10 \times 15 \times 20 \times 25 \times \dots \times 50$
 $5 \times 1 \times 5 \times 2 \times 5 \times 3 \times 5 \times 4 \dots \times 5 \times 10$
 $5^{10} (1 \times 2 \times 3 \times 4 \dots \times 10)$

The two will be less than the fives hence we need to count only the two's

1 to 10 no of 2's

$$\begin{array}{r} 2 \overline{) 10} \\ 2 \overline{) 5} \rightarrow \\ 2 \overline{) 2} \rightarrow \\ 2 \overline{) 1} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 2 \overline{) 10} \\ 2 \overline{) 5} \rightarrow \\ 2 \overline{) 2} \rightarrow \\ 2 \overline{) 1} \rightarrow \\ 0 \end{array}} \right\} 8$$

No. of 2's = $5 + 2 + 1 = 8$

Then No. of zeroes = 8

32.(b) $2 \times 4 \times 6 \times 8 \times 10 \dots \dots 200$
 $= 2 \times 1 \times 2 \times 2 \times 2 \times 2 \times 3 \dots \dots 2 \times 100$
 $= 2^{100} (1 \times 2 \times 3 \times \dots \times 100)$

We count No of 5

$$\begin{array}{r} 5 \overline{) 100} \\ 5 \overline{) 20} \rightarrow \\ 5 \overline{) 4} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 100} \\ 5 \overline{) 20} \rightarrow \\ 5 \overline{) 4} \rightarrow \\ 0 \end{array}} \right\} 24$$

200!

$$\begin{array}{r} 5 \overline{) 200} \\ 5 \overline{) 40} \rightarrow \\ 5 \overline{) 8} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 200} \\ 5 \overline{) 40} \rightarrow \\ 5 \overline{) 8} \rightarrow \\ 5 \overline{) 1} \rightarrow \\ 0 \end{array}} \right\} 49$$

No of 5's = 24

Then No. of zeroes = 24

33.(b) $1 \times 3 \times 5 \times 7 \dots \dots 99 \times 2^6$

Here No. of 5 is more than no. of 2 then count the number of 2

No. of 2's = 6

Now No. of zero = 6

34.(a) $2^{222} \times 5^{555}$

No. of 2's = 222

No. of 5's = 555

No. of 2's are less than Number of 5's

Pair (2's & 5's) = 222

No. of zero = 222

35.(d) $10 + 100 + 1000 + \dots \dots 1000000000$

$$\begin{array}{r} 10 \\ 100 \\ 1000 \\ \dots \dots \dots \\ 1000000000 \\ \hline 1111111110 \end{array}$$

This there is only one zero at the end of result

36.(b) $10^1 \times 10^2 \times 10^3 \times 10^4 \dots \dots 10^{10}$
 $10^{(1+2+3+\dots+10)} = 10^{55}$

$$\therefore 1+2+3+\dots+10 = \frac{10(10+1)}{2} = 55$$

No. of zero = 55

37.(b) $2^1 \times 5^2 \times 2^3 \times 5^4 \times 2^5 \times 5^6 \times 2^7 \times 5^8$
 $\times 2^9 \times 5^{10}$

$$\Rightarrow 2^{(1+3+5+7+9)} \times 5^{(2+4+6+8+10)}$$

$$\Rightarrow 2^{25} \times 5^{55}$$

Number of 2's are less than the Number of 5's

= Pair of $(2 \times 5) = 25$

No of zero = 25

38.(a) $(3^{123} - 3^{122} - 3^{121}) (2^{121} - 2^{120} - 2^{119})$

$$\Rightarrow 3^{121} (3^2 - 3^1 - 3^0) 2^{119} (2^2 - 2^1 - 2^0)$$

$$\Rightarrow (3^{121})(2^{119}) (9-3-1) (4-2-1)$$

$$\Rightarrow (3^{121})(2^{119}) (5) (1)$$

$$= 5^1 \times 2^{119} \times 3^{121}$$

No. of 5's = 1

No. of 2's = 119

Pair of (2 & 5) = 1

No. of zero = 1

39.(b) $(8^{123} - 8^{122} - 8^{121}) (3^{223} - 3^{222} - 3^{221})$

$$\rightarrow 8^{121} (8^2 - 8^1 - 1) 3^{221} (3^2 - 3^1 - 1)$$

$$\rightarrow 8^{121} (64 - 8 - 1) 3^{221} (9 - 3 - 1)$$

$$\rightarrow 8^{121} \times 55 \times 3^{221} \times 5$$

$$= 8^{121} \times 3^{221} \times 5 \times 11 \times 5$$

$$= (2^3)^{121} \times 3^{221} \times 5^2 \times 11$$

$$= 11 \times 5^2 \times 2^{363} \times 3^{221}$$

No. of 2's = 363

No. of 5's = 2

Pair of (2 & 5) = 2

No. of zero = 2

40.(a) $5^1 \times 1 \times 5^1 \times 2 \times 5^1 \times 3 \dots \dots 5^1 \times 15$
 $= 5^{15} (1 \times 2 \times 3 \dots \dots 15)$

each term multiple of 5 So power of 5's more than 2 then count the number of 2 from 1 to 15.

$$\begin{array}{r} 2 \overline{) 15} \\ 2 \overline{) 7} \rightarrow \\ 2 \overline{) 3} \rightarrow \\ 2 \overline{) 1} \rightarrow \\ 0 \end{array} \left. \vphantom{\begin{array}{r} 2 \overline{) 15} \\ 2 \overline{) 7} \rightarrow \\ 2 \overline{) 3} \rightarrow \\ 2 \overline{) 1} \rightarrow \\ 0 \end{array}} \right\} 11$$

number of (2 and 5) pairs = 11

then number of zeroes = 11

FACTOR

Factor → A number which divides a given number exactly is called factor (or divisor) of that given number and the given number is called a multiple of that number.

Ex. 1, 2, 4, and 8 are factors of 8 because 8 is perfectly divisible of 1, 2, 4 and 8

Factors and Multiple

Ex. Factors of 35 = 1, 5, 7, 35

Ex. Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Same,

Multiple of 2 = 2, 4, 6, 8, 10,

Multiple of 7 = 7, 14, 21, 28, 35

* 1 is a factor of every number

* every number is a factor of itself

* every number, except 1 has at least 2 factor

* every number has infinite number of its multiples

* every number is a multiple of itself

Number of Factors

Let N be the composite number and a, b, c, ... be its prime factors and p, q, r be the indices (or powers) of a, b, c, i.e. if N can be expressed as $N = a^p \cdot b^q \cdot c^r$ then total number of factors of N = $(p + 1) \times (q + 1) \times (r + 1)$

If a is even prime factor, b and c are odd prime factors

The number of even factors
= $(P) \times (q + 1) \times (r + 1)$

The number of odd factors
= $(1) \times (q + 1) \times (r + 1)$

Ex.1 Find the total number of factors of 8.

Sol. $8 = 1, 2, 4$ and 8 are Perfectly divisible
So number of factors = 4

* This method is easy for smaller number but for larger number its a problem So use for alternate method

Alternate

$$8 = 2 \times 2 \times 2 = 2^3$$

Number of Total factors

$$= 3 + 1 = 4$$

Ex.2 Find the total number of factors of 240

$$\begin{array}{l|l} \text{Sol.} & 240 \\ 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$

$$= 2^4 \times 3^1 \times 5^1$$

Total Factors

$$= (4 + 1) \times (1 + 1) \times (1 + 1)$$

$$= 5 \times 2 \times 2 = 20$$

Ex.3 Find the total number of factors of 500.

$$\begin{array}{l|l} \text{Sol.} & 500 \\ 2 & 250 \\ 2 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$$500 = 2 \times 2 \times 5 \times 5 \times 5$$

$$= 2^2 \times 5^3$$

$$\text{No. of factors} = (2 + 1) \times (3 + 1)$$

$$= 3 \times 4 = 12$$

Number of even Factor

Ex.4. Find the number of even factors of 24.

Sol. Factor of 24 = 1, 2, 3, 4, 6, 8, 12, 24
Even Factor of 24 = 2, 4, 6, 8, 12, 24,

So,

Total number of even Factor of 24 = 6

Alternate

$$\begin{array}{l|l} 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

$$24 = 2^3 \times 3^1$$

$$\begin{aligned} \text{Number of even factor} &= 3 \times (1 + 1) \\ &= 3 \times 2 = 6 \end{aligned}$$

Ex.5 Find the number of even factors of 60.

$$\begin{array}{l|l} \text{Sol.} & 60 \\ 2 & 30 \\ 2 & 15 \\ 3 & 5 \\ 5 & 1 \end{array}$$

$$60 = 2 \times 2 \times 3 \times 5$$

$$= 2^2 \times 3^1 \times 5^1$$

$$\begin{aligned} \text{No. of even factor} &= 2 \times (1 + 1) \times (1 + 1) \\ &= 2 \times 2 \times 2 = 8 \end{aligned}$$

No. of odd factor

Ex.6 Find the number of odd factors of 40.

Sol. 40 = 1, 2, 4, 5, 8, 10, 20, 40
Odd factors = 1, 5
Number of odd factors = 2

Alternate

$$\begin{array}{l|l} 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ 5 & 5 \\ & 1 \end{array}$$

$$40 = 2 \times 2 \times 2 \times 5$$

$$= 2^3 \times 5^1$$

$$\begin{aligned} \text{No. of odd Factors} &= 1 \times (1 + 1) \\ &= 1 \times 2 \\ &= 2 \end{aligned}$$

Ex.7 Find the number of factors, number of even factors and number of odd factors of 180

Sol. $180 = 2 \times 2 \times 3 \times 3 \times 5$
 $= 2^2 \times 3^2 \times 5^1$

Total Number of factors

$$\begin{aligned} &= (2 + 1) \times (2 + 1) \times (1 + 1) \\ &= 3 \times 3 \times 2 = 18 \end{aligned}$$

Number of even factors

$$= 2(2+1) \times (1+1) \\ = 2 \times 3 \times 2 = 12$$

Number of odd factors

$$= 1 \times (2+1) \times (1+1) \\ = 1 \times 3 \times 2 = 6$$

Ex.8 Find the number of factors, number of even factors and number of odd factors of 360.

Sol.

$$\begin{array}{r|l} 2 & 360 \\ 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \\ = 2^3 \times 3^2 \times 5^1$$

Total number of factors

$$= (3+1) \times (2+1) \times (1+1) \\ = 4 \times 3 \times 2 = 24$$

Number of even factors

$$= 3 \times (2+1) \times (1+1) \\ = 3 \times 3 \times 2 = 18$$

Number of odd factors

$$= 1 \times (2+1) \times (1+1) \\ = 3 \times 2 = 6$$

Ex.9 Find the number of factors, number of even factors and number of odd factors of 100

Sol.

$$\begin{array}{r|l} 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

$$100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$$

Total no. of Factor

$$= (2+1) \times (2+1) = 3 \times 3 = 9$$

$$\text{No. of even factor} = 2 \times (2+1) \\ = 2 \times 3 = 6$$

$$\text{No. of odd factor} = 1 \times (2+1) \\ = 1 \times 3 = 3$$

Sum of factors

Let N be the composite number and a, b, c,... be its prime factors and p, q, r be the indices (or powers) of a, b, c, i.e. if N can be expressed as $N = a^p \cdot b^q \cdot c^r$

then the sum of all the divisors (or factors) of N

$$= (a^0 + a^1 + a^2 + \dots + a^p) \times (b^0 + b^1 + b^2 + \dots + b^q) \times (c^0 + c^1 + c^2 + \dots + c^r)$$

If a is even prime factor and b and c odd prime factors then

$$\blacksquare \text{ Sum of even factors} = (a^1 + a^2 + \dots + a^p) \times (b^0 + b^1 + b^2 + \dots + b^q) \times (c^0 + c^1 + c^2 + \dots + c^r)$$

$$\blacksquare \text{ Sum of odd factor} = (a^0) \times (b^0 + b^1 + b^2 + \dots + b^q) \times (c^0 + c^1 + c^2 + \dots + c^r)$$

Ex10. Find the sum of all factors of 8.

Sol. factors of 8 = 1, 2, 4, 8

$$\text{Sum of factors} = 1 + 2 + 4 + 8 \\ = 15$$

This method is easy for smaller number but for larger number its a problem So use for alternate method

Alternate

$$8 = 2^3$$

$$\text{sum of all factors} = (2^0 + 2^1 + 2^2 + 2^3) \\ = 1 + 2 + 4 + 8 = 15$$

$$(a^0 = 1, \text{ where } a = \text{real number})$$

Ex.11 find the sum of all factors, sum of even factors and sum of odd factors of 24.

Sol. factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

$$\text{sum of factors} = 1 + 2 + 3 + 4 + 6 + 8 + 12 + 24 = 60$$

sum of even factors

$$= 2 + 4 + 6 + 8 + 12 + 24 = 56$$

$$\text{Sum of odd factors} = 1 + 3 = 4$$

Alternate

$$\begin{array}{r|l} 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array} \quad 24 = 2^3 \times 3^1$$

sum of all factors

$$= (2^0 + 2^1 + 2^2 + 2^3) \times (3^0 + 3^1)$$

$$= (1 + 2 + 4 + 8) \times (1 + 3)$$

$$= 15 \times 4 = 60$$

Sum of even factors

$$= (2^1 + 2^2 + 2^3) \times (3^0 + 3^1)$$

$$= (2 + 4 + 8) \times (1 + 3)$$

$$= 14 \times 4 = 56$$

Sum of odd factors

$$= (2^0) \times (3^0 + 3^1) = 1 \times 4 = 4$$

Ex12. find the sum of all factors, sum of even factors and sum of odd factors of 360.

Sol.

$$\begin{array}{r|l} 2 & 360 \\ 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array} \quad 360 = 2^3 \times 3^2 \times 5^1$$

sum of all factors

$$= (2^0 + 2^1 + 2^2 + 2^3) \times (3^0 + 3^1 + 3^2) \\ \times (5^0 + 5^1)$$

$$= 15 \times 13 \times 6 = 1170$$

Sum of even factors

$$= (2^1 + 2^2 + 2^3) \times (3^0 + 3^1 + 3^2) \\ \times (5^0 + 5^1)$$

$$= 14 \times 13 \times 6 = 1092$$

sum of odd factors

$$= 2^0 \times (3^0 + 3^1 + 3^2) \times (5^0 + 5^1)$$

$$= 1 \times 13 \times 6 = 78$$

Ex13. find the sum of all factors, sum of even factors and sum of odd factors of 100.

$$\begin{array}{r|l} 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array} \quad 100 = 2^2 \times 5^2$$

Sum of all factors

$$= (2^0 + 2^1 + 2^2) \times (5^0 + 5^1 + 5^2)$$

$$= 7 \times 31 = 217$$

Sum of even factors

$$= (2^1 + 2^2) \times (5^0 + 5^1 + 5^2)$$

$$= 6 \times 31 = 186$$

Sum of odd factors

$$= (2^0) \times (5^0 + 5^1 + 5^2)$$

$$= 1 \times 31 = 31$$

Prime Factorisation

Prime Factorisation : If a natural number is expressed as the product of prime numbers (factors) then the factorisation of the number is called its prime factorisation.

(i) **72**

$$\begin{array}{r|l} 2 & 72 \\ 2 & 36 \\ 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$72 = 2^3 \times 3^2$$

number of prime factors = 3 + 2 = 5

(ii) **540**

$$\begin{array}{r|l} 2 & 540 \\ 2 & 270 \\ 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$$

$$= 2^2 \times 3^3 \times 5^1$$

No. of prime factor = $2 + 3 + 1$
 $= 6$
 (iii) find the number of prime factor
 $2^3 \times 5^7 \times 21^4 \times 10^8$

Sol. $2^3 \times 5^7 \times 21^4 \times 10^8$
 $2^3 \times 5^7 \times (3 \times 7)^4 \times (2 \times 5)^8$
 $2^3 \times 5^7 \times 3^4 \times 7^4 \times 2^8 \times 5^8$

$2^{11} \times 3^4 \times 5^{15} \times 7^4$
 Total No. of prime factors
 $= 11 + 4 + 15 + 4 = 34$

Ex14. The Number of prime Factors In the expression $6^4 \times 8^6 \times 10^8 \times 12^{10}$ is
 (a) 48 (b) 64
 (c) 72 (d) 80

Sol. $6^4 \times 8^6 \times 10^8 \times 12^{10}$
 $\Rightarrow (2 \times 3)^4 \times (2 \times 2 \times 2)^6 \times (2 \times 5)^8 \times (2 \times 2 \times 3)^{10}$
 $\Rightarrow 2^4 \times 3^4 \times 2^6 \times 2^8 \times 5^8 \times (2^2 \times 3)^{10}$
 $\Rightarrow 2^4 \times 3^4 \times 2^{18} \times 2^8 \times 5^8 \times 2^{20} \times 3^{10}$
 $\Rightarrow 2^{4+18+8+20} \times 3^{4+10} \times 5^8$
 $= 2^{50} \times 3^{14} \times 5^8$
 Total No. of prime factor
 $= 50 + 14 + 8 = 72$

EXERCISE

- Find the number of Factors of 1728
 (a) 28 (b) 29 (c) 30 (d) 31
- Find the Number of Factor of 1420
 (a) 12 (b) 13 (c) 14 (d) 15
- Find the Number of Divisors of 10800
 (a) 30 (b) 60 (c) 120 (d) 180
- Find the No. of Prime Factor of 240.
 (a) 4 (b) 5 (c) 6 (d) 8
- Find the No. of prime factor.
 $(30)^{26} \times (25)^{51} \times (12)^{23}$
 (a) 249 (b) 250 (c) 255 (d) 260
- Find the No. of Prime Factor
 $(30)^{15} \times (22)^{11} \times (15)^{24}$
 (a) 110 (b) 115 (c) 120 (d) 125
- Find the No. of Prime Factor 180
 (a) 4 (b) 5 (c) 6 (d) 7

- Find the No. of Prime Factor of 536
 (a) 4 (b) 5 (c) 6 (d) 3
- Find the No. of prime Factor of 1044
 (a) 4 (b) 5 (c) 10 (d) 9
- Find The No. of prime factor of $(56)^{20} \times (36)^{31} \times (42)^{13} \times (13)^{21}$
 (a) 240 (b) 242 (c) 264 (d) 248
- Find the total Number of Prime Factors of
 $2^{17} \times 6^{31} \times 7^{55} \times 10^{11} \times 11^{10} \times 21^{12}$
 (a) 142 (b) 144 (c) 140 (d) 146
- Find the prime Factors 210
 (a) 3 (b) 4 (c) 5 (d) 6
- Find the sum of odd factors of 544
 (a) 16 (b) 18 (c) 20 (d) 22
- For the Number 2450 find
 (i) Number of all factors
 (ii) Number of even factors
 (iii) Number of odd factors

- (a) 18,9,9 (b) 18,10,8
 (c) 18,8,10 (d) 18,12,6
- For the Number 760
 (i) The sum and Number of all factors
 (ii) The Sum and Number of even factors
 (iii) The Sum and Number of odd factors
- For The Number 96
 (i) Sum and number of all factors
 (ii) The sum and Number of even factors
 (iii) The sum and Number of odd factors
- For the Number 270
 (i) The sum & Number of all Factor
 (ii) The sum & Number of even factor
 (iii) The sum & Number of odd Factor

ANSWER KEY

1. (a)	3. (b)	5. (a)	7. (b)	9. (b)	11. (c)	13. (b)
2. (a)	4. (c)	6. (b)	8. (a)	10. (c)	12. (b)	14. (a)

SOLUTION

- (a)

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

 $1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$
 $= 2^6 \times 3^3$
 No. of factors = $(6 + 1) \times (3 + 1)$

- (a)

2	1420
2	710
5	355
71	71
	1

 $1420 = 2 \times 2 \times 5 \times 71$
 $= 2^2 \times 5^1 \times 71^1$
 No. of factors
 $= (2 + 1) \times (1 + 1) \times (1 + 1)$
 $= 3 \times 2 \times 2 = 12$

- (b)

2	10800
2	5400
2	2700
2	1350
3	675
3	225
3	75
5	25
5	5
	1

 $10800 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$
 $= 2^4 \times 3^3 \times 5^2 = 2^4 \times 3^3 \times 5^2$

$$\begin{aligned}\text{No. of factors} &= (4+1)(3+1)(2+1) \\ &= 5 \times 4 \times 3 = 60\end{aligned}$$

$$\begin{array}{r|l} 4. & (c) \quad 2 \mid 240 \\ & 2 \mid 120 \\ & 2 \mid 60 \\ & 2 \mid 30 \\ & 3 \mid 15 \\ & 5 \mid 5 \\ & 1 \end{array}$$

$$\begin{aligned}240 &= 2 \times 2 \times 2 \times 3 \times 5 \\ &= 2^4 \times 3^1 \times 5^1\end{aligned}$$

$$\text{No. of prime factor} = 4+1+1=6$$

$$\begin{aligned}5. & (a) (30)^{26} \times (25)^{51} \times (12)^{23} \\ & \text{Break The form of prime factor} \\ \Rightarrow & (2^1 \times 3^1 \times 5^1)^{26} \times (5 \times 5)^{51} \times (2 \times 2 \times 3)^{23} \\ \Rightarrow & 2^{26} \times 3^{26} \times 5^{26} \times 5^{102} \times 2^{46} \times 3^{23} \\ \Rightarrow & 2^{26+46} \times 3^{26+23} \times 5^{26+102} \\ \Rightarrow & 2^{72} \times 3^{49} \times 5^{128}\end{aligned}$$

$$\text{No. of prime factors}$$

$$\Rightarrow 72 + 49 + 128 = 249$$

$$\begin{aligned}6. & (b) (30)^{15} \times (22)^{11} \times (15)^{24} \\ \Rightarrow & (2 \times 3 \times 5)^{15} \times (2 \times 11)^{11} \times (3 \times 5)^{24} \\ \Rightarrow & 2^{15} \times 3^{15} \times 5^{15} \times 2^{11} \times 11^{11} \times 3^{24} \times 5^{24} \\ \Rightarrow & 2^{15+11} \times 3^{15+24} \times 5^{15+24} \times 11^{11} \\ \Rightarrow & 2^{26} \times 3^{39} \times 5^{39} \times 11^{11}\end{aligned}$$

$$\text{No. of Prime factor}$$

$$26 + 39 + 39 + 11 = 115$$

$$\begin{array}{r|l} 7. & (b) \quad 2 \mid 180 \\ & 2 \mid 90 \\ & 3 \mid 45 \\ & 3 \mid 15 \\ & 5 \mid 5 \\ & 1 \end{array}$$

$$\begin{aligned}180 &= 2 \times 2 \times 3 \times 3 \times 5 \\ &= 2^2 \times 3^2 \times 5^1\end{aligned}$$

$$\text{No. of prime Factor} = 2+2+1 = 5$$

$$\begin{array}{r|l} 8. & (a) \quad 2 \mid 536 \\ & 2 \mid 268 \\ & 2 \mid 134 \\ & 67 \mid 67 \\ & 1 \end{array}$$

$$536 = 2 \times 2 \times 2 \times 67 = 2^3 \times 67^1$$

$$\text{No. of prime factor} = 3+1 = 4$$

$$\begin{array}{r|l} 9. & (b) \quad 2 \mid 1044 \\ & 2 \mid 522 \\ & 3 \mid 261 \\ & 3 \mid 87 \\ & 29 \mid 29 \\ & 1 \end{array}$$

$$\begin{aligned}1044 &= 2 \times 2 \times 3 \times 3 \times 29 \\ &= 2^2 \times 3^2 \times 29^1\end{aligned}$$

$$\text{No. of prime factor}$$

$$= 2 + 2 + 1 = 5$$

$$\begin{aligned}10. & (c) (56)^{20} \times (36)^{31} \times (42)^{13} \times (13)^{21} \\ \Rightarrow & (2 \times 2 \times 2 \times 7)^{20} \times (2^2 \times 3^2)^{31} \times (2 \times 3 \times 7)^{13} \times (13)^{21} \\ \Rightarrow & (2^3 \times 7)^{20} \times (2^{62} \times 3^{62}) \times (2 \times 3 \times 7)^{13} \times (13)^{21}\end{aligned}$$

$$\Rightarrow 2^{60} \times 7^{20} \times 2^{62} \times 3^{62} \times 2^{13} \times 3^{13} \times 7^{13} \times 13^{21}$$

$$\Rightarrow 2^{60+62+13} \times 3^{62+13} \times 7^{20+13} \times 13^{21}$$

$$\Rightarrow 2^{135} \times 3^{75} \times 7^{33} \times 13^{21}$$

$$\text{Number of prime factors}$$

$$= 135 + 75 + 33 + 21 = 264$$

$$\begin{aligned}11. & (c) 2^{17} \times 6^{31} \times 7^{5} \times 10^{11} \times 11^{10} \times 21^{12} \\ \Rightarrow & 2^{17} \times (2 \times 3)^{31} \times 7^5 \times (2 \times 5)^{11} \times 11^{10} \times (3 \times 7)^{12} \\ \Rightarrow & 2^{17} \times 2^{31} \times 3^{31} \times 7^5 \times 2^{11} \times 5^{11} \times 11^{10} \times 3^{12} \times 7^{12} \\ \Rightarrow & 2^{17+31+11} \times 3^{31+12} \times 5^{11} \times 7^{5+12} \times 11^{10} \\ \Rightarrow & 2^{59} \times 3^{43} \times 5^{11} \times 7^{17} \times 11^{10}\end{aligned}$$

$$\text{Total No. of Prime Factors}$$

$$= 59 + 43 + 11 + 17 + 10$$

$$= 140$$

$$\begin{array}{r|l} 12. & (b) \quad 2 \mid 210 \\ & 3 \mid 105 \\ & 5 \mid 35 \\ & 7 \mid 7 \\ & 1 \end{array}$$

$$210 = 2^1 \times 3^1 \times 5^1 \times 7^1$$

$$= 1 + 1 + 1 + 1 = 4$$

$$\begin{aligned}13. & (b) 544 = 2 \times 2 \times 2 \times 2 \times 2 \times 17 \\ & = 2^5 \times 17^1\end{aligned}$$

$$\text{Sum of odd factors}$$

$$= (2^0) \times (17^0 + 17^1)$$

$$= 1 \times (1 + 17)$$

$$= 1 \times 18 = 18$$

$$\begin{array}{r|l} 14. & (a) \quad 2 \mid 2450 \\ & 5 \mid 1225 \\ & 5 \mid 245 \\ & 7 \mid 49 \\ & 7 \mid 7 \\ & 1 \end{array}$$

$$2450 = 2 \times 5 \times 5 \times 7 \times 7$$

$$2450 = 2^1 \times 5^2 \times 7^2$$

$$\begin{aligned}\text{Number of Factor} &= (1+1)(2+1)(2+1) \\ &= 2 \times 3 \times 3 = 18\end{aligned}$$

$$\begin{aligned}\text{Number of even Factor} &= 1 \times (2+1) \times (2+1) \\ &= 1 \times 3 \times 3 = 9\end{aligned}$$

$$\begin{aligned}\text{Number of odd factor} &= 1(2+1) \times (2+1) \\ &= 3 \times 3 = 9\end{aligned}$$

$$\begin{array}{r|l} 15. & 2 \mid 760 \\ & 2 \mid 380 \\ & 2 \mid 190 \\ & 5 \mid 95 \\ & 19 \mid 19 \\ & 1 \end{array}$$

$$\begin{aligned}760 &= 2 \times 2 \times 2 \times 5 \times 19 \\ &= 2^3 \times 5^1 \times 19^1\end{aligned}$$

$$(i) \text{ Number of factor}$$

$$= (3+1) \times (1+1) \times (1+1)$$

$$= 4 \times 2 \times 2 = 16$$

$$\text{Sum of factor}$$

$$= (2^0 + 2^1 + 2^2 + 2^3) \times (5^0 + 5^1) \times (19^0 + 19^1)$$

$$= (1+2+4+8) \times (1+5) \times (1+19)$$

$$= 15 \times 6 \times 20 = 1800$$

$$(ii) \text{ Number of even factor}$$

$$= 3 \times (1+1) \times (1+1)$$

$$= 3 \times 2 \times 2 = 12$$

$$\text{Sum of even factor}$$

$$= (2^1 + 2^2 + 2^3) \times (5^0 + 5^1) \times (19^0 + 19^1)$$

$$= 14 \times 6 \times 20 = 1680$$

$$(iii) \text{ Number of odd factors}$$

$$= 1 \times (1+1) \times (1+1)$$

$$= 1 \times 2 \times 2 = 4$$

$$\text{Sum of odd factors}$$

$$= (2^0) \times (5^0 + 5^1) \times (19^0 + 19^1)$$

$$= 1 \times 6 \times 20 = 120$$

$$16. 96 = 2 \times 2 \times 2 \times 2 \times 3$$

$$= 2^5 \times 3^1$$

$$(i) \text{ Number of all factor}$$

$$= (5+1) \times (1+1) = 6 \times 2 = 12$$

$$\text{Sum of all factor}$$

$$= (2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5) \times (3^0 + 3^1)$$

$$= (1+2+4+8+16+32) \times (1+3)$$

$$= 63 \times 4 = 252$$

$$(ii) \text{ Number of even factor}$$

$$= 5 \times (1+1) = 5 \times 2 = 10$$

$$\text{Sum of even factor}$$

$$= (2^1 + 2^2 + 2^3 + 2^4 + 2^5) \times (3^0 + 3^1)$$

$$= (2+4+8+16+32) \times (1+3)$$

$$= 62 \times 4 = 248$$

$$(iii) \text{ Number of odd factor}$$

$$= 1 \times (1+1) = 1 \times 2 = 2$$

$$\text{Sum of odd factor}$$

$$= (2^0) \times (3^0 + 3^1)$$

$$= 1 \times 4 = 4$$

17.Sol.
$$\begin{array}{r|l} 2 & 270 \\ 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

$$270 = 2 \times 3 \times 3 \times 3 \times 5$$

$$= 2^1 \times 3^3 \times 5^1$$

(i) Number of all factor

$$= (1+1) \times (3+1) \times (1+1)$$

$$= 2 \times 4 \times 2 = 16$$

Sum of all factor

$$= (2^0 + 2^1) \times (3^0 + 3^1 + 3^2 + 3^3) \times (5^0 + 5^1)$$

$$= 3 \times 40 \times 6 = 720$$

(ii) Number of even factor

$$= 1 \times (3+1) \times (1+1)$$

$$= 1 \times 4 \times 2 = 8$$

Sum of even factor

$$= (2^1) \times (3^0 + 3^1 + 3^2 + 3^3) \times (5^0 + 5^1)$$

$$= 2 \times 40 \times 6 = 480$$

Number of odd factors

$$= 1 \times (3+1) \times (1+1)$$

$$= 4 \times 2 = 8$$

Sum of odd factors

$$= 2^0 \times (3^0 + 3^1 + 3^2 + 3^3) \times (5^0 + 5^1)$$

$$= 1 \times 40 \times 6 = 240$$



DIVISIBILITY

Rule of Divisibility

- * **Divisibility by 2** → If Last digit of the number is divisible by 2

Divisibility by 4 → If Last two digits of the number are divisible by 4

Divisibility by 8 → If Last three digits of the number are divisible by 8

Divisibility by 16 → If Last four digits of the number are divisible by 16

Divisibility by 32 → If Last five digits of the number are divisible by 32

- * **Divisibility of 3** → All such numbers the Sum of whose digits are divisible by 3

Divisibility of 9 → All such numbers the Sum of whose digits are divisible by 9

- * **Divisibility by 6** → A number is divisible by 6 If it is simultaneously divisible by 2 and 3

- * **Divisibility by 5** → If Last digit (0 and 5) is divisible by 5

Divisibility by 25 → If Last two digits of the number are divisible by 25

Divisibility by 125 → If Last three digits of the number are divisible by 125

- * **Divisibility by 7** → Double the last digit and subtract it from the remaining leading truncated number. If the result is divisible by 7, then so was the original number.

- * **Divisibility by 11** → The difference of the sum of the digits in the odd places and the sum of digits in the even places is '0' or multiple of 11 is divisible

- * **Divisibility by 3, 7, 11, 13, 21, 37 and 1001** →

- (i) If any number is made by repeating a digit 6 times the number will be divisible by 3, 7, 11, 13, 21, 37 and 1001 etc.

- (ii) A six digit number if formed by repeating a three digit number; for example, 256, 256 or 678, 678 etc. Any number of this form is always exactly divisible by 7, 11, 13, 1001 etc.

Some important points

- (a) If a is divisible by b then ac is also divisible by b.
 (b) If a is divisible by b and b is divisible by c then a is divisible by c.
 (c) If n is divisible by d and m is divisible by d then (m + n) and (m - n) are both divisible by d. This has an important implication. Suppose 48 and 528 are both divisible by 8. Then (528 + 48) as well as (528 - 48) are divisible by 8

Ex.1: Check to see if 203 is divisible by 7

$$\begin{array}{r|l} 20 & 3 \\ -6 & \times 2 \\ \hline 14 & \end{array}$$

Step I. Double the last digit = $3 \times 2 = 6$

Step II. Subtract that from the rest of the Number = $20 - 6 = 14$

Step III. Check to see if the difference is divisible by 7. 14 is divisible by 7 therefore 203 is also divisible by 7

Ex.2: Check to see if 68734 is divisible by 7

$$\begin{array}{r|l} 6873 & 4 \\ -8 & \times 2 \\ \hline 686 & 5 \\ -10 & \times 2 \\ \hline 67 & 6 \\ -12 & \times 2 \\ \hline 55 & \end{array}$$

55 is not divisible by 7 So, 68734 is not divisible by 7

Ex.3: Check to see if 24983 is divisible by 7

$$\begin{array}{r|l} 2498 & 3 \\ -6 & \times 2 \\ \hline 249 & 2 \\ -4 & \times 2 \\ \hline 24 & 5 \\ -10 & \times 2 \\ \hline 14 & \end{array}$$

14 is divisible by 7, therefore 24983 is also divisible by 7

Ex.4: Check to see if 65432577 is divisible by 7

Sol. When any number is made of more than five digits then we check divisibility by 7 another rule

Step I. First for we make pair of 3 digits from right side (last)

65 432 577

Step II. Add alternate pairs
 $= 65 + 577 = 642$

Step III. Subtract from remaining (3rd) pair = $642 - 432 = 210$

If difference is divisible by 7 therefore number is also divisible by 7

Here difference = 210

210 is divisible by 7. Therefore 65432577 will be divisible by 7.

Note:- We can use First rule of divisibility by 7 but when a number has more than 5 digits this rule is easy for solve problem.

Ex.5: Check to see if 23756789765 is divisible by 7

Sol. 23 756 789 765

Step I. Add alternate pair
 $765 + 756 = 1521$
 $23 + 789 = 812$

Step II. Subtract pairs
 $1521 - 812 = 709$

709 is not divisible by 7 therefore 23756789765 is not divisible by 7

Ex.6: If 5432*7 is divisible by 9, then the digit in place of * is
(a) 0 (b) 1 (c) 6 (d) 9

Sol. (c) $\frac{5+4+3+2+x+7}{9} = \frac{21+x}{9}$

Put the value of 'x'. So the number is completely divisible by 9. Put $x = 6$

$= \frac{21+6}{9} = \frac{27}{9} = '0'$ remainder

Property: A number is completely divisible by 9 if the sum of the digits of the number is completely divisible by 9 and give no remainder.

Ex.7: When 335 is added to 5A7, the result is 8B2. 8B2 is divisible by 3. What is the largest possible value of A?
(a) 8 (b) 2 (c) 1 (d) 4

Sol.
$$\begin{array}{r} 5 \ A \ 7 \\ + 3 \ 3 \ 5 \\ \hline 8 \ B \ 2 \end{array}$$

$\Rightarrow A \rightarrow 1, 2, 3, 4, 5 \ \&$

$B \rightarrow 5, 6, 7, 8, 9$

8B2 is exactly $\therefore 8 + B + 2 =$ multiple of 3

$\therefore B = 5 \text{ or } 8 \Rightarrow A = 1 \text{ or } 4$

Ex.8: If * is a digit such that 5824* is divisible by 11, then * equals :
(a) 2 (b) 3 (c) 5 (d) 6

Sol. (c)
$$\begin{array}{ccccccc} 5 & 8 & 2 & 4 & * \\ \hline \end{array}$$

 $\Rightarrow 5 + 2 + * = 8 + 4$
 $7 + * = 12$
 $* = 12 - 7 = 5$

Property: A number will be exactly divisible by 11 when the difference of the sum of odd place digits and even place digits is zero or divisible by 11.

Ex.9: Both the end digits of a 99 digit number N are 2. N is divisible by 11 then all the middle digits are:
(a) 1 (b) 2 (c) 3 (d) 4

Sol. (d) A number is divisible by 11 if the difference of the sum of digits at odd and even places by either zero or multiple of 11.

If the middle digit be 4, then 24442 or 244442 etc are divisible by 11.

Alternate:-

2.....2

This number has 99 digits. First (1st) and last (99th) term is 2 (given) middle terms (2nd to 98th) is assume 4. 3rd to 98th term = (4 4)

difference between odd and even place of terms (3rd to 98th) = 0

Remaining Terms

1st, 2nd and 99th (last)

Here,

1st term = 2

2nd terms = 4

Last terms = 2

Difference of odd and even place of the remaining term
(2 + 2) - 4 = 0

So, If the middle digit be 4, then 24442 or 244442 etc are divisible by 11.

Ex.10: Both the end digits of a 100 digit number N are 2. N is divisible by 11 then all the middle digits are:
(a) Only 4 (b) Only 2
(c) Only 3 (d) Any digit

Sol. (d) A number is divisible by 11 if the difference of the sum of digits at odd and even places by either zero or multiple of 11.

If the middle digit be any digit then 211112 or 23333332 etc are divisible by 11.

Alternate:-

2.....2

This number has 100 digits. First (1st) and last (100th) term is 2 (given) middle terms (2nd to 99th) is assume any digit.

2nd to 99th term = (n n)

difference between odd and even places (2nd to 99th) = 0

Remaining Terms

1st, and 100th (last)

Here,

1st term = 2

Last terms = 2

Difference of odd and even place of the remaining terms
(2 - 2) = 0

So, If the middle digit be any digit, then (2.....2) is divisible by 11.

Ex.11: If the number 243x 51 is divisible by 9 then the value of the digit marked as x would be:
(a) 3 (b) 1 (c) 2 (d) 4

Sol. (a) 243x 51 is divisible by 9 divisibility of 9 = sum of digit divisible by 9

$= 2 + 4 + 3 + x + 5 + 1 = \frac{15+x}{9}$

x would be $3 = \frac{18}{9}$

So, $x = 3$

Ex.12. $2^{71} + 2^{72} + 2^{73} + 2^{74}$ is divisible by

(a) 9 (b) 10 (c) 11 (d) 13

Sol.(b) Expression

$= 2^{71} (1 + 2 + 4 + 8)$

$= 2^{71} \times 15 = 2^{71} \times 3 \times 5$

Which is exactly divisible by 10.

Ex.13. A 4 digit number is formed by repeating a 2-digit number such of 2525, 3232, etc. Any number of this form is always exactly divisible by :
(a) 7 (b) 11 (c) 13
(d) Smallest 3-digit prime number

Sol.(d) Let the unit digit be x and ten's digit be y.

$\therefore$ Number

$= 1000y + 100x + 10y + x$

$= 1010y + 101x = 101 (10y + x)$

Clearly, this number is divisible by 101, which is the smallest three-digit prime number.

Ex. 14: A six digit number is formed by repeating a three digit number; for example, 256, 256 or 678, 678 etc. Any number of this form is always exactly divisible by:
(a) 7 only (b) 11 only
(c) 13 only (d) 1001

Sol. (d) The number (xyzxyz) can be written, after given corresponding weightage of the places at which the digits occur, as $100000x + 10000y + 1000z + 100 + 10y + z$
 $= 100100x + 10010y + 1001z$
 $= 1001 (100x + 10y + z)$
Since 1001 is a factor, the number is divisible by 1001. As the number is divisible by 1001, it will also be divisible by all three namely, 7, 11 and 13 and not by only one of these because all three are factors of 1001.

Ex.15: Which of the following number will always divide a six-digit number of the form xyxyxy (where $1 \leq x \leq 9, 1 \leq y \leq 9$)?

- (a) 1010 (b) 10101
(c) 11011 (d) 11010

Sol. (b) Number = $xy\ xy\ xy$
 $= xy \times 10000 + xy \times 100 + xy$
 $= xy (10000 + 100 + 1)$
 $= xy \times 10101$

Ex.16. 47 is added to the product of 71 and an unknown number. The new number is divisible by 7 giving the quotient 98. The unknown number is a multiple of

- (a) 2 (b) 5 (c) 7 (d) 3

Sol. (d) Let the unknown number be x .

$$\therefore 71 \times x + 47 = 98 \times 7$$

$$\Rightarrow 71x = 686 - 47 = 639$$

$$\Rightarrow x = \frac{639}{71} = 9 = 3 \times 3$$

Ex.17. When an integer K is divided by 3, the remainder is 1, and when $K + 1$ is divided by 5, there remainder is 0. Of the following, a possible value of K is

- (a) 62 (b) 63 (c) 64 (d) 65

Sol. (c) Take option (d)

When 64 is divided by 3, remainder = 1

When 65 is divided by 5, remainder = 0

Ex.18. If n is a whole number greater than 1, then $n^2 (n^2 - 1)$ is always divisible by:

- (a) 16 (b) 12 (c) 10 (d) 8

Sol. (b) $n^2 (n^2 - 1) = n^2 (n+1) (n-1)$

Now, we put values $n = 2, 3, \dots$
When $n = 2$

$\therefore n^2 (n^2 - 1) = 4 \times 3 \times 1 = 12$, which is a multiple of 12

When $n = 3$.

$n^2 (n^2 - 1) = 9 \times 4 \times 2 = 72$,
Which is also a multiple of 12.
etc.

Ex.19. If n is even, $(6^n - 1)$ is divisible by

- (a) 37 (b) 35 (c) 30 (d) 6

Sol. (b) When $n = 2$.

$$6^n - 1 = 6^2 - 1 = 36 - 1 = 35$$

When, n = an even number,
 $a^n - b^n$ is always divisible by $(a^2 - b^2)$.

Relation among divisor, dividend quotient and remainder :

$$\rightarrow 27 \text{ is divided by } 6 \text{ then } \frac{27}{6}$$

$$\begin{array}{r} \text{dividend} \\ 6 \overline{) 27} \quad (4 \rightarrow \text{Quotient} \\ \underline{24} \\ 3 \rightarrow \text{Remainder} \end{array}$$

$$6 \times 4 + 3 = 27$$

$$\text{divisor} \times \text{quotient} + \text{remainder} = \text{dividend}$$

Ex.20: In a problem involving division, the divisor is eight times the quotient and four times the remainder. If the remainder is 12, then the dividend is:

- (a) 300 (b) 288 (c) 512 (d) 524

Sol. (a) Remainder = 12

$$\text{Divisor} = 4 \times 12 = 48$$

$$\text{Quotient} = \frac{48}{8} = 6$$

$$\text{Dividend} = \text{divisor} \times \text{quotient} + \text{remainder} = 48 \times 6 + 12$$

$$= 288 + 12 = 300$$

Ex.21: The divisor is 25 times the quotient and 5 times the remainder. If the quotient is 16, the dividend is:

- (a) 6400 (b) 6480

- (c) 400 (d) 480

Sol. (b) Dividend = divisor $\times$ quotient + Remainder
According to Question

$$\text{Divisor} = 16 \times 25 = 5 \times R$$

$$\Rightarrow R = \frac{1}{5} \times 16 \times 25$$

Dividend

$$= [(16 \times 25) \times 16] + \frac{1}{5} \times 16 \times 25$$

$$= [16 \times 25 \times 16] + 80 = 6480$$

Ex.22: In a division problem, the divisor is 4 times the quotient and 3 times the remainder. If remainder is 4, the dividend is:

- (a) 36 (b) 40 (c) 12 (d) 30

Sol. (b)

$$\begin{array}{r} \text{Divisor} \overline{) \text{Dividend}} \quad (\text{Quotient} \\ \underline{} \\ \text{Remainder} \end{array}$$

According to the question

$$\begin{array}{r} 12 \overline{) \text{Dividend}} \quad (3 \\ \underline{} \\ 4 \end{array}$$

$$\text{Dividend is } (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

$$= (12 \times 3) + 4$$

$$= 40$$

Ex.23: In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, the dividend is:

Sol. Remainder = 46

$$\text{Divisor} = 5 \times 46 = 230$$

$$\text{Quotient} = \frac{230}{10} = 23$$

$$\text{Dividend} = \text{divisor} \times \text{quotient} + \text{remainder}$$

$$= 230 \times 23 + 46$$

$$= 5290 + 46 = 5336$$

Ex.24: On dividing 397246 by a certain number, the quotient is 865 and the remainder is 211. Find the divisor.

$$\text{Sol. Divisor} = \frac{397246 - 211}{865} = 459$$

Ex.25: A number when divided by 14 leaves a remainder of 8, but when the same number is divided by 7, it will leave the remainder:

- (a) 3 (b) 2 (c) 1

- (d) Can't be determined

Sol. $14 \overline{) N} \quad x$

$$\begin{array}{r} \text{8} \rightarrow \text{Remainder} \\ N = 14x + 8 \end{array}$$

According to the question,

$$\begin{array}{r} 0 \quad 1 \\ \uparrow \quad \uparrow \\ \frac{N}{7} = \frac{14x+8}{7} \end{array}$$

$$\text{Remainder} = \frac{8}{7} = 1$$

Ex.26: If a number is divided by 102 and leaves remainder 91. If this number is divided by 17 the remainder

Sol.

$$\begin{array}{r} \text{dividend} \\ 102 \overline{) M} \quad (P \rightarrow \text{Quotient} \\ \underline{} \\ 91 \rightarrow \text{Remainder} \end{array}$$

We know that

$$\boxed{\text{divisor} \times \text{quotient} + \text{remainder} = \text{dividend}}$$

$$M = 102 \times P + 91$$

$$\text{Now } M = \frac{17 \times 6 \times P + 91}{17}$$

$$M = \frac{17 \times 6P}{17} + \frac{91}{17}$$

$$R = \frac{91}{17} \Rightarrow R = 6$$

Note: If we have to find remainder of those term which divide previous term we will take remainder of it and divide by this term and we have to get.

$$\frac{91}{17} \quad R = 6$$

Ex.27: If a number is divided by 84 and leaves remainder 37. If this number is divided by 12.

Sol. Then the remainder 84 is divisible by 12

$$\text{So, remainder} = \frac{37}{12} = 1$$

Ex.28: A number when divided by 899 gives a remainder 63. If the same number is divided by 29, the remainder will be:

- (a) 10 (b) 5 (c) 4 (d) 2

Sol. (b) $\frac{\text{Remainder}}{29} = \frac{63}{29}$
 $\Rightarrow \text{remainder} = 5$

Ex.29: A number when divided by 296 gives a remainder 75. When the same number is divided by 37 the remainder will be

- (a) 1 (b) 2 (c) 8 (d) 11

Sol. (a) $\frac{\text{Remainder}}{37} = \frac{75}{37}$
 remainder = 1

Ex.30: A number being divided by 52 gives remainder 45. If the number is divided by 13, the remainder will be:

- (a) 5 (b) 6 (c) 12 (d) 7

Sol. (b) since 13 is factor of 52. So divide its remainder by 13

$$\text{Remainder} = \frac{45}{13} = 6$$

Ex.31: If A number is divided by 225 a remainder at 70. But when a square of the number is divided by 15. What is the remainder?

Sol. $225 \overline{) N} \quad Q$
 $\underline{70}$

$$N = 225Q + 70$$

$$\text{Square of number} = N^2$$

$$= (225Q + 70)^2$$

$$\text{Then} = \begin{array}{cc} 0 & 10 \\ \uparrow & \uparrow \\ (225Q + 70) & \\ \hline & 15 \end{array}$$

$$= \frac{(10)^2}{15} = \frac{100}{15}$$

$$\boxed{\text{Remainder} = 10}$$

Alternate:-

$$\frac{(\text{Remainder})^2}{15} = \frac{(70)^2}{15}$$

$$\begin{array}{cc} 10 & \\ \uparrow & \\ (70)^2 & = \frac{(10)^2}{15} = \frac{100}{15} \end{array}$$

Remainder = 10

Ex.32: If a number is divided 36 and leaves remainder 23. If cube of this number is divided by 12. Then what is the remainder.

Sol. $36 \overline{) N} \quad Q$
 $\underline{23}$
 $N = 36Q + 23$
 cube of number
 $= N^3 = (36Q + 23)^3$

$$\begin{array}{cc} 0 & -1 \\ \uparrow & \uparrow \\ \text{Now, } (36Q + 23)^3 & \\ \hline & 12 \end{array}$$

$$= \frac{(-1)^3}{12} = \frac{-1}{12}$$

$$\text{Remainder} = 12 - 1 = 11$$

Alternate:-

$$\text{Remainder} = \frac{(-1)^3}{12} = \frac{-1}{12}$$

$$= \frac{(-1)^3}{12} = \frac{-1}{12}$$

$$\text{Remainder} = 12 - 1 = 11$$

Ex.33: Two number when divided by 17. Leave remainder 13 and 11 respectively if the sum of those two numbers is divided by 17 the remainder will be

Sol. N_1 (First Number) = $17x + 13$
 N_2 (Second no.) = $17y + 11$

$$\frac{(N_1 + N_2)}{17} = \frac{17(x + y)}{17} + \frac{13 + 11}{17}$$

$$\text{Remainder} = \frac{24}{17} = 7$$

Ex.34: When a number is divided certain divisor, remainder is 35 but another no. is divided by the same divisor remainder is 27. If the sum of both number is divided by the same certain divisor remainder is 20. Find the certain divisor

Sol. $N_1 = Dx + 35 \dots (i)$

$N_2 = Dy + 27 \dots (ii)$

Here N_1 = First no.

N_2 = Second no.

D = certain divisor

x & y = Quotient

(i) + (ii)

According to the question

$$\frac{N_1 + N_2}{D} = \frac{D(x + y) + 62}{D}$$

Here divisor is same

$$\text{Then Remainder} = D \overline{) 62} \quad 1$$

$$\underline{- 42}$$

$$20$$

Remainder = 20

Quotient = 1

Dividend = 62

Divisor = $62 - 20 \times 1 = 42$

$$\boxed{\text{Divisor} = 42}$$

Alternate:

$\frac{N_1}{D}$	$\frac{N_2}{D}$	$\frac{N_1 + N_2}{D}$
$\downarrow$	$\downarrow$	$\downarrow$
R_1	R_2	R_3
$\boxed{D = R_1 + R_2 - R_3}$		

$$\text{Then divisor} = 35 + 27 - 20 = 42$$

Successive Division : If the quotient in a division is further used as a dividend for the next divisor and again the latest obtained divisor is used as a dividend for another divisor and so on, then it is called then " successive division" i.e, if we divide 150 by 4, we get 37 as quotient and 2 as a remainder then if 37 it divided by another divisor say 5 then we get 7 as a quotient and 2 remainder and again if we divide 7 by another divisor

say 3 we get 2 as quotient and 1 as a remainder i.e., we can represent it as following

$$\begin{array}{r|l} 4 & 150 \\ 5 & 37 \rightarrow 2 \\ 3 & 7 \rightarrow 2 \\ & 2 \rightarrow 1 \end{array} \left. \vphantom{\begin{array}{r|l} 4 & 150 \\ 5 & 37 \rightarrow 2 \\ 3 & 7 \rightarrow 2 \\ & 2 \rightarrow 1 \end{array}} \right\} \text{Remainder}$$

Now you can see that the quotient obtained in the first division behaves as a dividend for another divisor 5. Once again the quotient 7 is treated as a dividend for the next divisor 3. Thus it is clear from the above discussion as

Dividend	Divisor	Quotient	Remainder
150	4	37	2
37	5	7	2
7	3	2	1

So the 150 is successively divided by 4, 5, and 3 the corresponding remainders are 2, 2 and 1.

Ex.35: The least possible number when successively divided by 2, 5, 4, 3 gives respective remainders of 1, 1, 3, 1 is :
(a) 372 (b) 275 (c) 273 (d) 193

Sol. The problem can be expressed as

$$\begin{array}{r|l} 2 & A \\ 5 & B \rightarrow 1 \\ 4 & C \rightarrow 1 \\ 3 & D \rightarrow 3 \\ & E \rightarrow 1 \end{array} \left. \vphantom{\begin{array}{r|l} 2 & A \\ 5 & B \rightarrow 1 \\ 4 & C \rightarrow 1 \\ 3 & D \rightarrow 3 \\ & E \rightarrow 1 \end{array}} \right\} \text{Remainder}$$

So it can be solved as
 $(((((E \times 3) + 1)4 + 3)5 + 1)2 + 1) = A$
 (where A is the required number)

So for the least possible number $E = 1$ (the least positive integer)
 then $A = (((((1 \times 3) + 1) \times 4 + 3)5 + 1)2 + 1)$

[Since at $E = 0$, we get a two digit number]

So it can be solved as

$$\begin{array}{r|l} 2 & 193 & 1 \\ 5 & 96 & 1 \\ 4 & 19 & 3 \\ 3 & 4 & 1 \\ & 1 & \end{array}$$

$$D = 1 \times 3 + 1 = 4$$

$$C = 4 \times 4 + 3 = 19$$

$$B = 19 \times 5 + 1 = 96$$

$$A = 96 \times 2 + 1 = 193$$

So **Number = 193**

Alternate :

$$\begin{array}{r} 2 \xrightarrow{+} 1 \\ \swarrow \times \\ 5 \xrightarrow{+} 1 \\ \swarrow \times \\ 4 \xrightarrow{+} 3 \\ \uparrow \times \\ 3 \xrightarrow{+} 1 \end{array}$$

$$\text{Step 1. } (1 + 3) \times 4 = 16$$

$$\text{Step 2. } (16 + 3) \times 5 = 95$$

$$\text{Step 3. } (95 + 1) \times 2 = 192$$

$$\text{Step 4. } (192 + 1) = \mathbf{193}$$

Or

$$\text{Number} = (((1 + 3) \times 4 + 3) \times 5 + 1) \times 2 + 1$$

$$= ((16 + 3) \times 5 + 1) \times 2 + 1$$

$$= 96 \times 2 + 1$$

$$\text{Number} = 193$$

Ex.36: A number when divided successively by 4 and 5 leaves remainders 1 and 4 respectively. When it is successively divided by 5 and 4 the respective remainder will be

Sol. The least number x in this case will be determined as follows

$$\begin{array}{r|l} 4 & X \\ 5 & y-1 \\ & 1-4 \end{array}$$

$$y = 5 \times 1 + 4 = 9$$

$$X = 4 \times y + 1 = 4 \times 9 + 1 = 37$$

Now

$$\begin{array}{r|l} 5 & 37 \\ 4 & 7-2 \\ & 1-3 \end{array}$$

Here, the respective remainder are 2, 3

Alternate:-

Successive

Divisor Remainder

$$\begin{array}{r} 4 \xrightarrow{+} 1 \\ \uparrow \times \\ 5 \xrightarrow{+} 4 \end{array}$$

$$\text{Number} = (4 + 5) \times 4 + 1 = 36 + 1 = 37$$

$$\begin{array}{r|l} 5 & 37 \\ 4 & 7-2 \\ & 1-3 \end{array}$$

Remainder = 2, 3

Alternate II.

$$\begin{array}{r|l} 4 & 37 & 1 \\ 5 & 9 & 4 \\ & 1 & \end{array}$$

$$1 \times 5 + 4 = 9$$

$$9 \times 4 + 1 = 37$$

$$\text{Number} = 37$$

Now, divided by 5 and 4 successively

$$\begin{array}{r|l} 5 & 37 & 7 \\ & 35 & \\ & 2 & \rightarrow \text{Remainder} \end{array}$$

$$\begin{array}{r|l} 4 & 7 & 1 \\ & 4 & \\ & 3 & \rightarrow \text{Remainder} \end{array}$$

Remainder = 2, 3

Ex.37: Find the smallest no. which one successive divided 5, 3 and 7 give remainder 2, 1 and 2 respectively

Sol.

$$\begin{array}{r|l} 5 & 142 & 2 \\ 3 & 28 & 1 \\ 7 & 9 & 2 \\ & 1 & \end{array}$$

$$1 \times 7 + 2 = 9$$

$$9 \times 3 + 1 = 28$$

$$28 \times 5 + 2 = 142$$

$$\text{Number} = 142$$

Alternate:-

$$\begin{array}{r} 5 \xrightarrow{+} 2 \\ \swarrow \times \\ 3 \xrightarrow{+} 1 \\ \uparrow \times \\ 7 \xrightarrow{+} 2 \end{array}$$

$$[(2 + 7) \times 3 + 1] \times 5 + 2$$

$$= (28 \times 5) + 2$$

$$\text{Number} = 142$$

Ex.38: A least number when successively divided by 2, 3, 5 it leaves the respective remainder 1, 2 and 3. What will be the remainder if this number will be divided by 7 ?

Sol.

$$\begin{array}{r|l} 2 & 53 & 1 \\ 3 & 26 & 2 \\ 5 & 8 & 3 \\ & 1 & \end{array}$$

Step. I $5 \times 1 + 3 = 8$

Step. II $8 \times 3 + 2 = 26$

Step. III $26 \times 2 + 1 = 53$

So the least number = 53

According to the question,
53 is divided by 7 then
remainder = 4

Ex.39: Find the smallest no. which
when successive divided by
4, 5 and 6 give remainder 2,
1 and 1. Also find sequence
of remainder if the sequence
of divisor is reverse.

Sol.

4	146	2
5	36	1
6	7	1
	1	

$$6 \times 1 + 1 = 7$$

$$7 \times 5 + 1 = 36$$

$$36 \times 4 + 2 = 146$$

$$\text{Number} = 146$$

According to the question,

Now divisor is 6, 5 and 4

Then successive remainder

6	146
5	24 - 2
4	4 - 4
	1 - 0

Remainder = 2, 4 and 0

Ex.40: A number when divided
successively by 6, 7 and 8, it
leaves the respective
remainders of 3, 5 and 4, what
will be the last remainder
when such a least possible
number is divided
successively by 8, 7 and 6.

Sol.

6	537	3
7	89	5
8	12	4
	1	

Step. I $1 \times 8 + 4 = 12$

Step. II $12 \times 7 + 5 = 89$

Step. III $89 \times 6 + 3 = 537$

least number = 537

Now we divide 537
successively by 8, 7 and 6.

8	537
7	67 - 1
6	9 - 4
	1 - 3

So, 3 is the last Remainder.

Ex.41: A number when divided by 3
leaves a remainder 1. When
the quotient is divided by 2.
It leaves a remainder 1. What
will be the remainder when
the number is divided by 6?

Sol.

3	+	1
×		
2	+	1

$$\text{Number} = ((1+2) \times 3) + 1$$

$$= 9 + 1 = 10$$

According to question,

$$\text{Remainder} = \frac{10}{6} = 4$$

Ex.42: A number divided by 13 leaves
a remainder 1 and if the

quotient is divided by 5. We got
a remainder of 3. What will be
the remainder if the number
is divided by 65?

Sol.

13	+	1
×		
5	+	3

$$\text{Number} = [(3+5) \times 13] + 1$$

$$= 8 \times 13 + 1 = 105$$

According to the question,

$$\text{Remainder} = \frac{105}{65} = 40$$

BINOMIAL THEOREM

* **Statement of the theorem:-**

According to the theorem, it is possible to expand any power of $x + y$
into a sum of the form

$$(x + y)^n = {}^n C_0 x^n y^0 + {}^n C_1 x^{n-1} y^1 + {}^n C_2 x^{n-2} y^2 + \dots + {}^n C_{n-1} x^1 y^{n-1} + {}^n C_n x^0 y^n$$

Where each $\binom{n}{k}$ is a
specific positive integer
known as **binomial
coefficient**. (When an
exponent is zero, the
corresponding power
expression is taken to be 1
and this multiplicative factor
is often omitted from the
term. Hence one often sees
the right side written as

$$\binom{n}{0} x^n + \dots)$$

This formula
is also referred to as the
**binomial formula or the
binomial identity**. Using
summation notation, it can
be written as

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k =$$

$$\sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$

The final expression follows
from the previous one by the
symmetry of x and y in the
first expression, and by
comparison it follows that
the sequence of binomial
coefficients in the formula
is symmetrical. A simple
variant of the binomial

formula is obtained by
substituting 1 for y , so that
it involves only a single
variable. In this form, the
formula reads

$$(1 + x)^n = {}^n C_0 x^0 + {}^n C_1 x^1 + {}^n C_2 x^2 + \dots + {}^n C_{n-1} x^{n-1} + {}^n C_n x^n$$

or equivalently

$$(1 + x)^n = \sum_{k=0}^n \binom{n}{k} x^k.$$

Ex. (i) $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$,
(ii) $(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Some important points

1. The powers of x start at n and decrease by 1 in each term until they reach 0 (with $\{1\}$ often unwritten);
2. The powers of y start at 0 and increase by 1 until they reach n ;
3. The n^{th} row of pascal's Triangle will be the coefficients of the expanded binomial when the terms are arranged in this way;
4. The number of terms in the expansion before like terms are combined is the sum of the coefficients and is equal to 2^n , and
5. there will be **$(n + 1)$ terms** in the expression after

combining like terms in the expansion.

The binomial theorem can be applied to the powers of any binomial. for example.

$$(x+2)^3 = x^3 + 3x^2(2) + 3x(2)^2 + 2^3 = x^3 + 6x^2 + 12x + 8.$$

* For a binomial involving subtraction, the theorem can be applied by using the form $(x-y)^n = (x+(-y))^n$. This has the effect of changing the sign of every other term in expansion:

$$(x-y)^3 = (x+(-y))^3 = x^3 + 3x^2(-y) + 3x(-y)^2 + (-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

* $(a^n + b^n)$ is always divisible by $(a+b)$ when $n \rightarrow$ odd power
HINT

$$a^3+b^3 = (a+b)(a^2+ab+b^2)$$

Ex.43: Which of the following number will not completely divide the $(29)^{37} + (17)^{37}$?

- (a) 2 (b) 11 (c) 23 (d) 46

Sol. (b) $(29^{37} + 17^{37})$, $(29 + 17) = 46$ Completely divisible by $46 = 1, 2, 23, 46$
This will be completely divisible by all the factors of 46
So 11 will not divide the given number.

Ex.44: Which of the following will not completely divide $(3^{41} + 7^{82})$?

- (a) 4 (b) 52 (c) 17 (d) 26

Sol. (c) $3^{41} + 7^{82}$
 $\Rightarrow$ (Equalising the power)
 $\Rightarrow 3^{41} + (7^2)^{41}$
 $\Rightarrow 3^{41} + 49^{41}$, $3 + 49 =$ Completely Divisible by 52
 $52 = 1, 2, 4, 13, 26, 52$
So, 17 is not the factor of 52 hence this number will be completely divisible by 17

Ex.45. $(49)^{15} - 1$ is exactly divisible by:

- (a) 50 (b) 51 (c) 29 (d) 8

Sol.(d) $x^n - a^n$ is exactly divisible by $(x-a)$ if n is odd.
 $\therefore (49)^{15} - (1)^{15}$ is exactly divisible by $49-1 = 48$, that is a multiple of 8.

Ex.46: Which of the following completely divide

$$(29^{47} + 23^{47} + 17^{47})$$

- (a) 21 (b) 22 (c) 23 (d) 24

Sol. (c) $\frac{29^{47} + 17^{47} + 23^{47}}{23}$
 $29^{47} + 17^{47}$ will be completely divisible by 46 or its factor (2 and 23) and 23^{47} is com-

(a^3+b^3) $n \rightarrow \text{odd}$	(a^n-b^n) $n \rightarrow \text{odd}$	(a^n-b^n) $n \rightarrow \text{even}$	(a^n+b^n) $n \rightarrow \text{even}$
$(a+b)$ is perfectly divisible by $(a+b)$	$(a-b)$ is perfectly divisible by $(a-b)$	$(a-b)$ is perfectly divisible by $(a+b), (a-b)$	It can't be determined
$\frac{(a^3+b^3)}{(a+b)(a^2+ab+b^2)}$	$\frac{(a^3-b^3)}{(a-b)(a^2+b^2+ab)}$	$\frac{(a^2-b^2)}{(a+b)(a-b)}$	$a^2+b^2 \dots\dots$

pletely divisible by 23 so 23 will completely divide this number

$(a^n - b^n)$ is always divisible by $(a-b)$ where $n \rightarrow$ odd power

Hint

$$(a^3+b^3) = (a-b)(a^2+ab+b^2)$$

$(a^n - b^n)$ is always completely divisible by $(a-b)$, $(a+b)$ where $n \rightarrow$ (even power)

Hint

$$(a^2-b^2) = (a-b)(a+b)$$

Ex.47 Which of the following will not divide $23^{10} - 1024$ completely.

- (a) 3 (b) 5 (c) 7 (d) 4

Sol. 1024 is the value of 2^{10}

and

$23^{10} - 2^{10} \rightarrow (23-2)$ and $(23+2)$ is completely divisible
 $(23-2) = 21 = 1, 3, 7, 21$
 $(23+2) = 25 = 1, 5, 25$
Hence this number is not divisible by 4.

EXERCISE

- In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 48, the dividend is:
(a) 808 (b) 5008
(c) 5808 (d) 8508
- The divisor is 321, the quotient 11 and the remainder 260. Find the dividend.
(a) 3719 (b) 3971
(c) 3791 (d) 3179
- In a division sum, the divisor is 5 times the remainder and the quotient is 6 times the remainder which is 73. What

is the dividend?

- (a) 169943 (b) 159963
(c) 159943 (d) 159953

- The sum of 20 odd natural number is equal to:
(a) 210 (b) 300 (c) 400 (d) 240
- When a number is divided by 56, the remainder obtained is 29. What will be the remainder when the number is divided by 8?
(a) 4 (b) 5 (c) 3 (d) 7
- A number when divided successively by 4 and 5 leave the remainder 1 and 4

respectively. When it is successively divided by 5 and 4 the respective remainders will be:

- (a) 4,1 (b) 3,2 (c) 2,3 (d) 1,2

7. $4^{61} + 4^{62} + 4^{63} + 4^{64}$ is divisible by:

- (a) 3 (b) 10 (c) 11 (d) 13

8. $(3^{25} + 3^{26} + 3^{27} + 3^{28})$ is divisible by:

- (a) 11 (b) 16 (c) 25 (d) 30

9. The least number, which must be added to 6709 to make it exactly divisible by 9, is

- (a) 5 (b) 4 (c) 7 (d) 2

10. If $78*3945$ is divisible by 11 where $*$ is a digit, then $*$ is equal to :
(a) 1 (b) 0 (c) 3 (d) 5
11. When a number is divided by 357 the remainder is 39. If same number is divided by 17, the remainder will be :
(a) 0 (b) 3 (c) 5 (d) 11
12. A number when divided by 6 leaves remainder 3. When the square of the same number is divided by 6, the remainder is :
(a) 0 (b) 1 (c) 2 (d) 3
13. When a number is divided by 893, the remainder is 193. What will be remainder when it is divided by 47 ?
(a) 3 (b) 5 (c) 25 (d) 33
14. A number divided by 13 leaves a remainder 1 and if the quotient, thus obtained, is divided by 5, we get a remainder of 3. What will be the remainder if the number is divided by 65 ?
(a) 28 (b) 16 (c) 18 (d) 40
15. Which of the following number is NOT divisible by 18 ?
(a) 54036 (b) 50436
(c) 34056 (d) 65043
16. If n is an integer, then $(n^3 - n)$ is always divisible by :
(a) 4 (b) 5 (c) 6 (d) 7
17. A 4 digit number is formed by repeating a 2 digit number such as 2525, 3232, etc. Any number of this form is always exactly divisible by:
(a) 7 only (b) 11 only
(c) 13 only (d) Smallest 3 digit prime number
18. If two numbers are each divided by the same divisor, the remainders are respectively 3 and 4. If the sum of the two numbers be divided by the same divisor, the remainder is 2. The divisor is :
(a) 9 (b) 7 (c) 5 (d) 3
19. A number when divided by 5 leaves remainder 3. What is the remainder when the square of the same number is divided by 5 ?
(a) 1 (b) 2 (c) 3 (d) 4
20. If the number 4 8 3 2 7 * 8 is divisible by 11, then the missing digit (*) is
(a) 5 (b) 3 (c) 2 (d) 1
21. A number, when divided by 136, leaves remainder 36. If the same number is divided by 17, the remainder will be
(a) 9 (b) 7 (c) 3 (d) 2
22. Two numbers, when divided by 17, leaves remainder 13 and 11 respectively. If the sum of those two numbers is divided by 17, the remainder will be :
(a) 13 (b) 11 (c) 7 (d) 4
23. A number, when divided by 221, leaves a remainder 64. What is the remainder if the same number is divided by 13?
(a) 0 (b) 1 (c) 11 (d) 12
24. When a number is divided by 387, the remainder obtained is 48. If the same number is divided by 43, the remainder obtained will be ?
(a) 0 (b) 3 (c) 5 (d) 35
25. When two number are separately divided by 33, the remainders are 21 and 28 respectively. If the sum of the two number is divided by 33, the remainder will be ?
(a) 10 (b) 12 (c) 14 (d) 16
26. $(2^{71} + 2^{72} + 2^{73} + 2^{74})$ is divisible by :
(a) 9 (b) 10 (c) 11 (d) 13
27. When 'n' is divisible by 5 the remainder is 2. What is the remainder when n^2 is divided by 5 ?
(a) 2 (b) 3 (c) 1 (d) 4
28. A number when divided by 49 leaves 32 as remainder. The number when divided by 7 will have the remainder as:
(a) 4 (b) 3 (c) 2 (d) 5
29. When a number is divided by 36, the remainder is 19. What will be the remainder when the number is divided by 12 ?
(a) 7 (b) 5 (c) 3 (d) 0
30. In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, then the dividend is :
(a) 4236 (b) 4306
(c) 4336 (d) 5336
31. When a number is divided by 24, the remainder is 16. The remainder when the same number is divided by 12 is
(a) 3 (b) 4 (c) 6 (d) 8
32. The expression $8^n - 4^n$, where n is a natural number is always divisible by
(a) 15 (b) 18 (c) 36 (d) 48
33. $(4^{61} + 4^{62} + 4^{63})$ is divisible by
(a) 3 (b) 11 (c) 13 (d) 17
34. When an integer K is divided by 3, the remainder is 1, and when $K + 1$ is divided by 5, the remainder is 0. Of the following, a possible value of K is:
(a) 62 (b) 63 (c) 64 (d) 65
35. A number when divided by 91 gives a remainder 17. When the same number is divided by 13, the remainder will be :
(a) 0 (b) 4 (c) 6 (d) 3
36. A number when divided by 280 leaves 115 as remainder. When the same number is divided by 35, the remainder is:
(a) 15 (b) 10 (c) 20 (d) 17
37. A certain number when divided by 175 leaves a remainder 132. When the same number is divided by 25, the remainder is:
(a) 6 (b) 7 (c) 8 (d) 9
38. Which one of the following will completely divide by $5^{71} + 5^{72} + 5^{73}$
(a) 150 (b) 160 (c) 155 (d) 30
39. Which of the following numbers will always divide a six-digit number of the form $xyxyxy$ (where $1 \leq x \leq 9, 1 \leq y < 9$)?
(a) 1010 (b) 10101
(c) 11011 (d) 11010
40. A positive integer when divided by 425 gives remainder 45. When the same number is divided by 17, the remainder will be
(a) 5 (b) 2 (c) 11 (d) 13
41. A number x when divided by 289 leaves 18 as the remainder. The same number when divided by 17 leaves y as a remainder. The value of y is
(a) 5 (b) 2 (c) 3 (d) 1
42. When n is divided by 6, the remainder is 4. When $2n$ is divided by 6, the remainder is:
(a) 2 (b) 0 (c) 4 (d) 1
43. In a division sum, the divisor is 3 times the quotient and 6 times the remainder. If the remainder is 2, then the dividend is :
(a) 50 (b) 48 (c) 36 (d) 28

44. In a division sum, the divisor is 12 times the quotient and 5 times the remainder. If the remainder is 36, then the dividend is :
 (a) 2706 (b) 2726
 (c) 2736 (d) 2262
45. For any integral value of n , $3^{2n} + 9n + 5$ when divided by 3 will leave the remainder
 (a) 1 (b) 2
 (c) 0 (d) 5
46. The quotient when 10^{100} is divided by 5^{75} is :
 (a) 10^{25} (b) 2^{75}
 (c) $2^{75} \times 10^{25}$ (d) $2^{25} \times 10^{75}$
47. The remainder obtained when $23^3 + 31^3$ is divided by 54
 (a) 0 (b) 1
 (c) 3 (d) C.N.D
48. $(19^5 + 21^5)$ is divisible by
 (a) Only 10
 (b) Only 20
 (c) Both 10 & 20
 (d) Neither 10 nor 20
49. If $(17)^{41} + (29)^{41}$ is divided by 23. Find the remainder
 (a) 1 (b) 6 (c) 0 (d) 12
50. If $(3)^{41} + (7)^{82}$ always divisible by
 (a) 10 (b) 49 (c) 52 (d) 44
51. If $m^n - n^m = (m + n)$; $(m, n) \in$ prime numbers, then what can be said about m and n :
 (a) m, n are only even integers
 (b) m, n are only odd integers
 (c) m is even and n is odd
 (d) none of these

ANSWER KEY

1. (c)	6. (c)	11. (c)	16. (c)	21. (d)	26. (b)	31. (b)	36. (b)	41. (d)	46. (c)
2. (c)	7. (b)	12. (d)	17. (d)	22. (c)	27. (d)	32. (d)	37. (b)	42. (a)	47. (a)
3. (c)	8. (d)	13. (b)	18. (c)	23. (d)	28. (a)	33. (a)	38. (c)	43. (a)	48. (c)
4. (c)	9. (a)	14. (d)	19. (d)	24. (c)	29. (a)	34. (c)	39. (b)	44. (c)	49. (c)
5. (b)	10. (d)	15. (d)	20. (d)	25. (d)	30. (d)	35. (b)	40. (c)	45. (b)	50. (c)
									51. (c)

SOLUTION

1. (c) Remainder = 48
 Divisor = $48 \times 5 = 240$
 Quotient = $\frac{240}{10} = 24$
 Dividend = $240 \times 24 + 48$
 = $5760 + 48$
 = 5808
2. (c) Dividend = Divisor $\times$ Quotient + Remainder
 = $321 \times 11 + 260$
 = $3531 + 260 = 3791$
3. (c) Remainder = 73
 Quotient = $6 \times 73 = 438$
 Divisor = $5 \times 73 = 365$
 Dividend = $365 \times 438 + 73$
 = 159943
4. (c) 1, 3, 5, 7 20th term
 $a = 1, d = 2, n = 20$

$$\text{sum} = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{20}{2} [2 \times 1 + (20 - 1)2]$$

$$= 10[2 \times 1 + 19 \times 2] = 400$$
- Alternate :**
 The sum of first n odd natural numbers = $n^2 = 20^2 = 400$
5. (b) $\frac{\text{Remainder}}{8} = \frac{29}{8}$
 Remainder = 5
6. (c) Number is divided successively
 Remainder

$$\begin{array}{r} 4 \overline{) 37} \quad 1 \\ 5 \overline{) 9} \quad 4 \\ \hline 1 \end{array}$$

$$5 \times 1 + 4 = 9$$

$$9 \times 4 + 1 = 37$$
 Number is 37
7. (b) $4^{61} + 4^{62} + 4^{63} + 4^{64}$
 = $4^{61}(4^0 + 4^1 + 4^2 + 4^3)$
 = $4^{61}(1 + 4 + 16 + 64)$
 = $4^{61} \times 85$
 = $4^{60} \times 4 \times 85$
 = $4^{60} \times 340$
 = $4^{60} \times 34 \times 10$
 Now, check with option
 Only, check with option
 Only 10 can divide this.
8. (d) $(3^{25} + 3^{26} + 3^{27} + 3^{28})$
 = $3^{35}(3^0 + 3^1 + 3^2 + 3^3)$
 = $3^{25}(1 + 3 + 9 + 27)$
 = $3^{25} \times 40 = 3^{24} \times 120$
 Now, check with option
 Only, check with option
 Only 30 can divide this.
9. (a) 6709
 $\Rightarrow 6 + 7 + 0 + 9 = 22$
 $[9 - (\text{divisibility property})]$
 Sum of digits must be divisible by 9
 So $22 + 5 = 27$ is divisible by 9
 5 is answer
10. (d) $\overline{78} * \overline{3945}$
 Odd place : $7 + * + 9 + 5 = 21 + *$
 Even place : $8 + 3 + 4 = 15$
 $(21 + *) - (15) = \text{either } 11 \text{ or } 0$
 $(21 + *) - 15 = 11$
 $21 + * = 26$
 $* = 5$
11. (c) $\frac{\text{Remainder of number}}{17} = \frac{39}{17}$
 $\Rightarrow \text{remainder} = 5$
12. (d) Shortcut Method
 Let number is: 9 (Gives remainder 3 when divided by 6)
 Now $\frac{9^2}{6} = \frac{81}{6} \Rightarrow \text{Remainder} = 3$

$$13. (b) \frac{\text{Remainder of no.}}{47} = \frac{193}{47}$$

$$\Rightarrow \text{remainder} = 5$$

$$14. (d) \begin{array}{r} 13 \overline{) 1051} \\ \underline{5} \\ 8 \\ \underline{1} \\ 1 \end{array}$$

$$5 \times 1 + 3 = 8$$

$$13 \times 8 + 1 = 105$$

$$\text{remainder} = 105 \div 13$$

$$\text{Remainder} = 40$$

15. (d) A number will be divisible by 18 if it is divisible by 2 and 9

Clearly we can see 65043 is not divisible by 2. Because unit digit of 65043 is 3 so this will not be divisible by 18

16. (c) $(n^3 - n)$ and n is any integer. put $n = 2$ so, $2^3 - 2 = 6$

It will be always divisible by 6 (Put $n = 2, 3, 4, \dots$)

17. (d) Smallest 3 digit prime number is '101'

$xyxy$ is always divisible by 101

Hence, 101 Will be the divisor.

18. (c) **Shortcut Method**

divisor = Remainder 1 + Remainder 2 - Remainder 3

$$= 3 + 4 - 2 = 7 - 2 = 5$$

19. (d) Let no. be 8

$$\Rightarrow \frac{8^2}{5} = \frac{64}{5}$$

$$= 4 \text{ remainder}$$

Alternate:-

$$\begin{aligned} \text{Remainder} &= \frac{(\text{Remainder})^2}{5} \\ &= \frac{(3)^2}{5} = \frac{9}{5} = 4 \end{aligned}$$

20. (d) $\overline{48327} \times 8$

$$\text{odd place} \Rightarrow 4 + 3 + 7 + 8 = 22$$

$$\text{Even place} \Rightarrow 8 + 2 + * = 10 + *$$

Difference should be either zero or 11, 22, 33etc.

$$\Rightarrow 22 - (10 + *) = 11$$

$$22 - 10 - * = 11$$

$$12 - * = 11$$

$$* = 1$$

$$21. (d) \frac{\text{Remainder of no.}}{17} = \frac{36}{17}$$

$$\Rightarrow \text{remainder} = 2$$

22. (c) (dividend = divisor $\times$ quotient + remainder)

$$\text{First no.} = (17 \times n) + 13$$

$$\text{Let 'n'} = 1$$

$$\Rightarrow (17 \times 1) + 13$$

$$\Rightarrow 30$$

$$\text{Second no.} = (17 \times n) + 11$$

$$= (17 \times 1) + 11 = 28$$

According to question

$$\frac{30 + 28}{17} = \frac{58}{17} \Rightarrow \text{remainder} = 7$$

Alternate:-

$$\text{Divisor} = \text{Remainder 1} +$$

$$\text{Remainder 2} - \text{Remainder 3}$$

$$17 = 13 + 11 - \text{Remainder 3}$$

$$\text{Remainder 3} = 24 - 17 = 7$$

$$23. (d) \frac{\text{Remainder of no.}}{13} = \frac{64}{13}$$

$$\Rightarrow \text{remainder} = 12$$

$$24. (c) \frac{\text{Remainder of no.}}{43} = \frac{48}{43}$$

$$\Rightarrow \text{remainder} = 5$$

$$25. (d) \text{first no.} = (33 \times n) + 21$$

$$\text{Let no.} = 1$$

$$= (33 \times 1) + 21 = 54$$

$$\text{Second no.} = (33 \times n) + 28$$

$$= (33 \times 1) + 28 = 61$$

According to question

$$\frac{54 + 61}{33} = \frac{115}{33}$$

$$\Rightarrow 16 \text{ Remainder}$$

Alternate:-

$$\text{Divisor} = \text{Remainder 1} +$$

$$\text{Remainder 2} - \text{Remainder 3}$$

$$33 = 21 + 28 - \text{Remainder 3}$$

$$\text{Remainder 3} = 16$$

$$26. (b) (2^{71} + 2^{72} + 2^{73} + 2^{74})$$

$$= 2^{71}(2^0 + 2^1 + 2^2 + 2^3)$$

$$= 2^{71}(1 + 2 + 4 + 8)$$

$$= 2^{71} \times 15 = 2^{70} \times 30$$

It is divisible by 10

$$27. (d) \frac{n}{5} \Rightarrow \text{remainder } 2$$

If we put $n = 7$ Then it satisfies above situation

So $n = 7$

$$\frac{n^2}{5} = \frac{7^2}{5} = \frac{49}{5} \Rightarrow \text{remainder} = 4$$

$$28. (a) \frac{\text{remainder of no.}}{7} = \frac{32}{7}$$

$$\Rightarrow \text{Remainder} = 4$$

$$29. (a) \frac{\text{remainder of no.}}{12} = \frac{19}{12}$$

$$\Rightarrow \text{remainder} = 7$$

30. (d)

$$\begin{array}{ccc} \text{Quotient} & : & \text{Divisor} & : & \text{Remainder} \\ 1 & : & 10 & : & 5 \end{array}$$

$$\begin{array}{ccc} & : & 5 & : & 1 \\ & : & 10 & : & 2 \end{array}$$

$$\begin{array}{ccc} \downarrow \times 23 & & \downarrow \times 23 & & \downarrow \times 23 \\ 23 & : & 230 & : & 46 \end{array}$$

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

$$= (230 \times 23) + 46 = 5336$$

$$31. (b) \frac{\text{Remainder of no.}}{12} = \frac{16}{12}$$

$$= 4 \text{ is remainder}$$

32. (d) $8^n - 4^n$

$n = 1, 2, 3, \dots, (n \text{ is a natural number})$

Put, $n = 2$,

$$\text{expression} = 8^2 - 4^2 = 64 - 16 = 48$$

$$\therefore 8^n - 4^n \text{ is divisible by } 48$$

48 is completely divisible by 4

so 8^n is divisible 4

$$33. (a) (4^{61} + 4^{62} + 4^{63})$$

$$= 4^{61}(4^0 + 4^1 + 4^2)$$

$$= 4^{61}(1 + 4 + 16) = 4^{61} \times 21$$

Now check the options

Only 3 divides it. So '3' is answer

34. (c) Always do these types of question by options to save time Pick up the option and follow the question instruction take option (c)

$$64 \Rightarrow \text{Divide 3 it gives remainder 1}$$

Now add 1 to 64

$$\frac{65}{5} \Rightarrow \text{remainder '0' it satisfies}$$

So, $k = 64$ this is answer

$$35. (b) \frac{\text{Remainder of no.}}{13} = \frac{17}{13}$$

$$\text{remainder} = 4$$

$$36. (b) \frac{\text{Remainder of no.}}{35} = \frac{115}{35}$$

$$\text{Remainder} = 10$$

$$37. (b) \frac{\text{Remainder of no.}}{25} = \frac{132}{25}$$

$$\text{remainder} = 7$$

$$\begin{aligned} 38. (c) 5^{71} + 5^{72} + 5^{73} \\ = 5^{71} (5^0 + 5^1 + 5^2) \\ = 5^{71} (1 + 5 + 25) \\ = 5^{71} \times 31 = 5^{70} \times 155 \end{aligned}$$

Check with option,

So 155 is answer

$$\begin{aligned} 39. (b) \text{Number} &= xyxyxy \\ &= xy \times 10000 + xy \times 100 + xy \\ &= xy (10000 + 100 + 1) \\ &= xy(10101) \end{aligned}$$

Hence, option (B) will divide answer

Alternate:

You can assume (121212, 343434.....) any number divisible by option, So that number is divisible by exactly that's the answer

$$40. (c) \frac{\text{Remainder of no.}}{17} = \frac{45}{17}$$

$$\Rightarrow \text{remainder} = 11$$

$$41. (d) \frac{\text{Remainder of no.}}{17} = \frac{18}{17}$$

$$\Rightarrow \text{remainder} = 1$$

$$42. (a) \frac{n}{6} = \text{remainder } 4$$

$$\text{If } n = 10 \Rightarrow \frac{10}{6}$$

$$\Rightarrow \text{remainder} = 4 \text{ (matched) } n = 10$$

$$2n = 2 \times 10 \Rightarrow \frac{20}{6}$$

$$\Rightarrow \text{remainder} = 2$$

Note : Always put value in these type of questions.

43. (a)

Remainder : Divisor : Quotient

$$3 : 1$$

$$1 : 6$$

$$\begin{array}{ccc} 1 & : & 6 \\ \downarrow \times 2 & & \downarrow \times 2 \\ 2 & : & 12 \end{array}$$

Actual $\rightarrow 2$

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient})$$

$$+ \text{remainder}$$

$$= (12 \times 4) + 2 = 50$$

44. (c)

Remainder : Divisor : Quotient

$$12 : 1$$

$$1 : 5$$

$$\begin{array}{ccc} 12 & : & 60 \\ \downarrow \times 3 & & \downarrow \times 3 \\ 36 & : & 180 \end{array}$$

$$\text{Dividend} = (\text{divisor} \times \text{quotient}) + \text{Remainder}$$

$$= (180 \times 15) + 36$$

$$= 2736$$

$$45. (b) 3^{2n} + 9n + 5$$

$$\text{Put } n = 1$$

$$\Rightarrow 3^{2 \times 1} + 9 \times 1 + 5$$

$$\Rightarrow 9 + 9 + 5 \Rightarrow 23$$

$$\Rightarrow \frac{23}{3} \Rightarrow \text{remainder} = 2$$

Note: value of n can be 1,2,3,4,.....

$$46. (c) 10^{100} \div 5^{75}$$

$$\frac{2^{100} \times 5^{100}}{5^{75}} = 2^{100} \times 5^{25} = 2^{25} \cdot 2^{75} \cdot 5^{25}$$

$$= 2^{75} \times 10^{25}$$

47. (a) We know that $(a^n + b^n)$ is always divisible $(a + b)$ then.

where $n \rightarrow$ odd power

$(23^3 + 31^3)$ is Always divisible by $(23 + 31) = 54$

So remainder is '0'

48. (c) $(a^n + b^n)$, is always divisible by $(a + b)$

When $n \rightarrow$ odd power

$$(19 + 21) = 40$$

Factor of 40 (1, 2, 4, 5, 10, 20, 40) is divisible by $(19^5 + 21^5)$ then options 10 & 20 is divisible

49. (c) $(a^n + b^n)$, is always divisible $(a + b)$

When n is odd power

Then,

$(17^{41} + 29^{41})$ is always divisible by $(17 + 29) = 46$

factor of 46 (1, 2, 23, 46)

So, $(17^{41} + 29^{41})$ is perfectly divisible by 23

hence, Remainder '0'

$$50. (c) 3^{41} + 7^{82}$$

Equalising the power

$$3^{41} + (7^2)^{41} = 3^{41} + 49^{41}$$

$3^{41} + 49^{41}$ is always divisible $(3 + 49) = 52$

So 52 is divisible by $(3^{41} + 7^{82})$

$$51. (c) m^n - n^m = m + n$$

Consider $m = 2$ and $n = 5$, then

$$2^5 - 5^2 = 5 + 2$$

$$7 = 7$$

Thus option (a) and (b) are wrong and option (c) is correct.



REMAINDER THEOREM

Ex:- What remainders can be possible when 25 is divided by 7

$$\begin{array}{r} +4 \swarrow \quad \nwarrow -3 \\ 7 \overline{)25} \\ \underline{21} \\ +4 \end{array} \rightarrow \text{Actual Remainder}$$

$$\begin{array}{r} 7 \overline{)25} (4 \\ \underline{-28} \\ -3 \end{array} \rightarrow \text{Negative Remainder}$$

Remainder is always positive but some times we use negative remainder for our convenience if 25 is divided by 7 then actual remainder will be + 4 but - 3 can be used for convenience for actual remainder multiple of 7, less than 25 is 21 hence actual remainder will be + 4 and for negative remainder we have to see the multiple of 7 greater than 25, which is 28 so - 3 will be the remainder

Ex:- What will be the remainder when 37 is divided by 9

$$\begin{array}{r} 9 \overline{)37} (4 \\ \underline{-36} \\ +1 \end{array} \rightarrow \text{Actual Remainder}$$

$$\begin{array}{r} 9 \overline{)37} (5 \\ \underline{-45} \\ -8 \end{array} \rightarrow \text{Negative Remainder}$$

When 37 is divided 9, then the multiple of 9 smaller than 37 is 36. Hence actual remainder will be +1. If we want a negative remainder we have to see the multiple of 9 greater than 37 which is 45, hence - 8 will be the negative Remainder.

$$\begin{array}{r} +1 \swarrow \quad \nwarrow -8 \\ 9 \overline{)37} \end{array}$$

$$\begin{array}{r} +6 \swarrow \quad \nwarrow -1 \\ 7 \overline{)55} \end{array}$$

$$\begin{array}{r} +5 \swarrow \quad \nwarrow -1 \\ 6 \overline{)167} \end{array}$$

* '0' is the smallest divisible number when 0 is divided by any number always remainder will be 0

$$\begin{array}{r} 0 \quad 7 \overline{)0} (0 \\ \underline{-0} \\ 0 \end{array}$$

* when 0 is divided by 7, then 0th multiple of 7 is (7 × 0 = 0) then 0 is subtracted from 0, we will get zero.

Ex:- When 45 is divided by 14 then

$$\begin{array}{r} 14 \overline{)45} (3 \\ \underline{-42} \\ +3 \end{array} \rightarrow \text{Actual Remainder}$$

$$\begin{array}{r} 14 \overline{)45} (4 \\ \underline{-56} \\ -11 \end{array} \rightarrow \text{Negative Remainder}$$

$$\begin{array}{r} +3 \swarrow \quad \nwarrow -11 \\ 14 \overline{)45} \end{array}$$

$$\begin{array}{r} +1 \swarrow \quad \nwarrow -7 \\ 8 \overline{)73} \end{array}$$

$$\begin{array}{r} +3 \swarrow \quad \nwarrow -1 \\ 4 \overline{)111} \end{array}$$

$$\begin{array}{r} 0 \\ 100 \overline{)0} (0 \\ \underline{0} \\ 0 \end{array} \rightarrow \text{Remainder}$$

$$\begin{array}{r} +13 \swarrow \quad \nwarrow -2 \\ 15 \overline{)13} \end{array}$$

$$15 \overline{)13} (0 \quad \text{or} \quad 15 \overline{)13} (1$$

* When 13 is divided by 15, then the multiple of 15 which is less than 13 is 0. which is 0th multiple of 15. Hence actual remainder will be +13 and for the negative remainder we have to see the multiple of 15 which should be greater than 13, Now 15 is the multiple of 15 greater than 13, so remainder will be - 2

$$\begin{array}{r} +2 \swarrow \quad \nwarrow -1 \\ 3 \overline{)2} \end{array}$$

$$\begin{array}{r} 3 \overline{)2} (0 \\ \underline{-0} \\ +2 \end{array} \rightarrow \text{Actual Remainder}$$

$$\begin{array}{r} 3 \overline{)2} (1 \\ \underline{-3} \\ -1 \end{array} \rightarrow \text{Negative Remainder}$$

Ex:-

$$\begin{array}{r} +4 \swarrow \quad \nwarrow -3 \\ 7 \overline{)4} \end{array}$$

$$7 \overline{)4} (0$$

$$7 \overline{)4} (1$$

TYPE - 1

Ex.1 what will be the remainder when 23×34 is divided by 9

Sol. $\frac{23 \times 34}{9}$

when 23 is divided by 9 the remainder is

$$\begin{array}{r} +5 \quad -4 \\ \swarrow \quad \searrow \\ 23 \\ \hline 9 \end{array}$$

When 34 is divided by 9, the remainder is

$$\begin{array}{r} +7 \quad -2 \\ \swarrow \quad \searrow \\ 34 \\ \hline 9 \end{array}$$

then $\begin{array}{r} +5 \quad +7 \\ \uparrow \quad \uparrow \\ 23 \quad \times \quad 34 \\ \hline 9 \end{array} = 5 \times 7 = 35$

The sign will be the same between remainders as in the process. For Ex (23 × 34). Here we see that the sign b/w 23 and 34 is (×), So, the sign b/w remainders will be (×). If the product of remainder is greater than divisor, we have to divide it again to get the remainder

In this process when 23 is divided by 9, remainder +5 has been used and when 34 is divided by 9 remainder +7 has been used we can see that the sign between the process is (×), then the product of remainders is (5 × 7) = 35,

Which is greater than 9. Now again we have to divide 35 by 9 we will get + 8 or - 1 as remainder. If the remainder is negative (-1) it should be deducted from divisor, so we will get positive (+ve) remainder)

$$\begin{array}{r} +5 \quad +7 \\ \swarrow \quad \searrow \\ 23 \quad \times \quad 34 \\ \hline 9 \end{array} = \frac{5 \times 7}{9}$$

$$\begin{array}{r} +8 \quad -1 \\ \swarrow \quad \searrow \\ 35 \\ \hline 9 \end{array} = \frac{35}{9} = +8 \text{ (Remainder)} \text{ or } 9 - 1 = 8$$

Alternate II

$$\frac{-4 \quad -2}{23 \times 34} = \frac{-4 \times -2}{9} = 8$$

$$\begin{array}{r} +5 \quad -4 \quad +7 \quad -2 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 23 \quad \times \quad 34 \\ \hline 9 \end{array}$$

Now this time we have used negative remainder. If 23 is divided by 9, the remainder will be -4 and if 34 is divided by 9, the remainder will be -2 As there is (×) sign in the process, the product of the remainders is (+8) As the product is less than divisor so there is no need to divide it again.

Alternate III

$$\begin{array}{r} -4 \quad +7 \\ \swarrow \quad \searrow \\ 23 \quad \times \quad 34 \\ \hline 9 \end{array}$$

$$= -4 \times 7 = -28$$

After dividing 23 by 9 remainder - 4 has taken and after dividing 34 by 9, remainder (+7) has taken. Now the product of the remainders are (-28). We will neglect the (-ve) sign and again will get the remainder by dividing first process. After that we will put (-ve) sign. If the remainder is negative, then we will get (+ve) remainder by adding divisor into it.

$$\begin{array}{r} -4 \quad +7 \\ \swarrow \quad \searrow \\ 23 \quad \times \quad 34 \\ \hline 9 \end{array} = -28$$

Neglecting (-ve) sign

$$\begin{array}{r} +1 \\ \uparrow \\ = \frac{28}{9} \end{array} \text{ Now dividing by general process}$$

$$= +1 \text{ (Again putting (-ve) sign)} = -1 = 9 - 1$$

$$\text{(To get (+ve) remainder)} = 8$$

Same Remainder in each process

Ex.2 What will be the remainder when 43 × 83 is divided by 21?

Sol.

$$\begin{array}{r} 43 \times 83 \\ 21 \end{array} \begin{array}{l} +1 \quad -20 \\ \swarrow \quad \searrow \end{array} \begin{array}{r} 43 \\ \hline 21 \end{array} \begin{array}{l} +20 \quad -1 \\ \swarrow \quad \searrow \end{array} \begin{array}{r} 83 \\ \hline 21 \end{array}$$

Whether remainder is +ve or negative smaller remainder should be used for the easier calculation. If 43 is divided by 21, the smaller remainder will be (+1) and If 83 is divided by 21 the smaller remainder will be -1,

$$\begin{array}{r} +1 \quad -1 \\ \uparrow \quad \uparrow \\ 43 \quad \times \quad 83 \\ \hline 21 \end{array} = \frac{1 \times -1}{21} = -1 + 21 = 20$$

Ex.3 What will be the remainder

$$\text{when } \frac{121 + 93}{8}$$

$$\begin{array}{r} +1 \quad -7 \\ \swarrow \quad \searrow \\ 121 \\ \hline 8 \end{array} \begin{array}{r} +5 \quad -3 \\ \swarrow \quad \searrow \\ 93 \\ \hline 8 \end{array}$$

By using smaller remainders

$$\begin{array}{r} +1 \quad -3 \\ \uparrow \quad \uparrow \\ 121 \quad + \quad 93 \\ \hline 8 \end{array} = \frac{1 - 3}{8} = -2 = 8 - 2 = 6$$

In this operation we have used (+ve) sign. So the same sign (+) will be used b/w the remainders. (1 - 3) = (-2) the remainder is (-ve). So, to get actual remainder we have to add 8 hence actual remainder will be 6.

Ex.4 What will be the remainder

$$\text{when } \frac{130 + 147}{11}$$

Sol. $\begin{array}{r} +9 \quad -2 \\ \swarrow \quad \searrow \\ 130 \\ \hline 11 \end{array} \begin{array}{r} +4 \quad -7 \\ \swarrow \quad \searrow \\ 147 \\ \hline 11 \end{array}$

By using smaller remainder

$$\begin{array}{r} -2 \quad +4 \\ \uparrow \quad \uparrow \\ 130 \quad + \quad 147 \\ \hline 11 \end{array} = \frac{-2 + 4}{11} = 2$$

So, remainder is 2

$$\begin{array}{ccccccc} & +2 & & -1 & & -2 & & -1 & & -1 \\ & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \underline{127} & \times & \underline{139} & \times & \underline{12653} & \times & \underline{79} & \times & \underline{18769} \\ & & & & & & & & & 5 \end{array}$$
$$\Rightarrow \frac{2 \times -1 \times -2 \times -1 \times -1}{5} = \frac{4}{5} = 4$$
$$\begin{array}{ccccccc} +2 & -1 & & -2 & -1 & & -1 \\ \uparrow & \uparrow & & \uparrow & \uparrow & & \uparrow \\ \underline{127} & + \underline{139} & + \underline{12653} & + \underline{79} & + \underline{18769} & & \\ \hline & & 5 & & & & \end{array}$$

$$= \frac{\begin{array}{cccc} +5 & +1 & -1 & -1 \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 195 & \times 1958 & \times 1975 & \times 170 \end{array}}{19}$$

$$= \frac{5 \times 1 \times -1 \times -1}{19} = 5$$

$$\begin{array}{ccccccccc} & -1 & & -3 & & +2 & & +2 & & +1 \\ & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\ 17 & \underline{50} & \times & 17 & \underline{48} & \times & 17 & \underline{53} & \times & \underline{70} & \times & \underline{35} \\ \hline & & & & & & & & & & & 17 \end{array}$$

$$\begin{array}{ccccccccc} & -1 & & -3 & & +1 & & +2 & & +1 \\ & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\ 1750 & + & 1748 & + & 1752 & + & 70 & + & 35 \\ \hline & & & & 17 & & & & & \end{array}$$

Hence remainder is 0

$\lfloor \rightarrow$	It is sign of Factorial.
$\lfloor 1 \rightarrow 1$	
$\lfloor 2 \rightarrow$	$1 \times 2 = 2$
$\lfloor 3 \rightarrow$	$1 \times 2 \times 3 = 6$
$\lfloor 4 \rightarrow$	$1 \times 2 \times 3 \times 4 = 24$
$\lfloor 5 \rightarrow$	$1 \times 2 \times 3 \times 4 \times 5 = 120$
$\lfloor 0 \rightarrow$	value is 1

$$\underline{\underline{1}} = \frac{1}{10} \begin{matrix} +1 \swarrow \searrow -9 \\ 1 \end{matrix}$$

$$\underline{12} = 1 \times 2 = \frac{2}{10} \begin{matrix} +2 \swarrow \nearrow -8 \\ 2 \end{matrix}$$

$$\underline{13} = 1 \times 2 \times 3 = \frac{6}{10} \begin{matrix} +6 \swarrow \searrow -4 \\ 6 \end{matrix}$$

$$\underline{4} = 1 \times 2 \times 3 \times 4 = \frac{24}{10}$$

$$5 = 5 \times 4 \times 3 \times 2 \times 1 = \frac{120}{10}$$

=remainder = 0

$$\underline{6} = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = \frac{720}{10}$$

$$= \text{Remainder} = 0$$
$$\begin{array}{ccccccccccc} +1 & +2 & -4 & +4 & 0 & 0 & & & & 0 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & & & & \uparrow \\ \underline{1} & +\underline{2} & +\underline{3} & +\underline{4} & +\underline{5} & +\underline{6} & +\dots\dots\dots & +\underline{1} & 000 \\ \hline & & & & 10 & & & & & \end{array}$$

$$= \frac{1+2-4+4}{10} = \frac{3}{10} = \boxed{3}$$

$$\underline{1} = 1 = \frac{1}{12}$$

$$\underline{12} = 1 \times 2 = \frac{2}{12}$$

$$\underline{12} = 1 \times 2 \times 3 = \frac{6}{12}$$

$$\Rightarrow \frac{1+2+6}{12} = \frac{9}{12} = 9$$

$$\begin{array}{cccccccccccccccc} +1 & +2 & -3 & -3 & +3 & 0 & 0 & & & 0 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & & & \uparrow \\ \underline{1} & +\underline{2} & +\underline{3} & +\underline{4} & +\underline{5} & +\underline{6} & +\underline{7} & \dots & +\underline{1000} \\ \hline & & & & & & & & & & 9 \end{array}$$

Hence this number is divisible by 9

- * The number is divided by 10 to get unit digit
- * The number is divided by 100 (10^2) to get last two digits
- * The number is divided by (10^3) to get last three digit
- * This process will continues as it is

$$\frac{123 \times 13999 \times 497 \times 73 \times 96}{100 \times 25} 24$$

In such type of process we simplify the operation firstly. The number by which we simplify the operation, the same number is multiplied in the last. In this case 96 and 100 are simplified. So, we multiply by 4 in the last

$$\begin{array}{ccccccc} -2 & -1 & -3 & -2 & -1 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 123 \times 1399 \times 497 \times 73 \times 24 \\ \hline & & & & 25 \end{array}$$

$$= \frac{-12}{25} = -12$$

$$\Rightarrow 25 - 12 = 13$$

To get last two digit we multiply it by 4.

$$13 \times 4 = 52$$

So, the last two digit is 52 (5 and 2)

* divisibility of 25 $\rightarrow$ last 2 digits divisible by 25

Ex. 14 $39 \times 55 \times 57 \times 24 \times 13872 \times 9871$ Find the last two digits

$$\text{Sol. } = \frac{39 \times 55 \times 57 \times 24 \times 13872 \times 9871}{100 \times 25}$$

Simplifying two times by 4 and 5. So, to get last two digit we have to multiply 20 (4×5)

$$\begin{array}{ccccccc} -1 & +1 & +2 & +1 & +2 & +1 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 39 \times 11 \times 57 \times 6 \times 13872 \times 9871 \\ \hline & & & & & & 5 \end{array}$$

* divisibility of 5 $\rightarrow$ last 1 digit divisible by 5

$$= \frac{-1 \times 1 \times 2 \times 1 \times 2 \times 1}{5} = \frac{-4}{5} = -4$$

$$= 5 - 4 = 1$$

So, actual last two digits

$$1 \times 20 = 20 \text{ (2 and 0)}$$

Ex. 15 $173 \times 192 \times 99 \times 96$ find the last two digits

$$\text{Sol. } = \frac{173 \times 192 \times 99 \times 96}{100 \times 25}$$

Simplifying by 4

$$\begin{array}{cccc} -2 & -8 & -1 & -1 \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 173 \times 192 \times 99 \times 24 \\ \hline & & & & 25 \end{array}$$

$$= \frac{-2 \times -8 \times -1 \times -1}{25} = \frac{16}{25} = 16$$

So, Actual last two digit

$$= 16 \times 4 = 64 \text{ (6 and 4)}$$

Ex. 16 $87 \times 92 \times 194 \times 44$ Find the last two digits ?

$$\text{Sol. } = \frac{87 \times 92 \times 194 \times 44}{100 \times 25}$$

Simplifying by 4

$$\begin{array}{cccc} -13 & -2 & -6 & -6 \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 87 \times 23 \times 194 \times 44 \\ \hline & & & & 25 \end{array}$$

$$\frac{-13 \times -2 \times -6 \times -6}{25}$$

$$\frac{+1 \times +11}{25}$$

$$\frac{26 \times 36}{25}$$

$$\frac{1 \times 11}{25} = 11$$

So, Actual last two digits

$$= 11 \times 4 = 44$$

Ex. 17 What will be the remainder when 25 is divided by 13

$$\text{Sol. } \begin{array}{c} 12 \\ \swarrow \quad \searrow \\ 25 \\ \downarrow \\ 13 \\ \hline 12 \end{array} \quad \begin{array}{c} -1 \\ \downarrow \\ 13 \\ \hline 12 \end{array}$$

Remainder is always positive

Ex. 18 Find the remainder $\frac{(25)^{48}}{13}$?

$$\begin{array}{c} -1 \\ \uparrow \\ (25)^{48} \\ \hline 13 \end{array} = (-1)^{48} = 1$$

In such type of operations we try to get the multiple of divisor near to the dividends actual number So, that the difference b/w then will be 1. In this case the multiple of 13 near to 25 is 26. and the difference b/w 25 and 26 is 1 and power of even So, the remainder will be (+ve)

Ex. 19 $\frac{(36)^{13}}{7}$ Find the remainder?

$$\text{Sol. } \begin{array}{c} +1 \\ \uparrow \\ (36)^{13} \\ \hline 7 \end{array} = (+1)^{13} = 1$$

In this operation the multiple of 7 near to 36 is 35 and the difference between 36 and 35 is 1

Ex. 20 $\frac{2^{18}}{9}$ Find the remainder?

Sol. In such type of Operations, the power is simplified in such a way that the difference b/w divisor and the number made by breaking of power is minimum So, the number near to 9 should be 8 or 10

$$\begin{array}{c} 8 \\ \swarrow \quad \searrow \\ 9 \quad 10 \end{array}$$

$$\begin{array}{c} -1 \\ \uparrow \\ \frac{2^{3 \times 6}}{9} = \frac{(2^3)^6}{9} = \frac{(8)^6}{9} \\ = \frac{(-1)^6}{9} = \frac{1}{9} = 1 \end{array}$$

So Remainder = 1

Ex. 21 What will be the remainder when 2^{21} is divided by 9

$$\text{Sol. } \frac{2^{21}}{9} = \frac{(2^3)^7}{9} = \frac{(8)^7}{9} = \frac{(-1)^7}{9} = \frac{-1}{9} = 9 - 1 = 8$$

Ex. 22 What will be the remainder when 2^{22} is divided by 9

$$\text{Sol. } \frac{2^{22}}{9} = \frac{(2^3)^7 \times 2}{9} = \frac{(8)^7 \times 2}{9} = \frac{(-1)^7 \times 2}{9}$$

$$\Rightarrow \frac{-1 \times 2}{9} = \frac{-2}{9} = -2$$

$$\Rightarrow 9 - 2 = 7$$

Ex. 23 What will be the remainder when $(35)^{37}$ is divided by 9 ?

$$\text{Sol. } \frac{(35)^{37}}{9}$$

The multiple of 9 near to 35 is 36

$$\begin{array}{c} -1 \\ \uparrow \\ (35)^{37} \\ \hline 9 \end{array} = \frac{(1)^{37}}{9} = -1$$

$$\text{Remainder} = 9 - 1 = 8$$

Ex. 24 What will be the remainder when 7^{40} is divided by 400

Sol.

$7^1 = 7$
$7^2 = 49$
$7^3 = 343$
$7^4 = 2401$

$$\frac{7^{40}}{400} = \frac{(7^4)^{10}}{400}$$

$$\Rightarrow \frac{(2401)^{10}}{400} = \frac{(1)^{10}}{400} = 1$$

(power has broken in such a way that $7^4 = 2401$, which is near to the 2400 a multiple of 400)

Ex.25 What will be the remainder when 2^{42} is divided by 33

Sol. $\frac{2^{42}}{33}$ $33 \begin{matrix} \swarrow 32 \\ \searrow 34 \end{matrix}$
32 and 34 are near to the 33 the difference is 1. Hence Power is to be broken in such a way that we can get 32 and 34

$$\begin{matrix} 2^1 = 2 \\ 2^2 = 4 \\ 2^3 = 8 \\ 2^4 = 16 \\ 2^5 = 32 \end{matrix}$$

$$= \frac{2^2 \times 2^{40}}{33} = \frac{4 \times (2^5)^8}{33}$$

$$\Rightarrow \frac{\begin{matrix} +4 & -1 \\ \uparrow & \uparrow \\ 4 & \times (32)^8 \end{matrix}}{33}$$

$$= \frac{+4 \times (-1)^8}{33} = \frac{4 \times 1}{33} = 4$$

remainder = 4

Ex.26 What will be the remainder when 3^{55} is divided by 82

Sol. $\begin{matrix} 3^1 = 3 \\ 3^2 = 9 \\ 3^3 = 27 \\ 3^4 = 81 \end{matrix}$

$$\frac{3^{55}}{82} \Rightarrow \frac{3^3 \times 3^{52}}{82} \quad 82 \begin{matrix} \swarrow 81 \\ \searrow 83 \end{matrix}$$

$$\Rightarrow \frac{3^3 \times (3^4)^{13}}{82}$$

$$= \frac{\begin{matrix} +27 & -1 \\ \uparrow & \uparrow \\ 27 & \times (81) \end{matrix}}{82} = \frac{27 \times -1}{82} = -27$$

remainder = $82 - 27 = 55$

Ex.27 What will be the remainder when 2^{68} is divided by 65 ?

Sol.

$$\begin{matrix} & 64 \\ 65 & \swarrow \searrow \\ & 66 \\ \boxed{2^6 = 64} \end{matrix}$$

$$\frac{2^{68}}{65}$$

$$= \frac{2^2 \times (2^6)^{11}}{65}$$

$$= \frac{\begin{matrix} +4 & -1 \\ \uparrow & \uparrow \\ 4 & \times (64)^{11} \end{matrix}}{65} \Rightarrow \frac{+4 \times (-1)^{11}}{65}$$

$$\Rightarrow \frac{4 \times -1}{65} = \frac{-4}{65}$$

remainder = $65 - 4 = 61$

Ex.28 What will be the remainder when 4^{19} is divided by 33

Sol. $\frac{4^{19}}{33}$ $33 \begin{matrix} \swarrow 32 \\ \searrow 34 \end{matrix}$
 $4^{19} = (2^2)^{19} = 2^{38}$
So $\frac{2^{38}}{33} \therefore \boxed{2^5 = 32}$

$$\Rightarrow \frac{2^3 \times 2^{35}}{33} = \frac{8 \times (2^5)^7}{33}$$

$$\Rightarrow \frac{\begin{matrix} +8 & -1 \\ \uparrow & \uparrow \\ 8 & \times (32)^7 \end{matrix}}{33} = \frac{+8 \times (-1)^7}{33}$$

$$= \frac{8 \times -1}{33} = -8$$

remainder = $33 - 8 = \boxed{25}$

TYPE - 2

Ex.29 When 20 is divided by 8 the remainder will be

Sol. $\begin{array}{r} 8 \overline{)20} 2 \\ \underline{-16} \\ 4 \end{array} \rightarrow \text{Remainder}$

$$= \frac{20^5}{8 \times 2} = \frac{\begin{matrix} +1 \\ \uparrow \\ 20 \end{matrix}}{2} = \frac{5}{2} = 1$$

When 20 is divided by 8 we get '4' remainder.

If $\frac{20}{8}$ is simplified by 4 we get $\frac{5}{2}$.
Now 5 is divided by 2 we get remainder 1, In means that the divisor should be multiplied by remainder to get actual remainder

$$= \frac{5}{2} = 1 \times 2 = 4$$

(Actual remainder)

Ex.30 What will be the remainder when 2^{35} is by 10 ?

Sol. $\frac{2^{35}}{10} = \frac{2^{35}}{2 \times 5} = \frac{2 \times 2^{34}}{2 \times 5}$

This Fraction is simplified by 2

$$= \frac{2^{34}}{5} = \frac{(2^2)^{17}}{5}$$

$$\begin{matrix} -1 \\ \uparrow \\ (4)^{17} \end{matrix}$$

$$= \frac{(-1)^{17}}{5} = -1 = 5 - 1 = 4$$

Actual Remainder = $4 \times 2 = 8$

As this number was simplified by 2, So to get actual remainder we have to multiply it by 2

Ex.31 What will be the remainder when 5^{500} is divided by 500

Sol. $\frac{5^{500}}{500} = \frac{5^3 \times 5^{497}}{125 \times 4}$

$$= \frac{5^3 \times 5^{497}}{5^3 \times 4}$$

5^3 = simplifying by 125

$$\Rightarrow \frac{\begin{matrix} +1 \\ \uparrow \\ (5)^{497} \end{matrix}}{4} = \frac{(+1)^{497}}{4} = 1$$

Actual Remainder = $1 \times 125 = 125$

Ex.32 What will be the remainder when 37^{100} is divided by 7 ?

Sol. $\frac{(37)^{100}}{7} = \frac{(+2)^{100}}{7} = \frac{2 \times (2^3)^{33}}{7}$

$$2^3 = 8$$

2^{100} remainder is far greater than 7, So, we have to divide remainder again.

$$\begin{aligned} \Rightarrow \frac{2^2 \times 2^1}{7} &= \frac{2 \times (2)^{33}}{7} \\ &= \frac{2 \times 1}{7} = \frac{2}{7} \end{aligned}$$

So, Remainder = 2

Cyclicity:- Happening again and again in the same order or period

Ex.33 Find the remainder when 11^{77} is divided by 7

Sol. $\frac{11^{77}}{7}$

The Remainder when 11^1 is divided by 7 = $\frac{11}{7} = 4$

$$11^2 = \frac{11 \times 11}{7} = \frac{4 \times 4}{7} = \frac{16}{7} = 2$$

$$11^3 = \frac{11 \times 11 \times 11}{7} = \frac{4 \times 4 \times 4}{7} = \frac{64}{7} = 1$$

$$11^4 = \frac{11 \times 11 \times 11 \times 11}{7} = \frac{4 \times 4 \times 4 \times 4}{7} = 4$$

$$11^5 = \frac{11 \times 11 \times 11 \times 11 \times 11}{7} = \frac{4 \times 4 \times 4 \times 4 \times 4}{7} = 2$$

$$11^6 = \frac{11 \times 11 \times 11 \times 11 \times 11 \times 11}{7} = \frac{4 \times 4 \times 4 \times 4 \times 4 \times 4}{7} = 1$$

$$11^6 = \frac{11 \times 11 \times 11 \times 11 \times 11 \times 11}{7} = \frac{64 \times 64}{7} = \frac{1 \times 1}{7} = 1$$

you are seeing that after three steps the cycle of remainders is repeating, which is generally known as 'Pattern method'. So break the power of multiple of 3

$$\frac{(11)^{77}}{7} = \frac{(11)^{75} \times (11)^2}{7}$$

$$= \frac{(11^3)^{25} \times 121}{7}$$

$$\frac{(1)^{25} \times 2}{7} = \frac{1 \times 2}{7} = 2$$

Ex.34 Find the remainder when 5^{135} is divided by 7.

Sol. The Remainder when 5^1 is divided by 7 = $\frac{5^1}{7} = R = 5$.

$$\frac{5^2}{7} = \frac{25}{7} = R = 4$$

$$\frac{5^3}{7} = \frac{125}{7} = R = 6$$

$$\frac{5^4}{7} = \frac{5 \times 5 \times 5 \times 5}{7}$$

$$= \frac{25 \times 25}{7} = \frac{9}{7} = R = 2$$

$$= \frac{5^5}{7} = \frac{5 \times 5 \times 5 \times 5 \times 5}{7}$$

$$= \frac{25 \times 25 \times 5}{7} = \frac{9 \times 5}{7} = R = 3$$

$$= \frac{5^6}{7} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{7}$$

$$= \frac{25 \times 25 \times 25}{7} = \frac{-27}{7} = -6$$

$$\therefore R = -6 + 7 = 1$$

$$\frac{5^7}{7} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{7}$$

$$= \frac{25 \times 25 \times 25 \times 5}{7} = \frac{9 \times 6}{7} = 5$$

$$\frac{5^8}{7} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{7}$$

$$= \frac{25 \times 25 \times 25 \times 25}{7} = \frac{9 \times 9}{7} = 4$$

So we see that the cyclic period of remainder is 6, since after 6 steps the remainder starts repeating. Now we divide the power by 6

$$= \frac{5^{135}}{7} = \frac{(5^6)^{22} \times 5^3}{7}$$

$$= \frac{(1)^{22} \times 6}{7} = \frac{6}{7}$$

$$= \frac{6}{7}$$

Remainder = 6

Ex.35 Find the Remainder when 143^{321} is divided by 5

Sol. when 143 is divided by 5 we get remainder 3, thus 143^{321} is divided by 5 then remainder 3^{321} . This remainder is very large, so again divided

$$\frac{(143)^{321}}{5} = \frac{(3)^{321}}{5}$$

The remainder when 3^1 is divided by 5 = $\frac{3^1}{5} = R = 3$

$$= \frac{3^1}{5} = R = 3$$

$$\frac{3^2}{5} = \frac{9}{5} = R = 4$$

$$\frac{3^3}{5} = \frac{27}{5} = R = 2$$

$$\frac{3^4}{5} = \frac{81}{5} = R = 1$$

$$\frac{3^5}{5} = \frac{243}{5} = R = 3$$

$$\frac{3^6}{5} = \frac{729}{5} = R = 4$$

$$\frac{3^{321}}{5} = \frac{(3^4)^{80} \times 3}{5}$$

$$= \frac{(3^4)^{80} \times 3}{5}$$

$$= \frac{(+1)^{80} \times 3}{5} = \frac{1 \times 3}{5} = 3$$

Remainder = 3

Ex.36 find the remainder when 3^{6773} divide by 80

Sol.
$$= \frac{3^{6773}}{80}$$

we know that $= 3^4 = 81$

$$\frac{3^{6773}}{80} = \frac{3^{6772} \times 3^1}{80}$$

$$= \frac{(3^4)^{1693} \times 3^1}{80}$$

$$= \frac{\overset{+1}{\uparrow} (81)^{1693} \times 3^1}{80}$$

$$= \frac{(1)^{1693} \times 3}{80} = \frac{1 \times 3}{80} = 3$$

Hence Remainder = 3

Ex.37 Find the Remainder of $(32^{32})^{32}$ when divided by 7.

Sol.
$$\frac{(32^{32})^{32}}{7}$$

$$= \frac{(4^{32})^{32}}{7}$$

$$4 = 2^2$$

$$\Rightarrow \frac{(2^{2 \times 32})^{32}}{7}$$

$$\Rightarrow \frac{(2^{64})^{32}}{7}$$

$$\therefore 2^3 = 8$$

$$= \frac{(2^{63} \times 2^1)^{32}}{7}$$

$$= \frac{\overset{+1}{\uparrow} ((2^3)^{21} \times 2^1)^{32}}{7} = \frac{((8)^{21} \times 2^1)^{32}}{7}$$

$$= \frac{(1 \times 2)^{32}}{7} = \frac{2^{32}}{7}$$

$$\text{Again } \therefore 2^3 = 8$$

$$\frac{2^{30} \times 2^2}{7} = \frac{(2^3)^{10} \times 4}{7}$$

$$\overset{+1}{\uparrow} \frac{(8)^{10} \times 4}{7}$$

$$= \frac{(1)^{10} \times 4}{7} = \frac{1 \times 4}{7}$$

Remainder = 4

Ex.38 What will be the remainder when $[48 + (62)^{117}]$ is divided by 9?

Sol.
$$\overset{+3}{\uparrow} \overset{-1}{\uparrow} \frac{48 + (62)^{117}}{9}$$

$$= \frac{+3 + (-1)^{117}}{9} = \frac{3-1}{9} = \frac{2}{9} = 2$$

Hence Remainder = 2

Ex.39 when $[51 + (67)^{99}]$ is divided by 68, find the remainder

Sol.
$$\overset{-17}{\uparrow} \overset{-1}{\uparrow} \frac{51 + (67)^{99}}{68}$$

$$\Rightarrow \frac{-17 + (-1)^{99}}{68} = \frac{-17-1}{68} = \frac{-18}{68}$$

$$= -18$$

$$\text{Remainder} = 68 - 18 = 50$$

Remainder of Algebraic Function

When $F(x)$ is divided by $(x-a)$ the remainder is $F(a)$
 $\therefore (x-a)$ is a factor of $F(x)$
 then $f(a) = 0$

Ex.40 Is $(x-2)$ a factor of $f(x)$
 $x^2 + x - 5$?

Sol. $(x-2) = 0$

$$\boxed{x=2}$$

x value $f(x)$

$$F(2) = (2)^2 + (2) - 5$$

$$= 4 + 2 - 5$$

$$= 6 - 5 = 1 \neq 0$$

$(x-2)$ is not a factor of

$$x^2 - x + 5$$

If $F(2) = 0$, we can say $(x-2)$, it is a factor of $f(x)$

Ex.41 $x^{29} - x^{26} - x^{23} + 1$

(a) $(x-1)$ but not $(x+1)$

(b) $(x+1)$ but not $(x-1)$

(c) both $(x+1)$ & $(x-1)$

(d) Neither $(x+1)$ not $(x-1)$

Sol. (c) If $(x-1)$, is a factor then, $f(x) = 0$, and $x-1 = 0$
 $x = 1$

$$f(1) = 0$$

$$f(x) = x^{29} - x^{26} - x^{23} + 1$$

$$f(1) = 1 - 1 - 1 + 1 = 0$$

$$f(1) = 0,$$

we can say $(x-1)$ is a factor of $f(x)$

$$x+1 = 0$$

$$x = -1$$

$$x^{29} - x^{26} - x^{23} + 1$$

$$-1 - 1 + 1 + 1 = 0$$

$(x+1)$ is a factor of $f(x)$

Both $(x+1)$ & $(x-1)$ is a factor of $x^{29} - x^{26} - x^{23} + 1$

Ex.42 If $(x-2)$ is a factor of Polynomial $x^2 + kx + 4$. Find the value of k .

Sol. $(x-2)$ is a factor of $x^2 + kx + 4$ when $(x-2) = 0$

$$x = 2$$

$$f(2) = (2)^2 + 2k + 4 = 0$$

$$2k = -8$$

$$k = -4$$

Ex.43 If $(x+1)$ & $(x-1)$ are the Factor of the Polynomial $ax^3 + bx^2 + 3x + 5$. find the value of a and b

Sol. If $(x-1)$ is factor of $f(x)$ then,

$$x-1 = 0$$

$$x = 1$$

$$f(x) = ax^3 + bx^2 + 3x + 5$$

$$f(1) = a(1)^3 + b(1)^2 + 3(1) + 5 = 0$$

$$a + b = -8 \dots (i)$$

If $(x+1)$, is a factor of $f(x)$

Then,

$$(x+1) = 0$$

$$x = -1$$

$$f(-1) = a(-1)^3 + b(-1)^2 + 3(-1) + 5 = 0$$

$$-a + b - 3 + 5 = 0$$

$$-a + b = -2$$

$$a - b = 2 \dots (ii)$$

from (i) & (ii)

$$\boxed{a=-3}, \boxed{b=-5}$$

Ex.44 Find the remainder when $x^3 + 5x^2 + 7$ is divided by $(x-2)$

Sol. $x-2 = 0$

$$x = 2$$

$$f(x) = x^3 + 5x^2 + 7$$

$$f(2) = (2)^3 + 5(2)^2 + 7$$

$$= 8 + 20 + 7 = 35$$

Remainder = 35

Ex.45 Find the remainder when $x^2 - 7x + 15$ is divided by $x-3$

Sol. $x-3 = 0$

$$x = 3$$

Put the value of $x = 3$

$$F(x) = x^2 - 7x + 15$$

$$F(3) = (3)^2 - 7(3) + 15$$

$$= 9 - 21 + 15 = 3$$

Remainder 3

Ex.46 $x^{51} + 16$ when divided by $x + 1$ find the Remainder.

Sol. $(x + 1) = 0$

$$x = -1$$

$$f(x) = x^{51} + 16$$

$$f(-1) = (-1)^{51} + 16 = -1 + 16 = 15$$

$$\text{Remainder} = 15$$

Ex.47 If $x^2 + 4x + k$ when divided by $x - 2$ leave remainder $2x$. find the value of k .

Sol. $x^2 + 4x + k$

$$x - 2 = 0$$

$$x = 2$$

$$f(x) = 2x$$

$$f(2) = 2 + 2 = 4$$

$$f(2) = (2)^2 + 4 \times 2 + k = 4$$

$$4 + 8 + k = 4$$

$$K = -8$$

TYPE - 3

Ex.48 777777..... 129 Times is divided by 37 the remainder will be ?

Sol. If any number is made by repeating a digit 6 times the number will be divisible by 7, 11, 13 and 37.

So, 777777 126 times is divisible by 37 because 126 is the multiple of 6. So, the remaining three digits will be divided by 37 to get the remainder

$$\Rightarrow \frac{777777777 \dots 126 \text{ Times}, 777}{37}$$

$$\begin{array}{r} 37 \overline{) 777} \left(21 \quad \frac{777}{37} = 0 \right. \\ \underline{74} \\ 37 \\ \underline{37} \\ \times \end{array} \quad \text{Remainder} = 0$$

Hence, the number is divisible by 37.

Ex.49 When 4444444444 is divided by 13 the remainder will be ?

Sol. 4 is repeating 9 times in this number. As we know that any number repeating 6 times is divisible by 13. So the remaining three digits will be divided by 13 to get the remainder

$$\frac{4444444,444}{13} = \frac{444}{13}$$

$$\begin{array}{r} = 13 \overline{) 444} \left(34 \right. \\ \underline{39} \\ 54 \\ \underline{52} \\ 2 \rightarrow \text{Remainder} \end{array}$$

Ex.50 What will be the remainder when 123456789 is divided by 8 ?

Sol. (Divisibility Rule)

$2^1 = 2 \rightarrow$ Last digit divisible by 2

$2^2 = 4 \rightarrow$ Last two digits divisible by 4

$2^3 = 8 \rightarrow$ Last three digits divisible by 8

$2^4 = 16 \rightarrow$ Last four digits divisible by 16

$2^5 = 32 \rightarrow$ Last five digits divisible by 32

So, for the divisibility of 8 the last three digits of the number should be divisible by 8. In this way we get 5 as the remainder

$$\begin{array}{r} 123456789 \\ \underline{8} \end{array}$$

$$\begin{array}{r} 8 \overline{) 789} \left(98 \right. \\ \underline{64} \\ 149 \\ \underline{128} \\ 21 \rightarrow \text{Remainder} \end{array}$$

So, the remainder is 5.

Ex.51 What will be remainder when 12345678910111213141516 divided by 16.

Sol. For the divisibility of 16, the last four digits of the number should be divisible by 16. In this way we get 12 as remainder

$$\begin{array}{r} 12345678910111213141516 \\ \underline{16} \end{array}$$

$$\begin{array}{r} 16 \overline{) 1516} \left(94 \right. \\ \underline{144} \\ 76 \\ \underline{64} \\ 12 \rightarrow \text{Remainder} \end{array}$$

Hence the remainder is 12.

Ex.52 $10^1 + 10^2 + 10^3 + \dots + 10^{99} + 10^{100}$ when divided by 6, the remainder will be ?

Sol. $10^1 + 10^2 + 10^3 + \dots + 10^{99} + 10^{100}$

$$\begin{array}{r} +4 \\ \uparrow \\ 10 \\ \downarrow \\ 6 \end{array} \rightarrow \frac{10}{6} = \text{Remainder} = 4$$

$$\begin{array}{r} +4 +4 \\ \uparrow \uparrow \\ 10 + 10^2 \\ \downarrow \\ 6 \end{array} = \frac{4+4}{6} = \frac{8}{6}$$

$$\text{Remainder} = 8 - 6 = 2$$

$$\begin{array}{r} +4 +4 +4 \\ \uparrow \uparrow \uparrow \\ 10^1 + 10^2 + 10^3 \\ \downarrow \\ 6 \end{array} = \frac{4+4+4}{6} = \frac{12}{6}$$

$$\text{Remainder} = 0$$

The remainder will be zero (0) after each three numbers. So the remainder is 0 upto the 99th term. So the remaining 10^{100} term will be divided by 6 to get the remainder

$$\begin{array}{r} +4 +4 +4 +4 +4 +4 \\ \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \\ 10^1 + 10^2 + 10^3 + 10^1 + 10^2 + 10^3 \dots \\ +4 +4 \\ \uparrow \uparrow \\ 10^{99} + 10^{100} \\ \downarrow \downarrow \\ 6 \end{array}$$

$$\begin{array}{r} +4 \\ \uparrow \\ 10^{100} \\ \downarrow \\ 6 \end{array} = 4$$

Hence, the remainder is 4.

Ex.53 What will be the remainder when $10^1 + 10^2 + 10^3 + \dots + 10^{32}$ is divided by 6 ?

$$\begin{array}{r} +4 +4 +4 +4 +4 +4 \\ \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \\ 10^1 + 10^2 + 10^3 + \dots + 10^{30} + 10^{31} + 10^{32} \\ \downarrow \\ 6 \end{array}$$

0 will be the remainder of each three terms. So, 0 will be the remainder up to 30th term

$$\begin{array}{r} +4 +4 \\ \uparrow \uparrow \\ 10^{31} + 10^{32} \\ \downarrow \\ 6 \end{array}$$

$$= \frac{4+4}{6} = \frac{8}{6} = 2$$

Hence remainder is 2

EXERCISE

1. Find the Remainder when $77 \times 85 \times 73$ is divided by 9
(a) 1 (b) 2 (c) 4 (d) 7
2. Find the Remainder when $273 + 375 + 478 + 657 + 597$ is divided by 25
(a) 5 (b) 10 (c) 9 (d) 8
3. Find the Remainder when $1330 \times 1356 \times 1363 \times 1368 \times 1397$ is divided by 13
(a) 7 (b) 9 (c) 11 (d) 8
4. Find the Remainder when $2327 + 2372 + 2394 + 4624 + 4650$ is divided by 23
(a) 12 (b) 14 (c) 13 (d) 10
5. Find the Remainder when 67^{32} is divided by 68
(a) 67 (b) 66 (c) 1 (d) 0
6. Find the Remainder when 99^{99} is divided by 100
(a) 99 (b) 98 (c) 1 (d) 3
7. Find the Remainder 197^{130} is divided by 196
(a) 1 (b) 195 (c) 7 (d) 5
8. Find the Remainder 6^{36} is divided by 215
(a) 214 (b) 6 (c) 5 (d) 1
9. Find the Remainder 75^{7575} is divided by 37
(a) 1 (b) 36 (c) 3 (d) 7
10. Find the Remainder 43^{197} is divided by 7
(a) 42 (b) 41 (c) 1 (d) 6
11. Find the Remainder when 17^{200} is divided by 18
(a) 17 (b) 16 (c) 1 (d) 4
12. Find the Remainder when $(12^{13} + 23^{13})$ is divided by 11
(a) 2 (b) 1 (c) 0 (d) 3
13. Find the remainder when $(7^{19} + 2)$ is divided by 6
(a) 3 (b) 1 (c) 5 (d) 2
14. Find the Remainder when 3^{21} is divided by 5 is
(a) 3 (b) 2 (c) 1 (d) 4
15. Find the Remainder when 2^{31} is divided by 5
(a) 1 (b) 2 (c) 3 (d) 4
16. Find the Remainder when 2^{591} is divided by 255
(a) 225 (b) 128 (c) 127 (d) 64
17. Find the Remainder when 51^{203} is divided by 7
(a) 4 (b) 2 (c) 1 (d) 6
18. The Remainder when $(2)^{243}$ is divided by 3^2 is
(a) 8 (b) 4 (c) 10 (d) None of these
19. Find the Remainder when $(59)^{28}$ is divided by 7
(a) 2 (b) 4 (c) 6 (d) 1
20. Find the Remainder when 41^{77} is divided by 17
(a) 2 (b) 1 (c) 6 (d) 4
21. Find the Remainder when 2^{49} is divided by 7
(a) 1 (b) 2 (c) 3 (d) 4
22. Find the Remainder when $(51^{203} + 2^{49})$ is divided by 17
(a) 4 (b) 5 (c) 6 (d) None of these
23. Find the Remainder when 1234567891011121314 is divided by 8
(a) 4 (b) 2 (c) 6 (d) 3
24. Find the Remainder when 41424344454647484950 is divided by 16
(a) 2 (b) 12 (c) 6 (d) 8
25. Find the Remainder when 21222324252627282930 is divided by 8
(a) 5 (b) 2 (c) 3 (d) 4
26. Find the Remainder when 919293949596979899 is divided by 16
(a) 3 (b) 13 (c) 11 (d) 8
27. Find the Remainder when 313233343536373839 is divided by 4
(a) 1 (b) 2 (c) 3 (d) N.O.T.
28. Find the Remainder when 1234.... 41 digits is divided by 8
(a) 1 (b) 2 (c) 3 (d) 4
29. Find the Remainder when 1234..... 81 digits is divided by 16
(a) 13 (b) 8 (c) 1 (d) 7
30. Find the Remainder when 8^{77} is divided by 17
(a) 8 (b) 9 (c) 13 (d) 7
31. Find the Remainder when $1 + 2 + 3 + 4 + \dots + 100$ is divided by 5 is
(a) 0 (b) 1 (c) 2 (d) 3
32. Find the Remainder when $1 + 2 + 3 + 4 + \dots + 100$ is divided by 6 is
(a) 3 (b) 4 (c) 2 (d) 1
33. Find the Remainder when $1 + 2 + 3 + 4 + \dots + 50$ is divided by 12 is
(a) 2 (b) 8 (c) 7 (d) 9
34. Find the Remainder when 9^{111} is divided by 11
(a) 2 (b) 9 (c) 7 (d) 6
35. Find the Remainder when 5^{2450} is divided by 126
(a) 5 (b) 25 (c) 125 (d) 1
36. Find the Remainder when 40^{1012} is divided by 7
(a) 5 (b) 4 (c) 3 (d) 2
37. Find the Remainder when $10^1 + 10^2 + 10^3 + \dots + 10^{100}$ is divided by 6
(a) 4 (b) 6 (c) 2 (d) 3
38. Find the Remainder when $10^1 + 10^2 + 10^3 + \dots + 10^{1000} + 10^{1001}$ is divided by 6
(a) 4 (b) 6 (c) 2 (d) 3
39. Find the Remainder when 666666 134 times is divided by 13
(a) 1 (b) 3 (c) 11 (d) 9
40. Find the Remainder when 555555 244 times is divided by 37
(a) 18 (b) 5 (c) 36 (d) 0
41. Find the Remainder when 777777 363 times is divided by 11
(a) 0 (b) 7 (c) 1 (d) 3
42. Find the Remainder when 888888 184 times is divided by 37
(a) 1 (b) 8 (c) 36 (d) 7
43. Find the Remainder when 999999999 is divided by 13
(a) 8 (b) 11 (c) 5 (d) 12
44. Find the Remainder when 7^{99} is divided by 2400
(a) 1 (b) 49 (c) 343 (d) 7
45. Find the Remainder when 3^{1989} is divided by 7
(a) 2 (b) 6 (c) 4 (d) 5
46. Find the Remainder when 54^{124} is divided by 17
(a) 4 (b) 5 (c) 3 (d) 15
47. Find the Remainder when 21^{875} is divided by 17
(a) 8 (b) 13 (c) 16 (d) 9
48. Find the Remainder when 83^{261} is divided by 17
(a) 13 (b) 9 (c) 8 (d) 2

49. Find the Remainder when $(32^{32})^{32}$ is divided by 9
(a) 4 (b) 7 (c) 1 (d) 2
50. Find the Remainder when $(32^{32})^{32}$ is divided by 7
(a) 4 (b) 7 (c) 2 (d) 1
51. Find the Remainder when $(33^{34})^{35}$ is divided by 7
(a) 5 (b) 4 (c) 6 (d) 2
52. Find the Remainder when $888^{222} + 222^{888}$ is divided by 5
(a) 0 (b) 1 (c) 3 (d) 4
53. Find the Remainder when $2222^{5555} + 5555^{2222}$ is divided by 7
(a) 0 (b) 2 (c) 4 (d) 5
54. Find the Remainder when 50^{5152} is divided by 11
(a) 6 (b) 4 (c) 7 (d) 3
55. The Remainder when $(20)^{23}$ is divided by 17 is
(a) 11 (b) 3 (c) 6 (d) Can't determine
56. If $(x - 2)$ is a factor of $(x^2 + 3qx - 2q)$, then the value of q is :
(a) 2 (b) -2 (c) -1 (d) 1
57. If $x^3 + 6x^2 + 4x + k$ is exactly divisible by $(x + 2)$ then value of k is : -
(a) -6 (b) -7 (c) -8 (d) -10
58. Value of k for which $(x - 1)$ is a factor of $(x^3 - k)$ is :
(a) -1 (b) 1 (c) 8 (d) -8
59. If $x^{100} + 2x^{99} + k$ is divisible by $(x + 1)$, then the value of k is:
(a) 1 (b) -3 (c) 2 (d) -2
60. If $(x^3 - 5x^2 + 4p)$ is divisible by $(x + 2)$, then the value of p is
(a) 7 (b) -2 (c) 3 (d) -7
61. If $(x - a)$ is a factor of $(x^3 - 3x^2 + 2a^2x + b)$, then the value of b is:
(a) 0 (b) 2 (c) 1 (d) 3
62. If $x^3 + 3x^2 + 4x + k$ contains $(x + 6)$ as a factor, the value of k is:
(a) 66 (b) 33 (c) 132 (d) 36
63. If $(x + 2)$ and $(x - 1)$ are the factors of $(x^3 + 10x^2 + mx + n)$, the values of m and n are :
(a) $m = 5, n = -3$
(b) $m = 17, n = -8$
(c) $m = 7, n = -18$
(d) $m = 23, n = -19$
64. On dividing $(x^3 - 6x + 7)$ by $(x + 1)$, then remainder is :
(a) 2 (b) 12 (c) 0 (d) 7
65. If $(x^5 - 9x^2 + 12x - 14)$ is divided by $(x - 3)$, the remainder is :
(a) 184 (b) 56 (c) 2 (d) 1
66. When $(x^4 - 3x^3 + 2x^2 - 5x + 7)$ is divided by $(x - 2)$, then remainder is :
(a) 3 (b) -3 (c) 2 (d) 0
67. If $5x^3 + 5x^2 - 6x + 9$ is divided by $(x + 3)$, then remainder is :
(a) 135 (b) -135 (c) 63 (d) -63
68. If $(x^{11} + 1)$ is divided by $(x + 1)$, then remainder is :
(a) 2 (b) 0 (c) 11 (d) 12
69. If $2x^3 + 5x^2 - 4x - 6$ is divided by $2x + 1$, then remainder is :
(a) $-\frac{13}{3}$ (b) 3
(c) -3 (d) 6
70. If $x^3 + 5x^2 + 10k$ leaves remainder $-2x$ when divided by $x^2 + 2$, then the value of k is:
(a) -2 (b) -1 (c) 1 (d) 2

ANSWER KEY

1. (b)	8. (d)	15. (c)	22. (d)	29. (a)	36. (d)	43. (b)	50. (a)	57. (c)	64. (b)
2. (a)	9. (a)	16. (b)	23. (b)	30. (b)	37. (a)	44. (c)	51. (b)	58. (b)	65. (a)
3. (b)	10. (c)	17. (a)	24. (c)	31. (d)	38. (c)	45. (b)	52. (a)	59. (a)	66. (b)
4. (b)	11. (c)	18. (a)	25. (b)	32. (a)	39. (a)	46. (a)	53. (a)	60. (a)	67. (d)
5. (c)	12. (a)	19. (b)	26. (c)	33. (d)	40. (b)	47. (b)	54. (d)	61. (a)	68. (b)
6. (a)	13. (a)	20. (c)	27. (c)	34. (b)	41. (b)	48. (d)	55. (a)	62. (c)	69. (c)
7. (a)	14. (a)	21. (b)	28. (a)	35. (b)	42. (b)	49. (a)	56. (c)	63. (c)	70. (c)

SOLUTION

1. (b)
$$\begin{array}{r} +5 \quad +4 \quad +1 \\ 77 \times 85 \times 73 \\ \hline 9 \end{array}$$

$$= \frac{5 \times 4 \times 1}{9} = \frac{20}{9} = 2$$

2. (a)
$$\begin{array}{r} -2 \quad 0 \quad +3 \quad +7 \quad -3 \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ 273 + 375 + 478 + 657 + 597 \\ \hline 25 \end{array}$$

$$\frac{-2 + 0 + 3 + 7 - 3}{25} = \frac{5}{25} = 5$$

3. (b)
$$\begin{array}{r} +4 \quad +4 \quad -2 \quad +3 \quad +6 \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ 1330 \times 1356 \times 1363 \times 1368 \times 1397 \\ \hline 13 \end{array}$$

$$\frac{4 \times 4 \times -2 \times 3 \times 6}{13} = \frac{16 \times -36}{13}$$

* Avoid '-' (Negative) sign. Normally divided 36 by 13 remainder = -3. Now use '-' (Negative) sign

$R = (-3) = 3$

$$\begin{array}{r} +3 \quad +3 \\ \uparrow \quad \uparrow \\ 16 \times -36 \\ \hline 13 \end{array} = \frac{9}{13} = R = 9$$

4. (b)
$$\begin{array}{r} +4 \quad +3 \quad +2 \quad +1 \quad +4 \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ 2327 + 2372 + 2394 + 4624 + 4650 \\ \hline 23 \end{array}$$

$$\frac{4 + 3 + 2 + 1 + 4}{23} = \frac{14}{23}$$

 $R = 14$

$$5. (c) \frac{67^{32}}{68} = \frac{(-1)^{32}}{68} = \frac{1}{68}$$

$$R = 1$$

$$6. (a) \frac{99^{99}}{100} = \frac{(-1)^{99}}{100} = \frac{-1}{100}$$

$$R = 100 - 1 = 99$$

$$7. (a) \frac{197^{130}}{196} = \frac{(+1)^{130}}{196} = \frac{1}{196}$$

$$R = 1$$

$$8. (d) \frac{6^{36}}{215}$$

$\therefore$ We know that $6^3 = 216$

$\therefore$ So break the power multiple 3

$$= \frac{(6^3)^{12}}{215} = \frac{(216)^{12}}{215}$$

$$= \frac{(+1)^{12}}{215} = \frac{1}{215}$$

$$R = 1$$

$$9. (a) \frac{75^{7575}}{37} = \frac{(+1)^{7575}}{37} = \frac{1}{37}$$

$$R = 1$$

$$10. (c) \frac{43^{197}}{7} = \frac{1^{197}}{7} = \frac{1}{7}$$

$$R = 1$$

$$11. (c) \frac{17^{200}}{18} = \frac{(-1)^{200}}{18} = \frac{1}{18}$$

$$R = 1$$

$$12. (a) \frac{12^{13} + 23^{13}}{11}$$

$$= \frac{(+1)^{13} + (+1)^{13}}{11} = \frac{1+1}{11} = \frac{2}{11}$$

$$R = 2$$

$$13. (a) \frac{7^{19} + 2}{6} = \frac{(+1)^{19} + 2}{6}$$

$$= \frac{1+2}{6} = \frac{3}{6}$$

$$R = 3$$

$$14. (a) \frac{3^{21}}{5}$$

$$\therefore 3^2 = 9$$

Break The power multiple of 2 form

$$= \frac{(3^2)^{10} \times 3^1}{5} = \frac{(9)^{10} \times 3}{5}$$

$$= \frac{(-1)^{10} \times 3}{5}$$

$$= \frac{1 \times 3}{5} = \frac{3}{5}$$

$$R = 3$$

$$15. (c) \frac{2^{31}}{5}$$

$$\therefore 2^2 = 4$$

$$\frac{(2^2)^{15} \times 2^1}{5} = \frac{(4)^{15} \times 2^1}{5}$$

$$= \frac{(-1)^{15} \times 2}{5} = \frac{-1 \times 2}{5} = \frac{-2}{5}$$

$$R = 5 - 2 = 3$$

$$16. (b) \frac{2^{591}}{255}$$

$$\therefore 2^8 = 256$$

$$\text{Now } \frac{(2^8)^{73} \times 2^7}{255}$$

$$= \frac{(256)^{73} \times 128}{255}$$

$$= \frac{(1)^{73} \times 128}{255} = \frac{128}{255}$$

$$\text{Remainder} = 128$$

$$17. (a) \frac{51^{203}}{7} = \frac{(2)^{203}}{7}$$

$$\therefore 2^3 = 8$$

$$= \frac{(2^3)^{67} \times 2^2}{7} = \frac{(8)^{67} \times 4}{7}$$

$$\Rightarrow \frac{(+1)^{67} \times 4}{7}$$

$$\Rightarrow \frac{1 \times 4}{7} = \frac{4}{7}$$

$$R = 4$$

$$18. (a) \frac{2^{243}}{3^2} = \frac{2^{243}}{9}$$

$$\therefore 2^3 = 8$$

$$= \frac{(2^3)^{81}}{9} = \frac{(8)^{81}}{9} = \frac{(-1)^{81}}{9} = \frac{-1}{9}$$

$$\text{Remainder} = 9 - 1 = 8$$

$$19. (b) \frac{59^{28}}{7} = \frac{3^{28}}{7} = \frac{(3^3)^9 \times 3}{7}$$

$$\therefore 3^3 = 27$$

$$\Rightarrow \frac{(27)^9 \times 3}{7}$$

$$\Rightarrow \frac{(-1)^9 \times 3}{7} = \frac{-1 \times 3}{7} = \frac{-3}{7}$$

$$R = 7 - 3 = 4$$

$$20. (c) \frac{41^{77}}{17} = \frac{7^{77}}{17} = \frac{(7^2)^{38} \times 7^1}{17}$$

$$\Rightarrow \frac{(49)^{38} \times 7}{17} = \frac{(-2)^{38} \times 7}{17}$$

$\therefore$ There will be no effect of -ve sign because the power is even

$$\rightarrow \frac{2^{38} \times 7}{17} = \frac{(2^4)^9 \times 2^2 \times 7}{17}$$

$$\therefore 2^4 = 16$$

$$\rightarrow \frac{(16)^9 \times 4 \times 7}{17} = \frac{(-1)^9 \times 28}{17} = \frac{-1 \times 28}{17}$$

$$\begin{array}{c} -6 \\ \uparrow \\ = \frac{-28}{17} = \frac{(28)}{17} \text{ (Avoid -ve sign)} \end{array}$$

Now use -ve sign
 $R = -(-6) = 6$

21. (b) $\frac{2^{49}}{7} \quad \because 2^3 = 8$

$$\Rightarrow \frac{(2^3)^{16} \times 2^1}{7} = \frac{(8)^{16} \times 2}{7}$$

$$= \frac{(+1)^{16} \times 2}{7} = \frac{1 \times 2}{7} = \frac{2}{7}$$

$$R = 2$$

22. (d) $\frac{0}{17}$
 $\uparrow$
 $\frac{51^{203} + 2^{49}}{17}$

51 is divisible by 17 So $(51)^{203}$ is divisible by 17 then remainder '0', Now only divide 2^{49}

$$= \frac{2^{49}}{17} = \frac{(2^4)^{12} \times 2^1}{17} = \frac{(16)^{12} \times 2}{17}$$

$$= \frac{(-1)^{12} \times 2}{17} = \frac{1 \times 2}{17} = \frac{2}{17}$$

$$R = 2$$

23. (b) $\frac{1234567891011121314}{8}$

$\because$ divisibility by 8 $\rightarrow$ The Last Three digits are divisible by 8
 So Now last 3 digits 314 divide by 8 we get remainder

$$= \frac{314}{8}, R = 2$$

24. (c) $\frac{41424344 \dots 4950}{16}$

divisibility by 16 $\rightarrow$ The last Four digits are divisible by 16
 No : Last '4' digits 4 9 5 0

$$\frac{4950}{16} = R = 6$$

25. (b) $\frac{21222324252627282930}{8}$

Last '3' digits 930

$$\text{Remainder} = \frac{930}{8}$$

$$R = 2$$

26. (c) $\frac{919293949596979899}{16}$

Last '4' digits 9 8 9 9

$$\begin{array}{r} \text{Remainder} = 16 \left) \begin{array}{r} 9899 \\ 96 \\ \hline 29 \\ 16 \\ \hline 139 \\ 128 \\ \hline 11 \end{array} \right. \end{array}$$

$$\text{Remainder} = 11$$

27. (c) $\frac{313233 \dots 3839}{4}$

divisibility by 4 $\rightarrow$ The last '2' digits divisible by 4

Last '2' digits 39

$$R = \frac{39}{4}$$

$$R = 3$$

28. (a) $\frac{12345 \dots 41 \text{ digits}}{8}$

From 1 to 9 = 9 digits

Remainder = 41 - 9 = 32 digits

$$\text{Number} = \frac{32}{2} = 16$$

1, 2, 3, 4, 9/10 11 41 digits

Total Number = 9 + 16 = 25

1 2 3 4 23 24 25

Last '3' digits = 425

$$\text{Remainder} = \frac{425}{8}$$

$$R = 1$$

29. (a) $\frac{1234 \dots 81 \text{ digits}}{16}$

from 1 to 9 = 9 digits

Remainder digits = 81 - 9 = 72 digits

$$\text{Number} = \frac{72}{2} = 36$$

1, 2, 3, 9/10 11 .. 81 digits

Total Number = 9 + 36 + 45

1, 2, 3, 43 44 45

Last '4' digits 4445 divide by 16 we get remainder

$$R = \frac{4445}{16} = 13$$

$$\begin{array}{r} 16 \left) \begin{array}{r} 4445 \\ 32 \\ \hline 124 \\ 112 \\ \hline 125 \\ 112 \\ \hline 13 \end{array} \right. \end{array}$$

13 $\rightarrow$ Remainder

30. (b) $\frac{8^{77}}{17}$

$$\because 8^3 = 512$$

$$\Rightarrow \frac{(8^3)^{25} \times 8^2}{17} = \frac{(512)^{25} \times 64}{17}$$

$$\Rightarrow \frac{(2)^{25} \times 64}{17}$$

$$= \frac{(2^4)^6 \times 2^1 \times 64}{17}$$

$$= \frac{(16)^6 \times 2 \times 64}{17} = \frac{(-1)^6 \times 2 \times -4}{17}$$

$$= \frac{1 \times 2 \times -4}{17} = \frac{-8}{17}$$

$$R = 17 - 8 = 9$$

31. (d) $\frac{1+2+3+\dots+100}{5}$

$$\begin{array}{c} +1 \\ \uparrow \\ \underline{1} = 1 = \frac{1}{5}, R = 1 \end{array}$$

$$\begin{array}{c} +2 \\ \uparrow \\ \underline{2} = 1 \times 2 = \frac{2}{5}, R = 2 \end{array}$$

$$\begin{array}{c} +1 \\ \uparrow \\ \underline{3} = 1 \times 2 \times 3 = \frac{6}{5}, R = 1 \end{array}$$

$$\begin{array}{c} -1 \\ \uparrow \\ \underline{4} = 1 \times 2 \times 3 \times 4 = \frac{24}{5}, R = -1 \end{array}$$

$$\underline{5} = 5 \times 4 \times 3 \times 2 \times 1 = \frac{120}{5}, R = 0$$

5, 6, 7 100 is all perfect divisible by 5. So remainder '0'

$$\begin{array}{cccccccc} +1 & +2 & +1 & -1 & 0 & 0 & 0 & 0 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \underline{1+2+3+4+5+6+\dots+99+100} \end{array}$$

$$\Rightarrow \frac{1+2+1-1}{5} = \frac{3}{5}$$

$$R = 3$$

$$32. (a) \frac{1+2+3+\dots+100}{6}$$

$$1 = 1 = \frac{1}{6} = R = 1$$

$$2 = 1 \times 2 = \frac{2}{6} = R = 2$$

$$3 = 1 \times 2 \times 3 = \frac{6}{6} = R = 0$$

$$4 = 4 \times 3 \times 2 \times 1 = \frac{24}{6} = R = 0$$

$$= \frac{1+2+3+4+5+\dots+100}{6}$$

$$\frac{1+2}{6} = \frac{3}{6}$$

$$R = 3$$

$$33. (d) \frac{1+2+3+\dots+50}{12}$$

$$1 = 1 = \frac{1}{12} = R = 1$$

$$2 = 1 \times 2 = \frac{2}{12} = R = 2$$

$$3 = \frac{1 \times 2 \times 3}{12} = \frac{6}{12} = R = 6$$

$$4 = \frac{1 \times 2 \times 3 \times 4}{12} = \frac{24}{12} = R = 0$$

$$5 = \frac{1 \times 2 \times 3 \times 4 \times 5}{12} = \frac{120}{12} = R = 0$$

$$= \frac{1+2+3+4+5+\dots+50}{6}$$

$$\frac{1+2+6}{12} = \frac{9}{12} = 9$$

$$R = 9$$

$$34. (b) \frac{9^{111}}{11} = \frac{(9)^{111}}{11}$$

$$= \frac{(-2)^{111}}{11} = -\frac{(2)^{111}}{11}$$

Avoid -ve sign

$$\frac{2^{111}}{11} = \frac{(2^5)^{22} \times 2^1}{11}$$

$$\because 2^5 = 32$$

$$= \frac{(32)^{22} \times 2}{11}$$

$$= \frac{(-1)^{22} \times 2}{11} = \frac{1 \times 2}{11} = \frac{2}{11}$$

Now use -ve sign

$$R = -2$$

$$\text{Actual Remainder} = 11 - 2 = 9$$

$$35. (b) \frac{5^{2450}}{126}$$

$$\because 5^3 = 125$$

$$\frac{(5^3)^{816} \times 5^2}{126} = \frac{(125)^{816} \times 25}{126}$$

$$= \frac{(-1)^8 \times 25}{126} = \frac{1 \times 25}{126} = \frac{25}{126}$$

$$R = 25$$

$$36. (d) \frac{(40)^{1012}}{7} = \frac{(-2)^{1012}}{7}$$

-ve sign will be no effect because power is even

$$= \frac{(2)^{1012}}{7}$$

$$\because 2^3 = 8$$

$$= \frac{(2^3)^{337} \times 2^1}{7}$$

$$= \frac{(8)^{337} \times 2}{7} = \frac{(+1)^{337} \times 2}{7}$$

$$= \frac{1 \times 2}{7} = \frac{2}{7}$$

$$R = 2$$

$$37. (a) \frac{10^1 + 10^2 + 10^3 + 10^4 + \dots + 10^{100}}{6}$$

$$10^1 = \frac{10}{6} = R = 4$$

$$10^1 + 10^2 = \frac{10 + 100}{6} = \frac{110}{6} = R = 2$$

$$10^1 + 10^2 + 10^3 = \frac{10 + 100 + 1000}{6} = \frac{1110}{6} = \frac{12}{6} = R = 0$$

Remainder = 0

'0' will be the remainder of each three terms

So '0' will be the remainder of 99th term.

$$\Rightarrow \frac{10^{100}}{6} \quad R = 4$$

* **10ⁿ is divided by 6. We always get remainder 4, where n = natural number**

$$38. (c) \frac{10^1 + 10^2 + 10^3 + \dots + 10^{1000} + 10^{1001}}{6}$$

$$10^1 = \frac{10}{6} \quad R = 4$$

$$10^1 + 10^2 = \frac{10 + 100}{6} = \frac{110}{6} = R = 2$$

$$10^1 + 10^2 + 10^3 = \frac{10 + 100 + 1000}{6} = \frac{1110}{6} = \frac{12}{6} = R = 0$$

$$R = 0$$

'0' will be remainder of each three terms so '0' will be the remainder of 999th.

$$\frac{10^{1000} + 10^{1001}}{6} = \frac{4 + 4}{6} = \frac{8}{6}$$

$$R = 2$$

$$39. (a) \frac{666666 \dots 134 \text{ times}}{13}$$

- * If any digit is made by repeating a 6 times. The Number will be divisible by 3, 7, 11, 13, 37, 39. So, 666666 132 times is divisible by 13. because 132 is the multiple of 6. So, The Remaining 2 digits will be. divided by 13 to get the Remainder

$$= \frac{666666 \dots 132 \text{ times } 66}{13}$$

$$= \frac{66}{13} = R = 1$$

40. (b) $\frac{555555 \dots 244 \text{ times}}{37}$

555555 240 times is divisible by 37 because any digit is made by repeating 6 times. The number will be divisible by 37.

$$\frac{555555 \dots 240 \text{ times } 5555}{37}$$

$$\text{Remainder } \frac{5555}{37}$$

$$37 \overline{) 5555} \begin{array}{r} 15 \\ \underline{185} \\ 185 \\ \underline{185} \\ 5 \end{array} \rightarrow \text{Remainder}$$

$$R = 5$$

41. (b) $\frac{777777 \dots 363 \text{ Times}}{11}$

$$= \frac{777777 \dots 360 \text{ Times } 777}{11}$$

$$= \frac{777}{11}$$

$$\text{Remainder} = 7$$

42. (b) $\frac{888888 \dots 184 \text{ times}}{37}$

$$\Rightarrow \frac{888888 \dots 180 \text{ times } 8888}{37}$$

$$\Rightarrow \frac{8888}{37}$$

$$37 \overline{) 8888} \begin{array}{r} 24 \\ \underline{74} \\ 148 \\ \underline{148} \\ 0 \end{array} \rightarrow \text{Remainder}$$

$$\therefore \text{So Remainder} = 8$$

43. (b) $\frac{999999999}{13}$

6 times '9' is divisible by 13, remainder will be 0, Remaining digits divide by 13 we get Remainder

$$\frac{999}{13}$$

$$R = 11$$

44. (c) $\frac{7^{99}}{2400}$

$$\begin{array}{l} 7^1 = 7 \\ 7^2 = 49 \\ 7^3 = 343 \\ 7^4 = 2401 \end{array}$$

Break the power multiple of '4' form

$$\Rightarrow \frac{(7^4)^{24} \times 7^3}{2400}$$

$$\Rightarrow \frac{(2401)^{24} \times 343}{2400}$$

$$\Rightarrow \frac{(2401)^{24} \times 343}{2400} = \frac{(+1)^{24} \times 343}{2400}$$

$$= \frac{1 \times 343}{2400} = \frac{343}{2400}$$

$$R = 343$$

45. (b) $\frac{3^{1989}}{7}$

$$\therefore 3^3 = 27$$

$$\frac{(3^3)^{663}}{7} = \frac{(27)^{663}}{7}$$

$$= \frac{(-1)^{663}}{7} = -1$$

$$R = 7 - 1 = 6$$

46. (a)

$$\frac{54^{124}}{17} = \frac{(54)^{124}}{17}$$

$$= \frac{(3)^{124}}{17}$$

$$\begin{array}{l} 3^1 = 3 \\ 3^2 = 9 \\ 3^3 = 27 \\ 3^4 = 81 \end{array}$$

$$\therefore 3^4 = 81$$

$$= \frac{(3^4)^{31}}{17} = \frac{(81)^{31}}{17}$$

$$= \frac{(-4)^{31}}{17} = -\frac{(4)^{31}}{17}$$

$$\text{Avoid -ve sign.} = \frac{4^{31}}{17}$$

$$\therefore 4^2 = 16$$

$$= \frac{(4^2)^{15} \times 4^1}{17} = \frac{(16)^{15} \times 4}{17}$$

$$= \frac{(-1)^{15} \times 4}{17} = \frac{-1 \times 4}{17} = \frac{-4}{17} = -4$$

Now use -ve sign = -(-4)

$$\text{Remainder} = 4$$

47. (b)

$$\frac{(21)^{875}}{17} = \frac{(21)^{875}}{17}$$

$$= \frac{(4)^{875}}{17} \therefore 4^2 = 16$$

$$= \frac{(4^2)^{437} \times 4^1}{17}$$

$$= \frac{(16)^{437} \times 4^1}{17} = \frac{(-1)^{437} \times 4}{17}$$

$$= \frac{-1 \times 4}{17} = \frac{-4}{17}$$

$$R = 17 - 4 = 13$$

48. (d)

$$\frac{83^{261}}{17} = \frac{(83)^{261}}{17}$$

Avoid -ve sign.

$$= \frac{2^{261}}{17} = \frac{(2^4)^{65} \times 2^1}{17}$$

$$\therefore 2^4 = 16$$

$$= \frac{(16)^{65} \times 2}{17} = \frac{(-1)^{65} \times 2}{17}$$

$$= \frac{-1 \times 2}{17} = -2$$

Now use sign -(-2) = 2
Remainder = 2

49. (a) $\frac{(32^{32})^{32}}{9}$

Cyclicity

$$\begin{array}{c} +5 \\ \uparrow \\ \frac{32^1}{9} = R = 5 \end{array}$$

$$\begin{array}{c} +5 \quad +5 \\ \uparrow \quad \uparrow \\ \frac{32^2}{9} = \frac{32 \times 32}{9} = \frac{5 \times 5}{9} = \frac{25}{9} = R = 7 \end{array}$$

$$\begin{array}{c} +5 \quad +5 \quad +5 \\ \uparrow \quad \uparrow \quad \uparrow \\ \frac{32^3}{9} = \frac{32 \times 32 \times 32}{9} = \frac{5 \times 5 \times 5}{9} \end{array}$$

$$= \frac{125}{9} = R = 8$$

$$\begin{array}{c} +5 \quad +5 \quad +5 \quad +5 \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ \frac{32^4}{9} = \frac{32 \times 32 \times 32 \times 32}{9} = \frac{5 \times 5 \times 5 \times 5}{9} \end{array}$$

$$\begin{array}{c} -2 \quad -2 \\ \uparrow \quad \uparrow \\ \frac{25 \times 25}{9} = \frac{-2 \times -2}{9} = \frac{4}{9}, R = 4 \end{array}$$

$$\begin{array}{c} +5 \quad +5 \quad +5 \quad +5 \quad +5 \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ \frac{32^5}{9} = \frac{32 \times 32 \times 32 \times 32 \times 32}{9} = \frac{25 \times 25 \times 5}{9} \end{array}$$

$$= \frac{-2 \times -2 \times 5}{9} = \frac{20}{9}, R = 2$$

$$\begin{array}{c} +5 \quad +5 \quad +5 \quad +5 \quad +5 \quad +5 \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ \frac{32^6}{9} = \frac{32 \times 32 \times 32 \times 32 \times 32 \times 32}{9} = \frac{25 \times 25 \times 25}{9} \end{array}$$

$$= \frac{-2 \times -2 \times -2}{9} = \frac{-8}{9}, R = 9 - 8 = 1$$

After this It repeated so,
Cyclicity = 6

So, $\frac{(32)^6}{9} R = 1$

Now $(32^{32})^{32} = [(32^6)^5 \times 32^2]^{32}$

$$= \frac{((1)^5 \times 32^2)^{32}}{9} = \frac{(32^2)^{32}}{9}$$

$$\therefore \frac{32^2}{9} = R = 7$$

(above explain In Solution)

$$= \frac{(7)^{32}}{9} = \frac{(7^2)^{16}}{9} = \frac{(49)^{16}}{9}$$

$$= \frac{4^{16}}{9} = \frac{(4^3)^5 \times 4^1}{9} = \frac{(64)^5 \times 4}{9}$$

$$\frac{(+1)^5 \times 4}{9} = \frac{1 \times 4}{9} = \frac{4}{9}$$

$$R = 4$$

50. (a) $\frac{(32^{32})^{32}}{7}$

When 32 is divided by 7 then
Remainder 4

So, 32^{32} is divided by 7
remainder = 4^{32}

$$= \frac{(4^{32})^{32}}{7}$$

$$4 = 2^2$$

$$\Rightarrow \frac{(2^{2 \times 32})^{32}}{7}$$

$$\Rightarrow \frac{(2^{64})^{32}}{7}$$

$$\therefore 2^3 = 8$$

$$= \frac{(2^{63} \times 2^1)^{32}}{7}$$

$$= \frac{((2^3)^{21} \times 2^1)^{32}}{7} = \frac{((8)^{21} \times 2^1)^{32}}{7}$$

$$= \frac{(1 \times 2)^{32}}{7} = \frac{2^{32}}{7}$$

Again $\therefore 2^3 = 8$

$$\frac{2^{30} \times 2^2}{7} = \frac{(2^3)^{10} \times 4}{7}$$

$$= \frac{(8)^{10} \times 4}{7} = \frac{(1)^{10} \times 4}{7} = \frac{1 \times 4}{7}$$

$$\text{Remainder} = 4$$

51. (b) $\frac{(33^{34})^{35}}{7}$

$$\text{we solve } \frac{33^{34}}{7} = \frac{(33)^{34}}{7}$$

$$= \frac{(-2)^{34}}{7}$$

No effect of -ve sign. Because
power is even.

$$= \frac{2^{34}}{7} = \frac{(2^3)^{11} \times 2^1}{7}$$

$$= \frac{(8)^{11} \times 2}{7} = \frac{(+1)^{11} \times 2}{7} = \frac{1 \times 2}{7}$$

Now :- $\frac{(33^{34})^{35}}{7} = \frac{(2)^{35}}{7} = \frac{(2^3)^{11} \times 2^2}{7}$

$$= \frac{(8)^{11} \times 4}{7} = \frac{1 \times 4}{7} = \frac{4}{7}$$

$$R = 4$$

52. (a) $\frac{888^{222} + 222^{888}}{5}$

$$= \frac{888^{222}}{5} + \frac{222^{888}}{5}$$

$$= \frac{3^{222}}{5} + \frac{2^{888}}{5}$$

$$= \frac{(3^4)^{55} \times 3^2}{5} + \frac{(2^4)^{222}}{5}$$

$$= \frac{1 \times 9}{5} + \frac{1}{5}$$

$$= \frac{4}{5} + \frac{1}{5} = \frac{4+1}{5} \Rightarrow \frac{5}{5}$$

Thus the remainder is zero.

Alternatively:

[To check the divisibility by 5
just see the sum of the unit
digits which is 10 (=4+6)]

$$\therefore 8^{222} \rightarrow 4(\text{units digit})$$

$$\text{and } 2^{888} \rightarrow 6(\text{units digit})$$

Hence it is divisible. So there
is no remainder]

$$\begin{aligned}
 53.(a) \quad & \frac{2222^{5555} + 5555^{2222}}{7} \\
 & \begin{array}{cc} +3 & +4 \\ \uparrow & \uparrow \\ \frac{2222^{5555}}{7} & + \frac{5555^{2222}}{7} \\ \frac{(3)^{5555}}{7} & + \frac{(4)^{2222}}{7} \\ \frac{(3)^{1851} \times 3^2}{7} & + \frac{(4)^{740} \times (4)^2}{7} \\ \frac{(27)^{1851} \times 9}{7} & + \frac{(64)^{740} \times 16}{7} \\ \frac{(-1)^{1851} \times 9}{7} & + \frac{(+1)^{740} \times 16}{7} \\ \frac{-9}{7} & + \frac{16}{7} \\ \frac{-9 + 16}{7} & = \frac{7}{7} \\ \text{Remainder} & = 0 \end{array}
 \end{aligned}$$

$$54. (d) \frac{(50^{51})^{52}}{11}, \text{ we break } = 50^{51}$$

$$\begin{aligned}
 & \begin{array}{c} 6 \\ \uparrow \\ (50^{51}) = (6)^{51} \\ \frac{(6)^{51}}{11} = \frac{(6)^{51}}{11} \end{array} \\
 & = \frac{(6^2)^{25} \times 6^1}{11} = \frac{(36)^{25} \times 6}{11} \\
 & \frac{(3)^{25} \times 6}{11} = \frac{(3^5)^5 \times 6}{11} \\
 & \because 3^5 = 243 \\
 & \begin{array}{c} +1 \\ \uparrow \\ \frac{(243)^5 \times 6}{11} = \frac{(1)^5 \times 6}{11} = \frac{6}{11} \end{array} \\
 & \text{Now } \frac{50^{51}}{11} = \frac{6}{11} \\
 & \because 50^{51 \times 52} = \frac{(6)^{52}}{11} \\
 & \begin{array}{c} +3 \\ \uparrow \\ \frac{(6^2)^{26}}{11} = \frac{(36)^{26}}{11} \end{array}
 \end{aligned}$$

$$\begin{aligned}
 & \begin{array}{c} +1 \\ \uparrow \\ \frac{(3)^{26}}{11} = \frac{(3^5)^5 \times 3^1}{11} \\ \frac{(243)^5 \times 3}{11} \\ \frac{(1)^5 \times 3}{11} = \frac{1 \times 3}{11} = \frac{3}{11} \\ \text{Remainder} = 3 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 55. (a) \quad & \begin{array}{c} +3 \\ \uparrow \\ \frac{(20)^{23}}{17} \rightarrow \frac{(3)^{23}}{17} \rightarrow \frac{(3^7)^3 \times 3^2}{17} \rightarrow \frac{(27)^3 \times 9}{17} \\ \frac{(10)^7 \times 9}{17} \rightarrow \frac{(10^2)^3 \times 10 \times 9}{17} \rightarrow \frac{(100)^3 \times 90}{17} \\ \frac{(-2)^3 \times 5}{17} = \frac{-8 \times 5}{17} = \frac{-40}{17} \end{array}
 \end{aligned}$$

$\therefore$ Avoid (-) sign normally divide 40 by 17, Remainder 6, Now use negative sign $R = -6$
 $\therefore R = 17 - 6 = 11$

$$\begin{aligned}
 56. (c) \quad & (x-2) \text{ is factor of } x^2 + 3qx - 2q \\
 & \text{So, } x-2 = 0 \Rightarrow x = 2 \\
 & \text{Put, } x = 2 \text{ in } x^2 + 3qx - 2q \text{ and equate to zero.} \\
 & \therefore (2)^2 + 3 \times q \times 2 - 2q = 0 \\
 & \Rightarrow 4 + 6q - 2q = 0 \Rightarrow 4 + 4q = 0 \\
 & 1 + q = 0 \Rightarrow q = -1
 \end{aligned}$$

$$\begin{aligned}
 57. (c) \quad & \text{Factor } \Rightarrow x + 2 = 0 \Rightarrow x = -2 \text{ (put)} \\
 & \therefore (-2)^3 + 6(-2)^2 + 4(-2) + k = 0 \\
 & \Rightarrow -8 + 24 - 8 + k = 0 \\
 & \Rightarrow k = -8
 \end{aligned}$$

$$\begin{aligned}
 58.(b) \quad & x - 1 = 0 \Rightarrow x = 1 \text{ (put)} \\
 & \Rightarrow 1^3 - k = 0 \\
 & \Rightarrow 1 - k = 0 \Rightarrow k = 1
 \end{aligned}$$

$$\begin{aligned}
 59.(a) \quad & x + 1 = 0 \Rightarrow x = -1 \text{ (put)} \\
 & \Rightarrow x^{100} + 2x^{99} + k = 0 \\
 & \Rightarrow (-1)^{100} + 2(-1)^{99} + k = 0 \\
 & \Rightarrow 1 - 2 + k = 0 \Rightarrow -1 + k = 0 \\
 & \Rightarrow k = 1
 \end{aligned}$$

$$\begin{aligned}
 60.(a) \quad & x + 2 = 0 \Rightarrow x = -2 \text{ (put)} \\
 & \Rightarrow x^3 - 5x^2 + 4p = 0 \\
 & \Rightarrow (-2)^3 - 5(-2)^2 + 4p = 0 \\
 & \Rightarrow -8 - 20 + 4p = 0 \\
 & \Rightarrow -28 + 4p = 0 \Rightarrow 4p = 28 \\
 & \Rightarrow p = 7
 \end{aligned}$$

$$\begin{aligned}
 61.(a) \quad & x - a = 0 \Rightarrow x = a \\
 & \Rightarrow x^3 - 3x^2a + 2a^2x + b = 0 \\
 & \Rightarrow a^3 - 3a^2a + 2a^2a + b = 0 \\
 & \Rightarrow a^3 - 3a^3 + 2a^3 + b = 0 \\
 & \Rightarrow b = 0
 \end{aligned}$$

$$\begin{aligned}
 62.(c) \quad & (x+6) \text{ is a factor of } f(x) \\
 & \text{The } f(x) = 0 \\
 & x + 6 = 0 \\
 & x = -6 \\
 & f(x) = x^3 + 3x^2 + 4x + k \\
 & f(-6) = (-6)^3 + 3(-6)^2 + 4(-6) + k = 0 \\
 & = -216 + 108 - 24 + k = 0 \\
 & k = 132
 \end{aligned}$$

$$\begin{aligned}
 63.(c) \quad & (x+2) \& (x-1) \text{ are the factor of } x^3 + 10x^2 + mx + n, \text{ so put } x = -2 \text{ and } x = 1 \text{ respectively} \\
 & \Rightarrow (-2)^3 + 10(-2)^2 + m(-2) + n = 0 \\
 & \Rightarrow -8 + 40 - 2m + n = 0 \\
 & \Rightarrow -2m + n + 32 = 0 \\
 & \Rightarrow -2m + n = -32 \quad \dots\dots(i) \\
 & \text{Put } x = 1 \\
 & \Rightarrow (1)^3 + 10(1)^2 + m \cdot 1 + n = 0 \\
 & \Rightarrow 1 + 10 + m + n = 0 \\
 & \Rightarrow m + n = -11 \quad \dots\dots(ii) \\
 & \text{solve (i) and (ii) we get} \\
 & \Rightarrow m = 7
 \end{aligned}$$

$$\begin{aligned}
 & \text{and} \\
 & \Rightarrow n = -18 \\
 64. (b) \quad & x + 1 = 0 \Rightarrow x = -1 \\
 & \text{put the value of } x = -1 \text{ in equation} \\
 & R = x^3 - 6x + 7 = (-1)^3 - 6(-1) + 7 \\
 & = -1 + 6 + 7 \\
 & \text{Remainder} = 12
 \end{aligned}$$

$$\begin{aligned}
 65. (a) \quad & x - 3 = 0 \Rightarrow x = 3 \\
 & \text{put of value of } x = 3 \text{ in } x^5 - 9x^2 + 12x - 14 \\
 & R = (3)^5 - 9(3)^2 + 12(3) - 14 \\
 & R = 243 - 81 + 36 - 14 \\
 & R = 279 - 95 \\
 & R = 184
 \end{aligned}$$

$$\begin{aligned}
 66. (b) \quad & x - 2 = 0 \Rightarrow x = 2 \\
 & \text{put the value of } x = 2 \text{ in } x^4 - 3x^3 + 2x^2 - 5x + 7 \\
 & R = 2^4 - 3(2)^3 + 2(2)^2 - 5(2) + 7 \\
 & R = 16 - 24 + 8 - 10 + 7 \\
 & R = 31 - 34 = -3
 \end{aligned}$$

$$\begin{aligned}
 67. (d) \quad & x + 3 = 0 \Rightarrow x = -3 \\
 & \text{put te value of } x = -3 \text{ in } 5x^3 + 5x^2 - 6x + 9 \\
 & R = 5(-3)^3 + 5(-3)^2 - 6(-3) + 9 \\
 & R = -135 + 45 + 18 + 9 \\
 & R = -135 + 72 = -63
 \end{aligned}$$

$$\begin{aligned}
 68. (b) \quad & x + 1 = 0 \Rightarrow x = -1 \\
 & \text{put the value of } x = -1 \text{ in } x^{11} + 1 \\
 & R = (-1)^{11} + 1 = -1 + 1 = 0
 \end{aligned}$$

$$69. (c) 2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$$

put the value of $x = -\frac{1}{2}$ in $2x^3 + 5x^2 - 4x - 6$

$$R = 2\left(-\frac{1}{2}\right)^3 + 5\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) - 6$$

$$R = -2 \times \frac{1}{8} + \frac{5}{4} + \frac{4}{2} - 6$$

$$R = -\frac{1}{4} + \frac{5}{4} + 2 - 6$$

$$R = \frac{4}{4} - 4 = -3$$

$$70. (c) x^2 + 2 = 0 \Rightarrow x^2 = -2$$

Put the value of $x^2 = -2$ in $x^3 + 5x^2 + 10k$

$$R = x^2 \cdot x + 5x^2 + 10k = -2x \text{ (given)}$$

$$\Rightarrow (-2) x + 5 \cdot (-2) + 10k = -2x$$

$$\Rightarrow -10 + 10k = 0$$

$$\Rightarrow k = 1$$



Maths By Rakesh Yadav Sir

ARITHMETIC PROGRESSION & GEOMETRIC PROGRESSION

Arithmetic Progression

A.P.:- Quantities are said to be in arithmetic progression when they increase or decrease by a common difference.

Some more examples of Arithmetic Progression are as follows

1, 2, 3, 4, 5, 6, 7, ...
 3, 7, 11, 15, 19, 23,
 19, 17, 15, 13, 11,
 -10, -4, 2, 8, 14, 20,
 40, 37.5, 35, 32.5, 30,
 5, 12, 19, 26, 33,
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

d = Common difference

$= a_2 - a_1 = 12 - 5 = 7$

$a_1 \rightarrow$ First Term

$a_2 \rightarrow$ Second Term

$$T_n = a + (n-1)d$$

$T_n \rightarrow$ nth term of A.P.

$a \rightarrow$ First Term of A.P.

$d \rightarrow$ common difference

$$L = a + (n-1)d$$

$L \rightarrow$ Last term/ nth term of A.P.

Sum of n term.

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\text{or } S_n = \frac{n}{2} [a + l]$$

Ex. 1 5, 12, 19, 26, T_{10}

(i) Find the value of t_{10}

(ii) Sum of 10 term.

Sol. 5, 12, 19, 26, T_{10}

$$d = 12 - 5 = 7$$

$$a = 5$$

$$n = 10$$

$$T_{10} = a + (n-1)d$$

$$= 5 + (10-1)7$$

$$= 5 + 63 = 68$$

$$(ii) S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{10}{2} [2 \times 5 + (10-1)7]$$

$$= 5 [10 + 63] = 5 \times 73 = 365$$

Ex. 2 Find the sum of the 10 terms of the following series -11, -8, -5, -2, ...

Sol. -11, -8, -5, -2, ...

$$a = -11, d = 3, n = 10$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{10} = \frac{10}{2} [2 \times (-11) + (9)3]$$

$$= 5 \times (-22 + 27) = 25$$

Ex. 3 If $t_1 + t_5 + t_{10} + t_{15} + t_{20} + t_{24} = 225$ (where $t_1 \rightarrow$ first term of A.P., $T_5 \rightarrow$ fifth term of A.P.)

find the sum of

$$t_1 + t_2 + t_3 + \dots + t_{24}$$

$$T_n = a + (n-1)d$$

Sol. $t_1 =$ first term $= a$

$$t_5 = a + (5-1)d = a + 4d$$

$$t_{10} = a + 9d$$

$$t_{15} = a + 14d$$

$$t_{20} = a + 19d$$

$$t_{24} = a + 23d$$

$$a + a + 4d + a + 9d + a + 14d + a +$$

$$19d + a + 23d = 225$$

$$6a + 69d = 225$$

$$2a + 23d = 75$$

$$S_{24} = \frac{24}{2} [2a + (24-1)d]$$

$$S_{24} = 12 [2a + 23d]$$

$$\text{Put value of } 2a + 23d = 75$$

$$S_{24} = 12 [75] = 900$$

Ex. 4 Find the 33rd term of the sequence.

3, 8, 9, 13, 15, 18, 21, 23, ...

(a) 93 (b) 992 (c) 105 (d) 83

Sol. (b) 3, 8, 9, 13, 15, 18, 21, 23, ...

The above series is a combination of two APS

The 1st A.P. (3, 9, 15, 21, ...) +
 2nd AP (8, 13, 18, ...)

33rd Term lies, In first A.P. which 17th term of first A.P. Then

$$T_{17} = a + (n-1)d$$

$$a = 3$$

$$d = 6$$

$$T_{17} = 3 + 16 \times 6$$

$$T_{17} = 99$$

We can say that

$$T_{33} = 99$$

Ex. 5 Find the sum to 100 terms of the series

$$1 + 4 + 6 + 5 + 11 + 6 + \dots$$

$$(a) 7400 \quad (b) 7550$$

$$(c) 7600 \quad (d) 7500$$

Sol. (c) $1 + 4 + 6 + 5 + 11 + 6 + \dots$

The series is a combination of two APS.

$(1 + 6 + 11 + \dots 50\text{th term}) +$

$(4 + 5 + 6 + \dots 50\text{th term})$

Sum of series

$$a_1 = 1 \quad a = 4$$

$$d_1 = 5 \quad d = 1$$

$$n = 50 \quad n = 50$$

$$S_{100} = \frac{50}{2} [2 \times 1 + 49 \times 5] + \frac{50}{2} [2 \times$$

$$4 + 49 \times 1]$$

$$= 25 [2 + 245] + 25 [8 + 49]$$

$$= 25 [247] + 25 [57]$$

$$= 25 [247 + 57]$$

$$= 25 \times 304 = 7600$$

Ex. 6 Find the value of

$1 - 2 - 3 + 2 - 3 - 4 + \dots$ up to 100 terms

$$(a) -626 \quad (b) -622$$

$$(c) -624 \quad (d) -628$$

Sol. (a) Series is a combination of three APS.

$(1 + 2 + 3 \dots 34 \text{ terms}) -$

$$a = 1$$

$$d = 1$$

$(2 + 3 + 4 \dots 33 \text{ terms}) -$

$$a = 2$$

$$d = 1$$

$(3 + 4 + 5 \dots 33 \text{ terms})$

$$a = 3$$

$$d = 1$$

Sum of series

$$\begin{aligned}
 &= \frac{34}{2} [2 \times 1 + 33 \times 1] - \frac{33}{2} [2 \times 2 \\
 &+ 32 \times 1] - \frac{33}{2} [2 \times 3 + 32 \times 1] \\
 &= 17 [2 + 33] - \frac{33}{2} [4 + 32] - \frac{33}{2} [6 + 32] \\
 &= 17 \times 35 - \frac{33}{2} \times 36 - \frac{33}{2} \times 38 \\
 &= 17 \times 35 - 33 \times 18 - 33 \times 19 \\
 &= 595 - 594 - 627 = -626
 \end{aligned}$$

Ex.7 If the sum of first 11 terms of A.P is equal to sum of first 19 terms of that A.P. find the sum of first 30 terms of that A.P.

Sol. According to question

$$S_{11} = S_{19}$$

$$\begin{aligned}
 \frac{11}{2} [2a + 10d] &= \frac{19}{2} [2a + 18d] \\
 22a + 110d &= 38a + 342d \\
 16a &= -232d \\
 2a &= -29d
 \end{aligned}$$

$$2a + 29d = 0$$

$$\begin{aligned}
 S_{30} &= \frac{30}{2} [2a + (30-1)d] \\
 &= 15 [2a + 29d]
 \end{aligned}$$

Put value

$$2a + 29d = 0$$

$$S_{30} = 15 \times 0 = 0$$

Sum of first 30 terms of that A.P = 0

Total No. of Term

$$= \frac{\text{last term} - \text{First term}}{\text{common difference}} + 1$$

Ex.8 Find the Total No. of terms Between. 300 to 600 which are exactly divisible by 4.

Sol. 304, 308, 312 596
 $d = 4$, $a = 304$, last terms = 596

$$\text{No. of Terms} = \frac{l - a}{d} + 1$$

$$= \frac{596 - 304}{4} + 1$$

$$= \frac{292}{4} + 1$$

$$= 73 + 1 = 74$$

Ex.9 The sum of the second and the fifth term of an AP is 8 and that of the third and the seventh term is 14. Find the eleventh term.

(a) 19 (b) 17 (c) 15 (d) 16

Sol. (a) $T_2 + T_5 = 8$

$$T_3 + T_7 = 14$$

$$T_2 = a + d$$

$$T_5 = a + 4d$$

$$T_2 + T_5 = a + d + a + 4d = 8$$

$$2a + 5d = 8 \quad \dots\dots\dots(i)$$

$$T_3 = a + 2d$$

$$T_7 = a + 6d$$

$$T_3 + T_7 = a + 2d + a + 6d = 14$$

$$2a + 8d = 14 \quad \dots\dots\dots(ii)$$

$$(ii) - (i)$$

$$(2a + 8d) - (2a + 5d) = 14 - 8$$

$$3d = 6$$

$$d = 2$$

Put the value d equation (ii)

$$2a + 8 \times 2 = 14$$

$$a = -1$$

$$T_{11} = a + 10d$$

$$= -1 + 10 \times 2$$

$$= -1 + 20$$

$$= 19$$

Ex.10 A number 20 is divided Into Four Parts that are in AP such that the product of the first and fourth is to the product of the second and third is 2 : 3. Find the Largest parts.

(a) 12 (b) 4 (c) 8 (d) 9

Sol. (c) Let the 4 terms of A.P.

$$a - 3d, a - d, a + d, a + 3d$$

$$(a - 3d) + (a + 3d) + a - d + a + d = 20$$

$$4a = 20$$

$$a = 5$$

$$\frac{(a - 3d)(a + 3d)}{(a - d)(a + d)} = \frac{2}{3}$$

$$\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{2}{3}$$

Put the value of a

$$\frac{25 - 9d^2}{25 - d^2} = \frac{2}{3}$$

$$75 - 27d^2 = 50 - 2d^2$$

$$25 = 25d^2$$

$$d^2 = 1$$

$$d = 1$$

$$a - 3d = 5 - 3 \times 1 = 2$$

$$a - d = 5 - 1 = 4$$

$$a + d = 5 + 1 = 6$$

$$a + 3d = 5 + 3 \times 1 = 8$$

$$\text{Largest part} = 8$$

Ex.11 How many term of an AP must be taken for their sum to be equal to 120 if its third term is 9 and the difference between the seventh and the second term is 20 ?

(a) 6 (b) 9 (c) 7 (d) 8

Sol. (d) $T_3 = a + 2d = 9$

$$T_7 - T_2 = (a + 6d) - (a + d) = 20$$

$$5d = 20$$

$$d = 4$$

$$T_3 = a + 2 \times 4 = 9$$

$$a = 1$$

$$120 = \frac{n}{2} [2 \times 1 + (n - 1)4]$$

$$120 = \frac{n}{2} \times 2 [1 + (n - 1)2]$$

$$120 = n [1 + 2n - 2]$$

$$120 = n (2n - 1)$$

Take Option (d) , Put the value

$$n = 8$$

$$= 8 (2 \times 8 - 1) = 120$$

$$\therefore n = 8$$

Useful Results

* If the same quantity be added to, or subtracted from, all the terms of an AP, the resulting terms will form an AP, but with the same common difference as before.

* If all the terms of an AP be multiplied or divided by the same quantity, the resulting terms will form an AP, but with a new common difference, which will be the multiplication/division of the old common difference. (as the case may be)

* If $a_1, a_2, a_3, a_4, a_5, \dots, a_n$ are in A.P then

the arithmetic mean (A.M)

$$= \left(\frac{a_1 + a_2 + a_3 + \dots + a_n}{n} \right)$$

* The arithmetic mean (A.M) of odd numbers of consecutive terms is the **middle most term** itself.

* If a and b are in A.P then

$$\text{A.M.} = \frac{a + b}{2}$$

- * If you have to assume 3 terms in AP, assume them as **a-d, a, a+d** or **a, a+d, a+2d**
For assuming 4 terms of an AP we use; **a-3d, a-d, a+d and a+3d**

For assuming 5 terms of an AP, take them as:

a-2d, a-d, a, a+d, a+2d.

Geometric Progression

Geometric Progression (G.P) GP

→ Quantities are said to be in geometric progression when they increase or decrease by a constant factor

Ex. 3, 6, 12, 24,

↓ ↓
a₁ a₂

Common ratio (r) = $\frac{a_2}{a_1}$

nth term of G.P →

$$T_n = ar^{n-1}$$

Where a = First terms of G.P

r = common ratio

n → no. of Term.

Sum of G.P

$$S_n = a \frac{(r^n - 1)}{r - 1} \quad \text{where } r > 1$$

or,

$$S_n = a \frac{(1 - r^n)}{1 - r} \quad \text{Where } r < 1$$

Ex. 1 3, 6, 12, 24 T₁₀

Find

(i) T₁₀ (10th term of G.P)

(ii) S₁₀ (Sum of First 10 terms of G.P)

Sol. 3, 6, 12, 24, T₁₀

$$r = \frac{6}{3} = 2$$

$$a = 3$$

$$T_{10} = 3 \cdot 2^{(10-1)} = 3 \times 2^9 \\ = 3 \times 512 = 1536$$

$$S_n = a \frac{r^n - 1}{r - 1} \quad r > 1$$

$$S_{10} = \frac{3(2^{10} - 1)}{2 - 1} = \frac{3(2^{10} - 1)}{1}$$

$$= 3 \times (1024 - 1)$$

$$= 3 \times 1023 = 3069$$

Sum of an Infinite geometric progression when r < 1

$$S_{\infty} = \frac{a}{1 - r}$$

Ex. 2 16, 8, 4, 2, 1, $\frac{1}{2}$ ∞

find sum of G.P

Sol. 16, 8, 4, 2 ∞

$$a = 16$$

$$r = \frac{8}{16} = \frac{1}{2}$$

$$r < 1$$

$$S_{\infty} = \frac{a}{1 - r} = \frac{16}{1 - \frac{1}{2}} = \frac{16}{\frac{1}{2}} = 32$$

Ex. 3 Find the value of

$$25^{\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty\right)}$$

Sol. First Solve this part

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty$$

It is an Infinite G.P.

$$a = \frac{1}{3}$$

$$r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3}$$

∴ where r < 1

$$S_{\infty} = \frac{a}{1 - r} = \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

Then

Put the value of

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty = \frac{1}{2}$$

$$= (25)^{\frac{1}{2}} = 5$$

Ex. 4 Find the Geometric mean of 3, 9, 27.

(a) 9 (b) 27 (c) 3 (d) 81

Sol. (a) G.M = $(a_1 \cdot a_2 \cdot a_3 \dots)^{1/3}$

$$= (3 \times 9 \times 27)^{1/3}$$

$$= (3^1 \times 3^2 \times 3^3)^{1/3}$$

$$= (3^6)^{1/3} = 3^2 = 9$$

Ex. 5 Find the G.M of 2, 4, 8, 16, 32

(a) 4 (b) 8 (c) 16 (d) 32

Sol. (b) $(2 \times 4 \times 8 \times 16 \times 32)^{1/5}$

$$= (2^1 \times 2^2 \times 2^3 \times 2^4 \times 2^5)^{1/5}$$

$$= (2^{1+2+3+4+5})^{1/5} = 2^{15 \times \frac{1}{5}}$$

$$= 2^3 = 8$$

Ex. 6 The seventh Term of a G.P 8 times the Fourth Term. What will be the first term when its fifth term is 48 ?

(a) 3 (b) 6 (c) 2 (d) 4

Sol. (a) T₇ = 8 T₄

$$ar^6 = 8 \times ar^3$$

$$r^3 = 8$$

$$r = 2$$

$$T_5 = ar^4 = 48$$

$$a \times (2)^4 = 48$$

$$a \times 16 = 48$$

First term **a = 3**

Ex. 7 What will be the sum of n terms of the series

8 + 88 + 888 + ?

(a) $\frac{8(10^{n+1} - 9n)}{81}$

(b) $\frac{8(10^{n+1} - 10 - 9n)}{81}$

(c) $8(10^{n-1} - 10)$

(d) $8(10^{n+1} - 10)$

Sol. (b) 8 + 88 + 888 + n
= 8 (1 + 11 + 111 + n)

$$= \frac{8}{9} [9 + 99 + 999 + \dots n]$$

$$= \frac{8}{9} [(10-1) + (100-1) + (1000-1) + \dots n]$$

$$= \frac{8}{9} [(10^1 + 10^2 + 10^3 + \dots 10^n) - (1 + 1 + 1 + \dots n)]$$

↓

It is a G.P

$$= \frac{8}{9} \left[10 \frac{(10^n - 1)}{10 - 1} - n \right]$$

$$= \frac{8}{9} \left[\frac{10^{n+1} - 10}{9} - n \right]$$

$$= \frac{8}{9} \left[\frac{10^{n+1} - 10 - 9n}{9} \right]$$

$$= \frac{8[10^{n+1} - 10 - 9n]}{81}$$

Ex. 8 (666n digit)² + (888n digit) is equal to

(a) $(10^n - 1) \frac{4}{9}$

(b) $(10^{2n} - 1) \frac{4}{9}$

(c) $\frac{4}{9} (10^n - 1)^{n-1} - 1$

(d) $\frac{4}{9} (10^n + 1)$

Sol. (b) $(666 \dots n \text{ digit})^2 + (888 \dots n \text{ digit})^2$

In this type question we take $n = 1$ then

$$(6)^2 + 8 = 44$$

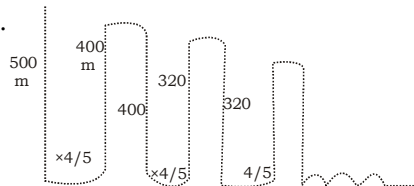
we take option (b) put the $n = 1$

$$(10^2 - 1) \frac{4}{9} = 99 \times \frac{4}{9} = 44$$

So, $(10^{2n} - 1) \frac{4}{9}$

Ex.9 A ball is thrown from a height of 500m on the ground. The ball bounce $4/5$ times of 1st every last bounce then calculate the total Distance the ball It stop.

Sol.



$$(500 + 400), (400 + 320), \dots$$

$$900, 720, \dots \infty$$

$$a = 900$$

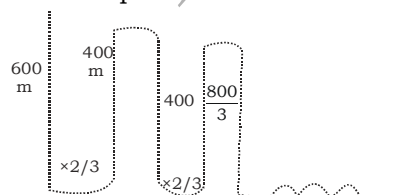
$$r = \frac{720}{900} = \frac{4}{5}$$

$$S_{\infty} = \frac{a}{1-r} = \frac{900}{1-\frac{4}{5}} = \frac{900}{1/5}$$

$$= 900 \times 5 = 4500 \text{ m}$$

Ex.10 A ball is thrown from a height of 600 m on the ground. the ball bounce $2/3$ times of Its every Last Bounce then calculate the Total Distance the ball till It stop.

Sol.



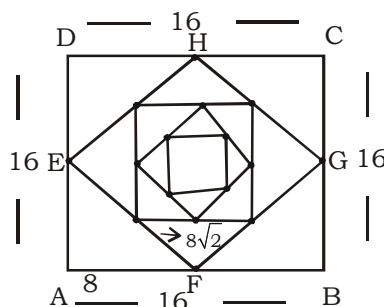
$$a = (600 + 400) = 1000$$

$$1000, 400 + \frac{800}{3}, \dots \quad r = \frac{2}{3}$$

$$S_{\infty} = \frac{1000}{1-\frac{2}{3}} = \frac{1000}{1/3} = 3000 \text{ m}$$

Ex.11 The side of a square is 16 cm Infinite squares are made by joining the mid points of the square. Calculate the area of all the squares made.

Sol.



Area of large square of ABCD $= (16)^2 = 256$

Square of made of mid-point (EFGH) $EF^2 = AE^2 + AF^2$

$$AE = \frac{16}{2} = 8 \text{ (E, midpoint of AD)}$$

$$AF = \frac{16}{2} = 8 \text{ (F, mid point of AB)}$$

$$EF = \sqrt{(8)^2 + (8)^2} = 8\sqrt{2}$$

$$\text{Area of EFGH} = (8\sqrt{2})^2 = 128$$

$$256 + 128 + \dots \infty$$

$$a = 256$$

$$r = 128/256 = 1/2$$

$$S_{\infty} = \frac{256}{1-\frac{1}{2}} = \frac{256}{1/2} = 512 \text{ cm}^2$$

* The figure made by the joining the mid-points of a square and the area of thus made square is half of the actual square

Ex.12 A number of Squares are described whose perimetres are in G.P. Then their sides will be In

(a) A.P (b) G.P

(c) H.P

(d) None of these

Sol. (b) Perimeter of square $= 4 \times \text{side}$

According to question

Let perimetres of square in G.P. 4, 8, 16, 32

Then sides of square

$$4a_1 = 4, \quad a_1 = 1$$

$$4a_2 = 8, \quad a_2 = 2$$

$$4a_3 = 16, \quad a_3 = 4$$

$$4a_4 = 32, \quad a_4 = 8$$

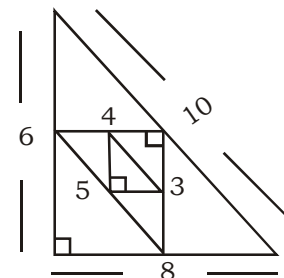
Where a_1, a_2, a_3, a_4 sides of square 1, 2, 4, 8

This is a G.P.

We can say that If perimeter of square In G.P. then their sides will be in G.P.

Ex.13 The sides of a right angle Δ are 6, 8 and 10 cm respectively. A new right angle Δ is made by joining the mid-points of all the sides. This process continues for infinite then calculate the area of all the Δ so made.

Sol.



Area of Large right angle tri-

$$\text{angle} = \frac{1}{2} \times 6 \times 8 = 24$$

$$\begin{array}{ccc} A_1 & A_2 & A_3 \\ \uparrow & \uparrow & \uparrow \\ 24 & 6 & 3/2 \dots \infty \end{array}$$

$A_1 = 24 \rightarrow$ Area of first right angle triangle

$A_2 = 6 \rightarrow$ Area of second right angle triangle

$$r = \frac{A_2}{A_1} = \frac{6}{24} = \frac{1}{4}$$

$$S_{\infty} = \frac{24}{1-\frac{1}{4}} = \frac{24}{3/4} = 24 \times \frac{4}{3} = 32 \text{ cm}^2$$

* The figure made by the joining the mid-points of a right angle triangle is right angle triangle and the area of thus made tri-

angle is $\frac{1}{4}$ of the actual triangle.

Ex.14 If A is the sum of the n terms of the series $1 + 1/4 + 1/16 + \dots$ and B is the sum of 2n terms

EXERCISE

1. Find t_{10} and s_{10} for the following series.
1, 8, 15, 22,
(a) 64,325 (c) 64,318
(c) 57,325 (d) 57,318
2. Find t_{20} and S_{20} for the following series
2, 8, 14, 20,
(a) 116,1172 (c) 110,1180
(c) 116,1180 (d) 110,1172
3. Find t_{24} and s_{24} for the following series.
3, 13, 23, 33,
(a) 233,2842 (b) 230,2832
(c) 230,2842 (d) 233,2832
4. Find t_{18} and s_{22} for the following series.
series - 3, 1, 5, 9,
(a) 65,858 (b) 60,850
(c) 60,868 (d) 65,850
5. Find t_{28} and s_{48} for the following series.
30, 33, 36, 39,
(a) 111,4834 (b) 111,4824
(c) 121,4824 (d) 121,4834
6. Find t_{30} and s_{30} for the following series.
If 36, 34, 32, 30,
(a) 22,210 (b) -22,210
(c) -22,220 (d) 22, -210
7. Find the t_{20} & S_{20} for the following series
2, 8, 32,
(a) $2^{39}, \frac{2}{3}(4^{20} - 1)$
(b) $2^{40}, \frac{4}{3}(4^{20} - 1)$
(c) $2^{38}, \frac{2}{3}(4^{21} - 1)$
(d) $2^{41}, \frac{2}{3}(4^{20} - 1)$
8. Find t_7 and s_7 for the following series.
1, 3, 9, 27,
(a) $729, \frac{1}{2}(3^7 - 1)$
(b) $243, \frac{1}{2}(3^8 - 1)$
(c) $729, \frac{1}{2}(3^7 + 1)$
(d) $243, \frac{1}{3}(3^8 + 1)$
9. Find t_{24} and s_{24} for the following series.
 $18, 9, \frac{9}{2}, \dots$
(a) $18\left(\frac{1}{2}\right)^{23}, 36\left(1 - \frac{1}{2^{22}}\right)$
(b) $18\left(\frac{1}{2}\right)^{22}, 36\left(1 - \frac{1}{2^{24}}\right)$
(c) $18\left(\frac{1}{2}\right)^{24}, 36\left(1 - \frac{1}{2^{24}}\right)$
(d) $18\left(\frac{1}{2}\right)^{23}, 36\left(1 - \frac{1}{2^{24}}\right)$
10. Find t_{30} and s_{30} for the following series.
64, 16, 4, 1,
(a) $\frac{1}{4^{26}}, \frac{64\left(1 - \left(\frac{1}{4}\right)^{30}\right)}{1 - \frac{1}{4}}$
(b) $\frac{1}{4^{26}}, \frac{64\left(1 - \left(\frac{1}{4}\right)^{30}\right)}{1 + \frac{1}{4}}$
(c) $\frac{1}{4^{26}}, \frac{64\left(1 + \left(\frac{1}{4}\right)^{30}\right)}{1 - \frac{1}{4}}$
(d) $\frac{1}{4^{26}}, \frac{64\left(1 + \left(\frac{1}{4}\right)^{30}\right)}{1 + \frac{1}{4}}$
11. Find the sum of all numbers divisible by 6 in 100 to 400
(a) 12450 (b) 12550
(c) 12400 (d) 12456
12. How many natural Numbers between 300 to 500 are multiple of 7?
(a) 29 (b) 28 (c) 27 (d) 30
13. Find the value of the Expression $1 - 6 + 2 - 7 + 3 - 8 + \dots$ to 100 terms
(a) -250 (b) -500
(c) -450 (d) -300
14. How many terms of the series -12, -9, -6, must be Taken that the sum may be 54?
(a) 15 (b) 14 (c) 18 (d) 12
15. How many terms are there In the A.P 20, 25, 30,130?
(a) 22 (b) 23 (c) 21 (d) 24
16. Find the 1st terms an A.P. whose 8th and 12th terms are respectively 39 & 59
(a) 5 (b) 6 (c) 4 (d) 3
17. There is an AP 1, 3, 5, which term of this AP is 55.?
(a) 27th (b) 26th (c) 25th (d) 28th
18. Find the 15th term of the sequence 20, 15, 10,
(a) -45 (b) -50 (c) -55 (d) 0
19. A number 15 is divided In three parts which are In A.P and The sum of their squares is 83. Find the Smallest No.
(a) 5 (b) 3 (c) 6 (d) 8
20. The sum of the first 16 terms of an A.P. whose first terms and third term are 5 and 15 respectively is
(a) 600 (b) 765 (c) 640 (d) 680
21. The Number of terms of the series $54 + 51 + 48 + \dots$ such that the sum is 513 is
(a) 18 (b) 19
(c) Both a and b (d) 15
22. A man receives ₹ 60 for the first week and ₹ 3 more each week than the preceding week. How much does he earn by the 20th week?
(a) ₹ 1770 (b) ₹ 1620
(c) ₹ 1890 (d) ₹ 1790
23. How many terms are there In the G.P 5, 20, 80, 320, 20480?
(a) 5 (b) 6 (c) 7 (d) 8
24. A boy agrees to work at the rate of one rupee on the first day, two rupees on the second day, four rupees on the third day and does on. How much will the boy get if he start working on the 1st of February and finishes on the 20th of February?
(a) 2^{20} (b) $2^{20} - 1$
(c) $2^{19} - 1$ (d) 2^{19}
25. If the Fifth term of a G.P. is 81 and first term is 16, what will be the 4th terms of the G.P?
(a) 36 (b) 18 (c) 54 (d) 24

26. The 4th and 10th term of a GP are $\frac{1}{3}$ and 243 respectively. Find the 2nd term.
(a) 3 (b) 1
(c) $\frac{1}{27}$ (d) $\frac{1}{9}$
27. The 7th and 21th term of an AP are 6 and -22 respectively. Find the 26th term.
(a) -34 (b) -32 (c) -12 (d) -10
28. The sum of 5 number in AP is 30 and the sum of their squares is 220. Which of the following is the third term?
(a) 5 (b) 6 (c) 8 (d) 10
29. The sum of the first four terms of an AP is 28 and sum of the first eight terms of the same AP is 88. Find the sum of the first 16 terms of the AP?
(a) 346 (b) 340 (c) 304 (d) 268
30. Find the number of terms of the series $\frac{1}{81}, \frac{-1}{27}, \frac{1}{9}, \dots, 729$
(a) 11 (b) 12 (c) 10 (d) 13
31. A man saves ₹ 100 in January 2014 and increases his saving by ₹ 50 every month over the previous month. What is the annual saving for the man in the year 2014 ?
(a) ₹ 4200 (b) ₹ 4500
(c) ₹ 4000 (d) ₹ 4100
32. What is the maximum sum of the terms in the Arithmetic progression
 $25, 24\frac{1}{2}, 24, \dots, 1, \frac{1}{2}$
(a) $637\frac{1}{2}$ (b) 625
(c) $662\frac{1}{2}$ (d) 650
33. An equilateral triangle is drawn by joining the midpoints of the sides of another equilateral triangle. A third equilateral triangle is drawn inside the second one joining the midpoints of the sides of the second equilateral triangle, and the process continues infinitely. Find the sum of the perimeters of all the equilateral triangles, if side of the largest equilateral triangle is 24 units.
(a) 288 units (b) 72 units
(c) 36 units (d) 144 units
34. Find the value of the expression $1 - 6 + 2 - 7 + 3 - 8 + \dots$ to 100 terms
(a) -250 (b) -500
(c) -450 (d) -300
35. Find the sum of the Integers between 1 and 200 that are multiples of 7 ?
(a) 2742 (b) 2842
(c) 2642 (d) 2546
36. After striking a floor a rubber ball rebounds $(\frac{7}{8})^{\text{th}}$ of the height from which it has fallen. Find the total distance that it travels before coming to rest, if it is gently dropped from a height of 420 meters.
(a) 2940 (b) 6300
(c) 1080 (d) 3360
37. Jack and Jill were playing mathematical puzzles with each other. Jill drew a square of sides 8 cm and then kept on drawing squares inside the squares by joining the mid points of the squares. She continued this process indefinitely. Jill asked jack to determine the sum of the areas of all the squares that she drew. If Jack answered correctly then what would be his answer ?
(a) 128 (b) 64 (c) 256 (d) 32
38. If the m^{th} term of an AP is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$ then find the sum to mn^{th} term
(a) $\frac{1}{4}(mn-1)$ (b) $\frac{1}{4}(mn+1)$
(c) $\frac{1}{2}(mn+1)$ (d) $\frac{1}{2}(mn-1)$
39. The first and the Last terms of an A.P. are 107 and 253. If there are five terms in this sequence, find the sum of sequence
(a) 1080 (b) 720
(c) 900 (d) 620
40. The sum of an infinite GP whose common ratio is numerically less than 1 is 32 and the sum of the first two terms is 24. What will be the third term ?
(a) 2 (b) 16 (c) 8 (d) 4
41. What will be the value of $n^{1/2} \cdot n^{1/4} \cdot n^{1/8} \cdot \dots \infty$
(a) n^2 (b) n (c) $n^{3/2}$ (d) n^3
42. Determine the first term of the geometric progeression, the sum of whose first term and third term is 40 and the sum of the second term and fourth term is 80.
(a) 12 (b) 8 (c) 16 (d) 4
43. It is G.P 32, 4, 8, n and 2 and G.M is 8, find the value of n
(a) 2 (b) 4 (c) 8 (d) 16
44. Find the Arithmetic mean of the following series
20, 23, 26, 29
(a) 49 (b) $\frac{59}{2}$ (c) $\frac{49}{2}$ (d) 59
45. If the arithmetic mean of the number $x_1, x_2, x_3, \dots, x_n$ is $\bar{x}$, then the arithmetic mean of the number $ax_1 + b, ax_2 + b, ax_3 + b, \dots, ax_n + b$, where a and b are two constants, would be ?
(a) $\bar{x}$ (b) $n\bar{x} + nb$
(c) $a\bar{x}$ (d) $a\bar{x} + b$
46. A boy draws n squares with sides 1, 2, 3, 4, 5, in inches. The average area covered by these n squares will be :
(a) $\left(\frac{n+1}{2}\right)$
(b) $\left(\frac{n+1}{2}\right)\left(\frac{2n+1}{3}\right)$
(c) $\left(\frac{n+1}{2}\right)\left(\frac{2n+1}{3}\right)^{-1}$
(d) $\left(\frac{n+1}{2}\right)^{-1}\left(\frac{2n+1}{3}\right)$

ANSWER KEY

1. (a)	6. (b)	11. (a)	16. (c)	21. (c)	26. (c)	31. (b)	36. (b)	41. (b)	46. (b)
2. (c)	7. (a)	12. (a)	17. (d)	22. (a)	27. (b)	32. (a)	37. (a)	42. (b)	
3. (d)	8. (a)	13. (a)	18. (b)	23. (c)	28. (d)	33. (d)	38. (c)	43. (d)	
4. (a)	9. (d)	14. (d)	19. (b)	24. (b)	29. (c)	34. (a)	39. (c)	44. (c)	
5. (b)	10. (a)	15. (b)	20. (d)	25. (c)	30. (a)	35. (b)	40. (d)	45. (d)	

1. (a) 1, 8, 15, 22,

This is A.P series

$$a = 1$$

$$d = (8 - 1) = 7$$

$$T_{10} = 1 + (10 - 1) 7 = 1 + 63 = \mathbf{64}$$

$$S_{10} = \frac{10}{2} [2 \times 1 + 9 \times 7]$$

$$= 5 [2 + 63] = 5 \times 65 = \mathbf{325}$$

2. (c) 2, 8, 14, 20

$$a = 2$$

$$d = 6$$

$$T_{20} = 2 + 19 \times 6 = \mathbf{116}$$

$$S_{20} = 10 [2 \times 2 + 19 \times 6]$$

$$= 10 [4 + 114] = \mathbf{1180}$$

3. (d) 3, 13, 23, 33

$$a = 3$$

$$d = 10$$

$$T_{24} = 3 + 23 \times 10 = \mathbf{233}$$

$$S_{24} = 12 [2 \times 3 + 23 \times 10]$$

$$= 12 [6 + 230]$$

$$= 12 \times 236 = \mathbf{2832}$$

4. (a) -3, 1, 5, 9,

$$a = -3$$

$$d = 1 - (-3) = 1 + 3 = 4$$

$$T_{18} = -3 + 17 \times 4 = \mathbf{65}$$

$$S_{22} = 11 [2 \times -3 + 21 \times 4]$$

$$= 11 [-6 + 84] = 78 \times 11 = \mathbf{858}$$

5. (b) 30, 33, 36, 39

$$a = 30$$

$$d = 3$$

$$T_{28} = 30 + 27 \times 3 = \mathbf{111}$$

$$S_{48} = 24 [2 \times 30 + 47 \times 3]$$

$$= 24 [60 + 141] = \mathbf{4824}$$

6. (b) 36, 34, 32, 30,

$$a = 36$$

$$d = (34 - 36) = -2$$

$$T_{30} = 36 + 29 \times -2$$

$$= 36 - 58 = \mathbf{-22}$$

$$S_{30} = 15 [2 \times 36 + 29 \times -2]$$

$$= 15 [72 - 58]$$

$$= 15 [14] = \mathbf{210}$$

7. (a) 2, 8, 32,

$$a = 2$$

$$r = \frac{8}{2} = 4$$

$$T_{20} = ar^{n-1} = 2(4)^{20-1} = 2 \times (4)^{19}$$

$$= 2 \times 2^{38} = \mathbf{2^{39}}$$

$$S_{20} = \frac{a(r^n - 1)}{r - 1} \quad (r > 1)$$

$$= \frac{2(4^{20} - 1)}{4 - 1} = \frac{2}{3}(4^{20} - 1)$$

8. (a) 1, 3, 9, 27

$$a = 1$$

$$r = \frac{3}{1} = 3$$

$$T_7 = 1(3)^{7-1} = 3^6 = \mathbf{729}$$

$$S_7 = \frac{1(3^7 - 1)}{3 - 1} = \frac{1}{2}(3^7 - 1)$$

9. (d) 18, 9, $\frac{9}{2}$,

$$a = 18$$

$$r = \frac{9}{18} = \frac{1}{2}$$

$$T_{24} = 18 \left(\frac{1}{2} \right)^{24-1} = 18 \left(\frac{1}{2} \right)^{23}$$

$$S_{24} = a \left(\frac{1 - r^n}{1 - r} \right) \quad r < 1$$

$$= \frac{18 \left[1 - \left(\frac{1}{2} \right)^{24} \right]}{1 - \frac{1}{2}}$$

$$= \frac{18 \left[1 - \left(\frac{1}{2} \right)^{24} \right]}{\frac{1}{2}} = \mathbf{36 \left[1 - \frac{1}{2^{24}} \right]}$$

10. (a) 64, 16, 4, 1

$$a = 64$$

$$d = \frac{16}{64} = \frac{1}{4}$$

$$T_{30} = 64 \left(\frac{1}{4} \right)^{30-1} = 64 \left(\frac{1}{4} \right)^{29}$$

$$= 4^3 \times \frac{1}{4^{29}} = \frac{1}{4^{26}}$$

$$S_{30} = \frac{64 \left(1 - \left(\frac{1}{4} \right)^{30} \right)}{1 - \frac{1}{4}} \quad r < 1$$

$$= \frac{64 \left(1 - \left(\frac{1}{4} \right)^{30} \right)}{1 - \frac{1}{4}}$$

11. (a) 1st Term = $a = 102$ (where is the 1st term greater than 100 that is divisible by 6)

Last term less than 400, which is divisible by 6 is 396.

then,

$$102 + 108 + 114 + \dots + 396$$

$$a = 102, d = 6$$

No. of term

$$= \frac{\text{Last term} - \text{First term}}{\text{difference}} + 1$$

$$= \frac{396 - 102}{6} + 1$$

$$= \frac{294}{6} + 1 = 49 + 1 = 50$$

$$S_{50} = \frac{50}{2} [2 \times 102 + 49 \times 6]$$

$$= 25 [204 + 294] = \mathbf{12450}$$

12. (a) First multiple of 7 term (300 to 500) = 301

Last multiple of 7 term (300 to 500) = 497

$$301, 308, \dots, 497$$

$$\text{No. of term} = \frac{497 - 301}{7} + 1$$

$$= \frac{196}{7} + 1 = 28 + 1 = 29$$

13. (a) 1 - 6 + 2 - 7 + 3 - 8 + to 100

(1 + 2 + 3 to 50 term)

[Where $a = 1, d = 1, n = 50$]

- (6 + 7 + 8 ... to 50 term)

[Where $a = 6, d = 1, n = 50$]

Both series are In A.P.

Use the formula for sum of an A.P.

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\begin{aligned}
 &= 25(2 \times 1 + 24 \times 1) - 25(2 \times 6 + 24 \times 1) \\
 &= 25(2 + 24) - 25(12 + 24) \\
 &= 25 \times 26 - 25 \times 36 \\
 &= 25(26 - 36) = 25 \times -10 \\
 &= -250
 \end{aligned}$$

14. (d) -12, -9, -6,

$$S_n = 54$$

$$a = -12$$

$$d = 3$$

$$n = ?$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$54 = \frac{n}{2} [2 \times -12 + (n-1)3]$$

$$108 = n [-24 + 3n - 3]$$

$$108 = n [3n - 27]$$

$$36 = n [n - 9]$$

Take option (d) $n = 12$, satisfy

$$= 12 \times 3 = 36$$

$$\text{So : } n = 12$$

15. (b) 20, 25, 30,130

$$a = 20$$

$$d = 5$$

$$\text{last term} = l = 130$$

$$l = a + (n-1)d$$

$$130 = 20 + (n-1)5$$

$$110 = (n-1)5$$

$$22 = n - 1$$

$$n = 23$$

16. (c) $T_8 = 39$

$$T_{12} = 59$$

$$T_8 = a + 7d = 39 \quad \dots (i)$$

$$T_{12} = a + 11d = 59 \quad \dots (ii)$$

$$(ii) - (i)$$

$$4d = 20$$

$$d = 5$$

Put the value of d (i) equation

$$a + 7 \times 5 = 39$$

$$a = 4$$

17. (d) 1, 3, 5,

$$a = 1$$

$$d = 2$$

$$n^{\text{th}} \text{ of AP} = 55$$

$$55 = 1 + (n-1)2$$

$$54 = (n-1)2$$

$$27 = n - 1$$

$$n = 28^{\text{th}}$$

18. (b) 20, 15, 10,

$$a = 20$$

$$d = -5$$

$$T_{15} = 20 + 14 \times -5$$

$$T_{15} = 20 - 70$$

$$T_{15} = -50$$

19. (b) Let three term In A.P.

$$a - d, a, a + d$$

$$\text{A.T.Q}$$

$$a - d + a + a + d = 15$$

$$3a = 15$$

$$a = 5$$

and

$$(a-d)^2 + (a)^2 + (a+d)^2 = 83$$

$$a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 83$$

$$3a^2 + 2d^2 = 83$$

$$\text{Put the value } a = 5$$

$$3 \times (5)^2 + 2d^2 = 83$$

$$75 + 2d^2 = 83$$

$$2d^2 = 8$$

$$d = 2$$

$$\text{Then } a - d = 5 - 2 = 3$$

$$a = 5$$

$$a + d = 5 + 2 = 7$$

$$\text{Smallest No.} = 3$$

20. (d) $a_1 = 5 = a \rightarrow (i)$

$$a_3 = 15 = a + 2d \rightarrow (ii)$$

$$(ii) - (i)$$

$$2d = 10$$

$$d = 5$$

$$5, 10, 15, \dots$$

$$a = 5$$

$$d = 5$$

$$n = 16$$

$$S_{16} = \frac{16}{2} [2 \times 5 + 15 \times 5] = 8[10 + 75]$$

$$= 680$$

21. (c) 54 + 51 + 48 +

$$a = 54, \quad d = -3$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$513 = \frac{n}{2} [2 \times 54 + (n-1)(-3)]$$

$$513 = \frac{n}{2} [108 - 3n + 3]$$

$$1026 = n(111 - 3n)$$

$$342 = n(37 - n)$$

This is a quadratic equation n has two values.

In this condition we help option.

Option Both $n = 18$, And $n = 19$,

Satisfy this equation.

Then (c) option.

22. (a) First week = $a = 60$

$$\text{second week} = 63$$

Therefore

$$60, 63, 66, \dots, 20^{\text{th}} \text{ week}$$

$$a = 60$$

$$d = 3$$

$$n = 20$$

$$S_{20} = 10 [2 \times 60 + 19 \times 3]$$

$$= 10 [120 + 57] = 1770$$

23. (c) 5, 20, 80, 320 20480

$$a = 5$$

$$r = \frac{20}{5} = 4$$

$$T_n = 20480$$

$$T_n = ar^{n-1}$$

$$20480 = 5(4)^{n-1}$$

$$4096 = (4)^{n-1}$$

$$2^{12} = 2^{2(n-1)}$$

$$2^{12} = 2^{2n-2}$$

same base comparison the power.

$$12 = 2n - 2$$

$$n = 7$$

24. (b) First day = 1

$$\text{IInd} = 2$$

$$\text{IIIrd} = 4$$

Boy does the work = 20 day

then 1, 2, 4, 8,

$$a = 1$$

$$r = 2$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad r > 1$$

$$= \frac{1(2^{20} - 1)}{2 - 1} = 2^{20} - 1$$

25. (c) $T_5 = ar^{5-1} = ar^4 = 81$

$$\therefore a = 16(\text{given})$$

$$16 \times r^4 = 81$$

$$r^4 = \frac{81}{16}$$

$$r = \frac{3}{2}$$

$$T_4 = ar^{4-1} = ar^3$$

$$= 16 \times \left(\frac{3}{2}\right)^3 = 16 \times \frac{27}{8}$$

$$T_4 = 54$$

26. (c) $T_4 = ar^3 = \frac{1}{3} \dots (i)$

$$T_{10} = ar^9 = 243 \dots (ii)$$

$$(ii), (i)$$

$$\frac{ar^9}{ar^3} = \frac{243}{1/3}$$

$$r^6 = 243 \times 3 = 729$$

$$r = 3$$

Put the value r First equation

$$a \times (3)^3 = \frac{1}{3}$$

$$a \times 27 = \frac{1}{3}$$

$$a = \frac{1}{81}$$

$$T_2 = ar = \frac{1}{81} \times 3 = \frac{1}{27}$$

$$27. (b) T_7 = a + 6d = 6 \dots\dots(i)$$

$$T_{21} = a + 20d = -22 \dots\dots(ii)$$

$$(ii) - (i)$$

$$(a + 20d) - (a + 6d) = -22 - 6$$

$$14d = -28$$

$$d = -2$$

Put the value d (i) equation

$$a + 6 \times -2 = 6$$

$$a - 12 = 6$$

$$a = 18$$

$$T_{26} = a + 25d = 18 + 25 \times -2$$

$$= 18 - 50 = -32$$

$$28. (d) \text{ Let the 5 number of A.P}$$

$$a - 2d, a - d, a, a + d, a + 2d$$

$$a - 2d + a - d + a + a + d + a + 2d$$

$$= 30$$

$$5a = 30$$

$$a = 6$$

$$(a - 2d)^2 + (a - d)^2 + a^2 + (a + d)^2 + (a + 2d)^2$$

$$= 220$$

$$a^2 + 4d^2 - 4ad + a^2 + d^2 - 2ad +$$

$$a^2 + a^2 + d^2 + 2ad + a^2 + 4d^2 +$$

$$4ad = 220$$

$$5a^2 + 10d^2 = 220$$

$$a^2 + 2d^2 = 44$$

$$\text{Put the value } a = 6$$

$$(6)^2 + 2d^2 = 44$$

$$36 + 2d^2 = 44$$

$$2d^2 = 8$$

$$d^2 = 4, d = 2$$

$$T_3 = a = 6$$

$$29. (c) S_4 = 28$$

$$S_8 = 88$$

$$S_4 = 2 [2a + 3d] = 28$$

$$S_8 = 4 [2a + 7d] = 88$$

$$2a + 3d = 14 \dots\dots(i)$$

$$2a + 7d = 22 \dots\dots(ii)$$

$$(ii) - (i)$$

$$4d = 8$$

$$d = 2$$

Put the value of d, equation (i)

$$2a + 3 \times 2 = 14$$

$$2a = 8$$

$$a = 4$$

$$S_{16} = 8 [2 \times 4 + 15 \times 2]$$

$$= 8 [8 + 30] = 8 \times 38 = 304$$

$$30. (a) a = \frac{1}{81}$$

$$d = \frac{-1/27}{1/81} = -3$$

$$T_n = ar^{n-1}$$

$$729 = \frac{1}{81} (-3)^{n-1}$$

$$729 \times 81 = (-3)^{n-1}$$

$$3^6 \times 3^4 = (-3)^{n-1}$$

$$3^{10} = (-3)^{n-1}$$

comparison

$$10 = n - 1$$

$$n = 11$$

$$31. (b) \text{ January Saves} = 100$$

$$\text{Febury} = 100 + 50 = 150$$

$$100 + 150 + \dots\dots\dots$$

Sum of 12 months saving

$$S_{12} = 6[2 \times 100 + 11 \times 50]$$

$$= 6 [200 + 550]$$

$$= 6 \times 750 = ₹ 4500$$

32. (a) The maximum sum of the Terms in the A.P. when all terms will be positive then.

A.P is

$$25, 24\frac{1}{2}, 24, 23\frac{1}{2}, \dots\dots\dots 1, \frac{1}{2}$$

$$\text{No. of term} = \frac{l - a}{d} + 1$$

$$\frac{1}{2} - 25$$

$$= \frac{2}{-1/2} + 1 = 49 + 1 = 50$$

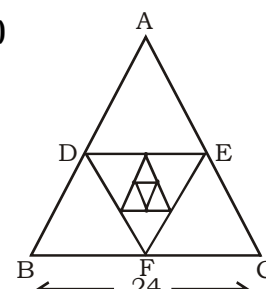
Sum of series

$$S_{50} = \frac{50}{2} [2 \times 25 + 49 \times -\frac{1}{2}]$$

$$= 25 [50 - 24.5] = 25 \times \frac{51}{2}$$

$$= 637\frac{1}{2}$$

33. (d)



$$\text{Perimeter of } \triangle ABC$$

$$= 3 \times \text{side} = 3 \times 24 = 72$$

$\triangle DEF$

$$\text{Perimeter } \triangle DEF = 3 \times 12 = 36$$

DE || BC, D & E mid point of AB & AC

$$\text{Then } DE = \frac{1}{2} BC$$

$$\text{Then } DF = \frac{1}{2} AC$$

$$EF = \frac{1}{2} AB$$

Therefore,

$$72, 36, 18, \dots\dots\dots \infty$$

$$a = 72, r = \frac{1}{2}$$

$$S_{\infty} = \frac{72}{1 - \frac{1}{2}} = 72 \times 2 = 144$$

$$34. (a) 1 - 6 + 2 - 7 + 3 - 8 + \dots \text{ to } 100 \text{ terms}$$

The above series is a combination of Two APs.

$$\Rightarrow (1 + 2 + 3 + \dots + 50 \text{ terms})$$

$$- (6 + 7 + 8 \dots\dots + 50 \text{ terms})$$

$$\Rightarrow 25 [2 \times 1 + 49 \times 1] - [25 (2 \times 6 + 49 \times 1)]$$

$$\Rightarrow 25 [51] - 25 \times 61$$

$$\Rightarrow 25 [51 - 61] = 25 \times -10 = -250$$

$$35. (b) \text{ Multiple of 7 from 1 to 200}$$

$$7, 14, \dots\dots\dots 196$$

$$\text{No. of Term} = \frac{l - a}{d} + 1$$

$$l = 196$$

$$a = 7$$

$$d = 7$$

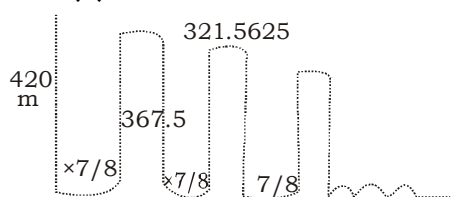
$$\text{No. of term} = \frac{196 - 7}{7} + 1$$

$$\text{No. of term} = \frac{189}{7} + 1 = 27 + 1 = 28$$

$$S_{28} = \frac{28}{2} [2 \times 7 + 27 \times 7]$$

$$= 14 [14 + 189] = \mathbf{2842}$$

36. (b)



$$(420+367.5), (367.5+321.5625), \dots$$

$$0$$

$$787.5, 689.0625, \dots$$

It is Infinite G.P

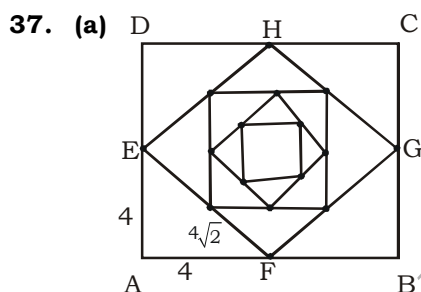
$$S_{\infty} = \frac{a}{1-r}$$

$$a = 787.5$$

$$r = \frac{7}{8}$$

$$S_{\infty} = \frac{787.5}{1 - \frac{7}{8}} = \frac{787.5}{1/8}$$

$$= 797.5 \times 8 = \mathbf{6300m.}$$



$$\text{Area of } ABCD = (8)^2 = 64$$

$$\text{Area of square of } EFGH$$

$$= (4\sqrt{2})^2 = 32$$

Same

$$64, 32, 16, \dots, \infty$$

It is a G.P.

$$S_{\infty} = \frac{a}{1-r}$$

$$\therefore a = 64, r = \frac{1}{2}$$

$$= \frac{64}{1 - \frac{1}{2}}$$

$$= \mathbf{128}$$

$$38. (c) T_m = a + (m-1)d = \frac{1}{n} \dots (i)$$

$$T_n = a + (n-1)d = \frac{1}{m} \dots (ii)$$

$$(i) - (ii)$$

$$(m-1)d - (n-1)d = \frac{1}{n} - \frac{1}{m}$$

$$d[m-1-n+1] = \frac{m-n}{mn}$$

$$d \times (m-n) = \frac{m-n}{mn}$$

$$d = \frac{1}{mn}$$

Put the value of 'd' equation (i)

$$a + (m-1) \times \frac{1}{mn} = \frac{1}{n}$$

$$a + m \times \frac{1}{mn} - \frac{1}{mn} = \frac{1}{n}$$

$$a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n}$$

$$a = \frac{1}{mn}$$

$$S_{mn} = \frac{mn}{2} [2a + (mn-1)d]$$

$$= \frac{mn}{2} \left[2 \times \frac{1}{mn} + (mn-1) \times \frac{1}{mn} \right]$$

$$= \frac{mn}{2} \times \frac{1}{mn} [2 + (mn-1)]$$

$$= \frac{1}{2} (2 + mn - 1)$$

$$= \frac{1}{2} (mn + 1)$$

39. (c) Let the 4 terms of A.P.

$$a - 2d, a - d, a, a + d, a + 2d$$

Sum of IInd and IVth term

$$= a - d + a + d$$

$$= \mathbf{2a}$$

A.T.Q.

$$I^{\text{st}} + V^{\text{th}} = 107 + 253 = 2a$$

$$a = 180$$

Sum of all 5 terms

$$2a + 2a + a = 5a = 5 \times 180$$

$$= \mathbf{900}$$

$$40. (d) S_{\infty} = \frac{a}{1-r} = 32$$

$$a = 32(1-r) \dots (i)$$

Let 2 Terms of G.P a, ar

$$a + ar = 24$$

$$a(1+r) = 24$$

$$a = \frac{24}{1+r}$$

$$\dots (ii)$$

put the value of a

$$\frac{24}{1+r} = 32(1-r)$$

$$24 = 32(1-r)(1+r)$$

$$\frac{3}{4} = (1-r^2)$$

$$r^2 = 1 - \frac{3}{4} = \frac{1}{4}$$

$$r = \frac{1}{2}$$

A.T.Q.

$$a + ar = 24$$

$$a(1+r) = 24$$

$$a \left(1 + \frac{1}{2} \right) = 24$$

$$a \times \frac{3}{2} = 24$$

$$a = 16$$

IIIrd Term of G.P = ar^2

$$= 16 \times \left(\frac{1}{2} \right)^2$$

$$= 16 \times \frac{1}{4} = \mathbf{4}$$

$$41. (b) n^{1/2}, n^{1/4}, n^{1/8}, \dots, \infty$$

$$n^{\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots, \infty \right)}$$

$$a = \frac{1}{2}, r = \frac{1}{4} = \frac{1}{2}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$S_{\infty} = \left(\frac{\frac{1}{2}}{1 - \frac{1}{2}} \right)$$

$$S_{\infty} = 1$$

$$n^{\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots, \infty \right)} = n^1 = n$$

42. (b) Let the 4 terms of G.P

a, ar, ar², ar³

A.T.Q.

$$a + ar^2 = 40, a(1 + r^2) = 40 \quad \dots(i)$$

$$ar + ar^3 = 80, ar(1 + r^2) = 80 \quad \dots(ii)$$

(i) / (ii)

$$\frac{a(1+r^2)}{ar(1+r^2)} = \frac{40}{80}$$

$$\frac{1}{r} = \frac{1}{2}$$

$$r = 2$$

Put the value r in equation (ii)

$$a + ar^2 = 40$$

$$a(1 + r^2) = 40$$

$$a(1 + 2^2) = 40$$

$$a(5) = 40$$

$$a = 8$$

43. (d) G.M = $(a_1 \times a_2 \times a_3 \times a_4 \times a_5)^{1/5}$

$$8 = (32 \times 4 \times 8 \times n \times 2)^{1/5}$$

$$8 = (2^5 \times 2^2 \times 2^3 \times n \times 2)^{1/5}$$

$$8 = (2^{10} \times 2 \times n)^{1/5}$$

$$8 = 2^{10 \times \frac{1}{5}} \times 2^{1/5} \times n^{1/5}$$

$$8 = 2^2 \times 2^{1/5} \times n^{1/5}$$

$$(2n)^{1/5} = 2$$

$$2n = 32$$

$$n = 16$$

$$\mathbf{44. (c)} \text{ A.M} = \frac{(a_1 + a_2 + a_3 + a_4)}{4}$$

$$20, 23, 26, 29,$$

$$\text{A.M} = \frac{(20 + 23 + 26 + 29)}{4}$$

$$= \frac{98}{4} = \frac{49}{2}$$

45. (d) AM of $ax_1 + b, ax_2 + b, ax_3 + b, \dots, ax_n + b$ is

$$= \frac{(ax_1 + b) + (ax_2 + b) + (ax_3 + b) + \dots + (ax_n + b)}{n}$$

$$= \frac{(ax_1 + ax_2 + ax_3 + \dots + ax_n) + (b + b + \dots + b)}{n}$$



$$= \frac{a(x_1 + x_2 + \dots + x_n) + nb}{n}$$

$$= \frac{a(x_1 + x_2 + x_3 + \dots + x_n)}{n} + \frac{nb}{n}$$

$$= a\bar{x} + b$$

46. (b) Average area

$$= \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n}$$

$$= \frac{\left(\frac{n(n+1)(2n+1)}{6} \right)}{n}$$

$$= \frac{(n+1)(2n+1)}{6}$$

$$= \frac{(n+1)}{2} \cdot \frac{(2n+1)}{3}$$

Hence, (b) is the correct option.

POWER, INDICES AND SURDS

- Before we proceed to exponents (Indices) and surds, it is proper to learn about Real numbers.

Number System

Natural Numbers: These are the numbers (1, 2, 3,etc) that are used for counting. In other words, all positive integers are natural numbers. The least natural number is 1 but there is no largest natural number.

The set of natural number is denoted by N.

Thus, $N = \{1, 2, 3, \dots\}$

- **Whole Numbers :** The set of numbers that includes all natural numbers and the number zero are called whole numbers. The set of whole numbers is denoted by W.

Thus, $W = \{0, 1, 2, 3, \dots\}$

Note : Whole numbers are also called as “Non-negative Integers”.

Integers : All the natural numbers, zero, and the negatives of natural numbers are called integers.

$I = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

- (i) Set of negative integers
= $\{-1, -2, -3, \dots\}$
- (ii) Set of non-negative integers
= $\{0, 1, 2, 3, \dots\}$
- (iii) Set of positive integers
= $\{1, 2, 3, \dots\}$
- (iv) Set of non-positive integers
= $\{0, 1, 2, 3, \dots\}$

Note: ‘0’ is definitely a non-negative integer as well as a non-positive integer.

- **Rational numbers :** The numbers which can be expressed in

the form of $\frac{p}{q}$, where p and q

are integers and $q \neq 0$ are called rational numbers and their set is denoted by Q.

Ex. $\frac{1}{4}, \frac{2}{5}, -\frac{3}{7}, 6$ (as $6 = \frac{6}{1}$) etc.

are rational numbers.

The set of rational numbers encloses the set of integers and fractions.

Representation of Rational Numbers as Decimals : The decimal form of a rational number is either terminating or non-terminating.

E.g. $\frac{17}{4} = 4.25, \frac{21}{5} = 4.2 \rightarrow$ terminating (or finite) decimal.

$\frac{16}{3} = 5.\bar{3}, \frac{2}{3} = 0.\bar{6} \rightarrow$ Non-terminating (or Recurring) decimal.

Note: If the denominator of a rational number has no prime factors other than 2 or 5, then and only then it is expressible as a terminating decimal.

- **Irrational numbers :** The numbers which when expressed in decimal form are neither terminating nor repeating decimals are called “Irrational numbers”.

e.g. $\sqrt{2}, \sqrt{3}, \sqrt{50}, \sqrt{7}, \pi$ etc

Note: The exact value of π is not $\frac{22}{7}$,

as $\frac{22}{7}$ is rational while π is irrational.

nal. $\frac{22}{7}$ is the approximate value of π . Similarly 3.14 is not an exact value of π .

- **Real numbers :** All rational and irrational numbers together form the set of real numbers, denoted by R.

Thus, every natural number, every whole number, every integer, every rational number and every irrational number is a real number.

Note:-

- (i) The sum (or difference) of a rational and an irrational number is irrational.

E.g. $(4+\sqrt{3}), (2-\sqrt{5}), (\frac{3}{2}-\sqrt{2}), 7+\pi$ etc.

are all irrational.

- (ii) The product of a rational and an irrational number is irrational, e.g. $4\sqrt{3}, -2\sqrt{5}$ etc. are all irrational.

- **Even and Odd numbers :** Integers divisible by 2 are called even numbers, while those which are not divisible by 2 are known as odd integers. Thus, $\dots, -6, -4, -2, 0, 2, 4, 6, \dots$ etc are even integers. And $-5, -3, -1, 1, 3, 5, \dots$ etc are odd integers.

- **Prime numbers:-** A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and itself.

E.g.: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, etc.

$\rightarrow$ 2 is the only even number which is prime.

- **Composite numbers:-** Composite numbers are the numbers greater than 1 which are not prime. e.g. 4, 6, 9, 14, 15, etc.

Note: 1 is neither prime nor composite. There are 25 prime numbers between numbers 1 & 100.

Test for Prime Numbers : Let x be a given number and let k be an integer very near to $\sqrt{x}$ s.t. $k > \sqrt{x}$.

If x is not divisible by any prime number less than k , then x is prime, otherwise, it is not prime.

E.g.: Check whether 571 is prime or not?

clearly, $24 > \sqrt{571}$

So, we divide 571 by each prime number less than 24 which are 2, 3, 5, 7, 11, 13, 17, 19 and 23 we find that 571 is not divisible by any of them. So, 571 is a prime number.

□ **Co-Prime Numbers** : Two numbers are co-prime, if their H.C.F (Highest common factor) is 1.

E.g. (2, 3), (3, 13), (5, 7) etc are co-prime numbers.

□ **Perfect Numbers** : If the sum of divisors of a number excluding N itself is equal to N , then N is called a perfect number.

E.g.: 6, 28, 496, 8128 etc.

For 6, divisors are 1, 2 and 3.

$$6 : 1 + 2 + 3 = 6$$

$$28 : 1 + 2 + 4 + 7 + 14 = 28$$

Note: The sum of the reciprocals of the divisors of a perfect number including that of its own is always equal to 2.

E.g. For 6, divisors are 1, 2 and 3.

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{6+3+2+1}{6} = \frac{12}{6} = 2$$

Type - I

Indices and Surds

⇒ Let n be a positive integer and a be real number, then:

$$a^n = \frac{a \times a \times a \times \dots \times a}{(n \text{ factors})}$$

a^n is called " n^{th} power of a " or " a raised to the power n " where, a is called the base and n is called index or exponent of the power a^n .

E.g. 3^2 = square of 3, 3^3 = cube of 3 etc.

⇒ **Laws of Indices:**

1. $a^m \times a^n = a^{m+n}$ where $a \neq 0$ and $(m, n) \in \mathbb{I}$

2. $a^m \times a^n \times a^p \dots = a^{m+n+p+\dots}$

$$3. \frac{a^m}{a^n} = \begin{cases} a^{m-n} & \text{if } m > n \\ \frac{1}{a^{n-m}} & \text{if } n > m \\ 1 & \text{if } m = n \end{cases}$$

$$4. (a^m)^n = a^{mn} = (a^n)^m$$

$$5. a^{m^n} = a^{m \times m \times \dots \times m} \text{ times } \neq (a^m)^n$$

$$6. (ab)^n = a^n b^n$$

$$7. \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$$

$$8. (-a)^n = \begin{cases} a^n, & \text{when } n \text{ is even} \\ -a^n, & \text{when } n \text{ is odd} \end{cases}$$

Remark:- These rules are also true when n is negative or fraction.

$$9. a^{-n} = a^{(-1)n} = (a^{-1})^n = \left(\frac{1}{a}\right)^n$$

$$= \frac{1}{a} \times \frac{1}{a} \times \frac{1}{a} \dots \dots \dots n \text{ times}$$

$$10. a^{p/q} = a^{1/q \times p} = (a^{1/q})^p \text{ is positive integer, } q \neq 0$$

$$= a^{1/q} \times a^{1/q} \times \dots \dots \dots p \text{ times}$$

⇒ If the index of a power is until (i.e. 1) then the value of the power is equal to its base, i.e.:

$$a^1 = a, 0^1 = 0$$

$$\bullet a^m = a^n \Rightarrow m = n \text{ when } a \neq 0, 1$$

$$\bullet a^m = b^m \Rightarrow a = b$$

Ex.1. Solve the following:

$$(i) (5)^3$$

$$\text{Sol. } 5^3 = 5 \times 5 \times 5 = 125$$

$$(ii) (-6)^4$$

$$\text{Sol. } (-6)^4 = (-6) \times (-6) \times (-6) \times (-6) = 1296$$

$$(iii) (-2)^5$$

$$\text{Sol. } (-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$$

Ex.2 Solve the following expression

$$(i) (32)^{-1/5}$$

$$\text{Sol. } (32)^{-1/5} = (2^5)^{-1/5} = 2^{5 \times -\frac{1}{5}} = 2^{-1} = \frac{1}{2}$$

$$(ii) \left(-\frac{1}{343}\right)^{-2/3}$$

$$\text{Sol. } \left(-\frac{1}{343}\right)^{-2/3} = \left(-\frac{1}{7^3}\right)^{-2/3} = (-7^{-3})^{-2/3} = (-7)^{-3 \times -2/3} = (-7)^2 = 49$$

$$(iii) 3^{-3} + (-3)^3$$

$$\text{Sol. } 3^{-3} + (-3)^3 = \frac{1}{3^3} + (-3)^3 = \frac{1}{27} - 27 = \frac{1-729}{27} = -\frac{728}{27}$$

Ex.3: Solve the following:

$$(2^2 + 2^3 + 2^{-2} + 2^{-3})$$

$$\text{Sol. } (2^2 + 2^3 + 2^{-2} + 2^{-3})$$

$$= \left(4 + 8 + \frac{1}{4} + \frac{1}{8}\right)$$

$$= 12 + \frac{3}{8} = \frac{99}{8}$$

Ex. 4 If $2^{2x-1} = \frac{1}{8^{(x-3)}}$, then $x = ?$

$$\text{Sol. } 2^{2x-1} = \frac{1}{8^{(x-3)}} \Rightarrow 2^{2x-1} = \frac{1}{2^{3(x-3)}}$$

$$\Rightarrow 2^{2x-1} = \frac{1}{2^{3x-9}}$$

$$\Rightarrow (2^{2x-1})(2^{3x-9}) = 1$$

$$\Rightarrow 2^{(2x-1)+(3x-9)} = 1$$

$$\Rightarrow 2^{5x-10} = 1 \Rightarrow 2^{5(x-2)} = 1$$

$$\Rightarrow 2^{5(x-2)} = 2^0$$

$$\Rightarrow x-2 = 0 \Rightarrow x = 2$$

Ex. 5 If $4^{2x+1} = 8^{x+3}$ then $x = ?$

$$\text{Sol. } 4^{2x+1} = 8^{x+3}$$

$$\Rightarrow (2^2)^{(2x+1)} = (2^3)^{(x+3)}$$

$$\Rightarrow 2^{4x+2} = 2^{3x+9}$$

$$\Rightarrow 4x+2 = 3x+9$$

($\because$ base in both side is same)

$$\Rightarrow x = 7$$

Ex. 6 If $3^{x-1} + 3^{x+1} = 90$, then $x = ?$

$$\text{Sol. } 3^{x-1} + 3^{x+1} = 90$$

$$\Rightarrow \frac{3^x}{3} + 3 \cdot 3^x = 90$$

$$\Rightarrow 3^x \left(\frac{1}{3} + 3\right) = 90$$

$$\Rightarrow 3^x \times \frac{10}{3} = 90$$

$$\Rightarrow \frac{3^x}{3} = 9$$

$$\Rightarrow 3^x = 27$$

$$\Rightarrow 3^x = 3^3$$

$$x = 3$$

TYPE - II

• **Surd:** If a is rational and n is a positive integer and $a^{1/n}$ = $\sqrt[n]{a}$ is

• Irrational, then $\sqrt[n]{a}$ is called a "surd of order n " or " n^{th} root of a " For the surd $\sqrt[n]{a}$, n is called the surd-index or the order of the surd and " a " is called the radicand. The symbol ' $\sqrt{}$ ' is called the surd sign or radical.

E.g. $\sqrt{5}$ is a surd of order 2 or square root of 5.

$\sqrt[3]{6}$ is a surd of order 3 or cube root of 6.

$\sqrt{6+\sqrt{5}}$ is not a surd as $6+\sqrt{5}$ is not a rational number.

Note: Every surd is an irrational number but every irrational number is not a surd.

→ In the surd $a^n\sqrt[n]{b}$, a and b are called factors of the surd.

(i) A surd which has unity as its rational factor (i.e. $a = 1$) is called “pure surd”.

E.g.: $\sqrt[4]{3}$, $\sqrt{2}$, $\sqrt[3]{3}$ etc.

(ii) A surd which has a rational factor other than unity, the other irrational, is called “mixed surd”. e.g. $3\sqrt{5}$, $2\sqrt{7}$, $5\sqrt[3]{7}$

→ If $\sqrt[n]{a}$ is a surd it implies.

(i) a is a rational number.

(ii) $\sqrt[n]{a}$ is an irrational number.

⇒ **Quadratic surd:** A surd of order 2 (i.e. $\sqrt{a}$) is called a quadratic surd.

E.g.: $\sqrt{2} = 2^{1/2}$ is a quadratic surd but $\sqrt{4} = 4^{1/2}$ is not a quadratic surd because $\sqrt{4} = 2$ is a rational number.

Therefore $\sqrt{4}$ is not a surd.

⇒ **Cubic Surd:** A surd of order 3 (i.e. $\sqrt[3]{a}$) is called a cubic surd.

E.g.: $\sqrt[3]{9}$ is a cubic surd but $\sqrt[3]{27} = 3$ is not a surd because $\sqrt[3]{27} = 3$ is a rational number.

⇒ **Quartic or Biquadratic surd:** A surd of order 4 (i.e. $\sqrt[4]{a}$) is called a quartic surd.

E.g.: $\sqrt[4]{3}$ is a quartic surd but $\sqrt[4]{81}$ is not a quartic surd.

Note: Each surd can be represented on the number line.

Important Formulae Based on Surds:

(i) $\sqrt[n]{a^n} = a$

(ii) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

(iii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ and $\frac{k\sqrt[n]{a}}{l\sqrt[n]{b}} = \frac{k}{l} \sqrt[n]{\frac{a}{b}}$

(iv) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

(v) $(\sqrt[n]{a^m}) = (a)^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m}$

(vi) $\sqrt{a} \times \sqrt{a} = a$

(vii) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

and k. $\sqrt[n]{a} \times \sqrt[l]{b} = kl.\sqrt[n]{a}.\sqrt[l]{b} = kl.\sqrt[n]{a^m b^n}$

(viii) $\sqrt{a^2 b} = a\sqrt{b}$

(ix) $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$

(x) $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$

(xi) $(\sqrt{a} + \sqrt{b}) \times (\sqrt{a} - \sqrt{b}) = a - b$, where a and b are positive rational numbers.

→ A surd is in its simplest form if:

(i) There is no factor which has n^{th} power of a rational number, under the radical sign whose index is n ,

(ii) There is no fraction under the radical sign, and

(iii) The index of the surd is the smallest possible.

E.g. The surd $\sqrt[4]{3 \times 5^4}$ is not in its simplest form since the number under the radical sign has factor 5^4 s.t. its index is equal to the order of the surd. Its simplest form:

$$\sqrt[4]{3 \times 5^4} = \sqrt[4]{3} \cdot \sqrt[4]{5^4} = (\sqrt[4]{3})(5) = 5 \cdot (\sqrt[4]{3})$$

⇒ **Similar or like Surds:** Surds having same irrational factors are called “similar or like surd”.

E.g.: $3\sqrt{3}$, $7\sqrt{3}$, $\frac{2}{5}\sqrt{3}$, $\sqrt{3}$ etc. are similar surds.

⇒ **Unlike surds:** Surds having non-common irrational factors are called “unlike surds”.

E.g.: $3\sqrt{3}$, $5\sqrt{2}$, $6\sqrt{7}$, etc. are unlike surds.

Ex. 7. $\left[\left(x + \frac{1}{y} \right)^a \left(x - \frac{1}{y} \right)^b \right] \div$

$$\left[\left(y + \frac{1}{x} \right)^a \left(y - \frac{1}{x} \right)^b \right] \text{ is equal to:}$$

Sol. $\left[\left(x + \frac{1}{y} \right)^a \left(x - \frac{1}{y} \right)^b \right] \div$

$$\left[\left(y + \frac{1}{x} \right)^a \left(y - \frac{1}{x} \right)^b \right]$$

$$\Rightarrow \frac{\left(x + \frac{1}{y} \right)^a \left(x - \frac{1}{y} \right)^b}{\left(y + \frac{1}{x} \right)^a \left(y - \frac{1}{x} \right)^b}$$

$$\Rightarrow \frac{\left(\frac{xy+1}{y} \right)^a \left(\frac{xy-1}{y} \right)^b}{\left(\frac{xy+1}{x} \right)^a \left(\frac{xy-1}{x} \right)^b}$$

$$\Rightarrow \frac{\frac{(xy+1)^a}{y^a} \cdot \frac{(xy-1)^b}{y^b}}{\frac{(xy+1)^a}{x^a} \cdot \frac{(xy-1)^b}{x^b}}$$

$$\Rightarrow \frac{(xy+1)^a (xy-1)^b}{y^{(a+b)}} \times$$

$$\frac{x^{(a+b)}}{(xy+1)^a (xy-1)^b}$$

$$\Rightarrow \frac{x^{a+b}}{y^{a+b}} = \left(\frac{x}{y} \right)^{a+b}$$

Ex. 8. If $x^{x^{3/2}} = (x^{3/2})^x$, then the value of x is:

Sol. $x^{x^{3/2}} = (x^{3/2})^x$

$$\Rightarrow x^{x^{3/2}} = x^{(3/2)x}$$

$$\Rightarrow x^{3/2} = \frac{3}{2}x$$

$$\Rightarrow x^{1/2} = \frac{3}{2} \Rightarrow x = \frac{9}{4}$$

Ex. 9 If $x^a = y^b = z^c$ and $y^2 = zx$ then

the value of $\frac{1}{a} + \frac{1}{c}$ is:

Sol. Let $x^a = y^b = z^c = k$

$$\Rightarrow x = k^{1/a}, y = k^{1/b}, z = k^{1/c}$$

Now,

$$\therefore y^2 = zx$$

$$\therefore (k^{1/b})^2 = (k^{1/c}) \cdot (k^{1/a})$$

$$\Rightarrow k^{2/b} = k^{\frac{1}{c} + \frac{1}{a}}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b}$$

Ex.10. $(a^{m-n})^l \times (a^{n-l})^m \times (a^{l-m})^n$

Sol. $(a^{m-n})^l \times (a^{n-l})^m \times (a^{l-m})^n$
 $= a^{ml-nl} \times a^{nm-lm} \times a^{ln-mn}$
 $= a^{ml-nl+nm-lm+ln-mn}$
 $= a^0 = 1$

Ex.11. If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of xyz is:

Sol. If $a^x = b$, $b^y = c$, $c^z = a$

$$\therefore a^x = b$$

$$\therefore (c^z)^x = b$$

$$\Rightarrow c^{zx} = b \Rightarrow (b^y)^{zx} = b$$

$$\Rightarrow b^{xyz} = b \Rightarrow xyz = 1$$

Ex.12. $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$ is

equal to:

Sol. $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a}$
 $= (x^{a-b})^{(a+b)} \times (x^{b-c})^{(b+c)} \times (x^{c-a})^{(c+a)}$
 $= x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$
 $= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$
 $= x^{a^2-b^2+b^2-c^2+c^2-a^2}$
 $= x^0 = 1$

Ex.13. The value of $\frac{1}{1+x^{b-a}+x^{c-a}}$

+ $\frac{1}{1+x^{c-b}+x^{a-b}}$ + $\frac{1}{1+x^{a-c}+x^{b-c}}$ is:

Sol. $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{c-b}+x^{a-b}}$
 $+ \frac{1}{1+x^{a-c}+x^{b-c}}$

$$\Rightarrow \frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^c}{x^b}+\frac{x^a}{x^b}}$$

$$+ \frac{1}{1+\frac{x^a}{x^c}+\frac{x^b}{x^c}}$$

$$\Rightarrow \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^c+x^a}$$

$$+ \frac{x^c}{x^c+x^a+x^b}$$

$$\Rightarrow \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1$$

Ex.14 $2^{2^x} = 16^{2^{3x}}$, then x is equal to

Sol. $2^{2^x} = 16^{2^{3x}}$

$$\Rightarrow 2^{2^x} = (2^4)^{2^{3x}}$$

$$\Rightarrow 2^{2^x} = 2^{4 \cdot 2^{3x}} = 2^{2^{3x+2}}$$

$$\Rightarrow 2^{2^x} = 2^{2^{3x+2}} \text{ (Since has base in both sides is equal)}$$

$$\Rightarrow 2^x = 2^{3x+2}$$

$$\Rightarrow x = 3x + 2$$

$$\Rightarrow 2x = -2$$

$$\Rightarrow x = -1$$

Ex.15. If $2^x = 4^y = 8^z$ and $xyz = 288$

find the value of $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z}$

Sol. $2^x = 4^y = 8^z$

$$2^x = 2^{2y} = 2^{3z}$$

[If $a^x = a^y = a^z$
then, $x = y = z$]

$$x = 2y = 3z$$

$$x = 3z$$

$$2y = 3z$$

$$y = \frac{3z}{2}$$

$$xyz = 288$$

Put the value of x,y in z form

$$3z \times \frac{3z}{2} \times z = 288$$

$$z^3 = 32 \times 2$$

$$z = 4$$

$$\text{Then } x = 3z = 4 \times 3 = 12$$

$$y = \frac{3z}{2} = \frac{3 \times 4}{2} = 6$$

So,

$$\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z}$$

Put the value x,y and z

$$\frac{1}{2 \times 12} + \frac{1}{4 \times 6} + \frac{1}{6 \times 4}$$

$$\frac{1}{24} + \frac{1}{24} + \frac{1}{24}$$

$$= 3 \times \frac{1}{24} = \frac{1}{8}$$

Ex.16: If $2^x = 3^y = 6^{-z}$, Find the value

$$\text{of } \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

Sol. $2^x = 3^y = 6^{-z} = k$

Then,

$$2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}, 6 = k^{\frac{1}{-z}}$$

We know that

$$2 \times 3 = 6$$

$$k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{\frac{1}{-z}}$$

Same base power will be add

$$k^{\frac{1}{x} + \frac{1}{y}} = k^{\frac{1}{-z}}$$

Now,

$$\frac{1}{x} + \frac{1}{y} = \frac{-1}{z}$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

Ex.17: If $\left(\frac{9}{4}\right)^x \times \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$, Find the value of x.

Sol. $\left(\frac{9}{4}\right)^x \times \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$

$$\left[a^x = \frac{1}{a^{-x}}\right]$$

$$\left(\frac{4}{9}\right)^{-x} \times \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{-2x} \times \left(\frac{2}{3}\right)^{3(x-1)} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{-2x+3x-3} = \left(\frac{2}{3}\right)^1$$

The power will be equal when the base is equal.

$$-2x + 3x - 3 = 1$$

$$x - 3 = 1$$

$$x = 4$$

Ex.18. If $2^x = 4^y = 8^z$,

$$\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7}, \text{ Find the}$$

value of z.

Sol. $2^x = 4^y = 8^z$,

$$2^x = 2^{2y} = 2^{3z}$$

When base is equal then power will be equal,

$$x = 2y = 3z$$

$$\text{So, } x = 3z, y = \frac{3z}{2}$$

$$\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7}$$

Put the value of x and y in z form

$$\frac{1}{2 \times 3z} + \frac{1}{4 \times \frac{3z}{2}} + \frac{1}{6z} = \frac{24}{7}$$

$$\frac{1}{6z} + \frac{1}{6z} + \frac{1}{6z} = \frac{24}{7}$$

$$\frac{1}{6z} \times 3 = \frac{24}{7}$$

$$\frac{1}{2z} = \frac{24}{7}$$

$$z = \frac{7}{48}$$

Ex.19: The value of expression

$$\frac{4^n \times 20^{m-1} \times 12^{m-n} \times 15^{m+n-2}}{16^m \times 5^{2m+n} \times 9^{m-1}} \text{ is:}$$

Sol.
$$\frac{4^n \times 20^{m-1} \times 12^{m-n} \times 15^{m+n-2}}{16^m \times 5^{2m+n} \times 9^{m-1}}$$

$$\Rightarrow \frac{2^{2n} \times 2^{2m-2} \times 5^{m-1} \times 2^{2m-2n} \times 3^{m-n}}{2^{4m} \times 5^{2m+n} \times 3^{2m-2}}$$

$$\Rightarrow 2^{2n+2m-2+2m-2n-4m} \times 3^{m-n+m+n-2-2m+2} \times 5^{m-1+m+n-2-2m-n}$$

$$\Rightarrow 2^{-2} \times 3^0 \times 5^{-3}$$

$$\Rightarrow \frac{1}{4} \times \frac{1}{125} = \frac{1}{500}$$

Ex.20: The value of Expression

$$\frac{(0.3)^{1/3} \cdot \left(\frac{1}{27}\right)^{1/4} \cdot (9)^{1/6} \cdot (0.81)^{2/3}}{(0.9)^{2/3} \cdot (3)^{-1/2} \cdot \left(\frac{1}{3}\right)^{-2} \cdot (243)^{-1/4}} \text{ is}$$

Sol.
$$\frac{(0.3)^{1/3} \cdot \left(\frac{1}{27}\right)^{1/4} \cdot (9)^{1/6} \cdot (0.81)^{2/3}}{(0.9)^{2/3} \cdot (3)^{-1/2} \cdot \left(\frac{1}{3}\right)^{-2} \cdot (243)^{-1/4}}$$

$$\Rightarrow \frac{\left(\frac{3}{10}\right)^{1/3} \left(\frac{1}{3}\right)^{3/4} (3)^{1/3} \left(\frac{81}{100}\right)^{2/3}}{\left(\frac{9}{10}\right)^{2/3} \left(\frac{1}{3}\right)^{1/2} \cdot (3)^2 (3)^{-5/4}}$$

Break the power of 3 & 10 separately

$$\Rightarrow \frac{3^{\frac{1}{3} + \frac{1}{3} - \frac{3}{4} + \frac{8}{3} - \frac{1}{3} - \frac{4}{3}} \times 10^{\frac{1}{3} - \frac{2}{3} - \frac{4}{3}}}{3^{\frac{4}{3} + 2 - \frac{1}{2} - \frac{5}{4} - \frac{2}{3}} \times 10^{-\frac{2}{3}}}$$

$$\Rightarrow \frac{3^{31/12} \times 10^{-\frac{5}{3}}}{3^{12} \times 10^{-\frac{2}{3}}}$$

$$\Rightarrow 3 \times 10^{-1} = \frac{3}{10} = 0.3$$

Ex.21: The value of expression

$$\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1} \left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}} \text{ is}$$

Sol.
$$\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{2^3}\right)^{-1} \left(\frac{3}{2}\right)^3 + \left(-\frac{1}{3}\right)^{-1}}$$

$$\Rightarrow \frac{1 - \left(\frac{1}{10}\right)^{-1}}{(3^{-1}) \left(2^3\right) \times 3^3 \times 2^{-3} + (-3)^1} = \frac{1-10}{(3^{-1}) \cdot 2^0 - 3} = \frac{-9}{9-3} = \frac{-9}{6} = \frac{-3}{2}$$

Ex.22: If $\frac{9^n \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$ then the value of (m - n) is:

Sol.
$$\frac{9^n \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$$

$$\Rightarrow \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{2n+2+n} - 3^{3n}}{8 \times 3^{3m}} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 8} = 3^{-3}$$

$$\Rightarrow \frac{3^{3n}(3^2 - 1)}{3^{3m} \times 8} = 3^{-3}$$

$$\Rightarrow 3^{3n-3m} = 3^{-3}$$

$$\Rightarrow 3n - 3m = -3$$

$$\Rightarrow m - n = 1$$

TYPE - III

Comparison of Surds: (i) If two surds are of the same order, then the one whose radicand is larger, is the larger of the two.

E.g.: $\sqrt[3]{19} > \sqrt[3]{15}$, $\sqrt{7} > \sqrt{5}$, $\sqrt[3]{9} > \sqrt[3]{7}$ etc.

(ii) If two surds are **distinct order**, we change them into the surds of the same order.

This order is the L.C.M. of the orders of the given surds.

E.g.: Which is larger $\sqrt{2}$ of $\sqrt[3]{3}$?

Sol. Given surds are of order 2 & 3 respectively whose L.C.M is 6.

Convert each into a surd of order 6, as show below :

$$\sqrt{2} = 2^{1/2} = 2^{\frac{1}{2} \times \frac{3}{3}} = 2^{3/6} = (2^3)^{1/6}$$

$$= (8)^{1/6} = \sqrt[6]{8}$$

$$= 3^{1/3} = 3^{\frac{1}{3} \times \frac{2}{2}} = 3^{2/6} = (9)^{1/6}$$

$$= \sqrt[6]{9}$$

Clearly, $\sqrt[6]{9} > \sqrt[6]{8}$, so $\sqrt[3]{3} > \sqrt{2}$

Ex.23: Which one is smaller out of $\sqrt[3]{2}$ and $\sqrt[4]{3}$?

Sol.
$$\sqrt[3]{2} = (2)^{1/3} = 2^{4/12} = (16)^{1/12}$$

$$\text{and } \sqrt[4]{3} = (3)^{1/4} = 3^{3/12} = (27)^{1/12}$$

(Taking the LCM of surd)

$$\text{Hence } \sqrt[3]{2} < \sqrt[4]{3}$$

Ex.24: Which one is greatest out of $\sqrt[3]{5}$, $\sqrt[4]{3}$, $\sqrt[5]{4}$?

Sol.
$$\sqrt[3]{5} = (5)^{1/3} = (5)^{20/60} = (5^{20})^{1/60} = \sqrt[60]{5^{20}}$$

$$\sqrt[4]{3} = (3)^{1/4} = (3)^{15/60} = (3^{15})^{1/60} = \sqrt[60]{3^{15}}$$

$$\sqrt[5]{4} = (4)^{1/5} = (4)^{12/60} = (4^{12})^{1/60} = \sqrt[60]{4^{12}}$$

$\sqrt[3]{5}$ is the greatest.

Ex.25: Which of the following relation is correct A = $\sqrt{2}$, B = $\sqrt[3]{3}$, C = $\sqrt[4]{4}$

Sol.
$$(2)^{1/2}, (3)^{1/3}, (4)^{1/4}$$

LCM 2, 3 and 4 = 12

convert each into a surd of order 12

$$(2)^{\frac{1}{2} \times \frac{6}{6}}, (3)^{\frac{1}{3} \times \frac{4}{4}}, (4)^{\frac{1}{4} \times \frac{3}{3}}$$

$$(2)^{\frac{6}{12}}, (3)^{\frac{4}{12}}, (4)^{\frac{3}{12}}$$

$$(2^6)^{1/12}, (3^4)^{1/12}, (4^3)^{1/12}$$

$$(64)^{1/12}, (81)^{1/12}, (64)^{1/12}$$

$$\text{clearly } (81)^{1/12} > (64)^{1/12} = (64)^{1/12}$$

$$\text{so, } (3)^{1/3} > (2)^{1/2} = (4)^{1/4}$$

$$\mathbf{B > A = C}$$

Ex.26: Arrange the following in descending order.

$$\sqrt{3} - \sqrt{2}, \sqrt{4} - \sqrt{3}, \sqrt{5} - \sqrt{4}, \sqrt{2} - 1$$

Sol. $\sqrt{3} - \sqrt{2} = \frac{\sqrt{3} - \sqrt{2}}{1} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}$

$$\Rightarrow \frac{3-2}{\sqrt{3} + \sqrt{2}} = \frac{1}{\sqrt{3} + \sqrt{2}}$$

Similarly, $\sqrt{4} - \sqrt{3} = \frac{1}{\sqrt{4} + \sqrt{3}},$

$$\sqrt{5} - \sqrt{4} = \frac{1}{\sqrt{5} + \sqrt{4}}$$

and $\sqrt{2} - 1 = \frac{1}{\sqrt{2} + 1}$

As we know, if the numerator is same then the fraction whose denominator is larger the fraction will be lower. Hence the correct order of descending is.

$$(\sqrt{2} - 1) > (\sqrt{3} - \sqrt{2}) > (\sqrt{4} - \sqrt{3}) > (\sqrt{5} - \sqrt{4})$$

Ex.27: Which is greater than

$$(\sqrt{8} - \sqrt{5}) \text{ or } (\sqrt{13} - \sqrt{10})$$

Sol. Rationalisation of Surds:

$$(\sqrt{8} - \sqrt{5}) \times \frac{(\sqrt{8} + \sqrt{5})}{(\sqrt{8} + \sqrt{5})} = \frac{3}{\sqrt{8} + \sqrt{5}}$$

$$(\sqrt{13} - \sqrt{10}) \times \frac{(\sqrt{13} + \sqrt{10})}{(\sqrt{13} + \sqrt{10})} = \frac{3}{\sqrt{13} + \sqrt{10}}$$

So, It is clear that

$$\Rightarrow \frac{3}{\sqrt{8} + \sqrt{5}} > \frac{3}{\sqrt{13} + \sqrt{10}}$$

Then $(\sqrt{8} - \sqrt{5}) > (\sqrt{13} - \sqrt{10})$

Ex.28: Arrange the following in ascending order.

$$\sqrt{8} + \sqrt{5}, \sqrt{6} + \sqrt{7}, \sqrt{9} + \sqrt{4}, \sqrt{11} + \sqrt{2}, \sqrt{10} + \sqrt{3}$$

Sol. In this type questions we use square method

So, $(\sqrt{8} + \sqrt{5})^2 = 13 + 2\sqrt{40}$

$$(\sqrt{6} + \sqrt{7})^2 = 13 + 2\sqrt{42}$$

$$(\sqrt{9} + \sqrt{4})^2 = 13 + 2\sqrt{36}$$

$$(\sqrt{11} + \sqrt{2})^2 = 13 + 2\sqrt{22}$$

$$(\sqrt{10} + \sqrt{3})^2 = 13 + 2\sqrt{30}$$

So, It is cleared that

$$13 + 2\sqrt{22} < 13 + 2\sqrt{30} < 13 + 2\sqrt{36} <$$

$$13 + 2\sqrt{40} < 13 + 2\sqrt{42}$$

Then

$$\sqrt{11} + \sqrt{2} < \sqrt{10} + \sqrt{3} < \sqrt{9} + \sqrt{4}$$

$$< \sqrt{8} + \sqrt{5} < \sqrt{6} + \sqrt{7}$$

Ex.29: Arrange the following in descending order.

$$\sqrt{8} - \sqrt{5}, \sqrt{6} - \sqrt{7}, \sqrt{9} - \sqrt{4},$$

$$\sqrt{11} - \sqrt{2}, \sqrt{10} - \sqrt{3}$$

Sol. We use square method

$$(\sqrt{8} - \sqrt{5})^2 = 13 - 2\sqrt{40}$$

$$(\sqrt{6} - \sqrt{7})^2 = 13 - 2\sqrt{42}$$

$$(\sqrt{9} - \sqrt{4})^2 = 13 - 2\sqrt{36}$$

$$(\sqrt{11} - \sqrt{2})^2 = 13 - 2\sqrt{22}$$

$$(\sqrt{10} - \sqrt{3})^2 = 13 - 2\sqrt{30}$$

It is cleared that

$$13 - 2\sqrt{22} > 13 - 2\sqrt{30} > 13 - 2\sqrt{36} >$$

$$13 - 2\sqrt{40} > 13 - 2\sqrt{42}$$

Then

$$\sqrt{11} - \sqrt{2} > \sqrt{10} - \sqrt{3} > \sqrt{9} - \sqrt{4}$$

$$> \sqrt{8} - \sqrt{5} > \sqrt{6} - \sqrt{7}$$

Ex.30: Which is greater than

$$(\sqrt{23} - \sqrt{21}) \text{ or } (\sqrt{21} - \sqrt{19})$$

Sol. Rationalisation of Surds:

$$(\sqrt{23} - \sqrt{21}) \times \frac{(\sqrt{23} + \sqrt{21})}{(\sqrt{23} + \sqrt{21})}$$

$$= \frac{2}{\sqrt{23} + \sqrt{21}}$$

$$(\sqrt{21} - \sqrt{19}) \times \frac{(\sqrt{21} + \sqrt{19})}{(\sqrt{21} + \sqrt{19})}$$

$$= \frac{2}{(\sqrt{21} + \sqrt{19})}$$

It is cleared that

$$\frac{2}{(\sqrt{21} + \sqrt{19})} > \frac{2}{\sqrt{23} + \sqrt{21}}$$

$$\text{Then } (\sqrt{21} - \sqrt{19}) > (\sqrt{23} - \sqrt{21})$$

Ex.31: Arrange the following in descending order.

$$2^{350}, 5^{200}, 3^{300}, 4^{250}$$

Sol. $2^{350}, 5^{200}, 3^{300}, 4^{250}$

Power in same form

$$(2^7)^{50}, (5^4)^{50}, (3^6)^{50}, (4^5)^{50}$$

$$(128)^{50}, (625)^{50}, (729)^{50}, (1024)^{50}$$

So,

$$4^{250} > 3^{300} > 5^{200} > 2^{350}$$

Ex.32: Arrange the following in descending order.

$$2^{72}, 5^{36}, 4^{48}, 3^{60}$$

Power in same form

$$(2^6)^{12}, (5^3)^{12}, (4^4)^{12}, (3^5)^{12}$$

$$(64)^{12}, (125)^{12}, (256)^{12}, (243)^{12}$$

Then

$$(256)^{12} > (243)^{12} > (125)^{12} > (64)^{12}$$

$$\therefore 4^{48} > 3^{60} > 5^{36} > 2^{72}$$

Ex.33: Arrange the following in descending order.

$$\sqrt[3]{3}, \sqrt[4]{4}, \sqrt[6]{6}, \sqrt[12]{12}$$

Sol. $\frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}$

$$\text{LCM of } 3, 4, 6 \text{ and } 12 = 12$$

Then

$$3^{\frac{1}{3} \times \frac{4}{4}}, 4^{\frac{1}{4} \times \frac{3}{3}}, 6^{\frac{1}{6} \times \frac{2}{2}}, 12^{\frac{1}{12}}$$

$$(3^4)^{\frac{1}{12}}, (4^3)^{\frac{1}{12}}, (6^2)^{\frac{1}{12}}, 12^{\frac{1}{12}}$$

It is cleared that

$$(81)^{\frac{1}{12}}, (64)^{\frac{1}{12}}, (36)^{\frac{1}{12}}, 12^{\frac{1}{12}}$$

$$(81)^{\frac{1}{12}} > (64)^{\frac{1}{12}} > (36)^{\frac{1}{12}} > 12^{\frac{1}{12}}$$

So,

$$3^{\frac{1}{3}} > 4^{\frac{1}{4}} > 6^{\frac{1}{6}} > 12^{\frac{1}{12}}$$

TYPE - IV

$\Rightarrow$ **Rationalisation of Surds:** If the product of two surds is rational, then each of them is called the (R.F.) rationalising factor of the other.

E.g.: $5\sqrt{7} \times \sqrt{7} = 5\sqrt{7 \times 7} = 5 \times 7 = 35$

$\therefore \sqrt{7}$ is a rationalising factor of $5\sqrt{7}$.

→ Rationalising factor (R.F) of the surd $\sqrt[n]{a}$ is $a^{1-\frac{1}{n}}$
 → R.F. of the surd $\sqrt{a} \pm \sqrt{b}$ is $\sqrt{a} \mp \sqrt{b}$.

E.g.: $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$
 $= (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$

∴ $(\sqrt{3} - \sqrt{2})$ is a R.F. of $(\sqrt{3} + \sqrt{2})$

Note: The R.F. of a given surd is not unique.

Ex.34: If $x = 7 + 4\sqrt{3}$ find the value

of $\frac{1}{x}$

Sol. $x = 7 + 4\sqrt{3}$

$$\frac{1}{x} = \frac{1}{7 + 4\sqrt{3}}$$

(Multiplying numerator and denominator by conjugate)

$$\frac{1}{x} = \frac{1}{7 + 4\sqrt{3}} \times \frac{7 - 4\sqrt{3}}{7 - 4\sqrt{3}}$$

$$= \frac{1}{x} = \frac{7 - 4\sqrt{3}}{(7)^2 - (4\sqrt{3})^2}$$

$$= \frac{1}{x} = \frac{7 - 4\sqrt{3}}{49 - 48}$$

$$= \frac{1}{x} = 7 - 4\sqrt{3}$$

Alternate:-

R.F. of the surd $\sqrt{a} \pm \sqrt{b}$ is $\sqrt{a} \mp \sqrt{b}$.

when difference of square of both number $\sqrt{a}$ and $\sqrt{b} = 1$

$$x = 7 + 4\sqrt{3}$$

$$\text{difference} = (7)^2 - (4\sqrt{3})^2 = 49 - 48 = 1$$

$$\text{Then } \frac{1}{x} = \frac{1}{7 + 4\sqrt{3}} = 7 - 4\sqrt{3}$$

Ex.35: If $x = \sqrt{3} - \sqrt{2}$, find the value

of $\frac{1}{x} = ?$

Sol. $x = \sqrt{3} - \sqrt{2}$

$$\text{Difference} = (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$$

$$\text{Then, } \frac{1}{x} = \sqrt{3} + \sqrt{2}$$

Ex.36: If $x = \frac{1}{5 - 2\sqrt{6}}$, Find the value

of $\frac{1}{x} = ?$

Sol. $x = \frac{1}{5 - 2\sqrt{6}}$

Difference of nos.

$$= (5)^2 - (2\sqrt{6})^2$$

$$= 25 - 24 = 1$$

$$\text{then, } \frac{1}{x} = \frac{1}{5 - 2\sqrt{6}} = 5 - 2\sqrt{6}$$

Note:- (i) If $x = \sqrt{3} + \sqrt{2}$

$$\text{then } \frac{1}{x} = \sqrt{3} - \sqrt{2}$$

(ii) If $x = 5 - 2\sqrt{6}$

$$\text{then } \frac{1}{x} = 5 + 2\sqrt{6}$$

(iii) If $x = \frac{1}{7 - 4\sqrt{3}}$

$$\text{then } \frac{1}{x} = \frac{1}{7 + 4\sqrt{3}}$$

- When the difference between two numbers is equal to 1 then reciprocal of this number only change their sign. ("+" change into "-", and "-" change into "+")

Ex.37: Find the value of $\frac{2 + \sqrt{3}}{2 - \sqrt{3}}$

Sol. $\frac{2 + \sqrt{3}}{2 - \sqrt{3}} = \frac{2 + \sqrt{3}}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$

(Multiplying numerator and denominator by conjugate)

$$= \frac{(2 + \sqrt{3})^2}{4 - 3} = \frac{4 + 3 + 4\sqrt{3}}{1}$$

$$= 7 + 4\sqrt{3}$$

Ex.38: The value of $\frac{\sqrt{5} - 2}{\sqrt{5} + 2}$ is:

Sol. $\frac{\sqrt{5} - 2}{\sqrt{5} + 2} = \frac{(\sqrt{5} - 2)}{(\sqrt{5} + 2)} \times \frac{(\sqrt{5} - 2)}{(\sqrt{5} - 2)}$

$$= \frac{(\sqrt{5} - 2)^2}{5 - 4}$$

$$= \frac{5 + 4 - 4\sqrt{5}}{1} = 9 - 4\sqrt{5}$$

Ex.39: Find the value of

$$\frac{5}{6\sqrt{6}} \times \frac{12\sqrt{30}}{25\sqrt{5}}$$

Sol. $\frac{5}{6\sqrt{6}} \times \frac{12\sqrt{30}}{25\sqrt{5}} = \frac{5}{6\sqrt{6}} \times \frac{12\sqrt{6}\sqrt{5}}{25\sqrt{5}} = \frac{2}{5}$

TYPE - V SOME USEFUL RESULTS

(a) If $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$

$$\text{then, } y = \frac{1 + \sqrt{1 + 4x}}{2}$$

Ex.40: $y = \sqrt{7 + \sqrt{7 + \sqrt{7 + \dots \infty}}}$, then which of the following is true?

- (a) $y = 3$ (b) $3 < y < 3.5$
 (b) $y = 7$ (d) Greater than 4

Sol. $y = \frac{1 + \sqrt{1 + 4x}}{2}$

Here, $x = 7$

$$\text{then } y = \frac{1 + \sqrt{1 + 4 \times 7}}{2}$$

$$= \frac{1 + \sqrt{29}}{2}$$

∴ $\sqrt{29}$ lies between 5 or 6

$$\text{So, } y = \frac{1 + 5}{2} = 3$$

$$\text{or, } y = \frac{1 + 6}{2} = 3.5$$

So, $3 < y < 3.5$ is correct.

Ex.41:

If $y = \sqrt{12 + \sqrt{12 + \sqrt{12 + \dots \infty}}}$, then $y = ?$

Sol. $y = \frac{1 + \sqrt{1+4 \times 12}}{2}$

$$y = \frac{1 + \sqrt{49}}{2}$$

$$= \frac{1+7}{2} = 4$$

Note:

if $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots \infty}}}$

and $x = n(n+1)$ then $y = (n+1)$

Alternate:

we have $x = 12 = 3 \times 4 = n(n+1)$

$$\therefore y = 4$$

Ex.42: If $\sqrt{20 + \sqrt{20 + \sqrt{20 + \dots \infty}}}$

then the value of $y = ?$

Sol. We have $x = 20 = 4 \times 5 = n(n+1)$

$$\therefore y = 5$$

(b) If $y = \sqrt{x - \sqrt{x - \sqrt{x - \dots \infty}}}$

$$\text{then } y = \frac{-1 + \sqrt{1+4x}}{2}$$

Ex.43: If $y = \sqrt{9 - \sqrt{9 - \sqrt{9 - \dots \infty}}}$

then which of the following is true?

- (a) $y = 3$ (b) $2.5 < y < 3$
(c) $y = 9$ (d) Greater than 4

Sol. $y = \frac{-1 + \sqrt{1+4x}}{2}$

Here, $x = 9$

$$\text{then } y = \frac{-1 + \sqrt{1+4 \times 9}}{2}$$

$$= \frac{-1 + \sqrt{37}}{2}$$

$\therefore \sqrt{37}$ lies between 6 or 7

$$\text{So, } y = \frac{-1+6}{2} = 2.5$$

$$\text{or, } y = \frac{-1+7}{2} = 3$$

So, $2.5 < y < 3$ is correct.

Ex.44: If $y = \sqrt{12 - \sqrt{12 - \sqrt{12 - \dots \infty}}}$, then the value of $y = ?$

Sol. $y = \frac{-1 + \sqrt{1+4 \times 12}}{2}$

$$y = \frac{-1 + \sqrt{49}}{2}$$

$$y = \frac{-1+7}{2} = 3$$

Note:

if $y = \sqrt{x - \sqrt{x - \sqrt{x - \dots \infty}}}$

and $x = n(n+1)$ then $y = n$

Alternate:

we have, $x = 12 = 3 \times 4 = n(n+1)$

$$\therefore y = 3.$$

Ex.45: If $y = \sqrt{42 - \sqrt{42 - \sqrt{42 - \dots \infty}}}$, the value of $y = ?$

Sol. We have, $x = 42 = 6 \times 7$

$$= n(n+1)$$

$$\therefore y = 6$$

(c) If $y = \sqrt{a \sqrt{a \sqrt{a \dots \infty}}}$

then $y = a$

Ex.46: $y = \sqrt{5 \sqrt{5 \sqrt{5 \dots \infty}}}$

Sol. $y = 5$

(d) If $y = \sqrt{a \sqrt{a \sqrt{a \dots n}}}$,

$$\text{then } y = a^{\frac{2^n - 1}{2^n}}$$

Ex.47: If $y = \sqrt{a \sqrt{a \sqrt{a \sqrt{a}}}}$, find the

value of y .

Sol. $n = 4$, then $y = a^{\frac{2^4 - 1}{2^4}}$

$$y = a^{\frac{15}{16}}$$

Ex.48: If $y = \sqrt{3 \sqrt{3 \sqrt{3 \sqrt{3 \sqrt{3}}}}}$, find

the value of y ?

Sol. $n = 5$

$$\text{Then, } y = 3^{\frac{2^5 - 1}{2^5}}$$

$$y = 3^{\frac{31}{32}}$$

(e) I $y = \left(\sqrt[m]{\left(\sqrt[p]{a^x} \right)^z} \right)^n$

$$a^{\frac{x \times z \times n}{m \times p}}$$

Ex.49: If $\left(\sqrt[12]{\left(\sqrt[6]{\left(\sqrt[3]{5^4} \right)^8} \right)^9} \right)^{18} \times$

$$\sqrt[9]{\left(\sqrt[8]{\left(\sqrt[4]{5^3} \right)^6} \right)^{12}}$$

Sol. $5^{\frac{4 \times 8 \times 9 \times 18}{3 \times 6 \times 12}} \times 5^{\frac{3 \times 6 \times 12}{4 \times 8 \times 9}}$

$$5^{8 \times 3} \times 5^{\frac{3}{4}}$$

$$5^{24 + \frac{3}{4}} = 5^{\frac{99}{4}}$$

Ex.50: $\sqrt{2 \sqrt{4 \sqrt{2 \sqrt{4 \sqrt{2 \sqrt{4 \dots \infty}}}}}}$ will be equal to

Sol. Let $x = \sqrt{2 \sqrt{4 \sqrt{2 \sqrt{4 \sqrt{2 \sqrt{4 \dots \infty}}}}}}$

$$x = \sqrt{2 \sqrt{4x}} \text{ (Squaring both sides)}$$

$$x^2 = 2 \sqrt{4x} \text{ (Cubing both sides)}$$

$$x^6 = 2^3 \cdot 4x = 2^5 \cdot x \Rightarrow x^5 = 2^5$$

$$x = 2$$

TYPE - VI

Square-root of an Irrational number:

As we know that,

$$(a+b)^2 = (a^2 + b^2) + 2ab$$

$$\therefore (\sqrt{2} + \sqrt{3})^2 = \underbrace{5}_{(a^2+b^2)} + \underbrace{2\sqrt{6}}_{(2ab)}$$

$$\therefore 5 + 2\sqrt{6} = \underbrace{5 + 2\sqrt{2}\sqrt{3}}_{(2ab)}$$

$$\therefore a = \sqrt{2} \text{ \& } b = \sqrt{3}$$

$$\text{\& } a^2 + b^2 = 5$$

$$\therefore 5 + 2\sqrt{6} = (\sqrt{2} + \sqrt{3})^2$$

$$\Rightarrow a+b = \sqrt{5+2\sqrt{6}} = \sqrt{2} + \sqrt{3}$$

Examples:

(i) $\sqrt{7-4\sqrt{3}} = \sqrt{7-2 \times 2 \times \sqrt{3}}$
 $\downarrow \quad \downarrow$
 $a \quad b$

$$\text{\& } a^2 + b^2 = 2^2 + \sqrt{3}^2 = 7$$

$$= \sqrt{(2-\sqrt{3})^2} = (2-\sqrt{3})$$

(ii) $\sqrt{5 - \sqrt{21}}$ for making it $2ab$
 $(2ab)$

$$\sqrt{\frac{(5-\sqrt{21}) \times 2}{2}} = \sqrt{\frac{10-2 \times \sqrt{3} \times \sqrt{7}}{2}}$$

$$= \frac{1}{\sqrt{2}} \sqrt{(10-2 \times \sqrt{3} \times \sqrt{7})}$$

$$\left[\begin{array}{l} \therefore a = \sqrt{7} \text{ \& } b = \sqrt{3} \\ \therefore a^2 + b^2 = 10 \end{array} \right]$$

$$= \frac{1}{\sqrt{2}} \sqrt{(\sqrt{7}-\sqrt{3})^2} = \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}}$$

Ex.51: Find the square root of $(7+4\sqrt{3})$.

Sol. $\frac{7+4\sqrt{3}}{(a^2+b^2) (2ab)}$

$$= \frac{7+4\sqrt{3}}{4+3 \quad 2 \times 2\sqrt{3}}$$

$$= (2+\sqrt{3})^2$$

$$= \sqrt{(2+\sqrt{3})^2} = 2+\sqrt{3}$$

Ex.52: Find the square root of $13-4\sqrt{3}$

Sol. $\frac{13-4\sqrt{3}}{(a^2+b^2) (2ab)}$

$$= \frac{13-4\sqrt{3}}{(12+1) \quad 2 \times 2\sqrt{3} \times 1}$$

$$= (\sqrt{12}-1)^2$$

Square root of

$$\text{Then } \sqrt{(\sqrt{12}-1)^2} = \sqrt{12}-1$$

Ex.53: Find the square root of $76+10\sqrt{3}$

Sol. $\frac{76+10\sqrt{3}}{(75+1) \quad 2 \times 5\sqrt{3} \times 1}$

$$= (5\sqrt{3}+1)^2$$

Square root

$$\text{Then } = \sqrt{(5\sqrt{3}+1)^2}$$

$$= 5\sqrt{3}+1$$

Ex.54: Find the square of $33-4\sqrt{35}$

Sol. $\frac{33-4\sqrt{35}}{(28+5) \quad 2 \times 2\sqrt{7} \times \sqrt{5}}$

$$= (2\sqrt{7}-\sqrt{5})^2$$

Square root

$$\text{Then, } \sqrt{(2\sqrt{7}-\sqrt{5})^2}$$

$$2\sqrt{7}-\sqrt{5}$$

Ex.55: Find the square root of $139-80\sqrt{3}$.

Sol. $\frac{139-80\sqrt{3}}{(75+64) \quad 2 \times 5\sqrt{3} \times 8}$

$$= (5\sqrt{3}-8)^2$$

Square root

$$\text{Then, } \sqrt{(5\sqrt{3}-8)^2}$$

$$5\sqrt{3}-8$$

Ex.56: Find the square root of $8-4\sqrt{3}$

Sol. $8-4\sqrt{3}$

$$= 2(4-2\sqrt{3})$$

$$= \frac{2(4-2\sqrt{3})}{\begin{matrix} \swarrow & \searrow \\ 3+1 & \\ 2\sqrt{3} & \times & 1 \end{matrix}}$$

$$= 2(\sqrt{3}-1)^2$$

Square root of

$$\text{Then, } \sqrt{2(\sqrt{3}-1)^2}$$

$$= \sqrt{2}(\sqrt{3}-1)$$

Ex.57: If $x = \sqrt{-\sqrt{3} + \sqrt{3+8\sqrt{7+4\sqrt{3}}}}$, then find the value of x.

Sol. $x = \sqrt{-\sqrt{3} + \sqrt{3+8\sqrt{7+4\sqrt{3}}}}$

$$x = \sqrt{-\sqrt{3} + \sqrt{3+8\sqrt{(2+\sqrt{3})^2}}}$$

$$\Rightarrow x = \sqrt{-\sqrt{3} + \sqrt{3+8(2+\sqrt{3})}}$$

$$\Rightarrow x = \sqrt{-\sqrt{3} + \sqrt{3+16+8\sqrt{3}}}$$

$$\Rightarrow x = \sqrt{-\sqrt{3} + \sqrt{19+8\sqrt{3}}}$$

$$\Rightarrow x = \sqrt{-\sqrt{3} + \sqrt{(4+\sqrt{3})^2}}$$

$$\Rightarrow x = \sqrt{-\sqrt{3} + (4+\sqrt{3})}$$

$$x = 2$$

Ex.58: If $x = \frac{2\sqrt{10}}{7}$, find the value of

$$\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}$$

Sol. $1+x = 1 + \frac{2\sqrt{10}}{7} = \frac{7+2\sqrt{10}}{7}$

$$1+x = \left(\frac{\sqrt{5} + \sqrt{2}}{\sqrt{7}} \right)^2$$

Square root of both sides,

$$\sqrt{1+x} = \sqrt{\left(\frac{\sqrt{5} + \sqrt{2}}{\sqrt{7}} \right)^2}$$

$$\sqrt{1+x} = \left(\frac{\sqrt{5} + \sqrt{2}}{\sqrt{7}} \right)$$

Same as $\sqrt{1-x} = \left(\frac{\sqrt{5} - \sqrt{2}}{\sqrt{7}} \right)$

$$\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}$$

Put the value,

$$= \frac{\frac{\sqrt{5} + \sqrt{2}}{\sqrt{7}} + \frac{\sqrt{5} - \sqrt{2}}{\sqrt{7}}}{\frac{\sqrt{5} + \sqrt{2}}{\sqrt{7}} - \frac{\sqrt{5} - \sqrt{2}}{\sqrt{7}}}$$

$$= \frac{\sqrt{5} + \sqrt{2} + \sqrt{5} - \sqrt{2}}{\sqrt{5} + \sqrt{2} - \sqrt{5} + \sqrt{2}} = \frac{2\sqrt{5}}{2\sqrt{2}}$$

$$= \frac{\sqrt{5}}{\sqrt{2}}$$

Ex.59: If $x = 38+5\sqrt{3}$, Find the value of $\sqrt{x}$.

Sol. $x = 38+5\sqrt{3}$

Multiply and divided by 2

$$x = \frac{76+2 \times 5 \times \sqrt{3}}{2}$$

$$x = \left(\frac{5\sqrt{3}+1}{\sqrt{2}} \right)^2$$

$$\text{Then, } \sqrt{x} = \sqrt{\left(\frac{5\sqrt{3}+1}{\sqrt{2}} \right)^2}$$

$$\sqrt{x} = \frac{5\sqrt{3}+1}{\sqrt{2}}$$

Ex.60: If $x = \frac{\sqrt{3}}{2}$, Find the value of

$$\sqrt{1+x}$$

Sol. $1+x = 1 + \frac{\sqrt{3}}{2} = \frac{(2+\sqrt{3})}{2}$

Multiply and divided by 2

$$1+x = \frac{4+2\sqrt{3}}{4}$$

$$1+x = \left(\frac{\sqrt{3}+1}{2}\right)^2$$

Square Root of Both Sides

$$\text{Then, } \sqrt{1+x} = \sqrt{\left(\frac{\sqrt{3}+1}{2}\right)^2}$$

$$\sqrt{1+x} = \frac{\sqrt{3}+1}{2}$$

Ex.61: If $x = \frac{\sqrt{3}}{2}$, Find the value of

$$\frac{\sqrt{1+x}}{1+\sqrt{1+x}} + \frac{\sqrt{1-x}}{1-\sqrt{1-x}}$$

Sol. When $x = \frac{\sqrt{3}}{2}$

$$\text{Then, } \sqrt{1+x} = \frac{\sqrt{3}+1}{2}$$

(Explanation above question 61)

Put the value:

$$\Rightarrow \frac{\frac{\sqrt{3}+1}{2}}{1+\left(\frac{\sqrt{3}+1}{2}\right)} + \frac{\frac{\sqrt{3}-1}{2}}{1-\left(\frac{\sqrt{3}-1}{2}\right)}$$

$$\Rightarrow \frac{\frac{\sqrt{3}+1}{2}}{\frac{2+\sqrt{3}+1}{2}} + \frac{\frac{\sqrt{3}-1}{2}}{\frac{2-\sqrt{3}+1}{2}}$$

$$\Rightarrow \frac{\frac{\sqrt{3}+1}{2}}{(3+\sqrt{3})} + \frac{\frac{\sqrt{3}-1}{2}}{(3-\sqrt{3})}$$

$$\Rightarrow \frac{(\sqrt{3}+1)}{\sqrt{3}(\sqrt{3}+1)} + \frac{\sqrt{3}-1}{\sqrt{3}(\sqrt{3}-1)}$$

$$\Rightarrow \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Ex.62: If $x = \frac{\sqrt{3}}{2}$, Find the value of

$$\frac{1+x}{1+\sqrt{1+x}} + \frac{1-x}{1-\sqrt{1-x}}$$

Sol. When $x = \frac{\sqrt{3}}{2}$

$$\text{Then, } \sqrt{1+x} = \frac{\sqrt{3}+1}{2}$$

(Explanation above question 61)

Put the value:

$$\Rightarrow \frac{1+\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}+1}{2}} + \frac{1-\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}-1}{2}}$$

$$\Rightarrow \frac{\frac{2+\sqrt{3}}{2}}{\frac{2+\sqrt{3}+1}{2}} + \frac{\frac{2-\sqrt{3}}{2}}{\frac{2-\sqrt{3}+1}{2}}$$

$$\Rightarrow \frac{2+\sqrt{3}}{3+\sqrt{3}} + \frac{2-\sqrt{3}}{3-\sqrt{3}} = 1$$

$$\begin{aligned} &= \frac{(2+\sqrt{3})(3-\sqrt{3}) + (2-\sqrt{3})(3+\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})} \\ &= \frac{6-2\sqrt{3}+3\sqrt{3}-3+6+2\sqrt{3}-3\sqrt{3}-3}{(3)^2-(\sqrt{3})^2} \\ &= \frac{6}{9-3} = \frac{6}{6} = 1 \end{aligned}$$

TYPE - VII

Useful Result:-

(a) If $x = \frac{4\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$

$$\frac{x+2\sqrt{a}}{x-2\sqrt{a}} + \frac{x+2\sqrt{b}}{x-2\sqrt{b}} = 2$$

(b) If $x = \frac{2\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$

$$\frac{x+\sqrt{a}}{x-\sqrt{a}} + \frac{x+\sqrt{b}}{x-\sqrt{b}} = 2$$

Ex.63: If $x = \frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$, the value of

$$\frac{x+\sqrt{20}}{x-\sqrt{20}} + \frac{x+\sqrt{12}}{x-\sqrt{12}}$$

Sol. $x = \frac{2 \times 2 \times \sqrt{5} \times \sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$\Rightarrow \frac{x}{2\sqrt{5}} = \frac{2\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$\Rightarrow \frac{x}{\sqrt{20}} = \frac{2\sqrt{3}}{\sqrt{5}+\sqrt{3}} \text{ also}$$

$$\Rightarrow \frac{x}{\sqrt{12}} = \frac{2\sqrt{5}}{\sqrt{5}+\sqrt{3}}$$

Applying componendo & dividendo

$$\therefore \frac{x+\sqrt{20}}{x-\sqrt{20}} = \frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}} = -\frac{3\sqrt{3}+\sqrt{5}}{\sqrt{5}-\sqrt{3}}$$

$$\text{Similarly, } \frac{x+\sqrt{12}}{x-\sqrt{12}} = \frac{\sqrt{3}+3\sqrt{5}}{\sqrt{5}-\sqrt{3}}$$

$$\begin{aligned} &\frac{x+\sqrt{20}}{x-\sqrt{20}} + \frac{x+\sqrt{12}}{x-\sqrt{12}} \\ &= \frac{-3\sqrt{3}-\sqrt{5}+\sqrt{3}+3\sqrt{5}}{\sqrt{5}-\sqrt{3}} \end{aligned}$$

$$= \frac{2\sqrt{5}-2\sqrt{3}}{(\sqrt{5}-\sqrt{3})} = 2$$

Note: In these type of questions there is maximum possibility of having answer '2'.

EXERCISE

1. The value of $\sqrt{5+2\sqrt{6}} - \frac{1}{\sqrt{5+2\sqrt{6}}}$ is:

- (a) $2\sqrt{2}$ (b) $2\sqrt{3}$
(c) $1+\sqrt{5}$ (d) $\sqrt{5}-1$

2. Simplify : $\left(\frac{\frac{3}{2+\sqrt{3}} - \frac{2}{2-\sqrt{3}}}{2-5\sqrt{3}} \right)$

- (a) $\frac{1}{2} - 5\sqrt{3}$ (b) $2 - 5\sqrt{3}$
(c) 1 (d) 0

3. The value of $(243)^{0.16} \times (243)^{0.04}$ is equal to :

- (a) 0.16 (b) 3 (c) $\frac{1}{3}$ (d) 0.04

4. The simplification of

$$\frac{0.06 \times 0.06 \times 0.06 - 0.05 \times 0.05 \times 0.05}{0.06 \times 0.06 + 0.06 \times 0.05 + 0.05 \times 0.05}$$

- (a) 1 (b) 0.1
(c) 0.01 (d) 0.001

5. Simplify :

$$\frac{0.05 \times 0.05 \times 0.05 - 0.04 \times 0.04 \times 0.04}{0.05 \times 0.05 + 0.002 + 0.04 \times 0.04}$$

- (a) 1 (b) 0.1
(c) 0.01 (d) 0.001

6. Which one of the following is the least?

$$\sqrt{3}, \sqrt[3]{2}, \sqrt{2} \text{ and } \sqrt[3]{4}$$

- (a) $\sqrt{2}$ (b) $\sqrt[3]{2}$ (c) $\sqrt{3}$ (d) $\sqrt[3]{3}$

7. Which one of the following is the biggest?

$$\sqrt[3]{4}, \sqrt[4]{6}, \sqrt[6]{15}, \text{ and } \sqrt[12]{245}$$

- (a) $\sqrt[3]{4}$ (b) $\sqrt[4]{6}$
(c) $\sqrt[6]{15}$ (d) $\sqrt[12]{245}$

8. Simplify :

$$\left[\sqrt[3]{\sqrt[6]{5^9}} \right]^4 \left[\sqrt[3]{\sqrt[6]{5^9}} \right]^4$$

- (a) 5^2 (b) 5^4 (c) 5^8 (d) 5^{12}

9. If $3^{x+8} = 27^{2x+1}$, the value of x is :

- (a) 7 (b) 3 (c) -2 (d) 1

10. The simplified form of

$$\left(16^{3/2} + 16^{-3/2} \right) \text{ is :}$$

- (a) 0 (b) $\frac{4097}{64}$
(c) 1 (d) $\frac{16}{4097}$

11. $(0.01024)^{1/5}$ is equal to :

- (a) 4.0 (d) 0.04
(c) 0.4 (d) 0.00004

12. $(64)^{-2/3} \times \left(\frac{1}{4} \right)^{-2}$ is equal to :

- (a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{16}$

13. $\left(\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} \right)$ simplifies to :

- (a) $\sqrt{5} + \sqrt{6}$ (b) $2\sqrt{5} + \sqrt{6}$
(c) $\sqrt{5} - \sqrt{6}$ (d) $2\sqrt{5} - 3\sqrt{6}$

14. $\left(\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1} \right)$ simplifies to:

- (a) $2 - \sqrt{3}$ (b) $2 + \sqrt{3}$
(c) $16 - \sqrt{3}$ (d) $40 - \sqrt{3}$

15. $\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \right)^2 + \left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \right)^2$ is equal to:

- (a) 64 (b) 62 (c) 66 (d) 68

16. $(7.5 \times 7.5 + 37.5 + 2.5 \times 2.5)$ is equal to:

- (a) 100 (b) 80 (c) 60 (d) 30

17. $\left(\frac{8}{125} \right)^{-4/3}$ simplifies to :

- (a) $\frac{625}{16}$ (b) $\frac{625}{8}$
(c) $\frac{625}{32}$ (d) $\frac{16}{625}$

18. The value of

$$\sqrt{\frac{(\sqrt{12}-\sqrt{8})(\sqrt{3}+\sqrt{2})}{5+\sqrt{24}}} \text{ is :}$$

- (a) $\sqrt{6} - \sqrt{2}$ (b) $\sqrt{6} + \sqrt{2}$
(c) $\sqrt{6} - 2$ (d) $2 - \sqrt{6}$

19. Simplify :

$$\left[64^{\frac{2}{3}} \times 2^{-2} \div 8^0 \right]^{\frac{1}{2}}$$

- (a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

20. The value of $\frac{1}{\sqrt{(12-\sqrt{140})}} -$

$$\frac{1}{\sqrt{(8-\sqrt{60})}} - \frac{2}{\sqrt{10+\sqrt{84}}} \text{ is :}$$

- (a) 0 (b) 1 (c) 2 (d) 3

21. Which of the following number is the least?

$$(0.5)^2, \sqrt{0.49}, \sqrt[3]{0.008}, 0.23$$

- (a) $(0.5)^2$ (b) $\sqrt{0.49}$
(c) $\sqrt[3]{0.008}$ (d) 0.23

22. Arrange the following in descending order : $\sqrt[3]{4}, \sqrt{2}, \sqrt[4]{3}, \sqrt[4]{5}$

- (a) $\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[4]{3}$
(b) $\sqrt[3]{5} > \sqrt[3]{4} > \sqrt[4]{3} > \sqrt{2}$
(c) $\sqrt{2} > \sqrt[4]{5} > \sqrt[3]{4} > \sqrt[4]{3}$
(d) $\sqrt{2} > \sqrt[3]{4} > \sqrt[4]{5} > \sqrt[4]{3}$

23. The value of

$$\frac{(243)^{0.13} \times (243)^{0.07}}{(7)^{0.25} \times (49)^{0.075} \times (343)^{0.2}}$$

- (a) $\frac{3}{7}$ (b) $\frac{7}{3}$ (c) $1\frac{3}{7}$ (d) $2\frac{2}{7}$

24. The value of :

$$\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$$

- (a) 1 (b) 2 (c) 3 (d) 8

25. $\sqrt[3]{0.004096}$ is equal to

- (a) 4 (b) 0.4
(c) 0.04 (d) 0.004

26. $\frac{2.3 \times 2.3 \times 2.3 - 1}{2.3 \times 2.3 + 2.3 + 1}$ is equal to

- (a) 1.3 (b) 3.3 (c) 0.3 (d) 2.2

27. The ascending order of

$$(2.89)^{0.5}, 2-(0.5)^2, \sqrt{3} \text{ and } \sqrt[3]{0.008} \text{ is}$$

- (a) $2-(0.5)^2, \sqrt{3}, \sqrt[3]{0.008}, (2.89)^{0.5}$
(b) $\sqrt[3]{0.008}, (2.89)^{0.5}, \sqrt{3}, 2-(0.5)^2$
(c) $\sqrt[3]{0.008}, \sqrt{3}, (2.89)^{0.5}, 2-(0.5)^2$
(d) $\sqrt{3}, \sqrt[3]{0.008}, 2-(0.5)^2, (2.89)^{0.5}$

28. $\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}$ is equal to
(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) 2 (d) 3

29. The value of $(\sqrt[3]{3.5} + \sqrt[3]{2.5})$
 $\left\{(\sqrt[3]{3.5})^2 - \sqrt[3]{8.75} + (\sqrt[3]{2.5})^2\right\}$ is :
(a) 5.375 (b) 1
(c) 6 (d) 5

30. The value of
 $(3+2\sqrt{2})^{-3} + (3-2\sqrt{2})^{-3}$ is
(a) 189 (b) 180 (c) 108 (d) 198

31. $\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} - \frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} + \frac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}}$ is equal to:
(a) 0 (b) $2\sqrt{15}$
(c) $2\sqrt{10}$ (d) $2\sqrt{6}$

32. $\frac{(0.96)^3 - (0.1)^3}{(0.96)^2 + 0.096 + (0.1)^2}$ is simplified to
(a) 1.06 (b) 0.95
(c) 0.86 (d) 0.97

33. When $(4+\sqrt{7})$ is presented in the form of perfect square it will be equal to :
(a) $(2+\sqrt{7})^2$ (b) $\left(\frac{\sqrt{7}}{2} + \frac{1}{2}\right)^2$

(c) $\left\{\frac{1}{\sqrt{2}}(\sqrt{7}+1)\right\}^2$ (d) $(\sqrt{3}+\sqrt{4})^2$
34. The simplified form of
 $\frac{2}{\sqrt{7}+\sqrt{5}} + \frac{7}{\sqrt{12}-\sqrt{5}} - \frac{5}{\sqrt{12}-\sqrt{7}}$ is :
(a) 5 (b) 2 (c) 1 (d) 0

35. $\frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}}$
 $+ \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{9}}$ is
(a) $\sqrt{3}$ (b) $3\sqrt{3}$
(c) $3-\sqrt{3}$ (d) $5-\sqrt{3}$

36. Simplify :
 $\frac{1}{\sqrt{100}-\sqrt{99}} - \frac{1}{\sqrt{99}-\sqrt{98}}$
 $+ \frac{1}{\sqrt{98}-\sqrt{97}} - \frac{1}{\sqrt{97}-\sqrt{96}}$
 $+ \dots + \frac{1}{\sqrt{2}-\sqrt{1}}$
(a) 10 (b) 9 (c) 13 (d) 11

37. $\left[\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}} + \frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}\right]$
in simplified form equals to :
(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 0

38. $\{(-2)^{-2}\}^{(-2)}$ is equal to :
(a) 16 (b) 8 (c) -8 (d) -1

39. Which is the greatest among
 $(\sqrt{19}-\sqrt{17})$, $(\sqrt{13}-\sqrt{11})$, $(\sqrt{7}-\sqrt{5})$
and $(\sqrt{5}-\sqrt{3})$?
(a) $\sqrt{19}-\sqrt{17}$ (b) $\sqrt{13}-\sqrt{11}$
(c) $\sqrt{7}-\sqrt{5}$ (d) $\sqrt{5}-\sqrt{3}$

40. If $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, then
(x+y) equals:
(a) 8 (b) 16
(c) $2\sqrt{15}$ (d) $2(\sqrt{5}+\sqrt{3})$

41. $0.75 \times 0.75 - 2 \times 0.75 \times 0.25 + 0.25 \times 0.25$ is equal to
(a) 250 (b) 2500
(c) 2.5 (d) 0.25

42. $\left(3 + \frac{1}{\sqrt{3}} + \frac{1}{3+\sqrt{3}} + \frac{1}{\sqrt{3}-3}\right)$ is equal to
(a) 1 (b) 3
(c) $3+\sqrt{3}$ (d) $3-\sqrt{3}$

43. $\left[8 - \left\{\frac{9}{4^4} \sqrt{2.2^2}\right\}^{\frac{1}{2}}\right]$ is equal to
(a) 32 (b) 8 (c) 1 (d) 0

44. $\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{2\sqrt{6}}{\sqrt{3}+1} + \frac{2\sqrt{3}}{\sqrt{6}+2}$ is equal to
(a) 3 (b) 2 (c) 0 (d) $\sqrt{3}$

45. Greatest among the numbers
 $\sqrt[3]{9}$, $\sqrt{3}$, $\sqrt[4]{16}$, $\sqrt[5]{80}$ is
(a) $\sqrt[3]{9}$ (b) $\sqrt{3}$
(c) $\sqrt[4]{16}$ (d) $\sqrt[5]{80}$

46. Given that $\sqrt{3} = 1.732$, the value of
 $\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$ is :
(a) 4.899 (b) 2.551
(c) 1.414 (d) 1.732

47. $\sqrt{12+\sqrt{12+\sqrt{12+\dots}}}$ is equal to
(a) 3 (b) 4 (c) 6 (d) 2

48. If $a = \frac{\sqrt{3}}{2}$, then the value of
 $\sqrt{1+a} + \sqrt{1-a}$ is :
(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$
(c) $2+\sqrt{3}$ (d) $2-\sqrt{3}$

49. If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$, $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, the value of
 $\left(\frac{a^2+ab+b^2}{a^2-ab+b^2}\right)$ is
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

50. $\left(\frac{2}{\sqrt{5}+\sqrt{3}} - \frac{3}{\sqrt{6}-\sqrt{3}} + \frac{1}{\sqrt{6}+\sqrt{5}}\right)$ is
equal to
(a) $-2\sqrt{6}$ (b) $-2\sqrt{5}$
(c) $-2\sqrt{3}$ (d) 0

51. $(\sqrt{2} + \sqrt{7-2\sqrt{10}})$ is equal to
(a) $\sqrt{2}$ (b) $\sqrt{7}$
(c) $\sqrt{5}$ (d) $2\sqrt{5}$

52. If $x = 1 + \sqrt{2} + \sqrt{3}$, then the value of
 $\left(x + \frac{1}{x-1}\right)$ is
(a) $1+2\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $3+\sqrt{2}$ (d) $2\sqrt{3}-1$

53. If m and n ($n > 1$) are whole numbers such
that $m^n = 121$, the value of $(m-1)^{n+1}$ is
(a) 1 (b) 10
(c) 121 (d) 1000

54. $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} -$
 $\frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} = ?$
(a) 5 (b) 4 (c) 3 (d) 2

55. $\left(\frac{2}{\sqrt{6}+2} + \frac{1}{\sqrt{7}+\sqrt{6}} + \frac{1}{\sqrt{8}-\sqrt{7}} + 2-2\sqrt{2}\right)$ is
equal to
(a) 0 (b) $2\sqrt{2}$
(c) $\sqrt{2}$ (d) $2\sqrt{7}$

56. $\left[\left\{\left(-\frac{1}{2}\right)^2\right\}^{-2}\right]^{-1}$ is equal to :
(a) $\frac{1}{16}$ (b) 16 (c) $-\frac{1}{16}$ (d) -16

57. The greatest number among $2^{60}, 3^{48}, 4^{36}$ and 5^{24} is
(a) 2^{60} (b) 3^{48} (c) 4^{36} (d) 5^{24}

58. $\sqrt{3}\sqrt{3}\sqrt{3}\dots$ is equal to
(a) $\sqrt{3}$ (b) 3
(c) $2\sqrt{3}$ (d) $3\sqrt{3}$

59. The greatest among the numbers $\sqrt{0.09}, \sqrt[3]{0.064}, 0.5$ and $\frac{3}{5}$
(a) $\sqrt{0.09}$ (b) $\sqrt[3]{0.064}$
(c) 0.5 (d) $\frac{3}{5}$

60. The greatest of the following numbers $0.16, \sqrt{0.16}, (0.16)^2, 0.04$ is
(a) 0.16 (b) $\sqrt{0.16}$
(c) 0.04 (d) $(0.16)^2$

61. The greatest of the numbers $\sqrt[2]{8}, \sqrt[4]{13}, \sqrt[5]{16}, \sqrt[10]{41}$ is:
(a) $\sqrt[4]{13}$ (b) $\sqrt[5]{16}$
(c) $\sqrt[10]{41}$ (d) $\sqrt[2]{8}$

62. If $2^x = 3^y = 6^{-z}$ then $\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$ is equal to
(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{1}{2}$

63. $2\sqrt[3]{40} - 4\sqrt[3]{320} + 3\sqrt[3]{625} - 3\sqrt[3]{5}$ is equal to
(a) $-2\sqrt[3]{340}$ (b) 0
(c) $\sqrt[3]{340}$ (d) $\sqrt[3]{660}$

64. The value of $\sqrt{40 + \sqrt{9 \times 81}}$ is
(a) $\sqrt{111}$ (b) 9
(c) 7 (d) 11

65. If $\frac{(x - \sqrt{24})(\sqrt{75} + \sqrt{50})}{\sqrt{75} - \sqrt{50}} = 1$, then the value of x is
(a) $\sqrt{5}$ (b) 5
(c) $2\sqrt{5}$ (d) $3\sqrt{5}$

66. Evaluate $\sqrt{20} + \sqrt{12} + \sqrt[3]{729} - \frac{4}{\sqrt{5} - \sqrt{3}} - \sqrt{81}$
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) 0 (d) $2\sqrt{2}$

67. If a, b are rationals and $a\sqrt{2} + b\sqrt{3} = \sqrt{98} + \sqrt{108} - \sqrt{48} - \sqrt{72}$, then the values of a, b are respectively
(a) 1, 2 (b) 1, 3
(c) 2, 1 (d) 2, 3

68. Let $\sqrt[3]{a} = \sqrt[3]{26} + \sqrt[3]{7} + \sqrt[3]{63}$ then
(a) $a < 729$ but $a > 216$
(b) $a < 216$
(c) $a > 729$
(d) $a = 729$

69. $2 + \frac{6}{\sqrt{3}} + \frac{1}{2 + \sqrt{3}} + \frac{1}{\sqrt{3} - 2}$ equals to
(a) $+(2\sqrt{3})$ (b) $-(2 + \sqrt{3})$
(c) 1 (d) 2

70. If $\frac{4 + 3\sqrt{3}}{\sqrt{7} + 4\sqrt{3}} = A + \sqrt{B}$, then $B - A$ is
(a) -13 (b) $2\sqrt{13}$
(c) 13 (d) $3\sqrt{3} - \sqrt{7}$

71. The smallest among the numbers $2^{250}, 3^{150}, 5^{100}$ and 4^{200}
(a) 4^{200} (b) 5^{100}
(c) 3^{150} (d) 2^{250}

72. Find the value of $\sqrt{30 + \sqrt{30 + \sqrt{30 + \dots}}}$
(a) 5 (b) $3\sqrt{10}$
(c) 6 (d) 7

73. The value of $\sqrt[3]{2\sqrt[3]{4}\sqrt[3]{2}\sqrt[3]{4} \dots}$ is
(a) 2 (b) 2^2 (c) 2^3 (d) 2^5

74. $55^3 + 17^3 - 72^3 + 201960$ is equal to
(a) -1 (b) 0 (c) 1 (d) 17

75. The value of $\frac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}}$ is
(a) 3 (b) 9 (c) 6 (d) 12

76. The simplified value of $(\sqrt{3} + 1)(10 + \sqrt{12})(\sqrt{12} - 2)(5 - \sqrt{3})$ is
(a) 16 (b) 88 (c) 176 (d) 132

77. If $2^{n-1} + 2^{n+1} = 320$, then the value of n is
(a) 6 (b) 8 (c) 5 (d) 7

78. If $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2}$, then the value of a is
(a) 4 (b) 5 (c) 6 (d) 8

79. The value of $(3 + 2\sqrt{2})^{-3} + (3 - 2\sqrt{2})^{-3}$ is
(a) 198 (b) 180 (c) 108 (d) 189

80. A tap is dripping at a constant rate into a container. The level (L cm) of the water in the container is given by the equation $L = 2 - 2^t$, where t is time taken in hours. Then the level of water in the container at the level of water in the container at the start is
(a) 0 cm (b) 1 cm (c) 2 cm (d) 4 cm

81. $\sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + \sqrt{225}}}}} = ?$
(a) 8 (b) 4 (c) $1/2$ (d) 2

82. The Simplified value of $\frac{\sqrt{6} + 2}{\sqrt{2} + \sqrt{2} + \sqrt{3}} - \frac{\sqrt{6} - 2}{\sqrt{2} - \sqrt{2} - \sqrt{3}} - \frac{2\sqrt{2}}{2 + \sqrt{2}}$
(a) $2\sqrt{6}$ (b) 2 (c) $\sqrt{3}$ (d) 0

83. $\frac{6^2 + 7^2 + 8^2 + 9^2 + 10^2}{\sqrt{7 + 4\sqrt{3}} - \sqrt{4 + 2\sqrt{3}}}$ is equal to/
(a) 330 (b) 355
(c) 305 (d) 366

84. $(3x - 2y) : (2x + 3y) = 5 : 6$, then one of the value of $\left(\frac{\sqrt[3]{x} + \sqrt[3]{y}}{\sqrt[3]{x} - \sqrt[3]{y}}\right)^2$ is
(a) $\frac{1}{25}$ (b) 5 (c) $\frac{1}{5}$ (d) 25

85. The value of $\frac{1}{1 + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \frac{1}{\sqrt{4} + \sqrt{5}} + \frac{1}{\sqrt{5} + \sqrt{6}} + \frac{1}{\sqrt{6} + \sqrt{7}} + \frac{1}{\sqrt{7} + \sqrt{8}} + \frac{1}{\sqrt{8} + \sqrt{9}}$ is
(a) 2 (b) 4 (c) 0 (d) 1

86. The value of $\sqrt{72 + \sqrt{72 + \sqrt{72 + \dots}}}$ is
(a) 9 (b) 18 (c) 8 (d) 12

87. The value of $\frac{1}{1 + \sqrt{2} + \sqrt{3}} + \frac{1}{1 - \sqrt{2} + \sqrt{3}}$ is:
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) 1 (d) $4(\sqrt{3} + \sqrt{2})$

88. The value of the expression $\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}$ is
(a) 30 (b) 5
(c) 3 (d) 2

89. The value of $\frac{3\sqrt{7}}{\sqrt{5} + \sqrt{2}} - \frac{5\sqrt{5}}{\sqrt{2} + \sqrt{7}} + \frac{2\sqrt{2}}{\sqrt{7} + \sqrt{5}}$ is:
(a) 1 (b) 0
(c) $2\sqrt{3}$ (d) $\sqrt{7}$

90. If $11\sqrt{n} = \sqrt{112} + \sqrt{343}$, then the value of n is:
(a) 3 (b) 11 (c) 13 (d) 7

ANSWER KEY

1. (a)	10. (b)	19. (c)	28. (c)	37. (c)	46. (d)	55. (d)	64. (c)	73. (a)	82. (d)
2. (c)	11. (c)	20. (a)	29. (c)	38. (a)	47. (b)	56. (a)	65. (b)	74. (b)	83. (a)
3. (b)	12. (a)	21. (c)	30. (d)	39. (d)	48. (a)	57. (b)	66. (c)	75. (b)	84. (d)
4. (c)	13. (c)	22. (a)	31. (a)	40. (a)	49. (b)	58. (b)	67. (a)	76. (c)	85. (a)
5. (c)	14. (c)	23. (a)	32. (c)	41. (d)	50. (c)	59. (d)	68. (a)	77. (d)	86. (a)
6. (b)	15. (b)	24. (b)	33. (c)	42. (b)	51. (c)	60. (b)	69. (d)	78. (a)	87. (c)
7. (a)	16. (a)	25. (b)	34. (d)	43. (d)	52. (a)	61. (d)	70. (c)	79. (a)	88. (c)
8. (b)	17. (a)	26. (a)	35. (c)	44. (c)	53. (d)	62. (a)	71. (b)	80. (b)	89. (b)
9. (d)	18. (c)	27. (b)	36. (d)	45. (a)	54. (a)	63. (b)	72. (c)	81. (d)	90. (d)

SOLUTION

$$1. (a) \sqrt{5+2\sqrt{6}} - \frac{1}{\sqrt{5+2\sqrt{6}}}$$

$$\Rightarrow (\sqrt{3} + \sqrt{2}) - \frac{1}{(\sqrt{3} + \sqrt{2})}$$

$$\left[\sqrt{5+2\sqrt{6}} = \sqrt{(\sqrt{3} + \sqrt{2})^2} \Rightarrow \sqrt{3} + \sqrt{2} \right]$$

$$a^2 + b^2 + 2ab = (a+b)^2$$

$$\Rightarrow \sqrt{3} + \sqrt{2} - \left(\frac{1}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} \right)$$

$$\Rightarrow \sqrt{3} + \sqrt{2} - \left(\frac{\sqrt{3} - \sqrt{2}}{3 - 2} \right)$$

$$\Rightarrow \sqrt{3} + \sqrt{2} - \sqrt{3} + \sqrt{2} \Rightarrow 2\sqrt{2}$$

$$2. (c) \frac{3}{2+\sqrt{3}} - \frac{2}{2-\sqrt{3}}$$

$$\Rightarrow \frac{3(2-\sqrt{3}) - 2(2+\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$\frac{6 - 3\sqrt{3} - 4 - 2\sqrt{3}}{2 - 5\sqrt{3}}$$

$$\Rightarrow \frac{2 - 5\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})(2-5\sqrt{3})}$$

$$= \frac{2 - 5\sqrt{3}}{2 - 5\sqrt{3}} = 1$$

$$3. (b) (243)^{0.16} \times (243)^{0.04} \left[a^m \times a^n = a^{m+n} \right]$$

$$\Rightarrow (243)^{0.16+0.04}$$

$$\Rightarrow 243^{0.20}$$

$$\Rightarrow 243^{\frac{20}{100}}$$

$$\Rightarrow 243^{\frac{1}{5}} \Rightarrow \sqrt[5]{243} = 3$$

4. (c)

$$\frac{0.06 \times 0.06 \times 0.06 - 0.05 \times 0.05 \times 0.05}{0.06 \times 0.06 + 0.06 \times 0.05 + 0.05 \times 0.05}$$

$$\Rightarrow \frac{0.06^3 - 0.05^3}{0.06^2 + 0.06 \times 0.05 + 0.05^2}$$

$$\Rightarrow \frac{a^3 - b^3}{a^2 + ab + b^2}$$

$$\Rightarrow \frac{(a-b)(a^2 + ab + b^2)}{(a^2 + ab + b^2)}$$

$$\Rightarrow a - b$$

$$\text{So, } a = 0.06, \quad b = 0.05$$

$$\Rightarrow 0.06 - 0.05 \Rightarrow 0.01$$

5. (c) $\frac{0.05 \times 0.05 \times 0.05 - 0.04 \times 0.04 \times 0.04}{0.05 \times 0.05 + 0.05 \times 0.04 + 0.04 \times 0.04}$

$$\Rightarrow \frac{(0.05)^3 - (0.04)^3}{0.05^2 + 0.05 \times 0.04 + 0.04^2}$$

(Description: same as above question)

$$a = 0.05, \quad b = 0.04$$

$$\Rightarrow a - b$$

$$\Rightarrow 0.01$$

6. (b) $\sqrt{3}, \sqrt[3]{2}, \sqrt{2}, \sqrt[3]{4}$

$$\Rightarrow \frac{1}{3^{\frac{1}{2}}}, \frac{1}{2^{\frac{1}{3}}}, \frac{1}{2^{\frac{1}{2}}}, \frac{1}{4^{\frac{1}{3}}}$$

(take LCM of 3 & 2)

$$\Rightarrow \frac{3}{3^{\frac{3}{2}}}, \frac{2}{2^{\frac{2}{3}}}, \frac{3}{2^{\frac{3}{2}}}, \frac{2}{4^{\frac{2}{3}}}$$

$$\Rightarrow \sqrt[3]{3^3}, \sqrt[3]{2^2}, \sqrt[3]{2^3}, \sqrt[3]{4^2}$$

$$\Rightarrow \sqrt[3]{27}, \sqrt[3]{4}, \sqrt[3]{8}, \sqrt[3]{16}$$

$$\Rightarrow \sqrt[3]{2} \text{ (Least)}$$

7. (a) $\sqrt[3]{4}, \sqrt[4]{6}, \sqrt[6]{15}, \sqrt[12]{245}$

$$\Rightarrow \frac{1}{4^{\frac{1}{3}}}, \frac{1}{6^{\frac{1}{4}}}, \frac{1}{15^{\frac{1}{6}}}, \frac{1}{245^{\frac{1}{12}}}$$

(take LCM of 3, 4, 12 & 6)

$$\Rightarrow \frac{4}{4^{\frac{4}{12}}}, \frac{3}{6^{\frac{3}{12}}}, \frac{2}{15^{\frac{2}{12}}}, \frac{1}{245^{\frac{1}{12}}}$$

$$\Rightarrow \sqrt[12]{4^4}, \sqrt[12]{3^3}, \sqrt[12]{2^2}, \sqrt[12]{245}$$

$$\Rightarrow \sqrt[12]{256}, \sqrt[12]{216}, \sqrt[12]{225}, \sqrt[12]{245}$$

$$\Rightarrow \text{Biggest} = \sqrt[3]{4}$$

$$8. (b) \left[\sqrt[3]{\frac{6}{5^9}} \right]^4 \left[\sqrt[3]{\frac{6}{5^9}} \right]^4$$

$$\Rightarrow \left[5^{9 \times \frac{1}{6} \times \frac{1}{3}} \right]^4 \left[5^{9 \times \frac{1}{3} \times \frac{1}{6}} \right]^4$$

$$\Rightarrow \left[\frac{1}{5^{\frac{1}{2}}} \right]^4 \left[\frac{1}{5^{\frac{1}{2}}} \right]^4$$

$$\Rightarrow 5^2 \times 5^2$$

$$\Rightarrow 5^{2+2} = 5^4$$

9. (d) $3^x + 8 = 27^{2x+1}$

$$3^x + 8 = (3^3)^{2x+1}$$

$$3^x + 8 = 3^{6x+3}$$

$$x + 8 = 6x + 3$$

$$5x = 5, \quad x = 1$$

$$10. (b) \left(16^{\frac{3}{2}} + 16^{\frac{-3}{2}} \right) \Rightarrow \left(16^{\frac{3}{2}} + \frac{1}{16^{\frac{3}{2}}} \right)$$

$$\Rightarrow \left(4^{2 \times \frac{3}{2}} + \frac{1}{4^{2 \times \frac{3}{2}}} \right) \Rightarrow 4^3 + \frac{1}{4^3}$$

$$\Rightarrow \frac{4097}{64}$$

11. (c) $(0.01024)^{\frac{1}{5}}$

$$\Rightarrow (0.4^5)^{\frac{1}{5}}$$

$$\Rightarrow 0.4^{5 \times \frac{1}{5}} = 0.4$$

12. (a) $64^{\frac{-2}{3}} \times \left(\frac{1}{4} \right)^{-2}$

$$\Rightarrow (4^3)^{\frac{-2}{3}} \times \left(\frac{1}{4} \right)^{-2}$$

$$\Rightarrow 4^{-2} \times \left(\frac{1}{4}\right)^{-2}$$

$$\Rightarrow \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right)^{-2} = \left(\frac{1}{4}\right)^{2-2}$$

$$\Rightarrow \left(\frac{1}{4}\right)^0 = 1$$

$$13. (c) \left(\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} \right)$$

$$\Rightarrow \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3}) + (1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$$

$$\Rightarrow \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{(\sqrt{5})^2 - (\sqrt{3})^2}$$

$$\Rightarrow \frac{2\sqrt{5}-2\sqrt{6}}{2} \Rightarrow \frac{2(\sqrt{5}-\sqrt{6})}{2}$$

$$\Rightarrow (\sqrt{5}-\sqrt{6})$$

$$14. (c)$$

$$\Rightarrow \left(\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}} + \frac{\sqrt{3}-1}{\sqrt{3}+1} \right)$$

$$\Rightarrow \left(\frac{(2+\sqrt{3})^2 + (2-\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})} + \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \right)$$

$$\Rightarrow \left(\frac{4+3+4\sqrt{3}+4+3-4\sqrt{3}}{4-3} + \frac{(\sqrt{3}-1)^2}{3-1} \right)$$

$$\Rightarrow \left(14 + \frac{3+1-2\sqrt{3}}{2} \right)$$

$$\Rightarrow 14 + \frac{2(2-\sqrt{3})}{2}$$

$$\Rightarrow 14 + 2 - \sqrt{3} = 16 - \sqrt{3}$$

$$15. (b) \left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \right)^2 \Rightarrow \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5}-\sqrt{3})^2}$$

$$\Rightarrow \frac{5+3+2\sqrt{15}}{5+3-2\sqrt{15}}$$

$$\Rightarrow \frac{8+2\sqrt{15}}{8-2\sqrt{15}}$$

$$\Rightarrow \frac{4+\sqrt{15}}{4-\sqrt{15}}$$

$$\text{Similarly: } \left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \right)^2 = \frac{4-\sqrt{15}}{4+\sqrt{15}}$$

Thus, the expression.

$$\Rightarrow \frac{4+\sqrt{15}}{4-\sqrt{15}} + \frac{4-\sqrt{15}}{4+\sqrt{15}}$$

$$\Rightarrow \frac{16+15+8\sqrt{15}+16+15-8\sqrt{15}}{(16-15)}$$

$$\Rightarrow 62$$

$$16. (a) a = 7.5 \text{ and } b = 2.5$$

$$\Rightarrow a \times a + 2ab + b \times b$$

$$\Rightarrow a^2 + 2ab + b^2$$

$$\Rightarrow (a+b)^2 = (7.5 + 2.5)^2$$

$$\Rightarrow (10)^2 \Rightarrow 100$$

$$17. (a) \left(\frac{8}{125} \right)^{\frac{-4}{3}}$$

$$\Rightarrow \left(\frac{125}{8} \right)^{\frac{4}{3}} \Rightarrow \left[\left(\frac{5}{2} \right)^3 \right]^{\frac{4}{3}}$$

$$\Rightarrow \left(\frac{5}{2} \right)^4 \Rightarrow \frac{625}{16}$$

$$18. (c) \left[\frac{(\sqrt{12}-\sqrt{8})(\sqrt{3}+\sqrt{2})}{5+\sqrt{24}} \right]$$

$$\Rightarrow \frac{\sqrt{36+\sqrt{24}-\sqrt{24}-\sqrt{16}}}{5+\sqrt{24}}$$

$$\Rightarrow \frac{\sqrt{6-4}}{5+\sqrt{24}}$$

$$\Rightarrow \frac{\sqrt{\frac{2}{5+\sqrt{24}} \times \frac{5-\sqrt{24}}{5-\sqrt{24}}}}$$

$$\Rightarrow \frac{\sqrt{2(5-\sqrt{24})}}{25-24}$$

$$\Rightarrow \sqrt{2(5-2\sqrt{6})}$$

$$\Rightarrow \sqrt{2((\sqrt{3})^2 + (\sqrt{2})^2 - 2\sqrt{3} \times \sqrt{2})}$$

$$\Rightarrow \sqrt{2(\sqrt{3}-\sqrt{2})^2}$$

$$\Rightarrow \sqrt{2}(\sqrt{3}-\sqrt{2})$$

$$\Rightarrow \sqrt{6}-2$$

$$19. (c) \left[64^{\frac{2}{3}} \times 2^{-2} \div 8^0 \right]^{\frac{1}{2}}$$

$$\Rightarrow \left((4)^{3 \times \frac{2}{3}} \times \left(\frac{1}{2} \right)^2 \div 1 \right)^{\frac{1}{2}}$$

$$\Rightarrow \left(4^2 \times \frac{1}{4} \div 1 \right)^{\frac{1}{2}}$$

$$\Rightarrow \left(16 \times \frac{1}{4} \right)^{\frac{1}{2}} = \sqrt{4} = 2$$

$$20. (a) \frac{1}{\sqrt{(12-\sqrt{140})}} - \frac{1}{\sqrt{(8-\sqrt{60})}}$$

$$- \frac{2}{\sqrt{(10+\sqrt{84})}}$$

$$\Rightarrow \frac{1}{\sqrt{12-\sqrt{4 \times 35}}} - \frac{1}{\sqrt{8-\sqrt{4 \times 15}}}$$

$$- \frac{2}{\sqrt{10+\sqrt{4 \times 21}}}$$

$$\Rightarrow \frac{1}{\sqrt{12-2\sqrt{35}}} - \frac{1}{\sqrt{8-2\sqrt{15}}}$$

$$- \frac{2}{\sqrt{10+2\sqrt{21}}}$$

$$\Rightarrow \frac{1}{\sqrt{(\sqrt{7})^2 + (\sqrt{5})^2 - 2\sqrt{7}\sqrt{5}}} -$$

$$\frac{1}{\sqrt{(5)^2 + (\sqrt{3})^2 - 2\sqrt{5}\sqrt{3}}} -$$

$$\frac{2}{\sqrt{(\sqrt{7})^2 + (\sqrt{3})^2 + 2\sqrt{7}\sqrt{3}}}$$

$$\Rightarrow \frac{1}{\sqrt{(\sqrt{7}-\sqrt{5})^2}} - \frac{1}{\sqrt{(\sqrt{5}-\sqrt{3})^2}} -$$

$$\frac{2}{\sqrt{(\sqrt{7}+\sqrt{3})^2}}$$

$$\Rightarrow \frac{1}{\sqrt{7}-\sqrt{5}} - \frac{1}{\sqrt{5}-\sqrt{3}} - \frac{2}{\sqrt{7}+\sqrt{3}}$$

Rationalizing in above equation.

$$\Rightarrow \frac{1}{\sqrt{7}-\sqrt{5}} \times \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} - \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} - \frac{\sqrt{7}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$\Rightarrow \frac{\sqrt{7}+\sqrt{5}}{2} - \frac{(\sqrt{5}+\sqrt{3})}{2} - \frac{(\sqrt{7}-\sqrt{3})}{2}$$

$$\Rightarrow \frac{\sqrt{7}+\sqrt{5}-\sqrt{5}-\sqrt{3}-\sqrt{7}+\sqrt{3}}{2}$$

$$\Rightarrow 0$$

21. (c)

$$(0.5)^2, \sqrt{0.49}, \sqrt[3]{0.008}, 0.23$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$0.25 \quad 0.7 \quad \boxed{0.2} \quad 0.23$$

least

22. (a)

Descending order:

$$\sqrt[12]{256} > \sqrt[12]{125} > \sqrt[12]{64} > \sqrt[12]{9}$$

$$\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[5]{3}$$

23. (a) $\frac{(243)^{0.13} \times (243)^{0.07}}{7^{0.25} \times 49^{0.075} \times 343^{0.2}}$

$$\Rightarrow \frac{243^{0.13+0.07}}{7^{0.25} \times 7^{2 \times 0.075} \times 7^{3 \times 0.2}}$$

$$\Rightarrow \frac{3^{5 \times 0.20}}{7^{0.25+0.15+0.6}} \Rightarrow \frac{3^1}{7^1} = \frac{3}{7}$$

24. (b) $\sqrt{-\sqrt{3} + \sqrt{3+8\sqrt{7+4\sqrt{3}}}}$

$$\Rightarrow \sqrt{-\sqrt{3} + \sqrt{3+8\sqrt{4+3+2 \times 2 \times \sqrt{3}}}}$$

$$\Rightarrow \sqrt{-\sqrt{3} + \sqrt{3+8\sqrt{(2+\sqrt{3})^2}}}$$

$$\Rightarrow \sqrt{-\sqrt{3} + \sqrt{3+8(2+\sqrt{3})}}$$

$$\Rightarrow \sqrt{-\sqrt{3} + \sqrt{3+16+8\sqrt{3}}}$$

$$\Rightarrow \sqrt{-\sqrt{3} + \sqrt{(\sqrt{3})^2 + (4)^2 + 2 \times 4 \times \sqrt{3}}}$$

$$\Rightarrow \sqrt{-\sqrt{3} + \sqrt{(4+\sqrt{3})^2}}$$

$$\Rightarrow \sqrt{-\sqrt{3} + 4 + \sqrt{3}}$$

$$\Rightarrow \sqrt{4} = 2$$

25. (b) $\sqrt[3]{0.004096}$

$$\Rightarrow \sqrt{0.16} \quad (16^3 = 4096)$$

$$\Rightarrow \sqrt{0.4 \times 0.4}$$

$$\Rightarrow 0.4$$

26. (a) $\frac{2.3 \times 2.3 \times 2.3 - 1}{2.3 \times 2.3 + 2.3 + 1}$

$a = 2.3$
 $b = 1$

$$\Rightarrow \frac{a^3 - b^3}{a^2 + ab + b^2}$$

$$\Rightarrow \frac{(a-b)(a^2 + ab + b^2)}{(a^2 + ab + b^2)}$$

$$\Rightarrow 2.3 - 1 = 1.3$$

27. (b) $(2.89)^{0.5}, 2-(0.5)^2, \sqrt{3}, \sqrt[3]{0.008}$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$2.89^{1/2} \quad 2-0.25 \quad \sqrt{3} \quad 0.2$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$1.7 \quad 1.75 \quad 1.732 \quad 0.2$$

Ascending order :

$$0.2 < 1.7 < 1.732 < 1.75$$

$$\sqrt[3]{0.008} < 2.89^{0.5} < \sqrt{3} < 2-(0.5)^2$$

28. (c) $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}$

$$x^2 = 2 + \sqrt{2 + \sqrt{2 + \dots}}$$

$$x^2 = 2 + x$$

$$x^2 - x - 2 = 0$$

$$x^2 - 2x + x - 2 = 0$$

$$x(x-2) + 1(x-2) = 0$$

$$(x+1)(x-2) = 0$$

$$x = 2$$

Shortcut Method

When the question is in this form

i.e. $\sqrt{x + \sqrt{x + \sqrt{x + \dots}}}$

Then factor the $\begin{matrix} x \\ n1 \quad n2 \\ mi.diff. \end{matrix}$ $n1 > n2$

So n1 is answer

$$\sqrt{2 + \sqrt{2 + 2}}$$

$$\textcircled{2} \times 1$$

29. (c) $(\sqrt[3]{3.5} + \sqrt[3]{2.5})$

$$\left\{ (\sqrt[3]{3.5})^2 - \sqrt[3]{8.75} + (\sqrt[3]{2.5})^2 \right\}$$

$$x = \sqrt[3]{3.5}$$

$$y = \sqrt[3]{2.5}$$

$$\Rightarrow (x+y)(x^2 - xy + y^2)$$

$$\Rightarrow x^3 + y^3$$

$$\Rightarrow (\sqrt[3]{3.5})^3 + (\sqrt[3]{2.5})^3$$

$$\Rightarrow 3.5 + 2.5 = 6$$

30. (d) $(3+2\sqrt{2})^{-3} + (3-2\sqrt{2})^{-3}$

$$\Rightarrow \left(\frac{1}{3+2\sqrt{2}} \right)^3 + \left(\frac{1}{3-2\sqrt{2}} \right)^3$$

$$\Rightarrow \left(\frac{1}{(3+2\sqrt{2}) \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}} \right)^3 +$$

$$\left(\frac{1}{(3+2\sqrt{2}) \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}} \right)^3$$

$$\Rightarrow \left(\frac{3-2\sqrt{2}}{9-8} \right)^3 + \left(\frac{3+2\sqrt{2}}{9-8} \right)^3$$

$$\Rightarrow (3-2\sqrt{2})^3 + (3+2\sqrt{2})^3$$

$a = 3 - 2\sqrt{2}$
 $b = 3 + 2\sqrt{2}$

$$a^3 + b^3 \Rightarrow (a+b)(a^2 + b^2 - ab)$$

$$\Rightarrow (3-2\sqrt{2} + 3+2\sqrt{2})(17+17-1)$$

$$\Rightarrow (6)(33)$$

$$\Rightarrow 198$$

31. (a) $\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} - \frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} + \frac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}}$

$$\Rightarrow \frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} \times \frac{(\sqrt{3}-\sqrt{2})}{\sqrt{3}-\sqrt{2}} -$$

$$\left(\frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} \right) + \frac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}} \times \frac{(\sqrt{5}-\sqrt{3})}{\sqrt{5}-\sqrt{3}}$$

$$\Rightarrow \frac{\sqrt{15}-\sqrt{10}}{3-2} - \frac{3\sqrt{3}(\sqrt{5}-\sqrt{2})}{5-2}$$

$$+ \frac{2\sqrt{2}(\sqrt{5}-\sqrt{3})}{5-3}$$

$$= \sqrt{15} - \sqrt{10} - (\sqrt{15} - \sqrt{6}) + \sqrt{10} - \sqrt{6}$$

$$\Rightarrow \sqrt{15} - \sqrt{10} - \sqrt{15} + \sqrt{6} + \sqrt{10} - \sqrt{6}$$

$$\Rightarrow 0$$

32. (c) $\frac{0.96^3 - 0.1^3}{0.96^2 + 0.096 + 0.1^2}$

$$\Rightarrow a = 0.96$$

$$\Rightarrow b = 0.1$$

$$\Rightarrow \frac{a^3 - b^3}{a^2 + ab + b^2}$$

$$\Rightarrow \frac{(a-b)(a^2+b^2+ab)}{(a^2+ab+b^2)}$$

$$\Rightarrow a-b$$

$$\Rightarrow 0.96 - 0.1$$

$$= 0.86$$

33. (c) $4 + \sqrt{7}$

$$\Rightarrow \frac{8 + 2\sqrt{7}}{2}$$

$$\Rightarrow \frac{(\sqrt{7})^2 + 1 + 2 \cdot \sqrt{7} \cdot 1}{2}$$

$$\Rightarrow \frac{(\sqrt{7}+1)^2}{(\sqrt{2})^2} \Rightarrow \left\{ \frac{1}{\sqrt{2}}(\sqrt{7}+1) \right\}^2$$

34. (d) $\frac{2}{\sqrt{7}+\sqrt{5}} + \frac{7}{\sqrt{12}-\sqrt{5}} - \frac{5}{\sqrt{12}-\sqrt{7}}$

$$\Rightarrow \frac{2}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} +$$

$$\frac{7}{\sqrt{12}-\sqrt{5}} \times \frac{(\sqrt{12}+\sqrt{5})}{(\sqrt{12}+\sqrt{5})} -$$

$$\left(\frac{5}{\sqrt{12}-\sqrt{7}} \times \frac{\sqrt{12}+\sqrt{7}}{\sqrt{12}+\sqrt{7}} \right)$$

$$\Rightarrow \frac{2(\sqrt{7}-\sqrt{5})}{2} + \frac{7(\sqrt{12}+\sqrt{5})}{7} -$$

$$\left(\frac{5(\sqrt{12}+\sqrt{7})}{5} \right)$$

$$\Rightarrow \sqrt{7}-\sqrt{5} + \sqrt{12}+\sqrt{5} - \sqrt{12}-\sqrt{7}$$

$$\Rightarrow 0$$

35. (c) $\frac{1}{\sqrt{3}+\sqrt{4}}$

$$\Rightarrow \frac{1}{\sqrt{4}+\sqrt{3}} \times \frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}}$$

$$\Rightarrow \frac{\sqrt{4}-\sqrt{3}}{1}$$

$$\Rightarrow \sqrt{4}-\sqrt{3}$$

Similarly $\Rightarrow \frac{1}{\sqrt{4}+\sqrt{5}} = \sqrt{5}-\sqrt{4}$

$$\Rightarrow \frac{1}{\sqrt{5}+\sqrt{6}} \Rightarrow \sqrt{6}-\sqrt{5}$$

$$\Rightarrow \frac{1}{\sqrt{6}+\sqrt{7}} \Rightarrow \sqrt{7}-\sqrt{6}$$

$$\Rightarrow \frac{1}{\sqrt{7}+\sqrt{8}} = \sqrt{8}-\sqrt{7}$$

$$\Rightarrow \frac{1}{\sqrt{8}-\sqrt{9}} \Rightarrow \sqrt{9}-\sqrt{8}$$

Now put values

$$\Rightarrow \sqrt{4}-\sqrt{3}+\sqrt{5}-\sqrt{4}+\sqrt{6}-\sqrt{5}+\sqrt{7}-$$

$$\sqrt{6}+\sqrt{8}-\sqrt{7}+\sqrt{9}-\sqrt{8}$$

$$\Rightarrow \sqrt{9}-\sqrt{3} \Rightarrow 3-\sqrt{3}$$

36. (d) $\frac{1}{\sqrt{100}-\sqrt{99}} \times \frac{\sqrt{100}+\sqrt{99}}{\sqrt{100}+\sqrt{99}}$

$$\Rightarrow \frac{\sqrt{100}+\sqrt{99}}{1} \Rightarrow \sqrt{100}+\sqrt{99}$$

Similarly

$$\Rightarrow \frac{1}{\sqrt{99}-\sqrt{98}} \Rightarrow \sqrt{99}+\sqrt{98}$$

$$\Rightarrow \frac{1}{\sqrt{98}-\sqrt{97}} \Rightarrow \sqrt{98}+\sqrt{97} \dots \text{and}$$

soon

Now : expression:

$$\Rightarrow \sqrt{100}+\sqrt{99}-\sqrt{99}-\sqrt{98}+\sqrt{98}+\sqrt{97}$$

$$\dots + \sqrt{2}+1$$

$$\Rightarrow \sqrt{100}+1 \Rightarrow 10+1=11$$

37. (c) $\left[\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}} + \frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}} \right]$

$$\Rightarrow \frac{1}{(\sqrt{2}+\sqrt{3})-(\sqrt{5})} \times \frac{(\sqrt{2}+\sqrt{3})+(\sqrt{5})}{(\sqrt{2}+\sqrt{3})+(\sqrt{5})}$$

$$\Rightarrow \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{2+3+2\sqrt{6}-5} \Rightarrow \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{2\sqrt{6}}$$

Similarly

$$\Rightarrow \frac{1}{(\sqrt{2}-\sqrt{3})-(\sqrt{5})} + \frac{(\sqrt{2}-\sqrt{3})+(\sqrt{5})}{(\sqrt{2}-\sqrt{3})+(\sqrt{5})}$$

$$\Rightarrow \frac{\sqrt{2}-\sqrt{3}+\sqrt{5}}{-2\sqrt{6}}$$

Now put the value in question

$$\Rightarrow \frac{(\sqrt{2}+\sqrt{3}+\sqrt{5})}{(2\sqrt{6})} - \frac{(\sqrt{2}-\sqrt{3}+\sqrt{5})}{(2\sqrt{6})}$$

$$\Rightarrow \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}-\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}}$$

$$\Rightarrow \frac{2\sqrt{3}}{2\sqrt{6}} \Rightarrow \frac{1}{\sqrt{2}}$$

38. (a) $\left\{ (-2)^{(-2)} \right\}^{-2}$

$$\Rightarrow \frac{1}{\left\{ (-2)^{(-2)} \right\}^2} \Rightarrow \frac{1}{(-2)^{-4}}$$

$$\Rightarrow (-2)^4 = 16$$

39. (d) $(\sqrt{19}-\sqrt{17}) \Rightarrow (\sqrt{19}-\sqrt{17}) \times$

$$\frac{(\sqrt{19}+\sqrt{17})}{\sqrt{19}+\sqrt{17}} \Rightarrow \frac{19-17}{\sqrt{19}+\sqrt{17}} = \frac{2}{\sqrt{19}+\sqrt{17}}$$

Similarly $(\sqrt{13}-\sqrt{11}) \Rightarrow \frac{2}{\sqrt{13}+\sqrt{11}}$

$$\boxed{(\sqrt{5}-\sqrt{3})} \Rightarrow \boxed{\frac{2}{\sqrt{5}+\sqrt{3}}}$$

Largest + (Because, Same Numerator is divided by Smallest denominator)

40. (a) $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} \Rightarrow \frac{(\sqrt{5}+\sqrt{3})^2}{2}$

$\Rightarrow$ Similarly

$$y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \Rightarrow \frac{(\sqrt{5}-\sqrt{3})^2}{2}$$

$$\Rightarrow x+y$$

$$\Rightarrow \frac{5+3+2\sqrt{15}+5+3-2\sqrt{15}}{2}$$

$$\Rightarrow \frac{16}{2} = 8$$

41. (d) $0.75 = a, \quad 0.25 = b$

$$\Rightarrow a \times a - 2 \times a \times b + b \times b$$

$$\Rightarrow a^2 - 2ab + b^2 \Rightarrow (a-b)^2$$

$$\Rightarrow (0.75 - 0.25)^2$$

$$\Rightarrow (0.50)^2 = 0.2500$$

42. (b) $3 + \frac{1}{\sqrt{3}} + \frac{1}{3+\sqrt{3}} + \frac{1}{\sqrt{3}-3}$

$$\Rightarrow 3 + \frac{1}{\sqrt{3}} + \frac{1}{3+\sqrt{3}} - \frac{1}{3-\sqrt{3}}$$

$$\Rightarrow 3 + \frac{1}{\sqrt{3}} + \left[\frac{3-\sqrt{3}-3-\sqrt{3}}{9-3} \right]$$

$$\Rightarrow 3 + \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{3} \Rightarrow 3 + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} = 3$$

$$\begin{aligned}
 43. (d) & \left[8 - \left(\frac{4^{\frac{9}{4}} \sqrt{2 \times 2^2}}{2\sqrt{2^{-2}}} \right)^{\frac{1}{2}} \right] \\
 & \Rightarrow \left[8 - \left(\frac{2^{2 \times \frac{9}{4}} \sqrt{2^{1+2}}}{2\sqrt{\frac{1}{4}}} \right)^{1/2} \right] \\
 & \Rightarrow \left[8 - \left(\frac{2^{9/2} \cdot 2^{3/2}}{2 \times \frac{1}{2}} \right)^{1/2} \right] \\
 & \Rightarrow \left[8 - \left(2^{12/2} \right)^{1/2} \right] \\
 & \Rightarrow \left[8 - \left(2^{6 \times \frac{1}{2}} \right) \right] \\
 & \Rightarrow [8 - 8] = 0 \text{ Ans.}
 \end{aligned}$$

$$\begin{aligned}
 44. (c) & \frac{3\sqrt{2}}{\sqrt{6} + \sqrt{3}} - \frac{2\sqrt{6}}{\sqrt{3} + 1} + \frac{2\sqrt{3}}{\sqrt{6} + 2} \\
 & \Rightarrow \left(\frac{3\sqrt{2}}{\sqrt{6} + \sqrt{3}} \times \frac{(\sqrt{6} - \sqrt{3})}{(\sqrt{6} - \sqrt{3})} \right) - \left(\frac{2\sqrt{6}}{\sqrt{3} + 1} \times \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)} \right) \\
 & \quad + \left(\frac{2\sqrt{3}}{\sqrt{6} + 2} \times \frac{(\sqrt{6} - 2)}{(\sqrt{6} - 2)} \right) \\
 & \Rightarrow \frac{3\sqrt{2}(\sqrt{6} - \sqrt{3})}{3} - \left(\frac{2\sqrt{6}(\sqrt{3} - 1)}{2} \right) \\
 & \quad + \frac{2\sqrt{3} \times (\sqrt{6} - 2)}{2} \\
 & \Rightarrow \sqrt{12} - \sqrt{6} - \sqrt{18} + \sqrt{6} + \sqrt{18} - 2\sqrt{3} \\
 & \Rightarrow \sqrt{12} - 2\sqrt{3} \\
 & \Rightarrow 2\sqrt{3} - 2\sqrt{3} = 0
 \end{aligned}$$

45. (a)

$$\begin{array}{cccc}
 \sqrt[3]{9} & \sqrt{3} & \sqrt[4]{16} & \sqrt[6]{80} \\
 \downarrow & \downarrow & \downarrow & \downarrow \\
 9^{1/3} & 3^{1/2} & 16^{1/4} & 80^{1/6} \\
 \downarrow & \downarrow & \downarrow & \downarrow \\
 9^{1/12} & 3^{6/12} & 16^{3/12} & 80^{2/12} \\
 \downarrow & \downarrow & \downarrow & \downarrow \\
 12\sqrt[4]{9^4} & 12\sqrt[4]{27^2} & 12\sqrt[4]{16^3} & 12\sqrt[4]{80^2}
 \end{array}$$

Square of 81 is largest. So Ans $\sqrt[3]{9}$

$$\begin{aligned}
 46. (d) & \frac{3 + \sqrt{6}}{5\sqrt{3} - 2\sqrt{12} - \sqrt{32} + \sqrt{50}} \\
 & \Rightarrow \frac{3 + \sqrt{6}}{5\sqrt{3} - 2 \times 2\sqrt{3} - 4\sqrt{2} + 5\sqrt{2}} \\
 & \Rightarrow \frac{3 + \sqrt{6}}{5\sqrt{3} - 4\sqrt{3} - 4\sqrt{2} + 5\sqrt{2}} \\
 & \Rightarrow \frac{3 + \sqrt{6}}{\sqrt{3} + \sqrt{2}} \Rightarrow \frac{\sqrt{3}(\sqrt{3} + \sqrt{2})}{\sqrt{3} + \sqrt{2}} \\
 & \Rightarrow \sqrt{3} = 1.732
 \end{aligned}$$

$$\begin{aligned}
 47. (b) & \sqrt{12 + \sqrt{12 + \sqrt{12 + \dots}}} \\
 & \quad \swarrow \quad \searrow \\
 & \quad \textcircled{4} \times 3
 \end{aligned}$$

$$\begin{aligned}
 48. (a) & a = \frac{\sqrt{3}}{2} \\
 & \Rightarrow a + 1 = \frac{\sqrt{3}}{2} + 1 \\
 & \Rightarrow \frac{\sqrt{3} + 2}{2} \\
 & \Rightarrow \frac{4 + 2\sqrt{3}}{4} = \frac{(\sqrt{3} + 1)^2}{4} \\
 & a + 1 = \frac{(\sqrt{3} + 1)^2}{4} \\
 & \Rightarrow \sqrt{a + 1} = \sqrt{\frac{(\sqrt{3} + 1)^2}{4}}
 \end{aligned}$$

$$\sqrt{1 + a} = \frac{\sqrt{3} + 1}{2}$$

Similarly,

$$\sqrt{1 - a} = \frac{\sqrt{3} - 1}{2}$$

put values :

$$\begin{aligned}
 & \frac{\sqrt{3} + 1}{2} + \frac{\sqrt{3} - 1}{2} \\
 & \Rightarrow \frac{\sqrt{3} + 1 + \sqrt{3} - 1}{2} \\
 & = \frac{2\sqrt{3}}{2} \\
 & \Rightarrow \sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 49. (b) & a + b = \frac{\sqrt{5} + 1}{\sqrt{5} - 1} + \frac{\sqrt{5} - 1}{\sqrt{5} + 1} \\
 & \Rightarrow \frac{(\sqrt{5} + 1)^2 + (\sqrt{5} - 1)^2}{(\sqrt{5} - 1)(\sqrt{5} + 1)} \\
 & \Rightarrow \frac{2[(\sqrt{5})^2 + 1]}{5 - 1} = \frac{2(5 + 1)}{4} = 3 \\
 & a \cdot b = \frac{\sqrt{5} + 1}{\sqrt{5} - 1} \times \frac{\sqrt{5} - 1}{\sqrt{5} + 1} = 1
 \end{aligned}$$

put value in expression

$$\begin{aligned}
 & \Rightarrow \frac{a^2 + ab + b^2}{a^2 - ab + b^2} = \frac{(a + b)^2 - ab}{(a + b)^2 - 3ab} \\
 & = \frac{3^2 - 1}{3^2 - 3} = \frac{9 - 1}{9 - 3} = \frac{4}{3}
 \end{aligned}$$

50. (c)

$$\begin{aligned}
 & \left[\frac{2}{\sqrt{5} + \sqrt{3}} - \frac{3}{\sqrt{6} - \sqrt{3}} + \frac{1}{\sqrt{6} + \sqrt{5}} \right] \\
 & \Rightarrow \frac{2}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}} - \frac{3}{\sqrt{6} - \sqrt{3}} \times \frac{\sqrt{6} + \sqrt{3}}{\sqrt{6} + \sqrt{3}} + \frac{1}{\sqrt{6} + \sqrt{5}} \times \frac{\sqrt{6} - \sqrt{5}}{\sqrt{6} - \sqrt{5}} \\
 & \Rightarrow \frac{2(\sqrt{5} - \sqrt{3})}{5 - 3} - \frac{3(\sqrt{6} + \sqrt{3})}{6 - 3} + \frac{\sqrt{6} - \sqrt{5}}{6 - 5} \\
 & \Rightarrow \sqrt{5} - \sqrt{3} - \sqrt{6} - \sqrt{3} + \sqrt{6} - \sqrt{5} \\
 & \Rightarrow -2\sqrt{3}
 \end{aligned}$$

$$51. (c) \quad (\sqrt{2} + \sqrt{7 - 2\sqrt{10}})$$

$$\begin{aligned}
 & \Rightarrow \sqrt{2} + \sqrt{(5)^2 + (\sqrt{2})^2 - 2\sqrt{5}\sqrt{2}} \\
 & \Rightarrow \sqrt{2} + \sqrt{(\sqrt{5} - \sqrt{2})^2} \\
 & \Rightarrow \sqrt{2} + \sqrt{5} - \sqrt{2} \Rightarrow \sqrt{5}
 \end{aligned}$$

$$52. (a) \quad x = 1 + \sqrt{2} + \sqrt{3}$$

$$\begin{aligned}
 & \Rightarrow x + \frac{1}{x - 1} \\
 & \Rightarrow 1 + \sqrt{2} + \sqrt{3} + \frac{1}{1 + \sqrt{2} + \sqrt{3} - 1} \\
 & \Rightarrow 1 + \sqrt{2} + \sqrt{3} + \frac{\sqrt{3} - \sqrt{2}}{3 - 2} \\
 & \Rightarrow 1 + 2\sqrt{3}
 \end{aligned}$$

53. (d) $m^n = 121 = 11^2$

$\Rightarrow m = 11$

$\Rightarrow n = 2$

$\Rightarrow (m-1)^{n+1}$

$\Rightarrow (11-1)^{2+1} \Rightarrow 10^3 \Rightarrow 1000$

54. (a) $\frac{1}{3-\sqrt{8}}$

$\Rightarrow \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} \Rightarrow \frac{3+\sqrt{8}}{9-8}$

$\Rightarrow 3+\sqrt{8}$

Similarly,

$\Rightarrow \frac{1}{\sqrt{8}-\sqrt{7}} = \sqrt{8} + \sqrt{7}$

$\Rightarrow \frac{1}{\sqrt{7}-\sqrt{6}} \Rightarrow \sqrt{7} + \sqrt{6}$

$\Rightarrow \frac{1}{\sqrt{6}-\sqrt{5}} = \sqrt{6} + \sqrt{5}$

$\Rightarrow \frac{1}{\sqrt{5}-2} = \sqrt{5} + 2$

Put value in question

$\Rightarrow (3+\sqrt{8}) - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6})$

$-(\sqrt{6}+\sqrt{5}) + (\sqrt{5}+2)$

$\Rightarrow 3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$

$\Rightarrow 3+2=5$

55. (d)

$\frac{2}{\sqrt{6}+2} + \frac{1}{\sqrt{7}+\sqrt{6}} + \frac{1}{\sqrt{8}-\sqrt{7}} + 2 - 2\sqrt{2}$

$\Rightarrow \frac{2}{\sqrt{6}+2} \times \frac{\sqrt{6}-2}{\sqrt{6}-2} + \frac{1}{\sqrt{7}+\sqrt{6}} \times \frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}-\sqrt{6}}$

$+ \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} + 2 - 2\sqrt{2}$

$\Rightarrow \frac{2 \times (\sqrt{6}-2)}{6-4} + \frac{(\sqrt{7}-\sqrt{6})}{7-6} + \frac{\sqrt{8}+\sqrt{7}}{8-7}$

$+ 2 - 2\sqrt{2}$

$\Rightarrow \sqrt{6}-2 + \sqrt{7}-\sqrt{6} + \sqrt{8}+\sqrt{7} + 2 - 2\sqrt{2}$

$\Rightarrow \sqrt{6}-2 + \sqrt{7}-\sqrt{6} + 2\sqrt{2} + \sqrt{7} + 2 - 2\sqrt{2}$

$\Rightarrow 2\sqrt{7}$

56. (a)

$\left[\left\{ \left(-\frac{1}{2} \right)^2 \right\}^{-2} \right]^{-1}$

$\Rightarrow \left\{ \left(-\frac{1}{2} \right)^2 \right\}^{-2 \times -1}$

$\Rightarrow \left(-\frac{1}{2} \right)^{2 \times 2} \Rightarrow \left(-\frac{1}{2} \right)^4 \Rightarrow \frac{1}{16}$

57. (b) $2^{60} \rightarrow (2^5)^{12} \rightarrow (32)^{12}$

$\Rightarrow 3^{48} \rightarrow (3^4)^{12} \rightarrow (81)^{12}$ (Greatest)

$\Rightarrow 4^{36} \rightarrow (4^3)^{12} \rightarrow (64)^{12}$

$\Rightarrow 5^{24} \rightarrow (5^2)^{12} \rightarrow (25)^{12}$

58. (b) $\sqrt{3}\sqrt{3}\sqrt{3} \dots \dots \dots$

Shortcut method

$\Rightarrow$ When the question is from

$\Rightarrow \sqrt{n}\sqrt{n}\sqrt{n} \dots \dots \dots \infty$

$\Rightarrow$ So n is answer

$\Rightarrow 3$

59. (d) $\sqrt{0.09} \quad \sqrt[3]{0.064} \quad 0.5 \quad \frac{3}{5}$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $0.3 \quad 0.4 \quad 0.5 \quad 0.6$

60. (b) $0.16 \quad \sqrt{0.16} \quad (0.16)^2 \quad 0.04$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $0.16 \quad 0.40 \quad 0.0256 \quad 0.04$

61. (d) $\sqrt[2]{8} \quad \sqrt[4]{13} \quad \sqrt[5]{16} \quad \sqrt[10]{41}$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $8^{1/2} \quad 13^{1/4} \quad 16^{1/5} \quad 41^{1/10}$
 $8^{10/20} \quad 13^{5/20} \quad 16^{4/20} \quad 41^{2/20}$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $20\sqrt[10]{8} \quad 20\sqrt[13]{5} \quad 20\sqrt[16]{4} \quad 20\sqrt[41]{2}$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $20\sqrt[64]{5} \quad 20\sqrt[13]{5} \quad 20\sqrt[16]{4} \quad 20\sqrt[41]{2}$

Greatest = $\sqrt[2]{8}$

62. (a) $2^x = 3^y = 6^z = k$

$\Rightarrow 2 = k^{1/x}; 3 = k^{1/y}; 6 = k^{-1/z}$

$\therefore 2 \times 3 = 6$

$k^{1/x} \times k^{1/y} = k^{-1/z}$

$\frac{1}{x} + \frac{1}{y} = -\frac{1}{z}$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

63. (b) $2\sqrt[3]{40} \Rightarrow 2 \times \sqrt[3]{2 \times 2 \times 2 \times 5}$

$\Rightarrow 2 \times 2 \sqrt[3]{5}$

$\Rightarrow 4\sqrt[3]{5}$

$\Rightarrow 4\sqrt[3]{320}$

$\Rightarrow 4 \times \sqrt[3]{4 \times 4 \times 4 \times 5}$

$\Rightarrow 4 \times 4 \sqrt[3]{5}$

$\Rightarrow 16 \sqrt[3]{5}$

$\Rightarrow 3\sqrt[3]{625}$

$\Rightarrow 3 \times \sqrt[3]{5 \times 5 \times 5 \times 5} \Rightarrow 3 \times 5 \sqrt[3]{5}$

$\Rightarrow 15 \sqrt[3]{5}$

Now put the value in question

$\Rightarrow 4\sqrt[3]{5} - 16 \sqrt[3]{5} + 15 \sqrt[3]{5} - 3 \sqrt[3]{5}$

$\Rightarrow 19 \sqrt[3]{5} - 19 \sqrt[3]{5}$

$\Rightarrow 0$

64. (c) $\sqrt{40 + \sqrt{9 \times 81}}$

$\Rightarrow \sqrt{40 + \sqrt{9 \times 9}}$

$\Rightarrow \sqrt{40 + 9} \Rightarrow \sqrt{49} \Rightarrow 7$

65. (b) $\frac{(x - \sqrt{24})(\sqrt{75} + \sqrt{50})}{\sqrt{75} - \sqrt{50}} = 1$

$\Rightarrow (x - \sqrt{24}) = \frac{\sqrt{75} - \sqrt{50}}{\sqrt{75} + \sqrt{50}}$

$\Rightarrow (x - \sqrt{24}) = \frac{(\sqrt{75} - \sqrt{50})^2}{75 - 50}$

$\Rightarrow (x - \sqrt{24}) = \frac{75 + 50 - 2\sqrt{75}\sqrt{50}}{25}$

$\Rightarrow (x - \sqrt{24}) = \frac{125 - 2 \times 5\sqrt{3} \times 5\sqrt{2}}{25}$

$\Rightarrow (x - \sqrt{24}) = \frac{125 - 50\sqrt{6}}{25}$

$\Rightarrow (x - \sqrt{24}) = \frac{25(5 - 2\sqrt{6})}{25}$

$\Rightarrow x - 2\sqrt{6} = 5 - 2\sqrt{6}$
 $\Rightarrow x = 5$

66. (c) $\sqrt{20} + \sqrt{12} + \sqrt[3]{729} - \frac{4}{\sqrt{5}-\sqrt{3}} - \sqrt{81}$

$\Rightarrow 2\sqrt{5} + 2\sqrt{3} + 9 - \left(\frac{4}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} \right) - 9$

$\Rightarrow 2\sqrt{5} + 2\sqrt{3} + 9 - \left(\frac{4(\sqrt{5}+\sqrt{3})}{2} \right) - 9$

$\Rightarrow 2\sqrt{5} + 2\sqrt{3} + 9 - 2\sqrt{5} - 2\sqrt{3} - 9 \Rightarrow 0$

67. (a) $a\sqrt{2} + b\sqrt{3}$

$= \sqrt{98} + \sqrt{108} - \sqrt{48} - \sqrt{72}$

$\Rightarrow \sqrt{7 \times 7 \times 2} + \sqrt{3 \times 3 \times 3 \times 2 \times 2}$

$-\sqrt{2 \times 2 \times 2 \times 2 \times 3} - \sqrt{3 \times 3 \times 2 \times 2 \times 2}$

$$\Rightarrow 7\sqrt{2} + 6\sqrt{3} - 4\sqrt{3} - 6\sqrt{2}$$

$$a\sqrt{2} + b\sqrt{3} = 1\sqrt{2} + 2\sqrt{3}$$

$$\begin{aligned} a &= 1 \\ b &= 2 \end{aligned}$$

68. (a) $\sqrt[3]{a} = \sqrt[3]{26} + \sqrt[3]{7} + \sqrt[3]{63}$
Take round figure

$$\Rightarrow \sqrt[3]{a} < \sqrt[3]{27} + \sqrt[3]{8} + \sqrt[3]{64}$$

$$\Rightarrow \sqrt[3]{a} < 3 + 2 + 4$$

$$\Rightarrow \sqrt[3]{a} < 9$$

$$\Rightarrow a < 9^3$$

$$\Rightarrow a < 729$$

Option A is answer

69. (d) $2 + \frac{6}{\sqrt{3}} + \frac{1}{2 + \sqrt{3}} + \frac{1}{\sqrt{3} - 2}$

$$\Rightarrow 2 + \frac{2 \times 3\sqrt{3}}{\sqrt{3} \times \sqrt{3}} + \frac{1}{2 + \sqrt{3}} - \frac{1}{2 - \sqrt{3}}$$

$$\Rightarrow 2 + 2\sqrt{3} + \left(\frac{(2 - \sqrt{3}) - (2 + \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} \right)$$

$$\Rightarrow 2 + 2\sqrt{3} + \left(\frac{2 - \sqrt{3} - 2 - \sqrt{3}}{4 - 3} \right)$$

$$\Rightarrow 2 + 2\sqrt{3} - 2\sqrt{3} \Rightarrow 2$$

70. (c) $\frac{4 + 3\sqrt{3}}{\sqrt{7 + 4\sqrt{3}}} = A + \sqrt{B}$

$$\Rightarrow \sqrt{7 + 4\sqrt{3}}$$

$$\Rightarrow \sqrt{2^2 + (\sqrt{3})^2 + 2 \times 2\sqrt{3}}$$

$$\Rightarrow \sqrt{(2 + \sqrt{3})^2}$$

$$\Rightarrow (2 + \sqrt{3})$$

$$\Rightarrow \frac{4 + 3\sqrt{3}}{2 + \sqrt{3}} = A + \sqrt{B}$$

$$\Rightarrow \frac{4 + 3\sqrt{3}}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = A + \sqrt{B}$$

$$\Rightarrow \frac{(4 + 3\sqrt{3})(2 - \sqrt{3})}{4 - 3} = A + \sqrt{B}$$

$$\Rightarrow 8 - 4\sqrt{3} + 6\sqrt{3} - 9 = A + \sqrt{B}$$

$$\Rightarrow 2\sqrt{3} - 1 = A + \sqrt{B}$$

$$A = -1 \text{ and } \sqrt{B} = 2\sqrt{3}$$

$$B = 2\sqrt{3} \times 2\sqrt{3} = 12$$

$$B - A = 12 - (-1) = 13$$

71. (b) $2^{250} \rightarrow (2^5)^{50} \rightarrow (32)^{50}$

$$\Rightarrow 3^{150} \rightarrow (3^3)^{50} \rightarrow (27)^{50}$$

$$\Rightarrow 5^{100} \rightarrow (5^2)^{50} \rightarrow (25)^{50} \text{ (Smallest)}$$

$$\Rightarrow 4^{200} \rightarrow (4^4)^{50} \rightarrow (256)^{50}$$

72. (c)

$$\sqrt{30 + \sqrt{30 + \sqrt{30} \dots}}$$

$$\textcircled{6} \times 5$$

73. (a) $x = \sqrt{2\sqrt{4\sqrt{2\sqrt{3\sqrt{4}} \dots}}}$

$$\Rightarrow \text{Squaring both sides}$$

$$\Rightarrow x^2 = 2\sqrt{4\sqrt{2\sqrt{3\sqrt{4}} \dots}}$$

Now cubing both sides

$$x^6 = 8 \times 4x$$

$$\Rightarrow x^5 = 32$$

$$\Rightarrow x^5 = 2^5$$

$$\Rightarrow x = 2$$

74. (b) $a = 55, \quad b = 17$
 $c = -72$
 $a + b + c = 55 + 17 - 72 = 0$
 $\therefore a^3 + b^3 + c^3 - 3abc = 0$
If $(a + b + c) = 0$
answer = 0

75. (b) $\frac{243^{n/5} \times 3^{2n+1}}{9^n \times 3^{n-1}}$

$$\Rightarrow \frac{3^{5 \times n/5} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}}$$

$$\Rightarrow \frac{3^n \times 3^{2n+1}}{3^{2n} \times 3^{n-1}}$$

$$\Rightarrow \frac{3^{n+2n+1}}{3^{2n+n-1}} \Rightarrow \frac{3^{3n+1}}{3^{3n-1}}$$

$$\Rightarrow 3^{(3n+1) - (3n-1)} \Rightarrow 3^{3n+1-3n+1}$$

$$\Rightarrow 3^2 = 9$$

76. (c) $(\sqrt{3} + 1)(10 + \sqrt{12})(\sqrt{12} - 2)(5 - \sqrt{3})$

$$\Rightarrow (\sqrt{3} + 1)(10 + 2\sqrt{3})(2\sqrt{3} - 2)(5 - \sqrt{3})$$

$$\Rightarrow (\sqrt{3} + 1) \times 2(5 + \sqrt{3}) \times 2(\sqrt{3} - 1)(5 - \sqrt{3})$$

$$\Rightarrow 4(\sqrt{3} + 1)(\sqrt{3} - 1)(5 + \sqrt{3})(5 - \sqrt{3})$$

$$\Rightarrow 4[(\sqrt{3})^2 - 1^2][(5)^2 - (\sqrt{3})^2]$$

$$\Rightarrow 4 \times 2 \times 22 \Rightarrow 176$$

77. (d) $2^{n-1} + 2^{n+1} = 320$

$$\Rightarrow 2^{n-1}(1 + 2^2) = 320$$

$$\Rightarrow 2^{n-1}(1 + 2^2) = 320$$

$$\Rightarrow 2^{n-1} \times 5 = 320$$

$$\Rightarrow 2^{n-1} = \frac{320}{5} = 64$$

$$\Rightarrow (2)^{n-1} = (2)^6$$

$$\Rightarrow n = 7$$

78. (a) $5\sqrt{5} \times 5^3 \div 5^{-3/2} = 5^{a+2}$

$$\Rightarrow 5^1 \times 5^{1/2} \times 5^3 \div 5^{-3/2} = 5^{a+2}$$

$$\Rightarrow 5^{1+1/2+3-(-3/2)} = 5^{a+2}$$

$$\Rightarrow 5^{1+1/2+3+3/2} = 5^{a+2}$$

$$\Rightarrow 5^6 = 5^{a+2}$$

$$\Rightarrow a + 2 = 6$$

$$\Rightarrow a = 4$$

79. (a) $(3 + 2\sqrt{2})^{-3} + (3 - 2\sqrt{2})^{-3}$

$$\Rightarrow \left(\frac{1}{3 + 2\sqrt{2}} \right)^3 + \left(\frac{1}{3 - 2\sqrt{2}} \right)^3$$

$$\Rightarrow (3 - 2\sqrt{2})^3 + (3 + 2\sqrt{2})^3$$

$$\Rightarrow 2 \times 27 + 6 \times 3 \times (2\sqrt{2})^2$$

$$\Rightarrow 2 \times 27 + 18 \times 8$$

$$\Rightarrow 54 + 144 \Rightarrow 198$$

80. (b) At the start $t = 0^\circ$
 $L = 2 - 2^\circ$
 $\Rightarrow 2 - 1 = 1 \text{ cm}$

81. (d) $\frac{\sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + \sqrt{225}}}}}}{\sqrt[3]{8}}$

$$\Rightarrow \frac{\sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{169}}}}}{2}$$

$$\Rightarrow \frac{\sqrt{10 + \sqrt{25 + \sqrt{121}}}}{2}$$

$$\Rightarrow \frac{\sqrt{10 + \sqrt{36}}}{2}$$

$$\Rightarrow \frac{\sqrt{16}}{2} \Rightarrow \frac{4}{2} \Rightarrow 2$$

82. (d)

$$\frac{\sqrt{6} + 2}{\sqrt{2} + \sqrt{2 + \sqrt{3}}} - \frac{\sqrt{6} - 2}{\sqrt{2} - \sqrt{2 - \sqrt{3}}} - \frac{2\sqrt{2}}{2 + \sqrt{2}}$$

$$\Rightarrow \frac{\sqrt{6} + 2}{\sqrt{2} + \frac{\sqrt{3} + 1}{\sqrt{2}}} - \frac{\sqrt{6} - 2}{\sqrt{2} - \frac{\sqrt{3} - 1}{\sqrt{2}}} - \frac{2}{\sqrt{2} + 1}$$

$$\Rightarrow \frac{(\sqrt{6} + 2)\sqrt{2}}{2 + \sqrt{3} + 1} - \frac{(\sqrt{6} - 2)\sqrt{2}}{2 - \sqrt{3} + 1} - \frac{2}{\sqrt{2} + 1}$$

$$= \frac{\sqrt{2}}{\sqrt{3}} \left[\frac{\sqrt{6}+2}{(\sqrt{3}+1)} - \frac{\sqrt{6}-2}{(\sqrt{3}-1)} \right] -$$

$$\frac{2}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$\frac{\sqrt{2}}{\sqrt{3}} \left[\frac{\sqrt{18}+2\sqrt{3}-\sqrt{6}-2-\sqrt{18}+2\sqrt{3}-\sqrt{6}+2}{3-1} \right]$$

$$- \frac{2(\sqrt{2}-1)}{2-1}$$

$$\Rightarrow \frac{\sqrt{2}}{\sqrt{3}} \left[\frac{2(-\sqrt{6}+2\sqrt{3})}{2} \right] - 2(\sqrt{2}-1)$$

$$\Rightarrow -\sqrt{3} \times \sqrt{2} \times \frac{\sqrt{2}}{\sqrt{3}} + 2\sqrt{3} \times \frac{\sqrt{2}}{\sqrt{3}} - 2(\sqrt{2}-1)$$

$$\Rightarrow -2 + 2\sqrt{2} - 2\sqrt{2} + 2 = 0$$

$$83. (a) \frac{6^2+7^2+8^2+9^2+10^2}{\sqrt{7+4\sqrt{3}}-\sqrt{4+2\sqrt{3}}}$$

$$\Rightarrow \frac{6^2+7^2+8^2+9^2+10^2}{\sqrt{(2+\sqrt{3})^2}-\sqrt{(\sqrt{3}+1)^2}}$$

$$\Rightarrow \frac{6^2+7^2+8^2+9^2+10^2}{2+\sqrt{3}-\sqrt{3}-1}$$

$$\Rightarrow 6^2+7^2+8^2+9^2+10^2$$

$$\Rightarrow 36+49+64+81+100 \Rightarrow 330 \text{ Ans.}$$

$$84. (d) \frac{3x-2y}{2x+3y} = \frac{5}{6}$$

$$18x-12y=10x+15y$$

$$8x=27y$$

$$\frac{x}{y} = \frac{27}{8}$$

$$\left[\frac{\sqrt[3]{x} + \sqrt[3]{y}}{\sqrt[3]{x} - \sqrt[3]{y}} \right]^2$$

$$\left(\frac{\sqrt[3]{27} + \sqrt[3]{8}}{\sqrt[3]{27} - \sqrt[3]{8}} \right)^2$$

$$\Rightarrow \left(\frac{3+2}{3-2} \right)^2 = (5)^2 = 25 \text{ Ans.}$$

85. (a)

$$\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \frac{1}{\sqrt{5}+\sqrt{4}} +$$

$$\frac{1}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{7}+\sqrt{6}} + \frac{1}{\sqrt{8}+\sqrt{7}} + \frac{1}{\sqrt{9}+\sqrt{8}}$$

After Rationalizing

$$= \frac{(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+(\sqrt{5}-\sqrt{4})}{(\sqrt{6}-\sqrt{5})} + 89.(b)$$

$$(\sqrt{6}-\sqrt{5})$$

$$(\sqrt{7}-\sqrt{6})+(\sqrt{8}-\sqrt{7})+(\sqrt{9}-\sqrt{8})$$

$$= \sqrt{9}-1 = 3-1 = 2$$

86. (a)

$$\sqrt{72+\sqrt{72+\sqrt{72+\dots}}}$$

$$\begin{matrix} 9 & \times & 8 \end{matrix}$$

$$87. (c) \frac{1}{1+\sqrt{2}+\sqrt{3}} + \frac{1}{1-\sqrt{2}+\sqrt{3}}$$

$$\Rightarrow \frac{1}{1+\sqrt{3}+\sqrt{2}} + \frac{1}{1+\sqrt{3}-\sqrt{2}}$$

$$\Rightarrow \frac{1+\sqrt{3}-\sqrt{2}+1+\sqrt{3}+\sqrt{2}}{(1+\sqrt{3})^2-(\sqrt{2})^2}$$

$$\Rightarrow \frac{2+2\sqrt{3}}{4+2\sqrt{3}-2} \Rightarrow \frac{2+2\sqrt{3}}{2+2\sqrt{3}}$$

$$\Rightarrow 1$$

$$88. (c) \sqrt{6+\sqrt{6+\sqrt{6+\dots+\infty}}}$$

(2, 3) are the factor of 6.

If there is '+' in ' $\sqrt{\quad}$ ' Answer is Highest value.

If there is '-' in ' $\sqrt{\quad}$ ', Answer is lowest value.

Alternate $\rightarrow$

$$x = \sqrt{6+\sqrt{6+\sqrt{6+\dots+\infty}}} \quad (\text{squaring both side})$$

$$x^2 = 6 + \sqrt{6+\sqrt{6+\dots+\infty}}$$

$$x^2 = 6 + x$$

$$\left[\because \sqrt{6+\sqrt{6+\dots+\infty}} = x \right]$$

$$x^2 - x - 6 = 0$$

$$x^2 - 3x + 2x - 6 = 0$$

$$x(x-3) + 2(x-3) = 0$$

$$(x+2)(x-3)$$

$$x \neq 2, \& x = 3$$

So, Answer is = 3

$$\frac{3\sqrt{7}}{\sqrt{5}+\sqrt{2}} - \frac{5\sqrt{5}}{\sqrt{2}+\sqrt{7}} + \frac{2\sqrt{2}}{\sqrt{7}+\sqrt{5}}$$

$$= \frac{3\sqrt{7}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} - \frac{5\sqrt{5}}{\sqrt{7}+\sqrt{2}} \times$$

$$\frac{\sqrt{7}-\sqrt{2}}{\sqrt{7}-\sqrt{2}} + \frac{2\sqrt{2}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$

$$= \frac{3\sqrt{7}(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2} - \frac{5\sqrt{5}(\sqrt{7}-\sqrt{2})}{(\sqrt{7})^2-(\sqrt{2})^2}$$

$$+ \frac{2\sqrt{2}(\sqrt{7}-\sqrt{5})}{(\sqrt{7})^2-(\sqrt{5})^2}$$

$$= \sqrt{35}-\sqrt{14}-\sqrt{35}+\sqrt{10}+\sqrt{14}-\sqrt{10} = 0$$

$$90. (d) 11\sqrt{n} = \sqrt{112} + \sqrt{343}$$

$$11\sqrt{n} = \sqrt{2 \times 2 \times 2 \times 2 \times 7} + \sqrt{7 \times 7 \times 7}$$

$$11\sqrt{n} = 4\sqrt{7} + 7\sqrt{7}$$

$$11\sqrt{n} = 11\sqrt{7}$$

$$\sqrt{n} = \sqrt{7}$$

$$n = 7$$

SIMPLIFICATION

- Simplification:**

In simplification an expression, we must remove the brackets strictly in the order $()$, $\{\}$, $[\]$ and then we must apply the operations:- Of, division, Multiplication, Addition and Subtraction.

- Remember:-** 'BODMAS' where B stands for bracket, O for of; D for division; M for multiplication, A for addition and S for Subtraction strictly in this order.

Note: 'Of' means multiplication.

- Division Algorithm:-**

Dividend = (Divisor $\times$ Quotient) + Remainder

- Modulus or Absolute value :** The absolute value of a real number X is denoted by the symbol $|x|$ and is defined as -

$$|x| = \begin{cases} x, & \text{if } x > 0 \\ -x, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \end{cases}$$

E.x : $|5| = 5$, $|-5| = -(-5) = 5$

Note: In multiplication and division, when both the numbers carry similar sign, we get positive sign in the result, otherwise we get negative sign in the result i.e.

$$\begin{array}{ll} (+) \times (+) & = + \\ (+) \times (-) & = - \\ (-) \times (+) & = - \\ (-) \times (-) & = + \\ (+) \div (+) & = + \\ (+) \div (-) & = - \\ (-) \div (+) & = - \\ (-) \div (-) & = + \end{array}$$

Important terms

- Identity element of Addition:**

'0' (zero) is called identity element of addition as Addition of '0' in any number does not affect that number.
e.g. $x + 0 = x$ ($x \in \mathbb{Q}$)

- Identity element of**

Multiplication: '1' is called identity element of multiplication as multiplication of '1' in any number does not affect that number.

e.g. $x \times 1 = x$

- Inverse Element of Addition/ Negative element of Addition/ Additive Inverse:**

The number is called "Additive inverse" of a certain number, when it is added to the certain number and result becomes '0' (zero).

E.x. (i) $x + (-x) = 0$

Here $(-x)$ is Additive inverse of x .

(ii) $(9) + (-9)$ is Additive inverse of '9'

- Inverse Element of Multiplication/Reciprocal Element/Multiplicative Inverse:**

The number is called "Multiplicative inverse" of a certain number, when the product of number and multiplicative inverse is 1.

E.x. $x \times \frac{1}{x} = 1$

Here, $\frac{1}{x}$ is multiplicative inverse of ' x '

TYPE - I

Ex.1. The value of $9\frac{8}{9} \times 9$ is:

Sol. $9\frac{8}{9} \times 9$

$$\left(9 + \frac{8}{9}\right) \times 9$$

$$9 \times 9 + \frac{8}{9} \times 9$$

$$81 + 8 = 89$$

Ex.2. The value of $\left(99\frac{7}{9}\right)9$ is:

Sol. $\left(99\frac{7}{9}\right)9$
 $\left(99 + \frac{7}{9}\right) \times 9$

$$99 \times 9 + \frac{7}{9} \times 9$$

$$(100 - 1) \times 9 + 7$$

$$900 - 9 + 7$$

$$= 900 - 2 = 898$$

Ex.3. The value of $\frac{1}{5} + 99\frac{44}{45} \times 9$ is:

Sol. $\frac{1}{5} + 99\frac{44}{45} \times 9$

$$\frac{1}{5} + \left(99 + \frac{44}{45}\right) \times 9$$

$$\frac{1}{5} + \left[\frac{99 \times 45 + 44}{45}\right] \times 9$$

$$\frac{1}{5} + \left[\frac{(100 - 1)45 + 44}{45}\right] \times 9$$

$$\frac{1}{5} + \frac{4500 - 45 + 44}{5}$$

$$\frac{1}{5} + \frac{4500 - 1}{5}$$

$$\frac{1}{5} + \frac{4500}{5} - \frac{1}{5} = 900$$

Ex.4. The value of $\frac{1}{8} + 999\frac{791}{792} \times 99$

Sol. $\frac{1}{8} + 999\frac{791}{792} \times 99$

$$= \frac{1}{8} + \left(999 + \frac{791}{792}\right)99$$

$$\begin{aligned}
&= \frac{1}{8} + 999 \times 99 + \frac{791}{792} \times 99 \\
&= \frac{1}{8} + (1000 - 1)99 + \frac{791}{8} \\
&= \frac{792}{8} + 99000 - 99 \\
&= 99 + 99000 - 99 = 99000
\end{aligned}$$

TYPE - II

Series base question:-

Ex.5. The value of

$$\begin{aligned}
&999 \frac{1}{7} + 999 \frac{2}{7} + 999 \frac{3}{7} + \\
&999 \frac{4}{7} \dots\dots\dots 999 \frac{6}{7} \\
\text{Sol. } &999 \frac{1}{7} + 999 \frac{2}{7} + 999 \frac{3}{7} + \\
&999 \frac{4}{7} \dots\dots\dots 999 \frac{6}{7} \\
&= 999 + \frac{1}{7} + 999 + \frac{2}{7} + \\
&999 + \frac{3}{7} \dots\dots\dots 999 + \frac{6}{7} \\
&= 999 \times 6 + \frac{1}{7} + \frac{2}{7} + \frac{3}{7} \dots\dots \frac{6}{7} \\
&5994 + \frac{1+2+3+4+5+6}{7} \\
&5994 + \frac{21}{7} = 5994 + 3 \\
&= 5997
\end{aligned}$$

Ex.6. The value of $3\frac{1}{3} + 33\frac{1}{3} + \dots$

$$\begin{aligned}
&333333\frac{1}{3} \text{ is:} \\
\text{Sol. } &3\frac{1}{3} + 33\frac{1}{3} + \dots\dots\dots 333333\frac{1}{3} \\
&= 3 + \frac{1}{3} + 33 + \frac{1}{3} \dots\dots\dots \\
&333333 + \frac{1}{3} \\
&= 3 + 33 + 333 + 3333 + 33333 \\
&+ 333333 + \frac{1}{3} \times 6 \\
&= 370368 + 2 \\
&= 370370
\end{aligned}$$

Ex.7. The value of $9 + 99 + 999 + \dots\dots + 20^{\text{th}}$ term is:

Sol. $9 + 99 + 999 + \dots\dots + 20^{\text{th}}$ term
 $= (10^1 - 1) + (10^2 - 1) + (10^3 - 1) + \dots\dots (10^{20} - 1)$
 $= (10^1 + 10^2 + 10^3 \dots\dots 10^{20}) - (1 + 1 + \dots\dots 20 \text{ times})$
 we use G.P formula:

$$\begin{aligned}
S_n &= \frac{a(r^n - 1)}{r - 1} \\
&= \frac{10(10^{20} - 1)}{10 - 1} - 20 \\
&= \frac{10(10^{20} - 1)}{9} - 20
\end{aligned}$$

Ex.8. The value of $\left[1 - \frac{1}{n+1}\right] +$

$$\left[1 - \frac{2}{n+1}\right] + \dots\dots + \left[1 - \frac{n}{n+1}\right]$$

is:

$$\begin{aligned}
\text{Sol. } &\left[1 - \frac{1}{n+1}\right] + \left[1 - \frac{2}{n+1}\right] + \dots\dots + \\
&\left[1 - \frac{n}{n+1}\right] \\
&= 1 - \frac{1}{n+1} + 1 - \frac{2}{n+1} \dots\dots 1 - \frac{n}{n+1} \\
&= (1 + 1 + \dots\dots n \text{ times}) - \frac{1}{n+1} \\
&(1 + 2 + 3 \dots\dots n) \\
&\therefore \text{Sum of } n \text{ natural no.} = \frac{n(n+1)}{2}
\end{aligned}$$

$$= n - \frac{1}{n+1} \times \frac{n(n+1)}{2}$$

$$= n - \frac{n}{2} = \frac{n}{2}$$

Ex.9. The value of $\left(2 - \frac{1}{3}\right) \times$

$$\left(2 - \frac{3}{5}\right) \times \left(2 - \frac{5}{7}\right) \dots\dots \left(2 - \frac{999}{1001}\right)$$

$$\begin{aligned}
\text{Sol. } &\left(2 - \frac{1}{3}\right) \times \left(2 - \frac{3}{5}\right) \times \left(2 - \frac{5}{7}\right) \\
&\dots\dots \left(2 - \frac{999}{1001}\right) \\
&= \frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \dots\dots\dots \frac{1003}{1001} = \frac{1003}{3}
\end{aligned}$$

Ex.10 The value of $999 \frac{92}{99} \times 99$ is:

$$\text{Sol. } 999 \frac{92}{99} \times 99$$

$$\left(999 + \frac{92}{99}\right) 99$$

$$\begin{aligned}
&999 \times 99 + \frac{92}{99} \times 99 \\
&(1000 - 1) \times 99 + 92 \\
&99000 - 99 + 92 \\
&99000 - 7 = 98993
\end{aligned}$$

Ex.11 Find the value of

$$\frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \dots\dots\dots + \frac{19}{8100}$$

$$\begin{aligned}
\text{Sol. } &\frac{3}{4} + \frac{5}{36} + \frac{7}{144} \dots\dots + \frac{19}{8100} \\
&\begin{array}{ccccccc}
\swarrow & \searrow & \swarrow & \searrow & \swarrow & \searrow & \swarrow \\
1 \times 9 & 4 \times 9 & 9 \times 16 & & & & 81 \times 100
\end{array} \\
&= \frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{9} + \frac{1}{9} - \frac{1}{16} \dots\dots \frac{1}{81} - \frac{1}{100} \\
&= 1 - \frac{1}{100} = \frac{100 - 1}{100} = \frac{99}{100}
\end{aligned}$$

Ex.12. Find the value of

$$\begin{aligned}
&\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \left(1 - \frac{1}{5^2}\right) \\
&\dots\dots \times \left(1 - \frac{1}{9^2}\right)
\end{aligned}$$

$$\begin{aligned}
\text{Sol. } &\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \\
&\left(1 - \frac{1}{5^2}\right) \dots\dots \times \left(1 - \frac{1}{9^2}\right)
\end{aligned}$$

$$\therefore A^2 - B^2 = (A + B)(A - B)$$

$$\begin{aligned}
&\left(1 - \frac{1}{2}\right) \left(1 + \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 + \frac{1}{3}\right) \\
&\left(1 - \frac{1}{4}\right) \left(1 + \frac{1}{4}\right) \dots\dots \left(1 - \frac{1}{9}\right) \\
&\left(1 + \frac{1}{9}\right)
\end{aligned}$$

$$= \frac{1}{2} \times \frac{3}{2} \times \frac{2}{3} \times \frac{4}{3} \times \frac{3}{4} \times \frac{5}{4}$$

$$\dots\dots \frac{8}{9} \times \frac{10}{9}$$

$$\frac{1}{2} \times \frac{10}{9} = \frac{5}{9}$$

Ex.13. Find the value of

$$\frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \dots$$

34 terms

Sol.

$$\frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \dots 34 \text{ terms}$$

After 6 Terms series repeat so, then

$$S_6 = \frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = 0$$

In the series sum of 6 terms is 0. so sum of 30 terms (multiple of 6) will be 0

$$S_{30} = 0$$

Now, solve the remaining 4 term

$$\frac{1}{2} + \frac{1}{3} - \frac{1}{4} - \frac{1}{2} = \frac{1}{12}$$

Ex.14. Find the value of $\left(1 - \frac{1}{4}\right)$

$$\left(1 + \frac{1}{3}\right) \left(1 + \frac{2}{3}\right) \left(1 - \frac{2}{5}\right) \left(1 + \frac{6}{7}\right)$$

$$\left(1 - \frac{12}{13}\right)$$

Sol. $\left(1 - \frac{1}{4}\right) \left(1 + \frac{1}{3}\right) \left(1 + \frac{2}{3}\right) \left(1 - \frac{2}{5}\right)$

$$\left(1 + \frac{6}{7}\right) \left(1 - \frac{12}{13}\right)$$

$$= \frac{3}{4} \times \frac{4}{3} \times \frac{5}{3} \times \frac{3}{5} \times \frac{13}{7} \times \frac{1}{13} = \frac{1}{7}$$

Ex.15. Find the value of $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 \dots 90^2$

Sol. $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2 \dots$

$$\dots 89^2 + 90^2$$

$$(1-2)(1+2) + (3-4)(3+4) + (5-6)(5+6) + (7-8)(7+8) \dots (89-90)(89+90)$$

$$= (-1) \times (3) + (-1) \times (7) + (-1) \times (11) + (-1) \times (15) \dots (-1) \times (179)$$

$$= -(3+7+11+15 \dots 179)$$

$$a = 3, d = 4$$

$$\text{No. of term} = \frac{179-3}{4} + 1$$

$$= 44 + 1 = 45$$

$$S_{45} = -\left(\frac{45}{2} [2 \times 3 + 44 \times 4]\right)$$

$$= -\frac{45}{2} [6 + 176]$$

$$= -\frac{45}{2} \times 182 = -4095$$

So,

The value of series = -4095

Ex.16 Find the value of $\frac{1}{12} + \frac{1}{20} +$

$$\frac{1}{30} + \dots \frac{1}{156}$$

Sol. $\frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \dots + \frac{1}{156}$

$$\frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6}$$

$$\dots \frac{1}{12} - \frac{1}{13}$$

$$= \frac{1}{3} - \frac{1}{13} = \frac{10}{39}$$

Ex.17. Find the value of

$$\frac{1}{10} + \frac{1}{40} + \frac{1}{88} \dots + \frac{1}{598}$$

Sol. $\frac{1}{10} + \frac{1}{40} + \frac{1}{88} \dots + \frac{1}{598}$

$$= \frac{1}{3} \left(\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} \dots \right)$$

$$\dots \frac{1}{23} - \frac{1}{26} \Big)$$

$$= \frac{1}{3} \left(\frac{1}{2} - \frac{1}{26} \right)$$

$$= \frac{1}{3} \left(\frac{12}{26} \right) = \frac{2}{13}$$

Alternate:-

$$\frac{1}{\text{diff. b/w two no.}} \left[\frac{1}{\text{First term}} - \frac{1}{\text{Last term}} \right]$$

$$\frac{1}{3} \left[\frac{1}{2} - \frac{1}{26} \right] = \frac{2}{13}$$

Ex.18. Find the value of

$$\frac{1}{45} + \frac{1}{117} + \dots + \frac{1}{3965}$$

Sol.

$$\frac{1}{45} + \frac{1}{117} + \frac{1}{221} \dots + \frac{1}{3965}$$

$$\frac{1}{5 \times 9} + \frac{1}{9 \times 13} + \frac{1}{13 \times 17} \dots + \frac{1}{61 \times 65}$$

$$\Rightarrow \frac{1}{4} \left(\frac{1}{5} - \frac{1}{9} + \frac{1}{9} - \frac{1}{13} \right)$$

$$+ \frac{1}{13} - \frac{1}{17} \dots \frac{1}{61} - \frac{1}{65} \Big)$$

$$\Rightarrow \frac{1}{4} \left[\frac{1}{5} - \frac{1}{65} \right]$$

$$= \frac{1}{4} \times \frac{12}{65} = \frac{3}{65}$$

TYPE - III

CONTINUED FRACTION

A continued fraction consists of the fractional denominators

Ex.19 The value of $\frac{1}{2 + \frac{1}{8 + \frac{1}{5}}}$ is:

$$= \frac{1}{2 + \frac{1}{41/5}}$$

$$= \frac{1}{2 + \frac{5}{41}} = \frac{1}{\frac{87}{41}} = \frac{41}{87}$$

Note: These fractions are solved starting from the bottom towards upside.

Ex.20. The value of

$$1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}}$$

Sol. $1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5}}}}$

$$\Rightarrow 1 + \frac{1}{2 + \frac{1}{3 + \frac{5}{21}}}$$

$$\Rightarrow 1 + \frac{1}{2 + \frac{1}{68/5}}$$

$$\Rightarrow 1 + \frac{1}{2 + \frac{5}{68}}$$

$$\Rightarrow 1 + \frac{1}{\frac{141}{68}} \Rightarrow 1 + \frac{68}{141}$$

$$x \Rightarrow \frac{209}{141}$$

Ex.21. The value of 2

$$+ \frac{3}{5 + \frac{2}{7 + \frac{6}{5 + \frac{2}{3}}}}$$

Sol. $2 + \frac{3}{5 + \frac{2}{7 + \frac{6}{5 + \frac{2}{3}}}}$

$$= 2 + \frac{3}{5 + \frac{2}{7 + \frac{6}{17/3}}}$$

$$= 2 + \frac{3}{5 + \frac{2}{7 + \frac{18}{17}}}$$

$$= 2 + \frac{3}{5 + \frac{2}{137/17}}$$

$$= 2 + \frac{3}{5 + \frac{34}{137}} = 2 + \frac{3}{\frac{719}{137}}$$

$$= 2 + \frac{411}{719} = 2 \frac{411}{719}$$

Ex 22. The value of

$$\frac{2}{2 + \frac{2}{3 + \frac{2}{3 + \frac{2}{3}}}} \text{ is :}$$

Sol. $\frac{2}{2 + \frac{2}{3 + \frac{2}{3 + \frac{2}{3}}}} \times 0.39$

$$= \frac{2}{2 + \frac{2}{3 + \frac{2}{11/3}}} \times 0.39$$

$$= \frac{2}{2 + \frac{2}{3 + \frac{6}{11}}} \times 0.39$$

$$= \frac{2}{2 + \frac{2}{39/11}} \times 0.39$$

$$= \frac{2}{2 + \frac{22}{39} \times 0.39}$$

$$= \frac{2}{2 + \frac{22}{39} \times \frac{39}{100}} = \frac{2}{2 + \frac{22}{100}}$$

$$= \frac{2}{\frac{222}{100}} = \frac{200}{222} = \frac{100}{111}$$

Ex23. The value of $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{6}}}$ is:

Sol. $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{6}}} = 1 + \frac{1}{1 + \frac{1}{7/6}}$

$$= 1 + \frac{1}{1 + \frac{6}{7}} = 1 + \frac{1}{13/7}$$

$$= 1 + \frac{7}{13} = \frac{20}{13} = 1 \frac{7}{13}$$

Ex24.

(a) The value of

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}}$$

Sol. $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1}}}}$

$$= 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}$$

$$= 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3}}} = 1 + \frac{1}{1 + \frac{1}{2}}$$

$$= \frac{1}{1 + \frac{1}{5}} = 1 + \frac{1}{1 + \frac{3}{5}} = 1 + \frac{1}{8/5}$$

$$= 1 + \frac{5}{8} = \frac{13}{8} = 1 \frac{5}{8}$$

(b) If $\frac{1}{a + \frac{1}{b + \frac{1}{c}}} = \frac{9}{26}$

Sol. Find the value of $a + b + c$
Convert the fraction same form of series

$$= \frac{1}{\frac{26}{9}} = \frac{1}{2 + \frac{8}{9}}$$

$$= \frac{1}{2 + \frac{1}{1 + \frac{1}{8}}}$$

Now, comparison both series

$$\frac{1}{a + \frac{1}{b + \frac{1}{c}}} = \frac{1}{2 + \frac{1}{1 + \frac{1}{8}}}$$

$$a = 2$$

$$b = 1$$

$$c = 8$$

$$\text{So, } a + b + c = 2 + 1 + 8 = 11$$

Alternate:-

$$\frac{26}{9} = 2 + \frac{8}{9}$$

$$a = 2$$

$$\frac{9}{8} = 1 + \frac{1}{8}$$

$$b = 1$$

$$\text{and } \frac{1}{c} = \frac{1}{8}$$

$$c = 8$$

$$\therefore a + b + c = 2 + 1 + 8 = 11$$

Ex.25. If $\frac{1}{a + \frac{1}{b + \frac{1}{c + \frac{1}{d}}}} = \frac{17}{60}$

Sol. Find the value of $a \times c + b + d$
Convert the fraction same form of series.

$$\frac{1}{\frac{60}{17}} = \frac{1}{3 + \frac{9}{17}} = \frac{1}{3 + \frac{1}{\frac{17}{9}}}$$

$$= \frac{1}{3 + \frac{1}{1 + \frac{8}{9}}}$$

$$= \frac{1}{3 + \frac{1}{1 + \frac{1}{\frac{9}{8}}}} = \frac{1}{3 + \frac{1}{1 + \frac{1}{\frac{9}{8}}}}$$

Comparison both series

$$\frac{1}{a + \frac{1}{b + \frac{1}{c + \frac{1}{d}}}} = \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\frac{9}{8}}}}}$$

$$a = 3, b = 1, c = 1 \text{ and } d = 8$$

$$\therefore a \times c + b + d = 3 \times 1 + 1 + 8 = 12$$

Alternate:-

$$\frac{17}{60}$$

$$\frac{60}{17} = 3 + \frac{9}{17}$$

$$a = 3$$

$$\frac{17}{9} = 1 + \frac{8}{9}$$

$$b = 1$$

$$\frac{9}{8} = 1 + \frac{1}{8}$$

$$c = 1$$

$$\text{and } \frac{1}{d} = \frac{1}{8}$$

$$\text{Then } d = 8$$

$$\text{Now } a \times c + b + d = 3 \times 1 + 1 + 8 = 12$$

TYPE - IV

Componendo and dividendo (C. & D) It is a theorem on proportions that allows for a quick way to perform calculations and **Reduce** the amount of expansions **needed**. It is particularly useful when dealing with equations involving fractions or rational functions.

Ex. $\frac{a}{b}, \frac{a+b}{a-b}, \frac{a+kb}{a-kb}$
If a, b, c and d are numbers such that b and d are non-

zero and $\frac{a}{b} = \frac{c}{d}$, then

Some Points

1. Componendo $\frac{a+b}{b} = \frac{c+d}{d}$
2. Dividendo $\frac{a-b}{b} = \frac{c-d}{d}$
3. for $k \neq \frac{a}{b}, \frac{a+kb}{a-kb} = \frac{c+kd}{c-kd}$
4. for $k \neq \frac{-b}{a}, \frac{a}{b} = \frac{a+kc}{b+kd}$

Ex.26. If $\frac{a}{b} = \frac{16}{3}$

Find the value $\frac{a+b}{a-b}$

Sol. If $\frac{a}{b} = \frac{c}{d}$

$$\text{Then } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$$\text{The value. } \frac{a}{b} = \frac{16}{3}$$

$$\frac{a+b}{a-b} = \frac{16+3}{16-3} = \frac{19}{13}$$

Ex.27. If $\frac{a}{b} = \frac{5}{3}$ Find $\frac{a+b}{a-b}$

Sol. $\frac{a}{b} = \frac{5}{3},$

use c & d

$$\frac{a+b}{a-b} = \frac{5+3}{5-3}$$

$$\frac{a+b}{a-b} = \frac{8}{2} = \frac{4}{1}$$

Ex.28. If $\frac{a+b}{a-b} = \frac{25}{12}$

Find the value of $\frac{a}{b}$

Sol. $\frac{a+b}{a-b} = \frac{25}{12}$

use c & d

$$\frac{(a+b)+(a-b)}{(a+b)-(a-b)} = \frac{25+12}{25-12}$$

$$\frac{2a}{2b} = \frac{37}{13}$$

$$\frac{a}{b} = \frac{37}{13}$$

Ex.29. $\frac{a+b}{a-b} = \frac{4}{1}$, Find the value of $\frac{a}{b}$

Sol. $\frac{a+b}{a-b} = \frac{4}{1}$

use C & D

$$\frac{a+b+a-b}{a+b-a+b} = \frac{4+1}{4-1}$$

$$\frac{2a}{2b} = \frac{5}{3},$$

$$\frac{a}{b} = \frac{5}{3}$$

Ex.30. If $\frac{x+\sqrt{5}}{x-\sqrt{5}} = 2$

Find the value of x

Sol. $\frac{x+\sqrt{5}}{x-\sqrt{5}} = \frac{2}{1}$

use C & D .

$$\frac{(x+\sqrt{5})+(x-\sqrt{5})}{(x+\sqrt{5})-(x-\sqrt{5})} = \frac{2+1}{2-1}$$

$$\frac{2x}{2\sqrt{5}} = \frac{3}{1}$$

$$\frac{x}{\sqrt{5}} = 3$$

$$x = 3\sqrt{5}$$

Ex.31. If $\frac{x^3+1}{x+1} = \frac{x^3-1}{x-1}$, ($x \neq 1, -1$)

Find the value of x

Sol. $\frac{x^3+1}{x+1} = \frac{x^3-1}{x-1}$

$$\frac{x^3+1}{x^3-1} = \frac{x+1}{x-1}$$

We use C & d

$$\frac{(x^3+1)+(x^3-1)}{(x^3+1)-(x^3-1)} = \frac{(x+1)+(x-1)}{(x+1)-(x-1)}$$

$$\frac{2x^3}{2} = \frac{2x}{2}$$

$$\frac{x^3}{1} = \frac{x}{1}$$

$$x^3 = x$$

$$x^3 - x = 0$$

$$x(x^2-1) = 0$$

However, since $x \neq 1, -1$,

So $x = 0$

Ex.32. If $\frac{2x+y}{2x-y} = \frac{5}{4}$, find the value

of $\frac{2x}{y}$

Sol. $\frac{2x+y}{2x-y} = \frac{5}{4}$

Use C & D

$$\frac{(2x+y)+(2x-y)}{(2x+y)-(2x-y)} = \frac{5+4}{5-4}$$

$$\Rightarrow \frac{4x}{2y} = \frac{9}{1}$$

$$\text{So, } \Rightarrow \frac{2x}{y} = 9$$

Ex.33. If $\frac{2x-y}{x+2y} = \frac{1}{2}$,

Find the value of $\frac{3x+y}{3x-y}$

Sol. $\frac{2x-y}{x+2y} = \frac{1}{2}$

$$4x-2y = x+2y$$

$$3x = 4y$$

$$\frac{3x}{y} = \frac{4}{1}$$

Use C & D

$$\frac{3x+y}{3x-y} = \frac{4+1}{4-1} = \frac{5}{3}$$

Ex.34. If $\frac{x^3+3x}{3x^2+1} = \frac{189}{61}$

Find the value of x

Sol. $\frac{x^3+3x}{3x^2+1} = \frac{189}{61}$

Use C & D

$$\Rightarrow \frac{x^3+3x+3x^2+1}{x^3+3x-3x^2-1} = \frac{189+61}{189-61}$$

$$= \frac{250}{128} = \frac{125}{64}$$

$$\Rightarrow \left(\frac{x+1}{x-1}\right)^3 = \frac{125}{64}$$

$$\left(\frac{x+1}{x-1}\right)^3 = \left(\frac{5}{4}\right)^3$$

$$\frac{x+1}{x-1} = \frac{5}{4}$$

$$4x+4 = 5x-5$$

$$x = 9$$

Ex.35. If $a+b = 1$

$$c+d = 1$$

$$a-b = \frac{d}{c},$$

find the value of c^2-d^2

Sol. $a+b = 1$ (i)

$$a-b = \frac{d}{c} \text{(ii)}$$

$$(i) / (ii)$$

$$\frac{a+b}{a-b} = \frac{1}{d/c}$$

$$\frac{a+b}{a-b} = \frac{c}{d}$$

Use. C & D

$$\frac{c+d}{c-d} = \frac{a}{b}$$

$$\therefore c+d = 1 \text{ (Given)}$$

$$\frac{1}{c-d} = \frac{a}{b}$$

$$\text{then, } (c-d) = \frac{b}{a}$$

$$\text{Now, } c^2-d^2 = (c+d)(c-d) \\ = 1 \times (c-d) = c-d$$

$$c^2-d^2 = \frac{b}{a}$$

Ex.36. If $x = \frac{\sqrt{m+3n} + \sqrt{m-3n}}{\sqrt{m+3n} - \sqrt{m-3n}}$

Find $3nx^2 + 3n$

Sol. $\frac{x}{1} = \frac{\sqrt{m+3n} + \sqrt{m-3n}}{\sqrt{m+3n} - \sqrt{m-3n}}$

Use C & D

$$\frac{x+1}{x-1} = \frac{\sqrt{m+3n}}{\sqrt{m-3n}}$$

square both sides

$$\left(\frac{x+1}{x-1}\right)^2 = \frac{m+3n}{m-3n}$$

Again use c & d

$$\frac{(x+1)^2 + (x+1)^2}{(x-1)^2 - (x-1)^2} = \frac{(m+3n) + (m-3n)}{(m+3n) - (m-3n)}$$

$$\frac{2(x^2+1)}{2 \times 2x} = \frac{2m}{6n}$$

$$\frac{x^2+1}{2x} = \frac{m}{3n}$$

$$3x^2n+3n = 2mx$$

Ex.37. If $(a+b) : \sqrt{ab} = 4:1$ $a > b$

Find $a : b$

Sol. $\frac{a+b}{\sqrt{ab}} = \frac{4}{1}$

multiply 2 both denominator sides

$$\frac{a+b}{2\sqrt{ab}} = \frac{4}{1 \times 2}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{2}{1}$$

Use C & D

$$\frac{a+b+2\sqrt{ab}}{a+b-\sqrt{ab}} = \frac{2+1}{2-1}$$

$$\frac{(\sqrt{a} + \sqrt{b})^2}{(\sqrt{a} - \sqrt{b})^2} = \frac{3}{1}$$

$$\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{\sqrt{3}}{1}$$

Again use C & D

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Square both sides

$$\frac{a}{b} = \frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)^2}$$

$$\frac{a}{b} = \frac{4+2\sqrt{3}}{4-2\sqrt{3}}$$

$$\frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

Ex.38. If $a+b : \sqrt{ab} = 4:1$

find the value a

Sol. $a+b = 4$,

$$\sqrt{ab} = 1$$

then, square both sides

$$ab = 1$$

$$b = \frac{1}{a}$$

Now, $a+b = 4$

Put the value b

$$a + \frac{1}{a} = 4$$

$$a^2 - 4a + 1 = 0$$

$$ax^2 + bx + c$$

we use,

$$\therefore \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$= \frac{4 \pm \sqrt{16 - 4}}{2}$$

$$= \frac{4 \pm \sqrt{12}}{2}$$

$$= \frac{4 \pm 2\sqrt{3}}{2}$$

Take positive then,

$$a = 2 + \sqrt{3}$$

Ex.39. If $x = \frac{\sqrt[3]{m+1} + \sqrt[3]{m-1}}{\sqrt[3]{m+1} - \sqrt[3]{m-1}}$ Find

the value of $x^2 + 3x - 3mx^2 - m$

Sol. $\frac{x}{1} = \frac{\sqrt[3]{m+1} + \sqrt[3]{m-1}}{\sqrt[3]{m+1} - \sqrt[3]{m-1}}$

Use C & D

$$\frac{x+1}{x-1} = \frac{\sqrt[3]{m+1}}{\sqrt[3]{m-1}}$$

cube both sides

$$\frac{(x+1)^3}{(x-1)^3} = \frac{m+1}{m-1}$$

$$\left(\frac{a+b}{a-b} \right)^3 = \frac{a^3 + b^3 + 3a^2b + 3b^2a}{a^3 - b^3 - 3a^2b + 3b^2a}$$

$$\frac{(x^3 + 3x) + (3x^2 + 1)}{(x^3 + 3x) - (3x^2 + 1)}$$

$$= \frac{m+1}{m-1}$$

Again use C & D

$$\frac{x^3 + 3x}{3x^2 + 1} = \frac{m}{1}$$

$$3mx^2 + m = x^2 + 3x$$

$$x^2 + 3x - 3mx^2 - m = 0$$

Ex.40. If $x = \frac{2ab}{b^2 + 1}$ and $b > 1$

Find $\frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}}$

Sol.

$$\frac{x}{a} = \frac{2b}{b^2 + 1}$$

Use C & D

$$\frac{a}{x} = \frac{b^2 + 1}{2b}$$

$$\frac{a+x}{a-x} = \frac{b^2 + 1 + 2b}{b^2 + 1 - 2b}$$

$$\frac{a+x}{a-x} = \left(\frac{b+1}{b-1} \right)^2$$

$$\frac{\sqrt{a+x}}{\sqrt{a-x}} = \frac{b+1}{b-1}$$

Again use C & D

$$\text{Then, } \frac{(\sqrt{a+x}) + (\sqrt{a-x})}{(\sqrt{a+x}) - (\sqrt{a-x})} = \frac{b}{1}$$

Ex.41. If $x = \frac{a-b}{a+b}$, $y = \frac{b-c}{b+c}$,

$$z = \frac{c-a}{c+a}$$

Find $\frac{1+x}{1-x} \times \frac{1+y}{1-y} \times \frac{1+z}{1-z}$

Sol.

$$x = \frac{a-b}{a+b}$$

$$\frac{1}{x} = \frac{a+b}{a-b}$$

Use C & D

$$\frac{1+x}{1-x} = \frac{a}{b} \quad \dots\dots(I)$$

Same $\frac{1+y}{1-y} = \frac{b}{c} \quad \dots\dots(II)$

$$\frac{1+z}{1-z} = \frac{c}{a} \quad \dots\dots(III)$$

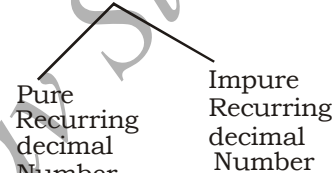
$$(I) \times (II) \times (III)$$

$$\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a} = 1$$

$$\text{So, } \frac{1+x}{1-x} \times \frac{1+y}{1-y} \times \frac{1+z}{1-z} = 1$$

TYPE - V

Recurring number



1. Pure recurring decimals :

These are recurring decimals where the recurrence starts immediately after the decimal point.

Ex: $0.4444 \dots\dots = 0.\overline{4}$

$$3.232323\dots = 3.\overline{23}$$

$$0.564564564 = 0.\overline{564}$$

2. Impure recurring decimals:

Unlike pure recurring decimals, in these decimals, the recurrence occurs after a certain number of digits in the decimal.

Ex: $0.43542542\dots\dots = 0.435\overline{42}$

$$0.546666\dots\dots = 0.54\overline{6}$$

recurring number

Pure recurring decimal number convert into fraction

$$0.333\dots = 0.\overline{3} = \frac{3}{9}$$

$$0.444\dots = 0.\overline{4} = \frac{4}{9}$$

$$0.454545\dots = 0.\overline{45} = \frac{45}{99}$$

$$3.232323\dots = 3.\overline{23}$$

$$= 3 + \frac{23}{99} = 3\frac{23}{99}$$

$$5.564564564 = 5.\overline{564}$$

$$= 5 + \frac{564}{999}$$

* the number of 9's in this group equals the number of digits in the recurring part of the decimal.

Impure recurring decimal number convert into fraction

$$0.4565656\ldots = 0.4\overline{56} = \frac{456-4}{990}$$

expanding the meaning

Step - 1

Subtract the non-recurring initial part of the decimal (in this case, it is 4) from the number formed by writing down the starting digits of the decimal value upto the digit where the recurring decimals are written for the first time; (456 - 4)

Step - 2

As many 9's as the number of digits in the recurring part of the decimal. (in this case, since the recurring part '56' has 2 digits, we write down 2 9's.) These nines have to be followed by as many zeroes as the number of digits in the non recurring part of the decimal value. (In this case, the non recurring part of the decimal value is '4'. Since, 4 has 1 digit, attach one zeroes to the two nines to get the number to divide the result of the first step.)

$$= \frac{456-4}{990}$$

42. Convert to fraction.

- (i) 0.22222.....
- (ii) 0.5555.....
- (iii) 0.444747.....
- (iv) 0.456456456.....
- (v) 0.57333333.....
- (vi) 0.72666666.....
- (vii) 9.868686.....
- (viii) 0.783333.....
- (ix) 0.67777.....
- (x) 4.717171.....
- (xi) 5.00 $\overline{73}$
- (xii) 0.87373.....

Sol. (i) $0.222\ldots = 0.\overline{2} = \frac{2}{9}$

(ii) $0.555\ldots = 0.\overline{5} = \frac{5}{9}$

(iii) $0.474747\ldots = 0.\overline{47} = \frac{47}{99}$

(iv) $0.456456\ldots = 0.\overline{456} = \frac{456}{999} = \frac{152}{333}$

(v) $0.57333\ldots = 0.57\overline{3} = \frac{573-57}{900}$

$$= \frac{516}{900} = \frac{43}{75}$$

(vi) $0.7266\ldots = 0.72\overline{6} = \frac{726-72}{900}$

$$= \frac{654}{900} = \frac{109}{150}$$

(vii) $9.868686\ldots = 9 + 0.\overline{86} = 9 + \frac{86}{99}$

(viii) $0.783333\ldots = 0.78\overline{3} = \frac{783-78}{900}$

$$= \frac{705}{900} = \frac{47}{60}$$

(ix) $0.6777\ldots = 0.6\overline{7} = \frac{67-6}{90} = \frac{61}{90}$

(x) $4.7171\ldots = 4.\overline{71} = 4 + \frac{71}{99} = 4\frac{71}{99}$

(xi) $5.00\overline{73} = 5 + \frac{73-00}{9900} = 5 + \frac{73}{9900}$

$$= 5\frac{73}{9900}$$

(xii) $0.8737373\ldots = 0.8\overline{73} = \frac{873-8}{990}$

$$= \frac{865}{900} = \frac{173}{180}$$

Ex. 43

$$0.56777\ldots$$

Sol $0.56\overline{7}$

$$= \frac{567-56}{900} = \frac{511}{900}$$

Ex. 44

$$0.43542542\ldots$$

Sol $0.4354\overline{2}$

$$= \frac{43542-43}{99900} = \frac{43499}{99900}$$

Ex. 45 Find the value of $\sqrt[3]{0.037}$

Sol. $0.03\overline{7} = \frac{37}{999} = \frac{1}{27}$

$$\sqrt[3]{0.03\overline{7}} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$$

$$= 0.333\ldots = 0.\overline{3}$$

Ex. 46 Find the value of

$$0.\overline{37} + 8.\overline{56} + 1.\overline{23}$$

Sol. $0.\overline{37} + 8.\overline{56} + 1.\overline{23}$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\frac{37}{99} + 8 + \frac{56}{99} + 1 + \frac{23}{99}$$

$$= 8 + 1 + \frac{37}{99} + \frac{56}{99} + \frac{23}{99}$$

$$= 9 + \frac{37+56+23}{99}$$

$$= 9 + \frac{116}{99}$$

$$= 9 + 1 + \frac{17}{99}$$

$$= 10 + 0.\overline{17} = 10.\overline{17}$$

Ex. 47 Find the value of

$$2.8\overline{56} + 3.\overline{74} + 5.875\overline{6}$$

Sol. $2.8\overline{56} + 3.\overline{74} + 5.875\overline{6}$

I	II	III
$\begin{array}{ccc} \times & \times & \times \\ 2.8 & 5 & 6 \end{array}$	$\begin{array}{cc} \times & \times \\ 5 & 6 \end{array}$	$\begin{array}{ccc} \times & \times & \times \\ 5 & 6 & 5 \end{array}$
$\begin{array}{ccc} 3.7 & 4 & 7 \end{array}$	$\begin{array}{cc} 4 & 7 \end{array}$	$\begin{array}{ccc} 4 & 7 & 4 \end{array}$
$\begin{array}{ccc} 5.8 & 7 & 5 \end{array}$	$\begin{array}{cc} 6 & 6 \end{array}$	$\begin{array}{ccc} 6 & 6 & 6 \end{array}$
$\begin{array}{ccc} 12.4 & 7 & 9 \end{array}$	$\begin{array}{cc} 7 & 0 \end{array}$	$\begin{array}{ccc} 7 & 0 & 5 \end{array}$

$$= 12.479\overline{70}$$

Where

I. → maximum digits non recurring
[where (5.875 $\overline{6}$) has 3 max. non recurring digit (875)]

II. → LCM of recurring digits of number (where 2, 2 and 1 recurring digits)

III. → atleast 3 digits

Type - VI

Number of Digits

Ex. 48 How many digits are required to write the counting number from 1 to 50?

Sol. No. × digit = total digit

$$\text{From 1 to 9} = 9 \times 1 = 9$$

$$\text{From 10 to 50} = 41 \times 2 = 82$$

$$\text{total digit} = 9 + 82 = 91$$

* There are 9 numbers from 1 to 9, and each number has 1 digit hence, the total number of digits are (1×9) = 9. In the same way there are 41 number (50 - 10 + 1) from 10 upto 50, and the number of digits are (41 × 2) = 82.

Hence,
The total number of digits
(9 + 82) = 91

Ex.49 How many digits are required to write the counting number from 1 to 672?

Sol. No. × digit = total digit
From 1 to 9 = 9 × 1 = 9
From 10 to 99 = 90 × 2 = 180
From 100 to 672 = 573 × 3 = 1719
total digits = 1908

Ex.50 How many digits are required to write the counting number from 1 to 8756?

Sol. No. × digit = total digit
From 1 to 9 = 9 × 1 = 9

From 10 to 99 = 90 × 2 = 180

From 100 to 999 = 900 × 3 = 2700

From 1000 to 8756 = 7757 × 4 = 31028

total digit = 9 + 180 + 2700 + 31028 = 33917

Ex.51 Calculate the number of digits in the product of $4^{11111} \times 5^{22222}$

Sol. $4 = 2^2$
 $= (2^2)^{11111} \times (5)^{22222} (a^m \times b^m)$
 $= (ab)^m$
(By equalising power)
 $= (2)^{22222} \times (5)^{22222} = (10)^{22222}$
So,
Number of digits = 22222 + 1 = 22223

No.	Digit
10^1	= $\frac{1}{0}$ 2 digit
10^2	= $\frac{1}{00}$ 3 digit
10^3	= $\frac{1}{000}$ 4 digit

Ex. 52 Calculate the number of digits in the product of $8^{232} \times 25^{348}$

Sol. $8^{232} \times 25^{348}$
 $(2^3)^{232} \times (5^2)^{348}$
(By equalising power)
 $(2)^{696} \times (5)^{696}$
 $= (10)^{696}$
number of digits
= 696 + 1 = 697

SOLUTION

1. Simplify : $1 + \frac{1}{1 + \frac{2}{2 + \frac{3}{1 + \frac{4}{1 + \frac{5}{5}}}}}$

- (a) $1\frac{11}{17}$ (b) $1\frac{5}{7}$
(c) $1\frac{6}{17}$ (d) $1\frac{21}{17}$

2. Simplify : $1 + \frac{2}{1 + \frac{3}{1 + \frac{4}{1 + \frac{5}{5}}}}$

- (a) $\frac{7}{4}$ (b) $\frac{4}{7}$ (c) $\frac{7}{5}$ (d) $\frac{3}{7}$

3. Simplify : $\frac{\frac{5}{3} \times \frac{7}{51} \text{ of } \frac{17}{5} - \frac{1}{3}}{\frac{2}{9} \times \frac{5}{7} \text{ of } \frac{28}{5} - \frac{2}{3}}$

- (a) $\frac{1}{2}$ (b) 4 (c) 2 (d) $\frac{1}{4}$

4. Assume that

$\sqrt{13} = 3.605$ (approximately)

$\sqrt{130} = 11.40$ (approximately)

find the value of $\sqrt{1.3} + \sqrt{1300} + \sqrt{0.013}$:

- (a) 36.164 (b) 36.304
(c) 37.304 (d) 37.164

5. On simplification of

$\frac{(2.644)^2 - (2.356)^2}{0.288}$

- (a) 1 (b) 4 (c) 5 (d) 6

6. What is the square root of 0.09 ?

- (a) 0.3 (b) 0.03
(c) 0.003 (d) 3.0

7. Find the value of

$\frac{(0.75)^3}{1 - 0.75} + [0.75 + (0.75)^2 + 1]$

- (a) 4 (b) 1 (c) 2 (d) 0.25

8. Find the value of

$\sqrt{x} \div \sqrt{441} = 0.02$

- (a) 1.64 (b) 2.64
(c) 1.764 (d) 0.1764

9. By which smallest number should 5808 be multiplied so that it becomes a perfect square ?

- (a) 2 (b) 7 (c) 11 (d) 3

10. $\frac{\sqrt[3]{8}}{\sqrt{16}} \div \sqrt{\frac{100}{49}} \times \sqrt[3]{125}$ is equal to :

- (a) 7 (b) $1\frac{3}{4}$ (c) $\frac{7}{100}$ (d) $\frac{4}{7}$

11. By which smallest number 1323 must be multiplied, so that it becomes a perfect cube ?

- (a) 2 (b) 3 (c) 5 (d) 7

12. When simplified, the expression

$(100)^{\frac{1}{2}} \times (0.001)^{\frac{1}{3}} - (0.0016)^{\frac{1}{4}} \times 3^0 + \left(\frac{5}{4}\right)^{-1}$

is equal to :

- (a) 1.6 (b) 0.8 (c) 1.0 (d) 0

13. When $\left(\frac{1}{2} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}\right)$ is divided

by $\left(\frac{2}{5} - \frac{5}{9} + \frac{3}{5} - \frac{7}{18}\right)$, the result is:

- (a) $5\frac{1}{10}$ (b) $2\frac{1}{18}$

- (c) $3\frac{1}{6}$ (d) $3\frac{3}{10}$

14. The square root of $(272^2 - 128^2)$ is:

- (a) 256 (b) 200 (c) 240 (d) 144

15. One-third of the square root of which number is 0.001?

- (a) 0.0009 (b) 0.000001
(c) 0.00009

(d) None of the above

16. $\sqrt[3]{\frac{72.9}{0.4096}}$ is equal to :

- (a) 0.5625 (b) 5.625
(c) 182 (d) 13.6

17. The value of $\frac{1}{3 + \frac{1}{2 - \frac{1}{\frac{7}{9}}}} + \frac{17}{22}$

- (a) $\frac{12}{22}$ (b) $\frac{22}{5}$ (c) $\frac{5}{22}$ (d) 1

18. If $x = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}$, then the

value of $2x + \frac{7}{4}$ is :

- (a) 3 (b) 4 (c) 5 (d) 6

19. Simplify : $\frac{19}{43} + \frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4}}}}$:

- (a) 1 (b) $\frac{19}{43}$ (c) $\frac{43}{19}$ (d) $\frac{38}{43}$

20. Simplify:

$$8\frac{1}{2} - \left[3\frac{1}{4} + \left\{ 1\frac{1}{4} - \frac{1}{2} \left(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6} \right) \right\} \right]$$

- (a) $4\frac{1}{2}$ (b) $4\frac{1}{6}$ (c) $9\frac{1}{2}$ (d) $\frac{2}{9}$

21. If $\frac{50}{*} = \frac{*}{12\frac{1}{2}}$, then the value of *

is:

- (a) $\frac{25}{2}$ (b) $\frac{4}{25}$ (c) 4 (d) 25

22. Find the sum of the following :

$$\frac{1}{9} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72}$$

- (a) $\frac{1}{2}$ (b) 0 (c) $\frac{1}{9}$ (d) 1

23. The value of $25 - 5[2 + 3(2 - 2(5 - 3) + 5) - 10] \div 4$

- (a) 5 (b) 23.25
(c) 23.75 (d) 25

24. If $x = \frac{1}{2 + \frac{1}{2}}$ then $\frac{1}{x} = ?$

- (a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) $\frac{3}{5}$ (d) $\frac{1}{2}$

25. $\frac{9}{20} - \left[\frac{1}{5} + \left\{ \frac{1}{4} + \left(\frac{5}{6} - \frac{1}{3} + \frac{1}{2} \right) \right\} \right]$ is equal to

- (a) 0 (b) 1 (c) $\frac{9}{20}$ (d) $\frac{9}{10}$

26. The value of

$$\sqrt{\frac{(0.1)^2 + (0.01)^2 + (0.009)^2}{(0.01)^2 + (0.001)^2 + (0.0009)^2}}$$

- (a) 10^2 (b) 10
(c) 0.1 (d) 0.01

27. The value of

$$\sqrt{\frac{(0.03)^2 + (0.21)^2 + (0.065)^2}{(0.003)^2 + (0.021)^2 + (0.0065)^2}}$$

- (a) 0.1 (b) 10 (c) 10^2 (d) 10^3

28. If $(102)^2 = 10404$, then the value of $\sqrt{104.04} + \sqrt{1.0404} + \sqrt{0.010404}$ is equal to

- (a) 0.306 (b) 0.0306
(c) 11.122 (d) 11.322

29. If $\sqrt{4096} = 64$, then the value of $\sqrt{40.96} + \sqrt{0.4096} + \sqrt{0.004096} + \sqrt{0.00004096}$ up to two place of decimals is:

- (a) 7.09 (b) 7.10
(c) 7.11 (d) 7.12

30. The least number that must be subtracted from 63522 to make the result a perfect square is :

- (a) 18 (b) 20 (c) 24 (d) 30

31. By which smallest number should 20184 be multiplied so that it becomes a perfect square?

- (a) 2 (b) 3 (c) 5 (d) 6

32. If $2 = x + \frac{1}{1 + \frac{1}{3 + \frac{1}{4}}}$, then the

value of x is :

- (a) $\frac{18}{17}$ (b) $\frac{21}{17}$ (c) $\frac{13}{17}$ (d) $\frac{12}{17}$

33. Simplify:

$$\left[\left(1 + \frac{1}{10 + \frac{1}{10}} \right) \times \left(1 + \frac{1}{10 + \frac{1}{10}} \right) - \left(1 - \frac{1}{10 + \frac{1}{10}} \right) \times \left(1 - \frac{1}{10 + \frac{1}{10}} \right) \right]$$

$$+ \left[\left(1 + \frac{1}{10 + \frac{1}{10}} \right) + \left(1 - \frac{1}{10 + \frac{1}{10}} \right) \right]$$

- (a) $\frac{100}{101}$ (b) $\frac{90}{101}$

- (c) $\frac{20}{101}$ (d) $\frac{101}{100}$

34. $\frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110} = ?$

- (a) $\sqrt{2} \frac{2}{27}$ (b) $\frac{1}{9}$

- (c) $\frac{5}{27}$ (d) $\frac{6}{55}$

35. The value of $1 \div [1 + 1 \div \{1 + 1 \div (1 + 1 \div 2)\}]$

- (a) 1 (b) $\frac{5}{8}$ (c) 2 (d) $\frac{1}{2}$

36. The simplification of $3.\overline{36} - 2.\overline{05} + 1.\overline{33}$ equals :

- (a) 2.60 (b) $2.\overline{61}$
(c) 2.64 (d) $2.\overline{64}$

37. $(0.2 \times 0.2 + 0.01)(0.1 \times 0.1 + 0.02)^{-1}$

- (a) $\frac{5}{3}$ (b) $\frac{41}{12}$ (c) $\frac{41}{4}$ (d) $\frac{9}{5}$

38. The value of

$$\sqrt{5 + \sqrt{11 + \sqrt{19 + \sqrt{29 + \sqrt{49}}}}}$$

- (a) 3 (b) 9 (c) 7 (d) 5

39. The value of $\sqrt[3]{\frac{7}{875}}$ is equal to

- (a) $\frac{1}{3}$ (b) $\frac{1}{15}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

40. $\frac{2}{2 + \frac{2}{3 + \frac{2}{3 + \frac{2}{3}}}}$ is simplified to

- (a) $\frac{1}{3}$ (b) 2
(c) 6 (d) None of these

41. $\frac{-\frac{1}{2} - \frac{2}{3} + \frac{4}{5} - \frac{1}{3} + \frac{1}{5} + \frac{3}{4}}{\frac{1}{2} + \frac{2}{3} - \frac{4}{5} + \frac{1}{3} - \frac{1}{5} - \frac{3}{4}}$ is simplified to

- (a) $-\frac{10}{3}$ (b) $-\frac{3}{10}$
(c) 1 (d) -2

42. The simplification of $(0.\overline{63} + 0.\overline{37} + 0.\overline{80})$ yields the result is:

- (a) $1.\overline{80}$ (b) $1.\overline{81}$
(c) $1.\overline{79}$ (d) 1.80

43. The square root of $0.\overline{4}$ is :

- (a) $0.\overline{8}$ (b) $0.\overline{6}$
(c) $0.\overline{7}$ (d) $0.\overline{9}$

44. The square root of

$$(7 + 3\sqrt{5})(7 - 3\sqrt{5})$$

- (a) 4 (b) $\sqrt{5}$ (c) $3\sqrt{5}$ (d) 2

45. $\left(4\frac{11}{15} + \frac{15}{71}\right)^2 - \left(4\frac{11}{15} - \frac{15}{71}\right)^2$ is equal to:

- (a) 1 (b) 2 (c) 3 (d) 4

46. $\frac{13}{48}$ is equal to

- (a) $\frac{1}{3 + \frac{1}{1 + \frac{1}{16}}}$ (b) $\frac{1}{2 + \frac{1}{1 + \frac{1}{8}}}$
(c) $\frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{8}}}}$ (d) $\frac{1}{3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4}}}}$

47. $\left(\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \frac{1}{9.11} + \frac{1}{11.13} + \frac{1}{13.15}\right)$ is equal to

- (a) $\frac{2}{45}$ (b) $\frac{2}{25}$ (c) $\frac{7}{45}$ (d) $\frac{2}{15}$

48. The number of digits in the square root of 625686734489 is

- (a) 4 (b) 5 (c) 6 (d) 7

49. There are some boys and girls in a room. The square of the number of the girls is less than the square of the number of boys by 28. If there were two more girls, the number of boys would have been the same as that of the girls. The total number of the boys and girls in the room are

- (a) 56 (b) 14 (c) 10 (d) 7

50. If $\sqrt[2]{0.014 \times 0.14x} = 0.014 \times 0.14 \sqrt[2]{y}$.

find the value of $\frac{x}{y}$,

- (a) 0.000196 (b) 0.00196
(c) 0.0196 (d) 0.196

51. $\left\{\frac{(0.1)^2 - (0.01)^2}{0.0001}\right\} + 1$ is equal to

- (a) 1010 (b) 110
(c) 101 (d) 100

52. $\frac{(100-1)(100-2)(100-3)\dots(100-200)}{100 \times 99 \times 98 \times \dots \times 3 \times 2 \times 1}$ is equal to

- (a) $\frac{100}{99 \times 98 \times 97 \times \dots \times 3 \times 2 \times 1}$
(b) $-\frac{1}{99 \times 98 \times 97 \times \dots \times 3 \times 2 \times 1}$
(c) 0
(d) $-\frac{2}{99 \times 98 \times 97 \times \dots \times 3 \times 2 \times 1}$

53. The value of $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}}$

- (a) $\frac{21}{13}$ (b) $\frac{17}{3}$ (c) $\frac{34}{21}$ (d) $\frac{8}{5}$

54. The value of $\frac{2\frac{1}{3} - 1\frac{2}{11}}{3 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3}}}}$

- (a) $\frac{38}{109}$ (b) $\frac{109}{38}$ (c) 1 (d) $\frac{116}{109}$

55. $\sqrt{8 + \sqrt{57 + \sqrt{38 + \sqrt{108 + \sqrt{169}}}}}$ = ?

- (a) 4 (b) 6 (c) 8 (d) 10

56. If the number p is 5 more than q and the sum of the squares of p and q is 55, then the product of p and q is

- (a) 10 (b) -10 (c) 15 (d) -15

57. $\frac{4\frac{2}{7} - \frac{1}{2}}{3\frac{1}{2} + 1\frac{1}{7}} \div \frac{1}{2 + \frac{1}{2 + \frac{1}{5 - \frac{1}{5}}}}$ is equal to

- (a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{3}$

58. If $\left[4 - \frac{5}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{4}}}}\right]^{th}$ part of a

$$\left[4 - \frac{5}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{4}}}}\right]^{th}$$

journey takes ten minutes,

then to complete $\frac{3}{5}^{th}$ of that journey. it will take

- (a) 80 minutes (b) 50 minutes
(c) 48 minutes (d) 60 minutes

59. $\sqrt{\frac{4\frac{1}{7} - 2\frac{1}{4}}{3\frac{1}{2} + 1\frac{1}{7}} \div \frac{1}{2 + \frac{1}{2 + \frac{1}{5 - \frac{1}{5}}}}}$ is equal to

- (a) 1 (b) 4 (c) 3 (d) 2

60. $\frac{1}{1 + 2^{a-b}} + \frac{1}{1 + 2^{b-a}}$ is equal to

- (a) a - b (b) b - a
(c) 1 (d) 0

61. Find the sum of

$$\left(1 - \frac{1}{n+1}\right) + \left(1 - \frac{2}{n+1}\right) + \left(1 - \frac{3}{n+1}\right) + \dots + \left(1 - \frac{n}{n+1}\right)$$

- (a) n (b) $\frac{1}{2}n$
(c) $(n+1)$ (d) $\frac{1}{2}(n+1)$

62. The square root of $33 - 4\sqrt{35}$ is :

- (a) $\pm(2\sqrt{7} + \sqrt{5})$ (b) $\pm(\sqrt{7} + 2\sqrt{5})$
(c) $\pm(\sqrt{7} - 2\sqrt{5})$ (d) $\pm(2\sqrt{7} - \sqrt{5})$

63. What number must be added to the expression $16a^2 - 12a$ to make it a perfect square ?

- (a) $\frac{9}{4}$ (b) $\frac{3}{2}$ (c) $\frac{13}{2}$ (d) 16

64. If $x [-2\{-4(-a)\}] + 5 [-2\{-2(-a)\}] = 4a$, then $x = ?$

- (a) -2 (b) -3 (c) -4 (d) -5

65. If $a = 64$ and $b = 289$, then the

value of $(\sqrt{a} + \sqrt{b} - \sqrt{b} - \sqrt{a})^{\frac{1}{2}}$ is

- (a) $2^{1/2}$ (b) 2
(c) 4 (d) -2

66. The simplified value of

$$\frac{3\sqrt{2}}{\sqrt{3} + \sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6} + \sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}}$$

- (a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

- (c) $\sqrt{3} - \sqrt{2}$ (d) 0

67. The value of $\frac{4 - \sqrt{0.04}}{4 + \sqrt{0.4}}$ is close to

- (a) 0.4 (b) 0.8 (c) 1.0 (d) 1.4

68. If $\sqrt{0.05 \times 0.5 \times a} = 0.5 \times 0.05 \times$

$\sqrt{b}$, then $\frac{a}{b}$ is equal to

- (a) 0.0025 (b) 0.025
(c) 0.25 (d) 0.00025

69. The value of $(1001)^3$ is

- (a) 1003003001
(b) 100303001
(c) 100300301
(d) 103003001

70. Find the Value of

$$5.\bar{6} + 7.\bar{3} + 8.\bar{7} + 6.\bar{1}$$

- (a) $27.\bar{8}$ (b) $27.\bar{7}$
(c) $27.\bar{08}$ (d) $27.\bar{07}$

71. Find the value of $6.\bar{74} + 7.\bar{32}$

- (a) $14.\bar{06}$ (b) $14.\bar{07}$
(c) $13.\bar{06}$ (d) $13.\bar{07}$

72. Which of the following number greatest of all ?

0.9, $0.\bar{9}$, $0.0\bar{9}$, $0.\bar{09}$

- (a) 0.9 (b) $0.\bar{9}$
(c) $0.0\bar{9}$ (d) $0.\bar{09}$

73. $0.\bar{001}$ is equal to

- (a) $\frac{1}{1000}$ (b) $\frac{1}{999}$
(c) $\frac{1}{99}$ (d) $\frac{1}{9}$

74. $1.\bar{27}$ in the form $\frac{p}{q}$ is equal to

- (a) $\frac{127}{100}$ (b) $\frac{73}{100}$
(c) $\frac{14}{11}$ (d) $\frac{11}{14}$

75. $8.3\bar{1} + 0.\bar{6} + 0.00\bar{2}$ is equal to

- (a) $8.\bar{912}$ (b) $8.9\bar{12}$
(c) $8.97\bar{9}$ (d) $8.9\bar{79}$

76. The difference of $5.\bar{76}$ & $2.\bar{3}$

- (a) $2.\bar{54}$ (b) $3.\bar{73}$
(c) $3.\bar{46}$ (d) $3.\bar{43}$

77. The value of $(0.\bar{63} + 0.\bar{37})$

- (a) 1 (b) $\frac{100}{99}$
(c) $\frac{99}{100}$ (d) $\frac{100}{33}$

ANSWER KEY

1. (a)	9. (d)	17. (d)	25. (a)	33. (c)	41. (b)	49. (b)	57. (c)	65. (a)	73. (b)
2. (a)	10. (b)	18. (c)	26. (b)	34. (d)	42. (b)	50. (b)	58. (c)	66. (d)	74. (c)
3. (c)	11. (d)	19. (d)	27. (b)	35. (b)	43. (b)	51. (d)	59. (a)	67. (b)	75. (c)
4. (c)	12. (a)	20. (a)	28. (d)	36. (d)	44. (d)	52. (c)	60. (c)	68. (b)	76. (d)
5. (c)	13. (a)	21. (d)	29. (c)	37. (a)	45. (d)	53. (c)	61. (b)	69. (a)	77. (b)
6. (a)	14. (c)	22. (a)	30. (a)	38. (a)	46. (d)	54. (a)	62. (d)	70. (a)	
7. (a)	15. (d)	23. (c)	31. (d)	39. (d)	47. (d)	55. (a)	63. (a)	71. (b)	
8. (d)	16. (b)	24. (b)	32. (b)	40. (d)	48. (c)	56. (c)	64. (b)	72. (b)	

SOLUTION

1. (a) According to the question,

$$1 + \frac{1}{1 + \frac{2}{2 + \frac{3}{1 + \frac{4}{5}}}}$$

$$\Rightarrow 1 + \frac{1}{1 + \frac{2}{2 + \frac{3}{5 + 4}}}$$

$$\Rightarrow 1 + \frac{1}{1 + \frac{2}{2 + \frac{3 \times 5}{9}}}$$

$$\Rightarrow 1 + \frac{1}{1 + \frac{2}{18 + 15}}$$

$$\Rightarrow 1 + \frac{1}{1 + \frac{18}{33}}$$

$$\Rightarrow 1 + \frac{1}{\frac{33 + 18}{33}} \Rightarrow 1 + \frac{33}{51}$$

$$\Rightarrow \frac{51 + 33}{51} \Rightarrow \frac{84}{51} \Rightarrow 1 \frac{11}{17}$$

2. (a) According to the question,

$$1 + \frac{2}{1 + \frac{3}{1 + \frac{4}{1 + \frac{5}{5 + 4}}}}$$

$$\Rightarrow 1 + \frac{2}{1 + \frac{15}{9 + 15}}$$

$$\Rightarrow \frac{24 + 18}{24} \Rightarrow \frac{42}{24} \Rightarrow \frac{7}{4}$$

3. (c) According to the question,

$$\frac{5}{3} \times \frac{7}{51} \text{ of } \frac{17}{5} - \frac{1}{3}$$

$$\frac{2}{9} \times \frac{5}{7} \text{ of } \frac{28}{5} - \frac{2}{3}$$

$$\Rightarrow \frac{\frac{5}{3} \times \frac{7}{51} \times \frac{17}{5} - \frac{1}{3}}{\frac{2}{9} \times \frac{5}{7} \times \frac{28}{5} - \frac{2}{3}}$$

$$\Rightarrow \frac{\frac{5}{3} \times \frac{7}{15} - \frac{1}{3}}{\frac{2}{9} \times 4 - \frac{2}{3}} \Rightarrow \frac{\frac{7}{9} - \frac{1}{3}}{\frac{8}{9} - \frac{2}{3}}$$

$$\Rightarrow \frac{\frac{7 - 3}{9}}{\frac{8 - 6}{9}} \Rightarrow \frac{4}{2} = 2$$

4. (c) According to the question,

$$\sqrt{1.3} + \sqrt{1300} + \sqrt{0.013}$$

$$\Rightarrow \sqrt{\frac{130}{100}} + \sqrt{1300} + \sqrt{\frac{130}{10000}}$$

$$\Rightarrow \frac{11.4}{10} + 36.05 + \frac{11.4}{100}$$

$$\Rightarrow 1.14 + 36.05 + 0.114$$

$$\Rightarrow 37.304$$

5. (c) According to the question,

$$\frac{(2.644)^2 - (2.356)^2}{0.288}$$

$$\Rightarrow \frac{(2.644 + 2.356)(2.644 - 2.356)}{0.288}$$

$$[\because a^2 - b^2 = (a+b)(a-b)]$$

$$\Rightarrow \frac{5 \times 0.288}{0.288} = 5$$

6. (a) According to the question,

$$\text{Square root of } 0.09 = \sqrt{0.09} = 0.3$$

7. (a) According to the question,

$$\Rightarrow \frac{(0.75)^3}{1 - 0.75} + [0.75 + (0.75)^2 + 1]$$

$$\Rightarrow \frac{(0.75)^3}{1 - 0.75} + \frac{1^3 - (0.75)^3}{1 - 0.75}$$

$$[\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]$$

$$\Rightarrow \frac{(0.75)^3 + 1 - (0.75)^3}{0.25}$$

$$\Rightarrow \frac{1}{0.25} \Rightarrow 4$$

8. (d) According to the question,

$$\sqrt{x} \div \sqrt{441} = 0.02$$

$$\Rightarrow \frac{\sqrt{x}}{\sqrt{441}} = 0.02$$

$$\Rightarrow \frac{\sqrt{x}}{21} = \frac{2}{100}$$

$$\Rightarrow \sqrt{x} = \frac{42}{100}$$

Squaring both sides.

$$\Rightarrow x = 0.1764$$

9. (d) According to the question,

3	5808
2	1936
2	968
2	484
2	242
11	121
11	11
	1

Factors are:

$$[3] 2, 2, 2, 2, 11, 11$$

Smallest number is = 3

10. (b) According to the question,

$$\frac{\sqrt[3]{8}}{\sqrt{16}} \div \sqrt{\frac{100}{49}} \times \sqrt[3]{125}$$

$$\Rightarrow \frac{2}{4} \div \frac{10}{7} \times 5$$

$$\Rightarrow \frac{2}{4} \times \frac{7}{10} \times 5$$

$$\Rightarrow \frac{7}{4} = 1 \frac{3}{4}$$

11. (d) According to the question,

3	1323
3	441
3	147
7	49
7	7
	1

Factors are: $\widehat{3, 3, 3}, \widehat{7, 7}, \widehat{7}$

$\therefore$ Smallest number is = 7

12. (a) According to the question,

$$(100)^{\frac{1}{2}} \times (0.001)^{\frac{1}{3}} - (0.0016)^{\frac{1}{4}} \times 3^0 + \left(\frac{5}{4}\right)^{-1}$$

$$\Rightarrow 10 \times 0.1 - 0.2 \times 1 + \frac{4}{5}$$

$$\Rightarrow 10 \times 0.1 - 0.2 + \frac{4}{5}$$

$$\Rightarrow 1 - \frac{2}{10} + \frac{4}{5}$$

$$\Rightarrow \frac{5-1+4}{5}$$

$$\Rightarrow \frac{8}{5} = 1.6$$

13. (a) According to the question,

$$\Rightarrow \frac{\frac{1}{2} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}}{\frac{2}{5} - \frac{5}{9} + \frac{3}{5} - \frac{7}{18}}$$

$$\Rightarrow \frac{\frac{30-15+12-10}{60}}{\frac{36-50+54-35}{90}}$$

$$\Rightarrow \frac{17}{60} \times \frac{90}{5} = 5 \frac{1}{10}$$

14. (c) According to the question,

$$\Rightarrow \sqrt{(272)^2 - (128)^2}$$

$$\Rightarrow \sqrt{(272+128)(272-128)}$$

$$[\because a^2 - b^2 = (a+b)(a-b)]$$

$$\Rightarrow \sqrt{400 \times 144}$$

$$\Rightarrow 20 \times 12 = 240$$

15. (d) According to the question,

$$\frac{1}{3} \sqrt{x} = 0.001$$

$$\sqrt{x} = 0.003$$

Squaring both sides

$$x = 0.000009$$

$$\therefore \text{Number is} = 0.000009$$

16. (b) According to the question,

$$\Rightarrow \sqrt[3]{\frac{72.9}{0.4096}}$$

$$\Rightarrow \sqrt[3]{\frac{729 \times 10000}{4096 \times 10}}$$

$$\Rightarrow \sqrt[3]{\frac{729}{4096} \times 1000}$$

$$\Rightarrow \frac{9}{16} \times 10$$

$$\Rightarrow 5.625$$

17. (d) According to the question,

$$\Rightarrow \frac{1}{3 + \frac{1}{2 - \frac{1}{\frac{7}{9}}}} + \frac{17}{22}$$

$$\Rightarrow \frac{1}{3 + \frac{1}{2 - \frac{9}{7}}} + \frac{17}{22}$$

$$\Rightarrow \frac{1}{3 + \frac{7}{5}} + \frac{17}{22}$$

$$\Rightarrow \frac{5}{15+7} + \frac{17}{22}$$

$$\Rightarrow \frac{5}{22} + \frac{17}{22} \Rightarrow \frac{22}{22} = 1$$

18. (c) According to the question,

$$\text{If } x = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}$$

$$\Rightarrow x = 1 + \frac{1}{1 + \frac{1}{1 + \frac{2}{3}}}$$

$$\Rightarrow x = 1 + \frac{1}{1 + \frac{3}{5}}$$

$$\Rightarrow x = 1 + \frac{5}{8}$$

$$\Rightarrow x = \frac{13}{8}$$

$$\therefore \text{Value of } 2x + \frac{7}{4} \text{ is}$$

$$\Rightarrow 2 \times \frac{13}{8} + \frac{7}{4}$$

$$\Rightarrow \frac{13}{4} + \frac{7}{4}$$

$$\Rightarrow \frac{20}{4} = 5$$

19. (d) According to the question,

$$\Rightarrow \frac{19}{43} + \frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4}}}}$$

$$\Rightarrow \frac{19}{43} + \frac{1}{2 + \frac{1}{3 + \frac{4}{5}}}$$

$$\Rightarrow \frac{19}{43} + \frac{1}{2 + \frac{5}{19}}$$

$$\Rightarrow \frac{19}{43} + \frac{19}{43} \Rightarrow \frac{38}{43}$$

20. (a) According to the question,

$$\Rightarrow 8\frac{1}{2} - \left[3\frac{1}{4} + \left\{ 1\frac{1}{4} - \frac{1}{2} \left(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6} \right) \right\} \right]$$

$$\Rightarrow \frac{17}{2} - \left[\frac{13}{4} + \left\{ \frac{5}{4} - \frac{1}{2} \left(\frac{3}{2} - \frac{1}{3} - \frac{1}{6} \right) \right\} \right]$$

$$\Rightarrow \frac{17}{2} - \left[\frac{13}{4} + \left\{ \frac{5}{4} - \frac{1}{2} \left(\frac{9-2-1}{6} \right) \right\} \right]$$

$$\Rightarrow \frac{17}{2} - \left[\frac{13}{4} + \left\{ \frac{5}{4} - \frac{1}{2} \times 1 \right\} \right]$$

$$\Rightarrow \frac{17}{2} - \left[\frac{13}{4} + \frac{5-2}{4} \right]$$

$$\Rightarrow \frac{17}{2} - \left[\frac{13}{4} + \frac{3}{4} \right]$$

$$\Rightarrow \frac{17}{2} - \frac{16}{4}$$

$$\Rightarrow \frac{34-16}{4} = \frac{18}{4}$$

$$\Rightarrow \frac{9}{2} = 4\frac{1}{2}$$

21. (d) According to the question,

$$\Rightarrow \frac{50}{x} = \frac{x}{12\frac{1}{2}}$$

$$\Rightarrow 50 \times \frac{25}{2} = x^2$$

$$\Rightarrow x^2 = 25 \times 25$$

$$\therefore x = 25$$

22. (a) According to the question,

$$\Rightarrow \frac{1}{9} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72}$$

$$\begin{array}{cccccccc} & \wedge & & \wedge & & \wedge & & \wedge \\ 2 & 3 & 3 & 4 & 4 & 5 & 5 & 6 & 6 & 7 & 7 & 8 & 8 & 9 \end{array}$$

$$\Rightarrow \frac{1}{9} + \frac{1}{2} + \frac{1}{3} + \frac{1}{3} + \frac{1}{4} + \frac{1}{4} + \frac{1}{5} + \frac{1}{5} + \frac{1}{6} + \frac{1}{6} + \frac{1}{7}$$

$$+ \frac{1}{7} - \frac{1}{8} + \frac{1}{8} - \frac{1}{9} \Rightarrow \frac{1}{2}$$

23. (c) According to the question.

$$\Rightarrow 25 - 5[2 + 3(2 - 2(5 - 3) + 5) - 10] \div 4$$

$$\Rightarrow 25 - 5[2 + 3(2 - 2 \times 2 + 5) - 10] \div 4$$

$$\Rightarrow 25 - 5[2 + 3(2 - 4 + 5) - 10] \div 4$$

$$\Rightarrow 25 - 5[2 + 3 \times 3 - 10] \div 4$$

$$\Rightarrow 25 - 5[11 - 10] \div 4$$

$$\Rightarrow 25 - \frac{5}{4}$$

$$\Rightarrow \frac{100 - 5}{4}$$

$$\Rightarrow \frac{95}{4} = 23.75$$

24. (b) $x = \frac{1}{2 + \frac{1}{2}} = \frac{2}{5} \Rightarrow \frac{1}{x} = \frac{5}{2}$

25. (a) According to the question

$$\Rightarrow \frac{9}{20} - \left[\frac{1}{5} + \left\{ \frac{1}{4} + \left(\frac{5}{6} - \frac{1}{3} + \frac{1}{2} \right) \right\} \right]$$

$$\Rightarrow \frac{9}{20} - \left[\frac{1}{5} + \left\{ \frac{1}{4} + \left(\frac{5}{6} - \frac{5}{6} \right) \right\} \right]$$

$$\Rightarrow \frac{9}{20} - \left[\frac{1}{5} + \left\{ \frac{1}{4} + 0 \right\} \right]$$

$$\Rightarrow \frac{9}{20} - \left[\frac{1}{5} + \frac{1}{4} \right]$$

$$\Rightarrow \frac{9}{20} - \frac{9}{20} = 0$$

26. (b) According to the question,

$$\Rightarrow \sqrt{\frac{(0.1)^2 + (0.01)^2 + (0.009)^2}{(0.01)^2 + (0.001)^2 + (0.0009)^2}}$$

$$\Rightarrow \sqrt{\frac{0.01 + 0.0001 + 0.000081}{0.0001 + 0.000001 + 0.00000081}}$$

$$\Rightarrow \sqrt{\frac{0.010181}{0.00010181}}$$

$$\Rightarrow \sqrt{100} = 10$$

27. (b) According to the question,

$$\Rightarrow \sqrt{\frac{(0.03)^2 + (0.21)^2 + (0.065)^2}{(0.003)^2 + (0.021)^2 + (0.0065)^2}}$$

$$\Rightarrow \sqrt{\frac{0.0009 + 0.0441 + 0.004225}{0.000009 + 0.000441 + 0.00004225}}$$

$$\Rightarrow \sqrt{\frac{0.049225}{0.00049225}} \Rightarrow \sqrt{100} = 10$$

28. (d) According to the question,

$$\Rightarrow \sqrt{104.04} + \sqrt{1.0404} + \sqrt{0.010404}$$

$$\Rightarrow \sqrt{\frac{10404}{100}} + \sqrt{\frac{10404}{10000}} + \sqrt{\frac{10404}{1000000}}$$

$$\Rightarrow \frac{102}{10} + \frac{102}{100} + \frac{102}{1000}$$

$$\Rightarrow 10.2 + 1.02 + 0.102 = 11.322$$

29. (c) According to the question,

$$\Rightarrow \sqrt{40.96} + \sqrt{0.4096} + \sqrt{0.004096} + \sqrt{0.00004096}$$

$$\Rightarrow \sqrt{\frac{4096}{100}} + \sqrt{\frac{4096}{10000}} + \sqrt{\frac{4096}{1000000}} + \sqrt{\frac{4096}{100000000}}$$

$$\Rightarrow \frac{64}{10} + \frac{64}{100} + \frac{64}{1000} + \frac{64}{10000}$$

$$\Rightarrow 6.4 + 0.64 + 0.064 + 0.0064 = 7.11$$

30. (a) According to the question,
As we know that the square of 252 is which is near the value of 63522

$$\therefore 63522 - x = 63504$$

$$x = 18$$

31. (d) According to the question,

4	20184
3	5046
2	1682
29	841
29	29
	1

Factors are $\widehat{2}, \widehat{2}, \widehat{3}, \widehat{2}, 29, 29$

$\therefore$ It should be multiplied by = 6

32. (b) According to the question,

$$\text{If } 2 = x + \frac{1}{1 + \frac{1}{3 + \frac{1}{4}}}$$

$$\Rightarrow 2 = x + \frac{1}{1 + \frac{1}{12 + 1}}$$

$$\Rightarrow 2 = x + \frac{1}{1 + \frac{1}{13}}$$

$$\Rightarrow 2 = x + \frac{13}{17}$$

$$\Rightarrow x = 2 - \frac{13}{17}$$

$$\Rightarrow x = \frac{34 - 13}{17} \Rightarrow x = \frac{21}{17}$$

33. (c) According to the question

$$\left[\left(1 + \frac{1}{10 + \frac{1}{10}} \right) \times \left(1 + \frac{1}{10 + \frac{1}{10}} \right) - \left(1 - \frac{1}{10 + \frac{1}{10}} \right) \times \left(1 - \frac{1}{10 + \frac{1}{10}} \right) \right]$$

$$\div \left[\left(1 + \frac{1}{10 + \frac{1}{10}} \right) + \left(1 - \frac{1}{10 + \frac{1}{10}} \right) \right]$$

$$\text{Let } 1 + \frac{1}{10 + \frac{1}{10}} = \frac{111}{101} = a$$

$$1 - \frac{1}{10 + \frac{1}{10}} = \frac{91}{101} = b$$

$$\Rightarrow \frac{a^2 - b^2}{a + b} \quad [\because a^2 - b^2 = (a + b)(a - b)]$$

$$\Rightarrow \frac{(a + b)(a - b)}{a + b}$$

$$\Rightarrow (a - b)$$

$$\Rightarrow \frac{111}{101} - \frac{91}{101}$$

$$\Rightarrow \frac{20}{101}$$

34. (d) According to the Question

$$\Rightarrow \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} + \frac{1}{110}$$

$$\Rightarrow \frac{1}{5} + \frac{1}{6} + \frac{1}{6} + \frac{1}{7} + \frac{1}{7} + \frac{1}{8} + \frac{1}{8} + \frac{1}{9} + \frac{1}{9} + \frac{1}{10} + \frac{1}{10} + \frac{1}{11}$$

$$\Rightarrow \frac{1}{5} - \frac{1}{11} \Rightarrow \frac{11 - 5}{55} \Rightarrow \frac{6}{55}$$

35. (b) According to the question,

$$\begin{aligned} & 1 \div \left[1 + 1 \div \left\{ 1 + 1 \div (1 + 1 \div 2) \right\} \right] \\ \Rightarrow & 1 \div \left[1 + 1 \div \left\{ 1 + 1 \div \left(1 + \frac{1}{2} \right) \right\} \right] \\ \Rightarrow & 1 \div \left[1 + 1 \div \left\{ 1 + 1 \div \frac{3}{2} \right\} \right] \\ \Rightarrow & 1 \div \left[1 + 1 \div \left\{ 1 + \frac{2}{3} \right\} \right] \\ \Rightarrow & 1 \div \left[1 + 1 \div \frac{5}{3} \right] \\ \Rightarrow & 1 \div \left[1 + \frac{3}{5} \right] \\ \Rightarrow & 1 \div \left[\frac{8}{5} \right] \Rightarrow \frac{5}{8} \end{aligned}$$

36. (d) According to question,

$$\begin{aligned} & 3.\overline{36} - 2.\overline{05} + 1.\overline{33} \\ \Rightarrow & 3.363636... - 2.050505... + 1.333333... \\ \Rightarrow & 2.646464... \\ \Rightarrow & 2.\overline{64} \end{aligned}$$

37. (a) According to the question,

$$\begin{aligned} & (0.2 \times 0.2 + 0.01)(0.1 \times 0.1 + 0.02)^{-1} \\ \Rightarrow & \frac{0.2 \times 0.2 + 0.01}{0.1 \times 0.1 + 0.02} \\ \Rightarrow & \frac{0.04 + 0.01}{0.01 + 0.02} \\ \Rightarrow & \frac{0.05}{0.03} = \frac{5}{3} \end{aligned}$$

38. (a) According to the question,

$$\begin{aligned} & \sqrt{5 + \sqrt{11 + \sqrt{19 + \sqrt{29 + \sqrt{49}}}}} \\ \Rightarrow & \sqrt{5 + \sqrt{11 + \sqrt{19 + \sqrt{29 + 7}}}} \\ \Rightarrow & \sqrt{5 + \sqrt{11 + \sqrt{19 + \sqrt{36}}}} \\ \Rightarrow & \sqrt{5 + \sqrt{11 + \sqrt{19 + 6}}} \\ \Rightarrow & \sqrt{5 + \sqrt{11 + \sqrt{25}}} \\ \Rightarrow & \sqrt{5 + \sqrt{11 + 5}} \\ \Rightarrow & \sqrt{5 + \sqrt{16}} \\ \Rightarrow & \sqrt{9} = 3 \end{aligned}$$

39. (d) According to the question,

$$\begin{aligned} & \Rightarrow \sqrt[3]{\frac{7}{875}} \\ & \Rightarrow \sqrt[3]{\frac{1}{125}} \Rightarrow \frac{1}{5} \end{aligned}$$

40. (d) According to the question,

$$\begin{aligned} & \frac{2}{2 + \frac{2}{3 + \frac{2}{3 + \frac{2}{3}}}} \times 0.39 \\ \Rightarrow & \frac{2}{2 + \frac{2}{3 + \frac{6}{11}}} \times 0.39 \\ \Rightarrow & \frac{2}{2 + \frac{22}{39}} \times 0.39 \end{aligned}$$

41. (b) According to the question,

$$\begin{aligned} & -\frac{1}{2} - \frac{2}{3} + \frac{4}{5} - \frac{1}{3} + \frac{1}{5} + \frac{3}{5} \\ \Rightarrow & \frac{1}{2} - \frac{2}{3} - \frac{4}{3} + \frac{1}{3} - \frac{1}{5} - \frac{4}{5} \\ & \frac{-30 - 40 + 48 - 20 + 12 + 45}{15 + 20 - 40 + 10 - 6 - 24} \\ \Rightarrow & \frac{60}{30} \\ \Rightarrow & \frac{15}{60} \times \frac{-30}{25} \Rightarrow \frac{-3}{10} \end{aligned}$$

42. (b) According to the question,

$$\begin{aligned} & \Rightarrow 0.\overline{63} + 0.\overline{37} + 0.\overline{80} \\ \Rightarrow & 0.6363... + 0.3737... + 0.8080... \\ \Rightarrow & 1.\overline{81} \end{aligned}$$

43. (b) According to the question,

$$\begin{aligned} & \Rightarrow \sqrt{0.4} \\ & \Rightarrow \sqrt{\frac{4}{9}} \\ & \Rightarrow \frac{2}{3} \\ & \Rightarrow 0.666666... \\ & \Rightarrow 0.\overline{6} \end{aligned}$$

44. (d) According to the question,

$$\begin{aligned} & \Rightarrow \sqrt{(7 + 3\sqrt{5})(7 - 3\sqrt{5})} \\ & \Rightarrow \sqrt{49 - 45} \Rightarrow \sqrt{4} = 2 \end{aligned}$$

45. (d) According to the question,

$$\begin{aligned} & \Rightarrow \left(4\frac{11}{15} + \frac{15}{71} \right)^2 - \left(4\frac{11}{15} - \frac{15}{71} \right)^2 \\ \Rightarrow & \left(4\frac{11}{15} + \frac{15}{71} + 4\frac{11}{15} - \frac{15}{71} \right) \left(4\frac{11}{15} + \frac{15}{71} - 4\frac{11}{15} + \frac{15}{71} \right) \\ \Rightarrow & \left(2 \times 4\frac{11}{15} \right) \left(2 \times \frac{15}{71} \right) \\ \Rightarrow & \left(2 \times \frac{71}{15} \times 2 \times \frac{15}{71} \right) \Rightarrow 4 \end{aligned}$$

46. (d) According to the question,

$$\begin{aligned} & \Rightarrow \frac{1}{3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4}}}} \Rightarrow \frac{1}{3 + \frac{1}{1 + \frac{4}{9}}} \\ \Rightarrow & \frac{1}{3 + \frac{9}{13}} \Rightarrow \frac{13}{48} \text{ Satisfied} \end{aligned}$$

47. (d) According to the question,

$$\begin{aligned} & \Rightarrow \left(\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \frac{1}{9.11} + \frac{1}{11.13} + \frac{1}{13.15} \right) \\ \Rightarrow & \frac{1}{2} \left(\frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \frac{1}{7} - \frac{1}{9} + \frac{1}{9} - \frac{1}{11} + \frac{1}{11} - \frac{1}{13} + \frac{1}{13} - \frac{1}{15} \right) \\ \Rightarrow & \frac{1}{2} \left[\frac{1}{3} - \frac{1}{15} \right] \Rightarrow \frac{1}{2} \left[\frac{5-1}{15} \right] \\ \Rightarrow & \frac{1}{2} \times \frac{4}{15} = \frac{2}{15} \end{aligned}$$

48. (c) According to the question,

$$\Rightarrow \sqrt{\overline{625686734489}} = 6$$

NOTE: For counting the digits of square root we make pairs first. Then the digits will be equal to number of pairs.

49. (b) Let the number of boys = x
the number of girls = y

According to the question,

$$\Rightarrow x^2 - y^2 = 28 \dots\dots\dots(i)$$

$$\Rightarrow x = y + 2$$

$$\Rightarrow x - y = 2 \dots\dots\dots(ii)$$

From eq. (i)

$$\Rightarrow x^2 - y^2 = 28$$

$$\Rightarrow (x + y)(x - y) = 28$$

$$\Rightarrow (x + y) \times 2 = 28$$

$$\Rightarrow x + y = 14$$

50. (b) According to the question,

$$\Rightarrow \sqrt[2]{0.014 \times 0.14x} = 0.014 \times 0.14 \sqrt[2]{y}$$

Squaring both sides

$$\Rightarrow 0.014 \times 0.14x = (0.014)^2 \times (0.14)^2 \times y$$

$$\Rightarrow \frac{x}{y} = 0.014 \times 0.14$$

$$\Rightarrow \frac{x}{y} = 0.00196$$

51. (d) According to the question

$$\Rightarrow \left[\frac{(0.1)^2 - (0.01)^2}{0.0001} \right] + 1$$

$$\Rightarrow \left[\frac{(0.1 + 0.01)(0.1 - 0.01)}{0.0001} \right] + 1$$

$$\Rightarrow \frac{0.11 \times 0.09}{0.0001} + 1$$

$$\Rightarrow 99 + 1 = 100$$

52. (c) According to the question

$$\Rightarrow \frac{(100-1)(100-2)(100-3)\dots\dots\dots(100-200)}{100 \times 99 \times 98 \times \dots\dots\dots \times 3 \times 2 \times 1}$$

$$\Rightarrow \frac{99 \times 98 \times 97 \times \dots\dots\dots 1 \times 0 \times (-1) \times (-2) \dots\dots\dots (-100)}{100 \times 99 \times 98 \times \dots\dots\dots 3 \times 2 \times 1}$$

$$= 0$$

53. (c) According to the question

$$\Rightarrow 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{2}{1 + \frac{3}{1}}}}}}$$

$$\Rightarrow 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{3}{5}}}} \Rightarrow 1 + \frac{1}{1 + \frac{1}{1 + \frac{5}{8}}}$$

$$\Rightarrow 1 + \frac{1}{1 + \frac{8}{13}} \Rightarrow 1 + \frac{13}{21} \Rightarrow \frac{34}{21}$$

54. (a) According to the question

$$= \frac{2\frac{1}{3} - 1\frac{2}{11}}{3 + \frac{1}{3 + \frac{1}{3}}} \Rightarrow \frac{7\frac{13}{33} - \frac{11}{11}}{3 + \frac{1}{3 + \frac{1}{10}}}$$

$$\Rightarrow \frac{77 - 39}{33 + \frac{10}{33}} \Rightarrow \frac{38}{33} \times \frac{33}{109}$$

$$\Rightarrow \frac{38}{109}$$

55. (a) According to the question

$$\Rightarrow \sqrt{8 + \sqrt{57 + \sqrt{38 + \sqrt{108 + \sqrt{169}}}}}$$

$$\Rightarrow \sqrt{8 + \sqrt{57 + \sqrt{38 + \sqrt{108 + 13}}}}$$

$$\Rightarrow \sqrt{8 + \sqrt{57 + \sqrt{38 + \sqrt{121}}}}$$

$$\Rightarrow \sqrt{8 + \sqrt{57 + \sqrt{38 + 11}}}$$

$$\Rightarrow \sqrt{8 + \sqrt{57 + \sqrt{49}}}$$

$$\Rightarrow \sqrt{8 + \sqrt{57 + 7}} \Rightarrow \sqrt{8 + \sqrt{64}}$$

$$\Rightarrow \sqrt{8 + 8} \Rightarrow \sqrt{16} = 4$$

56. (c) According to the question

$$p - q = 5 \dots\dots(i)$$

$$p^2 + q^2 = 55 \dots\dots(ii)$$

$$\therefore (p - q)^2 = p^2 + q^2 - 2pq$$

$$(5)^2 = 55 - 2pq$$

$$25 = 55 - 2pq$$

$$2pq = 30 \Rightarrow pq = 15$$

57. (c) Take a first part

$$\frac{4\frac{2}{7} - \frac{1}{2}}{\frac{1}{3} + 1\frac{1}{7}} = \frac{30 - \frac{1}{2}}{\frac{7}{2} + \frac{8}{7}} = \frac{60 - 7}{14} = \frac{53}{14}$$

Second part

$$\frac{1}{2 + \frac{1}{2 + \frac{1}{5 - \frac{1}{5}}}} = \frac{1}{2 + \frac{1}{2 + \frac{1}{\frac{24}{5}}}}$$

$$= \frac{1}{2 + \frac{1}{2 + \frac{5}{24}}} = \frac{1}{2 + \frac{1}{\frac{53}{24}}} = \frac{1}{2 + \frac{24}{53}}$$

$$= \frac{1}{\frac{130}{53}} = \frac{53}{130}$$

According to question,

$$= \frac{53}{65} \div \frac{53}{130}$$

$$= \frac{53}{65} \times \frac{130}{53} = \frac{130}{65} = 2$$

$$58. (c) 4 - \frac{5}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{4}}}}$$

$$= 4 - \frac{5}{1 + \frac{1}{3 + \frac{1}{\frac{9}{4}}}}$$

$$= 4 - \frac{5}{1 + \frac{1}{3 + \frac{4}{9}}} = 4 - \frac{5}{1 + \frac{1}{\frac{31}{9}}}$$

$$= 4 - \frac{5}{1 + \frac{9}{31}} = 4 - \frac{5}{\frac{40}{31}}$$

$$= 4 - \frac{5 \times 31}{40} = 4 - \frac{31}{8} = \frac{1}{8}$$

According to question

$$\frac{1}{8} \text{ part} = 10 \text{ minutes}$$

$$1 \text{ part} = 80 \text{ minutes}$$

$$\frac{3}{5} \text{ part} = 80 \times \frac{3}{5} = 48 \text{ minutes}$$

59. (a) Take first part

$$= \frac{4\frac{1}{7} - 2\frac{1}{4}}{3\frac{1}{2} + 1\frac{1}{7}} = \frac{29 - \frac{9}{4}}{\frac{7}{2} + \frac{8}{7}}$$

$$\frac{116-63}{\frac{28}{49+16}} = \frac{53}{28} \times \frac{14}{65} = \frac{53}{130}$$

Take second part

$$\begin{aligned} \frac{1}{2+\frac{1}{2+\frac{1}{5-\frac{1}{5}}}} &= \frac{1}{2+\frac{1}{2+\frac{1}{\frac{25-1}{5}}}} \\ &= \frac{1}{2+\frac{1}{2+\frac{5}{24}}} = \frac{1}{2+\frac{5}{24}} \\ &= \frac{1}{\frac{2}{1}+\frac{5}{24}} = \frac{1}{\frac{106+24}{24}} = \frac{53}{130} \end{aligned}$$

According to the question

$$\sqrt{\frac{53}{130} \div \frac{53}{130}} = \sqrt{\frac{53}{130} \times \frac{130}{53}} = \sqrt{1} = 1$$

$$60. (c) \frac{1}{1+2^{a-b}} + \frac{1}{1+2^{b-a}}$$

$$\begin{aligned} &= \frac{1}{1+2^{a-b}} + \frac{1}{1+2^{-(a-b)}} \\ &= \frac{1}{1+2^{a-b}} + \frac{2^{a-b}}{2^{a-b}+1} = \frac{1+2^{a-b}}{1+2^{a-b}} = 1 \end{aligned}$$

$$\begin{aligned} 61. (b) &\left(1 - \frac{1}{n+1}\right) + \left(1 - \frac{2}{n+1}\right) + \left(1 - \frac{3}{n+1}\right) + \dots \\ &\dots + \left(1 - \frac{n}{n+1}\right) \\ &= \left(\frac{n+1-1}{n+1}\right) + \left(\frac{n+1-2}{n+1}\right) + \left(\frac{n+1-3}{n+1}\right) + \dots \\ &+ \left(\frac{n+1-n}{n+1}\right) \\ &= \frac{n}{n+1} + \frac{n-1}{n+1} + \frac{n-2}{n+1} + \dots + \frac{1}{n+1} \\ &= \frac{1}{n+1} (n+(n-1)+(n-2)+\dots+1) \\ &\left\{1+2+3+\dots+n = \frac{n(n+1)}{2}\right\} \\ &= \frac{1}{n+1} \left(\frac{n(n+1)}{2}\right) = \frac{n}{2} \end{aligned}$$

$$\begin{aligned} 62. (d) &33 - 4\sqrt{35} \\ &= 33 - 2 \times 2\sqrt{35} \\ &= 33 - 2 \times 2 \times \sqrt{7} \times \sqrt{5} \\ &= 28 + 5 - 2 \times 2\sqrt{7} \times \sqrt{5} \\ &= (2\sqrt{7})^2 + (\sqrt{5})^2 - 2 \times 2\sqrt{7} \times \sqrt{5} \\ &= (2\sqrt{7} - \sqrt{5})^2 \\ \sqrt{33-4\sqrt{35}} &= \pm \sqrt{(2\sqrt{7} - \sqrt{5})^2} \\ &= \pm (2\sqrt{7} - \sqrt{5}) \end{aligned}$$

$$63. (a) (a-b)^2 = a^2 + b^2 - 2ab$$

$$16a^2 - 12a = (4a)^2 - 2 \times 4a \times \frac{3}{2} + \left(\frac{3}{2}\right)^2$$

$$\text{Number be added} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\begin{aligned} 64. (b) &x[-2\{-4(-a)\}] + 5[-2\{-2(-a)\}] \\ &= 4a \\ &x[-2\{4a\}] + 5[-2\{2a\}] = 4a \\ &x[-8a] + 5[-4a] = 4a \\ &-8ax - 20a = 4a \\ &-8ax = 24a \\ &x = -\frac{24a}{8a} \\ &x = -3 \end{aligned}$$

$$65. (a) a = 64, b = 289$$

$$\sqrt{a} = 8, \sqrt{b} = 17$$

$$\begin{aligned} \therefore &(\sqrt{a} + \sqrt{b} - \sqrt{b} - \sqrt{a})^{\frac{1}{2}} \\ &= (\sqrt{8+17} - \sqrt{17-8})^{\frac{1}{2}} \\ &= (\sqrt{25} - \sqrt{9})^{\frac{1}{2}} = (5-3)^{\frac{1}{2}} = 2^{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} 66. (d) &\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} \\ &= \frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} \times \frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} \\ &+ \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} \end{aligned}$$

$$\begin{aligned} &= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{3} - \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{4} \\ &+ \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{1} \\ &= 2\sqrt{3} - \sqrt{6} - 3\sqrt{2} + \sqrt{6} + 3\sqrt{2} - 2\sqrt{3} \\ &= 0 \end{aligned}$$

$$67. (b) \frac{4-\sqrt{0.04}}{4+\sqrt{0.4}} = \frac{4-0.2}{4+0.6} = \frac{3.8}{4.6} = \frac{38}{46} = \frac{19}{23} = 0.8$$

$$68. (b) \sqrt{0.05 \times 0.5 \times a} = 0.5 \times 0.05 \times \sqrt{b}$$

Squaring both sides

$$0.05 \times 0.5 \times a = 0.5 \times 0.5 \times 0.05 \times b$$

$$a = 0.5 \times 0.05 \times b$$

$$\frac{a}{b} = \frac{5 \times 5}{10 \times 100}$$

$$\frac{a}{b} = \frac{1}{40} = 0.025$$

$$\begin{aligned} 69. (a) &(1001)^3 \\ &= 1001 \times 1001 \times 1001 \\ &= 1002001 \times 1001 \\ &= 1003003001 \end{aligned}$$

$$70. (a) 5.\bar{6} + 7.\bar{3} + 8.\bar{7} + 6.\bar{1}$$

$$= 5 + \frac{6}{9} + 7 + \frac{3}{9} + 8 + \frac{7}{9} + 6 + \frac{1}{9}$$

$$= 26 + \left(\frac{6+3+7+1}{9}\right)$$

$$= 26 + \frac{17}{9}$$

$$= 26 + 1 + \frac{8}{9} = 27\frac{8}{9} = 27.\bar{8}$$

$$71. (b) 6.\overline{74} + 7.\overline{32}$$

$$= 6 + \frac{74}{99} + 7 + \frac{32}{99}$$

$$= 13 + \frac{74+32}{99} = 13 + \frac{106}{99}$$

$$= 13 + 1 + \frac{7}{99} = 14\frac{7}{99}$$

$$= 14.\overline{07}$$

72. (b) $0.9 = 0.9$

$0.\overline{9} = 0.999 \dots\dots$

$0.0\overline{9} = 0.09999\dots\dots$

$0.\overline{09} = 0.090909\dots\dots$

$0.\overline{9}$ is the greatest of all

73. (b) $0.\overline{001} = \frac{1}{999}$

74. (c) $1.\overline{27} = 1 + \frac{27}{99}$

$= 1 + \frac{3}{11} = \frac{14}{11}$

75. (c) $8.3\overline{1} + 0.\overline{6} + 0.00\overline{2}$

$\begin{array}{r} \text{x x} \\ 8.31 \\ \hline 0.66 \\ \hline 0.00 \\ \hline 8.97 \end{array}$	$\begin{array}{r} \text{x} \\ 1 \\ \hline 6 \\ \hline 2 \\ \hline 9 \end{array}$	$\begin{array}{r} \text{x x x} \\ 111 \\ \hline 666 \\ \hline 222 \\ \hline 999 \end{array}$	$= 8.979999$
			$= 8.97\overline{9}$

76. (d) $(5.\overline{76}) - (2.\overline{3})$

$= \left(5 + \frac{76}{99}\right) - \left(2 + \frac{3}{9}\right)$

$= 5 + \frac{76}{99} - 2 - \frac{3}{9}$

$= 3 + \left(\frac{76}{99} - \frac{3}{9}\right)$

$= 3.\overline{43}$

77. (b) $0.\overline{63} + 0.\overline{37}$

$= \frac{63}{99} + \frac{37}{99} = \frac{100}{99}$



LINEAR EQUATIONS IN TWO VARIABLES

- Linear Equations in Two Variables:** An equation of the form $ax + by + c = 0$ where $a, b, c \in \mathbb{R}$ (real numbers) and $a \neq 0, b \neq 0$

and x, y are variables is called a linear equation in two variables.

Examples : Each of the following equations is a linear equation :

- (i) $4x + 7y = 13$
- (ii) $2x - 5y = 36$
- (iii) $\sqrt{3}x - \sqrt{7}y = 2$

The condition $a \neq 0, b \neq 0$, is often denoted by $a^2 + b^2 \neq 0$

Note: The graph of a linear equation $ax + by + c = 0$, is a straight line.

Solution of linear equation : Any pair of values of x and y which satisfy the equation $ax + by + c = 0$, is called its solution.

E.g.: show that $x = 2$ and $y = 1$ is a solution of $2x + 5y = 9$

Sol: Substituting $x = 2$ and $y = 1$ in the given equation, we get LHS = $2 \times 2 + 5 \times 1 = 9 = \text{RHS}$

$\therefore x = 2, y = 1$ is a solution of $2x + 5y = 9$

- System of Linear Equations:**

Consistent System :- A system consisting of two simultaneous linear equations is said to be consistent, if it has at least one solution.

Inconsistent System : A system consisting of two simultaneous linear equations is said to be inconsistent, if it has no solution at all.

E.g.: Consider the system of equations: $x + y = 9$ & $3x + 3y = 5$. Clearly, there are no values of x and y which may simultaneously satisfy the given equations. So, the system given above is inconsistent.

Conditions for Solvability :

The system of equations $a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$ has :

- (i) a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(ii) an infinite number of solutions,

$$\text{if } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

- (iii) no solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Homogenous System of Equations:

The system of equations $a_1x + b_1y = 0; a_2x + b_2y = 0$ has

only solution $x = 0, y = 0$, when

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

- (ii) an infinite number of solutions

$$\text{when } \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

- The graphs of $a_1x + b_1y + c_1 = 0, a_2x + b_2y + c_2 = 0$ will be :
 - (i) Parallel, if the system has no solution ;
 - (ii) Coincident, if the system has infinite number of solutions ;
 - (iii) Intersecting, if the system has a unique solution.

EXERCISE

- If $11x - 13 = -2x + 78$, then $x = ?$
(a) 7 (b) 8 (c) 6 (d) 4
- If $2x + 3y = 29$ and $y = x + 3$, what is the value of x ?
(a) 5 (b) 6 (c) 4 (d) 7
- If $2x + 3y = 5$ and $x = -2$, then the value of y is :
(a) $\frac{1}{3}$ (b) 3 (c) 1 (d) 9
- The value of $x + y$ in the solution of the equations $\frac{x}{4} + \frac{y}{3} = \frac{5}{12}$ and $\frac{x}{2} + y = 1$
(a) $\frac{1}{2}$ (b) 2 (c) $\frac{5}{2}$ (d) $\frac{3}{2}$
- If $2x + 3y = 12$ and $3x - 2y = 5$, then x and y must have the values :
(a) 2 and 3 (b) 2 and -3
(c) 3 and -2 (d) 3 and 2
- The equations $ax + b = 0$ and $cx + d = 0$ are consistent, if :
(a) $ad = bc$ (b) $ad + bc = 0$
(c) $ab - cd = 0$ (d) $ab + cd = 0$
- The equations $2x + y = 5$ and $x + 2y = 4$ are
(a) consistent and have infinitely many solutions
(b) consistent and have a unique solution.
(c) inconsistent
(d) none of these
- The cost of 2 sarees and 4 shirts is Rs. 16000 while 1 saree and 6 shirts cost the same. The cost of 12 shirts is :
(a) Rs. 12,000 (b) Rs. 24,000
(c) Rs. 48,000
(d) Can't be determined
- The system of equations $kx - y = 2$ and $6x - 2y = 3$ has a unique solution when :
(a) $K = 0$ (b) $K \neq 0$
(c) $K = 3$ (d) $K \neq 3$
- The value of y in the solution of the equation $2^{x+y} = 2^{x-y} = \sqrt{8}$ is :
(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

11. The solutions of the equations $\frac{3x-y+1}{3} = \frac{2x+y+2}{5} = \frac{3x+2y+1}{6}$ is
 (a) $x = 2, y = 1$ (b) $x = 1, y = 1$
 (c) $x = -1, y = -1$ (d) $x = 1, y = 2$
12. If $x + 2y \leq 3, x > 0$ and $y > 0$, then one of the solutions is :
 (a) $x = -1, y = 2$
 (b) $x = 2, y = 1$
 (c) $x = 1, y = 1$
 (d) $x = 0, y = 0$
13. A purse contains 25 paise and 10 paise coins. The total amount in the purse is ₹ 8.25. If the number of 25 paise coins is one-third of the number of 10 paise coins in the purse, then the total number of coins in the purse:
 (a) 30 (b) 40 (c) 45 (d) 60
14. The value of k for which the system of equations $x + 2y = 5, 3x + ky + 15 = 0$ has no solution, is:
 (a) 6 (b) -6 (c) 2 (d) 4
15. The equations $2x - 5y = 9$ and $8x - 20y = 36$ have :
 (a) no common solution
 (b) exactly one common solution
 (c) exactly two common solutions
 (d) more than two common solutions
16. The difference between two numbers is 5 and the difference between their squares is 65. The larger number is :
 (a) 9 (b) 10 (c) 11 (d) 12
17. The number of solutions of the equations $x + \frac{1}{y} = 2$ and $2xy - 3y = -2$ is :
 (a) 0 (b) 1 (c) 2
 (d) None of these
18. If $2^a + 3^b = 17$ and $2^{a+2} - 3^{b+1} = 5$, then:
 (a) $a = 2, b = 3$ (b) $a = -2, b = 3$
 (c) $a = 2, b = -3$ (d) $a = 3, b = 2$
19. The solution to the system of equations $|x + y| = 1$ and $x - y = 0$ is given by:
 (a) $x = y = \frac{1}{2}$ (b) $x = y = -\frac{1}{2}$
 (c) $x = y = \frac{1}{2}$ or $x = y = -\frac{1}{2}$
 (d) $x = 1, y = 0$

ANSWER KEY

1. (a)	3. (b)	5. (d)	7. (b)	9. (d)	11. (b)	13. (d)	15. (d)	17. (d)	19. (c)
2. (c)	4. (d)	6. (a)	8. (b)	10. (a)	12. (c)	14. (a)	16. (a)	18. (d)	

SOLUTION

1. (a) $11x - 13 = -2x + 78$

$$\Rightarrow 11x + 2x = 78 + 13$$

$$\Rightarrow 13x = 91$$

$$\Rightarrow x = \frac{91}{13} = 7$$

2. (c) Putting $y = x + 3$ in $2x + 3y = 29$, we get,

$$2x + 3(x + 3) = 29 \Rightarrow 2x + 3x + 9 = 29$$

$$\Rightarrow 5x = 29 - 9 = 20 \Rightarrow x = \frac{20}{5} = 4$$

3. (b) Putting $x = -2$ in $2x + 3y = 5$, we get ;

$$-4 + 3y = 5 \Rightarrow 3y = 5 + 4 = 9$$

$$\Rightarrow y = \frac{9}{3} = 3$$

4. (d) Given equations are :

$$3x + 4y = 5 \quad \text{.....(i) and}$$

$$x + 2y = 2 \quad \text{.....(ii)}$$

$$(i) - 2 \times (ii): x = 5 - 4 = 1$$

$$\therefore \text{from (ii)} \quad 2y = 2 - x = 2 - 1 = 1$$

$$\Rightarrow y = \frac{1}{2}$$

$$\therefore x + y = 1 + \frac{1}{2} = \frac{3}{2}$$

5. (d) $2x + 3y = 12$ (i)

$3x - 2y = 5$ (ii)

(i) $\times 2$ + (ii) $\times 3$, we get ; $x = 3$

putting $x = 3$ in (i), we get $2 \times 3 + 3y = 12$

$$\Rightarrow 3y = 6 \Rightarrow y = 2$$

$$\therefore x = 3 \text{ and } y = 2$$

6. (a) The equations are consistent if

$$\frac{a}{c} = \frac{b}{d}$$

i.e. $ad = bc$

7. (b) $2x + y = 5$ (i)

$x + 2y = 4$ (ii)

On solving we get, $x = 2, y = 1$

Thus (b) is true

8. (b) Let cost of 1 saree = Rs. x &

cost of 1 shirt = Rs. y

$$\therefore 2x + 4y = 16000 \quad \text{.....(i)}$$

$$\text{and } x + 6y = 16000 \quad \text{.....(ii)}$$

Multiplying (ii) by 2 and subtracting (i) from it, we get,

$$8y = 16000 \Rightarrow y = 2000$$

$$\therefore \text{cost of 12 shirts} = (\text{Rs. } 2000 \times 12) = \text{Rs. } 24000$$

9. (d) For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{k}{6} \neq \frac{-1}{-2} \Rightarrow k \neq \left(6 \times \frac{1}{2}\right) \Rightarrow k \neq 3$$

10. (a) $2^{x+y} = 2^{x-y} = \sqrt{8} = 2^{3/2}$

$$\Rightarrow x + y = \frac{3}{2} \quad \text{.....(i)}$$

$$\text{and } x - y = \frac{3}{2} \quad \text{.....(ii)}$$

(i) - (ii)

$$2y = 0 \Rightarrow y = 0$$

11. (b) $\frac{3x-y+1}{3} = \frac{2x+y+2}{5}$

$$\Rightarrow 5(3x - y + 1) = 3(2x + y + 2)$$

$$\Rightarrow 9x - 8y = 1 \quad \dots\dots(i)$$

$$\text{and } \frac{3x - y + 1}{3} = \frac{3x + 2y + 1}{6}$$

$$= 2(3x - y + 1) = (3x + 2y + 1)$$

$$\Rightarrow 3x - 4y = -1 \quad \dots\dots(ii)$$

$$(i) - 2 \times (ii) :-$$

$$(9 - 6)x - 8y + 8y = 1 - (-2) \Rightarrow x = 1$$

$$\text{putting } x = 1 \text{ in (i) we get, } 9 \times 1 - 8y = 1$$

$$\Rightarrow 8y = 8 \Rightarrow y = 1$$

$$\therefore x = 1, y = 1.$$

12. (c) Here we will go through options.

in option (a) $x < 0$ and

in option (d) $x = 0$

hence (a) and (d) can't be the required answer because both does not satisfy the given condition i.e. $x > 0$.

Now option (b) $x = 2, y = 1$, then $x + 2y = 2 + 2(1) = 4$ which is > 3 clearly, values of option (b) do not satisfy $x + 2y \leq 3$

option (c) $x = 1, y = 1$, then $x + 2y = 1 + 2 = 3 \leq 3$

So, $x = 1, y = 1$ is one of the solutions.

13. (d) Let the number of 25 paise coins be x & that of 10 paise coins be y , then:

$$\frac{25}{100}x + \frac{10}{100}y = 8.25$$

$$\Rightarrow 5x + 2y = 165 \quad \dots\dots(i)$$

$$\text{and } x = \frac{1}{3}y \Rightarrow y = 3x \quad \dots\dots(ii)$$

putting $y = 3x$ in (i), we get :

$$5x + 6x = 165 \Rightarrow 11x = 165 \Rightarrow x = 15$$

$$\therefore \text{ from (ii), } y = 3x = 3 \times 15 = 45$$

$\therefore$ Total number of coins in the purse

$$= x + y = 15 + 45 = 60$$

$$14.(a) a_1x + b_1y + c_1 = 0$$

$a_2x + b_2y + c_2 = 0$ will have no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{1}{3} = \frac{2}{k} \Rightarrow k = 6$$

15.(d) The given equations are $2x - 5y = 9$ and $8x - 20y = 36 \Rightarrow 2x - 5y = 9$

Thus, there is one equation in two variables. So, the given equations have an infinite number of solutions.

16.(a) Let the numbers be x and y .

Then,

$$x - y = 5 \text{ and } x^2 - y^2 = 65$$

$$\therefore \frac{x^2 - y^2}{x - y} = \frac{65}{5} \Rightarrow x + y = 13$$

solving $x - y = 5$ and $x + y = 13$, we get;

$$x = 9 \text{ and } y = 4$$

$\therefore$ larger number = 9

17.(d)

$$x + \frac{1}{y} = 2 \Rightarrow \frac{1}{y} = 2 - x \Rightarrow y = \frac{1}{2 - x} \quad \dots\dots(i)$$

$$\text{and } 2xy - 3y = -2$$

$$\Rightarrow y(2x - 3) = -2 \quad \dots\dots(ii)$$

$$\text{putting } y = \frac{1}{2 - x} \text{ in (ii)}$$

$$\frac{2x - 3}{2 - x} = -2 \Rightarrow 2x - 3 = -4 + 2x. \text{ this}$$

$$\text{gives } 1 = 0$$

This is impossible So, there is no solution.

$$18.(b) 2^a + 3^b = 17 \text{ and}$$

$$2^{a+2} - 3^{b+1} = 5 \Rightarrow 2^2 \cdot 2^a - 3 \cdot 3^b = 5$$

$$\Rightarrow 4 \cdot 2^a - 3 \cdot 3^b = 5$$

let $2^a = x$ & $3^b = y$ then

$$x + y = 17 \quad \dots\dots(i)$$

$$4x - 3y = 5 \quad \dots\dots(ii)$$

$3 \times (i) + (ii)$, we get

$$7x = 56 \Rightarrow x = 8 \Rightarrow 2^a = 8 = 2^3 \Rightarrow$$

$$\boxed{a = 3}$$

putting $x = 8$ in (i), we get

$$y = 17 - 8 = 9 \Rightarrow 3^b = 9 = 3^2 \Rightarrow$$

$$\boxed{b = 2}$$

$$\therefore a = 3 \text{ and } b = 2.$$

19.(c) Note that $|a| = 1$ means $a = 1$ or $a = -1$

So, $|x + y| = 1 \Rightarrow x + y = 1$ or $-(x + y) = 1$

$$\Rightarrow (x + y) = -1$$

solving $x + y = 1, x - y = 0$, we get

$$x = \frac{1}{2} \text{ and } y = \frac{1}{2}$$

solving $x + y = -1, x - y = 0$, we get $x = -1/2$ and $y = -1/2$

$$\therefore x = y = \pm \frac{1}{2}$$



POLYNOMIALS

- Polynomials** : An expression of the form $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, where $a_n \neq 0$, is called a polynomial in x of degree n .

Here $a_0, a_1, a_2, \dots, a_n$ are real numbers and each power of x is a non-negative integer.

e.g.

- (i) $2x + 7$ is a polynomial in x of degree 1.
- (ii) $2y^2 - 5y + 7$ is a polynomial in y of degree 2.
- (iii) $3u^3 + \frac{3}{7}u^2 - 8u + \sqrt{7}$ is a polynomial in u of degree 3.
- (iv) $5t^4 - \frac{2}{7}t^3 + \sqrt{3}t^2 + \frac{3}{8}$ is a polynomial in t of degree 4.

- (v) $(\sqrt{x}+5), \frac{1}{x+3}, \frac{5}{x^2-3x+1}$ etc.

are not polynomials.

- Polynomials of Various Degrees :**

- (1) **Linear Polynomial** : A polynomial of degree 1 is called a linear polynomial.

A linear polynomial is of the form $p(x) = ax + b$, where $a \neq 0$
e.g.

$$(3x - 7), (\sqrt{2}x + 5), \left(x - \frac{7}{3}\right) \text{ etc.}$$

- (2) **Quadratic Polynomial** : A polynomial of degree 2 is called a quadratic polynomial.

It is of the form $p(x) = ax^2 + bx + c$, where $a \neq 0$

E.g. $(2x^2 + 7x - 9), (3x^2 - \sqrt{2}x + 7), (y^2 - 7y + \sqrt{5})$ etc.

Nature of Roots

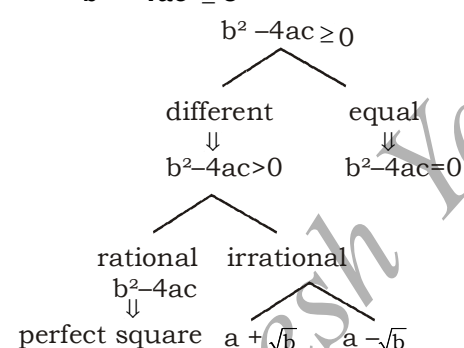
The value of x at which value of equation will be zero.

- Roots are imaginary :**

$$b^2 - 4ac \leq 0$$

- Roots are real:**

$$b^2 - 4ac \geq 0$$



Sum & product of root:-

Let there are two roots named α & β , then

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \& \quad \beta$$

$$= \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Sum of root:-

$$\alpha + \beta = \frac{-b}{a}$$

Product of root:

$$\alpha \beta = \frac{c}{a}$$

then, $ax^2 + bx + c = 0$ can be written as:

$$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\Rightarrow x^2 - \left(\frac{-b}{a}\right)x + \frac{c}{a} = 0$$

$$x^2 - (\text{sum of root})x + \text{product of root} = 0$$

- If the roots α & β be reciprocal to each other then $a = c$.

- If the two roots α & β be equal in magnitude and opposite in sign, then $b = 0$

- If a, b, c are rational number and $a + \sqrt{b}$ is one root of the quadratic equation, then the other root must be conjugate $a - \sqrt{b}$ and viceversa

Ex.1 Find the Quadratic equation whose one root is $3 + \sqrt{3}$

Sol. If one root is $3 + \sqrt{3}$ then second root will be $3 - \sqrt{3}$

Sum of root

$$= (3 + \sqrt{3}) + (3 - \sqrt{3}) = 6$$

Product of root

$$= (3 + \sqrt{3})(3 - \sqrt{3}) = 6$$

using,

$$x^2 (\text{sum of root}) x + (\text{product of root}) = 0$$

$$\Rightarrow x^2 - 6x + 6 = 0$$

Ex.2: Two roots of equation $2x^2 - 7x + 12 = 0$ are

$$\alpha \& \beta \text{ then, find } \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = ?$$

Sol. $2x^2 - 7x + 12 = 0$

On comparing with standard equation $ax^2 + bx + c = 0$
 $a = 2, b = -7, \& c = 12$

$$\alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{7}{2}$$

$$\alpha \beta = \frac{c}{a} \Rightarrow \alpha \beta = 6$$

$$\Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha \beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha \beta}{\alpha \beta}$$

$$= \frac{\left(\frac{7}{2}\right)^2 - 2 \times 6}{6}$$

$$= \frac{\frac{49}{4} - 12}{6} = \frac{49 - 48}{4 \times 6} = \frac{1}{24}$$

Ex.3: Find the product of the root of the equation $x^2 - \sqrt{3} = 0$

Sol. On comparing this equation with $ax^2 + bx + c = 0$

$$a = 1, b = 0 \text{ \& } c = -\sqrt{3}$$

$$\text{Product of root } \alpha \beta = \frac{c}{a} = -\sqrt{3}$$

- **Byquadratic Polynomial :** A polynomial of degree 4 is called a biquadratic polynomial. It is of the form $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ where $a \neq 0$

E.g. $(3x^4 + 7x^3 - 4x^2 + 6x + 11)$, $(4t^4 - 7t^3 + 6t^2 - 11t + 9)$ etc.

(3) Cubic Polynomial : A polynomial of degree 3 is called a cubic polynomial.

It is of the form $P(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$

E.g. $(4x^3 - 2x^2 + 7x + 9)$, $(2\sqrt{2}y^3 - 5y^2 - 8)$ etc.

Value of a Polynomial at a given point:

If $P(x)$ is a polynomial in x and if α is any real number, then the value obtained by putting $x = \alpha$ in $P(x)$ is called the value of $P(x)$ at $x = \alpha$

The value of $P(x)$ at $x = \alpha$ is denoted by $p(\alpha)$.

e.g. Let $p(x) = 3x^2 - 2x + 7$. then

$$\begin{aligned} p(2) &= (3 \times 2^2 - 2 \times 2 + 7) \\ &= (12 - 4 + 7) \\ &= 15 \end{aligned}$$

$$\begin{aligned} p(-1) &= [3 \times (-1)^2 - 2(-1) + 7] \\ &= (3 + 2 + 7) = 12 \end{aligned}$$

Zeros of a Polynomial : A real number α is called a zero of the polynomial $p(x)$, if $p(\alpha) = 0$

Note : 1. If α and β are the zeros of $p(x) = ax^2 + bx + c$, $a \neq 0$, then.

$$(i) \quad \alpha + \beta = -\frac{b}{a}$$

$$(ii) \quad \alpha \beta = \frac{c}{a}$$

(2) A quadratic polynomial whose zeros are α and β is given by

$$p(x) = \{x^2 - (\alpha + \beta)x + \alpha \beta\}$$

(3) If α , β and γ are the zeros of $p(x) = ax^3 + bx^2 + cx + d$, then,

$$(i) \quad \alpha + \beta + \gamma = -\frac{b}{a}$$

$$(ii) \quad (\alpha \beta + \beta \gamma + \gamma \alpha) = \frac{c}{a}$$

$$(iii) \quad \alpha \beta \gamma = -\frac{d}{a}$$

(4) A cubic polynomial whose zeros are α , β and γ is given by $p(x)$

$$= \{x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha \beta + \beta \gamma + \gamma \alpha)x - \alpha \beta \gamma\}$$

Factor Theorem : The Condition that $(x - a)$ is a factor of a polynomial $f(x)$, if and only if $f(a) = 0$

Thus, $(x - a)$ is a factor of $f(x) \Leftrightarrow f(a) = 0$.

Remarks : (i) $(x + a)$ is a factor of polynomial $p(x)$ if and only if $p(-a) = 0$

(ii) $(ax - b)$ is a factor of a polynomial $p(x)$, if $p\left(\frac{b}{a}\right) = 0$

(iii) $(ax + b)$ is a factor of a polynomial $p(x)$, if $p\left(-\frac{b}{a}\right) = 0$

(iv) $(x - a)(x - b)$ are factors of a polynomial $p(x)$ if $p(a) = 0$ and $p(b) = 0$.

• **Remainder Theorem :** If a polynomial $f(x)$ of degree $n \geq 1$, is divided by $(x - a)$, then the remainder is $f(a)$.

e.g. Let $f(x) = x^3 + 3x^2 - 5x + 4$ be divided by $(x - 1)$. Find the remainder.

$$\begin{aligned} \text{Sol. Remainder} &= f(1) \\ &= 1^3 + 3 \times 1^2 - 5 \times 1 + 4 = 3 \end{aligned}$$

Important Results :

(i) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n .

(ii) $(x^n + a^n)$ is divisible by $(x + a)$ only when n is odd.

(iii) $(x^n - a^n)$ is divisible by $(x + a)$ only for even values of n .

(iv) $(x^n + a^n)$ is never divisible by $(x - a)$

• **H.C.F & L.C.M of Polynomials :**

Divisor : A polynomial $p(x)$ is called a divisor of another polynomial $f(x) = p(x) \cdot g(x)$ for some polynomial $g(x)$.

• **H.C.F. or (G.C.D.) of Polynomials :** A polynomial $h(x)$ is called the H.C.F. or G.C.D of two or more given polynomials, if $h(x)$ is a polynomial of highest degree dividing each one of the given polynomials.

• **Remark :** The coefficient of highest degree term in H.C.F is always taken as positive.

e.g. What is the HCF of $(x + 3)^2(x - 2)^3$ and $(x - 1)(x + 3)(x - 2)^2$?

Sol. $p(x) = (x + 3)^2(x - 2)^3$

$$q(x) = (x - 1)(x + 3)(x - 2)^2$$

We see that $(x + 3)(x - 2)^2$ is such a polynomial that is a common divisor and whose degree is highest among all common divisors.

• **L.C.M. of Polynomials :** A polynomial $p(x)$ is called the L.C.M. of two or more given polynomials, if it is a polynomial of smallest degree which is divided by each one of the given polynomials.

e.g. Find the L.C.M of $(x - 3)(x + 4)^2$ and $(x - 3)^3(x + 4)$:

Sol : $p(x) = (x - 3)(x + 4)^2$

$$q(x) = (x - 3)^3(x + 4)$$

we make a polynomial by taking each factor of $p(x)$ and $q(x)$.

If a factor is common in both, then we take the factor which has highest degree in $p(x)$ and $q(x)$.

$$\therefore \text{LCM} = (x - 3)^3(x + 4)^2$$

Note : For any two polynomials $p(x)$ and $q(x)$

$$p(x) \times q(x) = (\text{Their H.C.F.}) \times (\text{Their L.C.M.})$$

• **Factorisation of Polynomials**

: To express a given polynomial as the product of polynomials, each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorisation.

• **Formulae for Factorisation:**

- (i) $(x + y)^2 = x^2 + y^2 + 2xy$
 (ii) $(x - y)^2 = x^2 + y^2 - 2xy$

- (iii) $(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$
 (iv) $(x + y)^2 - (x - y)^2 = 4xy$
 (v) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
 (vi) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
 (vii) $x^2 - y^2 = (x + y)(x - y)$
 (viii) $(x^3 + y^3) = (x + y)(x^2 + y^2 - xy)$
 (ix) $(x^3 - y^3) = (x - y)(x^2 + y^2 + xy)$
 (x) $(x + y + z)^2 = (x^2 + y^2 + z^2 + 2(xy + yz + zx))$
 (xi) $(x^3 + y^3 + z^3 - 3xyz) = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$$= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

(xii) $x^2 + y^2 + z^2 - xy - yz - zx$

$$= \frac{1}{2}[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

(xiii) $x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$

EXERCISE

1. If $f(x)$ is divided by $(3x + 5)$, the remainder is :

(a) $f\left(\frac{3}{5}\right)$ (b) $f\left(-\frac{3}{5}\right)$

(c) $f\left(\frac{5}{3}\right)$ (d) $f\left(-\frac{5}{3}\right)$

2. If $(x^{11} + 1)$ is divided by $(x + 1)$, the remainder is :

- (a) 0 (b) 2
 (c) 11 (d) 12

3. When $(x^4 - 3x^3 + 2x^2 - 5x + 7)$ is divided by $(x - 2)$, the remainder is :

- (a) 3 (b) -3 (c) 2 (d) 0

4. If $(x - 2)$ is a factor of $(x^2 + 3qx - 2q)$, then the value of q is :

- (a) 2 (b) -2 (c) 1 (d) -1

5. The value of λ for which the expression $x^3 + x^2 - 5x + \lambda$ will be divisible by $(x - 2)$ is :

- (a) 2 (b) -2 (c) -3 (d) 4

6. If $(x + 1)$ and $(x - 2)$ be the factors of $x^3 + (a + 1)x^2 - (b - 2)x - 6$, then the value of a and b will be :

- (a) 2 and 8 (b) 1 and 7
 (c) 5 and 3 (d) 3 and 7

7. The polynomial $(x^4 - 5x^3 + 5x^2 - 10x + 24)$ has a factor as :

- (a) $x + 4$ (b) $x - 2$
 (c) $x + 2$
 (d) None of these

8. $(x^{29} - x^{25} + x^{13} - 1)$ is divisible by:

- (a) both $(x - 1)$ & $(x + 1)$
 (b) $(x - 1)$ but not by $(x + 1)$
 (c) $(x + 1)$ but not by $(x - 1)$

- (d) neither $(x - 1)$ nor $(x + 1)$
9. The value of expression $(9x^2 + 12x + 7)$ for $x = -\frac{4}{3}$ is :

(a) 7 (b) 0 (c) -7 (d) 18

10. When $(x^3 - 2x^2 + px - q)$ is divided by $(x^2 - 2x - 3)$, the remainder is $(x - 6)$. The values of P and q are :

- (a) $p = -2, q = -6$
 (b) $p = 2, q = -6$
 (c) $p = -2, q = 6$
 (d) $p = 2, q = 6$

11. If $(x - a)$ is a factor of $(x^3 - 3x^2a + 2a^2x + b)$, then the value of b is :

- (a) 0 (b) 2 (c) 1 (d) 3

12. If $x^{100} + 2x^{99} + K$ is divisible by $(x + 1)$, then the value of K is :

- (a) -3 (b) 2 (c) -2 (d) 1

13. If the polynomial $f(x)$ is such that $f(-1) = 0$, then a factor of $f(x)$ is :

- (a) -1 (b) $x - 1$
 (c) $x + 1$ (d) $-1 - x$

14. If $x^3 + 5x^2 + 10K$ leaves remainder $-2x$ when divided by $x^2 + 2$, then the value of k is :

- (a) -2 (b) 1 (c) -1 (d) 2

15. Which of the following is a polynomial?

(a) $x^2 - 3x + 2\sqrt{x} + 7$

(b) $\sqrt{x} - \frac{1}{\sqrt{x}}$

(c) $x^{7/2} - x + x^{3/2}$

- (d) None of these

16. If α and β are the zeros of $x^2 + 3x + 7$, then the value of $(\alpha + \beta)$ is :

- (a) -3 (b) 3 (c) 7 (d) -7

17. If α and β are the zeros of $2x^2 + 3x - 10$, then the value of $\alpha\beta$ is:

- (a) $-\frac{5}{2}$ (b) 5 (c) -5 (d) $-\frac{3}{2}$

18. If common factor of $x^2 + bx + c$ and $x^2 + mx + n$ is $(x + a)$, then the value of a is :

(a) $\frac{c - n}{b - m}$ (b) $\frac{c - n}{b + m}$

(c) $\frac{c - n}{m - b}$ (d) $\frac{c + 1}{b - m}$

19. $(x^4 + 5x^3 + 6x^2)$ is equal to :

- (a) $x(x + 3)(x^2 + 2)$
 (b) $x^2(x + 3)(x + 2)$
 (c) $x^2(x - 2)(x - 3)$
 (d) $x(x^2 + 3)(x + 2)$

20. The factors of $(x^4 + 625)$ are :

- (a) $(x^2 - 25)(x^2 + 25)$
 (b) $(x^2 + 25)(x^2 + 25)$
 (c) $(x^2 - 10x + 25)(x^2 + 5x + 24)$
 (d) do not exist

21. The factors of $(x^4 + 4)$ are :

- (a) $(x^2 + 2)^2$
 (b) $(x^2 + 2)(x^2 - 2)$
 (c) $(x^2 + 2x + 2)(x^2 - 2x + 2)$
 (d) None of these

22. $(x + y)^3 - (x - y)^3$ can be factorized as :

- (a) $2y(3x^2 + y^2)$
 (b) $2x(3x^2 + y^2)$
 (c) $2y(3y^2 + x^2)$
 (d) $2x(x^2 + 3y^2)$

23. The H.C.F. of $x^2 - xy - 2y^2$ and $2x^2 - xy - y^2$ is :
 (a) $(x + y)$
 (b) $(x - y)$
 (c) $(2x - 3y)$
 (d) None of these
24. The H.C.F. of $(x^3 + x^2 + x + 1)$ and $(x^4 - 1)$ is:
 (a) $(x^2 - 1)(x^2 + 1)$
 (b) $(x + 1)(x^2 - 1)$
 (c) $(x + 1)(x^2 + 1)$
 (d) $(x^2 + 1)(x + 1)(x^3 + 1)$
25. The L.C.M of the polynomials X and Y, where $X = (x + 3)^2(x - 2)(x + 1)^2$ and $Y = (x + 1)^2(x + 3)(x + 4)$ is given by :
 (a) $(x - 2)(x + 4)(x + 3)^2(x + 1)^2$
 (b) $(x + 1)(x - 2)(x + 3)(x + 4)$
 (c) $(x - 2)(x + 1)(x + 3)^2(x + 4)$
 (d) $(x - 2)(x + 1)^2(x + 3)(x + 4)$
26. The L.C.M of $(x + 2)^2(x - 2)$ and $(x^2 - 4x - 12)$ is :
 (a) $(x + 2)(x - 2)$
 (b) $(x + 2)(x - 2)(x - 6)$
 (c) $(x + 2)(x - 2)^2$
 (d) $(x + 2)^2(x - 2)(x - 6)$
27. The H.C.F. of $(x^2 - 4)$, $(x^2 - 5x - 6)$ and $(x^2 + x - 6)$ is :
 (a) 1
 (b) $(x - 2)$
 (c) $(x + 2)$
 (d) $(x^2 + x - 6)$
28. The H.C.F of $2(x^2 - y^2)$ and $5(x^3 - y^3)$ is :
 (a) $2(x^2 - y^2)$ (b) $(x - y)$
 (c) $(x + y)$ (d) $(x^2 + y^2)$
29. The L.C.M of $(2x^2 - 3x + 2)$ and $(x^3 - 4x^2 + 4x)$ is :
 (a) $x(2x^2 + 1)(x^2 + 2)$
 (b) $x(2x + 1)(x - 2)^2$
 (c) $x(2x^2 + 1)(x - 1)^2$
 (d) $x(2x + 1)(x^2 - 1)$
30. The L.C.M of $(a^3 + b^3)$ and $(a^4 - b^4)$ is :
 (a) $(a^3 + b^3)(a^2 + b^2)(a - b)$
 (b) $(a^3 + b^3)(a + b)(a^2 + b^2)$
 (c) $(a + b)(a^2 + ab + b^2)(a^3 + b^3)$
 (d) $(a^3 + b^3)(a^2 - b^2)(a - b)$
31. If Polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 2x + a$ are divided by $(x - 2)$, the same remainder are obtained. Find the value of a :
 (a) 3 (b) -9 (c) -3 (d) -5
32. If the polynomial $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is divided by $(x - 1)$ and $(x + 1)$, the remainders are 5 and 19 respectively. The values of a and b are:
 (a) $a = 8, b = 5$
 (b) $a = 5, b = 6$
 (c) $a = 6, b = 8$
 (d) $a = 5, b = 8$
33. Factorise : $(x^8 + x^4y^4 + y^8)$
 (a) $(x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$
 (b) $(x^2 + xy - y^2)(x^4 - x^4y^4 + y^4)$
 (c) $(x^2 + xy + y^2)^2(x^4 - x^2y^2 + y^4)$
 (d) $(x^2 - xy + y^2)^2(x^4 - x^4y^4 - y^4)$
34. Factorise : $\left(x^6 + \frac{y^6}{27}\right)$
 (a) $\left(x^2 + \frac{y^2}{3}\right)\left(x^4 + \frac{x^2y^2}{3} + \frac{x^2y^6}{9}\right)$
 (b) $\left(x^2 + \frac{y^2}{3}\right)\left(x^4 - \frac{x^2y^2}{3} + \frac{y^4}{9}\right)$
 (c) $\left(x^2 - \frac{y^2}{3}\right)\left(x^4 - \frac{x^2y^2}{3} + \frac{x^2y^4}{9}\right)$
 (d) $\left(x^2 - \frac{y^2}{3}\right)\left(x^4 - \frac{x^2y^2}{3} + \frac{y^4}{9}\right)$
35. Factorise : $(x^4 + x^2 + 25)$
 (a) $(x^2 + 3x + 5)(x^2 + 3x - 5)$
 (b) $(x^2 + 5 + 3x)(x^2 + 5 - 3x)$
 (c) $(x^2 + x + 5)(x^2 - x + 5)$
 (d) None of these

ANSWER KEY

1. (d)	5. (b)	9. (a)	13. (c)	17. (c)	21. (c)	24. (c)	27. (a)	30. (a)	33. (a)
2. (a)	6. (b)	10. (c)	14. (b)	18. (a)	22. (a)	25. (a)	28. (b)	31. (c)	34. (b)
3. (b)	7. (b)	11. (a)	15. (d)	19. (b)	23. (d)	26. (d)	29. (b)	32. (d)	35. (b)
4. (d)	8. (b)	12. (d)	16. (a)	20. (d)					

SOLUTION

1.(d) $3x + 5 = 0 \Rightarrow x = -\frac{5}{3}$

So, remainder is $f\left(-\frac{5}{3}\right)$

2.(a) Remainder = $f(-1)$
 $= (-1)^{11} + 1$
 $= -1 + 1 = 0$

3.(b) Remainder = $f(2)$
 $= 2^4 - 3(2)^3 + 2(2)^2 - 5 \times 2 + 7$
 $= 16 - 24 + 8 - 10 + 7 = -3$

4.(d) Since $(x - 2)$ is a factor of $f(x)$
 $= x^2 + 3qx - 2q$
 $\therefore f(2) = 0 \Rightarrow 2^2 + 3q \times 2 - 2q = 0$

4q = -4 $\Rightarrow q = -1$
 5.(b) $(x - 2)$ is a factor of polynomial

$f(x) = x^3 + x^2 - 5x + \lambda$
 $\therefore f(2) = 0 \Rightarrow 2^3 + 2^2 - 5 \times 2 + \lambda = 0$

$\Rightarrow 12 - 10 + \lambda = 0 \Rightarrow \lambda = -2$

6.(b) Since $(x + 1)$ & $(x - 2)$ are the factors of
 $f(x) = x^3 + (a + 1)x^2 - (b - 2)x - 6$

$\therefore f(-1) = 0$ and $f(2) = 0$
 or $-1 + (a + 1) - (b - 2) - 6 = 0$
 and $8 + 4(a + 1) - (b - 2) \times 2 -$

$6 = 0$

or $a + b = 8$ (i)

and $2a - b = -5$ (ii)

(i) + (ii) $3a = 3 \Rightarrow a = 1$

From equation (i)

$b = 8 - 1 = 7$

$\therefore a = 1$ & $b = 7$

7.(b) Since $x = 2$ makes the given expression zero, so, $(x - 2)$ is its factor.

8.(b) Since $x = 1$ makes $x^{29} - x^{25} + x^{13} - 1$ zero, so $(x - 1)$ is its factor. And $x = -1$ does not make it zero so $(x + 1)$ is not its factor.

9.(a) $f(x) = 9x^2 + 12x + 7$

$$\therefore f\left(-\frac{4}{3}\right) = 9\left(-\frac{4}{3}\right)^2 + 12\left(-\frac{4}{3}\right) + 7$$

$$= 16 - 16 + 7 = 7$$

10.(c)

$$\begin{array}{r} x^2 - 2x - 3 \overline{) x^3 - 2x^2 + px - q} \\ \underline{x^3 - 2x^2 - 3x} \\ + px - q \end{array}$$

$$(p+3)x - q \leftarrow \text{remainder}$$

$$\therefore (p+3)x - q = x - 6$$

$$\therefore p+3 = 1 \text{ and } q = 6$$

$$\text{or } p = -2 \text{ and } q = 6$$

11.(a) let $f(x) = x^3 - 3x^2a + 2a^2x + b$

$$\therefore (x-a) \text{ is a factor of } f(x)$$

$$\therefore f(a) = 0 \Rightarrow a^3 - 3a^3 + 2a^3 + b = 0$$

$$\Rightarrow b = 0$$

12.(d) $\therefore x^{100} + 2x^{99} + k$
 $= f(x)$ (let) is divisible by $(x+1)$

$$\therefore f(-1) = 0$$

$$\Rightarrow 1 - 2 + k = 0 \Rightarrow k = 1$$

13.(c) Since $x = -1$ makes $f(x)$ zero,
 So $(x+1)$ is its factor.

14.(b)

$$\begin{array}{r} x+5 \overline{) x^2 + 2x^3 + 5x^2 + 10k} \\ \underline{x^3 + 2x} \\ 5x^2 - 2x + 10k \end{array}$$

$$\begin{array}{r} 5x^2 - 2x + 10k \\ \underline{5x^2} \\ -2x + 10k - 10 \end{array}$$

$$-2x + 10k - 10 \leftarrow \text{Remainder}$$

$$\text{but given, remainder} = -2x$$

$$\therefore -2x + 10k - 10 = -2x$$

$$\Rightarrow 10k = 10$$

$$\Rightarrow k = 1$$

15.(d) For polynomial, each power of x must be a non-negative integer.

16.(a) $\alpha + \beta = \frac{b}{a} = \frac{3}{1} = -3$

17.(c) $\alpha\beta = \frac{c}{a} = \frac{10}{2} = -5$

18.(a) Let $f(x) = x^2 + bx + c$
 and $g(x) = x^2 + mx + n$
 $\therefore (x+a)$ is a common factor of $f(x)$ and $g(x)$
 $\therefore f(-a) = 0$ and $g(-a) = 0$
 or $a^2 - ba + c = 0$ and $a^2 - ma + n = 0$
 $\Rightarrow a^2 = ab - c \dots (i)$ and $a^2 = ma - n \dots (ii)$

$$\therefore \text{from (i) and (ii)}$$

$$ab - c = ma - n$$

$$\Rightarrow a(b-m) = c-n \text{ or } a = \frac{c-n}{b-m}$$

19.(b) $x^4 + 5x^3 + 6x^2 = x^2(x^2 + 5x + 6)$

$$= x^2(x^2 + 3x + 2x + 6)$$

$$= x^2[x(x+3) + 2(x+3)]$$

$$= x^2(x+3)(x+2)$$

20.(b) Do not exist

21.(c) $x^4 + 4 = (x^2)^2 + (2)^2 + 4x^2 - 4x^2$

$$= (x+2)^2 - (2x)^2$$

$$= (x^2 + 2x + 2)(x^2 - 2x + 2)$$

22.(a) Using formulae, $a^3 - b^3$

$$= (a-b)(a^2 + b^2 + ab)$$

$$\therefore (x+y)^3 - (x-y)^3 = [(x+y) - (x-y)] + [(x+y)^2 + (x-y)^2 + (x+y)(x-y)]$$

$$= 2y[2(x^2 + y^2) + (x^2 - y^2)]$$

$$= 2y(3x^2 + y^2)$$

23.(d) $x^2 - xy - 2y^2 = (x^2 - y^2) - (xy + y^2)$

$$= (x+y)(x-y) - y(x+y)$$

$$= (x+y)(x-y-y)$$

$$= (x+y)(x-2y)$$

$$2x^2 - xy - y^2 = (x^2 - xy) + (x^2 - y^2)$$

$$= x(x-y) + (x+y)(x-y)$$

$$= (x-y)(x+x+y)$$

$$= (x-y)(2x+y)$$

Clearly, no factor is common,

So, H.C.F = 1

24.(c) $x^3 + x^2 + x + 1 = x^2(x+1) + 1$
 $(x+1)$

$$= (x+1)(x^2 + 1)$$

$$x^4 - 1 = (x^2 - 1)(x^2 + 1)$$

$$= (x+1)(x-1)(x^2 + 1)$$

$$\therefore \text{Required H.C.F} = (x+1)(x^2 + 1)$$

25.(a) $X = (x+3)^2(x-2)(x+1)^2$

$$Y = (x+1)^2(x+3)(x+4)$$

$$\text{So, LCM} = (x-2)(x+4)(x+3)^2(x+1)^2$$

26.(d) $x^2 - 4x - 12 = x^2 - 6x + 2x - 12$

$$= x(x-6) + 2(x-6)$$

$$= (x+2)(x-6)$$

$$\text{and other is } (x+2)^2(x-2)$$

$$\therefore \text{L.C.M} = (x+2)^2(x-2)(x-6)$$

27.(a) $x^2 - 4 = (x+2)(x-2)$

$$x^2 - 5x - 6 = x^2 - 6x + x - 6$$

$$= (x-6)(x+1)$$

$$\text{and } x^2 + x - 6 = x^2 + 3x - 2x - 6$$

$$= (x+3)(x-2)$$

Clearly, there is no common factor.

So, H.C.F = 1.

28.(b) $2(x^2 - y^2) = 2(x-y)(x+y)$
 and $5(x^3 - y^3) = 5(x-y)(x^2 + y^2 + xy)$

$$\therefore \text{H.C.F} = (x-y)$$

29.(b) $2x^2 - 3x + 2 = 2x^2 - 4x + x - 2 = 2x(x-2) + 1(x-2) = (x-2)(2x+1)$

$$x^3 - 4x^2 + 4x = x(x^2 - 4x + 4) = x(x-2)^2$$

$$\therefore \text{L.C.M} = x(x-2)^2(2x+1)$$

30.(a) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$$a^4 - b^4 = (a-b)(a+b)(a^2 + b^2)$$

$$\therefore \text{L.C.M} = (a-b)(a+b)(a^2 - ab + b^2)(a^2 + b^2)$$

$$= (a-b)(a^3 + b^3)(a^2 + b^2)$$

31.(c) $f(x) = 2x^3 + ax^2 + 3x - 5$

$$g(x) = x^3 + x^2 - 2x + a$$

By remainder theorem,

$$f(2) = 2(2)^3 + a(2)^2 + 3 \times 2 - 5$$

$$= 17 + 4a$$

$$\text{and, } g(2) = 2^3 + (2)^2 - 2 \times 2 + a$$

$$= 8 + a$$

$$\therefore 17 + 4a = 8 + a$$

$$\Rightarrow 3a = -9 \text{ or } a = -3$$

32.(d) By remainder theorem,

$$f(1) = 5 \dots (i) [\because x-1=0 \Rightarrow x=1]$$

$$\text{and } f(-1) = 19 \dots (ii) [\because x+1=0 \Rightarrow x=-1]$$

$$\text{Now, from (i) } 1 - 2 + 3 - a + b = 5$$

$$\text{or } b - a = 3 \dots (iii)$$

$$\text{from (ii) } 1 + 2 + 3 + a + b = 19$$

$$\text{or } a + b = 13 \dots (iv)$$

$$(iii) + (iv) \quad 2b = 16 \text{ or } b = 8$$

$$\text{Now from (iv), } a = 13 - 8 = 5$$

$$\therefore a = 5, b = 8$$

33.(a) $x^8 + x^4y^4 + y^8$

$$= x^8 + 2x^4y^4 + y^8 - x^4y^4$$

$$= (x^4 + y^4)^2 - (x^2y^2)^2$$

$$= (x^4 + y^4 + x^2y^2)(x^4 + y^4 - x^2y^2)$$

$$= [(x^2 + y^2)^2 - (xy)^2](x^4 - x^2y^2 + y^4)$$

$$= (x^2 + xy + y^2)(x^2 - xy + y^2)(x^4 - x^2y^2 + y^4)$$

34.(b) $x^6 + \frac{y^6}{27} = (x^2)^3 + \left(\frac{y^2}{3}\right)^3$

$$= \left(x^2 + \frac{y^2}{3}\right) \left(x^4 - \frac{x^2y^2}{3} + \frac{y^4}{9}\right)$$

35.(b) $x^4 + x^2 + 25 = (x^2)^2 + (5)^2 + 10x^2 - 9x^2$

$$= (x^2 + 5)^2 - (3x)^2$$

$$= (x^2 + 5 + 3x)(x^2 + 5 - 3x)$$

ALGEBRIC IDENTITIES

- An algebraic identity is an algebraic equation which is true for all values of the variable (s).

Important Formulae:

- $(a+b)^2 = a^2 + b^2 + 2ab$
- $(a-b)^2 = a^2 + b^2 - 2ab$
- $(a+b)^2 = (a-b)^2 + 4ab$
- $(a-b)^2 = (a+b)^2 - 4ab$
- $a^2 - b^2 = (a+b)(a-b)$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $(a^3+b^3) = (a+b)(a^2-ab+b^2)$
- $(a^3-b^3) = (a-b)(a^2+ab+b^2)$
- $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$
- $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
 $= \frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$

Note:-

$$a^3+b^3+c^3-3abc = 0,$$

$$\text{If } \{(i) a + b + c = 0 \text{ (} a \neq b \neq c \text{)}\}$$

or

$$(ii) a^2+b^2+c^2-ab-bc-ca = 0$$

$$12. a^2+b^2+c^2-ab-bc-ca = \frac{1}{2}$$

$$[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$13. \text{ If } ax^2 + bx + c =$$

$$0, \text{ then, } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Some Important Results

$$1. \text{ If } x + \frac{1}{x} = a, \text{ then}$$

$$(i) x^2 + \frac{1}{x^2} = a^2 - 2$$

$$(ii) x^4 + \frac{1}{x^4} = b^2 - 2 \text{ where}$$

$$b = a^2 - 2 \text{ 0 e.g. } x + \frac{1}{x} = 3,$$

Then,

$$x^2 + \frac{1}{x^2} = 3^2 - 2 = 7, \text{ and } x^4 + \frac{1}{x^4} = 49 - 2 = 47$$

$$2. \text{ If } x - \frac{1}{x} = a, \text{ then}$$

$$(i) x^2 + \frac{1}{x^2} = a^2 + 2$$

$$(ii) x^4 + \frac{1}{x^4} = b^2 - 2, \text{ where } b = a^2 + 2$$

$$\text{e.g. } x - \frac{1}{x} = 3, \text{ then}$$

$$x^2 + \frac{1}{x^2} = 3^2 + 2 = 11 \text{ and}$$

$$x^4 + \frac{1}{x^4} = 11^2 - 2 = 119.$$

$$3. \text{ If } x^4 + \frac{1}{x^4} = a, \text{ then}$$

$$(i) x^2 + \frac{1}{x^2} = \sqrt{a+2} = b$$

$$(ii) x + \frac{1}{x} = \sqrt{b+2}$$

$$(ii) x - \frac{1}{x} = \sqrt{b-2}$$

$$\text{e.g. } x^4 + \frac{1}{x^4} = 119$$

$$\therefore x^2 + \frac{1}{x^2} = \sqrt{119+2} = 11$$

$$x + \frac{1}{x} = \sqrt{11+2} = \sqrt{13}$$

$$x - \frac{1}{x} = \sqrt{11-2} = 3$$

$$4. \text{ If } x + \frac{1}{x} = 2, \text{ then } x = 1$$

$$5. \text{ If } x + \frac{1}{x} = -2, \text{ then } x = -1$$

$$6. x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$7. x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right)$$

$$8. \text{ If } x + \frac{1}{x} = 1, \text{ then } x^3 = -1$$

$$9. \text{ If } x + \frac{1}{x} = -1, \text{ then } x^3 = 1,$$

$$10. \text{ If } x + \frac{1}{x} = \sqrt{3} \text{ then } x^3 + \frac{1}{x^3} = 0 \Rightarrow x^6 = -1 \text{ or } x^6 + 1 = 0$$

$$11. \text{ If } ax + by = m \text{ and } bx - ay = n \text{ then, } (a^2 + b^2)(x^2 + y^2) = m^2 + n^2$$

Note : If the sum of squares of real numbers be zero, then each number is equal to zero i.e.

$$\text{if } (x-a)^2 + (y-b)^2 + (z-c)^2 = 0, \text{ then}$$

$$x - a = 0 \Rightarrow x = a,$$

$$y - b = 0 \Rightarrow y = b \text{ and}$$

$$z - c = 0 \Rightarrow z = c$$

$$\text{or if } x^2 + y^2 + z^2 = 0, \text{ then } x = 0, y = 0 \text{ \& } z = 0$$

Based on Increasing power

$$\text{Ex. 1 If } x + \frac{1}{x} = 3, \text{ find the value of}$$

$$x^2 + \frac{1}{x^2} = ?$$

$$\text{Sol. } x + \frac{1}{x} = 3$$

Squaring both sides,

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = (3)^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 9$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 9 - 2 = 7$$

$$(\text{If } x + \frac{1}{x} = a \text{ then } x^2 + \frac{1}{x^2} = a^2 - 2)$$

$$\text{Same } x^4 + \frac{1}{x^4} = (a^2 - 2)^2 - 2$$

Alternate:-

$$x + \frac{1}{x} = 3$$

$$x^2 + \frac{1}{x^2} = 3^2 - 2 = 7$$

Ex.2 If $x + \frac{1}{x} = 5$, find the (i) $x^2 + \frac{1}{x^2}$

(ii) $x^4 + \frac{1}{x^4}$

Sol.(i) $x + \frac{1}{x} = 5$

$$x^2 + \frac{1}{x^2} = 5^2 - 2 = 23$$

(ii) $x + \frac{1}{x} = 5$

$$x^2 + \frac{1}{x^2} = 23$$

Again squaring both sides

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (23)^2$$

$$x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2} = 529$$

$$x^4 + \frac{1}{x^4} = 529 - 2$$

$$x^4 + \frac{1}{x^4} = 527$$

Ex.3 If $x + \frac{1}{x} = 4$, find the (i) $x^2 + \frac{1}{x^2}$

(ii) $x^4 + \frac{1}{x^4}$

Sol.(i) $x + \frac{1}{x} = 4$

$$x^2 + \frac{1}{x^2} = 4^2 - 2 = 14$$

(ii) $x + \frac{1}{x} = 4$

$$x^2 + \frac{1}{x^2} = 14$$

Again squaring both sides

$$x^4 + \frac{1}{x^4} = 196 - 2$$

$$x^4 + \frac{1}{x^4} = 194$$

Ex.4 If $x + \frac{1}{x} = 6$, Find the value of

(i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$

(iii) $x^8 + \frac{1}{x^8}$

Sol. $x + \frac{1}{x} = 6$

$$x^2 + \frac{1}{x^2} = 6^2 - 2 = 34$$

$$x^4 + \frac{1}{x^4} = 34^2 - 2 = 1154$$

$$x^8 + \frac{1}{x^8} = 1154^2 - 2 = 1331714$$

Ex.5 If $x - \frac{1}{x} = 3$, Find the value of

(i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$

Sol. $x - \frac{1}{x} = 3$

squaring both side,

$$\left(x - \frac{1}{x}\right)^2 = 3^2$$

$$x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 9$$

$$x^2 + \frac{1}{x^2} = 9 + 2 = 11$$

squaring both sides,

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (11)^2$$

$$x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2} = 121$$

$$x^4 + \frac{1}{x^4} = 121 - 2 = 119$$

(If $x - \frac{1}{x} = a$ then $x^2 + \frac{1}{x^2} = a^2 + 2$

and $x^4 + \frac{1}{x^4} = (a^2 + 2)^2 - 2$)

Ex.6 If $x - \frac{1}{x} = 4$, Find the value of

(i) $x^2 + \frac{1}{x^2}$ (ii) $x^4 + \frac{1}{x^4}$

Sol. $x - \frac{1}{x} = 4$

$$x^2 + \frac{1}{x^2} = 4^2 + 2 = 18$$

$$x^4 + \frac{1}{x^4} = 18^2 - 2 = 322$$

Ex.7 If $x + \frac{1}{x} = 3$, Find the value of

$$x^3 + \frac{1}{x^3}$$

Sol. $x + \frac{1}{x} = 3$

Cube both sides,

$$\left(x + \frac{1}{x}\right)^3 = 3^3$$

$$x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x} \left(x + \frac{1}{x}\right) = 27$$

$\therefore$ Put the value of $x + \frac{1}{x} = 3$

$$x^3 + \frac{1}{x^3} + 3 \times 3 = 27$$

$$x^3 + \frac{1}{x^3} = 27 - 9 = 18$$

(If $x + \frac{1}{x} = a$ then $x^3 + \frac{1}{x^3} = a^3 - 3a$)

Alternate:

Here, $a = 3$

$$x^3 + \frac{1}{x^3} = 3^3 - 3 \times 3$$

$$x^3 + \frac{1}{x^3} = 18$$

Ex.8 If $x + \frac{1}{x} = 4$, Find the value of

$$x^3 + \frac{1}{x^3}$$

Sol. $x + \frac{1}{x} = 4$

$$x^3 + \frac{1}{x^3} = 4^3 - 3 \times 4 = 64 - 12 = 52$$

• **Same as:**

$\Rightarrow$ If $x + \frac{1}{x} = 3, 4, 5, 6$, then

$$x^3 + \frac{1}{x^3} = 18, 52, 110, 198$$

Ex.9 If $x - \frac{1}{x} = 4$, then the value of

$$x^3 - \frac{1}{x^3}$$

Sol. $x - \frac{1}{x} = 4$

Cube both sides,

$$\left(x - \frac{1}{x}\right)^3 = 4^3$$

$$x^3 - \frac{1}{x^3} - 3 \times x \times \frac{1}{x} \left(x - \frac{1}{x} \right) = 4^3$$

Put the value of $x - \frac{1}{x} = 4$

$$x^3 - \frac{1}{x^3} - 3 \times 4 = 64$$

$$x^3 - \frac{1}{x^3} = 64 + 12$$

$$x^3 - \frac{1}{x^3} = 76$$

(If $x - \frac{1}{x} = a$ then $x^3 - \frac{1}{x^3} = a^3 + 3a$)

Alternate:

Here, $a = 4$

$$x^3 - \frac{1}{x^3} = 4^3 + 3 \times 4 = 64 + 12 = 76$$

* Same as if $x - \frac{1}{x} = 3, 4, 5, 6$

Then $x^3 - \frac{1}{x^3} = 36, 76, 140, 234$

Ex.10 If $x + \frac{1}{x} = 5$ find the value of

$$x^5 + \frac{1}{x^5}$$

Sol. $x + \frac{1}{x} = 5$

$$x^2 + \frac{1}{x^2} = 5^2 - 2 = 23 \quad \dots(i)$$

$$x^3 + \frac{1}{x^3} = 5^3 - 3 \times 5 = 110 \quad \dots(ii)$$

(i) $\times$ (ii)

$$\left(x^2 + \frac{1}{x^2} \right) \left(x^3 + \frac{1}{x^3} \right) = 23 \times 110$$

$$x^5 + \frac{1}{x} + x + \frac{1}{x^5} = 2530$$

Put the value of $x + \frac{1}{x} = 5$

$$x^5 + \frac{1}{x^5} = 2530 - 5 = 2525$$

* If $x + \frac{1}{x} = a$

Then $x^5 + \frac{1}{x^5} = (a^2 - 2)(a^3 - 3a) - a$

Ex.11 If $x + \frac{1}{x} = 4$,

Find the value of $x^5 + \frac{1}{x^5}$

Sol. $x + \frac{1}{x} = 4$

$$x^2 + \frac{1}{x^2} = 4^2 - 2 = 14 \quad \dots(i)$$

$$x^3 + \frac{1}{x^3} = 4^3 - 3 \times 4 = 52 \quad \dots(ii)$$

(i) $\times$ (ii)

$$x^5 + \frac{1}{x^5} = 14 \times 52 - 4 = 724$$

Ex.12 If $x - \frac{1}{x} = 4$ find the value of

$$x^5 - \frac{1}{x^5}$$

Sol. $x - \frac{1}{x} = 4$

$$x^2 + \frac{1}{x^2} = 4^2 + 2 = 18$$

$\dots(i)$

$$x^3 - \frac{1}{x^3} = 4^3 + 3 \times 4 = 76 \quad \dots(ii)$$

(i) $\times$ (ii)

$$\left(x^2 + \frac{1}{x^2} \right) \left(x^3 - \frac{1}{x^3} \right) = 18 \times 76$$

$$x^5 - \frac{1}{x} + x - \frac{1}{x^5} = 18 \times 76$$

Put the value of $x - \frac{1}{x} = 4$

$$x^5 - \frac{1}{x^5} = 18 \times 76 - \left(x - \frac{1}{x} \right)$$

$$= 1368 - 4 = 1364$$

* If $\left(x - \frac{1}{x} \right) = a$, then

$$x^5 - \frac{1}{x^5} = (a^2 + 2)(a^3 + 3a) - a$$

Ex.13 If $x - \frac{1}{x} = 5$ then find the value of

$$x^5 - \frac{1}{x^5}$$

Sol. $x - \frac{1}{x} = 5$

$$x^2 + \frac{1}{x^2} = 5^2 + 2 = 27 \quad \dots(i)$$

$$x^3 - \frac{1}{x^3} = 5^3 + 3 \times 5 = 140 \quad \dots(ii)$$

(i) $\times$ (ii)

$$x^5 - \frac{1}{x^5} = 27 \times 140 - 5 = 3775$$

Ex.14 If $x + \frac{1}{x} = 3$, Find the value of

$$x^6 + \frac{1}{x^6}$$

Sol. $x + \frac{1}{x} = 3$

$$x^3 + \frac{1}{x^3} = 3^3 - 3 \times 3 = 18$$

Squaring both side,

$$\left(x^3 + \frac{1}{x^3} \right)^2 = 18^2$$

$$x^6 + \frac{1}{x^6} + 2 \left(x^3 \right) \left(\frac{1}{x^3} \right) = 324$$

$$x^6 + \frac{1}{x^6} = 324 - 2$$

$$x^6 + \frac{1}{x^6} = 322$$

* $x + \frac{1}{x} = a$, then $x^6 + \frac{1}{x^6} = (a^3 - 3a)^2 - 2$

Ex.15. If $x + \frac{1}{x} = 3$, Find the value of

$$x^7 + \frac{1}{x^7}$$

Sol. $x + \frac{1}{x} = 3$

$$x^2 + \frac{1}{x^2} = 3^2 - 2 = 7$$

$$x^4 + \frac{1}{x^4} = 7^2 - 2 = 47 \quad \dots(i)$$

$$x^3 + \frac{1}{x^3} = 3^3 - 3 \times 3 = 18 \quad \dots(ii)$$

Multiply (i) and (ii)

$$\left(x^3 + \frac{1}{x^3} \right) \times \left(x^4 + \frac{1}{x^4} \right)$$

$$= 47 \times 18$$

$$\Rightarrow x^7 + x + \frac{1}{x} + \frac{1}{x^7} = 18 \times 47$$

Put the value of $x + \frac{1}{x} = 3$

$$x^7 + \frac{1}{x^7} = 18 \times 47 - 3$$

$$x^7 + \frac{1}{x^7} = 843$$

$$\begin{aligned} * \quad x + \frac{1}{x} &= a, \text{ then } x^7 + \frac{1}{x^7} \\ &= ((a^2 - 2)^2 - 2) \times (a^3 - 3a) - a \end{aligned}$$

Ex.16 If $x - \frac{1}{x} = 4$, Find the value of

$$x^7 - \frac{1}{x^7}.$$

Sol. $x - \frac{1}{x} = 4$

$$x^2 + \frac{1}{x^2} = 4^2 + 2 = 18$$

$$x^4 + \frac{1}{x^4} = 18^2 - 2 = 322 \quad \dots(i)$$

$$x^3 - \frac{1}{x^3} = 4^3 + 3 \times 4 = 76 \quad \dots(ii)$$

Multiply (i) and (ii)

$$\begin{aligned} &\left(x^4 + \frac{1}{x^4}\right) \times \left(x^3 - \frac{1}{x^3}\right) \\ &= 322 \times 76 \end{aligned}$$

$$x^7 - x + \frac{1}{x} - \frac{1}{x^7} = 24472$$

$$x^7 - \frac{1}{x^7} = 24472 + \left(x - \frac{1}{x}\right)$$

$$x^7 - \frac{1}{x^7} = 24472 + 4 = \mathbf{24476}$$

$$\begin{aligned} * \quad x - \frac{1}{x} &= a, \text{ then } x^7 - \frac{1}{x^7} \\ &= ((a^2 + 2)^2 - 2) \times (a^3 + 3a) + a \end{aligned}$$

Ex.17. If $x + \frac{1}{x} = 4$, find the value of

$$x^2 - \frac{1}{x^2}$$

Sol. $x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)$

$$x + \frac{1}{x} = 4$$

$$x^2 + \frac{1}{x^2} = 4^2 - 2 = 14$$

Subtract 2 both side

$$x^2 + \frac{1}{x^2} - 2 = 14 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 12$$

$$x - \frac{1}{x} = \sqrt{12}$$

We know that,

$$x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)$$

$$x^2 - \frac{1}{x^2} = 4 \sqrt{12}$$

Ex.18. If $x - \frac{1}{x} = 3$ find the value of x^3

$$+ \frac{1}{x^3}$$

Sol. $x - \frac{1}{x} = 3$

$$x^2 + \frac{1}{x^2} = 3^2 + 2 = 11$$

Adding 2 both sides

$$x^2 + \frac{1}{x^2} + 2 = 11 + 2$$

$$\left(x + \frac{1}{x}\right)^2 = 13$$

$$x + \frac{1}{x} = \sqrt{13}$$

Now,

$$x^3 + \frac{1}{x^3} = (\sqrt{13})^3 - 3 \times \sqrt{13}$$

$$x^3 + \frac{1}{x^3} = 13\sqrt{13} - 3\sqrt{13}$$

$$x^3 + \frac{1}{x^3} = 10\sqrt{13}$$

Ex.19 If $x - \frac{1}{x} = 4$, Find the value of

$$x^6 - \frac{1}{x^6}$$

Sol: $x - \frac{1}{x} = 4$

$$x^2 + \frac{1}{x^2} = 4^2 + 2 = 18$$

Adding 2 both sides,

$$x^2 + \frac{1}{x^2} + 2 = 18 + 2$$

$$\left(x + \frac{1}{x}\right)^2 = 20$$

$$x + \frac{1}{x} = \sqrt{20}$$

$$x^3 + \frac{1}{x^3} = (\sqrt{20})^3 - 3 \times \sqrt{20}$$

$$x^3 + \frac{1}{x^3} = 20\sqrt{20} - 3\sqrt{20}$$

$$x^3 + \frac{1}{x^3} = 17\sqrt{20} \quad \dots(i)$$

When $x - \frac{1}{x} = 4$

Then,

$$x^3 - \frac{1}{x^3} = 4^3 + 3 \times 4 = 76 \quad \dots(ii)$$

Multiply (i) and (ii)

$$\left(x^3 + \frac{1}{x^3}\right) \times \left(x^3 - \frac{1}{x^3}\right)$$

$$= 17\sqrt{20} \times 76$$

$$x^6 - \frac{1}{x^6} = 1292\sqrt{20}$$

Ex.20 If $x^2 + \frac{1}{x^2} = 27$, Find the value

of $x + \frac{1}{x}$

Sol. $x^2 + \frac{1}{x^2} = 27$

Adding 2 both sides,

$$x^2 + \frac{1}{x^2} + 2 = 27 + 2$$

$$\left(x + \frac{1}{x}\right)^2 = 29$$

$$x + \frac{1}{x} = \sqrt{29}$$

Ex21. If $x^2 + \frac{1}{x^2} = 31$, find the value

of $x - \frac{1}{x}$

Sol. $x^2 + \frac{1}{x^2} = 31$

Subtract 2 both sides,

$$\left(x - \frac{1}{x}\right)^2 = 29$$

$$x - \frac{1}{x} = \sqrt{29}$$

Ex.22 If $x^4 + \frac{1}{x^4} = 23$, find the value

of $x + \frac{1}{x}$

Sol. $x^4 + \frac{1}{x^4} = 23$

By adding (2) both side,
We get,

$$x^4 + \frac{1}{x^4} + 2 = 23 + 2$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (5)^2$$

$$x^2 + \frac{1}{x^2} = 5 \quad \dots(i)$$

Adding 2 both side
We get,

$$x^2 + \frac{1}{x^2} + 2 = 5 + 2$$

$$\left(x + \frac{1}{x}\right)^2 = 7$$

$$x + \frac{1}{x} = \sqrt{7}$$

Ex.23 If $x^4 + \frac{1}{x^4} = 194$, find the value

of $x^3 + \frac{1}{x^3}$

Sol. $x^4 + \frac{1}{x^4} = 194$

By adding (2) both side,
We get,

$$x^4 + \frac{1}{x^4} + 2 = 194 + 2$$

$$x^4 + \frac{1}{x^4} + 2 = 196$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (14)^2$$

$$x^2 + \frac{1}{x^2} = 14$$

Again adding (2) both side
We get,

$$x^2 + \frac{1}{x^2} + 2 = 14 + 2$$

$$\left(x + \frac{1}{x}\right)^2 = (4)^2$$

$$x + \frac{1}{x} = 4$$

Now,

$$x^3 + \frac{1}{x^3} = 4^3 - 3 \times 4$$

$$x^3 + \frac{1}{x^3} = 52$$

Ex.24 If $x + \frac{1}{x} = 5$, find the value of

$$x^2 - \frac{1}{x^2}$$

Sol. $x + \frac{1}{x} = 5$

$$x^2 + \frac{1}{x^2} - 5^2 - 2 = 23$$

Subtract 2 both sides,

$$x^2 + \frac{1}{x^2} - 2 = 23 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 21$$

$$x - \frac{1}{x} = \sqrt{21}$$

We know that,

$$\begin{aligned} x^2 - \frac{1}{x^2} &= \left(x - \frac{1}{x}\right) \times \left(x + \frac{1}{x}\right) \\ &= 5\sqrt{21} \end{aligned}$$

Ex.25 If $x^4 + \frac{1}{x^4} = 322$, Find the value

of $x^3 - \frac{1}{x^3}$

Sol. $x^4 + \frac{1}{x^4} = 322$

By adding (2) both side,
We get,

$$x^4 + \frac{1}{x^4} + 2 = 322 + 2$$

$$x^4 + \frac{1}{x^4} + 2 = 324$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (18)^2$$

$$x^2 + \frac{1}{x^2} = 18 \quad \dots(i)$$

Again subtracting (2) from both side,

We get,

$$x^2 + \frac{1}{x^2} - 2 = 18 - 2$$

$$x^2 + \frac{1}{x^2} - 2 = 16$$

$$\left(x - \frac{1}{x}\right)^2 = (4)^2$$

$$x - \frac{1}{x} = 4$$

Now,

$$x^3 - \frac{1}{x^3} = 4^3 + 3 \times 4$$

$$x^3 - \frac{1}{x^3} = 76$$

Ex.26 If $2x + \frac{1}{7x} = 4$, Find the

value of $49x^2 + \frac{1}{4x^2}$

Sol. $2x + \frac{1}{7x} = 4$

Multiply by $\frac{7}{2}$ both sides

$$\frac{7}{2} \left(2x + \frac{1}{7x}\right) = 4 \times \frac{7}{2}$$

$$7x + \frac{1}{2x} = 14$$

Squaring both sides,

$$\left(7x + \frac{1}{2x}\right)^2 = (14)^2$$

$$\begin{aligned} 49x^2 + \frac{1}{4x^2} + 2 \times 7x \times \frac{1}{2x} \\ = 196 \end{aligned}$$

$$49x^2 + \frac{1}{4x^2} = 196 - 7$$

$$49x^2 + \frac{1}{4x^2} = 189$$

Ex.27 If $4x + \frac{1}{3x} = 5$, Find the

value of $9x^2 + \frac{1}{16x^2}$

Sol. $4x + \frac{1}{3x} = 5$

Multiply by $\frac{3}{4}$ both sides

$$\frac{3}{4} \left(4x + \frac{1}{3x} \right) = 5 \times \frac{3}{4}$$

$$3x + \frac{1}{4x} = \frac{15}{4}$$

Squaring both side,

$$\left(3x + \frac{1}{4x} \right)^2 = \left(\frac{15}{4} \right)^2$$

$$9x^2 + \frac{1}{16x^2} + 2 \times 3x \times \frac{1}{4x} = \frac{225}{16}$$

$$9x^2 + \frac{1}{16x^2} = \frac{225}{16} - \frac{3}{2}$$

$$9x^2 + \frac{1}{16x^2} = \frac{201}{16}$$

Ex.28 If $a = x + \frac{1}{x}$ and $b = x - \frac{1}{x}$, find

the value of $a^4 - 2a^2b^2 + b^4$

(a) 10 (b) 4 (c) 16 (d) 8

Sol. $a^4 - 2a^2b^2 + b^4 = (a^2 - b^2)^2$

$$= ((a+b)(a-b))^2$$

$$a + b = 2x \quad \dots(i)$$

$$a - b = \frac{2}{x} \quad \dots(ii)$$

From (i) and (ii)

$$a^4 - 2a^2b^2 + b^4$$

$$= \left(2x \times \frac{2}{x} \right)^2 = 16$$

Alternate:

Let $x = 1$

Then,

$$a = 2, \quad b = 0$$

Put the value of a and b in equation,

$$a^4 - 2a^2b^2 + b^4$$

Then,

$$= 2^4 - 2 \times 2^2 \times 0 + 0^4 = 16$$

Option c is correct.

Ex.29 If $x+y+z=7$, $x^2+y^2+z^2=19$

Find $xy + yz + zx = ?$

Sol. $(x+y+z)^2 =$

$$x^2 + y^2 + z^2 + 2(xy + yz + zx)$$

$$(7)^2 = 19 + 2(xy + yz + zx)$$

$$49 - 19 = 2(xy + yz + zx)$$

$$30 = 2(xy + yz + zx)$$

$$15 = xy + yz + zx$$

Ex.30 If $x^2 + 2 = 2x$, Find $x^4 - x^3 + x^2 + 5$

Sol. $x^2 + 2 = 2x$

Squaring both sides,

$$(x^2 + 2)^2 = (2x)^2$$

$$x^4 + 4 + 4x^2 = 4x^2$$

$$x^4 = -4$$

$$x^2 + 2 = 2x$$

$$x^2 = 2x - 2$$

$$x^2 = 2(x - 1)$$

$$x - 1 = \frac{x^2}{2}$$

$$x^4 - x^3 + x^2 + 5$$

Put the value of $x^4 = -4$

$$-4 - x^2(x - 1) + 5$$

$$1 - x^2(x - 1)$$

Put the value of $x - 1$

$$1 - \frac{x^2 \times x^2}{2}$$

$$1 - \frac{x^4}{2} = 1 + \frac{4}{2} = 3$$

Ex.31 If $a^4 + a^2b^2 + b^4 = 8$ and

$a^2 + ab + b^2 = 4$ find the value of ab .

Sol. $a^4 + a^2b^2 + b^4 = 8$

$$a^4 + b^4 = 8 - a^2b^2 \quad \dots(i)$$

$$a^2 + b^2 = 4 - ab$$

Squaring both sides,

$$(a^2 + b^2)^2 = (4 - ab)^2$$

$$a^4 + b^4 + 2a^2b^2 = 16 + a^2b^2 - 8ab$$

From equation (i),

$$8 - a^2b^2 + 2a^2b^2 = 16 + a^2b^2 - 8ab$$

$$8 = 8ab$$

$$ab = 1$$

Alternate:-

We use formula,

$$a^4 + a^2b^2 + b^4 =$$

$$(a^2 + ab + b^2)(a^2 - ab + b^2)$$

$$8 = 4 \times (a^2 - ab + b^2)$$

$$(a^2 - ab + b^2) = 2 \quad \dots(i)$$

$$(a^2 + ab + b^2) = 4 \text{ (As Given)} \dots(ii)$$

(ii) - (i)

$$2ab = 2$$

$$ab = 1$$

Ex.32 If $a^4 + a^2b^2 + b^4 = 12$, $a^2 - ab + b^2 = 4$, Find the value of ab .

Sol. $a^4 + a^2b^2 + b^4 =$

$$(a^2 + ab + b^2)(a^2 - ab + b^2)$$

$$12 = 4 \times (a^2 + ab + b^2)$$

$$(a^2 + ab + b^2) = 3 \quad \dots(i)$$

$$(a^2 - ab + b^2) = 4 \quad \dots(ii)$$

(i) - (ii)

$$2ab = -1$$

$$ab = -\frac{1}{2}$$

Ex.33 If $x = a^2 + b^2$, $y = \sqrt{2}ab$, find

$$\frac{a^4 + b^4}{a^2 - ab\sqrt{2} + b^2}$$

$$(a) \ x + y \quad (b) \ x - y$$

$$(c) \ xy \quad (d) \ 2xy$$

Sol. $x = a^2 + b^2$

Squaring both side,

$$x^2 = (a^2 + b^2)^2$$

$$x^2 = a^4 + b^4 + 2a^2b^2 \quad \dots(i)$$

$$y = \sqrt{2}ab$$

$$y^2 = 2a^2b^2 \quad \dots(ii)$$

From (i) and (ii)

$$x^2 = a^4 + b^4 + y^2$$

$$x^2 - y^2 = a^4 + b^4$$

Now,

Put the value of

$$\frac{a^4 + b^4}{a^2 - ab\sqrt{2} + b^2}$$

$$[(x - y) = a^2 + b^2 - 2\sqrt{2}ab]$$

$$= \frac{x^2 - y^2}{x - y}$$

$$\frac{(x+y)(x-y)}{(x-y)} = (x+y)$$

Ex.34 If $x + y = 1$, $x^4 + y^4 = -1$, Find $x^2y^2 - 2xy$

Sol. $x + y = 1$
 Squaring both side
 $(x + y)^2 = (1)^2$
 $x^2 + y^2 + 2xy = 1$
 $x^2 + y^2 = 1 - 2xy$
 Again Squaring both sides,
 $(x^2 + y^2)^2 = (1 - 2xy)^2$
 $x^4 + y^4 + 2x^2y^2 = 1 + 4x^2y^2 - 4xy$
 Put the value of
 $x^4 + y^4 = -1$
 $-1 + 2x^2y^2 = 1 + 4x^2y^2 - 4xy$
 $-1 = 1 + 2x^2y^2 - 4xy$
 $-2 = 2(x^2y^2 - 2xy)$
 $x^2y^2 - 2xy = -1$

Ex.35 If $x + y + z = 3$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$,
 $x^2 + y^2 + z^2 = 6$, Find $xyz = ?$

Sol. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$
 $xy + yz + zx = 2xyz$
 $(x + y + z)^2 =$
 $x^2 + y^2 + z^2 + 2(xy + yz + zx)$
 $9 = 6 + 2(2xyz)$
 $3 = 4xyz$
 $xyz = \frac{3}{4}$

Ex.36 If $\frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 1$ &

$\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 0$ Find

$\frac{p^2}{x^2} + \frac{q^2}{y^2} + \frac{r^2}{z^2} = ?$

Sol. Let $a = \frac{p}{x}$, $b = \frac{q}{y}$ and $c = \frac{r}{z}$
 Then, $a + b + c = 1$
 & $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$
 $\Rightarrow ab + bc + ca = 0$
 $\Rightarrow (a + b + c)^2 =$

$a^2 + b^2 + c^2 + 2(ab + bc + ca)$
 $1 = a^2 + b^2 + c^2 + 2(0)$
 $a^2 + b^2 + c^2 = 1$

Hence, $\frac{p^2}{x^2} + \frac{q^2}{y^2} + \frac{r^2}{z^2} = 1$

Ex.37 If $x^3 + y^3 = 0$ find $x + y = ?$

(a) $\sqrt{3xy}$ (b) $\sqrt{2xy}$
 (c) $3xy$ (d) $\sqrt{4xy}$

Sol. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
 $\therefore x^3 + y^3 = 0$ (As given)
 $(x + y)^3 = 3xy(x + y)$
 $(x + y)^2 = 3xy$
 $x + y = \sqrt{3xy}$

Ex.38 If $x^4 + y^4 = x^2y^2$ find $x^6 + y^6$.

Sol. $x^6 + y^6 = (x^2)^3 + (y^2)^3$
 $= (x^2 + y^2)(x^4 + y^4 - x^2y^2)$
 $= (x^2 + y^2)(x^2y^2 - x^2y^2)$
 $= 0$

Ex.39 If $(x - a)(x - b) = 1$ and $a - b = 5$

$= 0$ find $(x - a)^3 - \frac{1}{(x - a)^3} = ?$

(a) 125 (b) -125
 (c) 0 (d) 140

Sol. $(x - a)(x - b) = 1$

$\Rightarrow (x - b) = \frac{1}{(x - a)}$

$(x - a)(x - b) = 1$ (i)
 $\therefore a - b + 5 = 0$
 $-b = -a - 5$

Put the value $(-b)$ In equation (i)

$(x - a)(x - a - 5) = 1$

let $(x - a) = M$

$M(M - 5) = 1$

$M - 5 = \frac{1}{M}$

$M - \frac{1}{M} = 5$

Now,

$M^3 - \frac{1}{M^3} = 5^3 + 3 \times 5$

$M^3 - \frac{1}{M^3} = 140$

Put the value of $M = x - a$

So,

$(x - a)^3 - \frac{1}{(x - a)^3} = 140$

Ex.40 If $x^2 + x = 5$ find the value of

$(x + 3)^3 + \frac{1}{(x + 3)^3}$

Sol. Let $(x + 3) = m$

$x + 3 = m$

$x = m - 3$

Put the value of 'x'

$x^2 + x = 5$

$\Rightarrow (m - 3)^2 + (m - 3) = 5$
 $\Rightarrow m^2 + 9 - 6m + m - 3 = 5$
 $\Rightarrow m^2 - 5m + 1 = 0$
 $\Rightarrow m^2 + 1 = 5m$

$\Rightarrow m + \frac{1}{m} = 5$

Now,

$m^3 + \frac{1}{m^3} = 5^3 - 3 \times 5$

$m^3 + \frac{1}{m^3} = 110$

Put the value of $m = x + 3$

Then,

$(x + 3)^3 + \frac{1}{(x + 3)^3} = 110$

Ex.41 If $x(x - 3) = -1$ find the value of $x^3(x^3 - 18)$

Sol. $x(x - 3) = -1$

$x - 3 = \frac{-1}{x}$

$x + \frac{1}{x} = 3$

$x^3 + \frac{1}{x^3} = 3^3 - 3 \times 3 = 18$

$x^3 - 18 = \frac{-1}{x^3}$ (i)

Now,

$x^3(x^3 - 18)$

From Equation (i)

$= x^3 \times \frac{-1}{x^3} = -1$

$x^3(x^3 - 18) = -1$

Ex.42 If $(a + b)^2 = 21 + c^2$, $(b + c)^2 = 32 + a^2$ and $(c + a)^2 = 28 + b^2$, find $a + b + c = ?$

Sol. $(a + b)^2 - c^2 = 21$

$\Rightarrow (a + b + c)(a + b - c) = 21$... (i)

$(b + c)^2 - a^2 = 32$

$\Rightarrow (b + c + a)(b + c - a) = 32$.. (ii)

$(c + a)^2 - b^2 = 28$

$\Rightarrow (c + a + b)(c + a - b) = 28$.. (iii)

Adding all three equations:-

$(a + b + c)[(a + b + c) + (b + c + a) + (c + a - b)] = 81$

$\Rightarrow (a + b + c)^2 = 81$

$\Rightarrow a + b + c = 9$

$$\begin{aligned}
 & a^3 + b^3 + c^3 - 3abc \\
 &= \frac{1}{2} (a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] \\
 &= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) \\
 \text{(i) If } (a+b+c) &= 0 \\
 &\text{then } a^3 + b^3 + c^3 - 3abc = 0 \\
 &a^3 + b^3 + c^3 = 3abc \\
 \text{(ii) If } a^3 + b^3 + c^3 - 3abc &= 0 \\
 &a, b \text{ and } c \text{ are distinct no} \\
 &\text{then } a+b+c = 0 \\
 \text{(iii) } a^3 + b^3 + c^3 - 3abc &= 0 \\
 &a, b \text{ and } c \text{ all are +ve integer no} \\
 &\text{then } a=b=c \\
 \text{(iv) } a^2 + b^2 + c^2 - ab - bc - ca &= 0 \\
 &a^2 + b^2 + c^2 = ab + bc + ca \\
 &\text{then } a=b=c
 \end{aligned}$$

Ex.43 If $a+b+c=0$, then the value of $a^3 + b^3 + c^3$ is:

- (a) 0 (b) abc
(c) $3abc$
(d) None of these

Sol. $\therefore a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

or $a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

Hence (c) is the correct option.

Ex.44 If $a^3 + b^3 + c^3 - 3abc = 0$ and $a+b+c \neq 0$

Which statement is true

- (a) $a>b>c$ (b) $a=b=c$
(c) $a>b<c$ (d) $a<b<c$

Sol.(b) We know that,

$$\begin{aligned}
 & a^3 + b^3 + c^3 - 3abc \\
 &= (a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]
 \end{aligned}$$

$\therefore a^3 + b^3 + c^3 - 3abc = 0$ (As given)

When $a+b+c \neq 0$

$$\begin{aligned}
 & \text{Then } (a-b)^2 + (b-c)^2 + (c-a)^2 \\
 &= 0
 \end{aligned}$$

$$(a-b)^2 = 0$$

$$(a-b) = 0$$

$$\text{Then } a = b$$

$$\text{Same } b = c$$

$$c = a$$

So, **(a = b = c)**

Ex.45 If $a^3 + b^3 + c^3 - 3abc = 0$, $a+b+c \neq 0$ and a, b & c are natural number find the possible value of $a+b+c$

- (a) 4 (b) 8 (c) 5 (d) 12

Sol.(d) We know that in this condition $a=b=c$ and given a, b and c are natural no. we take option (d) because 12 is divide 3 equal natural part

$$\frac{12}{3} = 4$$

$$a = b = c = 4$$

Then 12 is possible $a+b+c$

Ex.46 If $a^3 + b^3 + c^3 - 3abc = 0$, $a+b+c \neq 0$ and a, b & c are natural number find the possible value of $a \times b \times c$

- (a) 4 (b) 8 (c) 5 (d) 12

Sol.(b) We know that in this condition $a=b=c$ and given a, b and c are natural no. we take option (b) because $8 = 2 \times 2 \times 2$

We can say that **8** is possible value of $a \times b \times c$

Ex.47 Find the value of

$$\frac{(x^2 - y^2)^3 + (y^2 - z^2)^3 + (z^2 - x^2)^3}{(x-y)^3 + (y-z)^3 + (z-x)^3}$$

Sol. Let, $a = x^2 - y^2$

$$b = y^2 - z^2$$

$$c = z^2 - x^2$$

$$a + b + c = 0$$

$$\text{Then, } a^3 + b^3 + c^3 = 3abc$$

$$\text{Thus, } p = x - y$$

$$q = y - z$$

$$r = z - x$$

$$p + q + r = 0$$

$$\text{Then, } p^3 + q^3 + r^3 = 3pqr$$

A.T.Q.

$$\frac{(x^2 - y^2)^3 + (y^2 - z^2)^3 + (z^2 - x^2)^3}{(x-y)^3 + (y-z)^3 + (z-x)^3}$$

$$\begin{aligned}
 &= \frac{3(x^2 - y^2)(y^2 - z^2)(z^2 - x^2)}{3(x-y)(y-z)(z-x)} \\
 &\therefore \mathbf{(a^2 - b^2) = (a+b)(a-b)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{(x+y)(x-y)(y+z)(y-z)(z+x)(z-x)}{(x-y)(y-z)(z-x)} \\
 &= \mathbf{(x+y)(y+z)(z+x)}
 \end{aligned}$$

Ex.48 If $a+b+c=0$ Find the value of $a^3 + b^3 + c^3 + 3abc$

- (a) 0 (b) 1
(c) abc (d) $6abc$

Sol.(d) $(a+b+c) = 0$

$$\text{Then } a^3 + b^3 + c^3 = 3abc$$

$$\text{So, } a^3 + b^3 + c^3 + 3abc$$

$$= 3abc + 3abc$$

$$= \mathbf{6abc}$$

Ex.49 If $x = 1.235$

$$y = 3.422$$

$$z = 4.377$$

$$\text{Find } x^3 + y^3 - z^3 + 3xyz$$

Sol. $x + y = z$

$$(x) + (y) + (-z) = 0$$

$$(x)^3 + (y)^3 + (-z)^3 = 3(x)(y)(-z)$$

$$x^3 + y^3 - z^3 = -3xyz$$

$$\therefore x^3 + y^3 - z^3 + 3xyz = \mathbf{0}$$

Ex.50 If $a^2 + b^2 = c^2$, Find the value of

$$\frac{a^6 + b^6 - c^6}{a^2 b^2 c^2}$$

Sol. $a^2 + b^2 = c^2$

Cube both side

$$(a^2 + b^2)^3 = (c^2)^3$$

$$a^6 + b^6 + 3a^2 b^2 (a^2 + b^2) = c^6$$

$$a^6 + b^6 + 3a^2 b^2 c^2 = c^6$$

$$a^6 + b^6 - c^6 = -3a^2 b^2 c^2$$

A.T.Q.

$$\frac{a^6 + b^6 - c^6}{a^2 b^2 c^2}$$

$$\begin{aligned}
 &= \frac{-3a^2 b^2 c^2}{a^2 b^2 c^2} \\
 &= \mathbf{-3}
 \end{aligned}$$

Ex.51 If $a^{1/3} + b^{1/3} = c^{1/3}$

Which statement is true

- (a) $a^3 + b^3 - c^3 = 3abc$
(b) $a^3 + b^3 - c^3 + 3abc = 0$
(c) $(a+b-c)^3 - 27abc = 0$
(d) $(a+b-c)^3 + 27abc = 0$

Sol.(d) $(a)^{1/3} + (b)^{1/3} + (-c)^{1/3} = 0$

If $(a+b+c) = 0$

Then $a^3 + b^3 + c^3 = 3abc$

$$(a) + (b) + (-c) = 3(a)^{1/3} (b)^{1/3} (-c)^{1/3}$$

$$a + b - c = -3a^{1/3} b^{1/3} c^{1/3}$$

Cube both side

$$(a+b-c)^3 = (-3a^{1/3} b^{1/3} c^{1/3})^3$$

$$(a+b-c)^3 = -27abc$$

$$\therefore \mathbf{(a+b-c)^3 + 27abc = 0}$$

Ex.52 If $x+y+z=2s$

Find the value of $(s-x)^3 + (s-y)^3 + 3(s-x)(s-y)z$

- (a) y^3 (b) x^3 (c) z^3 (d) 0

Sol.(c) $x+y+z=2s$

$$x+y+z = s+s$$

$$s-x+s-y-z=0$$

$$(s-x) + (s-y) + (-z) = 0$$

$$(s-x)^3 + (s-y)^3 + (-z)^3 = 3(s-x)(s-y)(-z)$$

$$(s-x)^3 + (s-y)^3 - z^3 = -3(s-x)(s-y)z$$

$$(s-x)^3 + (s-y)^3 + 3(s-x)(s-y)z = \mathbf{z^3}$$

Ex.53 Find the value of

$$\sqrt[3]{2 \times 333^3 + 334^3 - 3 \times 333^2 \times 334}$$

Sol. $\sqrt[3]{2 \times 333^3 + 334^3 - 3 \times 333^2 \times 334}$
After describing

$$= \sqrt[3]{333^3 + 333^3 + 334^3 - 3 \times 333 \times 333 \times 334}$$

$$\begin{aligned}
 & a^3 + b^3 + c^3 - 3abc \\
 &= \frac{1}{2} (a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]
 \end{aligned}$$

$$= \sqrt{\frac{1}{2}(333+333+334)[(333-333)^2 + (333-334)^2 + (334-333)^2]}$$

$$= \sqrt[3]{\frac{1}{2}(1000)(0+1+1)}$$

$$= \sqrt[3]{\frac{1}{2} \times 1000 \times 2} = 10$$

Ex.54 If $a = 20$, $b = 25$, $c = 15$, Find

$$\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca}$$

Sol. $\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca} = \frac{(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)}{a^2 + b^2 + c^2 - ab - bc - ca}$

$$\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ca}$$

A.T.Q.

$$= \frac{(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)}{a^2 + b^2 + c^2 - ab - bc - ca}$$

$$= a + b + c$$

Put the value

$$= 20 + 25 + 15 = 60$$

Ex.55 If $a + b + c = 3$ and $a^2 + b^2 + c^2 = 6$,

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1, \text{ Find } abc$$

Sol. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$

$$\frac{ab + bc + ca}{abc} = 1$$

$$ab + bc + ca = abc$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2abc$$

$$(3)^2 = 6 + 2abc$$

$$9 - 6 = 2abc$$

$$abc = \frac{3}{2}$$

Ex.56 If $a + b + c = 15$

$$\text{and } a^2 + b^2 + c^2 = 83$$

$$\text{find } a^3 + b^3 + c^3 - 3abc$$

Sol.

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(15)^2 = 83 + 2(ab + bc + ca)$$

$$225 - 83 = 2(ab + bc + ca)$$

$$ab + bc + ca = 71$$

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)$$

$$[a^2 + b^2 + c^2 - (ab + bc + ca)]$$

$$a^3 + b^3 + c^3 - 3abc = 15(83 - 71)$$

$$= 15 \times 12 = 180$$

Ex.57 Find $(a - b)^3 + (b - c)^3 + (c - a)^3 = ?$

Sol. $\therefore (a - b) + (b - c) + (c - a) = 0$

$$\therefore (a - b)^3 + (b - c)^3 + (c - a)^3$$

$$= 3(a - b)(b - c)(c - a)$$

Ex.58 The value of $a^3 + b^3 + c^3 - 3abc$,

Where $a = 87$, $b = -126$ and $c = 39$ is:

(a) 0

(b) 1259

(c) -48

(d) None of these

Sol. $\therefore a + b + c = 0$

$$\therefore a^3 + b^3 + c^3 - 3abc = 0$$

Thus (a) is the correct option.

Ex.59 Find the value of $a^3 + b^3 + c^3 - 3abc$ If $a + b + c = 12$ and $ab + bc + ac = 47$.

Sol. $\therefore a + b + c = 12$

$$\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ac) = 144$$

$$\Rightarrow a^2 + b^2 + c^2 + 2 \times 47 = 144$$

$$\Rightarrow a^2 + b^2 + c^2 = 50$$

$$\text{Now, since } a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= 12(50 - 47) = 12 \times 3 = 36$$

Ex.60 If $a = 997$, $b = 999$ and $c = 996$ find the value of $a^3 + b^3 + c^3 - 3abc$

Sol.

$$\frac{a^3 + b^3 + c^3 - 3abc}{2} = \frac{1}{2} (a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$$

$$= \frac{1}{2} (999 + 997 + 996) \left[\frac{(999 - 997)^2 + (997 - 996)^2 + (996 - 999)^2}{2} \right]$$

$$= \frac{1}{2} \times 2992 \times 14$$

$$= 2992 \times 7 = 20944$$

TYPE IV

$$* a^2 + b^2 + c^2 - ab - bc - ca$$

$$= \frac{1}{2} [(a - b)^2 + (b - c)^2 + (c - a)^2]$$

When,

$$a^2 + b^2 + c^2 - ab - bc - ca = 0$$

$$a^2 + b^2 + c^2 = ab + bc + ca$$

Then $a = b = c$

Ex.61 If $a = 99$, $b = 97$, and $c = 96$

Find the value of

$$a^2 + b^2 + c^2 - ab - bc - ca$$

Sol. $a^2 + b^2 + c^2 - ab - bc - ca$

$$= \frac{1}{2} [(a - b)^2 + (b - c)^2 + (c - a)^2]$$

$$= \frac{1}{2} [(99 - 97)^2 + (97 - 96)^2 + (96 - 99)^2]$$

$$= \frac{1}{2} [(2)^2 + (1)^2 + (-3)^2]$$

$$= \frac{1}{2} [4 + 1 + 9] = 7$$

Ex.62 If $a = 556$, $b = 558$ and $c = 561$

Find the value of

$$a^2 + b^2 + c^2 - ab - bc - ca$$

Sol. $a^2 + b^2 + c^2 - ab - bc - ca$

$$= \frac{1}{2} [(a - b)^2 + (b - c)^2 + (c - a)^2]$$

$$= \frac{1}{2} [(556 - 558)^2 + (558 - 561)^2 + (561 - 556)^2]$$

$$= \frac{1}{2} [4 + 9 + 25] = \frac{1}{2} \times 38 = 19$$

Ex.63 If $a^2 = b + c$, $b^2 = c + a$

$$c^2 = a + b$$

$$\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c}$$

Here a , b and c non zero number

$$a^2 = b + c$$

Divide by 3 both sides

$$\frac{a^2}{a} = \frac{b+c}{a}$$

Then,

$$a = \frac{b+c}{a}$$

$$\text{Thus, } b = \frac{c+a}{b}$$

$$c = \frac{a+b}{c}$$

Now put the value of a , b and c

$$\Rightarrow \frac{1}{1 + \frac{b+c}{a}} + \frac{1}{1 + \frac{c+a}{b}} + \frac{1}{1 + \frac{a+b}{c}}$$

$$\Rightarrow \frac{a}{a+b+c} + \frac{b}{b+c+a} + \frac{c}{a+b+c}$$

$$\Rightarrow \frac{a+b+c}{a+b+c} = 1$$

Alternate:

We put the value

$$a = b = c = 2$$

$$a^2 = b + c = (2)^2 = 2 + 2$$

$$4 = 4$$

$$\text{Then } \frac{1}{1+2} + \frac{1}{1+2} + \frac{1}{1+2}$$

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

Ex.64 If $\frac{x^2+y^2+z^2}{xy+yz+zx} = 1$, then find

$$\text{the value of } \frac{x+y}{z} + \frac{y+z}{x} + \frac{z+x}{y}$$

Sol. $\frac{x^2+y^2+z^2}{xy+yz+zx} = 1$

$$x^2 + y^2 + z^2 = xy + yz + zx$$

$$\text{In this condition } x = y = z$$

$$\text{Put the value of } x = y = z = 1$$

$$\text{Then, } \frac{x+y}{z} + \frac{y+z}{x} + \frac{z+x}{y}$$

$$= \frac{1+1}{1} + \frac{1+1}{1} + \frac{1+1}{1}$$

$$= 2 + 2 + 2 = 6$$

Ex.65 If $x^2 + y^2 + z^2 = xy + yz + zx$, find the value of

$$\frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{y+x}$$

Sol. $x^2 + y^2 + z^2 = xy + yz + zx$

Here,

$$x = y = z$$

$$\text{Let put the value of } x = y = z = 1$$

Now,

$$\frac{x}{y+z} + \frac{y}{z+x} + \frac{z}{y+x}$$

$$= \frac{1}{1+1} + \frac{1}{1+1} + \frac{1}{1+1}$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

Ex.66 If $a + b + c = 0$, Find the value of

$$\frac{1}{a^2+b^2-c^2} + \frac{1}{b^2+c^2-a^2} + \frac{1}{c^2+a^2-b^2}$$

Sol. $a+b+c = 0$

$$a+b = -c$$

Squaring Both side

$$(a+b)^2 = (-c)^2$$

$$a^2+b^2+2ab = c^2$$

$$a^2+b^2-c^2 = -2ab \quad \dots(I)$$

$$\text{Same, } b+c = -a$$

$$b^2+c^2-a^2 = -2bc \quad \dots(II)$$

$$\text{Thus, } c+a = -b$$

$$c^2+a^2-b^2 = -2ac \quad \dots(III)$$

Put the value

$$\frac{1}{a^2+b^2-c^2} + \frac{1}{b^2+c^2-a^2} + \frac{1}{c^2+a^2-b^2}$$

$$\Rightarrow \frac{1}{-2ab} + \frac{1}{-2bc} + \frac{1}{-2ab}$$

$$\frac{a+b+c}{-2abc} = \frac{0}{-2abc} = 0$$

Alternate:-

We choose the value which we follow the condition.

$$\text{If } a+b+c = 0$$

$$a = 1, b = 1, c = -2$$

Put the value a, b, c

$$= \frac{1}{a^2+b^2-c^2} + \frac{1}{b^2+c^2-a^2} + \frac{1}{c^2+a^2-b^2}$$

$$= \frac{1}{(1)^2+(1)^2-(-2)^2} + \frac{1}{(1)^2+(-2)^2-(1)^2}$$

$$+ \frac{1}{(-2)^2+(1)^2-(1)^2}$$

$$= \frac{1}{1+1-4} + \frac{1}{1+4-1} + \frac{1}{4+1-1}$$

$$= -\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 0$$

Ex.67 If $x+y+z = 0$ then the value

$$\text{of } \frac{x^2y^2+y^2z^2+z^2x^2}{x^4+y^4+z^4} \text{ is:}$$

$$(a) 0 \quad (b) 1/2$$

$$(c) 1 \quad (d) 2$$

Sol. $\because (x+y+z) = 0$

$$\Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0$$

$$\Rightarrow x^2 + y^2 + z^2 = -2(xy + yz + zx)$$

$$\Rightarrow (x^2 + y^2 + z^2)^2 = 4(xy + yz + zx)^2$$

$$\Rightarrow x^4 + y^4 + z^4 + 2(x^2y^2 + y^2z^2 + z^2x^2) = 4[x^2y^2 + y^2z^2 + z^2x^2 + 2xyz(x+y+z)]$$

$$\therefore x^4 + y^4 + z^4 = 2(x^2y^2 + y^2z^2 + z^2x^2) \quad [\because (x+y+z) = 0]$$

$$\therefore \frac{x^2y^2 + y^2z^2 + z^2x^2}{x^4 + y^4 + z^4} = \frac{1}{2}$$

Hence (b) is the correct option.

Alternate:

$$x+y+z = 0$$

$$\text{Then, Let } x = 1, y = 1, z = -2$$

$$\frac{x^2y^2 + y^2z^2 + z^2x^2}{x^4 + y^4 + z^4}$$

Now,

$$\frac{(1^2) \times (1^2) + (1^2) \times (-2^2) + (-2^2) \times (1^2)}{(1^4) + (1^4) + (-2)^4}$$

$$\frac{1+4+4}{1+1+16} = \frac{9}{18} = \frac{1}{2}$$

Ex.68 If $pq+qr+rp = 0$, Find the value

$$\frac{p^2}{p^2-qr} + \frac{q^2}{q^2-pr} + \frac{r^2}{r^2-pq}$$

Sol. $pq + pr + rp = 0$

$$pq + rp = -qr$$

Same, $pq + qr = -pr$

$$qr + rp = -pq$$

$$\frac{p^2}{p^2-qr} + \frac{q^2}{q^2-pr} + \frac{r^2}{r^2-pq}$$

$$\Rightarrow \frac{p^2}{p^2+pq+rp} + \frac{q^2}{q^2+pq+qr}$$

$$+ \frac{r^2}{r^2+qr+rp}$$

$$= \frac{p^2}{p(p+q+r)} + \frac{q^2}{q(p+q+r)}$$

$$+ \frac{r^2}{r(p+q+r)}$$

$$= \frac{p}{p+q+r} + \frac{q}{p+q+r} + \frac{r}{p+q+r}$$

$$= \frac{p+q+r}{p+q+r} = 1$$

Alternate:

$$\text{Put the value } p = -1, q = 2$$

$$r = 2$$

So,

$$pq + qr + rp$$

$$= -1 \times 2 + 2 \times 2 + 2 \times -1 = 0$$

$$= \frac{p^2}{p^2-pq} + \frac{q^2}{a^2-pr} + \frac{r^2}{r^2-pq}$$

$$= \frac{(-1)^2}{(-1)^2 - (2 \times 2)} + \frac{(2)^2}{(2)^2 - (-1 \times 2)}$$

$$+ \frac{(2)^2}{(2)^2 - (-1 \times 2)}$$

$$= \frac{1}{1-4} + \frac{4}{4+2} + \frac{4}{4+2}$$

$$= \frac{-1}{3} + \frac{4}{6} + \frac{4}{6}$$

$$= \frac{-2+4+4}{6} = \frac{6}{6} = 1$$

Ex.69 If $p \times q \times r = 1$ Find $\frac{1}{1+p+q^{-1}}$

$$+ \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}}$$

Sol. $\frac{1}{1+p+\frac{1}{q}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+\frac{1}{pq}+\frac{1}{p}}$

$$(\because Pqr = 1, r = \frac{1}{pq})$$

$$\frac{q}{q+pq+1} + \frac{1}{1+q+pq} + \frac{pq}{pq+1+q}$$

$$\frac{pq+1+q}{q+pq+1} = 1$$

Alternate

$$p = 1 = q = r$$

Then

$$= \frac{1}{1+1+1} + \frac{1}{1+1+1} + \frac{1}{1+1+1}$$

$$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

Ex.70 If $\frac{x-a^2}{b+c} + \frac{x-b^2}{c+a} + \frac{x-c^2}{a+b}$

$$= 4(a+b+c)$$

Find the value of x

- (a) $(a^2+b^2+c^2)$
- (b) $(a+b+c)^2$
- (c) $(a^2+b^2+c^2-ab-bc-ca)$
- (d) $(ab+bc+ca)$

Sol. We take option (B)

$$\text{Then } x = (a+b+c)^2$$

$$\frac{(a+b+c)^2 - a^2}{(b+c)} + \frac{(a+b+c)^2 - b^2}{(a+c)}$$

$$+ \frac{(a+b+c)^2 - c^2}{(a+b)}$$

$$\Rightarrow \frac{a^2 - b^2 = (a+b)(a-b)}{(a+b+c-a)(a+b+c+a)}$$

$$+ \frac{(a+b+c-b)(a+b+c+b)}{(a+c)}$$

$$+ \frac{(a+b+c-c)(a+b+c+c)}{(a+b)}$$

$$\Rightarrow \frac{(b+c)(2a+b+c)}{(b+c)} +$$

$$\frac{(a+c)(2b+a+c)}{(a+c)} +$$

$$\frac{(a+b)(2c+a+b)}{(a+b)}$$

$$\Rightarrow 2a+b+c+2b+a+c+2c+a+b$$

$$\Rightarrow 4a+4b+4c = 4(a+b+c)$$

L.H.S = R.H.S

$$\text{So, } x = (a+b+c)^2$$

Alternate:

$$\text{Let, } a = b = c = 1$$

Then

$$\frac{x-1}{2} + \frac{x-1}{2} + \frac{x-1}{2} = 4 \times 3$$

$$\frac{(x-1) \times 3}{2} = 4 \times 3$$

$$x-1 = 8$$

$$x = 9$$

Option (B)

$$x = (a+b+c)^2$$

$$= (1+1+1)^2$$

$$= (3)^2 = 9$$

So, It is proved

$$\therefore x = (a+b+c)^2$$

Ex.71 If $\frac{x-a^2}{b^2+c^2} + \frac{x-b^2}{c^2+a^2} + \frac{x-c^2}{a^2+b^2}$
= 3 Find the value of x

- (a) $a^2+b^2+c^2$
- (b) $a^2+b^2+c^2-ab-bc-ca$
- (c) $(a+b+c)^2$
- (d) $a^2+b^2+c^2+ab+bc+ca$

Sol. We take option (A) $a^2+b^2+c^2$

$$\frac{a^2+b^2+c^2-a^2}{b^2+c^2} + \frac{a^2+b^2+c^2-b^2}{c^2+a^2}$$

$$+ \frac{a^2+b^2+c^2-c^2}{a^2+b^2}$$

$$\Rightarrow \frac{b^2+c^2}{b^2+c^2} + \frac{c^2+a^2}{c^2+a^2} + \frac{a^2+b^2}{a^2+b^2}$$

$$1+1+1 = 3$$

L.H.S = R.H.S

$$\text{Then, } (x = a^2+b^2+c^2)$$

Ex.72 If $x + \frac{1}{y} = 1, y + \frac{1}{z} = 1$

Find the value of (i) $z + \frac{1}{x}$

$$(ii) \ x + y + z + \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

Sol. $x + \frac{1}{y} = 1$

$$x = 1 - \frac{1}{y}$$

$$x = \frac{y-1}{y}$$

$$y + \frac{1}{z} = 1$$

....(i)

$$\frac{1}{z} = 1 - y$$

$$z = \frac{1}{1-y}$$

....(ii)

From (i) & (ii)

$$(i) \ z + \frac{1}{x} = \frac{1}{1-y} + \frac{y}{y-1}$$

$$= \frac{1}{1-y} - \frac{y}{1-y} = \frac{1-y}{1-y} = 1$$

$$(ii) \ x + \frac{1}{y} + y + \frac{1}{z} + z + \frac{1}{x}$$

$$= 1 + 1 + 1 = 3$$

$$\text{So, } x + y + z + \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 3$$

Alternate:-

$$(i) \ x + \frac{1}{y} = 1$$

$$\text{Let, } x = \frac{1}{2}$$

Then $y = 2$

$$\text{So, } \frac{1}{2} + \frac{1}{2} = 1$$

$$\text{Now, } y + \frac{1}{z} =$$

$$\text{If } y = 2$$

$$\text{Then } z = -1$$

$$2 + \frac{1}{-1} = 2 - 1 = 1$$

$$\text{Now, } z + \frac{1}{x} =$$

Put the value z and x

$$-1 + \frac{1}{1/2} = -1 + 2 = 1$$

Ex.73 If $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = 1$

$$\text{Find } \frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b}$$

Sol. Divide and Multiply $a + b + c$

$$\text{Now, } \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$$

$$= \frac{1 \times (a+b+c)}{(a+b+c)}$$

$$\Rightarrow \left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \right) (a+b+c)$$

$$= a+b+c$$

$$\Rightarrow \frac{a(a+b+c)}{b+c} + \frac{b(a+b+c)}{c+a}$$

$$+ \frac{c(a+b+c)}{a+b} = a+b+c$$

$$\Rightarrow \frac{a^2 + a(b+c)}{b+c} + \frac{b^2 + b(a+c)}{c+a} +$$

$$\frac{c^2 + c(a+b)}{a+b} = a+b+c$$

$$\Rightarrow \frac{a^2}{b+c} + a + \frac{b^2}{c+a} + b + \frac{c^2}{a+b} + c$$

$$= a+b+c$$

$$\text{Then, } \frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b} = 0$$

Alternate:

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = 1$$

We break equation small step

Let, $a = 0, b = 1$

Put the value of a and b

Then,

$$\frac{0}{1+c} + \frac{1}{c+0} + \frac{c}{0+1} = 1$$

$$\frac{1}{c} + c = 1,$$

$$1 + c^2 = c$$

$$c^2 = c - 1$$

Again put the value a, b

$$\frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b}$$

$$\Rightarrow 0 + \frac{1}{c+0} + \frac{c^2}{0+1}$$

$$\Rightarrow \frac{1}{c} + c^2$$

Put the value of c^2

Then,

$$\text{So, } \left[\frac{1}{c} + c \right] - 1 = 1 - 1 = 0$$

Ex.74 If $\frac{b-c}{a} + \frac{a+c}{b} + \frac{a-b}{c} = 1$ And

$a-b+c \neq 0$ which statement is true

$$(a) \frac{1}{b} = \frac{1}{a} - \frac{1}{c} \quad (b) \frac{1}{a} = \frac{1}{b} - \frac{1}{c}$$

$$(c) \frac{1}{c} = \frac{1}{a} + \frac{1}{b} \quad (d) \text{N.O.T}$$

$$\text{Sol. } \frac{b-c}{a} + \frac{a+c}{b} + \frac{a-b}{c} = 1$$

We adding ± 1 in this equation

$$\frac{b-c}{a} - 1 + \frac{a+c}{b} - 1 + \frac{a-b}{c} + 1 = 1 - 1$$

$$\frac{b-c-a}{a} + \frac{a+c-b}{b} + \frac{a-b+c}{c} = 0$$

$$\frac{-(a-b+c)}{a} + \frac{(a-b-c)}{b} + \frac{(a-b+c)}{c}$$

$$= 0$$

$$\frac{-1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$

$$\frac{1}{b} = \frac{1}{a} - \frac{1}{c}$$

Alternate:

Let, $b = 1$

$c = 1$

Then,

Put the value of b and c

$$\frac{1-1}{a} + \frac{a+1}{1} + \frac{a-1}{1} = 1$$

$$a+1+a-1=1$$

$$a = \frac{1}{2}$$

$$\text{Option (A)} \frac{1}{b} = \frac{1}{a} - \frac{1}{c}$$

$$= \frac{1}{1/2} - \frac{1}{1} = 2 - 1 = 1$$

So, L.H.S = R.H.S

$$\frac{1}{b} = \frac{1}{a} - \frac{1}{c}$$

TYPE - V

When $a + \frac{1}{a} = 2$ then $a = 1$

$$\text{Proof } a + \frac{1}{a} = 2$$

$$a^2 + 1 = 2a$$

$$a^2 - 2a + 1 = 0$$

$$(a-1)^2 = 0$$

$$\text{So, } a = 1$$

Ex.75 If $a + \frac{1}{a} = 2$, find the value of

$$a^{100} + \frac{1}{a^{100}}$$

$$\text{Sol. } a + \frac{1}{a} = 2$$

Then, $a = 1$

Put the value of a ,

$$\Rightarrow 1^{100} + \frac{1}{1^{100}}$$

$$\Rightarrow 1 + \frac{1}{1} \Rightarrow 1 + 1 = 2$$

Ex.76 If $a + \frac{1}{a} = 2$, find the value of

$$(i) a^{50} + \frac{1}{a^{50}} \quad (ii) a^{49} - \frac{1}{a^{49}}$$

$$(iii) a^3 + a^2 + a + 1$$

$$\text{Sol. } a + \frac{1}{a} = 2$$

$a = 1$

$$(i) a^{50} + \frac{1}{a^{50}} = 1^{50} + \frac{1}{1^{50}}$$

$$= 1 + \frac{1}{1} = 2$$

$$(ii) a^{49} - \frac{1}{a^{49}} = 1^{49} - \frac{1}{1^{49}}$$

$$= 1 - \frac{1}{1} = 0$$

$$(iii) a^3 + a^2 + a + 1$$

$$1^3 + 1^2 + 1 + 1 = 4$$

Ex.77 If $p + \frac{1}{p-3} = 5$, find the value of :-

$$(i) (p-3)^{100} + \left(\frac{1}{p-3} \right)^{100}$$

$$(ii) p^2 - 3p + 4$$

$$\text{Sol.(i)} p + \frac{1}{p-3} = 5$$

Subtract 3 both sides,

$$(p-3) + \frac{1}{(p-3)} = 5 - 3$$

$$(p-3) + \frac{1}{(p-3)} = 2$$

Let $a = (p-3)$

Then,

$$a + \frac{1}{a} = 2$$

$$\therefore a = 1 = (P - 3)$$

Now,

$$p - 3 = 1$$

$$p = 4$$

$$(p-3)^{100} + \left(\frac{1}{p-3}\right)^{100}$$

$$1^{100} + \frac{1}{1^{100}} = 2$$

$$\begin{aligned} \text{(ii)} \quad & p^2 - 3p + 4 \\ & 4^2 - 3 \times 4 + 4 \\ & = 8 \end{aligned}$$

* When $a + \frac{1}{a} = -2$, then $a = -1$

Ex.78 If $a + \frac{1}{a} = -2$, Find the value

$$\text{of } a^{200} + \frac{1}{a^{200}}$$

$$\text{Sol. } a + \frac{1}{a} = -2$$

$$\text{Now, } a = -1$$

$$a^{200} + \frac{1}{a^{200}}$$

$$= (-1)^{200} + \frac{1}{(-1)^{200}}$$

$$= 1 + 1 = 2$$

Ex.79 If $a + \frac{1}{a} = -2$, find the value of

$$\text{(i)} \quad a^{99} + \frac{1}{a^{99}} \quad \text{(ii)} \quad a^{32} + \frac{1}{a^{31}}$$

$$\text{(iii)} \quad a^4 + a^3 + a^2 + a + 1$$

$$\text{Sol. } a + \frac{1}{a} = -2$$

$$\text{So, } a = -1$$

$$\text{(i)} \quad a^{99} + \frac{1}{a^{99}}$$

$$(-1)^{99} + \frac{1}{(-1)^{99}}$$

$$(-1) + (-1) = -2$$

$$\text{(ii)} \quad a^{32} + \frac{1}{a^{31}}$$

$$(-1)^{32} + \frac{1}{(-1)^{31}}$$

$$1 - 1 = 0$$

$$\begin{aligned} \text{(iii)} \quad & a^4 + a^3 + a^2 + a + 1 \\ & (-1)^4 + (-1)^3 + (-1)^2 + (-1) + (1) \\ & = 1 \end{aligned}$$

Ex.80 If $p + \frac{1}{p-3} = 1$, Find the value of

$$\text{(i)} \quad (P-3)^{42} + \frac{1}{(p-3)^{42}}$$

$$\text{(ii)} \quad (P-3)^{101} - \frac{1}{(p-3)^{101}}$$

$$\text{(iii)} \quad P^3 + 4P^2 + 5P + 1$$

$$\text{Sol. } p + \frac{1}{p-3} = 1$$

Subtract 3 both sides

$$(P-3) + \frac{1}{(p-3)} = 1 - 3$$

$$(P-3) + \frac{1}{(p-3)} = -2$$

So,

$$(P-3) = -1$$

$$P = -1 + 3 = 2$$

$$P = 2$$

$$\text{(i)} \quad (P-3)^{42} + \frac{1}{(p-3)^{42}}$$

$$(-1)^{42} + \frac{1}{(-1)^{42}}$$

$$= 1 + 1 = 2$$

$$\text{(ii)} \quad (P-3)^{101} - \frac{1}{(p-3)^{101}}$$

$$(-1)^{101} - \frac{1}{(-1)^{101}}$$

$$= (-1) - (-1)$$

$$= -1 + 1 = 0$$

$$\text{(iii)} \quad P^3 + 4P^2 + 5P + 1$$

Put the value of $P = 2$

$$2^3 + 4 \times 2^2 + 5 \times 2 + 1$$

$$= 8 + 16 + 10 + 1 = 35$$

TYPE - VI

(A). When $a + \frac{1}{a} = 1$,

or

$$a^2 - a + 1 = 0$$

$$\text{Then } a^3 = -1$$

or

$$a^3 + 1 = 0$$

$$\text{Proof } a + \frac{1}{a} = 1$$

Squaring both sides

$$a^2 + \frac{1}{a^2} = 1^2 - 2 = -1$$

$$a^2 + \frac{1}{a^2} = -1$$

Multiply a both sides

$$= a \left(a^2 + \frac{1}{a^2} \right) - 1 \times a$$

$$a^3 + \frac{1}{a} = -a$$

$$a^3 + \frac{1}{a} + a = 0$$

$$a^3 + 1 = 0$$

or

$$a^3 = -1$$

Ex.81 If $a + \frac{1}{a} = 1$, Find the value

$$\text{of } a^{15} + \frac{1}{a^{15}}$$

$$\text{Sol. } a + \frac{1}{a} = 1$$

Then,

$$a^3 = -1$$

$$a^{15} + \frac{1}{a^{15}} = (a^3)^5 + \frac{1}{(a^3)^5}$$

$$(-1)^5 + \frac{1}{(-1)^5} = (-1) + (-1)$$

$$= -2$$

Ex.82 If $a^2 - a + 1 = 0$, Find the value of

$$\text{(i)} \quad a^{36} + \frac{1}{a^{36}} \quad \text{(ii)} \quad a^{37} + \frac{1}{a^{37}}$$

$$\text{(iii)} \quad a^{38} + \frac{1}{a^{38}}$$

$$\text{Sol. } a^2 - a + 1 = 0$$

or

$$a + \frac{1}{a} = 1$$

Then,

$$a^3 = -1$$

$$\text{(i)} \quad a^{36} + \frac{1}{a^{36}}$$

$$(a^3)^{12} + \frac{1}{(a^3)^{12}}$$

$$= (-1)^{12} + \frac{1}{(-1)^{12}} = 1 + 1 = 2$$

(ii) $a^{37} + \frac{1}{a^{37}}$

Break the power multiple of 3

$$a^{36} \times a + \frac{1}{a^{36} \times a}$$

$$= (-1)^{12} \times a + \frac{1}{(-1)^{12} \times a}$$

$$= a + \frac{1}{a} = 1 \text{ (As Given)}$$

So,

$$a^{37} + \frac{1}{a^{37}} = 1$$

(iii) $a^{38} + \frac{1}{a^{38}}$

Break the power multiple of 3

$$a^{36} \times a^2 + \frac{1}{a^{36} \times a^2}$$

$$= (-1)^{12} \times a^2 + \frac{1}{(-1)^{12} \times a^2}$$

$$= a^2 + \frac{1}{a^2}$$

$$= a + \frac{1}{a} = 1 \text{ (As Given)}$$

Then,

$$a^2 + \frac{1}{a^2} = 1^2 - 2 = -1$$

$$a^{38} + \frac{1}{a^{38}} = -1$$

Ex.83 If $a + \frac{1}{a} = 1$ find the value of

$$a^{100} + a^{99} + a^{98} + a^{97} + a^{96} + a^{95}$$

Sol. $a + \frac{1}{a} = 1$

Then,

$$a^3 + 1 = 0$$

$$a^{100} + a^{99} + a^{98} + a^{97} + a^{96} + a^{95}$$

$$= a^{100} + a^{97} + a^{99} + a^{96} + a^{98} + a^{95}$$

$$= a^{97}(a^3 + 1) + a^{96}(a^3 + 1) + a^{95}$$

$$(a^3 + 1)$$

$$= a^{97} \times 0 + a^{96} \times 0 + a^{95} = 0$$

Note: When difference of the power is 3 then the value of both terms is = 0

Ex.84 If $\frac{x}{3} + \frac{3}{x} = 1$, Find the value of x^3

Sol. Let $a = \frac{x}{3}$

Then,

$$a + \frac{1}{a} = 1$$

We know that $a^3 = -1$

Put the value a

$$\left(\frac{x}{3}\right)^3 = -1$$

$$\frac{x^3}{27} = -1$$

$$x^3 = -27$$

(B). When $a + \frac{1}{a} = -1$

or

$$a^2 + a + 1 = 0$$

Then,

$$a^3 = 1$$

Proof :- $a + \frac{1}{a} = -1$

$$a^2 + \frac{1}{a^2} = (-1)^2 - 2 = -1$$

Multiply a both sides.

$$a\left(a^2 + \frac{1}{a^2}\right) = -1 \times a$$

$$a^3 + \frac{1}{a} = -a$$

$$a^3 + a + \frac{1}{a} = 0$$

$$a^3 - 1 = 0$$

$$a^3 = 1$$

Ex.85 If $a + \frac{1}{a} = -1$ Find the value of

(i) $a^{48} + \frac{1}{a^{48}}$ (ii) $a^{50} + \frac{1}{a^{50}}$

(iii) $a^{51} + \frac{1}{a^{51}}$

Sol. (i) $a^{48} + \frac{1}{a^{48}}$

$$1^{16} + \frac{1}{1^{16}} = 1 + 1 = 2$$

(ii) $a^{50} + \frac{1}{a^{50}}$

$$a^{48} \times a^2 + \frac{1}{a^{48} \times a^2}$$

$$= 1^{16} \times a^2 + \frac{1}{1^{16} \times a^2}$$

$$= a^2 + \frac{1}{a^2}$$

$$= a + \frac{1}{a} = -1$$

Then,

$$= a^2 + \frac{1}{a^2} = (-1)^2 - 2$$

$$= -1$$

$$\therefore a^{50} + \frac{1}{a^{50}} = -1$$

(iii) $a^{51} + \frac{1}{a^{51}}$

$$(a^3)^{17} + \frac{1}{(a^3)^{17}}$$

$$= 1^{17} + \frac{1}{1^{17}} = 1 + 1 = 2$$

Ex.86 If $a^2 + a + 1 = 0$ find the value of $(a^2 - a + 1)(a + 1 - a^2)$

Sol. $a^2 + a + 1 = 0$

or

$$a + \frac{1}{a} = -1$$

We know that

$$a^3 = 1$$

$$a^2 + 1 = -a \text{(i)}$$

$$a + 1 = -a^2 \text{(ii)}$$

$$(a^2 - a + 1)(a + 1 - a^2)$$

From Equation (i) and (ii)

$$(-a - a)(-a^2 - a^2)$$

$$= -2a \times -2a^2$$

$$= 4a^3 = 4 \times 1 = 4$$

Ex.87 If $a^2 + a + 1 = 0$, find the value of $a^5 + a^4 + 1$

Sol. $a^2 + a + 1 = 0$

or

$$a + \frac{1}{a} = -1$$

Then,

$$a^3 = 1$$

$$a^5 + a^4 + 1$$

$$a^3 \times a^2 + a^3 \times a + 1$$

Put the value of $a^3 = 1$

$$a^2 + a + 1 = 0 \text{ (As Given)}$$

Ex.88 If $x^{1/4} + \frac{1}{x^{1/4}} = 1$ find the value

of $x^{252} + \frac{1}{x^{252}}$

Sol. $x^{1/4} + \frac{1}{x^{1/4}} = 1$

Squaring both sides

$$x^{1/2} + \frac{1}{x^{1/2}} = 1^2 - 2 = -1$$

Again Squaring both side

$$x + \frac{1}{x} = (-1)^2 - 2 = -1$$

Then,

$$x + \frac{1}{x} = -1$$

$$x^3 = 1$$

Now,

$$x^{252} + \frac{1}{x^{252}}$$

$$(x^3)^{84} + \frac{1}{(x^3)^{84}}$$

$$(1)^{84} + \frac{1}{(1)^{84}}$$

$$= 1 + 1 = 2$$

TYPE - VII

When $x + \frac{1}{x} = \sqrt{3}$

Then

$$x^6 + 1 = 0$$

or

$$x^6 = -1$$

Proof $x + \frac{1}{x} = \sqrt{3}$

Cube both sides

$$x^3 + \frac{1}{x^3} = (\sqrt{3})^3 - 3 \times \sqrt{3}$$

$$x^3 + \frac{1}{x^3} = 0$$

Multiply x^3 both sides

$$x^6 + 1 = 0$$

$$x^6 = -1$$

Ex.89 If $x + \frac{1}{x} = \sqrt{3}$, Find the value of

$$(i) x^{90} + \frac{1}{x^{90}} \quad (ii) x^{96} + \frac{1}{x^{96}}$$

Sol. $x + \frac{1}{x} = \sqrt{3}$

$$\text{Then } x^6 = -1$$

$$(i) x^{90} + \frac{1}{x^{90}}$$

$$(x^6)^{15} + \frac{1}{(x^6)^{15}}$$

$$= (-1)^{15} + \frac{1}{(-1)^{15}}$$

$$= -1 - 1 = -2$$

$$(ii) x^{96} + \frac{1}{x^{96}}$$

$$(x^6)^{16} + \frac{1}{(x^6)^{16}}$$

$$= (-1)^{16} + \frac{1}{(-1)^{16}}$$

$$= 1 + 1 = 2$$

Ex.90 If $x + \frac{1}{x} = \sqrt{3}$ find the value of

$$(i) x^{92} + \frac{1}{x^{92}} \quad (ii) x^{93} + \frac{1}{x^{93}}$$

Sol. (i) $x^{92} + \frac{1}{x^{92}}$

Break the power multiple of 6

$$x^{90} \times x^2 + \frac{1}{x^{90} \times x^2}$$

$$= (-1)^{15} \times x^2 + \frac{1}{(-1)^{15} \times x^2}$$

$$= -(x^2 + \frac{1}{x^2})$$

$$x + \frac{1}{x} = \sqrt{3} \quad (\text{As Given})$$

Then,

$$x^2 + \frac{1}{x^2} = (\sqrt{3})^2 - 2$$

$$x^2 + \frac{1}{x^2} = 3 - 2 = 1$$

So,

$$x^{92} + \frac{1}{x^{92}} = -1$$

$$(ii) x^{93} + \frac{1}{x^{93}}$$

Break the power multiple of 6

$$x^{90} \times x^3 + \frac{1}{x^{90} \times x^3}$$

$$= (-1)^{15} \times x^3 + \frac{1}{(-1)^{15} \times x^3}$$

$$= -(x^3 + \frac{1}{x^3})$$

$$x + \frac{1}{x} = \sqrt{3} \quad (\text{As Given})$$

Then,

$$x^3 + \frac{1}{x^3} = (\sqrt{3})^3 - 3 \times \sqrt{3}$$

$$x^3 + \frac{1}{x^3} = 3\sqrt{3} - 3\sqrt{3} = 0$$

So,

$$x^{93} + \frac{1}{x^{93}} = 0$$

Ex.91 If $x + \frac{1}{x} = \sqrt{3}$ find the value of

$$(i) x^6 + 5$$

$$(ii) x^{102} + x^{96} + x^{101} + x^{95} + x^{100} + x^{94}$$

Sol. $x + \frac{1}{x} = \sqrt{3}$

Then,

$$x^6 + 1 = 0$$

$$(i) x^6 + 5$$

$$\frac{x^6 + 1}{0} + 4 = 4$$

$$(ii) x^{102} + x^{96} + x^{101} + x^{95} + x^{100} + x^{94}$$

$$x^{96}(x^6 + 1) + x^{95}(x^6 + 1) + x^{94}(x^6 + 1)$$

$$= x^{96} \times 0 + x^{95} \times 0 + x^{94} \times 0 = 0$$

Note: When difference of the power is 6 then the value of sum of both of terms value is 0.

TYPE - VIII

If $x+y = 0$ Then $x = -y$

or

$$x = 0, y = 0$$

If $x^2 + y^2 = 0$

Then $x^2 = 0, x = 0$

And $y^2 = 0, y = 0$

If $(x-1)^2 + (y-2)^2 = 0$ then we can say $x = 1$ and $y = 2$

Ex.92 If $(x+3)^2 + (y-5)^2 + (z+2)^2 = 0$

find the value $\sqrt{x+y+z}$

Sol. $(x+3)^2 + (y-5)^2 + (z+2)^2 = 0$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 0 & 0 & 0 \\ x = -3 & y = 5 & z = -2 \end{array}$$

$$\sqrt{x+y+z} = \sqrt{-3+5-2}$$

$$= 0$$

Ex.93 If $(a-4)^2 + (b-5)^2 + (c-3)^2 = 0$

find the value $\frac{a+b}{c}$

Sol. $(a - 4)^2 + (b - 5)^2 + (c - 3)^2 = 0$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 0 & 0 & 0 \\ a = 4 & b = 5 & c = 3 \end{array}$$

$$\frac{a+b}{c} = \frac{4+5}{3} = 3$$

Ex.94 If $x^2 + y^2 + z^2 + 4x + 2y + 5 = 0$
find the value $x^2 + y^3 + z^4$

Sol. $x^2 + y^2 + z^2 + 4x + 2y + 5 = 0$
 $x^2 + 4x + 4 + y^2 + 2y + 1 + z^2 = 0$
 $(x + 2)^2 + (y + 1)^2 + z^2 = 0$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 0 & 0 & 0 \\ x = -2 & y = -1 & z = 0 \\ x^2 + y^3 + z^4 = (-2)^2 + (-1)^3 + (0)^4 \\ = 4 - 1 + 0 = 3 \end{array}$$

Ex.95 If $a^2 + b^2 + c^2 = 2(a - 2b - 2c) - 9$
find the value of $a^3 + b^4 - c^2$

Sol. $a^2 + b^2 + c^2 = 2(a - 2b - 2c) - 9$
 $a^2 + b^2 + c^2 - 2a + 4b - 4c + 9 = 0$
 $a^2 - 2a + 1 + b^2 + 4b + 4 + c^2 - 4c + 4 = 0$
 $(a - 1)^2 + (b + 2)^2 + (c - 2)^2 = 0$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 0 & 0 & 0 \\ a = 1 & b = -2 & c = 2 \\ a^3 + b^4 - c^2 = (1)^3 + (-2)^4 - (2)^2 \\ = 1 + 16 - 4 = 13 \end{array}$$

Ex.96 If $5x^2 + 4xy + y^2 + 1 + 2x = 0$
find the value of x & y

Sol. $5x^2 + 4xy + y^2 + 1 + 2x = 0$
 $4x^2 + 4xy + y^2 + x^2 + 2x + 1 = 0$
 $(2x + y)^2 + (x + 1)^2 = 0$

$$\begin{array}{ccc} \downarrow & \downarrow \\ 0 & 0 \\ 2x = -y & x = -1 \\ \mathbf{x = -1}, \\ 2x = -y \\ 2 \times -1 = -y \\ \mathbf{y = 2} \end{array}$$

Ex.97 If $(x + y - z - 1)^2 + (y + z - x - 5)^2 + (z + x - y - 3)^2 = 0$

find $\sqrt{x + y + z}$

Sol. $(x + y - z - 1)^2 + (y + z - x - 5)^2 + (z + x - y - 3)^2 = 0$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 0 & 0 & 0 \\ x + y - z = 1 & y + z - x = 5 & z + x - y = 3 \end{array}$$

Adding all three equations

$$x + y - z + y + z - x + z + x - y = 1 + 5 + 3$$

$$x + y + z = 9$$

Then $\sqrt{x + y + z} = \sqrt{9} = 3$

TYPE - IX

Rationalising factor of the surd $x = \sqrt{a} \pm \sqrt{b}$ and

$$\frac{1}{x} = \sqrt{a} \mp \sqrt{b}.$$

Ex.98 If $x = \frac{1}{5 + 2\sqrt{6}}$, $y = \frac{1}{5 - 2\sqrt{6}}$

Find the value of $\frac{1}{x+1} + \frac{1}{y+1}$

Sol. $x = \frac{1}{5 + 2\sqrt{6}}$

Now, $\frac{1}{x} = \frac{1}{5 - 2\sqrt{6}} = y$

$$\frac{1}{x} = y$$

Put the value of y in equation

$$= \frac{1}{x+1} + \frac{1}{y+1}$$

$$= \frac{1}{x+1} + \frac{1}{\frac{1}{x}+1}$$

$$\frac{1}{x+1} + \frac{1 \times x}{1+x} = \frac{1+x}{1+x} = 1$$

Note:

If $xy = 1$ then $\frac{1}{1+x} + \frac{1}{1+y} = 1$

Ex.99 $x = (\sqrt{3} + \sqrt{2})^{-3}$,

$$y = (\sqrt{3} - \sqrt{2})^{-3}$$

Find the value of $(x + 1)^{-1} + (y + 1)^{-1}$

Sol: $x = (\sqrt{3} + \sqrt{2})^{-3}$

Then

$$\frac{1}{x} = (\sqrt{3} - \sqrt{2})^{-3} = y$$

$$\frac{1}{x} = y$$

Now,

$$(x + 1)^{-1} + (y + 1)^{-1}$$

$$= \frac{1}{x+1} + \frac{1}{y+1}$$

$$= \frac{1}{x+1} + \frac{1}{\frac{1}{x}+1}$$

$$= \frac{1}{x+1} + \frac{1 \times x}{1+x} = \frac{1+x}{1+x} = 1$$

Ex.100. If $x = 2 + \sqrt{3}$, find the value

of $\frac{x^6 + x^4 + x^2 + 1}{x^3}$

Sol. $x = 2 + \sqrt{3}$

$$\frac{1}{x} = \frac{1}{2 + \sqrt{3}} = 2 - \sqrt{3}$$

$$x + \frac{1}{x} = 4$$

Then,

$$x^3 + \frac{1}{x^3} = (4)^3 - 3 \times 4$$

$$= x^3 + \frac{1}{x^3} = 52$$

Now,

$$= \frac{x^6 + x^4 + x^2 + 1}{x^3}$$

$$= x^3 + x + \frac{1}{x} + \frac{1}{x^3}$$

$$= x^3 + \frac{1}{x^3} + x + \frac{1}{x}$$

Put the value

$$52 + 4 = 56$$

Ex101. If $x = 5 + 2\sqrt{6}$ Find the value

of $\frac{x^4 + \frac{1}{x^2}}{x^2 - 3x + 1}$

Sol. $x = 5 + 2\sqrt{6}$

$$\frac{1}{x} = 5 - 2\sqrt{6}$$

$$x + \frac{1}{x} = 10$$

Then $x^3 + \frac{1}{x^3} = (10)^3 - 3 \times 10$

$$x^3 + \frac{1}{x^3} = 970$$

Now,

x divide or nominator and denominator

$$\frac{x^4 + \frac{1}{x^2}}{x^2 - 3x + 1}$$

$$= \frac{\frac{x^4}{x} + \frac{1}{x^2 \times x}}{\frac{x^2}{x} - \frac{3x}{x} + \frac{1}{x}}$$

$$= \frac{x^3 + \frac{1}{x^3}}{x + \frac{1}{x} - 3}$$

$$= \frac{970}{10-3} = \frac{970}{7}$$

Ex.102. If $x = 7 + 4\sqrt{3}$, find the value

$$\text{of } \frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$$

Sol. $x = 7 + 4\sqrt{3}$

$$\frac{1}{x} = 7 - 4\sqrt{3}$$

$$x + \frac{1}{x} = 14$$

Then,

$$x^2 + \frac{1}{x^2} = (14)^2 - 2$$

$$x^2 + \frac{1}{x^2} = \mathbf{194}$$

x^2 divide or nominator and denominator

$$= \frac{\frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^2}}{\frac{x^4 + 1}{x^2}}$$

$$= \frac{\frac{x^4}{x^2} + \frac{3x^3}{x^2} + \frac{5x^2}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}}{\frac{x^4}{x^2} + \frac{1}{x^2}}$$

$$= \frac{x^2 + 3x + 5 + \frac{3}{x} + \frac{1}{x^2}}{x^2 + \frac{1}{x^2}}$$

$$= \frac{x^2 + \frac{1}{x^2} + 3\left(x + \frac{1}{x}\right) + 5}{x^2 + \frac{1}{x^2}}$$

$$= \frac{194 + 3 \times 14 + 5}{194} = \frac{241}{194}$$

Ex.103 If $x = 3 - 2\sqrt{2}$, $y = 3 + 2\sqrt{2}$

$$\text{Find the value of } \frac{x^2}{y} + \frac{y^2}{x}$$

Sol. $x = 3 - 2\sqrt{2}$

$$\frac{1}{x} = 3 + 2\sqrt{2}$$

Then, $\frac{1}{x} = y$

$$\frac{x^2}{y} + \frac{y^2}{x}$$

Put the value of $y = \frac{1}{x}$

$$\frac{x^2}{\frac{1}{x}} + \left(\frac{1}{x}\right)^2 \times \frac{1}{x}$$

$$x^3 + \frac{1}{x^3}$$

Now,

$$x + \frac{1}{x} = 6$$

$$x^3 + \frac{1}{x^3} = (6)^3 - 3 \times 6$$

$$x^3 + \frac{1}{x^3} = 198$$

Ex.104. If $x = 5 + 2\sqrt{6}$ and $xy = 1$,

Find the value of $\frac{x^2 + y^2 + 2xy}{x^3 + y^3 + 3xy}$

Sol. $x = 5 + 2\sqrt{6}$

$$\frac{1}{x} = 5 - 2\sqrt{6}$$

$$xy = 1$$

(As given)

Then,

$$y = \frac{1}{x} = 5 - 2\sqrt{6}$$

$$\frac{x^2 + y^2 + 2xy}{x^3 + y^3 + 3xy}$$

Put the value of $y = \frac{1}{x}$

$$x^2 + \frac{1}{x^2} + 2$$

$$x^3 + \frac{1}{x^3} + 3$$

Now,

$$x + \frac{1}{x} = 10$$

Then

$$x^2 + \frac{1}{x^2} = (10)^2 - 2$$

$$x^2 + \frac{1}{x^2} = 98$$

And,

$$x^3 + \frac{1}{x^3} = (10)^3 - 3 \times 10$$

$$x^3 + \frac{1}{x^3} = 970$$

Put the value in equation,

$$\frac{98 + 2}{970 + 3} = \frac{100}{973}$$

Ex.105. If $x + \frac{1}{x} = 10$, Find the value

$$\text{of } \frac{7x}{x^2 - 5x + 1}$$

Sol.

$$\frac{7x}{x^2 - 5x + 1}$$

x divide or nominator and denominator

$$\frac{\frac{7x}{x}}{x^2 - 5x + 1} = \frac{7}{x - 5 + \frac{1}{x}}$$

$$= \frac{7}{x + \frac{1}{x} - 5}$$

$$x + \frac{1}{x} = 10$$

(As Given)

$$\frac{7}{10 - 5} = \frac{7}{5}$$

Ex.106. $\frac{2p}{p^2 - 2p + 1} = \frac{1}{4}$, Find the

$$\text{value of } p + \frac{1}{p}$$

Sol. $\frac{2p}{p^2 - 2p + 1} = \frac{1}{4}$

$$8p = p^2 - 2p + 1$$

$$10p = p^2 + 1$$

divide by p both sides,

$$p + \frac{1}{p} = 10$$

Ex.107 If $x + \frac{a}{x} = 1$, find the value of

$$\frac{x^2 + x + a}{x^3 - x^2}$$

Sol. $x + \frac{a}{x} = 1$

$x^2 + a = x$ (i)

$x^2 - x = -a$ (ii)

$$\frac{x^2 + x + a}{x^3 - x^2}$$

From equation (i)

$$\frac{x + x}{x^3 - x^2}$$

$$= \frac{2}{x^2 - x}$$

From equation (ii)

$$\frac{x^2 + x + a}{x^3 - x^2} = \frac{-2}{a}$$

Type - X

Ex.108 If $\frac{a^2 - bc}{a^2 + bc} + \frac{b^2 - ac}{b^2 + ac} + \frac{c^2 - ab}{c^2 + ab} = 1$ Find

$$\frac{2a^2}{a^2 + bc} + \frac{2b^2}{b^2 + ac} + \frac{2c^2}{c^2 + ab}$$

Sol. $\frac{a^2 - bc}{a^2 + bc} + \frac{b^2 - ac}{b^2 + ac} + \frac{c^2 - ab}{c^2 + ab} = 1$

Added 1 every terms

$$\Rightarrow \frac{a^2 - bc}{a^2 + bc} + 1 + \frac{b^2 - ac}{b^2 + ac} + 1 + \frac{c^2 - ab}{c^2 + ab} + 1 = 1 + 3$$

$$\Rightarrow \frac{a^2 - bc + a^2 + bc}{a^2 + bc} + \frac{b^2 - ac + b^2 + ac}{b^2 + ac} + \frac{c^2 - ab + c^2 + ab}{c^2 + ab} = 4$$

$$\Rightarrow \frac{2a^2}{a^2 + bc} + \frac{2b^2}{b^2 + ac} + \frac{2c^2}{c^2 + ab} = 4$$

Note: In this type of question when based is same we add or subtract 1

Ex.109 If $\frac{1}{a-1} + \frac{2}{b-2} + \frac{3}{c-3} = 10$, Find the value of

$$\frac{a}{a-1} + \frac{b}{b-2} + \frac{c}{c-3}$$

Sol. $\frac{1}{a-1} + \frac{2}{b-2} + \frac{3}{c-3} = 10$

$$\frac{1}{a-1} + 1 + \frac{2}{b-2} + 1 + \frac{3}{c-3} + 1 = 10 + 1 + 1 + 1$$

$$\frac{a}{a-1} + \frac{b}{b-2} + \frac{c}{c-3} = 13$$

Ex.110 If $\frac{1}{x+1} + \frac{2}{x+2} + \frac{3}{x+3} \dots\dots$

$$\frac{1007}{x+1007} = 1249$$

find $\frac{x}{x+1} + \frac{x}{x+2} \dots\dots \frac{x}{x+1007}$

Sol. $\frac{1}{x+1} + \frac{2}{x+2} + \frac{3}{x+3} \dots\dots$

$$\frac{1007}{x+1007} = 1249$$

$$\Rightarrow \frac{1}{x+1} - 1 + \frac{2}{x+2} - 1 + \frac{3}{x+3} - 1 \dots\dots$$

$$\frac{x}{x+1007} - 1 = 1249 - 1007$$

$$\Rightarrow \frac{x}{x+1} + \frac{x}{x+2} \dots\dots \frac{x}{x+1007} = -242$$

Ex.111 If $x = 101$ find the value of

$$x(x^2 - 3x + 3)$$

Sol: $x(x^2 - 3x + 3)$

$$x^3 - 3x^2 + 3x$$

We add and subtract 1 then,

$$x^3 - 3x^2 + 3x - 1 + 1$$

$$= (x-1)^3 + 1$$

Put the value x

$$= (101-1)^3 + 1$$

$$= 100^3 + 1 = 1000001$$

Ex.112 If $x = 102$ find the value of $x(x^2 + 3x + 3)$

Sol: $x(x^2 + 3x + 3)$

$$x^3 + 3x^2 + 3x$$

We add and subtract 1 then,

$$x^3 + 3x^2 + 3x - 1 + 1$$

$$= (x+1)^3 - 1$$

Put the value x

$$= (102+1)^3 - 1 = 103^3 - 1$$

$$= 1092727 - 1 = 1092726$$

Ex.113 : If $\frac{1}{\sqrt[3]{25} - \sqrt[3]{5} + 1}$

$$= A\sqrt[3]{25} + B\sqrt[3]{5} + C$$

Find the value of $A + B + C$

Sol. $\frac{1}{\sqrt[3]{25} - \sqrt[3]{5} + 1}$

$$= A\sqrt[3]{25} + B\sqrt[3]{5} + C$$

$$\Rightarrow \frac{1}{\frac{2}{5^{\frac{2}{3}}} - \frac{1}{5^{\frac{1}{3}}} + 1} = A(5)^{\frac{2}{3}} + B(5)^{\frac{1}{3}} + C$$

$$\Rightarrow \left(\frac{1}{5^{\frac{2}{3}} + 1} \right) \text{ multiply and divided}$$

$$\Rightarrow \frac{1 \times \left(\frac{1}{5^{\frac{2}{3}} + 1} \right)}{\left(\frac{1}{5^{\frac{2}{3}} + 1} \right) \left[\left(\frac{1}{5^{\frac{2}{3}}} \right)^2 - \frac{1}{5^{\frac{2}{3}}} \times 1 + (1)^2 \right]}$$

$$(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$$

$$\Rightarrow \frac{\frac{1}{5^{\frac{2}{3}} + 1}}{\left(\frac{1}{5^{\frac{2}{3}}} \right)^2 + 1^3}$$

$$\Rightarrow \frac{\frac{1}{5^{\frac{2}{3}} + 1}}{\frac{1}{5} + 1} = \frac{\frac{1}{5^{\frac{2}{3}} + 1}}{6} = A(5)^{\frac{2}{3}} + B\left(5^{\frac{1}{3}}\right) + C$$

$$\Rightarrow \frac{5^{\frac{1}{3}}}{6} + \frac{1}{6} = A(5)^{\frac{2}{3}} + B\left(5^{\frac{1}{3}}\right) + C$$

Comparison of the terms

$$A = 0, B = \frac{1}{6} \text{ and } C = \frac{1}{6}$$

$$\text{Then, } A + B + C = 0 + \frac{1}{6} + \frac{1}{6} = 0$$

Ex.114 : If $\frac{1}{\sqrt[3]{16} + \sqrt[3]{4} + 1}$

$$= A\sqrt[3]{16} + B\sqrt[3]{4} + C$$

Find the value of $A + B + C$

Sol. $\frac{1}{\sqrt[3]{16} + \sqrt[3]{4} + 1} = A\sqrt[3]{16} + B\sqrt[3]{4} + C$

$$\Rightarrow \frac{1}{\frac{2}{4^{\frac{2}{3}}} - \frac{1}{4^{\frac{1}{3}}} + 1} = A(4)^{\frac{2}{3}} + B(4)^{\frac{1}{3}} + C$$

$$\Rightarrow \left(\frac{1}{4^{\frac{2}{3}} - 1} \right) \text{ multiply and divided}$$

$$\Rightarrow \frac{1 \times \left(\frac{1}{4^{\frac{2}{3}} - 1} \right)}{\left(\frac{1}{4^{\frac{2}{3}} - 1} \right) \left[\left(\frac{1}{4^{\frac{2}{3}}} \right)^2 + \frac{1}{4^{\frac{2}{3}}} \times 1 + (1)^2 \right]}$$

$$\frac{4^{\frac{1}{3}} - 1}{\left(4^{\frac{1}{3}}\right)^3 - 1^3}$$

$$\Rightarrow \frac{4^{\frac{1}{3}} - 1}{4 - 1} = \frac{4^{\frac{1}{3}} - 1}{3}$$

$$= A(4)^{\frac{2}{3}} + B\left(4^{\frac{1}{3}}\right) + C$$

$$\Rightarrow \frac{4^{\frac{1}{3}}}{3} - \frac{1}{3} = A(4)^{\frac{2}{3}} + B\left(4^{\frac{1}{3}}\right) + C$$

Comparison of the terms

$$A = 0, B = \frac{1}{3} \text{ and } C = -\frac{1}{3}$$

$$\text{Then, } A + B + C = 0 + \frac{1}{3} - \frac{1}{3} = 0$$

TYPE - XI

Ex.115 If $x = 11$, Find the value of $x^5 - 12x^4 + 13x^3 - 12x^2 + 12x - 5$

Sol. $x^5 - 12x^4 + 13x^3 - 12x^2 + 12x - 5$
Expandable from the equation
Then,
 $x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 5 + x^3$
 $= x^5 - 11x^4 - x^4 + 11x^3 + x^3$
 $= -11x^2 - x^2 + 11x + x - 5 + x^3$
Put the value of $x = 11$
 $= 11 - 5 + (11)^3 = 11 - 5 + 1331$
 $= 1337$

Ex.116 $x = 8$ find $x^4 - 9x^3 + 9x^2 - 9x$

Sol. $x^4 - 9x^3 + 9x^2 - 9x$
Expandable from the equation
Then,
 $x^4 - 8x^3 - x^3 + 8x^2 - 8x - x$
Put the value of $x = 8$
 $= -8$

Ex.117 If $x = 12$ find $x^4 - 13x^3 + 13x^2 - 12x + 11$

Sol. $x^4 - 13x^3 + 13x^2 - 12x + 11$
Expandable from the equation
Then,

$$\underbrace{x^4 - 12x^3 - x^3 + 12x^2 + x^2 - 12x + 11}_{\begin{matrix} 0 & 0 & 0 \end{matrix}} = 11$$

Ex.118 If $x = 12$ find $x^4 - 14x^3 + 13x^2 \div 12x + 11$

Sol. $x^4 - 14x^3 + 13x^2 \div 12x + 11$
Expandable from the equation
Then,

$$\underbrace{x^4 - 12x^3 - x^3 + 12x^2 + x^2 - 12x + 11 - x^3}_{\begin{matrix} 0 & 0 & 0 \end{matrix}}$$

Put the value of $x = 12$

$$\begin{aligned} \text{Then,} \\ &= 11 - 12^3 \\ &= 11 - 1728 = -1717 \end{aligned}$$

Ex.119 find the value of $\frac{1 + 7231 \times 7233}{7232}$

Sol. Let $7232 = x$
Then,
 $\frac{1 + (x-1) \times (x+1)}{x}$
 $= \frac{1 + x^2 - 1}{x} = x$

So,
 $x = 7232$

Ex.120 find the value of $\frac{4 + 673 \times 677}{675}$

Sol. Let $675 = x$
 $\Rightarrow \frac{4 + (x-2)(x+2)}{x}$
 $\Rightarrow \frac{4 + x^2 - 4}{x} = x$

Then,
 $x = 675$

Ex.121 Find the value of $\frac{16 + 9748 \times 9756}{9752}$

Sol. Let $9752 = x$
 $= \frac{16 + (x-4)(x+4)}{x}$
 $= \frac{16 + x^2 - 16}{x} = x$

So,
 $x = 9752$

Ex.122 What will be added in the product of numbers (30×36) That resulted will be perfect square number

Sol. (30×36)
Difference of the number
between $= 36 - 30$
 $= 6$

$$\text{Divided by 2 in difference} = \frac{6}{2} = 3$$

$$\begin{aligned} \text{Square of } 3 &= 3^2 = 9 \\ (30 \times 36) + 9 &= 1080 + 9 \\ &= 1089 \end{aligned}$$

1089 is perfect square of 33

So,
9 will be added the product of number

Note:- $\left(\frac{D}{2}\right)^2$ always added the product of number we will get perfect square integer number
Here $D = \text{Difference of numbers}$

Ex.123 What will be added in the product of numbers (174×182) That resulted will be perfect number

Sol. $D = 8$
 $\frac{D}{2} = \frac{8}{2} = 4$

$$\left(\frac{D}{2}\right)^2 = 4^2 = 16$$

So,
16 will be added the product of number

Maximum or minimum value of quadratic equation

Quadratic equation in general form

1. When $a > 0$ (In the equation $ax^2 + bx + c$) The expression gives minimum value

$$y = \frac{4ac - b^2}{4a}$$

$$\text{Max} = -\infty$$

2. When $a < 0$ (In the equation $ax^2 + bx + c$) The expression gives maximum value

$$= \frac{4ac - b^2}{4a}$$

$$\text{Min} = -\infty$$

Ex.124 Expression $5x^2 - 8x + 14$ what will be minimum value ?

Sol. $5x^2 - 8x + 14$
 $a > 0$

$$\text{Then, min value} = \frac{4ac - b^2}{4a}$$

$$\begin{aligned} \text{here,} \\ a &= 5, \quad b = -8, \quad c = 14 \end{aligned}$$

$$\text{Now, min} = \frac{4 \times 5 \times 14 - (-8)^2}{4 \times 5}$$

$$= \frac{280 - 64}{20} = \frac{216}{20}$$

$$\text{Min value} = \frac{54}{5}$$

Ex.125 Find the minimum value of $(x-2)(x-9)$

Sol. $(x-2)(x-9)$
 $= x^2 - 2x - 9x + 18$
 $= x^2 - 11x + 18$
Coefficient of x^2 is 1 which $1 > 0$
Then min value

$$= \frac{4 \times 1 \times 18 - (-11)^2}{4 \times 1} = \frac{72 - 121}{4}$$

$$\text{min value} = \frac{-49}{4}$$

Ex.126 Find the maximum value of $12 - 7x - x^2$

Sol. $12 - 7x - x^2$ or $-x^2 - 7x + 12$
Here $a < 0$
Coefficient of x^2 is -1 which less than 0
So,

$$\text{Max value} = \frac{4ac - b^2}{4a}$$

$$= a = -1, b = -7 \text{ and } c = 12$$

$$= \frac{4 \times -1 \times 12 - (-7)^2}{4 \times (-1)}$$

$$= \frac{-48 - 49}{-4} = \frac{-97}{-4} = \frac{97}{4}$$

Ex.127 Find the maximum value of $5 - 12x - 3x^2$

Sol. $5 - 12x - 3x^2$
or
 $-3x^2 - 12x + 5$
Here coefficient of x^2 is -3 which less than 0,
Now,
 $a = -3, b = -12, c = 5$

$$\text{So, max value} = \frac{4ac - b^2}{4a}$$

$$= \frac{4 \times -3 \times 5 - (-12)^2}{4 \times -3}$$

$$= \frac{-60 - 144}{-12}$$

$$= \frac{-204}{-12} = 17$$

(ii) a and b are two numbers

Then,

$$A.M = \frac{a+b}{2}$$

$$\text{and } G.M = \sqrt{ab}$$

Thus when a, b and c are three numbers

Then,

$$A.M = a+b+c$$

$$G.M = \sqrt[3]{abc}$$

Relation between A.M & G.M

$A.M \geq G.M$

When a, b, c are +ve Real number

m and $\frac{1}{m}$ are two number

$$A.M = \frac{M + \frac{1}{m}}{2}, G.M = \sqrt{M \times \frac{1}{m}}$$

$A.M \geq G.M$

$$\frac{m + \frac{1}{m}}{2} \geq \sqrt{m \times \frac{1}{m}}$$

$$\frac{m + \frac{1}{m}}{2} \geq 1$$

$$\frac{m + \frac{1}{m}}{2} \geq 2$$

When m +ve Real number.

Minimum value of $m + \frac{1}{m} = 2$

And

We can say minimum value

$$m^n + \frac{1}{m^n} = 2$$

When m is +ve Real number

Ex.128 If $x > 1$, find the minimum

$$\text{value of } f(x) = x^2 + \frac{1}{x^2 - 1} - 3$$

$$\text{Sol. } x^2 + \frac{1}{x^2 - 1} - 3$$

Add and subtract 1,

Now,

$$(x^2 - 1) + \frac{1}{(x^2 - 1)} - 3 + 1$$

Let $x^2 - 1 = m$

$$m + \frac{1}{m} - 2$$

Minimum value of $m + \frac{1}{m} = 2$

Then,

$$2 - 2 = 0$$

So, minimum value $F(x) = 0$

Ex.129 Find the minimum value of x^2

$$+ \frac{1}{x^2 + 1} - 4$$

Sol. Add and subtract

$$\text{Now, } (x^2 + 1) + \frac{1}{(x^2 + 1)} - 4 - 1$$

Let $x^2 + 1 = m$

$$\text{Then, } m + \frac{1}{m} - 5$$

Minimum value of $m + \frac{1}{m} = 2$

$$= 2 - 5 = -3$$

So, minimum value of $F(x) = -3$

Ex.130 Find the minimum value of

$$x^2 + \frac{1}{x^2 - 2} + 5, \text{ here } x > \sqrt{2}$$

$$\text{Sol. } x^2 + \frac{1}{x^2 - 2} + 5$$

$$(x^2 - 2) + \frac{1}{(x^2 - 2)} + 5 + 2$$

Let $x^2 - 2 = m$

$$\text{Now, } m + \frac{1}{m} + 7$$

Minimum value $= 7 + 2 = 9$

Ex.131 Find the maximum value of

$$\frac{x^4}{x^8 + 1}, \text{ where } x \text{ is Real number}$$

$$\text{Sol. } \frac{x^4}{x^8 + 1}$$

componendo and dividendo
divide by x^4

$$\frac{\frac{x^4}{x^4}}{\frac{x^8 + 1}{x^4}} = \frac{1}{x^4 + \frac{1}{x^4}}$$

$$\text{Min value} = x^4 + \frac{1}{x^4} = 2$$

Then, max value = 1

(iii) If $x+y$ will be given then xy will be maximum

When $x=y$

Ex.132 Find maximum value xy if $x+y = 20$

Sol. $x+y = 20$

For maximum $x = y$

Then $x = y = 10$

Max value $xy = 10 \times 10 = 100$

Ex.133 Find the maximum value xy , if $x + y = 25$

Sol. $x + y = 25$

For maximum $x = y$

$$\text{Then } x=y = \frac{25}{2}$$

$$\text{Max value} = xy \times \frac{25}{2} \times \frac{25}{2}$$

$$= \frac{625}{4}$$

(iv) If x, y will be given then $x + y$ will be minimum when $x = y$ here x & y (+ve) Real number

Ex.134 Find the minimum value of $x + y$ if $xy = 16$

Sol. $xy = 16$
For minimum value $x = y = z$
 $x = y = 4$
Min value $= x + y = 4 + 4 = 8$

Ex.135 Find the minimum value of $x + y + z$ if $xyz = 216$

Sol. $xyz = 216$
Min value $x = y = z$
 $x \times x \times x = x^3 = 216$

$$x = 6$$

$$x = y = z = 6$$

$$\text{Minimum value} = x + y + z$$

$$= 6 + 6 + 6 = 18$$

Ex.136 If $x + y + z = 18$, Find the Maximum value of $(x-1)(y-2)(z-3)$

Sol. For maximum
 $(x-1) = (y-2) = (z-3) = m$
 $x = m+1$
 $y = m+2$
 $z = m+3$
 $\therefore x+y+z = 18$ (As given)
Put the value x, y , and z
 $m+1 + m+2 + m+3 = 18$
 $3m = 12$
 $m = 4$
So, max value of
 $(x-1)(y-2)(z-3) = m \times m \times m$
 $= m^3$

Put the value m
Then
Max value $= (4)^3 = 64$

Ex.137 If $x+y+z = 24$, maximum value of

$$(x-1)(y-2)(z+3)$$

Sol. For max value $(x-1) = (y-2) = (z-3) = m$
 $x = m+1$
 $y = m+2$
 $z = m-3$
 $\therefore x+y+z = 24$
Put the value x, y and z
 $m+1 + m+2 + m-3 = 24$
 $3m = 24$
 $m = 8$
Max value $= (x+1)(y+2)(z-3)$
 $= m \times m \times m = m^3$
So, max value $= (8)^3 = 512$

EXERCISE

1. The value of

$$\left(1 + \frac{1}{x}\right) \left(1 + \frac{1}{x+1}\right) \left(1 + \frac{1}{x+2}\right) \left(1 + \frac{1}{x+3}\right) \text{ is:}$$

$$(a) 1 + \frac{1}{x+4} \quad (b) x+4$$

$$(c) \frac{1}{x} \quad (d) \frac{x+4}{x}$$

2. If $x = 7 - 4\sqrt{3}$, then the value of

$$\left(x + \frac{1}{x}\right) \text{ is:}$$

$$(a) 3\sqrt{3} \quad (b) 8\sqrt{3}$$

$$(c) 14 + 8\sqrt{3} \quad (d) 14$$

3. If $\frac{2a+b}{a+4b} = 3$, then find the value of

$$\frac{a+b}{a+2b}$$

$$(a) \frac{5}{9} \quad (b) \frac{2}{7} \quad (c) \frac{10}{9} \quad (d) \frac{10}{7}$$

4. If $A : B = \frac{1}{2} : \frac{3}{8}$ $B : C = \frac{1}{3} : \frac{5}{9}$ and $C : D$

$$= \frac{5}{6} : \frac{3}{4} \text{ then find the ratio of } A : B : C : D$$

$$(a) 6 : 4 : 8 : 10 \quad (b) 6 : 8 : 9 : 10$$

$$(c) 8 : 6 : 10 : 9 \quad (d) 4 : 6 : 8 : 10$$

5. If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$ then $\frac{a+b+c}{c}$ is equal to

$$(a) 0 \quad (b) 1 \quad (c) 2 \quad (d) 3$$

6. If $\frac{144}{0.144} = \frac{14.4}{x}$, then the value of x is

$$(a) 144 \quad (b) 14.4$$

$$(c) 1.44 \quad (d) 0.0144$$

7. If $1 < x < 2$, then the value of

$$\sqrt{(x-1)^2} + \sqrt{(x-3)^2} \text{ is}$$

$$(a) 1 \quad (b) -2$$

$$(c) 3 \quad (d) 2x - 4$$

8. Given that $10^{0.48} = x$, $10^{0.70} = y$ and $x^z = y^2$, then the value of z is close to

$$(a) 1.45 \quad (b) 1.88 \quad (c) 2.9 \quad (d) 3.7$$

9. If $47.2506 = 4A + 7B + 2C + \frac{5}{D} + 6E$, then the value of $5A + 3B + 6C + D + 3E$ is

$$(a) 53.6003 \quad (b) 53.603$$

$$(c) 153.6003 \quad (d) 213.0003$$

10. If $3^{x^2+7} = 250$, then x is equal to

$$(a) 5 \quad (b) 3 \quad (c) 2 \quad (d) 1$$

11. If $\frac{1}{4} \times \frac{2}{6} \times \frac{3}{8} \times \frac{4}{10} \times \frac{5}{12} \times \dots \times \frac{31}{64} = \frac{1}{2^x}$ the value of x is

$$(a) 31 \quad (b) 32 \quad (c) 36 \quad (d) 37$$

12. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ & $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$, then

$$\text{value of } x^2 + y^2 \text{ is:}$$

$$(a) 14 \quad (b) 13 \quad (c) 15 \quad (d) 10$$

13. If $x = 3 + 2\sqrt{2}$, then the value of

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) \text{ is:}$$

$$(a) 1 \quad (b) 2$$

$$(c) 2\sqrt{2} \quad (d) 3\sqrt{3}$$

14. If $p = 999$, then the value of

$$\sqrt[3]{p(p^2 + 3p + 3)} + 1 \text{ is}$$

$$(a) 1000 \quad (b) 999$$

$$(c) 998 \quad (d) 1002$$

15. If $x : y = 7 : 3$ then the value of $\frac{xy + y^2}{x^2 - y^2}$ is

$$(a) \frac{3}{4} \quad (b) \frac{4}{3} \quad (c) \frac{3}{7} \quad (d) \frac{7}{3}$$

16. If $x^{\sqrt{x}} = (x\sqrt{x})^x$, then x equals

$$(a) \frac{4}{9} \quad (b) \frac{2}{3} \quad (c) \frac{9}{4} \quad (d) \frac{3}{2}$$

17. If $a = 7$, $b = 5$ and $c = 3$, then the value of $a^2 + b^2 + c^2 - ab - bc - ca$ is

$$(a) 12 \quad (b) -12 \quad (c) 0 \quad (d) 8$$

18. $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} - 3 \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5} + \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$ is equal to

$$(a) \frac{2}{3} \quad (b) \frac{3}{4} \quad (c) \frac{47}{60} \quad (d) \frac{49}{60}$$

19. If $x = 7 - 4\sqrt{3}$, then $\sqrt{x} + \frac{1}{\sqrt{x}}$ is equal to:
 (a) 1 (b) 2 (c) 3 (d) 4
20. If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ & $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, then the value of $\frac{a^2+ab+b^2}{a^2-ab+b^2}$ is
 (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{3}$
21. If $a = 4.36$, $b = 2.39$ and $c = 1.97$, then the value of $a^3 - b^3 - c^3 - 3abc$ is
 (a) 3.94 (b) 2.39 (c) 0 (d) 1
22. If $\frac{3a+5b}{3a-5b} = 5$, then $a : b$ is equal to
 (a) 2:1 (b) 2:3 (c) 1:3 (d) 5:2
23. If $p : q = r : s = t : u = 2 : 3$, then $(mp+nr+ot) : (mq+ns+ou)$ equals :
 (a) 3:2 (b) 2:3 (c) 1:3 (d) 1:2
24. If $x : y = 3 : 4$, then $(7x+3y) : (7x-3y)$ is equal to :
 (a) 5 : 2 (b) 4 : 3
 (c) 11 : 3 (d) 37 : 19
25. For what value(s) of a is $x + \frac{1}{4}\sqrt{x} + a^2$ a perfect square ?
 (a) $\pm \frac{1}{18}$ (b) $\frac{1}{8}$ (c) $-\frac{1}{5}$ (d) $\frac{1}{4}$
26. If $a \neq b$, then which of the following statements is true?
 (a) $\frac{a+b}{2} = \sqrt{ab}$ (b) $\frac{a+b}{2} < \sqrt{ab}$
 (c) $\frac{a+b}{2} > \sqrt{ab}$ (d) All of the above
27. If x, y are two positive real number and $x^{1/3} = y^{1/4}$, then which of the following relations is true?
 (a) $x^3 = y^4$ (b) $x^3 = y$
 (c) $x = y^4$ (d) $x^{20} = y^{15}$
28. If $x = \frac{\sqrt{3}}{2}$, then $\frac{\sqrt{1+x}}{1+\sqrt{1+x}} + \frac{\sqrt{1-x}}{1-\sqrt{1-x}}$ is equal to
 (a) 1 (b) $2/\sqrt{3}$
 (c) $2 - \sqrt{3}$ (d) 2
29. If for non-zero, x , $x^2 - 4x - 1 = 0$, the value of $x^2 + \frac{1}{x^2}$ is
 (a) 4 (b) 10 (c) 12 (d) 18
30. $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)\left(x^2 + \frac{1}{x^2} + 1\right)$ is equal to
 (a) $x^6 + \frac{1}{x^6}$ (b) $x^8 + \frac{1}{x^8}$
 (c) $x^8 - \frac{1}{x^8}$ (d) $x^6 - \frac{1}{x^6}$
31. If $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7} + b$, then the value of a is
 (a) $\frac{11}{3}$ (b) $-\frac{4}{3}$
 (c) $\frac{4}{3}$ (d) $-\frac{4\sqrt{7}}{3}$
32. If $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$ then $c + \frac{1}{a}$ is equal to
 (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2
33. If $x = \sqrt{3} + \sqrt{2}$, then the value of $\left(x^3 + \frac{1}{x^3}\right)$ is
 (a) $6\sqrt{3}$ (b) $12\sqrt{3}$
 (c) $18\sqrt{3}$ (d) $24\sqrt{3}$
34. If $x + y = 7$, then the value of $x^3 + y^3 + 21xy$ is
 (a) 243 (b) 143 (c) 343 (d) 443
35. If $\frac{1}{x^3} + \frac{1}{y^3} = \frac{1}{z^3}$, then $\{(x+y-z)^3 + 27xyz\}$ equals :
 (a) -1 (b) 1 (c) 0 (d) 27
36. If $\frac{a}{b} + \frac{b}{a} = 1$, $a \neq 0, b \neq 0$ the value of $a^3 + b^3$ is
 (a) 0 (b) 1 (c) -1 (d) 2
37. If $p = 99$, then value of $p(p^2 + 3p + 3)$ is
 (a) 999 (b) 9999
 (c) 99999 (d) 999999
38. If $5\sqrt{x} + 12\sqrt{x} = 13\sqrt{x}$, then x is equal to
 (a) $\frac{25}{4}$ (b) 4 (c) 9 (d) 16
39. If x, y and z are real number such that $(x-3)^2 + (y-4)^2 + (z-5)^2 = 0$ then $(x+y+z)$ is equal to
 (a) -12 (b) 0 (c) 8 (d) 12
40. If $x = 3 + \sqrt{8}$, then $x^2 + \frac{1}{x^2}$ is equal to
 (a) 38 (b) 36 (c) 34 (d) 30
41. If $x - \frac{1}{x} = 4$, then $\left(x + \frac{1}{x}\right)$ is equal to
 (a) $5\sqrt{2}$ (b) $2\sqrt{5}$ (c) $4\sqrt{2}$ (d) $4\sqrt{5}$
42. If $4b^2 + \frac{1}{b^2} = 2$, then the value of $8b^3 + \frac{1}{b^3}$ is
 (a) 0 (b) 1 (c) 2 (d) 5
43. If $\left(\frac{3}{5}\right)^3 \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$, then x is equal to
 (a) -2 (b) 2 (c) -1 (d) 1
44. $\frac{\sqrt{3+x} + \sqrt{3-x}}{\sqrt{3+x} - \sqrt{3-x}} = 2$ then x is equal to
 (a) $\frac{5}{12}$ (b) $\frac{12}{5}$ (c) $\frac{5}{7}$ (d) $\frac{7}{5}$
45. If $x = \frac{\sqrt{3}}{2}$, then the value of $\left(\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}\right)$ is
 (a) $-\sqrt{3}$ (b) -1 (c) 1 (d) $\sqrt{3}$
46. If $\frac{\sqrt{x+4} + \sqrt{x-4}}{\sqrt{x+4} - \sqrt{x-4}} = 2$ then x is equal to
 (a) 2.4 (b) 3.2 (c) 4 (d) 5
47. If $x = (\sqrt{2} + 1)^{\frac{1}{3}}$, the value of $\left(x^3 - \frac{1}{x^3}\right)$ is
 (a) 0 (b) $-\sqrt{2}$ (c) +2 (d) $3\sqrt{2}$
48. If $\frac{x^2 - x + 1}{x^2 + x + 1} = \frac{3}{2}$, then the value of $\left(x + \frac{1}{x}\right)$ is
 (a) 4 (b) -5 (c) 6 (d) 8
49. If $x = 3 + \sqrt{8}$, then $x^2 + \frac{1}{x^2}$ is equal to
 (a) 38 (b) 36 (c) 34 (d) 30
50. If $x = 5 + 2\sqrt{6}$, then the value of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ is.
 (a) $2\sqrt{2}$ (b) $3\sqrt{2}$
 (c) $2\sqrt{3}$ (d) $3\sqrt{3}$
51. If $x = \sqrt{3} + \sqrt{2}$, then the value of $\left(x^2 + \frac{1}{x^2}\right)$ is :
 (a) 4 (b) 6 (c) 9 (d) 10

52. If $x + \frac{9}{x} = 6$, then the value of $\left(x^2 + \frac{9}{x^2}\right)$ is
(a) 8 (b) 9 (c) 10 (d) 12
53. If $2p + \frac{1}{p} = 4$, then value of $p^3 + \frac{1}{8p^3}$ is
(a) 4 (b) 5 (c) 8 (d) 15
54. If $a^4 + b^4 = a^2b^2$, then $(a^6 + b^6)$ equals
(a) 0 (b) 1
(c) $a^2 + b^2$ (d) $a^2b^4 + a^4b^2$
55. If $x + \frac{1}{x} = 3$, then the value of $\frac{x^3 + \frac{1}{x}}{x^2 - x + 1}$ is :
(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{11}{2}$
56. If $a + \frac{1}{a} + 1 = 0$ ($a \neq 0$) then the value of $(a^4 - a)$ is:
(a) 0 (b) 1 (c) 2 (d) -1
57. If $x = a + \frac{1}{a}$ and $y = a - \frac{1}{a}$, then the value of $x^4 + y^4 - 2x^2y^2$ is
(a) 24 (b) 18 (c) 16 (d) 12
58. If $a = 11$ and $b = 9$, then the value of $\left(\frac{a^2 + b^2 + ab}{a^3 - b^3}\right)$ is
(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{20}$ (d) 20
59. If $p = 101$, then the value of $\sqrt[3]{p(p^2 - 3p + 3)} - 1$ is
(a) 100 (b) 101
(c) 102 (d) 1000
60. If $x = 19$ and $y = 18$, then the value of $\frac{x^2 + y^2 + xy}{x^3 - y^3}$ is
(a) 1 (b) 37
(c) 324 (d) 361
61. If 50% of $(p - q) = 30\%$ of $(p + q)$, then $p : q$ is equal to
(a) 5 : 3 (b) 4 : 1
(c) 3 : 5 (d) 1 : 4
62. If $\frac{a}{3} = \frac{b}{2}$, then value of $\frac{2a+3b}{3a-2b}$ is
(a) $\frac{12}{5}$ (b) $\frac{5}{12}$ (c) 1 (d) $\frac{12}{7}$
63. If $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$
(a) 1 (b) 2 (c) 3 (d) 4
64. If $\frac{2x-y}{x+2y} = \frac{1}{2}$, then value of $\frac{3x-y}{3x+y}$ is :
(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) 1
65. If $x + \frac{1}{x} = 5$, then $\frac{2x}{3x^2 - 5x + 3}$ is equal to
(a) 5 (b) $\frac{1}{5}$ (c) 3 (d) $\frac{1}{3}$
66. If $\sqrt{1 - \frac{x^3}{100}} = \frac{3}{5}$, then x equals
(a) 2 (b) 4
(c) 16 (d) $(136)^{1/3}$
67. If $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = a+b\sqrt{6}$, then the values of a and b are respectively
(a) $\frac{9}{15}, \frac{4}{15}$ (b) $\frac{3}{11}, \frac{4}{33}$
(c) $\frac{9}{10}, \frac{2}{5}$ (d) $\frac{3}{5}, \frac{4}{15}$
68. If $\sqrt{1 + \frac{x}{961}} = \frac{32}{31}$, then the value of x is
(a) 63 (b) 61 (c) 65 (d) 64
69. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = 3$
then $\frac{2a^2 + 3c^2 + 4e^2}{2b^2 + 3d^2 + 4f^2} = ?$
(a) 2 (b) 3 (c) 4 (d) 9
70. If $2x + \frac{1}{3x} = 5$. Find the value of $\frac{5x}{6x^2 + 20x + 1}$.
(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{5}$ (d) $\frac{1}{7}$
71. If x varies inversely as $(y^2 - 1)$ and x is equal to 24 when $y = 10$, then the value of x when $y = 5$ is
(a) 99 (b) 12 (c) 24 (d) 100
72. If $x^2 + y^2 + 2x + 1 = 0$, then the value of $x^{31} + y^{35}$ is
(a) -1 (b) 0 (c) 1 (d) 2
73. If $\frac{x}{2x^2 + 5x + 2} = \frac{1}{6}$, then value of $\left(x + \frac{1}{x}\right)$ is:
(a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) -2
74. If a, b, c are real and $a^2 + b^2 + c^2 = 2$ ($a - b - c - 3$) then the value of $2a - 3b + 4c$ is
(a) -1 (b) 0 (c) 1 (d) 2
75. If $(3a+1)^2 + (b-1)^2 + (2c-3)^2 = 0$, then the value of $(3a+b+2c)$ is equal to;
(a) 3 (b) -1 (c) 2 (d) 5
76. The value of the expression $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-c)}$
(a) 0 (b) 3 (c) $\frac{1}{3}$ (d) 2
77. If $(a-3)^2 + (b-4)^2 + (c-9)^2 = 0$, then the value of $\sqrt{a+b+c}$ is :
(a) -4 (b) 4 (c) ± 4 (d) ± 2
78. If $1.5x = 0.04y$, then the value of $\frac{y^2 - x^2}{y^2 + 2xy + x^2}$ is
(a) $\frac{730}{77}$ (b) $\frac{73}{77}$ (c) $\frac{73}{770}$ (d) $\frac{74}{77}$
79. If $\frac{1}{a^3} = 11$, then the value of $a^2 - 331a$ is
(a) 1331331 (b) 1331000
(c) 1334331 (d) 1330030
80. If $x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} = 4$, then the value of $x^2 + y^2$ is
(a) 2 (b) 4 (c) 8 (d) 16
81. If $x^2 = y + z$, $y^2 = z + x$, $z^2 = x + y$, then the value of $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$ is
(a) -1 (b) 1 (c) 2 (d) 4
82. If $a^2 + b^2 = 2$ and $c^2 + d^2 = 1$ then the value of $(ad - bc)^2 + (ac + bd)^2$ is
(a) $\frac{4}{9}$ (b) $\frac{1}{2}$ (c) 1 (d) 2
83. If $x = \frac{4ab}{a+b}$ $a \neq b$, the value of $\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$ is
(a) a (b) b (c) 2ab (d) 2

84. If $m + \frac{1}{m-2} = 4$,
find the value of $(m-2)^2 + \frac{1}{(m-2)^2}$
(a) -2 (b) 0 (c) 2 (d) 4
85. If $a^2 + b^2 + 2b + 4a + 5 = 0$, then the value
of $\frac{a-b}{a+b}$ is
(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$
86. If $x-y = \frac{x+y}{7} = \frac{xy}{4}$, the numerical
value of xy is
(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$
87. If $x+y+z = 0$, then $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} = ?$
(a) $(xyz)^2$ (b) $x^2 + y^2 + z^2$
(c) 9 (d) 3
88. If $a+b+c = 0$, then the value of
 $\frac{1}{(a+b)(b+c)} + \frac{1}{(a+c)(b+a)}$
 $+ \frac{1}{(c+a)(c+b)}$
(a) 1 (b) 0 (c) -1 (d) -2
89. If $a+b+c = 0$, then the value of
 $\frac{a^2+b^2+c^2}{a^2-bc}$ is
(a) 0 (b) 1 (c) 2 (d) 3
90. If $x^2 + y^2 - 4x - 4y + 8 = 0$, then the value
of $x-y$ is
(a) 4 (b) -4 (c) 0 (d) 8
91. If $x = b+c-2a$, $y = c+a-2b$, $z = a+b-2c$,
then the value of $x^2 + y^2 - z^2 + 2xy$ is
(a) 0 (b) $a+b+c$
(c) $a-b+c$ (d) $a+b-c$
92. For real a, b, c if $a^2 + b^2 + c^2 = ab + bc$
 $+ca$, then value of $\frac{a+c}{b}$ is:
(a) 1 (b) 2 (c) 3 (d) 0
93. If $x + \frac{1}{x} = \sqrt{3}$ then the value of $x^{18} + x^{12} + x^6$
 $+ 1$ is
(a) 0 (b) 1 (c) 2 (d) 3
94. If for two real constants a and b the
expression $ax^3 + 3x^2 - 8x + b$ is exactly
divisible by $(x+2)$ and $(x-2)$
(a) $a=2, b=12$ (b) $a=12, b=2$
(c) $a=2, b=-12$ (d) $a=-2, b=12$
95. If $x^2 - 3x + 1 = 0$, then the value of
 $x^3 + \frac{1}{x^3}$ is
(a) 9 (b) 18 (c) 27 (d) 1
96. If $x + \frac{1}{4x} = \frac{3}{2}$, find the value of
 $8x^3 + \frac{1}{8x^3}$.
(a) 18 (b) 36 (c) 24 (d) 16
97. If $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$ ($x \neq 0, y \neq 0, x \neq y$)
then the value of $x^3 - y^3$ is
(a) 0 (b) 1 (c) -1 (d) 2
98. If $x = a(b-c)$, $y = b(c-a)$ and $z = c(a-b)$,
then $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3 = ?$
(a) $\frac{xyz}{3abc}$ (b) $3xyzabc$
(c) $\frac{3xyz}{abc}$ (d) $\frac{xyz}{abc}$
99. If $xy(x+y) = 1$, then the value of
 $\frac{1}{x^3y^3} - x^3 - y^3$ is:
(a) 0 (b) 1 (c) 3 (d) -2
100. If $x^4 + \frac{1}{x^4} = 119$ and $x > 1$, then the
value of $x^3 + \frac{1}{x^3}$ is
(a) $6\sqrt{13}$ (b) $8\sqrt{13}$
(c) $13\sqrt{13}$ (d) $10\sqrt{13}$
101. If $3x + \frac{1}{2x} = 5$, then the value of
 $8x^3 + \frac{1}{27x^3}$ is:
(a) $118\frac{1}{2}$ (b) $30\frac{10}{27}$
(c) 0 (d) 1
102. If $x + y = z$, then the expression $x^3 + y^3 - z^3 + 3xyz$ will be equal to :
(a) 0 (b) $3xyz$
(c) $-3xyz$ (d) z^3
103. If the sum of $\frac{a}{b}$ and its reciprocal is 1
and $a \neq 0, b \neq 0$, then the value of
 $a^3 + b^3$ is
(a) 2 (b) -1 (c) 0 (d) 1
104. If $x = 2^{-2/3} + 2^{2/3}$ then the value of $x^3 - 6x^2 + 18x + 18$ is
(a) 22 (b) 33 (c) 40 (d) 45
105. If $a^3 - b^3 - c^3 - 3abc = 0$, then
(a) $a = b = c$ (b) $a+b+c = 0$
(c) $a+c = b$ (d) $a = b+c$
106. If $a = 2.361, b = 3.263$ and $c = 5.624$, then
the value of $a^3 + b^3 - c^3 + 3abc$ is
(a) $(p-q)(q-r)^3 + (r-p)^3$
(b) $3(p-q)(q-r)(r-p)$
(c) 0 (d) 1
107. If $p = 124$, $\sqrt[3]{p(p^2 + 3p + 3)} + 1 = ?$
(a) 5 (b) 7 (c) 123 (d) 125
108. If $x + \frac{1}{x} = 2$ and x is real, then the
value of $x^{17} + \frac{1}{x^{19}}$ is
(a) 1 (b) 0 (c) 2 (d) -2
109. If $x : y = 3 : 4$, then the value of
 $\frac{5x-2y}{7x+2y} = ?$
(a) $\frac{7}{25}$ (b) $\frac{7}{23}$ (c) $\frac{7}{29}$ (d) $\frac{7}{17}$
110. If $x + y = 2z$ then the value of
 $\frac{x}{x-z} + \frac{z}{y-z}$ is
(a) 1 (b) 3 (c) $\frac{1}{2}$ (d) 2
111. If $a^3b = abc = 180$, a, b, c are positive
integers, then the value of b is
(a) 110 (b) 180 (c) 4 (d) 25
112. If a, b are rational number and
 $(a-1)\sqrt{2} + 3 = b\sqrt{2} + a$, the value of
 $(a+b)$ is
(a) -5 (b) 3 (c) -3 (d) 5
113. If $ax^2 + bx + c = a(x-p)^2$, then the rela-
tion among a, b, c would be
(a) $abc = 1$ (b) $b^2 = ac$
(c) $b^2 = 4ac$ (d) $2b = a+c$
114. If $a+b+c+d = 1$, then the maximum
value of
 $(1+a)(1+b)(1+c)(1+d)$ is
(a) 1 (b) $\left(\frac{1}{2}\right)^3$ (c) $\left(\frac{3}{4}\right)^3$ (d) $\left(\frac{5}{4}\right)^4$
115. If $a^2 + b^2 + c^2 + 3 = 2(a+b+c)$ then the
value of
 $(a+b+c)$ is
(a) 2 (b) 3 (c) 4 (d) 5
116. If $x - \frac{1}{x} = 5$, then $x^2 + \frac{1}{x^2}$ is :
(a) 5 (b) 25 (c) 27 (d) 23
117. If $x = 3 + 2\sqrt{2}$, then the value of
 $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$ is:
(a) 1 (b) 2
(c) $2\sqrt{2}$ (d) $3\sqrt{3}$

118. If $a + b + c = 0$, then the value of

$$\frac{a^2 + b^2 + c^2}{a^2 - bc} \text{ is}$$

- (a) 0 (b) 1 (c) 2 (d) 3

119. If $n = 7 + 4\sqrt{3}$, then the value of

$$\left(\sqrt{n} + \frac{1}{\sqrt{n}}\right) \text{ is:}$$

- (a) $2\sqrt{3}$ (b) 4
(c) -4 (d) $-2\sqrt{3}$

120. If $x = \sqrt{3} + \sqrt{2}$, then the value of

$$\left(x + \frac{1}{x}\right) \text{ is}$$

- (a) $2\sqrt{2}$ (b) $2\sqrt{3}$
(c) 2 (d) 3

121. If $p + q = 10$ and $pq = 5$, then the

numerical value of $\frac{p}{q} + \frac{q}{p}$ will be

- (a) 16 (b) 20 (c) 22 (d) 18

122. If $x = 3 + 2\sqrt{2}$ and $xy = 1$, then the value

$$\text{of } \frac{x^2 + 3xy + y^2}{x^2 - 3xy + y^2} \text{ is}$$

- (a) $\frac{30}{31}$ (b) $\frac{70}{31}$ (c) $\frac{35}{31}$ (d) $\frac{37}{31}$

123. If $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b}$, then

$$\text{अदि } \frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b} \text{ है, तो :}$$

$$(a) \frac{x-y}{b-a} = \frac{y-z}{c-b} = \frac{z-x}{a-c}$$

$$(b) \frac{x}{a} = \frac{y}{b} = \frac{z}{c}$$

$$(c) \frac{x-y}{c} = \frac{y-z}{b} = \frac{z-x}{a}$$

(d) None of the above is true

124. If $x - y = 2$, $xy = 24$, then the value of $(x^2 + y^2)$ is

- (a) 25 (b) 36 (c) 63 (d) 52

125. If the expression $\frac{x^2}{y^2} + tx + \frac{y^2}{4}$ is a per-

fect square, then the values of t is

- (a) ± 1 (b) ± 2 (c) 0 (d) ± 3

126. If $a = x + y$, $b = x - y$, $c = x + 2y$, then $a^2 + b^2 + c^2 - ab - bc - ca$ is

- (a) $4y^2$ (b) $5y^2$ (c) $6y^2$ (d) $7y^2$

127. If $x + \frac{1}{x} = 2$, $x \neq 0$ then value of

$$x^2 + \frac{1}{x^3} \text{ is equal to}$$

- (a) 1 (b) 2 (c) 3 (d) 4

128. If $\frac{a}{b} + \frac{b}{a} = 1$, $a \neq 0$, $b \neq 0$ the value of $a^3 + b^3$ is

- (a) 0 (b) 1 (c) -1 (d) 2

129. If $\left(x + \frac{1}{x}\right)^2 = 3$ then the value of $(x^{72} + x^{66} + x^{54} + x^{24} + x^6 + 1)$

- (a) 0 (b) 1 (c) 84 (d) 206

130. If $a + \frac{1}{a} = \sqrt{3}$, then the value of

$$a^6 - \frac{1}{a^6} + 2 \text{ will be}$$

- (a) 1 (b) 2 (c) $3\sqrt{3}$ (d) 5

131. If $x^3 + y^3 = 35$ and $x + y = 5$, then the

value of $\frac{1}{x} + \frac{1}{y}$ will be :

- (a) $\frac{1}{3}$ (b) $\frac{5}{6}$ (c) 6 (d) $\frac{2}{3}$

132. If $a^3 - b^3 = 56$ and $a - b = 2$ then value of $a^2 + b^2$ will be :

- (a) 48 (b) 20 (c) 22 (d) 5

133. If $(a^2 + b^2)^3 = (a^3 + b^3)^2$ then $\frac{a}{b} + \frac{b}{a} = ?$

- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $-\frac{1}{3}$ (d) $-\frac{2}{3}$

134. If $x + \frac{1}{x} = 5$, then the value of

$$\frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$$

- (a) $\frac{43}{23}$ (b) $\frac{47}{21}$ (c) $\frac{41}{23}$ (d) $\frac{45}{21}$

135. If x is real, $x + \frac{1}{x} \neq 0$ and

$$x^3 + \frac{1}{x^3} = 0, \text{ then the value of}$$

$$\left(x + \frac{1}{x}\right)^4 \text{ is}$$

- (a) 4 (b) 9 (c) 16 (d) 25

136. If $x + \frac{1}{x} = 3$, then the value of

$$\left(x^5 + \frac{1}{x^5}\right) \text{ is}$$

- (a) 322 (b) 126 (c) 123 (d) 113

137. If $m^4 + \frac{1}{m^4} = 119$, then $m - \frac{1}{m} = ?$

- (a) ± 3 (b) 4 (c) ± 2 (d) ± 1

138. If $x + y + z = 6$, then the value of $(x-1)^3 + (y-2)^3 + (z-3)^3$ is

- (a) $3(x-1)(y+2)(z-3)$
(b) $3(x+1)(y-2)(z-3)$
(c) $3(x-1)(y-2)(z+3)$
(d) $3(x-1)(y-2)(z-3)$

139. If $a + b + c = 6$, $a^2 + b^2 + c^2 = 14$ and $a^3 + b^3 + c^3 = 36$, then the value of abc is

- (a) 3 (b) 6 (c) 9 (d) 12

140. If $a + b = 1$ and $a^3 + b^3 + 3ab = k$, then the value of k is

- (a) 1 (b) 3 (c) 5 (d) 7

141. If $a = 34$, $b = c = 33$, then the value of $a^3 + b^3 + c^3 - 3abc$ is

- (a) 0 (b) 111 (c) 50 (d) 100

142. If $(2^x)(2^y) = 8$ and $(9^x)(3^y) = 81$, then (x, y) is:

- (a) (1,2) (b) (2,1) (c) (1,1) (d) (2,2)

143. The expression $x^4 - 2x^2 + k$ will be a perfect square when the value of k is

- (a) 2 (b) 1 (c) -1 (d) -2

$$144. \frac{\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} - 3 \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} - \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5} - \frac{1}{3} \cdot \frac{1}{5} \cdot \frac{1}{4} - \frac{1}{4} \cdot \frac{1}{5} \cdot \frac{1}{3}}{\frac{1}{3} \cdot \frac{1}{3} + \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{1}{5} - \left(\frac{1}{3} \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{5} + \frac{1}{4} \cdot \frac{1}{5}\right)}$$

- (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{47}{60}$ (d) $\frac{49}{60}$

145. If $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$, then the

value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is

- (a) 1 (b) 2 (c) 3 (d) 4

146. If a, b, c are real numbers and $a^2 + b^2 + c^2 = 2(a-b-c) - 3$ then the value of $2a - 3b + 4c$ is

- (a) -1 (b) 9 (c) 1 (d) 2

147. The value of the expression

$$\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} + \frac{(c-a)^2}{(a-b)(b-a)}$$

- (a) 0 (b) 3 (c) $\frac{1}{3}$ (d) 2

148. If $(x-3)^2 + (y-5)^2 + (z-4)^2 = 0$ then the value of

$$\frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{16} \text{ is}$$

- (a) 12 (b) 9 (c) 3 (d) 1

149. x varies inversely as square of y . Given that $y = 2$ for $x = 1$, the value of x for $y = 6$ will be equal to

- (a) 3 (b) 9 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$

150. If $x^2 - y^2 = 80$ and $x - y = 8$, then the average of x and y is

- (a) 2 (b) 3 (c) 4 (d) 5

151. The third proportional to $\left(\frac{x}{y} + \frac{y}{x}\right)$ and

$$\sqrt{x^2 + y^2} \text{ is}$$

- (a) xy (b) $\sqrt{xy}$
(c) $\sqrt[3]{xy}$ (d) $\sqrt[4]{xy}$

152. The value of $\frac{4+3\sqrt{3}}{7+4\sqrt{3}}$ is
 (a) $5\sqrt{3} - 8$ (b) $5\sqrt{3} + 8$
 (c) $8\sqrt{3} + 5$ (d) $8\sqrt{3} - 5$
153. If $x = \frac{4\sqrt{15}}{\sqrt{5} + \sqrt{3}}$, the value of $\frac{x + \sqrt{20}}{x - \sqrt{20}} + \frac{x + \sqrt{12}}{x - \sqrt{12}}$ is
 (a) 1 (b) 2 (c) $\sqrt{3}$ (d) $\sqrt{5}$
154. If $x = 5 - \sqrt{21}$, then the value of $\frac{\sqrt{x}}{\sqrt{32-2x} - \sqrt{21}}$ is
 (a) $\frac{1}{\sqrt{2}}(\sqrt{3} - \sqrt{7})$ (b) $\frac{1}{\sqrt{2}}(\sqrt{7} - \sqrt{3})$
 (c) $\frac{1}{\sqrt{2}}(\sqrt{7} + \sqrt{3})$ (d) $\frac{1}{\sqrt{2}}(7 + \sqrt{3})$
155. The value of $(x^{a+b})^{a-b} (x^{b+c})^{b-c} (x^{c+a})^{c-a}$
 (a) 1 (b) 2 (c) -1 (d) 0
156. If $\frac{x}{a} = \frac{1}{a} - \frac{1}{x}$, then the value of $x - x^2$ is :
 (a) -a (b) $\frac{1}{a}$ (c) a (d) $-\frac{1}{a}$
157. If $x + \frac{1}{x} = 99$, find the value of $\frac{100x}{2x^2 + 102x + 2}$
 (a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$
158. If $\frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0$ then the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is
 (a) 9 (b) 3 (c) 4 (d) 6
159. If $\frac{xy}{x+y} = a$, $\frac{xz}{x+z} = b$ and $\frac{yz}{y+z} = c$, where a, b, c are all non-zero numbers, then x equals to
 (a) $\frac{2abc}{ab+bc-ac}$ (b) $\frac{2abc}{ab+ac-bc}$
 (c) $\frac{2abc}{ac+bc-ab}$ (d) $\frac{2abc}{ab+bc+ac}$
160. If x and y are positive real numbers and $xy = 8$, then the minimum value of $2x + y$ is
 (a) 9 (b) 17 (c) 10 (d) 8
161. If the expression $x^2 + x + 1$ is written in the form $\left(x + \frac{1}{2}\right)^2 + q^2$, then the possible values of q are
 (a) $\pm \frac{1}{3}$ (b) $\pm \frac{\sqrt{3}}{2}$
 (c) $\pm \frac{2}{\sqrt{3}}$ (d) $\pm \frac{1}{2}$
162. If $a^2 - 4a - 1 = 0$, then value of $a^2 + \frac{1}{a^2} + 3a - \frac{3}{a}$ is
 (a) 25 (b) 30 (c) 35 (d) 40
163. One of the factors of the expression $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ is :
 (a) $4x + \sqrt{3}$ (b) $4x + 3$
 (c) $4x - 3$ (d) $4x - \sqrt{3}$
164. If $\sqrt{x} = \sqrt{3} - \sqrt{5}$, then the value of $x^2 - 16x + 6$ is
 (a) 0 (b) -2 (c) 2 (d) 4
165. If $x + y + z = 0$, then $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} = ?$
 (a) $(xyz)^2$ (b) $x^2 + y^2 + z^2$
 (c) 9 (d) 3
166. If यदि $a + b + c = 0$, then the value of $\left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right) \left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right)$
 (a) 8 (b) -3 (c) 9 (d) 0
167. If a, b, c are non-zero $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$, then the value of abc is :
 (a) -1 (b) 3 (c) -3 (d) 1
168. If $a + b + c = 2s$, then $\frac{(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2}{a^2 + b^2 + c^2}$
 (a) $a^2 + b^2 + c^2$ (b) 0
 (c) 1 (d) 2
169. If $x = 3 + 2\sqrt{2}$, the value of $x^2 + \frac{1}{x^2}$ is
 (a) 36 (b) 30 (c) 32 (d) 34
170. If $x\left(3 - \frac{2}{x}\right) = \frac{3}{x}$, then the value of $x^2 + \frac{1}{x^2}$ is
 (a) $2\frac{1}{9}$ (b) $2\frac{4}{9}$ (c) $3\frac{1}{9}$ (d) $3\frac{4}{9}$
171. If $x^2 - 3x + 1 = 0$, then the value of $x^2 + x + \frac{1}{x} + \frac{1}{x^2}$ is
 (a) 10 (b) 2 (c) 6 (d) 8
172. If $a^2 + b^2 = 5ab$, the value of $\left(\frac{a^2}{b^2} + \frac{b^2}{a^2}\right)$ is :
 (a) 32 (b) 16 (c) 23 (d) -23
173. If यदि $xy + yz + zx = 0$, then तो $\left(\frac{1}{x^2 - yz} + \frac{1}{y^2 - zx} + \frac{1}{z^2 - xy}\right) (x, y, z \neq 0)$
 (a) 3 (b) 1
 (c) $x + y + z$ (d) 0
174. If $a + b + c = 9$ (where a, b, c are real numbers), then the minimum value of $a^2 + b^2 + c^2$ is
 (a) 100 (b) 9 (c) 27 (d) 81
175. If $a^2 + b^2 + 4c^2 = 2(a + b - 2c) - 3$ and a, b, c are real, then the value of $(a^2 + b^2 + c^2)$ is
 (a) 3 (b) $3\frac{1}{4}$ (c) 2 (d) $2\frac{1}{4}$
176. Number of solutions of the two equations $4x - y = 2$ and $2x - 8y + 4 = 0$ is
 (a) zero (b) one
 (c) two (d) infinitely many
177. If $\frac{a}{b} = \frac{4}{5}$ and $\frac{b}{c} = \frac{15}{16}$, then $\frac{18c^2 - 7a^2}{45c^2 + 20a^2}$ is equal to
 (a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$
178. If $x \neq 0, y \neq 0$ and $z \neq 0$ and $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$ then the relation among x, y, z is
 (a) $x + y + z = 0$
 (b) $x + y = z$
 (c) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$
 (d) $x = y = z$

179. If $x = 3t$, $y = \frac{1}{2}(t+1)$, then the value of t for which $x = 2y$ is

- (a) 1 (b) $\frac{1}{2}$ (c) -1 (d) $\frac{2}{3}$

180. If $x^2 + \frac{1}{5}x + a^2$ is a perfect square, then a is

- (a) $\frac{1}{100}$ (b) $\pm \frac{1}{10}$ (c) $\frac{1}{10}$ (d) $-\frac{1}{10}$

181. Find the value of x for which the expression $2-3x-4x^2$ has the greatest value.

- (a) $-\frac{41}{16}$ (b) $\frac{3}{8}$ (c) $-\frac{3}{8}$ (d) $\frac{41}{16}$

182. The expression $x^4 - 2x^2 + k$ will be a perfect square if the value of k is

- (a) 1 (b) 0 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

183. If $\frac{5x}{2x^2+5x+1} = \frac{1}{3}$, then the value of

$$\left(x + \frac{1}{2x}\right)$$

- (a) 15 (b) 10 (c) 20 (d) 5

184. If $xy(x+y) = 1$, then the value of

$$\frac{1}{x^3y^3} - x^3 - y^3$$
 is:

- (a) 0 (b) 1 (c) 3 (d) -2

185. If $x > 1$ and $x^2 + \frac{1}{x^2} = 83$ then

$$x^3 - \frac{1}{x^3}$$
 is

- (a) 764 (b) 750 (c) 756 (d) 760

186. If $\left(a + \frac{1}{a}\right)^2 = 3$, then $a^3 + \frac{1}{a^3} = ?$

- (a) $2\sqrt{3}$ (b) 2 (c) $3\sqrt{3}$ (d) 0

187. If $\frac{x}{x^2-2x+1} = \frac{1}{3}$, then the value of

$$x^3 + \frac{1}{x^3}$$
 is:

- (a) 64 (b) 110 (c) 81 (d) 124

188. If $\left(x + \frac{1}{x}\right) = 4$, then the value of

$$x^4 + \frac{1}{x^4}$$
 is:

- (a) 64 (b) 194 (c) 81 (d) 124

189. If $x + y + z = 6$ and $x^2 + y^2 + z^2 = 20$ then the value of $x^3 + y^3 + z^3 - 3xyz$ is

- (a) 64 (b) 70 (c) 72 (d) 76

190. If $x = 1 - \sqrt{2}$, the value of $\left(x - \frac{1}{x}\right)^3$ is:

- (a) -8 (b) 8 (c) $2\sqrt{2}$ (d) 1

191. If $x = a - b$, $y = b - c$, $z = c - a$, then the numerical value of the algebraic expression $x^3 + y^3 + z^3 - 3xyz$ will be

- (a) $a + b + c$ (b) 0 (c) $4(a + b + c)$ (d) $3abc$

192. If $x = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$ and $y = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$, then the value of $x^3 + y^3$ is:

- (a) 950 (b) 730 (c) 650 (d) 970

193. If $2x + \frac{2}{x} = 3$, then the value of

$$x^3 + \frac{1}{x^3} + 2$$
 is

- (a) $-\frac{9}{8}$ (b) $-\frac{25}{8}$ (c) $\frac{7}{8}$ (d) 11

194. If $a + b + c = 15$ and $a^2 + b^2 + c^2 = 83$ then the value of $a^3 + b^3 + c^3 - 3abc$

- (a) 200 (b) 180 (c) 190 (d) 210

195. If $x + \frac{1}{x+1} = 1$, then $(x+1)^5 + \frac{1}{(x+1)^5}$

equals

- (a) 1 (b) 2 (c) 4 (d) 8

196. If $\frac{1}{a} + \frac{1}{b} = \frac{1}{a+b}$, then the value of

$$a^3 - b^3$$
 is

- (a) 0 (b) -1 (c) 1 (d) 2

197. If $a + b + c = 0$, then $a^3 + b^3 + c^3$ is equal to

- (a) $a + b + c$ (b) abc (c) $2abc$ (d) $3abc$

198. If $x = y = 333$ and $z = 334$, then the value of $x^3 + y^3 + z^3 - 3xyz$ is

- (a) 0 (b) 667 (c) 1000 (d) 2334

199. Out of the given responses one of the factors of

$$(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$$
 is

- (a) $(a + b)(a - b)$ (b) $(a + b)(a + b)$ (c) $(a - b)(a - b)$ (d) $(b - c)(b - c)$

200. If $a = \frac{b^2}{b-a}$ then the value of $a^3 + b^3$ is

- (a) $6ab$ (b) 0 (c) 1 (d) 2

201. If $p - 2q = 4$, then the value of $p^3 - 8q^3 - 24pq$ is

- (a) 2 (b) 0 (c) 3 (d) -1

202. If $x = -1$, then the value of

$$\frac{1}{x^{99}} + \frac{1}{x^{98}} + \frac{1}{x^{97}} + \frac{1}{x^{96}} + \frac{1}{x^{95}} + \frac{1}{x^{94}} + \frac{1}{x} - 1$$

- (a) 1 (b) 0 (c) -2 (d) -1

203. If $\frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$ and a, b, c are rational numbers then $a + b + c$ is equal to

- (a) 0 (b) 1 (c) 2 (d) 3

204. If $x = \sqrt[3]{2 + \sqrt{3}}$, then the value of

$$x^3 + \frac{1}{x^3}$$
 is

- (a) 8 (b) 9 (c) 2 (d) 4

205. If $x = \sqrt[3]{5} + 2$, then the value of $x^3 - 6x^2 + 12x - 13$

- (a) -1 (b) 1 (c) 2 (d) 0

206. The simplest form of the expression

$$\frac{p^2 - p}{2p^3 + p^2} + \frac{p^2 - 1}{p^2 + 3p} + \frac{p^2}{p + 1}$$

- (a) $2p^2$ (b) $\frac{1}{2p^2}$

- (c) $p + 3$ (d) $\frac{1}{p + 3}$

207. If $x + \frac{1}{x} = 2$, then the value of

$$\left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right)$$
 is

- (a) 20 (b) 4 (c) 8 (d) 16

208. If a, b, c be all positive integers then the least positive value of $a^3 + b^3 + c^3 - 3abc$ is.

- (a) 0 (b) 2 (c) 4 (d) 3

209. When $f(x) = 12x^3 - 13x^2 - 5x + 7$ is divided by $(3x + 2)$, then the remainder is

- (a) 2 (b) 0 (c) -1 (d) 1

210. If the equation $2x^2 - 7x + 12 = 0$ has two

roots α and β , then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

is

- (a) $\frac{7}{2}$ (b) $\frac{1}{24}$ (c) $\frac{7}{24}$ (d) $\frac{97}{24}$

211. If $x^3 + \frac{3}{x} = 4$ ($a^3 + b^3$) and $3x + \frac{1}{x^3} = 4$ ($a^3 + b^3$), then $a^2 - b^2$ is equal to

- (a) 4 (b) 0 (c) 1 (d) 2

212. The term to be added to $121a^2 + 64b^2$ to make a perfect square is

- (a) $176ab$ (b) $276a^2b$ (c) $178ab$ (d) $188b^2a$

213. If $a = 2 + \sqrt{3}$, then the value of

$$\left(a^2 + \frac{1}{a^2}\right)$$

- (a) 12 (b) 14 (c) 16 (d) 10

214. For what value(s) of k the expression

$$p + \frac{1}{4} + \sqrt{p} + k^2 \text{ is perfect square ?}$$

- (a) 0 (b) $\pm \frac{1}{4}$ (c) $\pm \frac{1}{8}$ (d) $\pm \frac{1}{2}$

215. The reciprocal of $x + \frac{1}{x}$ is

- (a) $\frac{x}{x^2 + 1}$ (b) $\frac{x}{x + 1}$
(c) $x - \frac{1}{x}$ (d) $\frac{1}{x} + x$

216. If a, b, c are positive and $a+b+c=1$,

then the least value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ is

- (a) 9 (b) 5 (c) 3 (d) 1

217. If $a(2 + \sqrt{3}) = b(2 - \sqrt{3}) = 1$, then the value of

$$\frac{1}{a^2 + 1} + \frac{1}{b^2 + 1}$$

- (a) -1 (b) 1 (c) 4 (d) 9

218. If $(2 + \sqrt{3})a = (2 - \sqrt{3})b = 1$ then the

value of $\frac{1}{a} + \frac{1}{b}$ is

- (a) 1 (b) 2 (c) $2\sqrt{3}$ (d) 4

219. If $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}$ ($a \neq b \neq c$), then the value of abc is

- (a) ± 1 (b) ± 2 (c) 0 (d) $\pm \frac{1}{2}$

220. If $(x-2)$ is a factor of $x^2 + 3Qx - 2Q$, then the value of Q is

- (a) 2 (b) -2 (c) 1 (d) -1

221. If $a + b = 12$, $ab = 22$, then $(a^2 + b^2)$ is equal to

- (a) 188 (b) 144 (c) 34 (d) 100

222. If $x = \sqrt{3} - \frac{1}{\sqrt{3}}$ and $y = \sqrt{3} + \frac{1}{\sqrt{3}}$,

then the value of $\frac{x^2}{y} + \frac{y^2}{x}$ is

- (a) $\sqrt{3}$ (b) $3\sqrt{3}$
(c) $16\sqrt{3}$ (d) $2\sqrt{3}$

223. If $x^2 + ax + b$ is a perfect square, then which one of the following relations between a and b is true

- (a) $a^2 = b$ (b) $a^2 = 4b$
(c) $b = 4a$ (d) $b^2 = a$

224. If $a + b + c + d = 4$, then find the

$$\text{value of } \frac{1}{(1-a)(1-b)(1-c)} +$$

$$\frac{1}{(1-b)(1-c)(1-d)} +$$

$$\frac{1}{(1-c)(1-d)(1-a)} +$$

$$\frac{1}{(1-d)(1-a)(1-b)}$$

- (a) 0 (b) 5 (c) 1 (d) 4

225. If $x - \frac{1}{x} = 1$, then the value of

$$\frac{x^4 - \frac{1}{x^2}}{3x^2 + 5x - 3}$$

- (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 0

226. If $x + y = 15$, then $(x-10)^3 + (y-5)^3$ is

- (a) 25 (b) 125
(c) 625 (d) 0

227. If $x^2 + \frac{1}{x^2} = 66$, then the value of

$$\frac{x^2 - 1 + 2x}{x} = ?$$

- (a) ± 8 (b) 10, -6
(c) 6, -10 (d) ± 4

228. If $a^2 + a + 1 = 0$, then the value of a^9 is

- (a) 2 (b) 3 (c) 1 (d) 0

229. If $x + \frac{2}{x} = 1$, then the value of

$$\frac{x^2 + x + 2}{x^2(1-x)} \text{ is}$$

- (a) 1 (b) -1 (d) 2 (d) -2

230. If $x = -2k$ and $y = 1 - 3k$, then for what value of k, will be $x = y$?

- (a) 0 (b) 1 (c) -1 (d) 2

231. Find the value of

$$\sqrt{(x^2 + y^2 + z)(x + y - 3z)} + \sqrt[3]{xy^3z^2}$$

when $x = +1$, $y = -3$, $z = -1$,

- (a) 1 (b) 0 (c) -1 (d) $\frac{1}{2}$

232. If $x + \frac{1}{x} = 2$, then the value of

$$\left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) \text{ is}$$

- (a) 20 (b) 4 (c) 8 (d) 16

233. If $x + \frac{1}{x} = 5$, then $x^6 + \frac{1}{x^6}$ is

- (a) 12098 (b) 12048
(c) 14062 (d) 12092

234. If $x^2 - 3x + 1 = 0$, then the value of

$$\frac{x^6 + x^4 + x^2 + 1}{x^3} \text{ will be}$$

- (a) 18 (b) 15 (c) 21 (d) 30

235. If x is a rational number and

$$\frac{(x+1)^3 - (x-1)^3}{(x+1)^2 - (x-1)^2} = 2, \text{ then the sum of}$$

numerator and denominator of x is:

- (a) 3 (b) 4 (c) 5 (d) 7

236. If $x = \sqrt{5} + 2$, then the value

$$\frac{2x^2 - 3x - 2}{3x^2 - 4x - 3} \text{ is equal to}$$

- (a) 0.1785 (b) 0.525
(c) 0.625 (d) 0.785

237. If $a = 2.234$, $b = 3.121$ and $c = -5.355$, then the value of $a^3 + b^3 + c^3 - 3abc$ is

- (a) -1 (b) 0 (c) 1 (d) 2

238. If $x^2 + y^2 + 1 = 2x$, then the value of $x^3 + y^5$ is

- (a) 2 (b) 0 (c) -1 (d) 1

239. If $3(a^2 + b^2 + c^2) = (a + b + c)^2$ then the relation between a, b and c is

- (a) $a = b = c$ (b) $a = b \neq c$
(c) $a < b < c$ (d) $a > b > c$

240. If $x(x-3) = -1$, then the value of $x^3(x^3 - 18)$ is

- (a) -1 (b) 2 (c) 1 (d) 0

241. The factors of $(a^2 + 4b^2 + 4ab - 4a - 8b - 8)$ are

- (a) $(a - 2b - 4)(a - 2b + 2)$
(b) $(a - b - 2)(a + 2b + 2)$
(c) $(a + 2b - 4)(a + 2b + 2)$
(d) $(a + 2b - 4)(a - 2b + 2)$

242. The value of

$$\frac{1}{a^2 + ax + x^2} - \frac{1}{a^2 - ax + x^2} + \frac{2ax}{a^4 + a^2x^2 + x^4}$$

is

- (a) 2 (b) 1 (c) -1 (d) 0

243. If $x = 11$, then the value of $x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$ is

- (a) 5 (b) 10 (c) 15 (d) 20

244. If $p = 99$, then the value of $p(p^2 + 3p + 3)$ is

- (a) 10000000 (b) 999000
(c) 99999 (d) 990000

245. Which one is not an example of an equality relation of two expressions in x:

- (a) $(x+3)^2 = x^2 + 6x + 9$
(b) $(x+2y)^3 = x^3 + 8y^3 + 6xy(x+2y)$
(c) $(x+2)^2 = x^2 + 2x + 4$
(d) $(x+3)(x-3) = x^2 - 9$

246. If $\left(a + \frac{1}{a}\right)^2 = 3$, then the value of $a^3 + \frac{1}{a^3}$ is
(a) 0 (b) 1 (c) 2 (d) 6
247. If $a + \frac{1}{a} = \sqrt{3}$, then the value of $a^{18} + a^{12} + a^6 + 1$ is
(a) 0 (b) 1 (c) 2 (d) 6
248. If $x = 997$, $y = 998$ and $z = 999$ then the value of $x^2 + y^2 + z^2 - xy - yz - zx$ is
(a) 0 (b) 1 (c) -1 (d) 3
249. If $x + \frac{1}{x} = 3$, then the value of $\frac{3x^2 - 4x + 3}{x^2 - x + 1}$ is
(a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{5}{3}$
250. If $x = 3 + 2\sqrt{2}$, then $\frac{x^6 + x^4 + x^2 + 1}{x^3}$ is equal to
(a) 216 (b) 192 (c) 198 (d) 204
251. If $a + b + c = 0$, then the value of $(a + b - c)^2 + (b + c - a)^2 + (c + a - b)^2$ is
(a) 0 (b) $8abc$ (c) $4(a^2 + b^2 + c^2)$ (d) $4(ab + bc + ca)$
252. If $p^3 + 3p^2 + 3p = 7$, then the value of $p^2 + 2p$ is
(a) 4 (b) 3 (c) 5 (d) 6
253. If $x = 2015$, $y = 2014$ and $z = 2013$, then the value of $x^2 + y^2 + z^2 - xy - yz - zx$ is
(a) 3 (b) 4 (c) 6 (d) 2
254. If $3a^2 = b^2 \neq 0$, then the value of $\frac{(a+b)^3 - (a-b)^3}{(a+b)^2 + (a-b)^2}$ is
(a) $\frac{3b}{2}$ (b) b (c) $\frac{b}{2}$ (d) $\frac{2b}{3}$
255. If $x > 1$ and $x + \frac{1}{x} = 2\frac{1}{12}$, then the value of $x^4 - \frac{1}{x^4}$ is
(a) $\frac{58975}{20736}$ (b) $\frac{59825}{20736}$ (c) $\frac{57985}{20736}$ (d) $\frac{57895}{20736}$
256. The value of $\frac{4x^3 - x}{(2x+1)(6x-3)}$ when $x = 9999$ is
(a) 1111 (b) 2222 (c) 3333 (d) 6666
257. If $a^3 + b^3 = 9$ and $a + b = 3$, then the value of $\frac{1}{a} + \frac{1}{b}$ is
(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) -1
258. If $t^2 - 4t + 1 = 0$, then the value of $t^3 + \frac{1}{t^3}$ is:
(a) 44 (b) 48 (c) 52 (d) 64
259. If $\sqrt[3]{a} + \sqrt[3]{b} = \sqrt[3]{c}$, then the simplest value of $(a+b-c)^3 + 27abc$ is
(a) -1 (b) 3 (c) -3 (d) 0
260. If $4x + 5y = 83$ and $3x : 2y = 21 : 22$, then $(y-x)$ equals
(a) 3 (b) 4 (c) 7 (d) 11
261. If $x = \sqrt[3]{a + \sqrt{a^2 + b^3}} + \sqrt[3]{a - \sqrt{a^2 + b^3}}$, then $x^3 + 3bx$ is equal to
(a) 0 (b) a (c) $2a$ (d) 1
262. If $\frac{x^{24} + 1}{x^{12}} = 7$ then the value of $\frac{x^{72} + 1}{x^{36}}$ is
(a) 343 (b) 433 (c) 432 (d) 322
263. If $P = 99$ then the value of $P(P^2 + 3P + 3)$ is
(a) 989898 (b) 998889 (c) 988899 (d) 999999
264. If $x = 2$ then the value of $x^3 + 27x^2 + 243x + 631$ is
(a) 1321 (b) 1233 (c) 1231 (d) 1211
265. If $x^2 + y^2 + z^2 = 2(x + z - 1)$, then the value of $x^3 + y^3 + z^3$ is
(a) -1 (b) 2 (c) 0 (d) 1
266. If $x + \frac{1}{x} = 1$, then the value of $\frac{2}{x^2 - x + 2}$ is
(a) $\frac{2}{3}$ (b) 2 (c) 1 (d) 4
267. If $x = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$ and $y = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$, then the value of $\frac{x^2 + xy + y^2}{x^2 - xy + y^2}$ is
(a) $\frac{65}{63}$ (b) $\frac{67}{65}$ (c) $\frac{69}{67}$ (d) $\frac{63}{61}$
268. If $4a - \frac{4}{a} + 3 = 0$, then the value of $a^3 - \frac{1}{a^3} + 3 = ?$
(a) $\frac{7}{16}$ (b) $\frac{3}{16}$ (c) $\frac{21}{64}$ (d) $\frac{21}{16}$
269. If $x = z = 225$ and $y = 226$ then the value of $x^2 + y^3 + z^3 - 3xyz$ is
(a) 765 (b) 676 (c) 674 (d) 576
270. If $x^2 + x = 5$ then the value of $\frac{1}{(x+3)^3} + \frac{1}{(x+3)^3}$ is
(a) 140 (b) 110 (c) 130 (d) 120
271. If $m = -4$, $n = -2$, then the value of $m^3 - 3m^2 + 3m + 3n + 3n^2 + n^3$ is
(a) 124 (b) -124 (c) 126 (d) -126
272. $2x - ky + 7 = 0$ and $6x - 12y + 15 = 0$ has no solution for:
(a) $k = -4$ (b) $k = 4$ (c) $k = 1$ (d) $k = -1$
273. If $x = 332$, $y = 333$, $z = 335$, then the value of $x^3 + y^3 + z^3 - 3xyz$ is
(a) 7000 (b) 8000 (c) 9000 (d) 10000
274. If $2 + x\sqrt{3} = \frac{1}{2 + \sqrt{3}}$, then the simplest value of x is:
(a) 1 (b) -2 (c) 2 (d) -1
275. If $m - 5n = 2$, then the value of $(m^3 - 125n^3 - 30mn)$ is:
(a) 6 (b) 7 (c) 8 (d) 9
276. If $x = \sqrt[3]{a} \sqrt[3]{b} \sqrt[3]{a} \sqrt[3]{b} \dots \dots \dots \infty$, then the value of x is:
(a) $\sqrt[3]{ab^3}$ (b) $\sqrt[3]{a^5b}$ (c) $\sqrt[3]{a^3b}$ (d) $\sqrt[3]{a^5b}$
277. If $x + \frac{1}{x} = 2$, then the value of $x^{12} - \frac{1}{x^{12}}$ is:
(a) -4 (b) 4 (c) 2 (d) 0
278. If $x + \frac{1}{x} = 1$, then the value of $\frac{x^2 + 3x + 1}{x^2 + 7x + 1}$ is:
(a) $\frac{1}{2}$ (b) $\frac{3}{7}$ (c) 2 (d) 3
279. If $x + (1/x) = 2$, then the value of $x^7 + (1/x^5)$ is:
(a) 2^5 (b) 2^{12} (c) 2 (d) 2^7
280. The term, that should be added to $(4x^2 + 8x)$ so that resulting expression be a perfect square, is:
(a) $2x$ (b) 2 (c) 1 (d) 4
281. If $999x + 888y = 1332$ and $888x + 999y = 555$ Then the value of $x + y$ is?
(a) 888 (b) 1 (c) 555 (d) 999

282. If $x = \frac{1}{2+\sqrt{3}}$, $y = \frac{1}{2-\sqrt{3}}$, then the value of $8xy(x^2 + y^2)$ is
(a) 112 (b) 194 (c) 290 (d) 196

283. If $a = \frac{\sqrt{x+2} + \sqrt{x-2}}{\sqrt{x+2} - \sqrt{x-2}}$, then the value of $a^2 - ax$ is
(a) 2 (b) 1 (c) 0 (d) -1

284. If $a + b = 1$, find the value of $a^3 + b^3 - ab - (a^2 - b^2)^2$
(a) 0 (b) 1 (c) -1 (d) 2

285. If $a - \frac{1}{a-3} = 5$, then the value of $(a-3)^3 - \frac{1}{(a-3)^3}$
(a) 7 (b) 14 (c) 2 (d) 5

286. $(3x-2y) : (2x+3y) = 5 : 6$, then one of the value of $\left(\frac{\sqrt[3]{x} + \sqrt[3]{y}}{\sqrt[3]{x} - \sqrt[3]{y}}\right)^2$ is
(a) $\frac{1}{25}$ (b) 5 (c) $\frac{1}{5}$ (d) 25

287. If $x - \sqrt{3} - \sqrt{2} = 0$ and $y - \sqrt{3} + \sqrt{2} = 0$, then value of $(x^3 - 20\sqrt{2}) - (y^3 + 2\sqrt{2})$
(a) 2 (b) 3 (c) 1 (d) 0

288. $3(a^2+b^2+c^2) = (a+b+c)^2$, then the relation between a, b and c is
(a) $a = b \neq c$ (b) $a \neq b \neq c$
(c) $a \neq b = c$ (d) $a = b = c$

289. $x = a^{\frac{1}{2}} + a^{-\frac{1}{2}}$, $y = a^{\frac{1}{2}} - a^{-\frac{1}{2}}$, then value of $(x^4 - x^2y^2 - 1) + (y^4 - x^2y^2 + 1)$
(a) 13 (b) 12 (c) 14 (d) 16

290. If $m = \sqrt{5+\sqrt{5+\sqrt{5+\dots}}}$
 $n = \sqrt{5-\sqrt{5-\sqrt{5-\dots}}}$
then among the following the relation between m & n holds is
(a) $m-n+1=0$ (b) $m+n+1=0$
(c) $m+n-1=0$ (d) $m-n-1=0$

291. If $\frac{3-5x}{2x} + \frac{3-5y}{2y} + \frac{3-5z}{2z} = 0$, then the value of $\frac{2}{x} + \frac{2}{y} + \frac{2}{z}$ is
(a) 20 (b) 10 (c) 5 (d) 15

292. If $2s = a + b + c$, then the value of $s(s-c) + (s-a)(s-b)$ is
(a) ab (b) 0
(c) abc (d) $\frac{a+b+c}{2}$

293. If $p + m = 6$ and $p^3 + m^3 = 72$, then the value of pm is
(a) 6 (b) 9 (c) 12 (d) 8

294. When x^m is multiplied by x^n , product is 1. The relation between m and n is
(a) $mn = 1$ (b) $m + n = 1$
(c) $m = n$ (d) $m = -n$

295. If $\frac{2p}{p^2 - 2p + 1} = \frac{1}{4}$, then the value of $\left(p + \frac{1}{p}\right)$ is
(a) 7 (b) 1 (c) $\frac{2}{5}$ (d) 10

296. If $x = 2$, $y = 1$ and $z = -3$, then $x^3 + y^3 + z^3 - 3xyz$ is equal to
(a) 6 (b) 0 (c) 2 (d) 8

297. $(x^3 + y^6)(x^3 - y^6)$ is equal to
(a) $x^6 - y^{12}$ (b) $x^9 - y^{16}$
(c) $x^6 + y^{12}$ (d) $x^9 + y^{36}$

298. The sum of $\frac{1}{x+y}$ and $\frac{1}{x-y}$ is
(a) $\frac{2y}{x^2 - y^2}$ (b) $\frac{2x}{x^2 - y^2}$
(c) $\frac{-2y}{x^2 - y^2}$ (d) $\frac{2x}{y^2 - x^2}$

299. If $x + y = 2a$, then the value of $\frac{a}{x-a} + \frac{a}{y-a}$ is
(a) 0 (b) -1 (c) 1 (d) 2

300. For real a, b, c if $a^2 + b^2 + c^2 = ab + bc + ca$, the value of $\frac{a+c}{b}$ is :
(a) 2 (b) 1 (c) 0 (d) 3

301. If $p^3 - q^3 = (p-q)\{(p-q)^2 - xpq\}$, then find the value of x is:
(a) -1 (b) 3 (c) 1 (d) -3

302. If $x + y + z = 6$ and $xy + yz + zx = 10$, then the value of $x^3 + y^3 + z^3 - 3xyz$ is:
(a) 36 (b) 40 (c) 42 (d) 48

303. If $\frac{x+1}{x-1} = \frac{a}{b}$ and $\frac{1-y}{1+y} = \frac{b}{a}$, then the value of $\frac{x-y}{1+xy}$ is:
(a) $\frac{a^2 - b^2}{ab}$ (b) $\frac{a^2 + b^2}{2ab}$
(c) $\frac{a^2 - b^2}{2ab}$ (d) $\frac{2ab}{a^2 - b^2}$

304. If $a^2 + b^2 + c^2 - ab - bc - ca = 0$ then a : b : c is:
(a) 1 : 2 : 1 (b) 2 : 1 : 1
(c) 1 : 1 : 2 (d) 1 : 1 : 1

305. If $x = a(b-c)$, $y = b(c-a)$, $z = c(a-b)$ then the value of $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3$ is:
(a) 3 (b) 2 (c) 1 (d) 0

- (a) $\frac{xyz}{abc}$ (b) 0

- (c) $\frac{3xyz}{abc}$ (d) $\frac{2xyz}{abc}$

306. If $\frac{x}{y} = \frac{a+2}{a-2}$, then the value of $\frac{x^2 - y^2}{x^2 + y^2}$

- (a) $\frac{2a}{a^2+2}$ (b) $\frac{4a}{a^2+4}$

- (c) $\frac{2a}{a^2+4}$ (d) $\frac{4a}{a^2+2}$

307. If $\frac{a}{b} + \frac{b}{a} = 2$, then the value of a - b is:

- (a) 2 (b) -1 (c) 0 (d) 1

308. If $x(x+y+z)=20$, $y(x+y+z)=30$, & $z(x+y+z)=50$, then the value of $2(x+y+z)$ is:

- (a) 20 (b) 10 (c) 15 (d) 18

309. If $x+y=4$, $x^2+y^2=14$ and $x > y$. Then the correct value of x and y is:

- (a) $2 - \sqrt{2}$, $\sqrt{3}$ (b) 3, 1

- (c) $2 + \sqrt{3}$, $2 - \sqrt{3}$ (d) $2 + \sqrt{3}$, $2\sqrt{2}$

310. If for non-zero x, $x^2 - 4x - 1 = 0$ the value of $x^2 + \frac{1}{x^2}$ is:

- (a) 4 (b) 10 (c) 12 (d) 18

311. If $a = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

- and $b = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then $\frac{a^2}{b} + \frac{b^2}{a}$ value of :

- (a) 1030 (b) 970

- (c) 1025 (d) 930

312. If $(2a-1)^2 + (4b-3)^2 + (4c+5)^2 = 0$ then

- the value of $\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2}$ is:

- (a) $3\frac{3}{8}$ (b) $2\frac{3}{8}$ (c) 0 (d) $1\frac{3}{8}$

313. If $\left(a + \frac{1}{a}\right)^2 = 3$, then find the value

- of $a^{30} + a^{24} + a^{18} + a^{12} + a^6 + 1$

- (a) 0 (b) 27 (c) 1 (d) -1

314. If $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$, then the value of $a^3 - b^3$ is:

- (a) 3 (b) 2 (c) 1 (d) 0

ANSWER KEY

1. (d)	33. (c)	65. (b)	97. (a)	129.(a)	161.(b)	193.(c)	225.(b)	257.(b)	286.(d)
2. (d)	34. (c)	66. (b)	98. (c)	130.(b)	162.(b)	194.(b)	226.(d)	258.(c)	287.(d)
3. (c)	35. (c)	67. (d)	99. (c)	131.(b)	163.(d)	195.(b)	227.(b)	259.(d)	288.(d)
4. (c)	36. (a)	68. (a)	100.(d)	132.(b)	164.(c)	196.(a)	228.(c)	260.(b)	289.(d)
5. (c)	37. (d)	69. (d)	101.(b)	133.(b)	165.(d)	197.(d)	229.(a)	261.(c)	290.(d)
6. (d)	38. (b)	70. (d)	102.(a)	134.(a)	166.(c)	198.(c)	230.(b)	262.(d)	291.(b)
7. (d)	39. (d)	71. (a)	103.(c)	135.(b)	167.(a)	199.(a)	231.(b)	263.(d)	292.(a)
8. (a)	40. (c)	72. (a)	104.(c)	136.(c)	168.(c)	200.(b)	232.(b)	264.(b)	293.(d)
9. (c)	41. (b)	73. (b)	105.(d)	137.(a)	169.(d)	201.(a)	233.(a)	265.(b)	294.(d)
10. (c)	42. (a)	74. (c)	106.(c)	138.(d)	170.(b)	202.(c)	234.(c)	266.(b)	295.(d)
11. (c)	43. (c)	75. (a)	107.(d)	139.(b)	171.(a)	203.(a)	235.(b)	267.(d)	296.(b)
12. (a)	44. (b)	76. (b)	108.(c)	140.(a)	172.(c)	204.(d)	236.(c)	268.(c)	297.(a)
13. (b)	45. (d)	77. (c)	109.(c)	141.(d)	173.(d)	205.(d)	237.(b)	269.(b)	298.(b)
14. (a)	46. (d)	78. (b)	110.(a)	142.(a)	174.(c)	206.(b)	238.(d)	270.(b)	299.(a)
15. (a)	47. (c)	79. (b)	111.(b)	143.(b)	175.(d)	207.(b)	239.(a)	271.(d)	300.(a)
16. (c)	48. (b)	80. (a)	112.(d)	144.(c)	176.(b)	208.(a)	240.(a)	272.(b)	301.(d)
17. (a)	49. (c)	81. (b)	113.(c)	145.(d)	177.(d)	209.(d)	241.(a)	273.(a)	302.(a)
18. (c)	50. (c)	82. (d)	114.(d)	146.(c)	178.(d)	210.(b)	242.(d)	274.(d)	303.(d)
19. (d)	51. (d)	83. (d)	115.(b)	147.(b)	179.(b)	211.(c)	243.(b)	275.(c)	304.(d)
20. (b)	52. (c)	84. (c)	116.(c)	148.(c)	180.(c)	212.(a)	244.(c)	276.(d)	305.(c)
21. (c)	53. (b)	85. (c)	117.(b)	149.(d)	181.(d)	213.(b)	245.(c)	277.(d)	306.(b)
22. (d)	54. (a)	86. (a)	118.(c)	150.(d)	182.(a)	214.(a)	246.(a)	278.(a)	307.(c)
23. (b)	55. (c)	87. (d)	119.(b)	151.(a)	183.(d)	215.(a)	247.(a)	279.(c)	308.(a)
24. (c)	56. (a)	88. (b)	120.(b)	152.(a)	184.(c)	216.(a)	248.(d)	280.(d)	309.(c)
25. (b)	57. (c)	89. (c)	121.(d)	153.(b)	185.(c)	217.(b)	249.(c)	281.(b)	310.(d)
26. (c)	58. (a)	90. (c)	122.(d)	154.(b)	186.(d)	218.(d)	250.(d)	282.(a)	311.(b)
27. (d)	59. (a)	91. (a)	123.(a)	155.(a)	187.(b)	219.(a)	251.(c)	283.(d)	312.(c)
28. (b)	60. (a)	92. (b)	124.(a)	156.(c)	188.(b)	220.(d)	252.(b)	284.(a)	313.(a)
29. (d)	61. (b)	93. (a)	125.(a)	157.(c)	189.(c)	221.(d)	253.(a)	285.(b)	314.(d)
30. (d)	62. (a)	94. (c)	126.(d)	158.(c)	190.(d)	222.(b)	254.(a)		
31. (b)	63. (d)	95. (b)	127.(b)	159.(c)	191.(b)	223.(b)	255.(a)		
32. (c)	64. (b)	96. (a)	128.(a)	160.(d)	192.(d)	224.(a)	256.(c)		

SOLUTION

$$1. (d) \left(\frac{1}{x} + 1 \right) \left(1 + \frac{1}{x+1} \right) \left(1 + \frac{1}{x+2} \right) \left(1 + \frac{1}{x+3} \right)$$

Taking L.C.M of each term.

$$\Rightarrow \left(\frac{x+1}{x} \right) \left(\frac{x+1+1}{x+1} \right) \left(\frac{x+2+1}{x+2} \right) \left(\frac{x+3+1}{x+3} \right)$$

$$\Rightarrow \frac{1}{x} \times (x+4) \Rightarrow \frac{x+4}{x}$$

$$2. (d) x = 7 - 4\sqrt{3}$$

$$\frac{1}{x} \Rightarrow \frac{1}{7 - 4\sqrt{3}}$$

By rationalisation

$$\frac{1}{x} = \frac{1}{7 - 4\sqrt{3}} \times \frac{7 + 4\sqrt{3}}{7 + 4\sqrt{3}}$$

$$= \frac{7 + 4\sqrt{3}}{49 - 48} = 7 + 4\sqrt{3}$$

$$\therefore x + \frac{1}{x} = 7 - 4\sqrt{3} + 7 + 4\sqrt{3} = 14$$

$$3. (c) \frac{2a+b}{a+4b} = 3 \quad (\text{given})$$

$$2a + b = 3(a+4b)$$

$$2a + b = 3a + 12b$$

$$\Rightarrow -a = 11b$$

$$a = -11b$$

$$\therefore \frac{a+b}{a+2b}$$

$$\Rightarrow \frac{-11b+b}{-11b+2b}$$

$$= \frac{-10b}{-9b} = \frac{10}{9}$$

$$4. (c) A : B = \frac{1}{2} : \frac{3}{8}$$

$$\Rightarrow 8 : 6$$

$$\Rightarrow 4 : 3$$

$$\Rightarrow B : C \Rightarrow \frac{1}{3} : \frac{5}{9}$$

$$\Rightarrow 9 : 15 \Rightarrow 3 : 5$$

$$\Rightarrow C : D \Rightarrow \frac{5}{6} : \frac{3}{4}$$

$$\Rightarrow 20 : 18 \Rightarrow 10 : 9$$

$$A : B : C : D$$

$$4 : 3 : 5 : 9$$

$$8 : 6 : 10 : 9$$

$$5. (c) \frac{a}{3} = \frac{b}{4} = \frac{c}{7} = k$$

$$\therefore a = 3k$$

$$b = 4k$$

$$c = 7k$$

$$\therefore \frac{a+b+c}{c} = \frac{3k+4k+7k}{7k} = 2$$

$$6. (d) \frac{144}{0.144} = \frac{14.4}{x}$$

$$\Rightarrow \frac{144 \times 1000}{144} = \frac{144}{x \times 10}$$

$$\Rightarrow 1000 = \frac{144}{10x}$$

$$\Rightarrow x = \frac{144}{1000 \times 10}$$

$$x = \frac{144}{10000} = 0.0144$$

$$7. (d) 1 < x < 2$$

$$\sqrt{(x-1)^2} + \sqrt{(x-3)^2}$$

(square root cancel with square)

$$\therefore x-1 + x-3 = 2x-4$$

$$8. (c) 10^{0.48} = x$$

$$10^{0.70} = y$$

$$\text{and } x^z = y^2$$

$$\therefore (10^{0.48})^z = (10^{0.70})^2$$

$$\Rightarrow 10^{0.48z} = 10^{1.40} \text{ (If } a^x = a^y, \text{ if base equal power are equal: } (x=y))$$

$$\therefore 0.48z = 1.40$$

$$z = \frac{140}{48} = \frac{35}{12} = 2.9$$

$$9. (c) 47.2506 = 4A + 7B + 2C + \frac{5}{D} + 6E$$

$$47.2506 = 4 \times 10 + 7 \times 1 + 2 \times 0.1000 + 5 \times 0.0100 + 0 + 6 \times 0.0001$$

$$\therefore A = 10 \quad B = 1 \quad C = 0.1000$$

$$D = \frac{1}{\frac{1}{100}} = 100, \quad E = 0.0001$$

$$\therefore 5A + 3B + 6C + D + 3E$$

$$= 5 \times 10 + 3 \times 1 + 6 \times 0.1 + 100 + 3 \times 0.0001$$

$$= 50 + 3 + 0.6 + 100 + 0.0003$$

$$= 153.6003$$

$$10. (c) 3^{x+3} + 7 = 250$$

$$3^{x+3} = 250 - 7$$

$$3^{x+3} = 243$$

$$3^{x+3} = 3^5$$

$$x+3 = 5$$

$$x = 2$$

$$11. (c) \frac{1}{4} \times \frac{2}{6} \times \frac{3}{8} \times \frac{4}{10} \times \frac{5}{12} \times \dots \times \frac{31}{64}$$

$$= \frac{1}{2^x}$$

$$\Rightarrow \left(\frac{1}{2}\right)^{30} \times \left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^{30+5} = \frac{1}{2^x}$$

$$\text{or } \frac{1}{2^{36}} = \frac{1}{2^x}$$

$$\therefore x = 36$$

$$12. (a) x = \frac{\sqrt{3}+1}{\sqrt{3}-1} \text{ and } y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$\Rightarrow \therefore x = \frac{1}{y}$$

$$x = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}+1)^2}{3-1}$$

$$= \frac{3+1+2\sqrt{3}}{2} = \frac{4+2\sqrt{3}}{2}$$

$$= (2+\sqrt{3})$$

$$x^2 = (2+\sqrt{3})^2 = 4+3+4\sqrt{3}$$

$$= 7+4\sqrt{3}$$

$$y^2 = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$y^2 = \frac{7-4\sqrt{3}}{49-48} = \frac{7-4\sqrt{3}}{1}$$

$$= 7-4\sqrt{3}$$

$$\therefore x^2 + y^2 = 7+4\sqrt{3} + 7-4\sqrt{3}$$

$$= 14$$

Alternate:-

$$x^2 + y^2 = x^2 + \frac{1}{x^2}$$

$$= \left(x + \frac{1}{x}\right)^2 - 2 \left(\because x = \frac{1}{y}\right)$$

$$= \left(\frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2 - 2$$

$$= \left[\frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{3-1}\right]^2 - 2$$

$$\therefore (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$= \left[\frac{2\left((\sqrt{3})^2 + (1)^2\right)}{2}\right]^2 - 2$$

$$= (3+1)^2 - 2 = 16 - 2 = 14$$

$$13. (b) x = 3 + 2\sqrt{2}$$

$$x = 2 + 1 + 2\sqrt{2} = (\sqrt{2}+1)^2$$

$$\sqrt{x} = \sqrt{2} + 1$$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}+1}$$

$$\Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

(हर का परिमेयकरण करने पर)

$$= \frac{\sqrt{2}-1}{1} = \sqrt{2}-1$$

$$\therefore \sqrt{x} - \frac{1}{\sqrt{x}}$$

$$= \sqrt{2} + 1 - (\sqrt{2}-1)$$

$$= \sqrt{2} + 1 - \sqrt{2} + 1 = 2$$

$$14. (a) P = 999$$

$$\sqrt[3]{p(p^2+3p+3)+1}$$

$$\Rightarrow \sqrt[3]{p^3+3p^2+3p+1}$$

$$\therefore \sqrt[3]{(p+1)^3}$$

$$= \sqrt[3]{(999+1)^3}$$

$$= \sqrt[3]{(1000)^3} = 1000$$

$$15. (a) x:y$$

$$7:3$$

$$\therefore \frac{xy+y^2}{x^2-y^2} = \frac{21+9}{49-9} = \frac{30}{40} = \frac{3}{4}$$

$$16. (c) x^{x\sqrt{x}} = (x\sqrt{x})^x$$

$$x^{x\sqrt{x}} = \left(x^{\frac{3}{2}}\right)^x$$

$$x^{x\sqrt{x}} = x^{\frac{3}{2}x}$$

(If bases are same then their power is also same)

$$\therefore x\sqrt{x} = \frac{3}{2}x \text{ or } \sqrt{x} = \frac{3}{2}$$

$$x = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$17. (a) a^2 + b^2 + c^2 - ab - bc - ca$$

$$= \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$= \frac{1}{2}[(7-5)^2 + (5-3)^2 + (3-7)^2]$$

$$= \frac{1}{2}(4+4+16) = \frac{24}{2} = 12$$

$$18. (c)$$

$$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} - 3 \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} + \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$$

$$\frac{1}{3} \times \frac{1}{3} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{5} - \left(\frac{1}{3} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{5} + \frac{1}{5} \times \frac{1}{3}\right)$$

$$A^3 + B^3 + C^3 - 3ABC = (A + B + C)(A^2 + B^2 + C^2 - AB - BC - CA)$$

$$\therefore \frac{\left(\frac{1}{3}\right)^3 + \left(\frac{1}{4}\right)^3 - 3 \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} + \left(\frac{1}{5}\right)^3}{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{5}\right)^2 - \frac{1}{3} \times \frac{1}{4} - \frac{1}{4} \times \frac{1}{5} - \frac{1}{5} \times \frac{1}{3}}$$

$$= \frac{\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right) \left[\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{5}\right)^2 - \frac{1}{3} \times \frac{1}{4} - \frac{1}{4} \times \frac{1}{5} - \frac{1}{5} \times \frac{1}{3}\right]}{\left[\left(\frac{1}{3}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{5}\right)^2 - \frac{1}{3} \times \frac{1}{4} - \frac{1}{4} \times \frac{1}{5} - \frac{1}{5} \times \frac{1}{3}\right]}$$

$$= \frac{20+15+12}{60} = \frac{47}{60}$$

19. (d) $x = 7 - 4\sqrt{3}$

$$= 4 + 3 - 4\sqrt{3}$$

$$= (2)^2 + (\sqrt{3})^2 - 2 \times 2 \times \sqrt{3}$$

$$= (2 - \sqrt{3})^2$$

$$\therefore [(a^2 + b^2 - 2ab = (a-b)^2)]$$

$$\Rightarrow x = (2 - \sqrt{3})^2$$

$$\sqrt{x} = 2 - \sqrt{3}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$= 2 + \sqrt{3}$$

$$\therefore \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$= 2 - \sqrt{3} + 2 + \sqrt{3} = 4$$

20. (b) $a = \frac{\sqrt{5} + 1}{\sqrt{5} - 1}$ $b = \frac{\sqrt{5} - 1}{\sqrt{5} + 1}$

$$\therefore a = \frac{1}{b}$$

$$a + b = a + \frac{1}{a}$$

$$\Rightarrow \frac{\sqrt{5} + 1}{\sqrt{5} - 1} + \frac{\sqrt{5} - 1}{\sqrt{5} + 1}$$

$$\Rightarrow \frac{5 + 1 + 2\sqrt{5} + 5 + 1 - 2\sqrt{5}}{(\sqrt{5})^2 - (1)^2}$$

$$\Rightarrow \frac{6 + 2\sqrt{5} + 6 - 2\sqrt{5}}{5 - 1} = \frac{12}{4} = 3$$

$$\therefore \frac{a^2 + ab + b^2}{a^2 - ab + b^2}$$

$$= \frac{a^2 + \frac{1}{a^2} + ab}{a^2 + \frac{1}{a^2} - ab}$$

$$\Rightarrow a + \frac{1}{a} = 3$$

$$a^2 + \frac{1}{a^2} = 9 - 2 = 7 \quad (ab = 1)$$

$$\therefore \frac{a^2 + \frac{1}{a^2} + ab}{a^2 + \frac{1}{a^2} - ab} = \frac{7 + 1}{7 - 1} = \frac{8}{6} = \frac{4}{3}$$

21. (c) $a = 4.36$ $b = 2.39$ $c = 1.97$

$$a - b - c$$

$$= 4.36 - 2.39 - 1.97$$

$$= 0$$

$$a^3 - b^3 - c^3 - 3abc$$

$$= \frac{1}{2} (a-b-c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$= 0$$

22. (d) $\frac{3a + 5b}{3a - 5b} = 5$

$$\Rightarrow 3a + 5b = 15a - 25b$$

$$\Rightarrow 12a = 30b$$

$$\Rightarrow 2a = 5b$$

$$\frac{a}{5} : \frac{b}{2}$$

23. (b) $p : q = r : s = t : u = 2 : 3$

$$\therefore \frac{mp + nr + ot}{mq + ns + ou}$$

$$\Rightarrow \frac{m \times 2x + n \times 2x + o \times 2x}{m \times 3x + n \times 3x + o \times 3x}$$

$$\Rightarrow \frac{2x(m + n + o)}{3x(m + n + o)} = \frac{2}{3}$$

$$\therefore mp + nr + ot : mq + ns + ou$$

24. (c) $x : y = 3 : 4$

$$\frac{7x + 3y}{7x - 3y} = \frac{y}{y} \left(\frac{7 \frac{x}{y} + 3}{7 \frac{x}{y} - 3} \right) = \frac{7 \times \frac{3}{4} + 3}{7 \times \frac{3}{4} - 3}$$

$$\frac{\frac{21}{4} + 3}{\frac{21}{4} - 3} = \frac{\frac{21 + 12}{4}}{\frac{21 - 12}{4}} = \frac{11}{3}$$

25. (b) $x + \frac{1}{4}\sqrt{x} + a^2$

$$= (\sqrt{x})^2 + 2 \times \frac{1}{8} \times \sqrt{x} + a^2$$

$$[(A^2 + 2BA + B^2) = (A + B)^2]$$

Here, $A = \sqrt{x}$ and $B = a$

$$B = \frac{1}{8} \quad \therefore a = \frac{1}{8}$$

(c) Given that $a \neq b$
Let $a = 16$, $b = 4$
 $\therefore$ by options

$$\text{So, } \frac{a+b}{2} = \frac{16+4}{2} = 10$$

$$\text{and } \sqrt{ab} = \sqrt{16 \times 4} = 8$$

$$\therefore \frac{a+b}{2} > \sqrt{ab}$$

$\therefore$ option (c) is correct.

27. (d) $x^{1/3} = y^{1/4}$

$$\Rightarrow \text{LCM of } 3, 4 = 12$$

$$\therefore (x^{1/3})^{12} \Rightarrow (y^{1/4})^{12}$$

$$x^4 = y^3$$

take power '5' on both sides

$$\Rightarrow (x^4)^5 = (y^3)^5$$

$$\Rightarrow x^{20} = y^{15}$$

28. (b) $x = \frac{\sqrt{3}}{2}$

$$\text{or } 1 + x = 1 + \frac{\sqrt{3}}{2} = \frac{2 + \sqrt{3}}{2}$$

$$= \frac{2(2 + \sqrt{3})}{2 \times 2}$$

(divides and multiply by 2)

$$\Rightarrow 1 + x = \frac{4 + 2\sqrt{3}}{4}$$

$$= \frac{1 + 3 + 2\sqrt{3}}{4}$$

$$= \frac{(1)^2 + (\sqrt{3})^2 + 2 \times 1 \times \sqrt{3}}{4}$$

$$1 + x \Rightarrow \frac{(1 + \sqrt{3})^2}{4}$$

$$\therefore \sqrt{1+x} = \frac{1 + \sqrt{3}}{2}$$

Similarly,

$$\sqrt{1-x} \Rightarrow \frac{\sqrt{3} - 1}{2}$$

$$\therefore \frac{\sqrt{1+x}}{1 + \sqrt{1+x}} + \frac{\sqrt{1-x}}{1 - \sqrt{1-x}}$$

$$= \frac{\frac{1 + \sqrt{3}}{2}}{1 + \frac{1 + \sqrt{3}}{2}} + \frac{\frac{\sqrt{3} - 1}{2}}{1 - \frac{\sqrt{3} - 1}{2}}$$

$$= \frac{1 + \sqrt{3}}{3 + \sqrt{3}} + \frac{\sqrt{3} - 1}{3 - \sqrt{3}}$$

$$= \frac{1 + \sqrt{3}}{\sqrt{3}(\sqrt{3} + 1)} + \frac{\sqrt{3} - 1}{\sqrt{3}(\sqrt{3} - 1)}$$

$$= \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

29. (d) $x^2 - 4x - 1 = 0$
 $x^2 - 1 = 4x$
 (divide x both sides)
 $x - \frac{1}{x} = 4$
 $x^2 + \frac{1}{x^2} - 2 = 16$
 $x^2 + \frac{1}{x^2} = 18$

30. (d) $\left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} - 1\right)$
 $\left(x^2 + \frac{1}{x^2} + 1\right)$
 $= \left(x + \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} - 1\right) \left(x - \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} + 1\right)$
 $\therefore$
 $\boxed{(A+B)(A^2-AB+B^2) = A^3+B^3}$
 $\boxed{(A-B)(A^2+AB+B^2) = A^3-B^3}$
 $= \left(x^3 + \frac{1}{x^3}\right) \left(x^3 - \frac{1}{x^3}\right) = \boxed{x^6 - \frac{1}{x^6}}$

31. (b) $\frac{\sqrt{7}-2}{\sqrt{7}+2} = a\sqrt{7} + b$
 $L.H.S = \frac{\sqrt{7}-2}{\sqrt{7}+2} \times \frac{\sqrt{7}-2}{\sqrt{7}-2}$
 (Rationalisation)
 $= \frac{(\sqrt{7}-2)^2}{(\sqrt{7})^2 - (4)} = \frac{7+4-4\sqrt{7}}{7-4}$
 $= \frac{11-4\sqrt{7}}{3}$
 $\frac{11}{3} - \frac{4}{3}\sqrt{7} = -\frac{4}{3}\sqrt{7} + \frac{11}{3}$
 $= a\sqrt{7} + b = R.H.S$
 Compare the coefficients of $\sqrt{7}$ and constant term
 $a = -\frac{4}{3}$
 $b = \frac{11}{3}$

32. (c) $a + \frac{1}{b} = 1, b + \frac{1}{c} = 1, c + \frac{1}{a} = ?$
 Put values,
 $a = \frac{1}{2} \quad b = 2 \quad c = -1$
 $c + \frac{1}{a} = -1 + \left(\frac{1}{\frac{1}{2}}\right) = -1 + 2 = 1$

Alternate:

$$\Rightarrow a + \frac{1}{b} = 1 \dots (i)$$

$$\Rightarrow a = 1 - \frac{1}{b} = \frac{b-1}{b}$$

$$\frac{1}{a} = \frac{b}{b-1} \Rightarrow b + \frac{1}{c} = 1$$

$$\frac{1}{c} = 1 - b, \quad \boxed{c = \frac{1}{1-b}}$$

$$\therefore c + \frac{1}{a} = \frac{1}{1-b} + \frac{b}{b-1}$$

$$= \frac{1}{1-b} - \frac{b}{1-b} = \frac{1-b}{1-b} = 1$$

33. (c) $x = \sqrt{3} + \sqrt{2}$
 $\therefore \frac{1}{x} = \sqrt{3} - \sqrt{2}$
 $x^3 + \frac{1}{x^3}$
 $\therefore x^3 = (\sqrt{3} + \sqrt{2})^3$
 $= (\sqrt{3})^3 + (\sqrt{2})^3 + 3 \times \sqrt{3} \times \sqrt{2} (\sqrt{3} + \sqrt{2})$
 $= 3\sqrt{3} + 2\sqrt{2} + 3\sqrt{6}(\sqrt{3} + \sqrt{2})$
 $= 3\sqrt{3} + 2\sqrt{2} + 9\sqrt{2} + 6\sqrt{3}$
 $x^3 = 9\sqrt{3} + 11\sqrt{2}$
 $\frac{1}{x^3} = 9\sqrt{3} - 11\sqrt{2}$
 $x^3 + \frac{1}{x^3} = 9\sqrt{3} + 11\sqrt{2} + 9\sqrt{3} - 11\sqrt{2}$
 $= 18\sqrt{3}$

Alternate:
 $x = \sqrt{3} + \sqrt{2}$
 $\frac{1}{x} = \sqrt{3} - \sqrt{2}$
 and $x + \frac{1}{x} = \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}$
 $= 2\sqrt{3}$
 $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$
 $= (2\sqrt{3})^3 - 3(2\sqrt{3})$
 $= 24\sqrt{3} - 6\sqrt{3} = 18\sqrt{3}$

34. (c) $x + y = 7$ (cubing both sides)
 $(x + y)^3 = (7)^3$
 $x^3 + y^3 + 3(x + y)xy = 343$
 $x^3 + y^3 + 21xy = 343$

35. (c) $x^{1/3} + y^{1/3} = z^{1/3}$
 (cubing both sides)
 $(x^{1/3} + y^{1/3})^3 = (z^{1/3})^3$

$$\Rightarrow x + y + 3x^{1/3} \cdot y^{1/3} \cdot (x^{1/3} + y^{1/3}) = z$$

$$\Rightarrow x + y - z + 3x^{1/3} \cdot y^{1/3} \cdot z^{1/3} = 0$$

$$\Rightarrow x + y - z = -3x^{1/3} y^{1/3} z^{1/3}$$
 (cubing again both sides)
 $(x + y - z)^3 = -27xyz$
 $(x + y - z)^3 + 27xyz = 0$

36. (a) $\frac{a}{b} + \frac{b}{a} = 1$
 $a \neq 0, \quad b \neq 0$
 $a^2 + b^2 = ab$
 $a^2 + b^2 - ab = 0$
 $(a + b)(a^2 + b^2 - ab) = (a + b) \times 0$
 [(multiply both sides by $(a + b)$)]
 $a^3 + b^3 = 0$

37. (d) $p = 99$
 $p(p^2 + 3p + 3)$
 $= p^3 + 3p^2 + 3p + 1 - 1$
 $= (p + 1)^3 - 1$
 $= (100)^3 - 1 = 1000000 - 1$
 $= 999999$

38. (b) $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$
 By option put $x = 4$
 $\Rightarrow 5^{\sqrt{4}} + 12^{\sqrt{4}} = 13^{\sqrt{4}}$
 $\Rightarrow 5^2 + 12^2 = 13^2$
 $\Rightarrow 169 = 169$
 hence, $x = 4$

39. (d) $(x - 3)^2 + (y - 4)^2 + (z - 5)^2 = 0$
 $\therefore (x - 3)^2 = 0 \quad x = 3$
 $(y - 4)^2 = 0 \quad y = 4$
 $(z - 5)^2 = 0 \quad z = 5$
 $(x + y + z) \Rightarrow 4 + 3 + 5 \Rightarrow 12$

40. (c) $x = 3 + \sqrt{8}$
 $\frac{1}{x} = \frac{1}{3 + \sqrt{8}}$
 $\frac{1}{x} = \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$
 $= \frac{3 - \sqrt{8}}{9 - 8} = 3 - \sqrt{8}$
 $x + \frac{1}{x} = 3 + \sqrt{8} + 3 - \sqrt{8} = 6$
 $x + \frac{1}{x} = 6$
 squaring both sides
 $x^2 + \frac{1}{x^2} + 2 = 36$
 $x^2 + \frac{1}{x^2} = 34$

$$41. \quad (b) \quad x - \frac{1}{x} = 4$$

$$x^2 + \frac{1}{x^2} - 2 = 16$$

(On Squaring)

$$\Rightarrow x^2 + \frac{1}{x^2} = 18$$

$$x^2 + \frac{1}{x^2} + 2 - 2 = 18$$

$$x^2 + \frac{1}{x^2} + 2 = 20$$

$$\left(x + \frac{1}{x}\right)^2 = 20$$

$$x + \frac{1}{x} = \sqrt{20}$$

$$= \sqrt{4 \times 5} = 2\sqrt{5}$$

$$42. \quad (a) \quad 4b^2 + \frac{1}{b^2} = 2$$

$$(2b)^2 + \left(\frac{1}{b}\right)^2 + 4 - 4 = 2$$

$$\left(2b + \frac{1}{b}\right)^2 - 4 = 2$$

$$\left(2b + \frac{1}{b}\right)^2 = 6$$

$$2b + \frac{1}{b} = \sqrt{6}$$

Take cube both sides

$$\left(2b + \frac{1}{b}\right)^3 = (\sqrt{6})^3$$

$$8b^3 + \frac{1}{b^3} + 3 \times 2b \times \frac{1}{b} \left(2b + \frac{1}{b}\right) = 6\sqrt{6}$$

$$8b^3 + \frac{1}{b^3} + 6\sqrt{6} = 6\sqrt{6}$$

$$8b^3 + \frac{1}{b^3} = 0$$

$$43. \quad (c) \quad \left(\frac{3}{5}\right)^3 \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$$

$$\Rightarrow \left(\frac{3}{5}\right)^{+3-6} = \left(\frac{3}{5}\right)^{2x-1}$$

$$\Rightarrow -3 = 2x - 1$$

$$\Rightarrow 2x = -2$$

$$\Rightarrow x = -1$$

$$44. \quad (b) \quad \frac{\sqrt{3+x} + \sqrt{3-x}}{\sqrt{3+x} - \sqrt{3-x}} = \frac{2}{1} \quad (\text{by c-d rule})$$

$$\Rightarrow \frac{\sqrt{3+x}}{\sqrt{3-x}} = \frac{2+1}{2-1} = \frac{3}{1}$$

$$\left(\frac{A}{B} = \frac{C}{D}\right) \Rightarrow \frac{A+B}{A-B} = \frac{C+D}{C-D}$$

$$\frac{\sqrt{3+x}}{\sqrt{3-x}} = 3$$

Squaring both sides

$$\frac{3+x}{3-x} = 9$$

$$3+x = 27-9x$$

$$10x = 24$$

$$x = \frac{24}{10} = \frac{12}{5}$$

$$45. \quad (d) \quad x = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}} \times \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$$

$$= \frac{(\sqrt{1+x} + \sqrt{1-x})^2}{(\sqrt{1+x})^2 - (\sqrt{1-x})^2}$$

$$= \frac{1+x+1-x+2\sqrt{1-x^2}}{1+x-1+x}$$

$$= \frac{2+2\sqrt{1-x^2}}{2x} = \frac{1+\sqrt{1-x^2}}{x}$$

$$= \frac{1+\sqrt{1-\frac{3}{4}}}{\frac{\sqrt{3}}{2}} \times 2$$

$$= \left(1 + \frac{1}{2}\right) \times 2 = \frac{3}{2} \times 2 = \sqrt{3}$$

$$46. \quad (d) \quad \frac{\sqrt{x+4} + \sqrt{x-4}}{\sqrt{x+4} - \sqrt{x-4}} = \frac{2}{1}$$

by C - D rule

$$\Rightarrow \frac{\sqrt{x+4}}{\sqrt{x-4}} = \frac{2+1}{2-1} = \frac{3}{1}$$

$$\Rightarrow \left(\frac{\sqrt{x+4}}{\sqrt{x-4}}\right)^2 = \left(\frac{3}{1}\right)^2$$

$$\Rightarrow \frac{x+4}{x-4} = 9 \text{ again C \& D rule}$$

$$\Rightarrow \frac{x}{4} = \frac{9+1}{9-1}$$

$$\Rightarrow \frac{x}{4} = \frac{10}{8}$$

$$\Rightarrow x = \frac{10}{8} \times 4 = 5$$

$$47. \quad (c) \quad x = (\sqrt{2}+1)^{\frac{1}{3}}$$

Take cube on both sides

$$\Rightarrow x^3 = \sqrt{2} + 1$$

$$\Rightarrow \frac{1}{x^3} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= \frac{\sqrt{2}-1}{1}$$

$$\frac{1}{x^3} = \sqrt{2} - 1$$

$$\Rightarrow x^3 - \frac{1}{x^3} = \sqrt{2} + 1 - \sqrt{2} + 1 = 2$$

$$48. \quad (b) \quad \frac{x^2 - x + 1}{x^2 + x + 1} = \frac{3}{2} \text{ Given}$$

$$\Rightarrow \frac{x \left\{ \left(x + \frac{1}{x}\right) - 1 \right\}}{x \left\{ \left(x + \frac{1}{x}\right) + 1 \right\}} = \frac{3}{2}$$

$$\text{Let } \left(x + \frac{1}{x} = y\right)$$

$$\Rightarrow \frac{\left(x + \frac{1}{x}\right) - 1}{\left(x + \frac{1}{x}\right) + 1} = \frac{3}{2}$$

$$\frac{y-1}{y+1} = \frac{3}{2}$$

$$\Rightarrow 2(y-1) = 3(y+1)$$

$$2y - 2 = 3y + 3$$

$$y = -2 - 3 = -5$$

$$\therefore x + \frac{1}{x} = -5$$

$$49. \quad (c) \quad x = 3 + \sqrt{8}$$

$$x^2 = 9 + 8 + 2 \times 3 \sqrt{8}$$

$$x^2 = 17 + 6\sqrt{8}$$

$$\frac{1}{x^2} = 17 - 6\sqrt{8}$$

$$x^2 + \frac{1}{x^2} = 17 + 6\sqrt{8} + 17 - 6\sqrt{8} = 34$$

50. (c) $x = 5 + 2\sqrt{6}$

$$x = 3 + 2 + 2\sqrt{3} \times \sqrt{2}$$

$$x = (\sqrt{3})^2 + (\sqrt{2})^2 + 2\sqrt{3} \times \sqrt{2}$$

$$x = (\sqrt{3} + \sqrt{2})^2$$

$$\sqrt{x} = \sqrt{3} + \sqrt{2}$$

Similarly

$$\Rightarrow \frac{1}{\sqrt{x}} = \sqrt{3} - \sqrt{2}$$

$$\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}$$

$$= 2\sqrt{3}$$

51. (d) $x = \sqrt{3} + \sqrt{2}$

$$x^2 = 3 + 2 + 2\sqrt{6}$$

$$x^2 = 5 + 2\sqrt{6}$$

$$\frac{1}{x^2} = 5 - 2\sqrt{6}$$

$$x^2 + \frac{1}{x^2}$$

$$= 5 + 2\sqrt{6} + 5 - 2\sqrt{6}$$

$$= 10$$

52. (c) $x + \frac{9}{x} = 6$

Take values of x

$$\text{Let } x = 3$$

$$3 + \frac{9}{3} = 6$$

Prove So, $x = 3$

$$\therefore x^2 + \frac{9}{x^2} = 9 + \frac{9}{9} = 10$$

Alternate:

$$x + \frac{9}{x} = 6$$

On squaring

$$\left(x + \frac{9}{x}\right)^2 = 36$$

$$x^2 + \frac{81}{x^2} + 2 \times \frac{9}{x} \times x = 36$$

$$x^2 + \frac{81}{x^2} - 18 = 0$$

$$\left(x - \frac{9}{x}\right)^2 = 0$$

$$\begin{aligned} x &= \frac{9}{x} \\ x^2 &= 9 \end{aligned}$$

$$\text{Hence } x^2 + \frac{9}{x^2} = 9 + \frac{9}{9} = 10$$

53. (b) $2p + \frac{1}{p} = 4$

Divide by 2

$$\frac{2p}{2} + \frac{1}{2p} = \frac{4}{2}$$

$$p + \frac{1}{2p} = 2$$

Take cube on both sides

$$\Rightarrow \left(p + \frac{1}{2p}\right)^3 = (2)^3$$

$$p^3 + \frac{1}{8p^3} + 3 \times p \times \frac{1}{2p} \left(p + \frac{1}{2p}\right) = 8$$

$$p^3 + \frac{1}{8p^3} + \frac{3}{2} \times 2 = 8$$

$$p^3 + \frac{1}{8p^3} = 8 - 3 = 5$$

54. (a) $a^6 + b^6 = (a^2)^3 + (b^2)^3$
 $= (a^2 + b^2)(a^4 - a^2b^2 + b^4)$
 $\therefore a^6 + b^6 = (a^2 + b^2) \times 0 = 0$

55. (c) $x + \frac{1}{x} = 3$ (Given)

$$\frac{x^3 + \frac{1}{x}}{x^2 - x + 1} \quad (\text{Divide by } x)$$

$$\Rightarrow \frac{\frac{x^3}{x} + \frac{1}{x^2}}{\frac{x^2}{x} - \frac{x}{x} + \frac{1}{x}} = \frac{x^2 + \frac{1}{x^2}}{x - 1 + \frac{1}{x}}$$

$$= \frac{x^2 + \frac{1}{x^2}}{x + \frac{1}{x} - 1}$$

$$\therefore x + \frac{1}{x} = 3$$

$$\therefore x^2 + \frac{1}{x^2} = 9 - 2 = 7$$

$$\therefore \frac{x^2 + \frac{1}{x^2}}{x + \frac{1}{x} - 1} = \frac{7}{3 - 1} = \frac{7}{2}$$

56. (a) $a + \frac{1}{a} + 1 = 0$

$$a + \frac{1}{a} = -1$$

Squaring both sides

$$\Rightarrow a^2 + \frac{1}{a^2} + 2 = 1$$

$$\Rightarrow a^2 + \frac{1}{a^2} = -1$$

$$\Rightarrow a^2 + 1 = \frac{-1}{a^2} \quad \dots (i)$$

$$\Rightarrow a + \frac{1}{a} = -1 \quad (\text{Given})$$

$$\therefore a^2 + 1 = -a \quad \dots (ii)$$

$$\Rightarrow -a = \frac{-1}{a^2} \quad \text{from equation (i)}$$

and (ii)

$$a^3 = 1$$

$$\therefore a^3 - 1 = 0$$

$$\Rightarrow a^4 - a = 0 \times a = 0$$

(Multiply a both sides)

57. (c) $x = a + \frac{1}{a}$

$$y = a - \frac{1}{a}$$

$$\therefore (x + y) = a + \frac{1}{a} + a - \frac{1}{a} = 2a$$

$$\therefore (x - y) = a + \frac{1}{a} - a + \frac{1}{a} = \frac{2}{a}$$

$$\therefore x^4 + y^4 - 2x^2y^2 = (x^2 - y^2)^2$$

$$\Rightarrow ((x + y)(x - y))^2$$

$$\Rightarrow \left(2a \times \frac{2}{a}\right)^2$$

$$= (4)^2 = 16$$

58. (a) $a = 11$
 $b = 9$

$$\Rightarrow \frac{a^2 + b^2 + ab}{a^3 - b^3}$$

$$\Rightarrow (a^3 - b^3) = ((a - b)(a^2 + ab + b^2))$$

$$\therefore \frac{a^2 + b^2 + ab}{(a - b)(a^2 + ab + b^2)}$$

$$= \frac{1}{a - b} = \frac{1}{11 - 9} = \frac{1}{2}$$

59. (a) $p = 101$

$$= \sqrt[3]{p(p^2 - 3p + 3) - 1}$$

$$= \sqrt[3]{p^3 - 3p^2 + 3p - 1}$$

$$\begin{aligned} \therefore [(p-1)^3 - p^3 - (1)^3 - 3p(p-1)] & 64. \\ = \sqrt[3]{(p-1)^3} & \\ = p-1 = 101-1 = 100 & \\ 60. (a) x = 19 \quad y = 18 & \end{aligned}$$

$$\begin{aligned} & \frac{x^2 + y^2 + xy}{x^3 - y^3} \\ & = \frac{x^2 + y^2 + xy}{(x-y)(x^2 + y^2 + xy)} \end{aligned}$$

$$= \frac{1}{x-y} = \frac{1}{19-18} = 1$$

$$61. (b) 50\% (p-q) = 30\% (p+q)$$

$$\frac{p-q}{2} = \frac{3}{10} (p+q)$$

$$\left[50\% = \frac{1}{2} \right]$$

$$\begin{aligned} \Rightarrow 5(p-q) &= 3(p+q) \\ \Rightarrow 5p - 5q &= 3p + 3q \\ \Rightarrow 2p &= 8q \\ \Rightarrow 1p &= 4q \\ \Rightarrow p : q & \\ \Rightarrow 4 : 1 & \end{aligned}$$

$$62. (a) \frac{a}{3} = \frac{b}{2} \Rightarrow \frac{a}{b} = \frac{3}{2}$$

$$\begin{aligned} \frac{2a+3b}{3a-2b} &= \frac{2 \times 3 + 3 \times 2}{3 \times 3 - 2 \times 2} \\ &= \frac{6+6}{9-4} = \frac{12}{5} \end{aligned}$$

$$63. (d) \frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$$

Add 3 both sides

$$\Rightarrow \frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} + 3 = 1 + 3$$

$$\Rightarrow \left(\frac{a}{1-a} + 1 \right) + \left(\frac{b}{1-b} + 1 \right) +$$

$$\left(\frac{c}{1-c} + 1 \right) = 4$$

$$\Rightarrow \left(\frac{a+1-a}{1-a} \right) + \left(\frac{b+1-b}{1-b} \right) +$$

$$\left(\frac{c+1-c}{1-c} \right) = 4$$

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 4$$

$$(b) \frac{2x-y}{x+2y} \times \frac{1}{2} \quad (\text{Cross Multiply})$$

$$\Rightarrow 4x - 2y = x + 2y$$

$$3x = 4y$$

$$x : y = 4 : 3$$

$$\Rightarrow \frac{3x-y}{3x+y} = \frac{3 \times 4 - 3}{3 \times 4 + 3}$$

$$= \frac{12-3}{12+3} = \frac{9}{15} = \frac{3}{5}$$

$$65. (b) x + \frac{1}{x} = 5$$

$$\therefore \frac{2x}{3x^2 - 5x + 3}$$

(Divide by x)

$$= \frac{\frac{2x}{x}}{\frac{3x^2}{x} - \frac{5x}{x} + \frac{3}{x}} = \frac{2}{3x + \frac{3}{x} - 5}$$

$$= \frac{2}{3\left(x + \frac{1}{x}\right) - 5} = \frac{2}{3 \times 5 - 5}$$

$$= \frac{2}{10} = \frac{1}{5}$$

$$66. (b) \sqrt{1 - \frac{x^3}{100}} = \frac{3}{5}$$

$$\Rightarrow 1 - \frac{x^3}{100} = \left(\frac{3}{5}\right)^2$$

$$\Rightarrow 1 - \frac{9}{25} = \frac{x^3}{100}$$

$$\Rightarrow \frac{16}{25} = \frac{x^3}{100}$$

$$\Rightarrow \frac{16 \times 100}{25} = x^3$$

$$\Rightarrow 16 \times 4 = x^3$$

$$\Rightarrow 4 = x$$

$$67. (d) \frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{48} + \sqrt{18}} = a + b\sqrt{6}$$

$$\Rightarrow \frac{4\sqrt{3} + 5\sqrt{2}}{\sqrt{16 \times 3} + \sqrt{9 \times 2}}$$

$$= \frac{4\sqrt{3} + 5\sqrt{2}}{4\sqrt{3} + 3\sqrt{2}}$$

$$\Rightarrow \frac{4\sqrt{3} + 5\sqrt{2}}{4\sqrt{3} + 3\sqrt{2}} \times \frac{4\sqrt{3} - 3\sqrt{2}}{4\sqrt{3} - 3\sqrt{2}}$$

$$= \frac{(4\sqrt{3} + 5\sqrt{2})(4\sqrt{3} - 3\sqrt{2})}{48 - 18}$$

$$= \frac{8\sqrt{6} + 18}{30} = \frac{8\sqrt{6}}{30} + \frac{18}{30}$$

$$= \frac{4}{15} \sqrt{6} + \frac{3}{5}$$

$$= \frac{3}{5} + \frac{4}{15} \sqrt{6}$$

$$\therefore \frac{3}{5} + \frac{4}{15} \sqrt{6} = a + b\sqrt{6}$$

By comparing coefficients of rational and irrational parts.

$$\Rightarrow a = \frac{3}{5} \quad b = \frac{4}{15}$$

$$\left(\frac{3}{5}, \frac{4}{15} \right)$$

$$68. (a) \sqrt{1 + \frac{x}{961}} = \frac{32}{31}$$

(Squaring both sides)

$$\Rightarrow 1 + \frac{x}{961} = \frac{1024}{961}$$

$$\Rightarrow \frac{961 + x}{961} = \frac{1024}{961}$$

$$x = 1024 - 961 = 63$$

$$69. (d) \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{3}{1}$$

$$\Rightarrow \frac{2 \times 9 + 3 \times 9 + 4 \times 9}{2 \times 1 + 3 \times 1 + 4 \times 1}$$

$$= \frac{18 + 27 + 36}{2 + 3 + 4} = \frac{81}{9} = 9$$

$$70. (d) 2x + \frac{1}{3x} = 5$$

$$\Rightarrow 6x^2 + 1 = 15x$$

$$\therefore \frac{5x}{6x^2 + 20x + 1}$$

$$= \frac{5x}{15x + 20x} = \frac{5x}{35x} = \frac{1}{7}$$

$$71. (a) x \propto \frac{1}{y^2 - 1} \quad (\text{Given})$$

$$x = k \times \frac{1}{y^2 - 1}$$

(k is constant)

Now $x = 24$ when $y = 10$ given

$$\Rightarrow 24 = k \times \frac{1}{(10)^2 - 1}$$

$$\Rightarrow 24 = \frac{k}{99}$$

$$k = 24 \times 99$$

$$\Rightarrow x = ?$$

$$y = 5$$

$$x = 24 \times 99 \times \frac{1}{25 - 1}$$

$$= 24 \times 99 \times \frac{1}{24} \quad x = 99$$

72. (a) $x^2 + y^2 + 2x + 1 = 0$

$$\Rightarrow x^2 + 2x + 1 + y^2 = 0$$

$$(x + 1)^2 + y^2 = 0$$

Hence both terms are squares and there addition is zero so, it can be possible only when both terms are zeros.

$$\therefore x + 1 = 0$$

$$\Rightarrow x = -1 \quad y = 0$$

$$\therefore x^{31} + y^{35}$$

$$= (-1)^{31} + (0)^{35} = -1$$

73. (b) $\frac{x}{2x^2 + 5x + 2} = \frac{1}{6}$

$$\frac{\frac{x}{x}}{2\frac{x^2}{x} + \frac{5x}{x} + \frac{2}{x}} = \frac{1}{6}$$

$$\Rightarrow \frac{1}{2x + \frac{2}{x} + 5} = \frac{1}{6}$$

$$\Rightarrow 2 \left(x + \frac{1}{x} \right) + 5 = 6$$

$$\Rightarrow 6 - 5 = 2 \left(x + \frac{1}{x} \right)$$

$$\Rightarrow 1 = 2 \left(x + \frac{1}{x} \right)$$

$$\Rightarrow x + \frac{1}{x} = \frac{1}{2}$$

74. (c) $a^2 + b^2 + c^2 = 2(a - b - c) - 3$

$$\Rightarrow a^2 + b^2 + c^2 = 2a - 2b - 2c - 3$$

$$\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2c + 1 + 1 + 1 = 0$$

$$\Rightarrow (a^2 - 2a + 1) + (b^2 + 2b + 1) + (c^2 + 2c + 1) = 0$$

$$\Rightarrow (a - 1)^2 + (b + 1)^2 + (c + 1)^2 = 0$$

$$a = 1$$

$$b = -1$$

$$c = -1$$

$$\therefore 2a - 3b + 4c$$

$$= 2 \times 1 - 3 \times (-1) + 4 \times (-1)$$

$$= 2 + 3 - 4 = 1$$

75. (a) $(3a + 1)^2 + (b - 1)^2 + (2c - 3)^2 = 0$

$$\Rightarrow (3a + 1)^2 = 0$$

$$\Rightarrow 3a = -1$$

$$\Rightarrow a = \frac{-1}{3}$$

$$(b - 1)^2 = 0 \Rightarrow b - 1 = 0$$

$$\Rightarrow b = 1$$

$$(2c - 3)^2 = 0$$

$$\Rightarrow c = \frac{3}{2}$$

$$\therefore 3a + b + 2c = 3 \times \frac{-1}{3} + 1 + \frac{3}{2} \times 2$$

$$= -1 + 1 + 3 = 3$$

76. (b) $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} +$

$$\frac{(c-a)^2}{(a-b)(b-c)}$$

Now

$$\Rightarrow \frac{(a-b)^2}{(b-c)(c-a)} \times \frac{(a-b)}{a-b}$$

Multiply divide by $(a-b)$ in Ist term

$$\Rightarrow (b-c) \text{ in IInd term}$$

$$\Rightarrow (c-a) \text{ in IIIrd term}$$

$$\Rightarrow \frac{(a-b)^2(a-b)}{(b-c)(c-a)(a-b)} +$$

$$\frac{(b-c)^2(b-c)}{(a-b)(b-c)(c-a)} +$$

$$\frac{(c-a)^2(c-a)}{(a-b)(b-c)(c-a)}$$

$$\Rightarrow \text{Let } a - b = x$$

$$b - c = y$$

$$c - a = z$$

$$\therefore x + y + z = 0$$

$$\therefore x^3 + y^3 + z^3 = 3xyz$$

$$\therefore (a-b)^3 + (b-c)^3 + (c-a)^3$$

$$= 3(a-b)(b-c)(c-a)$$

$$\therefore \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

77. (c) $(a-3)^2 + (b-4)^2 + (c-9)^2 = 0$

$$a - 3 = 0$$

$$a = 3$$

$$b - 4 = 0$$

$$b = 4$$

$$c - 9 = 0$$

$$c = 9$$

$$\therefore \sqrt{a+b+c} = \sqrt{3+4+9}$$

$$= \sqrt{16} = \pm 4$$

78. (b) $1.5x = 0.04y$

$$\Rightarrow \frac{x}{y} = \frac{0.04}{1.5} = \frac{4}{100} \times \frac{10}{15} = \frac{2}{75}$$

$$\therefore \frac{y^2 - x^2}{y^2 + x^2 + 2xy} = \frac{(y-x)(y+x)}{(y+x)^2}$$

$$\Rightarrow \frac{y-x}{y+x} = \frac{75-2}{75+2} = \frac{73}{77}$$

79. (b) $a^{1/3} = 11, a = 11^3 = 1331$

$$a^2 - 331a = a(a - 331)$$

$$= 1331(1331 - 331)$$

$$= 1331 \times 1000 = 1331000$$

80. (a) $x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} = 4$

Take $x = y = 1$

$$1 + 1 + \frac{1}{1} + \frac{1}{1} = 4$$

Hence

$$x^2 + y^2 = 1 + 1 = 2$$

81. (b) $x^2 = y + z$ (i)

$$x^2 + x = y + z + x$$

add x on both sides

$$x(x + 1) = x + y + z$$

$$\Rightarrow y^2 = x + z$$
(ii)

$$y^2 + y = x + y + z$$

add y on both sides

$$y(y + 1) = (x + y + z)$$

$$\Rightarrow z^2 = y + x$$
(ii)

$$z^2 + z = x + z + y$$

add z on both sides

$$z(z + 1) = x + y + z$$

$$\therefore x(x + 1) = x + y + z$$

$$\frac{x}{x + y + z} = \frac{1}{x + 1}$$

$$\frac{y}{x + y + z} = \frac{1}{y + 1}$$

$$\frac{z}{x + y + z} = \frac{1}{z + 1}$$

By adding them

$$= \frac{x}{x + y + z} + \frac{y}{x + y + z} +$$

$$\frac{z}{x + y + z} = \frac{1}{x + 1} + \frac{1}{y + 1} + \frac{1}{z + 1}$$

$$= \frac{x + y + z}{x + y + z} = \frac{1}{x + 1} + \frac{1}{y + 1} + \frac{1}{z + 1} = 1$$

Alternate:-

$$x = y = z = 2$$

$$\therefore \frac{1}{2+1} + \frac{1}{2+1} + \frac{1}{2+1} = \frac{1}{3} +$$

$$\frac{1}{3} + \frac{1}{3} = 1$$

82. (d) $a^2 + b^2 = 2$
 $c^2 + d^2 = 1$

Put values of a, b, c, d

Take $a = b = 1$

$c = 1$

$d = 0$

$\Rightarrow (ad - bc)^2 + (ac + bd)^2$

$\Rightarrow (0-1)^2 + (1+0)^2$

$\Rightarrow (-1)^2 + (1)^2 = 2$

83. (d) $x = \frac{4ab}{a+b}$

$\Rightarrow \frac{x}{2a} = \frac{2b}{a+b}$

$\frac{x+2a}{x-2a} = \frac{2b+a+b}{2b-a-b} = \frac{3b+a}{b-a}$

(By C - D rule)

$\Rightarrow$ again

$\frac{x}{2b} = \frac{2a}{a+b}$

$\frac{x+2b}{x-2b} = \frac{2a+a+b}{2a-a-b} = \frac{3a+b}{a-b}$

$\Rightarrow \frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b}$

$\Rightarrow \frac{3b+a-3a-b}{b-a}$

$\Rightarrow \frac{2b-2a}{b-a} = \frac{2(b-a)}{(b-a)} = 2$

84. (c) $m + \frac{1}{m-2} = 4$

$\Rightarrow (m-2) + \frac{1}{(m-2)} = 2$

$\Rightarrow$ Squaring both sides

$(m-2)^2 + \frac{1}{(m-2)^2} + 2 \times (m-2) \times$

$\frac{1}{(m-2)} = 4$

$(m-2)^2 + \frac{1}{(m-2)^2} = 2$

85. (c) $a^2 + b^2 + 2b + 4a + 5 = 0$

$a^2 + b^2 + 2b + 4a + 4 + 1 = 0$

$a^2 + 4a + 4 + b^2 + 2b + 1 = 0$

$(a+2)^2 + (b+1)^2 = 0$

$a+2=0 \quad a=-2$

$b+1=0 \quad b=-1$

$\frac{a-b}{a+b} \Rightarrow \frac{-2+1}{-2-1}$

$\Rightarrow \frac{-1}{-3} = \frac{1}{3}$

86. (a) $x - y = \frac{x+y}{7} = \frac{xy}{4} = k$ (let)

$x - y = k$ (i)

$x + y = 7k$ (ii)

$xy = 4k$ (iii)

$\therefore x - y = k$ (i)

$x + y = 7k$ (ii)

$\frac{x}{4k} = \frac{4k}{x}$

$y = 3k$

$\therefore xy \Rightarrow 4k \times 3k = 12k^2$

$12k^2 = 4k$

$k = \frac{1}{3} \therefore xy = 4k = \frac{4}{3}$

87. (d) $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} = 0$

$= \frac{x^2}{yz} \times \frac{x}{x} + \frac{y^2 \times y}{zx \times y} + \frac{z^2}{xy} \times \frac{z}{z} = 0$

$= \frac{x^3 + y^3 + z^3}{xyz} = 0$

$\therefore$ If $x + y + z = 0$

then, $x^3 + y^3 + z^3$

$\Rightarrow 3xyz$

$\therefore \frac{3xyz}{xyz} = 3$

88. (b) $a + b + c = 0$

$\frac{1}{(a+b)(b+c)} + \frac{1}{(a+c)(b+a)} +$

$\frac{1}{(c+a)(c+b)}$

$\Rightarrow \frac{(a+c) + (b+c) + (a+b)}{(a+b)(a+c)(b+c)}$

$\Rightarrow \frac{2(a+b+c)}{(a+b)(a+c)(b+c)} = 0$

$(\therefore a + b + c = 0)$

89. (c) $a + b + c = 0$

Assume values $a = 2 \quad b = -2 \quad c = 0$

$a + b + c = 2 - 2 + 0 = 0$ (satisfy)

$\therefore \frac{a^2 + b^2 + c^2}{a^2 - bc}$

$\Rightarrow \frac{4+4+0}{4-0} \Rightarrow \frac{8}{4} = 2$

Alternate:-

$a + b + c = 0$

$b + c = -a$

Squaring both sides

$(b + c)^2 = a^2$

$b^2 + c^2 + 2bc = a^2$

$b^2 + c^2 = a^2 - 2bc$

$\therefore \frac{a^2 + b^2 + c^2}{a^2 - bc}$

$\Rightarrow \frac{a^2 + a^2 - 2bc}{a^2 - bc}$

$\Rightarrow \frac{2a^2 - 2bc}{a^2 - bc} \Rightarrow \frac{2(a^2 - bc)}{a^2 - bc} = 2$

90. (c) $x^2 + y^2 - 4x - 4y + 8 = 0$

$x^2 + 4 - 4x + y^2 + 4 - 4y = 0$

$(x-2)^2 + (y-2)^2 = 0$

$x-2=0, \quad y-2=0$

$x=2, \quad y=2$

$\therefore x - y = 2 - 2 = 0$

91. (a) $x = b + c - 2a$

$y = c + a - 2b$

$z = a + b - 2c$

$x + y + z = (b + c - 2a) + (c + a - 2b) + (a + b - 2c)$

$\therefore$ Now

$= x^2 + y^2 + 2xy - z^2$

$= (x+y)^2 - z^2 \quad (A^2 - B^2 = (A+B)(A-B))$

$(x+y-z)(x+y+z)$

$=$ As we know $(x+y+z) = 0$

$\therefore x^2 + y^2 - z^2 + 2xy = 0 \times (x+y-z) = 0$

92. (b) $a^2 + b^2 + c^2 = ab + bc + ca$

take value $a = b = c = 2$

$\Rightarrow \frac{a+c}{b} = \frac{2+2}{2} = 2$

93. (a) $x + \frac{1}{x} = \sqrt{3}$ (take cube on both sides)

$\left(x + \frac{1}{x}\right)^3 = (\sqrt{3})^3$

$x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x} \left(x + \frac{1}{x}\right) = 3\sqrt{3}$

$\Rightarrow x^3 + \frac{1}{x^3} + 3(\sqrt{3}) = 3\sqrt{3}$

$x^3 + \frac{1}{x^3} = 0$

$\therefore x^6 = -1$

$\therefore x^{18} + x^{12} + x^6 + 1$

$= (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1 = 0$

94. (c) $ax^3 + 3x^2 - 8x + b$ is divisible

by $(x+2)$ and $(x-2)$

$\therefore (x+2)$ and $(x-2)$ are factors

$\therefore x + 2 = 0 \Rightarrow x = -2$

$x - 2 = 0 \Rightarrow x = 2$

Put $x = -2$

$\therefore a(-2)^3 + 3(-2)^2 - 8(-2) + b = 0$

$= -8a + 12 + 16 + b = 0$

$-8a + b + 28 = 0$

$-8a + b = -28$ (I)

and

Put $x = 2$

$$\Rightarrow a(2)^3 + 3(2)^2 - 8 \times 2 + b = 0$$

$$\Rightarrow 8a + 12 - 16 + b = 0$$

$$8a + b - 4 = 0$$

$$8a + b = 4$$

.....(II)

From equation (I) & (II)

$$\therefore -8a + b = -28$$

$$\frac{8a + b = 4}{2b = -24}$$

$$b = -12$$

$$a = 2$$

95. (b) $x^2 - 3x + 1 = 0$

$$x^2 + 1 = 3x$$

Divide by x

$$= \frac{x^2}{x} + \frac{1}{x}$$

$$\Rightarrow \frac{3x}{x}$$

$$x + \frac{1}{x} = 3$$

Cubing both sides

$$\Rightarrow x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x} \left(x + \frac{1}{x} \right) = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27$$

$$x^3 + \frac{1}{x^3} = 18$$

96. (a) $x + \frac{1}{4x} = \frac{3}{2}$

Multiply by 2 both sides

$$\therefore 2x + \frac{1}{2x} = 3$$

Take cube both sides

$$= \left(2x + \frac{1}{2x} \right)^3 = (3)^3$$

$$= 8x^3 + \frac{1}{8x^3} + 3 \times 2x \times \frac{1}{2x} \left(2x + \frac{1}{2x} \right)$$

$$= 27$$

$$= 8x^3 + \frac{1}{8x^3} + 3 \times 3 = 27$$

$$8x^3 + \frac{1}{8x^3} = 27 - 9 = 18$$

97. (a) $\frac{1}{x+y} = \frac{1}{x} + \frac{1}{y}$

$$\Rightarrow \frac{1}{x+y}$$

$$= \frac{x+y}{xy}$$

$$xy = (x+y)^2$$

$$x^2 + y^2 + 2xy = xy$$

$$x^2 + y^2 + xy = 0$$

$$\therefore x^3 - y^3 = (x-y)(x^2 + y^2 + xy)$$

$$(x^3 - y^3) = (x-y) \times 0 = 0$$

98. (c) $x = a(b-c)$

$$y = b(c-a)$$

$$z = c(a-b)$$

$$\text{Let } \frac{x}{a} = b-c \quad \frac{x}{a} = A$$

$$\frac{y}{b} = c-a \quad \frac{y}{b} = B$$

$$\frac{z}{c} = a-b \quad \frac{z}{c} = C$$

$$\therefore A+B+C = b-c+c-a+a-b=0$$

$$\therefore A^3 + B^3 + C^3 = 3ABC$$

$$\therefore \left(\frac{x}{a} \right)^3 + \left(\frac{y}{b} \right)^3 + \left(\frac{z}{c} \right)^3$$

$$= 3 \times \frac{x}{a} \times \frac{y}{b} \times \frac{z}{c}$$

$$= \frac{3xyz}{abc}$$

99. (c) $xy(x+y) = 1$

$$x+y = \frac{1}{xy}$$

Cubing both sides

$$\Rightarrow (x+y)^3 = \frac{1}{x^3 y^3}$$

$$x^3 + y^3 + 3xy(x+y) = \frac{1}{x^3 y^3}$$

$$x^3 + y^3 + 3 = \frac{1}{x^3 y^3} \left(x+y = \frac{1}{xy} \right)$$

$$\frac{1}{x^3 y^3} - x^3 - y^3 = 3$$

100. (d) $x^4 + \frac{1}{x^4} = 119 \quad x > 1$

$$\therefore x^4 + \frac{1}{x^4} + 2 = 119 + 2 = 121$$

$$\left(x^2 + \frac{1}{x^2} \right)^2 = (11)^2$$

$$x^2 + \frac{1}{x^2} = 11$$

$$x^2 + \frac{1}{x^2} + 2 = 11 + 2$$

$$\left(x + \frac{1}{x} \right)^2 = 13$$

$$x + \frac{1}{x} = \sqrt{13}$$

$\Rightarrow$ Taking cube both sides

$$x^3 + \frac{1}{x^3} + 3\sqrt{13} = (\sqrt{13})^3$$

$$x^3 + \frac{1}{x^3} + 3\sqrt{13} = 13\sqrt{13}$$

$$x^3 + \frac{1}{x^3} = 10\sqrt{13}$$

101. (b) $3x + \frac{1}{2x} = 5$

$\Rightarrow$ Multiply both sides by $\frac{2}{3}$

$$\therefore 3x \times \frac{2}{3} + \frac{1}{2} \times \frac{2}{3} = 5 \times \frac{2}{3}$$

$$2x + \frac{1}{3x} = \frac{10}{3}$$

$\therefore$ Taking cube on both side

$$8x^3 + \frac{1}{27x^3} + 3 \times 2x \times \frac{1}{3x} \left(2x + \frac{1}{3x} \right)$$

$$= \left(\frac{10}{3} \right)^3$$

$$8x^3 + \frac{1}{27x^3} + 2 \times \frac{10}{3} = \frac{1000}{27}$$

$$8x^3 + \frac{1}{27x^3} = \frac{1000}{27} - \frac{20}{3}$$

$$= \frac{1000 - 180}{27} = \frac{820}{27} = 30 \frac{10}{27}$$

102. (a) $x + y = z$

$$x + y - z = 0$$

If $a + b + c = 0$

$$\text{then } a^3 + b^3 + c^3 - 3abc = 0$$

$$\therefore x^3 + y^3 - z^3 = -3xyz$$

$$\therefore x^3 + y^3 - z^3 + 3xyz = 0$$

$$3xyz - 3xyz = 0$$

103. (c) $\frac{a}{b} + \frac{b}{a} = 1$

$$\therefore a^2 + b^2 = ab$$

$$\therefore a^2 + b^2 - ab = 0$$

$$\therefore a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$= (a+b) \times 0 = 0$$

104. (c) $x = 2 - 2^{1/3} + 2^{2/3}$
 $x - 2 = 2^{2/3} - 2^{1/3}$ (I)
 Take cube both sides
 $(x - 2)^3 = (2^{2/3} - 2^{1/3})^3$
 $x^3 - 8 - 6x(x - 2) = (2^{2/3})^3 - (2^{1/3})^3 - 3 \times 2^{2/3} \cdot 2^{1/3} (2^{2/3} - 2^{1/3})$
 $x^3 - 8 - 6x^2 + 12x = 2^2 - 2 - 3 \times 2 \frac{2+1}{3} (x - 2)$

From equation (I)
 $\therefore x^3 - 8 - 6x^2 + 12x = 4 - 2 - 3 \times 2 (x - 2)$
 $x^3 - 8 - 6x^2 + 12x = 2 - 6x + 12$
 $x^3 + 18x - 6x^2 - 8 - 14 = 0$
 $x^3 + 18x - 6x^2 - 22 = 0$
 $\therefore x^3 - 6x^2 + 18x + 18 = 22 + 18 = 40$

105. (d) $a^3 - b^3 - c^3 - 3abc = 0$
 $\therefore a - b - c = 0$
 $a = b + c$

106. (c) $a = 2.361$
 $b = 3.263$
 $c = 5.624$
 $a + b - c = 0$
 $2.361 + 3.263 - 5.624 = 0$
 $\therefore a^3 + b^3 - c^3 + 3abc \Rightarrow 0$

107. (d) $p = 124$
 $\sqrt[3]{p(p^2 + 3p + 3) + 1}$
 $= \sqrt[3]{p^3 + 3p^2 + 3p + 1}$
 $= \sqrt[3]{(p+1)^3} = \sqrt[3]{(125)^3} = 125$

108. (c) $x + \frac{1}{x} = 2$
 (assume $x = 1$, so, $1 + 1 = 2$)

$x^{17} + \frac{1}{x^{19}} = (1)^{17} + \frac{1}{(1)^{19}}$
 $= 1 + 1 = 2$

109. (c) $x : y = 3 : 4$

$\therefore \frac{5x - 2y}{7x + 2y} = \frac{5 \times 3 - 2 \times 4}{7 \times 3 + 2 \times 4}$
 $= \frac{15 - 8}{21 + 8} = \frac{7}{29}$

110. (a) $x + y = 2z$
 $x - z = z - y$
 $x - z \Rightarrow -(y - z)$ (i)

$\frac{x}{x - z} + \frac{z}{y - z} = \frac{x}{x - z} - \frac{z}{x - z}$
 $= \frac{x - z}{x - z} = 1$

111. (b) $a^3b = abc = 180$
 or $a = 1$, $b = 180$
 then $c = 1$
 $\therefore b = 180$

112. (d) $(a - 1)\sqrt{2} + 3 = b\sqrt{2} + a$
 Comparing Coefficient of $\sqrt{2}$ & constant terms.

$\therefore a = 3$
 $\therefore a - 1 = b$
 $3 - 1 = b$
 $b = 2$
 $a + b = 3 + 2 = 5$
 113. (c) $ax^2 + bx + c = a(x - p)^2$
 $ax^2 + bx + c = a(x^2 + p^2 - 2px)$
 $\Rightarrow ax^2 + bx + c = ax^2 + ap^2 - 2apx$
 Comparing coefficients of x^2 and x
 $\Rightarrow b = -2ap$

$\Rightarrow p = \frac{-b}{2a}$ (i)
 and $c = ap^2$

$\Rightarrow c = a \times \frac{b^2}{4a^2}$ (From (i))

$\boxed{4ac = b^2}$

114. (d) $a + b + c + d = 1$
 $(1 + a)(1 + b)(1 + c)(1 + d)$
 $\Rightarrow$ For maximum value a, b, c, d

$a = b = c = d = \frac{1}{4}$
 $= \left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{4}\right) = \left(\frac{5}{4}\right)^4$

115. (b) $a^2 + b^2 + c^2 + 3 = 2(a + b + c)$
 $a^2 + b^2 + c^2 + 3 = 2a + 2b + 2c$
 $a^2 - 2a + 1 + b^2 - 2b + 1 + c^2 - 2c + 1 = 0$
 $(a - 1)^2 + (b - 1)^2 + (c - 1)^2 = 0$
 $a = 1$
 $b = 1$
 $c = 1$
 $(a + b + c) \Rightarrow 1 + 1 + 1 = 3$

116. (c) $x - \frac{1}{x} = 5$
 $x^2 + \frac{1}{x^2} - 2 = 25$
 $\Rightarrow x^2 + \frac{1}{x^2} = 27$

117. (b) $x = 3 + 2\sqrt{2}$
 $x = 2 + 1 + 2\sqrt{2}$
 $(\sqrt{2})^2 + (1)^2 + 2 \times 1 \times \sqrt{2}$
 $x = (\sqrt{2} + 1)^2$

$\sqrt{x} = \sqrt{(\sqrt{2} + 1)^2}$
 $\sqrt{x} = \sqrt{2} + 1$
 $\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = \sqrt{2} - 1$
 $\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{2} + 1 - \sqrt{2} + 1 = 2$

118. (c) Put $a = b = 1$ and $c = -2$
 we get $a + b + c = 1 + 1 - 2 = 0$
 $0 = 0$ (satisfy)

$\therefore \frac{a^2 + b^2 + c^2}{a^2 - bc}$
 $= \frac{(1)^2 + (1)^2 + (-2)^2}{(1)^2 - (1)(-2)} = \frac{6}{3} = 2$

119. (b) $n = 7 + 4\sqrt{3}$
 $n = 4 + 3 + 4\sqrt{3}$
 $n = (2)^2 + (\sqrt{3})^2 + 2 \times 2 \times \sqrt{3}$
 $n = (2 + \sqrt{3})^2$

$\sqrt{n} = 2 + \sqrt{3}$
 $\therefore \frac{1}{\sqrt{n}} = 2 - \sqrt{3}$

$\sqrt{n} + \frac{1}{\sqrt{n}} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

120. (b) $x = \sqrt{3} + \sqrt{2}$

$\frac{1}{x} = \frac{1}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \sqrt{3} - \sqrt{2}$

$x + \frac{1}{x} = \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2} = 2\sqrt{3}$

121. (d) $p + q = 10$ (i)
 and $pq = 5$
 Squaring both sides of equation (i)
 $(p + q)^2 = (10)^2$
 $p^2 + q^2 + 2pq = 100$
 $p^2 + q^2 + 2 \times 5 = 100$
 $p^2 + q^2 = 90$
 Now,

$\therefore \frac{p}{q} + \frac{q}{p} = \frac{p^2 + q^2}{pq} = \frac{90}{5} = 18$

122. (d) $x = 3 + 2\sqrt{2}$, $xy = 1$, $y^2 = \frac{1}{x^2}$

$y = \frac{1}{x} = \frac{1}{3 + 2\sqrt{2}} = 3 - 2\sqrt{2}$

$\therefore x + \frac{1}{x} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6$

$\therefore x^2 + \frac{1}{x^2} = 36 - 2 = 34$

$= \frac{x^2 + 3xy + y^2}{x^2 - 3xy + y^2} = \frac{x^2 + \frac{1}{x^2} + 3}{x^2 + \frac{1}{x^2} - 3}$

$= \frac{34 + 3}{34 - 3} = \frac{37}{31}$

$$123. (a) \frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b} \text{ (given)}$$

$$\therefore \frac{x}{b+c} = \frac{y}{c+a}$$

$$\Rightarrow \frac{x}{b+c} = \frac{x-y}{(b+c)-(c+a)} = \frac{x-y}{b-a}$$

Similarly

$$\frac{y}{c+a} = \frac{z}{a+b}$$

$$\therefore \frac{y}{c+a} = \frac{y-z}{(c+a)-(a+b)} = \frac{y-z}{c-b} \text{ again}$$

$$\Rightarrow \frac{z}{a+b} = \frac{x}{b+c}$$

$$\Rightarrow \frac{z}{a+b} = \frac{z-x}{(a+b)-(b+c)} = \frac{z-x}{a-c}$$

$$\therefore \frac{x-y}{b-a} = \frac{y-z}{c-b} = \frac{z-x}{a-c}$$

$$124. (d) \begin{aligned} x-y &= 2, \quad xy = 24 \quad (\text{given}) \\ x^2 + y^2 - 2xy &= 4 \\ x^2 + y^2 - 2 \times 24 &= 4 \\ x^2 + y^2 &= 4 + 48 = 52 \end{aligned}$$

$$125. (a) \frac{x^2}{y^2} + tx + \frac{y^2}{4} \quad (\text{given})$$

To make it a perfect square it should be in the form
 $A^2 + 2AB + B^2 = (A+B)^2$

$$= \left(\frac{x}{y}\right)^2 + tx + \left(\frac{y}{2}\right)^2$$

$$= A^2 + 2AB + B^2$$

$$A = \frac{x}{y}, \quad B = \frac{y}{2} \quad \& \quad 2AB = tx$$

$$\text{So, } tx = 2 \times \frac{x}{y} \times \frac{y}{2}$$

$$tx = x$$

$$t = 1$$

$$126. (d) \begin{aligned} a &= x+y \\ b &= x-y \\ c &= x+2y \end{aligned}$$

$$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} ((a-b)^2 + (b-c)^2 + (c-a)^2)$$

$$= \frac{1}{2} ((x+y-x+y)^2 + (x-y-x-2y)^2 + (x+2y-x-y)^2)$$

$$= \frac{1}{2} ((2y)^2 + (-3y)^2 + y^2)$$

$$= \frac{1}{2} (4y^2 + 9y^2 + y^2)$$

$$= \frac{1}{2} (14y^2) = 7y^2$$

$$127. (b) \quad x + \frac{1}{x} = 2, \quad x \neq 0$$

$$\text{put } x = 1$$

$$1 + 1 = 2$$

$$\therefore x^2 + \frac{1}{x^3} = 1 + 1 = 2$$

$$128. (a) \quad \frac{a}{b} + \frac{b}{a} = 1$$

$$\frac{a^2 + b^2}{ab} = 1$$

$$\begin{aligned} a^2 + b^2 - ab &= 0 \\ \therefore a^3 + b^3 &= (a+b)(a^2 - ab + b^2) \\ &= (a+b) \times 0 = 0 \end{aligned}$$

$$129. (a) \quad \left(x + \frac{1}{x}\right)^2 = 3$$

$$x + \frac{1}{x} = \sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 0$$

$$\Rightarrow x^6 + 1 = 0$$

$$\Rightarrow x^6 = -1$$

$$\Rightarrow x^{72} + x^{66} + x^{54} + x^{24} + x^6 + 1$$

$$\begin{aligned} &= (x^6)^{12} + (x^6)^{11} + (x^6)^9 + (x^6)^4 \\ &+ x^6 + 1 \\ &\Rightarrow (-1)^{12} + (-1)^{11} + (-1)^9 + (-1)^4 + -1 + 1 \\ &\Rightarrow 1 - 1 - 1 + 1 - 1 + 1 = 0 \end{aligned}$$

$$130. (b) \quad a + \frac{1}{a} = \sqrt{3}$$

$$a^6 = -1$$

$$\therefore a^6 - \frac{1}{a^6} + 2 = -1 - \frac{1}{(-1)} + 2$$

$$= -1 + 1 + 2 = 2$$

$$131. (b) \quad x^3 + y^3 = 35$$

$$\Rightarrow x + y = 5$$

Take cube on both sides,

$$(x+y)^3 = (5)^3$$

$$x^3 + y^3 + 3xy(x+y) = 125$$

$$35 + 3xy(5) = 125$$

$$15xy = 125 - 35$$

$$15xy = 90$$

$$xy = 6$$

$$\therefore \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{5}{6}$$

$$132. (b) \quad a^3 - b^3 = 56$$

$$\Rightarrow a - b = 2$$

$$\Rightarrow a^3 - b^3 - 3ab(a-b) = (2)^3$$

(By cubing)

$$56 - 3ab \times 2 = 8$$

$$-6ab = 8 - 56$$

$$6ab = 48$$

$$\Rightarrow ab = 8$$

$$(a-b)^2 = 2$$

$$(a-b)^2 = a^2 + b^2 - 2ab = 4$$

$$= a^2 + b^2 = 4 + 2ab$$

$$a^2 + b^2 = 4 + 2 \times 8 = 20$$

$$133. (b) \quad (a^2 + b^2)^3 = (a^3 + b^3)^2$$

$$\Rightarrow a^6 + b^6 + 3a^2b^2(a^2 + b^2) = a^6 + b^6 + 2a^3b^3$$

$$\Rightarrow a^6 + b^6 + 3a^4b^2 + 3a^2b^4 = a^6 + b^6 + 2a^3b^3$$

$$\Rightarrow 3a^4b^2 + 3a^2b^4 = 2a^3b^3$$

$$\Rightarrow a^2b^2(3a^2 + 3b^2) = 2a^3b^3$$

$$3a^2 + 3b^2 = 2ab$$

$$3(a^2 + b^2) = 2ab$$

$$\Rightarrow \frac{a^2 + b^2}{ab} = \frac{2}{3}$$

$$\Rightarrow \frac{a}{b} + \frac{b}{a} = \frac{2}{3}$$

$$134. (a) \quad x + \frac{1}{x} = 5 \quad x^2 + \frac{1}{x^2} + 2 = 25$$

$$x^2 + \frac{1}{x^2} = 23$$

$$\text{now, } \frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$$

divided by x^2 ,

$$= \frac{\frac{x^4}{x^2} + \frac{3x^3}{x^2} + \frac{5x^2}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}}{\frac{x^4}{x^2} + \frac{1}{x^2}}$$

$$= \frac{x^2 + 3x + 5 + \frac{3}{x} + \frac{1}{x^2}}{x^2 + \frac{1}{x^2}}$$

$$= \frac{x^2 + \frac{1}{x^2} + 3\left(x + \frac{1}{x}\right) + 5}{x^2 + \frac{1}{x^2}}$$

$$= \frac{23 + 3(5) + 5}{23} = \frac{43}{23}$$

$$135. (b) \quad x^3 + \frac{1}{x^3} = 0$$

$$\left(x + \frac{1}{x}\right)^3 - 3x \times \frac{1}{x} \left(x + \frac{1}{x}\right) = 0$$

$$\left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = 0$$

$$\left(x + \frac{1}{x}\right)^3 = 3\left(x + \frac{1}{x}\right)$$

$$\left(x + \frac{1}{x}\right)^2 = 3$$

$$\left(\left(x + \frac{1}{x}\right)^2\right)^2 = (3)^2$$

$$\left(x + \frac{1}{x}\right)^4 = 9$$

136. (c) $x + \frac{1}{x} = 3$

(Squaring both sides)

$$x^2 + \frac{1}{x^2} = 7$$

On cubing both sides

$$x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x} \left(x + \frac{1}{x} \right) = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27$$

$$x^3 + \frac{1}{x^3} = 18$$

$$\therefore \left(x^3 + \frac{1}{x^3} \right) \left(x^2 + \frac{1}{x^2} \right) = 18 \times 7$$

$$\left(x^5 + \frac{1}{x^5} \right) + \left(x + \frac{1}{x} \right) = 126$$

$$\left(x^5 + \frac{1}{x^5} \right) + 3 = 126$$

$$\left(x^5 + \frac{1}{x^5} \right) = 123$$

137. (a) $m^4 + \frac{1}{m^4} = 119$

$$m^4 + \frac{1}{m^4} + 2 = 119 + 2$$

$$\left(m^2 + \frac{1}{m^2} \right)^2 = 121$$

$$m^2 + \frac{1}{m^2} = 11$$

$$m^2 + \frac{1}{m^2} - 2 = 11 - 2$$

$$\left(m - \frac{1}{m} \right)^2 = 9$$

$$m - \frac{1}{m} = \pm 3$$

138. (d) $x + y + z = 6$
 $(x-1)^3 + (y-2)^3 + (z-3)^3$

$$\therefore \text{ as } x + y + z = 6$$

Take values

$$x = 1, y = 2, z = 3$$

$$(1 + 2 + 3) = 6$$

$$\therefore (1-1)^3 + (2-2)^3 + (3-3)^3 = 0$$

Now assume values in options.

option 'd' satisfies the given relation.

Hence 'd' is correct.

139. (b) $a + b + c = 6$

$$= a^2 + b^2 + c^2 = 14$$

$$a^3 + b^3 + c^3 = 36$$

Put values as

$$a = 1, b = 2, c = 3$$

$$1 + 2 + 3 = 6$$

$$1 + 4 + 9 = 14$$

$$1 + 8 + 27 = 36$$

$$\therefore abc = 1 \times 2 \times 3 = 6$$

Alternate:

$$\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$36 = 14 + 2(ab + bc + ca)$$

$$(ab + bc + ca) = 11$$

$$\Rightarrow a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\Rightarrow 36 - 3abc = 6(14 - 11)$$

$$\Rightarrow 36 - 3abc = 6 \times 3$$

$$-3abc = 18 - 36$$

$$3abc = 18$$

$$abc = 6$$

140. (a) $a + b = 1$

By cubing

$$a^3 + b^3 + 3ab(a + b) = 1^3$$

$$a^3 + b^3 + 3ab = 1 \quad (a + b = 1)$$

$$a^3 + b^3 + 3ab = k$$

From above both equations

$$k = 1$$

141. (d) $a = 34, b = 33, c = 33$

$$\therefore a^3 + b^3 + c^3 - 3abc = (a + b + c) \times$$

$$\frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$= (34 + 33 + 33) \times \frac{1}{2}$$

$$[(34-33)^2 + (33-33)^2 + (33-34)^2]$$

$$= 100 \times \frac{1}{2} (1 + 0 + 1)$$

$$= 100 \times 1 = 100$$

142. (a) $2^x \cdot 2^y = 8 \Rightarrow 2^{x+y} = 2^3$

$$x + y = 3$$

$$9^x \cdot 3^y = 81$$

$$3^{2x} \cdot 3^y = 3^4$$

$$\Rightarrow 2x + y = 4$$

.....(ii)

from equation (i) and (ii),

$$\begin{array}{r} x + y = 3 \\ 2x + y = 4 \\ \hline -x = -1 \end{array}$$

$$x = 1$$

$$y = 2$$

$$\therefore (x, y) = (1, 2)$$

143. (b) $x^4 - 2x^2 + k$

$$\Rightarrow (A + B)^2 = A^2 + 2AB + B^2$$

$$(A - B)^2 = A^2 - 2AB + B^2$$

$$\Rightarrow (x^2)^2 - 2 \times x^2 + k$$

$$(A)^2 - 2 \times AB + B^2$$

$$\therefore A = x^2, B = -1$$

$$B^2 = K$$

$$(-1)^2 = K$$

$$K = 1$$

144. (c) $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} + \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} - 3 \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} + \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$
 $\frac{1}{3 \times 3 \times 3} + \frac{1}{4 \times 4 \times 4} - \left(\frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 3} \right)$

$$= \frac{\left(\frac{1}{3} \right)^3 + \left(\frac{1}{4} \right)^3 + \left(\frac{1}{5} \right)^3 - 3 \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5}}{\left(\frac{1}{3} \right)^2 + \left(\frac{1}{4} \right)^2 + \left(\frac{1}{5} \right)^2 - \left(\frac{1}{3} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{5} + \frac{1}{5} \times \frac{1}{3} \right)}$$

$$\therefore A^3 + B^3 + C^3 - 3ABC = (A^2 + B^2 + C^2 - AB - BC - CA)(A + B + C)$$

$$\text{Let } \frac{1}{3} = A$$

$$\frac{1}{4} = B$$

$$\frac{1}{5} = C$$

$$\Rightarrow \frac{(A+B+C)(A^2+B^2+C^2-AB-BC-CA)}{(A^2+B^2+C^2-AB-BC-CA)}$$

$$= A + B + C$$

$$\therefore A + B + C = \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$

$$= \frac{20 + 15 + 12}{60} = \frac{47}{60}$$

145. (d) $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$

Adding 3 on both sides

$$\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} + 3 = 1 + 3$$

$$\frac{a}{1-a} + 1 + \frac{b}{1-b} + 1 + \frac{c}{1-c} + 1 = 4$$

$$\Rightarrow \frac{a+1-a}{1-a} + \frac{b+1-b}{1-b} + \frac{c+1-c}{1-c} = 4$$

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 4$$

146. (c) $a^2 + b^2 + c^2 = 2(a - b - c) - 3$

$$\Rightarrow a^2 + b^2 + c^2 - 2a + 2b + 2c + 1 + 1 + 1 = 0$$

$$\Rightarrow a^2 - 2a + 1 + b^2 + 2b + 1 + c^2 + 2c + 1 = 0$$

$$\Rightarrow (a-1)^2 + (b+1)^2 + (c+1)^2 = 0$$

$$a = 1, b = -1, c = -1$$

$$\therefore 2a - 3b + 4c = 2 \times 1 - 3 \times -1 + 4 \times -1$$

$$= 2 + 3 - 4 = 1$$

147. (b) $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(a-b)(c-a)} +$

$$\frac{(c-a)^2}{(a-b)(b-c)}$$

$$\Rightarrow \frac{(a-b)^2}{(b-c)(c-a)} \times \frac{(a-b)}{(a-b)} + \frac{(b-c)^2}{(a-b)(c-a)} \times \frac{(b-c)}{(b-c)} + \frac{(c-a)^2}{(a-b)(b-a)} \times \frac{(c-a)}{(c-a)}$$

$$\Rightarrow \frac{(a-b)^3}{(b-c)(c-a)(a-b)} + \frac{(b-c)^3}{(a-b)(c-a)(b-c)} + \frac{(c-a)^3}{(a-b)(b-a)(c-a)}$$

$$= \frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(b-c)(c-a)(a-b)}$$

(If $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xy$)

$$= \frac{3(a-b)(b-c)(c-a)}{(b-c)(c-a)(a-b)} = 3$$

148. (c) $(x-3)^2 + (y-5)^2 + (z-4)^2 = 0$

$$\therefore \begin{aligned} (x-3)^2 &= 0 & x &= 3 \\ (y-5)^2 &= 0 & y &= 5 \\ (z-4)^2 &= 0 & z &= 4 \end{aligned}$$

$$\therefore \frac{x^2}{9} + \frac{y^2}{25} + \frac{z^2}{16} = \frac{9}{9} + \frac{25}{25} + \frac{16}{16} = 3$$

149. (d) $x \propto \frac{1}{y^2}$ (Inversely proportional)

$$x = \frac{k}{y^2}$$

(y = 2) for (x = 1) (Given)

$$\therefore 1 = \frac{k}{(2)^2} \Rightarrow 1 = \frac{k}{4}$$

$$k = 4$$

$\therefore$ For $y = 6$

$$x = \frac{4}{(6)^2} = \frac{1}{9} = \frac{1}{9}$$

150. (d) $x^2 - y^2 = 80$

$$\Rightarrow (x-y)(x+y) = 80$$

$$x - y = 8 \quad \dots\dots(I)$$

$$\therefore (x+y) \times 8 = 80$$

$$(x+y) = 10 \quad \dots\dots(II)$$

Now average of x and y

$$= \frac{x+y}{2} = \frac{10}{2}$$

$$= 5$$

151. (a) Third proportional of a and $b = \frac{b^2}{a}$

Third proportion of $\left(\frac{x}{y} + \frac{y}{x}\right)$ and

$$\sqrt{x^2 + y^2}$$

$$\frac{(\sqrt{x^2 + y^2})^2}{\frac{x}{y} + \frac{y}{x}} = \frac{x^2 + y^2}{xy} = xy$$

152. (a) $\frac{4+3\sqrt{3}}{7+4\sqrt{3}}$

(By Rationalization of denominator)

$$\frac{4+3\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{(4+3\sqrt{3})(7-4\sqrt{3})}{49-48}$$

$$\frac{4+3\sqrt{3}}{7-4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{(4+3\sqrt{3})(7-4\sqrt{3})}{-16\sqrt{3}-12 \times 3}$$

$$\frac{28+21\sqrt{3}}{28-36+5\sqrt{3}} = \frac{5\sqrt{3}+8}{-8+5\sqrt{3}}$$

153. (b) $x = \frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$

$$\frac{x+\sqrt{20}}{x-\sqrt{20}} + \frac{x+\sqrt{12}}{x-\sqrt{12}} = ?$$

$$\therefore x = \frac{4 \times \sqrt{5} \times \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{2 \times \sqrt{5} \times \sqrt{12}}{\sqrt{5} + \sqrt{3}}$$

or $\frac{2 \times \sqrt{20} \times \sqrt{3}}{\sqrt{5} + \sqrt{3}}$

$$\frac{x}{\sqrt{12}} = \frac{2\sqrt{5}}{\sqrt{5} + \sqrt{3}}$$

by C - D rule

$$\frac{x+\sqrt{12}}{x-\sqrt{12}} = \frac{2\sqrt{5}+\sqrt{5}+\sqrt{3}}{2\sqrt{5}-\sqrt{5}-\sqrt{3}}$$

$$= \frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$\frac{x}{\sqrt{20}} = \frac{2\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$\frac{x+\sqrt{20}}{x-\sqrt{20}} = \frac{2\sqrt{3}+\sqrt{5}+\sqrt{3}}{2\sqrt{3}-\sqrt{5}-\sqrt{3}}$$

$$= \frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}$$

$$\therefore \frac{x+\sqrt{12}}{x-\sqrt{12}} + \frac{x+\sqrt{20}}{x-\sqrt{20}}$$

$$= \frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}}$$

$$= \frac{3\sqrt{5}+\sqrt{3}-3\sqrt{3}-\sqrt{5}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{2\sqrt{5}-2\sqrt{3}}{\sqrt{5}-\sqrt{3}} = 2$$

154. (b) $x = 5 - \sqrt{21}$

$$2x = 10 - 2\sqrt{21} \quad \dots\dots(i)$$

$$\Rightarrow 2x = (\sqrt{7})^2 + (\sqrt{3})^2 - 2(\sqrt{7})(\sqrt{3})$$

$$\Rightarrow 2x = (\sqrt{7}-\sqrt{3})^2$$

$$\Rightarrow x = \frac{1}{2}(\sqrt{7}-\sqrt{3})^2$$

$$\Rightarrow \sqrt{x} = \frac{1}{\sqrt{2}} \sqrt{(\sqrt{7}-\sqrt{3})^2}$$

$$= \frac{1}{\sqrt{2}}(\sqrt{7}-\sqrt{3})$$

$$\therefore \frac{\sqrt{x}}{\sqrt{32-2x}-\sqrt{21}}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}(\sqrt{32-(10-2\sqrt{21})}-\sqrt{21})}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}(\sqrt{22+2\sqrt{21}}-\sqrt{21})}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}(\sqrt{(\sqrt{21}+1)^2}-\sqrt{21})}$$

$$= \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}(\sqrt{21}+1-\sqrt{21})}$$

$$= \frac{(\sqrt{7}-\sqrt{3})}{\sqrt{2}}$$

155. (a) $(x^{b+c})^{b-c} (x^{c+a})^{c-a} (x^{a+b})^{a-b} (x \neq 0)$

$$= x^{b^2-c^2} \cdot x^{c^2-a^2} \cdot x^{a^2-b^2}$$

$$x^{b^2-c^2+c^2-a^2+a^2-b^2} = x^0 = 1$$

156. (c) $\frac{x}{a} = \frac{1}{a} - \frac{1}{x}$

$$\frac{x}{a} - \frac{1}{a} = -\frac{1}{x}$$

$$\left(\frac{x-1}{a}\right) = -\frac{1}{x}$$

$$\frac{1-x}{a} = \frac{1}{x}$$

$$x(1-x) = a$$

$$x - x^2 = a$$

157. (c) $x + \frac{1}{x} = 99$

$$\therefore x^2 + 1 = 99x$$

$$2(x^2 + 1) = 2 \times 99x$$

$$2x^2 + 2 = 198x$$

$$= \frac{100x}{2x^2 + 2 + 102x}$$

$$= \frac{100x}{198x + 102x} = \frac{100x}{300x} = \frac{1}{3}$$

158. (c) $\frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0$

$$\Rightarrow \frac{4x}{x} - \frac{3}{x} + \frac{4y}{y} - \frac{3}{y} + \frac{4z}{z} - \frac{3}{z} = 0$$

$$= 4 - \frac{3}{x} + 4 - \frac{3}{y} + 4 - \frac{3}{z} = 0$$

$$12 - 3\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$$

$$-3\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = -12$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 4$$

159. (c) $\frac{xy}{x+y} = a, \frac{xz}{x+z} = b, \frac{yz}{y+z} = c$

Now

$$\Rightarrow \frac{x+y}{xy} = \frac{1}{a}, \frac{x+z}{xz} = \frac{1}{b}, \frac{y+z}{yz} = \frac{1}{c}$$

$$= \frac{1}{c}$$

$$\Rightarrow \frac{1}{y} + \frac{1}{x} = \frac{1}{a}, \frac{1}{z} + \frac{1}{x} = \frac{1}{b}, \frac{1}{z} + \frac{1}{y} = \frac{1}{c}$$

$$+ \frac{1}{y} = \frac{1}{c}$$

Now we have to find the value of x

$$\therefore \frac{1}{a} + \frac{1}{b} - \frac{1}{c} = \frac{1}{y} + \frac{1}{x} + \frac{1}{z} + \frac{1}{x} - \frac{1}{y} - \frac{1}{z}$$

$$\therefore \frac{1}{a} + \frac{1}{b} - \frac{1}{c} = \frac{2}{x}$$

$$\frac{bc + ac - ab}{abc} = \frac{2}{x}$$

$$x = \frac{2abc}{bc + ac - ab}$$

160. (d) $xy = 8$

So, $(x, y) = (1, 8)$

We have to Questions the options and check them

(8, 1)

(2, 4)

(4, 2)

$$\therefore 2x + y = 2 \times 1 + 8 = 10$$

$$2 \times 8 + 1 = 11$$

$$2 \times 2 + 4 = 8 \text{ minimum}$$

$$2 \times 4 + 2 = 10$$

Hence in this question we have all the options. So, take all positive factor otherwise we should have to take -ve values also.

$(x, y) = (1, 8)$

(8, 1)

(2, 4)

(4, 2)

(-1, -8)

(-8, -1)

(-2, -4)

(-4, -2)

161. (b) $x^2 + x + 1 = 0$ (i)

$$\left(x + \frac{1}{2}\right)^2 + q^2 = 0$$

$$\Rightarrow x^2 + \frac{1}{4} + 2 \times \frac{1}{2} \times x + q^2$$

$$= x^2 + x + \left(q^2 + \frac{1}{4}\right) = 0 \text{(ii)}$$

Comparing constant term of equation (i) and (ii)

$$\therefore q^2 + \frac{1}{4} = 1$$

$$= q^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$q = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$

162. (b) $a^2 - 4a - 1 = 0$

$$a^2 - 1 = 4a$$

$$a - \frac{1}{a} = 4$$

Squaring both sides

$$a^2 + \frac{1}{a^2} - 2 = 16$$

$$a^2 + \frac{1}{a^2} = 18$$

$$\therefore a^2 + \frac{1}{a^2} + 3\left(a - \frac{1}{a}\right)$$

$$= 18 + 3 \times 4 = 18 + 12 = 30$$

163. (d) $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$$4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$$

$$(4x - \sqrt{3})(\sqrt{3}x + 2) = 0$$

164. (c) $\sqrt{x} = \sqrt{3} - \sqrt{5}$

$$x = 3 + 5 - 2\sqrt{3}\sqrt{5}$$

$$x = 8 - 2\sqrt{5} \times \sqrt{3}$$

$$x - 8 = -2\sqrt{15}$$

(Squaring both sides)

$$x^2 + 64 - 16x = 60$$

$$x^2 + 4 - 16x = 0$$

$$x^2 + 6 - 16x = 2$$

165. (d) $x + y + z = 0$

$$\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} = ?$$

$$\frac{x^3}{xyz} + \frac{y^3}{yxz} + \frac{z^3}{zxy}$$

$$\Rightarrow \frac{x^3 + y^3 + z^3}{zxy}$$

$$x + y + z = 0$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 0$$

$$x^3 + y^3 + z^3 = 3xyz$$

$$\therefore \frac{3xyz}{zxy} = 3$$

166. (c) $a + b + c = 0$

Have values

$a = 1, b = 2, c = -3$

$$\Rightarrow \left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right) \left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right)$$

$$\Rightarrow \left(\frac{1+2}{-3} + \frac{2-3}{1} + \frac{-3+1}{2}\right) \left(\frac{1}{2-3} + \frac{2}{-3+1} + \frac{-3}{1+2}\right)$$

$$\Rightarrow (-1 - 1 - 1)(-1 - 1 - 1)$$

$$\Rightarrow -3 \times -3 = 9$$

167. (a) $a + \frac{1}{b} = 1, b + \frac{1}{c} = 1$

Values of a, b, c assume

$$a = \frac{1}{2}$$

$$b = 2$$

$$c = -1$$

$$\therefore abc = \frac{1}{2} \times 2 \times -1 = -1$$

168. (c) $a + b + c = 2s$

$\therefore$ let

$$a = 2$$

$$b = 1$$

$$c = 1$$

$$s = 2$$

$$\therefore \frac{(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2}{a^2 + b^2 + c^2}$$

$$\Rightarrow \frac{(2-2)^2 + (2-1)^2 + (2-1)^2 + 2^2}{2^2 + 1^2 + 1^2}$$

$$\Rightarrow \frac{0 + 1 + 1 + 4}{4 + 1 + 1} = \frac{6}{6} = 1$$

169. (d) $x = 3 + 2\sqrt{2}$

$$x^2 = (3 + 2\sqrt{2})^2$$

(Squaring both sides)

$$= 9 + 8 + 12\sqrt{2}$$

$$= 17 + 12\sqrt{2}$$

$$\frac{1}{x^2} = \frac{1}{17 + 12\sqrt{2}} \times \frac{17 - 12\sqrt{2}}{17 - 12\sqrt{2}}$$

$$= 17 - 12\sqrt{2}$$

$$\therefore \frac{1}{x^2} + x^2 = 17 + 12\sqrt{2} + 17 - 12\sqrt{2} = 34$$

170. (b) $x \left(3 - \frac{2}{x} \right) = \frac{3}{x}$

$$\Rightarrow 3x - 2 = \frac{3}{x}$$

$$\Rightarrow 3x - \frac{3}{x} = 2$$

$$\Rightarrow 3 \left(x - \frac{1}{x} \right) = 2$$

$$\Rightarrow x - \frac{1}{x} = \frac{2}{3}$$

Squaring both sides

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 = \frac{4}{9}$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \frac{4}{9} + 2 = 2\frac{4}{9}$$

171. (a) $x^2 - 3x + 1 = 0$

$$\Rightarrow x^2 + 1 = 3x$$

$$x + \frac{1}{x} = 3$$

Squaring both sides

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 9$$

$$x^2 + \frac{1}{x^2} = 7$$

$$\therefore x^2 + \frac{1}{x^2} + x + \frac{1}{x}$$

$$= 7 + 3 = 10$$

172. (c) $a^2 + b^2 = 5ab$

$$\Rightarrow \frac{a^2}{ab} + \frac{b^2}{ab} = 5$$

$$\Rightarrow \frac{a}{b} + \frac{b}{a} = 5$$

Squaring both sides

$$\left(\frac{a}{b} \right)^2 + \left(\frac{b}{a} \right)^2 + 2 \times \frac{a}{b} \times \frac{b}{a} = 25$$

$$\frac{a^2}{b^2} + \frac{b^2}{a^2} = 25 - 2 = 23$$

173. (d) $xy + yz + zx = 0$

$$\therefore xy + zx = -yz$$

$$\Rightarrow xy + yz = -zx$$

$$\Rightarrow yz + zx = -xy$$

$$\therefore \frac{1}{x^2 - yz} + \frac{1}{y^2 - zx} + \frac{1}{z^2 - xy}$$

Putting values of $-yz$, $-zx$, $-xy$ from above

$$\Rightarrow \frac{1}{x^2 + (xy + zx)} + \frac{1}{y^2 + (xy + yx)}$$

$$+ \frac{1}{z^2 + (yz + zx)}$$

$$\Rightarrow \frac{1}{x(x + y + z)} + \frac{1}{y(x + y + z)}$$

$$+ \frac{1}{z(x + y + z)}$$

$$\Rightarrow \frac{1}{(x + y + z)} \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)$$

$$\Rightarrow \frac{1}{(x + y + z)} \left(\frac{zy + xz + xy}{xyz} \right)$$

$$\Rightarrow \frac{1}{x + y + z} \times 0 = 0$$

174. (c) $a + b + c = 9$

For minimum value $a = b = c$

$$\Rightarrow 3a = 9$$

$$a = \frac{9}{3} = 3$$

For minimum value $a = b = c = 3$

$$a^2 + b^2 + c^2 = 3^2 + 3^2 + 3^2$$

$$\Rightarrow 9 + 9 + 9 \Rightarrow 27$$

175. (d) $a^2 + b^2 + 4c^2 = 2(a + b - 2c) - 3$

$$\Rightarrow a^2 + b^2 + 4c^2 - 2a - 2b + 4c + 3 = 0$$

$$\Rightarrow a^2 - 2a + 1 + b^2 - 2b + 1 + 4c^2 + 4c + 1 = 0$$

$$(a - 1)^2 + (b - 1)^2 + (2c + 1)^2 = 0$$

$$\therefore a - 1 = 0 \quad a = 1$$

$$b - 1 = 0 \quad b = 1$$

$$2c + 1 = 0 \quad c = -\frac{1}{2}$$

$$\therefore a^2 + b^2 + c^2 = 1 + 1 + \frac{1}{4} = 2 + \frac{1}{4}$$

$$\frac{1}{4} = \frac{9}{4} = 2\frac{1}{4}$$

176. (b) $4x - y = 2$

$$2x - 8y + 4 = 0$$

Note:-

For two linear equations

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where x and y are variable.

(i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then there will be unique solution.

(ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then infinite solution

(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then no solution.

$$\therefore \frac{4}{2} \neq \frac{-1}{-8}$$

So, the equations have only one solution

177. (d) $\frac{a}{b} = \frac{4}{5}$ and $\frac{b}{c} = \frac{15}{16}$

$$\Rightarrow \frac{a}{b} \times \frac{b}{c} = \frac{4}{5} \times \frac{15}{16} = \frac{3}{4}$$

$$\therefore \frac{a}{c} = \frac{3}{4}$$

$$\therefore \frac{18c^2 - 7a^2}{45c^2 + 20a^2} = \frac{c^2 \left(18 - 7 \frac{a^2}{c^2} \right)}{c^2 \left(45 + 20 \frac{a^2}{c^2} \right)}$$

$$= \frac{18 - 7 \left(\frac{a}{c} \right)^2}{45 + 20 \left(\frac{a}{c} \right)^2} = \frac{18 - 7 \times \frac{9}{16}}{45 + 20 \times \frac{9}{16}}$$

$$= \frac{18 - \frac{63}{16}}{45 + \frac{45}{4}} = \frac{225 \times 4}{16 \times 225} = \frac{1}{4}$$

178. (d) $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}$

Go through options 'd'

take $x = y = z$

$$\frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{x^2} = \frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{x^2}$$

$\therefore$ Option d is right

179. (b) $x = 3t$, (I)

$$y = \frac{1}{2} (t + 1)$$

$$x = 2y$$

$$\Rightarrow x = 2 \times \frac{1}{2} (t + 1)$$

$$x = t + 1 \quad \text{(II)}$$

$$\therefore 3t = t + 1 \text{ (from equation (i) and (ii))}$$

$$2t = 1$$

$$t = \frac{1}{2}$$

$$180. (c) \quad x^2 + \frac{1}{5}x + a^2$$

$$A^2 + 2 \times AB + B^2 = (A + B)^2$$

$$x^2 + 2 \times \frac{1}{10}x + a^2 = \left(x + \frac{1}{10}\right)^2$$

$$A = x$$

$$B = \frac{1}{10}$$

$$B = a = \frac{1}{10}$$

$$181. (d) \quad 2 - 3x - 4x^2 = 0$$

$$-4x^2 - 3x + 2 = 0$$

$$ax^2 + bx + c = 0$$

In quadratic equation

$$(i) \text{ When } a > 0$$

$$\text{Minimum value} = \frac{4ac - b^2}{4a}$$

$$(ii) \text{ When } a < 0$$

$$\text{Maximum value} = \frac{4ac - b^2}{4a}$$

$$\therefore \text{ In } -4x^2 - 3x + 2$$

$$a < 0$$

$$\therefore \text{ Maximum value}$$

$$= \text{Maximum value}$$

$$\frac{4 \times -4 \times 2 - (-3)^2}{4 \times -4}$$

$$= \frac{-32 - 9}{-16} = \frac{41}{16}$$

$$182. (a) \quad x^4 - 2x^2 + k$$

$$\Rightarrow (x^2)^2 - 2 \times x^2 \times 1 + (\sqrt{k})^2$$

$$\Rightarrow A^2 - 2 \times A \times B + (\sqrt{k})^2$$

$$\Rightarrow \sqrt{k} = \pm 1$$

$$k = 1$$

$$183. (d) \quad \frac{5x}{2x^2 + 5x + 1} = \frac{1}{3}$$

$$\frac{5}{\frac{2x^2}{x} + \frac{5x}{x} + \frac{1}{x}} = \frac{1}{3}$$

$$\frac{5}{2x + \frac{1}{x} + 5} = \frac{1}{3}$$

$$\left(2x + \frac{1}{x} + 5\right) = 15$$

$$2x + \frac{1}{x} = 10$$

Divide by 2 both sides

$$x + \frac{1}{2x} = \frac{10}{2} = 5$$

$$184. (c) \quad xy(x + y) = 1$$

$$x + y = \frac{1}{xy}$$

Cubing both sides

$$\Rightarrow (x + y)^3 = \frac{1}{x^3 y^3}$$

$$x^3 + y^3 + 3xy(x + y) = \frac{1}{x^3 y^3}$$

$$x^3 + y^3 + 3 = \frac{1}{x^3 y^3} \left(x + y = \frac{1}{xy}\right)$$

$$\frac{1}{x^3 y^3} - x^3 - y^3 = 3$$

$$185. (c) \quad x^2 + \frac{1}{x^2} = 83$$

Subtracting 2 from both sides

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 83 - 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 81$$

$$\Rightarrow x - \frac{1}{x} = 9$$

Take cube on both sides

$$x^3 - \frac{1}{x^3} - 3 \left(x - \frac{1}{x}\right) = 729$$

$$x^3 - \frac{1}{x^3} - 3 \times 9 = 729$$

$$x^3 - \frac{1}{x^3} = 729 + 27 = 756$$

$$186. (d) \quad \left(a + \frac{1}{a}\right)^2 = 3$$

$$a + \frac{1}{a} = \sqrt{3}$$

Take cube on both sides

$$\left(a + \frac{1}{a}\right)^3 = (\sqrt{3})^3$$

$$= a^3 + \frac{1}{a^3} + 3a \times \frac{1}{a} \left(a + \frac{1}{a}\right) = 3\sqrt{3}$$

$$= a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$$

$$= a^3 + \frac{1}{a^3} = 0$$

$$187. (b) \quad \frac{x}{x^2 - 2x + 1} = \frac{1}{3}$$

$$\Rightarrow \frac{1}{\frac{x^2}{x} - \frac{2x}{x} + \frac{1}{x}} = \frac{1}{3}$$

$$\Rightarrow \frac{1}{x + \frac{1}{x} - 2} = \frac{1}{3}$$

$$x + \frac{1}{x} - 2 = 3$$

$$\Rightarrow x + \frac{1}{x} = 3 + 2 = 5$$

Taking cube on both sides

$$\Rightarrow \left(x + \frac{1}{x}\right)^3 = (5)^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 5 = 125$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 110$$

$$188. (b) \quad x + \frac{1}{x} = 4$$

Squaring both sides

$$x^2 + \frac{1}{x^2} + 2 = 16$$

$$x^2 + \frac{1}{x^2} = 14$$

Squaring again

$$x^4 + \frac{1}{x^4} = 196 - 2 = 194$$

$$189. (c) \quad x + y + z = 6$$

$$x^2 + y^2 + z^2 = 20$$

$$\Rightarrow (x + y + z)^2 = (6)^2$$

$$\Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) = 36$$

$$\Rightarrow 20 + 2(xy + yz + zx) = 36$$

$$\Rightarrow 2(xy + yz + zx) = 16$$

$$xy + yz + zx = 8$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz$$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 6(20 - 8)$$

$$= 6 \times 12 = 72$$

$$190. (b) \quad x = 1 - \sqrt{2}$$

$$\frac{1}{x} = \frac{1}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}}$$

$$= \frac{1 + \sqrt{2}}{1 - 2} = \frac{1 + \sqrt{2}}{-1} = -(\sqrt{2} + 1)$$

$$\therefore x - \frac{1}{x} = 1 - \sqrt{2} + \sqrt{2} + 1 = 2$$

Take cube

$$\left(x - \frac{1}{x}\right)^3 = (2)^3 \Rightarrow \left(x - \frac{1}{x}\right)^3 = 8$$

$$\begin{aligned} 191. (b) \quad & x = a - b \\ & y = b - c \\ & z = c - a \\ & x + y + z = a - b + b - c + c - a = 0 \\ & \therefore x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = 0 \end{aligned}$$

$$192. (d) \quad x = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

$$y = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

$$\therefore x = \frac{1}{y}$$

$$y = \frac{1}{x} \Rightarrow xy = 1$$

$$\therefore x + y = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} + \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

$$\Rightarrow \frac{(\sqrt{3} - \sqrt{2})^2 + (\sqrt{3} + \sqrt{2})^2}{1}$$

$$\Rightarrow \frac{3 + 2 - 2\sqrt{6} + 3 + 2 + 2\sqrt{6}}{1} = 10$$

$$\Rightarrow (x + y)^3 = x^3 + y^3 + 3xy(x + y)$$

$$\Rightarrow (10)^3 = x^3 + y^3 + 3 \times 1(10)$$

$$\Rightarrow x^3 + y^3 = 1000 - 30 = 970$$

$$193. (c) \quad 2x + \frac{2}{x} = 3$$

$$\Rightarrow x + \frac{1}{x} = \frac{3}{2}$$

Taking cube on both sides

$$= \left(x + \frac{1}{x}\right)^3 = \left(\frac{3}{2}\right)^3$$

$$\left| \begin{array}{l} x + \frac{1}{x} = a \\ x^3 + \frac{1}{x^3} = a^3 - 3a \end{array} \right.$$

$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = \frac{27}{8}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times \frac{3}{2} = \frac{27}{8}$$

$$\Rightarrow x^3 + \frac{1}{x^3} = \frac{27}{8} - \frac{9}{2}$$

$$= \frac{27 - 36}{8} = \frac{-9}{8}$$

$$\therefore x^3 + \frac{1}{x^3} + 2 = \frac{-9}{8} + 2$$

$$= x^3 + \frac{1}{x^3} + 2 = \frac{-9 + 16}{8} = \frac{7}{8}$$

$$\begin{aligned} 194. (b) \quad & a + b + c = 15 \\ & a^2 + b^2 + c^2 = 83 \\ & \therefore (a + b + c)^2 = (15)^2 \end{aligned}$$

$$\Rightarrow a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 225$$

$$\Rightarrow 83 + 2(ab + bc + ca) = 225$$

$$\Rightarrow 2(ab + bc + ca) = 225 - 83 = 142$$

$$ab + bc + ca = 71$$

$$\Rightarrow a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\Rightarrow a^3 + b^3 + c^3 - 3abc = 15(83 - 71) = 15 \times 12 = 180$$

$$195. (b) \quad x + \frac{1}{x+1} = 1$$

adding (1) both sides

$$\therefore x + 1 + \frac{1}{x+1} = 1 + 1$$

$$\Rightarrow (x+1) + \left(\frac{1}{x+1}\right) = 2$$

$$\text{Put } x + 1 = 1$$

$$\text{and } \frac{1}{x+1} = 1$$

$$\therefore (x+1)^5 + \left(\frac{1}{x+1}\right)^5 = 1 + 1 = 2$$

$$196. (a) \quad \frac{1}{a} + \frac{1}{b} = \frac{1}{a+b}$$

$$\Rightarrow \frac{a+b}{ab} = \frac{1}{a+b}$$

$$\Rightarrow (a+b)^2 = ab$$

$$\Rightarrow a^2 + b^2 + 2ab = ab$$

$$a^2 + b^2 + ab = 0$$

$$a^3 - b^3 = (a-b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = 0$$

$$197. (d) \quad \text{If } a + b + c = 0$$

$$\text{then, } a^3 + b^3 + c^3 - 3abc = 0$$

$$a^3 + b^3 + c^3 = 3abc$$

$$198. (c) \quad x = y = 333, z = 334$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz$$

$$= \frac{1}{2}(x+y+z)((x-y)^2 + (y-z)^2 + (z-x)^2)$$

$$\Rightarrow \frac{1}{2}(333 + 333 + 334)[(333 + 333)^2 + (333 - 334)^2 + (334 - 333)^2]$$

$$\Rightarrow \frac{1}{2}(1000)(0 + 1 + 1) = 1000$$

$$199. (a) \quad (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$$

$$\text{Let } a^2 - b^2 = A$$

$$b^2 - c^2 = B$$

$$c^2 - a^2 = C$$

$$A + B + C = a^2 - b^2 + b^2 - c^2$$

$$+ c^2 - a^2 = 0$$

$$\therefore A^3 + B^3 + C^3 - 3ABC$$

$$= \frac{1}{2}(A + B + C)[(A - B)^2 + (B - C)^2 +$$

$$(C - A)^2]$$

$$A^3 + B^3 + C^3 - 3ABC = 0$$

$$A^3 + B^3 + C^3 = 3ABC$$

$$\text{Where } A = a^2 - b^2 \text{ etc.}$$

$$\therefore A^3 + B^3 + C^3 = 3 \times (a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$$

Hence $(a + b)(a - b)$ is a factor

$$200. (b) \quad a = \frac{b^2}{b-a}$$

$$\Rightarrow a(b-a) = b^2$$

$$ab - a^2 = b^2$$

$$a^2 + b^2 - ab = 0$$

$$\Rightarrow a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$$

$$\therefore a^3 + b^3 = 0$$

$$201. (b) \quad p - 2q = 4$$

Take cube on both sides

$$(p - 2q)^3 = (4)^3$$

$$p^3 - 8q^3 - 3p \times 2q(p - 2q) = 64$$

$$p^3 - 8q^3 - 6pq \times 4 = 64$$

$$p^3 - 8q^3 - 24pq = 64$$

$$p^3 - 8q^3 - 24pq - 64 = 0$$

$$202. (c) \quad x = -1$$

$$\frac{1}{x^{99}} + \frac{1}{x^{98}} + \frac{1}{x^{97}} + \frac{1}{x^{96}} + \frac{1}{x^{95}} + \frac{1}{x^{94}} +$$

$$\frac{1}{x} - 1$$

$$= \frac{1}{(-1)^{99}} + \frac{1}{(-1)^{98}} + \frac{1}{(-1)^{97}} + \frac{1}{(-1)^{96}} +$$

$$\frac{1}{(-1)^{95}} + \frac{1}{(-1)^{94}} + \frac{1}{(-1)} - 1$$

$$= -1 + 1 - 1 + 1 - 1 + 1 + \frac{1}{-1} - 1 = -2$$

$$203. (a) \quad \frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = a\sqrt[3]{4} + b\sqrt[3]{2} + c$$

$$\Rightarrow \frac{1}{\sqrt[3]{4} + \sqrt[3]{2} + 1} = \frac{1}{\left(\frac{1}{2^{\frac{1}{3}}}\right)^2 + \frac{1}{2^{\frac{1}{3}}} + (1)^2}$$

$$\Rightarrow (\therefore A^3 - B^3 = (A - B)(A^2 + AB + B^2))$$

$$\therefore \text{Put, } A = \frac{1}{2^{\frac{1}{3}}}, B = 1$$

$$\left(\frac{1}{2^{\frac{1}{3}}} - 1\right)$$

$$= \frac{\left(\frac{1}{2^{\frac{1}{3}}} - 1\right)}{\left(\left(\frac{1}{2^{\frac{1}{3}}} - 1\right)\left(\left(\frac{1}{2^{\frac{1}{3}}}\right)^2 + \frac{1}{2^{\frac{1}{3}}} + (1)^2\right)\right)}$$

$$= \frac{\left(\frac{1}{2^{\frac{1}{3}}} - 1\right)}{\left(\frac{1}{2^{\frac{1}{3}}}\right)^3 - (1)^3} = \left(\frac{1}{2^{\frac{1}{3}}} - 1\right)$$

$$\frac{1}{2^3} - 1 = a \left(\frac{2}{2^3} \right) + b \left(\frac{1}{2^3} \right) + c$$

(Comparing the terms)

$$a = 0$$

$$b = 1$$

$$c = -1$$

$$\therefore a + b + c = 0 + 1 - 1 = 0$$

204. (d) $x = \sqrt[3]{2 + \sqrt{3}}$

$$x^3 = 2 + \sqrt{3}$$

$$\frac{1}{x^3} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$\Rightarrow \frac{2 - \sqrt{3}}{4 - 3} = 2 - \sqrt{3}$$

$$\therefore x^3 + \frac{1}{x^3} = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$$

205. (d) $x = \sqrt[3]{5} + 2$

$$\Rightarrow x - 2 = \sqrt[3]{5}$$

Take cube on both sides

$$\Rightarrow (x - 2)^3 = (\sqrt[3]{5})^3$$

$$\Rightarrow x^3 - 8 - 3 \times 2 \times x [x - 2] = 5$$

$$\Rightarrow x^3 - 8 - 6x^2 + 12x = 5$$

$$\therefore x^3 - 6x^2 + 12x - 13 = 0$$

206. (b) $\frac{p^2 - p}{2p^3 + p^2} + \frac{p^2 - 1}{p^2 + 3p} + \frac{p^2}{p + 1}$

In such type of question assume values of p.

$$\therefore \text{Let } p = 1$$

$$\therefore \frac{1-1}{2+1} + \frac{1-1}{1+3} + \frac{1}{1+1}$$

$$= 0 + 0 + \frac{1}{2} = \frac{1}{2}$$

Now check options (b)

$$\frac{1}{2p^2} = \frac{1}{2}$$

Hence option (b) is Answer.

207. (b) $x + \frac{1}{x} = 2$

$$\Rightarrow \text{Put } x = 1$$

$$\therefore 1 + \frac{1}{1} = 2$$

$$2 = 2 \text{ (satisfy)}$$

$$\left(x^2 + \frac{1}{x^2} \right) \left(x^3 + \frac{1}{x^3} \right)$$

$$= (1 + 1) (1 + 1)$$

$$= 2 \times 2 = 4$$

208. (a) a, b, c, are +ve integers

So, minimum value is $a = b = c = 1$

$\therefore$ Putting the value of x in equation

$$a^3 + b^3 + c^3 - 3abc$$

$$= 1 + 1 + 1 - 3 \times 1 \times 1 \times 1 = 0$$

Hence minimum value is 0.

209. (d) $f(x) = 12x^3 - 13x^2 - 5x + 7$

If we divide $f(x)$ by

$$\Rightarrow (3x + 2) \text{ then}$$

$$3x + 2 = 0$$

$$x = -\frac{2}{3}$$

$$\therefore f\left(-\frac{2}{3}\right) = 12 \left(-\frac{2}{3}\right)^3 - 13 \left(-\frac{2}{3}\right)^2$$

$$- 5 \left(-\frac{2}{3}\right) + 7$$

$$= -12 \times \frac{8}{27} - \frac{52}{9} + \frac{10}{3} + 7$$

$$= \frac{-96 - 156 + 90 + 189}{27}$$

$$= \frac{-252 + 279}{27} = \frac{27}{27} = 1$$

210. (b) $2x^2 - 7x + 12 = 0$

roots are α, β

$$\therefore \alpha\beta = +\frac{c}{a}, \alpha + \beta = \frac{-b}{a} =$$

$$\therefore \alpha + \beta = +\frac{7}{2}, \alpha\beta = \frac{12}{2} = 6$$

$$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(\frac{7}{2}\right)^2 - 2 \times 6}{6} = \frac{\frac{49}{4} - 12}{6}$$

$$= \frac{49 - 48}{6 \times 4} = \frac{1}{24}$$

211. (c) $x^3 + \frac{3}{x} = 4 (a^3 + b^3)$

$$\text{and } 3x + \frac{1}{x^3} = 4 (a^3 + b^3)$$

$$\therefore x^3 + \frac{3}{x} = 3x + \frac{1}{x^3}$$

$$\Rightarrow x^3 - \frac{1}{x^3} = 3x - \frac{3}{x}$$

$$\left(x^3 - \frac{1}{x^3} \right) = 3 \left(x - \frac{1}{x} \right)$$

$$\Rightarrow \left(x - \frac{1}{x} \right) \left(x^2 + \frac{1}{x^2} + 1 \right)$$

$$= 3 \left(x - \frac{1}{x} \right)$$

$$x^2 + 1 + \frac{1}{x^2} = 3$$

$$x^2 + \frac{1}{x^2} = 2 \quad \boxed{x=1}$$

$$4(a^3 + b^3) = 1 + 3$$

$$\Rightarrow a^3 + b^3 = 1$$

$$\text{Let } a = 1 \quad b = 0$$

$$\therefore a^2 - b^2 = 1 - 0 = 1$$

or you can directly put the value of x also,

212. (a) $121a^2 + 64b^2$
 $= (11a)^2 + (8b)^2 + 2 \times 11a \times 8b$
 $= (11a + 8b)^2$

$\therefore$ So term added to make perfect square = 176 ab

213. (b) $a = 2 + \sqrt{3}$

$$\Rightarrow a^2 = (2 + \sqrt{3})^2$$

$$= 4 + 3 + 4\sqrt{3}$$

$$= 7 + 4\sqrt{3}$$

$$\Rightarrow \frac{1}{a^2} = \frac{1}{7 + 4\sqrt{3}}$$

$$= \frac{7 - 4\sqrt{3}}{(7 + 4\sqrt{3})(7 - 4\sqrt{3})}$$

$$= \frac{7 - 4\sqrt{3}}{1}$$

$$\therefore a^2 + \frac{1}{a^2} = 7 + 4\sqrt{3} + 7 - 4\sqrt{3} = 14$$

214. (a) $p + \frac{1}{4} + \sqrt{p} + k^2$

$$\Rightarrow p + \sqrt{p} + \left(k^2 + \frac{1}{4} \right)$$

$$\Rightarrow (\sqrt{p})^2 + 2 \times \frac{1}{2} \times \sqrt{p} + \left(k^2 + \frac{1}{4} \right)$$

$$\Rightarrow A^2 + 2 \times A \times B + B^2$$

$$\Rightarrow A = \sqrt{p} \quad B^2 = \left(k^2 + \frac{1}{4} \right)$$

$$B = \frac{1}{2}$$

$$\therefore k^2 + \frac{1}{4} = \left(\frac{1}{2} \right)^2$$

$$k^2 + \frac{1}{4} = \frac{1}{4}$$

$$k^2 = 0$$

$$k = 0$$

215. (a) Reciprocal of $\left(x + \frac{1}{x} \right)$

$$= \frac{1}{\left(x + \frac{1}{x} \right)} = \frac{x}{x^2 + 1}$$

216. (a) For minimum value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

$a = b = c$
 $a + b + c = 1$ (given)

$\therefore a = b = c = \frac{1}{3}$

$\frac{1}{a} = \frac{1}{b} = \frac{1}{c} = 3$

$\therefore$ Minimum value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$
 $= 3 + 3 + 3 = 9$

217. (b) $a(2 + \sqrt{3}) = b(2 - \sqrt{3}) = 1$

$a = \frac{1}{2 + \sqrt{3}}$ $b = \frac{1}{2 - \sqrt{3}}$

$\Rightarrow a = \frac{1}{b}$

$\Rightarrow \frac{1}{a^2 + 1} + \frac{1}{b^2 + 1}$

$\Rightarrow \frac{1}{\frac{1}{b^2} + 1} + \frac{1}{b^2 + 1}$

$\Rightarrow \frac{1}{\frac{1 + b^2}{b^2}} + \frac{1}{b^2 + 1}$

$\Rightarrow \frac{b^2}{b^2 + 1} + \frac{1}{b^2 + 1}$

$\Rightarrow \frac{b^2 + 1}{b^2 + 1} = 1$

218. (d) $(2 + \sqrt{3})a = (2 - \sqrt{3})b = 1$

$\Rightarrow \frac{1}{a} = (2 + \sqrt{3})$

By rationals

$\Rightarrow \frac{1}{b} = (2 - \sqrt{3})$

$\Rightarrow \frac{1}{a} + \frac{1}{b} = 2 - \sqrt{3} + 2 + \sqrt{3} = 4$

219. (a) $a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a}$

To save your time assume values of a, b, c according to equation.

Let $a = 2, b = -1$ & $c = \frac{1}{2}$

$2 + \frac{1}{-1} = -1 + \frac{1}{1/2} = \frac{1}{2} + \frac{1}{2}$

$= 1 = 1 = 1$

$\therefore abc = 2 \times -1 \times \frac{1}{2} = -1$

220. (d) $(x - 2)$ is a factor of $x^2 + 3Qx - 2Q$
for $(x - 2) = 0$
 $x^2 + 3Qx - 2Q = 0$
 $\Rightarrow 4 + 3 \times Q \times 2 - 2 \times Q = 0$

$\Rightarrow 4 + 6Q - 2Q = 0$

$4Q = -4$

$Q = -1$

221. (d) $a + b = 12$ (I)

$ab = 22$ (II)

Squaring both sides of equation (I)

$\Rightarrow a^2 + b^2 + 2ab = 144$

$a^2 + b^2 + 2 \times 22 = 144$

$a^2 + b^2 = 144 - 44 = 100$

222. (b) $x = \sqrt{3} - \frac{1}{\sqrt{3}}$ & $y = \sqrt{3} + \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{x^2}{y} + \frac{y^2}{x} = \frac{x^3 + y^3}{xy}$

$= \frac{(x + y)(x^2 - xy + y^2)}{xy}$

$\therefore x + y = \sqrt{3} - \frac{1}{\sqrt{3}} + \sqrt{3} + \frac{1}{\sqrt{3}} = 2\sqrt{3}$

$\therefore xy = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$

$= 3 - \frac{1}{3} = \frac{8}{3}$

$\Rightarrow \frac{(x + y)(x^2 + y^2 + 2xy - 2xy - xy)}{xy}$

$\Rightarrow \frac{(x + y)((x + y)^2 - 3xy)}{xy}$

$\Rightarrow \frac{2\sqrt{3}\left((2\sqrt{3})^2 - 3 \times \frac{8}{3}\right)}{\frac{8}{3}}$

$\Rightarrow \frac{2\sqrt{3}(12 - 8)}{\frac{8}{3}} \Rightarrow \frac{2 \times 3\sqrt{3}(4)}{8} = 3\sqrt{3}$

223. (b) $x^2 + ax + b$

$\Rightarrow x^2 + 2 \times \frac{1}{2} a \times x + (\sqrt{b})^2$

$\Rightarrow A^2 + 2 \times A \times B + B^2 = (A + B)^2$

$\therefore A = x, B = \frac{1}{2}a, B^2$

$= (\sqrt{b})^2, B = \sqrt{b}$

$\Rightarrow \left(x + \frac{1}{2}a\right)^2$ be perfect square

at $\sqrt{b} = \frac{1}{2}a, b = \frac{1}{4}a^2$

$\therefore a^2 = 4b$

224. (a) $a + b + c + d = 4$

$\frac{1}{(1-a)(1-b)(1-c)} + \frac{1}{(1-a)(1-b)(1-c)}$

$+ \frac{1}{(1-c)(1-d)(1-a)} + \frac{1}{(1-b)(1-c)(1-d)}$

Put $a = 0, b = 0$ and c

$= 2$ and $d = 2$

$a + b + c + d$

$= 0 + 0 + 2 + 2 = 4 = 4$ (satisfy)

$\frac{1}{(1-0)(1-0)(1-2)} + \frac{1}{(1-0)(1-2)(1-2)}$

$+ \frac{1}{(1-2)(1-2)(1-0)} + \frac{1}{(1-2)(1-0)(1-0)}$

$\Rightarrow \frac{1}{-1} + \left(\frac{1}{+1}\right) + \frac{1}{-1 \times -1} + \frac{1}{-1}$

$\Rightarrow -1 + 1 + 1 - 1 = 0$

225. (b) $x - \frac{1}{x} = 1$

$\Rightarrow \frac{x^4 - \frac{1}{x^2}}{3x^2 + 5x - 3}$

divide and multiply by x

$\Rightarrow \frac{\frac{x^4}{x} - \frac{1}{x^3}}{\frac{3x^2}{x} + \frac{5x}{x} - \frac{3}{x}}$

$= \frac{x^3 - \frac{1}{x^3}}{3x - \frac{3}{x} + 5} \Rightarrow \frac{x^3 - \frac{1}{x^3}}{3\left(x - \frac{1}{x}\right) + 5}$

$\Rightarrow x - \frac{1}{x} = 1$

Take cube on both sides

$\left(x - \frac{1}{x}\right)^3 = (1)^3$

$\Rightarrow x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 1$

$x^3 - \frac{1}{x^3} - 3 = 1$

$\Rightarrow x^3 - \frac{1}{x^3} = 4$

$= \frac{x^3 - \frac{1}{x^3}}{3\left(x - \frac{1}{x}\right) + 5} = \frac{4}{3 \times 1 + 5} = \frac{4}{8} = \frac{1}{2}$

226. (d) $x + y = 15$

$\Rightarrow x - 10 = 5 - y$

$$x - 10 = -(y - 5)$$

Take cube on both sides

$$\Rightarrow (x - 10)^3 = -(y - 5)^3$$

$$\Rightarrow (x - 10)^3 + (y - 5)^3 = 0$$

227. (b) $x^2 + \frac{1}{x^2} = 66$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 66 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 64$$

$$\left(x - \frac{1}{x}\right)^2 = (8)^2$$

$$\Rightarrow x - \frac{1}{x} = \pm 8$$

$$\Rightarrow \frac{x^2 - 1 + 2x}{x} = \frac{\frac{x^2}{x} - \frac{1}{x} + \frac{2x}{x}}{\frac{x}{x}}$$

$$\Rightarrow \frac{\left(x - \frac{1}{x}\right) + 2}{1}$$

When $x - \frac{1}{x} = +8$

Then $\left(x - \frac{1}{x}\right) + 2 = 8 + 2 = 10$

When $x - \frac{1}{x} = -8$

$$-8 + 2 = -6$$

$$\therefore (10, -6)$$

228. (c) $a^2 + a + 1 = 0$

$$\left[\begin{array}{l} a^3 + 1 = (a + 1)(a^2 - a + 1) \\ a^3 - 1 = (a - 1)(a^2 + a + 1) \end{array} \right]$$

$$\therefore (a^3 - 1) = (a - 1) \times 0$$

$$a^3 - 1 = 0$$

$$a^3 = 1$$

$$(a^3)^3 = 1^3$$

$$a^9 = 1$$

229. (a) $x + \frac{2}{x} = 1$

$$x^2 + 2 = x$$

$$x^2 - x = -2$$

$$x - x^2 = 2$$

$$\therefore \frac{x^2 + x + 2}{x^2(1-x)} = \text{divide \& multiply by } x$$

$$\Rightarrow \frac{\frac{x^2}{x} + \frac{x}{x} + \frac{2}{x}}{\frac{x^2}{x}(1-x)} = \frac{x + \frac{2}{x} + 1}{x(1-x)}$$

$$\Rightarrow \frac{x + \frac{2}{x} + 1}{x - x^2} = \frac{1 + 1}{2} = 1$$

230. (b) $y = 1 - 3k$ and $x = -2k$ (given)

$$\therefore \text{ For } x = y$$

$$-2k = 1 - 3k$$

$$k = 1$$

231. (b) $\sqrt{(x^2 + y^2 + z)(x + y - 3z)} + \sqrt[3]{xyz^2}$
 $x = 1$ $y = -3$ $z = -1$ (given)

$$\Rightarrow \sqrt{(1+9-1)(1-3+3)} + \sqrt[3]{1 \times (-3)^3 \times 1}$$

$$= 3 + (-3) = 0$$

232. (b) $x + \frac{1}{x} = 2$ (I)

Squaring both sides

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4$$

$$\Rightarrow x^2 + \frac{1}{x^2} = +2$$

Cubing equation (I)

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 8$$

$$x^3 + \frac{1}{x^3} + 6 = 8$$

$$x^3 + \frac{1}{x^3} = 2$$

$$\therefore \left(x^2 + \frac{1}{x^2}\right) \left(x^3 + \frac{1}{x^3}\right) = 2 \times 2 = 4$$

233. (a) $x + \frac{1}{x} = 5$

$\therefore$ Take cube on both sides

$$\Rightarrow \left(x + \frac{1}{x}\right)^3 = (5)^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 5 = 125$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 110$$

$\therefore$ Squaring both sides

$$\left(x^3 + \frac{1}{x^3}\right)^2 = (110)^2$$

$$\Rightarrow x^6 + \frac{1}{x^6} + 2 = 12100$$

$$x^6 + \frac{1}{x^6} = 12100 - 2 = 12098$$

234. (c) $x^2 - 3x + 1 = 0$

$$\Rightarrow x^2 + 1 = 3x$$

$$x + \frac{1}{x} = 3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27$$

$$x^3 + \frac{1}{x^3} = 18$$

$$\Rightarrow \frac{x^6 + x^4 + x^2 + 1}{x^3}$$

$$\Rightarrow \frac{x^6}{x^3} + \frac{x^4}{x^3} + \frac{x^2}{x^3} + \frac{1}{x^3}$$

$$\Rightarrow x^3 + \frac{1}{x^3} + \frac{1}{x} + x$$

$$\Rightarrow 18 + 3 = 21$$

235. (b) $\frac{(x+1)^3 - (x-1)^3}{(x+1)^2 - (x-1)^2} = 2$

$$\Rightarrow A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

$$A^2 - B^2 = (A - B)(A + B)$$

$$\Rightarrow \frac{(x+1-x+1)((x+1)^2 + (x-1)(x+1) + (x-1)^2)}{(x+1-x+1)(x+1+x-1)}$$

$$= 2$$

$$\Rightarrow \frac{(x^2+1+2x+x^2-1+x^2+1-2x)}{(2x)} = 2$$

$$\Rightarrow \frac{3x^2+1}{2x} = 2$$

$$\Rightarrow 3x^2 + 1 = 4x$$

$$3x^2 - 4x + 1 = 0$$

$$3x^2 - 3x - x + 1 = 0$$

$$3x(x-1) - 1(x-1) = 0$$

$$(3x-1)(x-1) = 0$$

$$\Rightarrow 3x - 1 = 0$$

$$x = \frac{1}{3}$$

$$\Rightarrow x - 1 = 0$$

$$\text{For } x = 1 = \frac{1}{1}$$

By adding numerator and denominator

$$1 + 1 = 2$$

No option is satisfied

$$\therefore x = \frac{1}{3}$$

$$1 + 3 = 4$$

236. (c) $x = \sqrt{5} + 2$

$$\Rightarrow \frac{1}{x} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$\Rightarrow \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$$

$$\Rightarrow x - \frac{1}{x} = \sqrt{5} + 2 - \sqrt{5} + 2 = 4$$

$$\therefore \frac{2x^2 - 3x - 2}{3x^2 - 4x - 3}$$

$$\begin{aligned} &= \frac{\frac{2x^2}{x} - \frac{2}{x} - \frac{3x}{x}}{\frac{3x^2}{x} - \frac{3}{x} - \frac{4x}{x}} \Rightarrow \frac{2x - \frac{2}{x} - 3}{3x - \frac{3}{x} - 4} \\ &= \frac{2\left(x - \frac{1}{x}\right) - 3}{3\left(x - \frac{1}{x}\right) - 4} \Rightarrow \frac{2 \times 4 - 3}{3 \times 4 - 4} \\ &= \frac{8 - 3}{12 - 4} = \frac{5}{8} = 0.625 \end{aligned}$$

237. (b) $a = 2.234$
 $b = 3.121$
 $c = -5.355$
 $\therefore a + b + c = 0$
 $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

238. (d) $x^2 + y^2 + 1 = 2x$
 $x^2 - 2x + 1 + y^2 = 0$
 $(x - 1)^2 + y^2 = 0$
 If $A^2 + B^2 = 0$

As powers are even
it can possible only
when $A = 0$ & $B = 0$

$\therefore x - 1 = 0$
 $x = 1$
 $y = 0$
 $\therefore x^3 + y^3 = 1 + 0 = 1$

239. (a) $3(a^2 + b^2 + c^2) = (a + b + c)^2$
 by options $a = b = c$
 $3(a^2 + a^2 + a^2) = 9a^2$
 $\Rightarrow 9a^2 = 9a^2$

240. (a) $x(x - 3) = -1$

$\Rightarrow (x - 3) = \frac{-1}{x}$

Taking cube on both sides

$\Rightarrow (x - 3)^3 = \left(\frac{-1}{x}\right)^3$

$\Rightarrow x^3 - 27 - 9x(x - 3) = \frac{-1}{x^3}$

$\Rightarrow x^3 - 27 - 9x \times -1 = \frac{-1}{x^3}$

$\Rightarrow x^3 - 27 + 9 = \frac{-1}{x^3}$

$\Rightarrow x^3 - 18 = \frac{-1}{x^3}$

$\Rightarrow x^3(x^3 - 18) = -1$

241. (a) $a^2 + 4b^2 + 4b - 4ab - 2a - 8$
 $= a^2 - 4ab + 4b^2 - 2a + 4b - 8$
 $= (a - 2b)^2 - 2(a - 2b) - 8$
 Put $t = a - 2b$
 $= t^2 - 2t - 8$
 $= t^2 - 4t + 2t - 8$
 $= t(t - 4) + 2(t - 4)$
 $= (t + 2)(t - 4)$
 $= (a - 2b - 4)(a - 2b + 2)$
 (Put the value of assume t)

242. (d) $\frac{1}{a^2 + ax + x^2} - \frac{1}{a^2 - ax + x^2}$
 $+ \frac{2ax}{a^4 + a^2x^2 + x^4}$

$= \frac{a^2 - ax + x^2 - a^2 - ax - x^2}{(a^2 + x^2 + ax)(a^2 + x^2 - ax)} + \frac{2ax}{a^4 + a^2x^2 + x^4}$

$= \frac{-2ax}{(a^2 + x^2)^2 - (ax)^2} + \frac{2ax}{a^4 + x^4 + a^2x^2}$

$= \frac{-2ax}{a^4 + x^4 + 2x^2a^2 - a^2x^2} + \frac{2ax}{a^4 + x^4 + a^2x^2}$

$= \frac{-2ax}{a^4 + x^4 + x^2a^2} + \frac{2ax}{a^4 + x^4 + a^2x^2} = 0$

243. (b) $x = 11$
 $x^5 - 12x^4 + 12x^3 - 12x^2 + 12x - 1$
 $= x^5 - 11x^4 - x^4 + 11x^3 + x^3 - 11x^2 - x^2 + 11x + x - 1$
 $= (11)^5 - 11 \times (11)^4 - (11)^4 + 11 \times (11)^3 + 11^3 - 11 \times (11)^2 - (11)^2 \times 11 + (11 \times 11) + (11) - 1$
 $= 0 - 0 + 0 + 0 + 11 - 1 = 10$

244. (c) $p = 99$
 $p(p^2 + 3p + 3)$
 $\Rightarrow p^3 + 3p^2 + 3p + 1 - 1$
 $(p + 1)^3 - 1$
 $\Rightarrow (99 + 1)^3 - 1$
 $\Rightarrow (100)^3 - 1$
 $\Rightarrow 1000000 - 1 = 999999$

245. (c) From option (c) LHS $(x + 2)^2 = x^2 + 4x + 4$
 RHS $= x^2 + 2x + 4$
 $\therefore \text{LHS} \neq \text{RHS}$

246. (a) $\left(a + \frac{1}{a}\right)^2 = 3$

$\Rightarrow a + \frac{1}{a} = \sqrt{3}$

Cube on both sides

$\left(a + \frac{1}{a}\right)^3 = (\sqrt{3})^3$

$\Rightarrow a^3 + \frac{1}{a^3} + 3\sqrt{3} = 3\sqrt{3}$

$\Rightarrow a^3 + \frac{1}{a^3} = 0$

247. (a) $a + \frac{1}{a} = \sqrt{3}$

Take cube on both sides

$\Rightarrow a^3 + \frac{1}{a^3} + 3a \times \frac{1}{a} \left(a + \frac{1}{a}\right) = (\sqrt{3})^3$

$\Rightarrow a^3 + \frac{1}{a^3} + 3 \times \sqrt{3} \Rightarrow 3\sqrt{3}$

$\Rightarrow a^3 + \frac{1}{a^3} = 0$

$\Rightarrow a^6 + 1 = 0$

$\Rightarrow a^6 = -1$

$= a^{18} + a^{12} + a^6 + 1$
 $= (-1)^3 + (-1)^2 - 1 + 1$
 $= -1 + 1 - 1 + 1 = 0$

248. (d) $x^2 + y^2 + z^2 - xy - xz - yz = \frac{1}{2} [(x - y)^2 + (y - z)^2 + (z - x)^2]$
 $= \frac{1}{2} [(997 - 998)^2 + (998 - 999)^2 + (999 - 997)^2]$
 $= \frac{1}{2} (1 + 1 + 4) = 3$

249. (c) $x + \frac{1}{x} = 3$

$\frac{3x^2 + 3 - 4x}{x^2 + 1 - x} = \frac{\frac{3x^2}{x^2} + \frac{3}{x} - 4}{\frac{x^2}{x^2} + \frac{1}{x} - \frac{x}{x}} = \frac{3\left(x + \frac{1}{x}\right) - 4}{\left(x + \frac{1}{x}\right) - 1}$

$= \frac{3 \times 3 - 4}{3 - 1} = \frac{9 - 4}{3 - 1} = \frac{5}{2}$

250. (d) $x = 3 + 2\sqrt{2}$

$\therefore \frac{1}{x} = 3 - 2\sqrt{2}$

$x + \frac{1}{x} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2}$

$x + \frac{1}{x} = 6$

$\left(x + \frac{1}{x}\right)^3 = (6)^3$

$x^3 + \frac{1}{x^3} + 3 \times 6 = 216$

$x^3 + \frac{1}{x^3} = 216 - 18 = 198$

$\therefore \frac{x^6 + x^4 + x^2 + 1}{x^3}$

$= x^3 + x + \frac{1}{x} + \frac{1}{x^3}$

$= x^3 + \frac{1}{x^3} + x + \frac{1}{x}$

$= 198 + 6 = 204$

251. (c) $(a + b - c)^2 + (b + c - a)^2 + (c + a - b)^2 = ?$

$\Rightarrow a + b + c = 0$ (given)

$\Rightarrow a + b = -c$

$\Rightarrow b + c = -a$

$$\begin{aligned} \Rightarrow a + c &= -b \\ \Rightarrow (a + b - c)^2 + (b + c - a)^2 + (c + a - b)^2 \\ \Rightarrow (-c - c)^2 + (-a - a)^2 + (-b - b)^2 \\ \Rightarrow (-2c)^2 + (-2a)^2 + (-2b)^2 \\ \Rightarrow 4c^2 + 4a^2 + 4b^2 \\ \Rightarrow 4(a^2 + b^2 + c^2) \end{aligned}$$

$$\begin{aligned} 252. (b) \quad p^3 + 3p^2 + 3p - 7 &= 0 \\ p^3 + 3p^2 + 3p + 1 &= 7 + 1 \\ (p + 1)^3 &= (2)^3 \\ p + 1 &= 2 \\ \Rightarrow p &= 1 \end{aligned}$$

$$\therefore p^2 + 2p = 1 + 2 = 3$$

$$\begin{aligned} 253. (a) \quad x &= 2015 \\ y &= 2014 \\ z &= 2013 \end{aligned}$$

$$= x^2 + y^2 + z^2 - xy - yz - zx = \frac{1}{2} [(x - y)^2 + (y - z)^2 + (z - x)^2]$$

$$= \frac{1}{2} [(2015 - 2014)^2 + (2014 - 2013)^2 + (2013 - 2015)^2]$$

$$= \frac{1}{2} (1 + 1 + 4) = 3$$

$$254. (a) \quad 3a^2 = b^2 \text{ (given)}$$

$$\frac{(a+b)^3 - (a-b)^3}{(a+b)^2 + (a-b)^2}$$

$$\Rightarrow \frac{a^3 + b^3 + 3ab(a+b) - (a^3 - b^3 - 3ab(a-b))}{a^2 + b^2 + 2ab + a^2 + b^2 - 2ab}$$

$$\Rightarrow \frac{2b^3 + 6a^2b}{2a^2 + 2b^2} \Rightarrow \frac{b^3 + 3a^2b}{a^2 + b^2}$$

$$\Rightarrow \frac{\frac{b^3 + b^3}{3} + b^2}{\frac{b^2}{3} + b^2} = \frac{2b^3}{b^2 \left(\frac{1}{3} + 1 \right)}$$

$$\Rightarrow \frac{2b}{\frac{4}{3}} = \frac{3b}{2}$$

$$255. (a) \quad x + \frac{1}{x} = 2 \quad \frac{1}{12} = \frac{25}{12}$$

$$x^2 + \frac{1}{x^2} + 2 = \frac{675}{144}$$

$$x^2 + \frac{1}{x^2} = \frac{625}{144} - 2$$

$$x^2 + \frac{1}{x^2} = \frac{625 - 288}{144}$$

$$x^2 + \frac{1}{x^2} = \frac{337}{144}$$

$$x^2 + \frac{1}{x^2} - 2 = \frac{337}{144} - 2$$

$$\left(x - \frac{1}{x} \right)^2 = \frac{337 - 288}{144} = \frac{49}{144}$$

$$x - \frac{1}{x} = \frac{7}{12}$$

$$\therefore \left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right) = \frac{25}{12} \times \frac{7}{12} = \frac{175}{144}$$

$$\therefore \left(x^2 - \frac{1}{x^2} \right) = \frac{175}{144}$$

$$\therefore x^4 - \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2} \right) \left(x^2 - \frac{1}{x^2} \right) =$$

$$\frac{175}{144} \times \frac{337}{144} = \frac{58975}{20736}$$

$$256. (c) \quad x = 9999$$

$$\frac{4x^3 - x}{(2x+1)(6x-3)} = \frac{x(4x^2-1)}{3(2x+1)(2x-1)}$$

$$= \frac{x(4x^2-1)}{3(4x^2-1)} = \frac{x}{3}$$

$$\therefore \frac{9999}{3} = 3333$$

$$257. (b) \quad a^3 + b^3 = 9$$

$$a + b = 3$$

$$\text{Assume values, } a = 2, \quad b = 1$$

$$\therefore (2)^3 + 1 = 9$$

$$2 + 1 = 3$$

$$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{2} + 1 = \frac{3}{2}$$

$$258. (c) \quad t^2 - 4t + 1 = 0$$

$$t^2 + 1 = 4t$$

$$\frac{t^2 + 1}{t} = \frac{4t}{t}$$

$$t + \frac{1}{t} = 4$$

$$[\text{take cube both sides}]$$

$$t + \frac{1}{t} + 3t \left(t + \frac{1}{t} \right) = 64$$

$$t^3 + \frac{1}{t^3} = 64 - 12 = 52$$

$$t^3 + \frac{1}{t^3} = 52$$

$$259. (d) \quad \sqrt[3]{a} + \sqrt[3]{b} = \sqrt[3]{c}$$

$$\text{Take cube both sides}$$

$$(\sqrt[3]{a} + \sqrt[3]{b})^3 = (\sqrt[3]{c})^3$$

$$\Rightarrow a + b + 3a^{\frac{1}{3}}b^{\frac{1}{3}} (\sqrt[3]{a} + \sqrt[3]{b}) = c$$

$$\Rightarrow a + b + 3a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}} = c$$

$$\Rightarrow a + b - c = -3a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}}$$

$$\text{Again take cube both sides}$$

$$\Rightarrow (a + b - c)^3 = -27abc$$

$$\Rightarrow (a + b - c)^3 + 27abc = 0$$

$$260. (b) \quad 4x + 5y = 83$$

$$3x : 2y = 21 : 22$$

$$x : y = 7 : 11$$

$$\text{let } x = 7 \text{ and } y = 11$$

$$y - x = 11 - 7$$

$$= 4$$

$$261. (c) \quad x = \sqrt[3]{a + \sqrt{a^2 + b^3}} + \sqrt[3]{a - \sqrt{a^2 + b^3}}$$

$$\text{Take cube on both sides}$$

$$x^3 = (a + \sqrt{a^2 + b^3}) +$$

$$(a - \sqrt{a^2 + b^3}) + 3(\sqrt{a + \sqrt{a^2 + b^3}})^{\frac{1}{3}}$$

$$(\sqrt{a - \sqrt{a^2 + b^3}})^{\frac{1}{3}}$$

$$(\sqrt[3]{a + \sqrt{a^2 + b^3}} + \sqrt[3]{a - \sqrt{a^2 + b^3}})$$

$$x^3 = 2a + 3(\sqrt{a^2 - (a^2 + b^3)})^{\frac{2}{3}}(x)$$

$$x^3 = 2a + 3(\sqrt{-b^3})^{\frac{2}{3}}(x)$$

$$x^3 = 2a + 3(-b^{\frac{3}{2}})^{\frac{2}{3}}(x)$$

$$x^3 = 2a - 3bx$$

$$x^3 + 3bx = 2a$$

$$262. (d) \text{ Given}$$

$$\frac{x^{24} + 1}{x^{12}} = 7$$

$$\frac{x^{24} + 1}{x^{12}} \Rightarrow \frac{x^{24}}{x^{12}} + \frac{1}{x^{12}}$$

$$\Rightarrow x^{12} + \frac{1}{x^{12}} = 7$$

$$\Rightarrow \text{Cubing both sides}$$

$$\Rightarrow \left(x^{12} + \frac{1}{x^{12}} \right)^3 = 7^3$$

$$\Rightarrow x^{36} + \frac{1}{x^{36}} + \frac{3 \times x^{12} \times 1}{x^{12}} \left(x^{12} + \frac{1}{x^{12}} \right)$$

$$= 343$$

$$\Rightarrow x^{36} + \frac{1}{x^{36}} + 3 \times 7 = 343$$

$$\Rightarrow x^{36} + \frac{1}{x^{36}} = 343 - 21$$

$$\Rightarrow x^{36} + \frac{1}{x^{36}} = \frac{x^{72} + 1}{x^{36}}$$

$$= 322$$

263. (d) Given $P = 99$
 find $P(P^2 + 3P + 3) = ?$
 to put value in equation
 $\Rightarrow 99 ((99)^2 + (3 \times 99) + 3)$
 $\Rightarrow (100 - 1) [(100 - 1)^2 + [3 \times (100 - 1) + 3]]$
 $\Rightarrow (100 - 1) [10000 + 1 - 200 + 300 - 3 + 3]$
 $\Rightarrow (100 - 1) (10000 + 100 + 1)$
 $\Rightarrow (100 - 1) (10101)$
 $\Rightarrow 99 \times 10101$
 $\Rightarrow 999999$

264. (b) Given, $x = 2$
 Find $x^3 + 27x^2 + 243x + 631$
 $\Rightarrow$ to put value $x = 2$
 $\Rightarrow 2^3 + 27 \times 2^2 + (243 \times 2) + 631$
 $\Rightarrow 8 + 108 + 486 + 631$
 $\Rightarrow 1233$

265. (b) Given, $x^2 + y^2 + z^2 = 2(x + z - 1)$
Find- $x^3 + y^3 + z^3 = ?$
 $\Rightarrow x^2 + y^2 + z^2 = 2(x + z - 1)$
 $\Rightarrow x^2 + y^2 + z^2 = 2x + 2z - 2$
 $\Rightarrow x^2 + y^2 + z^2 = 2x + 2z - 1 - 1$
 $\Rightarrow (x^2 + 1 - 2x) + y^2 + (z^2 + 1 - 2z) = 0$
 $\Rightarrow (x - 1)^2 + y^2 + (z - 1)^2 = 0$
 $\Rightarrow (x - 1)^2 = 0$
 $\Rightarrow x = 1$
 $\Rightarrow y^2 = 0$
 $\Rightarrow y = 0$
 $\Rightarrow (z - 1)^2 = 0$
 $\Rightarrow z = 1$
 Value substituted in question,
 $\Rightarrow x^3 + y^3 + z^3$
 $\Rightarrow 1^3 + 0 + 1^3$
 $\Rightarrow 2$

266. (b) Given, $x + \frac{1}{x} = 1$
 Find: $\frac{2}{x^2 - x + 2} = ?$

$$x + \frac{1}{x} = 1$$

$$x^2 + 1 = x$$

$$(x^2 - x) = -1$$

Putting value in,

$$\frac{2}{(x^2 - x) + 2} = 2$$

267. (d) Given,
 $x = \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}, y = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}}$

Find: $\frac{x^2 + y^2 + xy}{x^2 + y^2 - xy} = ?$

$$\Rightarrow \frac{x^2 + y^2 + 2xy - xy}{x^2 + y^2 - 2xy + xy}$$

$$\Rightarrow \frac{(x + y)^2 - xy}{(x - y)^2 + xy}$$

Now, $x + y = \frac{(\sqrt{5} - \sqrt{3})}{(\sqrt{5} + \sqrt{3})} + \frac{(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})}$

$$\Rightarrow x + y = \frac{(\sqrt{5} - \sqrt{3})^2 + (\sqrt{5} + \sqrt{3})^2}{\sqrt{5}^2 - \sqrt{3}^2}$$

$$\Rightarrow x + y = \frac{2(\sqrt{5}^2 + \sqrt{3}^2)}{2}$$

$$\Rightarrow x + y = 8 \dots\dots\dots(i)$$

Again, $x - y = \frac{(\sqrt{5} - \sqrt{3})}{(\sqrt{5} + \sqrt{3})} - \frac{(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})}$

$$= \frac{4 \times \sqrt{5} \times \sqrt{3}}{2}$$

$$\Rightarrow (x - y) = 2\sqrt{15} \dots\dots\dots(ii)$$

And, $xy = \frac{(\sqrt{5} - \sqrt{3})}{(\sqrt{5} + \sqrt{3})} \times \frac{(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})}$

$$xy = 1$$

Substitutes values in the question,

$$\Rightarrow \frac{(x + y)^2 - xy}{(x - y)^2 + xy}$$

$$\Rightarrow \frac{8^2 - 1}{(2\sqrt{15})^2 + 1} \Rightarrow \frac{63}{61}$$

268. (c) Given, $4a - \frac{4}{a} + 3 = 0$

Find: $a^3 - \frac{1}{a^3} + 3 = ?$

$$\Rightarrow 4a - \frac{4}{a} = -3$$

$$\Rightarrow a - \frac{1}{a} = \frac{-3}{4}$$

$$\Rightarrow \left(a - \frac{1}{a}\right)^3 = \left(\frac{-3}{4}\right)^3$$

[Cubing both sides]

$$\Rightarrow a^3 - \frac{1}{a^3} - 3a \times \frac{1}{a} \left(a - \frac{1}{a}\right) = \frac{-27}{64}$$

$$\Rightarrow a^3 - \frac{1}{a^3} - 3 \times \left(\frac{-3}{4}\right) = \frac{-27}{64}$$

$$\Rightarrow a^3 - \frac{1}{a^3} = \frac{-27}{64} - \frac{9}{4}$$

$$\Rightarrow a^3 - \frac{1}{a^3} + 3 = \frac{-27}{64} - \frac{9}{4} + 3$$

$$\Rightarrow \frac{192 - 171}{64} \Rightarrow a^3 - \frac{1}{a^3} + 3 = \frac{21}{64}$$

269. (b) According to the question,

$$\therefore x = z = 225$$

$$y = 226$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz = ?$$

As we know,

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)$$

$$[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

$$= \frac{1}{2} [225 + 225 + 226] [(225 - 226)^2 + (226 - 225)^2 + (225 - 225)^2]$$

$$= \frac{676}{2} \times [1 + 1 + 0] = 676$$

270. (b) $x^2 + x = 5$ then $(x + 3)^3 + \frac{1}{(x + 3)^3}$

Let $x + 3 = m$

$$x = m - 3$$

then $(m - 3)^2 + (m - 3) = 5$

$$m^2 + 9 - 6m + m - 3 = 5$$

$$m^2 - 5m + 6 = 5$$

$$m^2 - 5m = -1$$

$$m - 5 = \frac{-1}{m}$$

$$m + \frac{1}{m} = 5$$

then $m^3 + \frac{1}{m^3} = 125 - 15$

$$m^3 + \frac{1}{m^3} = 110$$

Here $m = x + 3$ then

$$(x + 3)^3 + \frac{1}{(x + 3)^3} = 110$$

271. (d) Given, $m = -4, n = -2$
 Find $m^3 - 3m^2 + 3m + 3n + 3n^2 + n^3$
 Putting value of m and n

$$\Rightarrow (-4)^3 - 3(-4)^2 + 3(-4) + 3(-2) + 3(-2)^2 + (-2)^3$$

$$\Rightarrow -64 - 48 - 12 - 6 + 12 - 8$$

$$\Rightarrow -64 - 60 - 2$$

$$\Rightarrow -126$$

272. (b) $2x - ky + 7 = 0$ (i)
 $6x - 12y + 15 = 0$ (ii)

There has no solution for

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{6} = \frac{-k}{-12}$$

$$\frac{1}{3} = \frac{k}{12} \Rightarrow k = 4$$

273. (a) Here, $x = 332$,

$$y = 333, z = 335$$

Find $x^3 + y^3 + z^3 - 3xyz$

$$= \frac{1}{2}(a + b + c)$$

$$[(a - b)^2 + (b - c)^2 + (c - a)^2]$$

$$= \left(\frac{332 + 333 + 335}{2}\right) [(333 - 332)^2 + (335 - 333)^2]$$

$$+ (335 - 332)^2] = \frac{1000}{2} (1^2 + 2^2 + 3^2)$$

$$= \frac{1000}{2} (14) = 7000$$

274. (d) If $2 + x\sqrt{3} = \frac{1}{2 + \sqrt{3}}$

Find $x = ?$

$$\Rightarrow 2 + \sqrt{3} x = \frac{1}{2 + \sqrt{3}}$$

$$\Rightarrow 2 + \sqrt{3} \times x = \frac{2 - \sqrt{3}}{1}$$

$$\Rightarrow 2 + x\sqrt{3} = 2 - \sqrt{3}$$

$$\Rightarrow x = -1$$

275. (c) Given $m - 5n = 2$
find $m^3 - 125n^3 - 30mn$

$$\Rightarrow m - 5n = 2$$

$$\Rightarrow (m - 5n)^3 = 2^3 \text{ (cubing both sides)}$$

$$\Rightarrow m^3 - 125n^3 - 3m \times 5n (m - 5n) = 8$$

$$\Rightarrow m^3 - 125n^3 - 15mn \times 2 = 8$$

$$\Rightarrow m^3 - 125n^3 - 30mn = 8$$

276. (d) Given

$$x = \sqrt[3]{a^3 b^3 \sqrt[3]{a^3 b^3} \dots \dots \dots} \dots \dots \dots (i)$$

on squaring both side

$$\Rightarrow x^2 = a^3 b^3 \sqrt[3]{a^3 b^3} \dots \dots \dots$$

On cubing both sides

$$\Rightarrow x^6 = a^3 b^3 \sqrt[3]{a^3 b^3 \sqrt[3]{a^3 b^3} \dots \dots \dots}$$

$$\Rightarrow x^6 = a^3 b^3 x \text{ from equation (i)}$$

$\Rightarrow$ On dividing above eq. by x we get

$$\Rightarrow \frac{x^6}{x} = \frac{a^3 b^3 x}{x}$$

$$\Rightarrow x^5 = a^3 b^3$$

$$\Rightarrow x = \sqrt[5]{a^3 b^3}$$

277. (d) Given: $x + \frac{1}{x} = 2 \dots \dots \dots (i)$

$$\text{The value of } x^{12} - \frac{1}{x^{12}} = ?$$

$$\Rightarrow \text{if } x = 1 \Rightarrow x + \frac{1}{x} = 2$$

$$1 + 1 = 2$$

$$\text{Then, } x^{12} - \frac{1}{x^{12}}$$

$$\Rightarrow 1^{12} - \frac{1}{1^{12}}$$

$$\Rightarrow 1 - 1 = 0$$

278. (a) Given: $x + \frac{1}{x} = 1 \dots \dots \dots (i)$

$$\text{Find } \frac{x^2 + 3x + 1}{x^2 + 7x + 1} = ?$$

From equation (i)

$$\Rightarrow x + \frac{1}{x} = 1$$

$$\Rightarrow x^2 + 1 = x$$

$$\Rightarrow \frac{(x^2 + 1) + 3x}{(x^2 + 1) + 7x}$$

$$\Rightarrow \frac{x + 3x}{x + 7x} \Rightarrow \frac{4x}{8x} = \frac{1}{2}$$

279. (c) $x + \frac{1}{x} = 2$

$$\text{Find } x^7 + \frac{1}{x^5} = 2$$

$$\Rightarrow x + \frac{1}{x} = 2 \Rightarrow \text{Let } x = 1$$

$$\downarrow \quad \downarrow$$

$$1 \quad 1$$

To, put value in question,

$$\Rightarrow x^7 + \frac{1}{x^5} \Rightarrow 1^7 + \frac{1}{1^5}$$

$$\Rightarrow 1 + 1 = 2$$

280. (d) Given expression,

$$\Rightarrow 4x^2 + 8x$$

Let P should be added,

$$\Rightarrow 4x^2 + 8x + P$$

$$\Rightarrow (2x)^2 + 2 \times (2x) \times 2$$

$$[(a+b)^2 = a^2 + b^2 + 2ab]$$

Term that should be added = $2^2 = 4$

$$P = 4$$

281. (b) $999x + 888y = 1332$

$$888x + 999y = 555$$

$$1887 \quad (x + y) = 1887$$

$$x + y = 1$$

282. (a) According to the question,

$$\Rightarrow x = \frac{1}{2 + \sqrt{3}}, \quad y = \frac{1}{2 - \sqrt{3}}$$

$$\Rightarrow x = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}},$$

$$y = \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$\Rightarrow x = 2 - \sqrt{3}, \quad y = 2 + \sqrt{3}$$

$$8xy (x^2 + y^2)$$

$$= 8(2 - \sqrt{3})(2 + \sqrt{3})[(2 - \sqrt{3})^2 + (2 + \sqrt{3})^2]$$

$$\Rightarrow 8 \times 1 [7 - 2\sqrt{3} + 7 + 2\sqrt{3}] = 112$$

283. (d) According to the question,

$$a = \frac{\sqrt{x+2} + \sqrt{x-2}}{\sqrt{x+2} - \sqrt{x-2}}$$

$$\text{Put } x = 2$$

$$a = \frac{\sqrt{2+2} + \sqrt{2-2}}{\sqrt{2+2} - \sqrt{2-2}}$$

$$a = \frac{\sqrt{4}}{\sqrt{4}} = 1$$

$$a^2 - ax = 1^2 - 1 \times 2 = 1 - 2 = -1$$

284. (a) Let $a = 0$
 $b = 1$

$$\Rightarrow a^3 + b^3 - ab - (a^2 - b^2)^2$$

$$\Rightarrow 0 + 1 - 0 - (0 - 1)^2$$

$$\Rightarrow 1 - 1 = 0$$

285. (b) $a - \frac{1}{(a-3)} = 5$

$$a - 3 - \frac{1}{a-3} = 5 - 3$$

$$(a-3) - \frac{1}{(a-3)} = 2$$

Cubing both sides

$$\left[(a-3) - \frac{1}{(a-3)} \right]^3 = 2^3$$

$$(a-3)^3 - \frac{1}{(a-3)^3} - 3(a-3) \times \frac{1}{(a-3)}$$

$$\left[(a-3) - \frac{1}{(a-3)} \right] = 8$$

$$(a-3)^3 - \frac{1}{(a-3)^3} - 3[2] = 8$$

$$(a-3)^3 - \frac{1}{(a-3)^3} = 8 + 6 = 14 \text{ Ans.}$$

286. (d) $\frac{3x-2y}{2x+3y} = \frac{5}{6}$

$$18x - 12y = 10x + 15y$$

$$8x = 27y$$

$$\frac{x}{y} = \frac{27}{8}$$

$$\left[\frac{\sqrt[3]{x}}{\sqrt[3]{x}} + \frac{\sqrt[3]{y}}{\sqrt[3]{y}} \right]^2$$

$$\left(\frac{\sqrt[3]{27}}{\sqrt[3]{27}} + \frac{\sqrt[3]{8}}{\sqrt[3]{8}} \right)^2$$

$$\Rightarrow \left(\frac{3+2}{3-1} \right)^2 = (5)^2 = 25$$

287. (d) According to the Question

$$x = \sqrt{3} + \sqrt{2}$$

$$y = \sqrt{3} - \sqrt{2}$$

$$\begin{aligned}
 & (x^3 - 20\sqrt{2}) - (y^3 + 2\sqrt{2}) \\
 & = [(\sqrt{3} + \sqrt{2})^3 - 20\sqrt{2} - (\sqrt{3} - \sqrt{2})^3 \\
 & \quad - 2\sqrt{2}] \\
 & = 3\sqrt{3} + 2\sqrt{2} + 9\sqrt{2} + 6\sqrt{3} - \\
 & \quad 20\sqrt{2} - 3\sqrt{3} + 2\sqrt{2} + 9\sqrt{2} - 6\sqrt{3} \\
 & \quad - 2\sqrt{2} \\
 & = 9\sqrt{3} - 9\sqrt{2} - 9\sqrt{3} + 9\sqrt{2} = 0
 \end{aligned}$$

288. (d) **SHORTCUT METHOD**
 Always do these types of question with the help of
 Put $a = b = c = 1$
 $3(1^2 + 1^2 + 1^2) = (a + b + c)^2$
 $3 = 3$ satisfied

So this is answer $\rightarrow a = b = c$

289. (d) According to the question,

$$\begin{aligned}
 x &= \sqrt{a} + \frac{1}{\sqrt{a}} \quad \& \quad y = \sqrt{a} - \frac{1}{\sqrt{a}} \\
 \therefore x^4 - x^2y^2 - 1 + y^4 - x^2y^2 + 1 \\
 &= x^4 - 2x^2y^2 + y^4 \\
 &= [x^2 - y^2]^2
 \end{aligned}$$

$$\left[\left(\sqrt{a} + \frac{1}{\sqrt{a}} \right)^2 - \left(\sqrt{a} - \frac{1}{\sqrt{a}} \right)^2 \right]^2$$

$$\begin{aligned}
 &= \left[a + \frac{1}{a} + 2 - a - \frac{1}{a} + 2 \right]^2 \\
 &= [4]^2 = 16
 \end{aligned}$$

290. (d) Let $m = \sqrt{5 + \sqrt{5 + \sqrt{5}}}$
 Factor = $(a) \times (a+1)$
 Here $m = a + 1$
 or $m - 1 = a$ (i)

Let $n = \sqrt{5 - \sqrt{5 - \sqrt{5}}}$
 Factor = $(a) \times (a+1)$
 Here $n = a$ (ii)
 From (i) & (ii)
 $m - 1 = n$
 or $m - n - 1 = 0$

$$291. (b) \frac{3-5x}{2x} + \frac{3-5y}{2y} + \frac{3-5z}{2z} = 0$$

$$\text{or } \frac{3}{2x} - \frac{5}{2} + \frac{3}{2y} - \frac{5}{2} + \frac{3}{2z} - \frac{5}{2} = 0$$

$$\text{or } \frac{3}{2x} + \frac{3}{2y} + \frac{3}{2z} = \frac{3 \times 5}{2}$$

$$\text{or } \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3 \times 5 \times 2}{2 \times 3}$$

$$\text{or } \frac{2}{x} + \frac{2}{y} + \frac{2}{z} = \frac{3 \times 5 \times 2 \times 2}{2 \times 3} = 10$$

292. (a) According to the question
 If $2S = a + b + c$

$$S = \frac{a+b+c}{2}$$

Let $a = 10, b = 10, c = 10$

$$\therefore S = \frac{10+10+10}{2}$$

$$S = \frac{30}{2} = 15$$

$\therefore S(S - C) + (S - a)(S - b)$
 $15(15 - 10) + (15 - 10)(15 - 10) = 75 + 25 = 100$
 Now check from option.
 Option (a) $ab = 10 \times 10 = 100$
 (Satisfied)

293. (d) $p + m = 6$ (i)

$$\begin{aligned}
 p^3 + m^3 &= 72 \\
 (p + m)(p^2 + m^2 - pm) &= 72 \\
 (p + m)[(p + m)^2 - 3pm] &= 72 \\
 [\because p^2 + m^2 = (p + m)^2 - 2pm] \\
 6[(6)^2 - 3pm] &= 72 \text{ from (i)} \\
 36 - 3pm &= 12 \\
 pm &= 8
 \end{aligned}$$

294. (d) $x^m \times x^n = 1$
 $x^{m+n} = x^0$ ($\because x^0 = 1$)
 $m + n = 0$
 $m = -n$

$$295. (d) \frac{2p}{p^2-2p+1} = \frac{1}{4}$$

$$\begin{aligned}
 \frac{2}{p-2+\frac{1}{p}} &= \frac{1}{4} \\
 p-2+\frac{1}{p} &= 8 \\
 (\text{Divide } p \text{ both in nu. \& de.})
 \end{aligned}$$

$$p + \frac{1}{p} - 2 = 8$$

$$p + \frac{1}{p} = 10$$

296. (b) According to the question,
 $x = 2, y = 1, z = -3$
 $\therefore x^3 + y^3 + z^3 - 3xyz = ?$
 As we know that
 $a + b + c = 0$ then $a^3 + b^3 + c^3 - 3abc = 0$
 $\therefore 2 + 1 - 3 = 0$
 $x^3 + y^3 + z^3 - 3xyz = 0$

297. (a) According to the question
 $\Rightarrow (x^3 + y^6)(x^3 - y^6)$
 $\Rightarrow x^6 + x^3y^6 - x^3y^6 - y^{12}$
 $\Rightarrow x^6 - y^{12}$

298. (b) According to the question,

$$\begin{aligned}
 &\Rightarrow \frac{1}{x+y} + \frac{1}{x-y} \\
 &\Rightarrow \frac{x-y+x+y}{x^2-y^2} \Rightarrow \frac{2x}{x^2-y^2}
 \end{aligned}$$

299. (a) Given, $x + y = 2a$ to

$$\text{Find } \frac{a}{(x-a)} + \frac{a}{(y-a)} = ?$$

$$\begin{aligned}
 \Rightarrow x + y &= 2a \\
 \downarrow \quad \downarrow \quad \downarrow \\
 3 \quad 1 \quad 2 \\
 \Rightarrow \text{Let } x &= 3, y = 1, a = 2
 \end{aligned}$$

$$\begin{aligned}
 &\Rightarrow \frac{a}{(x-a)} + \frac{a}{y-a} \\
 &\Rightarrow \frac{2}{(3-2)} + \frac{2}{(1-2)}
 \end{aligned}$$

$$\Rightarrow \frac{2}{1} + \frac{2}{-1} = 0$$

300. (a) $a^2 + b^2 + c^2 = ab + bc + ca$
 Let $a = b = c = 1$

$$\begin{aligned}
 &\Rightarrow a^2 + b^2 + c^2 = ab + bc + ca \\
 &\Rightarrow 1^2 + 1^2 + 1^2 = 1 \times 1 + 1 \times 1 + 1 \times 1 \\
 &\Rightarrow 3 = 3
 \end{aligned}$$

$$\Rightarrow \text{to find } \frac{a+c}{b} = ?$$

$$\Rightarrow \frac{1+1}{1} = 2$$

301. (d) $p^3 - q^3 = (p - q)[(p - q)^2 - xpq]$
 $\Rightarrow p^3 - q^3 = (p - q)[p^2 + q^2 - 2pq - xpq]$

$$\Rightarrow p^3 - q^3 = (p - q)[p^2 + q^2 - 2pq - (-3)pq]$$

$$\Rightarrow p^3 - q^3 = (p - q)(p^2 + q^2 + pq)$$

$$\Rightarrow \text{So, } x = -3$$

$$\Rightarrow \text{because } a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$$

302. (a) Given

$$\Rightarrow x + y + z = 6$$

$$\Rightarrow xy + yz + zx = 10$$

$$\text{To find } x^3 + y^3 + z^3 - 3xyz = ?$$

$$\Rightarrow \text{Using formula.}$$

$$\Rightarrow (x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$$

$$\Rightarrow 6^2 = x^2 + y^2 + z^2 + 2 \times 10$$

$$\Rightarrow 36 = x^2 + y^2 + z^2 + 20$$

$$\Rightarrow x^2 + y^2 + z^2 = 16$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz =$$

$$(x + y + z)[x^2 + y^2 + z^2 - xy - yz - zx]$$

$$= 6[16 - (xy + yz + zx)]$$

$$= 6[16 - 10]$$

$$= 6 \times 6 = 36$$

303. (d) Given:

$$\Rightarrow \frac{x+1}{x-1} = \frac{a}{b}$$

$$\Rightarrow \frac{x}{1} = \frac{a+b}{a-b}$$

(using componendo & dividendo)

$$\Rightarrow x = \frac{a+b}{a-b} \dots\dots\dots(i)$$

Again,

$$\Rightarrow \frac{1-y}{1+y} = \frac{b}{a}$$

$$\Rightarrow \frac{1+y}{1-y} = \frac{a}{b}$$

$$\Rightarrow \frac{1}{y} = \frac{a+b}{a-b}$$

$$\Rightarrow y = \frac{a-b}{a+b} \dots\dots\dots(i)$$

$$\Rightarrow \text{From question, } \frac{x-y}{1+xy}$$

$$\Rightarrow \frac{\frac{a+b}{a-b} - \frac{a-b}{a+b}}{1 + \left(\frac{a+b}{a-b}\right)\left(\frac{a-b}{a+b}\right)}$$

$$\Rightarrow \frac{(a+b)^2 - (a-b)^2}{(a^2 - b^2)(1+1)}$$

$$\Rightarrow \frac{4ab}{2(a^2 - b^2)} \Rightarrow \frac{2ab}{a^2 - b^2}$$

304. (d) Given
 $a^2 + b^2 + c^2 - ab - bc - ca = 0$
 to find $a : b : c = ?$
 $\Rightarrow$ According to the question,
 $a^2 + b^2 + c^2 - ab - bc - ca = 0$
 $\Rightarrow a^2 + b^2 + c^2 = ab + bc + ca$
 $\Rightarrow$ Let $a = b = c = 1$
 $\Rightarrow 1^2 + 1^2 + 1^2 = 1 \times 1 + (1 \times 1) + (1 \times 1)$
 $\Rightarrow 3 = 3$
 $\Rightarrow$ So ratio of $a : b : c = 1 : 1 : 1$

305. (c) $\frac{x}{a} = b - c$; $\frac{y}{b} = c - a$; $\frac{z}{c} = a - b$

$$\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 + \left(\frac{z}{c}\right)^3 = 3\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(\frac{z}{c}\right)$$

$$\left(\because \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0\right) = \frac{3xyz}{abc}$$

306. (b) $\frac{x}{y} = \frac{a+2}{a-2}$

$$\frac{x^2}{y^2} = \frac{(a+2)^2}{(a-2)^2}$$

Applying Componendo & Dividendo

$$\frac{x^2 - y^2}{x^2 + y^2} = \frac{(a+2)^2 - (a-2)^2}{(a+2)^2 + (a-2)^2} = \frac{8a}{2a^2 + 8} = \frac{4a}{a^2 + 4}$$

307. (c) $\frac{a}{b} + \frac{b}{a} = 2$

$$a^2 + b^2 = 2ab$$

$$(a - b)^2 = 0$$

$$a - b = 0$$

308. (a) Put $(x + y + z) = 10$

$$x = 2$$

$$y = 3$$

$$z = 5$$

$$x(x + y + z) = 20$$

$$2(10) = 20$$

Similarly other will satisfied

so value of $2(x + y + z)$

$$\Rightarrow 2(10) = 20$$

309. (c) $x^2 + y^2 = 14$

$$x + y = 4 \dots\dots(i)$$

squaring both sides

$$x^2 + y^2 + 2xy = 16$$

$$14 + 2xy = 16$$

$$2xy = 2$$

$$xy = 1$$

$$x^2 + y^2 = 14$$

subtrance $(2xy)$ from both sides.

$$x^2 + y^2 - 2xy = 14 - 2 \times 1$$

$$(x - y)^2 = 14 - 2 \times 1$$

$$x - y = \sqrt{12}$$

$$x - y = 2\sqrt{3} \dots\dots\dots(2)$$

solve equation (1) and (2)

$$y = 2 - \sqrt{3}$$

$$x = 2 + \sqrt{3}$$

310. (d) $x^2 - 4x - 1 = 0$

$$\Rightarrow x - \frac{1}{x} = 4$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 4^2$$

(squaring both sides)

$$\Rightarrow x^2 + \frac{1}{x^2} = 18$$

311. (b) **Given**

$$\therefore a = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}, b = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

$$\text{to find } \frac{a^2 + b^2}{b + a} = ?$$

$$\Rightarrow \frac{a^3 + b^3}{ab} = ?$$

$$\Rightarrow \frac{(a + b)^3 - 3ab(a + b)}{ab} = ?$$

$$\Rightarrow a + b = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} + \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

$$\Rightarrow \frac{(\sqrt{3} - \sqrt{2})^2 + (\sqrt{3} + \sqrt{2})^2}{\sqrt{3}^2 - \sqrt{2}^2}$$

$$\Rightarrow \frac{2(\sqrt{3}^2 + \sqrt{2}^2)}{1}$$

$$\Rightarrow 2 \times (5) \Rightarrow a + b = 10$$

Again

$$\Rightarrow a \times b = \frac{(\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2})} \times \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})}$$

$$\Rightarrow ab = 1$$

$$\Rightarrow \frac{(a + b)^3 - 3ab(a + b)}{ab}$$

$$\Rightarrow \frac{10^3 - 3 \times 1 \times 10}{1}$$

$$\Rightarrow 1000 - 30 = 970$$

312. (c) $(2a-1)^2 + (4b-3)^2 + (4c+5)^2 = 0$

$$\Rightarrow \frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2}$$

$$\Rightarrow \frac{\frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]}{a^2 + b^2 + c^2}$$

$$\Rightarrow \text{then, } 2a - 1 = 0$$

$$a = \frac{1}{2}$$

$$\Rightarrow 4b - 3 = 0$$

$$\Rightarrow b = \frac{3}{4}$$

$$\Rightarrow 4c + 5 = 0$$

$$\Rightarrow c = -\frac{5}{4}$$

$$\Rightarrow \frac{\frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]}{a^2 + b^2 + c^2}$$

$$\Rightarrow \frac{\frac{1}{2}\left(\frac{1}{2} + \frac{3}{4} - \frac{5}{4}\right)\left[\left(\frac{1}{2} - \frac{3}{4}\right)^2 + \left(\frac{3}{4} - \frac{5}{4}\right)^2 + \left(\frac{5}{4} - \frac{1}{2}\right)^2\right]}{\left(\frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)^2 + \left(\frac{5}{4}\right)^2}$$

$$\Rightarrow \frac{\frac{1}{2}\left(\frac{2+3-5}{4}\right)\left[\left(\frac{2-3}{4}\right)^2 + \left(\frac{8}{4}\right)^2 + \left(\frac{-5-2}{4}\right)^2\right]}{\frac{1}{4} + \frac{9}{16} + \frac{25}{16}}$$

$$\Rightarrow \frac{1}{2} \times 0 = 0 \quad (a + b + c = 0)$$

313. (a) $\left(a + \frac{1}{a}\right)^2 = 3$

$$a + \frac{1}{a} = \sqrt{3}$$

cube both sides

$$a^3 + \frac{1}{a^3} + 3 \times a \times \frac{1}{a} \left(a + \frac{1}{a}\right) = (\sqrt{3})^3$$

$$a^3 + \frac{1}{a^3} + 3\sqrt{3}$$

$$= 3\sqrt{3}$$

$$a^3 + \frac{1}{a^3} = 0$$

$$a^6 + 1 = 0$$

$$\Rightarrow a^{30} + a^{24} + a^{18} + a^{12} + a^6 + 1$$

$$= a^{24} (a^6 + 1) + a^{12} (a^6 + 1) + a^6 + 1$$

$$= a^{24} (0) + a^{12} (0) + 0 = 0$$

314. (d) $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$

$$\frac{1}{a+b} = \frac{b+a}{ab}$$

$$ab = a^2 + b^2 + 2ab$$

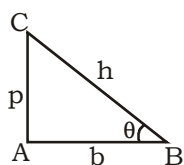
$$a^2 + b^2 + ab = 0$$

$$a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$$

$$= (a-b)(0) = 0$$

TRIGONOMETRY

1. Trigonometric Ratio:



To study different trigonometric ratio functions we will consider a right angled triangle. Suppose ABC is a right angled triangle with $\angle A = 90^\circ$.

We can obtain six different trigonometric ratio from the sides of these triangle. They are respectively

$$\frac{AC}{BC}, \frac{AB}{BC}, \frac{AC}{AB}, \frac{AB}{AC}, \frac{BC}{AB} \text{ and } \frac{BC}{AC}.$$

If $\angle B = \theta$ then these ratio are respectively called $\sin \theta$, $\cos \theta$, $\tan \theta$, $\cot \theta$, $\sec \theta$ and $\csc \theta$. Clearly for the given angle θ , AC (p) is perpendicular, AB (b) is base and BC (h) is hypotenuse. Hence six different trigonometric ratios are follows (see the given figure)

Trigonometric Ratios:-

$$\sin \theta = \frac{AC}{BC} = \frac{p}{h} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\cos \theta = \frac{AB}{BC} = \frac{b}{h} = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\tan \theta = \frac{AC}{AB} = \frac{p}{b} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\csc \theta = \frac{BC}{AC} = \frac{h}{p} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$

$$\sec \theta = \frac{BC}{AB} = \frac{h}{b} = \frac{\text{Hypotenuse}}{\text{Base}}$$

$$\cot \theta = \frac{AB}{AC} = \frac{b}{p} = \frac{\text{Base}}{\text{Perpendicular}}$$

Clearly $\sin \theta$ and $\csc \theta$ are reciprocals to each other. Similarly $\cos \theta$ and $\sec \theta$ are reciprocals to each other while $\tan \theta$ and $\cot \theta$ are reciprocals to each other.

Relations between Trigonometric Ratios :-

$$(i) \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\text{or } \csc \theta \times \sin \theta = 1$$

$$(ii) \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\text{or } \sec \theta \times \cos \theta = 1$$

$$(iii) \quad \cot \theta = \frac{1}{\tan \theta}$$

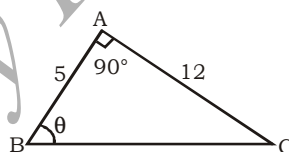
$$\text{or } \cot \theta \times \tan \theta = 1$$

$$(iv) \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

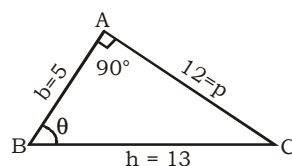
$$(v) \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

TYPE - 1

Ex.1 Write all the six t-ratios value in the given figure:



Sol. In $\triangle ABC$ is, a right angle triangle with $\angle A = 90^\circ$,



Let $AC = 12 = p$ and $AB = 5 = b$

Then from Pythagoras theorem,

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144} = \sqrt{169} = 13$$

Here side opposite to θ is AC which is p.

Side adjacent to θ is AB, which is b.

Side opposite to right angle is BC, which is hypotenuse h.

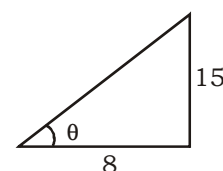
$$\therefore \sin \theta = \frac{p}{h} = \frac{12}{13}, \quad \csc \theta = \frac{h}{p} = \frac{13}{12}$$

$$\cos \theta = \frac{b}{h} = \frac{5}{13}, \quad \sec \theta = \frac{h}{b} = \frac{13}{5}$$

$$\tan \theta = \frac{p}{b} = \frac{12}{5}, \quad \cot \theta = \frac{b}{p} = \frac{5}{12}$$

Ex.2 If $15 \cot \theta = 8$ then calculate the remaining trigonometric ratio.

Sol.



$$\cot \theta = \frac{8}{15} = \frac{b}{p}$$

Let $b = 8k$ $p = 15k$

From pythagoras theorem, $h^2 = p^2 + b^2 = (15k)^2 + (8k)^2$

$$\text{or, } h^2 = 225k^2 + 64k^2 = 289k^2$$

$$\text{or, } h = \sqrt{289k^2} = 17k$$

$$\text{Hence, } \sin \theta = \frac{p}{h} = \frac{15k}{17k} = \frac{15}{17}$$

$$\cos \theta = \frac{b}{h} = \frac{8k}{17k} = \frac{8}{17}$$

$$\tan \theta = \frac{p}{b} = \frac{15k}{8k} = \frac{15}{8}$$

$$\sec \theta = \frac{h}{b} = \frac{17k}{8k} = \frac{17}{8}$$

$$\csc \theta = \frac{h}{p} = \frac{17k}{15k} = \frac{17}{15}$$

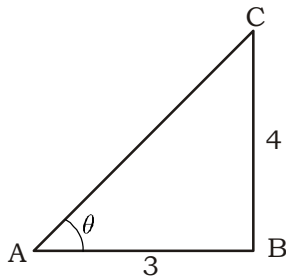
Ex.3 If $\tan \theta = \frac{4}{3}$, then $\cos \theta = ?$

$$(a) \quad \frac{4}{5} \quad (b) \quad \frac{3}{5}$$

$$(c) \quad \frac{3}{4} \quad (d) \quad \frac{1}{5}$$

Sol.(b) $\tan \theta = \frac{BC}{AB} = \frac{4}{3}$

$\therefore AC = \sqrt{(4)^2 + (3)^2} = 5$



$\therefore \cos \theta = \frac{AB}{AC} = \frac{3}{5}$

Ex.4 If $\tan \theta = \frac{4}{3}$, the value of

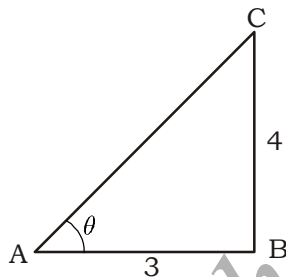
$\frac{1 - \sin \theta}{1 + \sin \theta}$ is:-

- (a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{9}$ (d) $\frac{1}{13}$

Sol.(c) $\tan \theta = \frac{4}{3} = \frac{BC}{AB}$

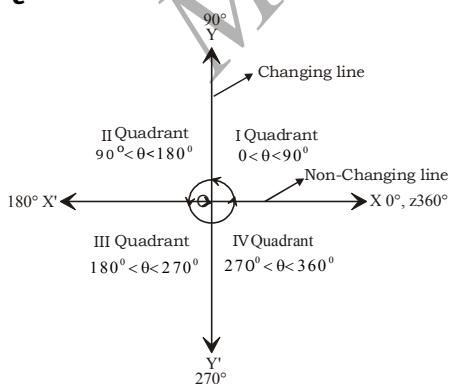
and $AC = \sqrt{(3)^2 + (4)^2} = 5$

$\therefore \sin \theta = \frac{BC}{AC} = \frac{4}{5}$



$\therefore \frac{1 - \sin \theta}{1 + \sin \theta} = \frac{1 - \frac{4}{5}}{1 + \frac{4}{5}} = \frac{1}{9}$

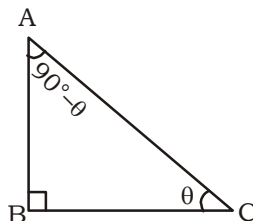
Quadrants:-



Let XOX' and YOY' be two mutually perpendicular lines. These lines divide the plane into four parts and each one of them is called a quadrant.

Complementary Angle.

For a given angle θ its complementary angle is $(90^\circ - \theta)$.



From definition,

$\sin \theta = \frac{\text{side opposite angle } \theta}{\text{hypotenuse}} = \frac{AB}{AC}$

and $\cos (90^\circ - \theta)$

$= \frac{\text{side along with angle } (90^\circ - \theta)}{\text{hypotenuse}} = \frac{AB}{AC}$

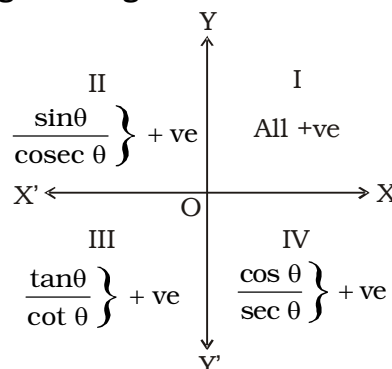
$\therefore \boxed{\sin \theta = \cos(90^\circ - \theta)}$

Similarly, we can prove that

$\therefore \boxed{\cos \theta = \sin(90^\circ - \theta)}$

$\Rightarrow 90^\circ, 270^\circ \dots$ (odd multiple of 90°) will be changed
 $\Rightarrow 0^\circ, 180^\circ, 360^\circ \dots$ (multiple of 180°) will not be changed
Change will be in following manner:
 $\sin \theta \rightarrow \cos \theta$ & $\cos \theta \rightarrow \sin \theta$
 $\tan \theta \rightarrow \cot \theta$ & $\cot \theta \rightarrow \tan \theta$
 $\sec \theta \rightarrow \csc \theta$ & $\csc \theta \rightarrow \sec \theta$

Signs of Trigonometric Ratios:-



Trigonometric Ratios of Allied Angles

(A) T-ratios of $(-\theta)$ in terms of those of θ :-

1. $\sin(-\theta) = -\sin \theta$
2. $\cos(-\theta) = \cos \theta$
3. $\tan(-\theta) = -\tan \theta$
4. $\cot(-\theta) = -\cot \theta$

5. $\sec(-\theta) = \sec \theta$

6. $\operatorname{cosec}(-\theta) = -\operatorname{cosec} \theta$

(B) T-ratios of $(90^\circ - \theta)$ in terms of those of θ :-

1. $\sin(90^\circ - \theta) = \cos \theta$

2. $\cos(90^\circ - \theta) = \sin \theta$

3. $\tan(90^\circ - \theta) = \cot \theta$

4. $\cot(90^\circ - \theta) = \tan \theta$

5. $\sec(90^\circ - \theta) = \operatorname{cosec} \theta$

6. $\operatorname{cosec}(90^\circ - \theta) = \sec \theta$

(C) T-ratios of $(90^\circ + \theta)$ in terms of those of θ :-

1. $\sin(90^\circ + \theta) = \cos \theta$

2. $\cos(90^\circ + \theta) = -\sin \theta$

3. $\tan(90^\circ + \theta) = -\cot \theta$

4. $\cot(90^\circ + \theta) = -\tan \theta$

5. $\sec(90^\circ + \theta) = -\operatorname{cosec} \theta$

6. $\operatorname{cosec}(90^\circ + \theta) = \sec \theta$

(D) T-ratios of $(180^\circ - \theta)$ in terms of those of θ :-

1. $\sin(180^\circ - \theta) = \sin \theta$

2. $\cos(180^\circ - \theta) = -\cos \theta$

3. $\tan(180^\circ - \theta) = -\tan \theta$

4. $\cot(180^\circ - \theta) = -\cot \theta$

5. $\sec(180^\circ - \theta) = -\sec \theta$

6. $\operatorname{cosec}(180^\circ - \theta) = \operatorname{cosec} \theta$

(E) T-ratios of $(180^\circ + \theta)$ in terms of those of θ :-

1. $\sin(180^\circ + \theta) = -\sin \theta$

2. $\cos(180^\circ + \theta) = -\cos \theta$

3. $\tan(180^\circ + \theta) = \tan \theta$

4. $\cot(180^\circ + \theta) = \cot \theta$

5. $\sec(180^\circ + \theta) = -\sec \theta$

6. $\operatorname{cosec}(180^\circ + \theta) = -\operatorname{cosec} \theta$

(F) T-ratios of $(270^\circ - \theta)$ in terms of those of θ :-

1. $\sin(270^\circ - \theta) = -\cos \theta$
2. $\cos(270^\circ - \theta) = -\sin \theta$
3. $\tan(270^\circ - \theta) = \cot \theta$
4. $\cot(270^\circ - \theta) = \tan \theta$
5. $\sec(270^\circ - \theta) = -\operatorname{cosec} \theta$
6. $\operatorname{cosec}(270^\circ - \theta) = -\sec \theta$

(G) T-ratios of $(270^\circ + \theta)$ in terms of those of θ :-

1. $\sin(270^\circ + \theta) = -\cos \theta$
2. $\cos(270^\circ + \theta) = \sin \theta$
3. $\tan(270^\circ + \theta) = -\cot \theta$
4. $\cot(270^\circ + \theta) = -\tan \theta$
5. $\sec(270^\circ + \theta) = \operatorname{cosec} \theta$
6. $\operatorname{cosec}(270^\circ + \theta) = -\sec \theta$

(H) T-ratios of $(360^\circ - \theta)$ in terms of those of θ :-

1. $\sin(360^\circ - \theta) = -\sin \theta$
2. $\cos(360^\circ - \theta) = \cos \theta$
3. $\tan(360^\circ - \theta) = -\tan \theta$
4. $\cot(360^\circ - \theta) = -\cot \theta$
5. $\sec(360^\circ - \theta) = \sec \theta$
6. $\operatorname{cosec}(360^\circ - \theta) = -\operatorname{cosec} \theta$

(I) T-ratios of $(360^\circ + \theta)$ in terms of those of θ :-

1. $\sin(360^\circ + \theta) = \sin \theta$
2. $\cos(360^\circ + \theta) = \cos \theta$
3. $\tan(360^\circ + \theta) = \tan \theta$
4. $\cot(360^\circ + \theta) = \cot \theta$
5. $\sec(360^\circ + \theta) = \sec \theta$
6. $\operatorname{cosec}(360^\circ + \theta) = \operatorname{cosec} \theta$

(J) T-ratios of $(n \times 360^\circ + \theta)$ in terms of those of θ :-

1. $\sin(n \times 360^\circ + \theta) = \sin \theta$
2. $\cos(n \times 360^\circ + \theta) = \cos \theta$
3. $\tan(n \times 360^\circ + \theta) = \tan \theta$
4. $\cot(n \times 360^\circ + \theta) = \cot \theta$

$$5. \sec(n \times 360^\circ + \theta) = \sec \theta$$

$$6. \operatorname{cosec}(n \times 360^\circ + \theta) = \operatorname{cosec} \theta$$

Value of some specific angle of trigonometrical (t)-ratio function.

We must learn the following table to solve the question based on trigonometrical (t)-ratio angle 0° , 30° , 45° , 60° , 90°

θ	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

TYPE - II

Ex.5 find the value of following

- (i) $\sin 120^\circ$
- (ii) $\cos 210^\circ$
- (iii) $\tan 570^\circ$
- (iv) $\cot 780^\circ$
- (v) $\sin 960^\circ$
- (vi) $\cos 1020^\circ$
- (vii) $\sec 1500^\circ$

Sol. (i) $\sin 120^\circ$
 $= \sin(90 + 30)^\circ$

$$(\sin(90 + \theta) = \cos \theta)$$

$$= \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Sol. (ii) $\cos 210^\circ$
 $= \cos(180 + 30)^\circ$

$$(\cos(180 + \theta) = -\cos \theta)$$

$$= -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

Sol. (iii) $\tan 570^\circ$
 $= \tan(540 + 30)^\circ$

(540° multiple of 180° , Then no change
 $\tan(540 + \theta) = \tan \theta$)

$$= \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Sol. (iv) $\cot 780^\circ$
 $= \cot(720 + 60)^\circ$

$$\therefore \cot(n \times 360 + \theta) = \cot \theta$$

$$= \cot(2 \times 360 + 60)^\circ$$

$$= \cot 60^\circ = \frac{1}{\sqrt{3}}$$

Sol. (v) $\sin 960^\circ$

$$= \sin(900 + 60)^\circ$$

$\therefore 900^\circ$ multiple of 180° , so no change of Trigonometry function.

$$= \sin(2 \times 360 + 180 + 60) = \sin(180 + 60) = -\sin 60^\circ$$

$$= -\frac{\sqrt{3}}{2} \left(\sin(180 + \theta) \right) = -\sin \theta$$

Sol. (vi) $\cos(1020)^\circ$

$$= \cos(1080 - 60)^\circ$$

1080 multiple of 180° , so no change In Trigonometry function.

$$= \cos(3 \times 360 - 60)^\circ$$

$$= \cos 60^\circ = \frac{1}{2}$$

Sol. (vii) $\sec(1500)^\circ$

$$= \sec(1440 + 60)^\circ$$

$$= \sec(4 \times 360 + 60)^\circ$$

$$\left(\sec(n \times 360 + \theta) \right) = \sec \theta$$

$$= \sec 60^\circ = 2$$

Ex.6

$$\frac{\cos(90^\circ + A) \cdot \sec(360^\circ - A) \cdot \tan(180^\circ - A)}{\sec(A - 720^\circ) \cdot \sin(A + 540^\circ) \cdot \cot(A - 90^\circ)} = ?$$

- (a) 0
- (b) 1
- (c) -1
- (d) None of these

Sol. (b)

$$\frac{\cos(90^\circ + A) \cdot \sec(360^\circ - A) \cdot \tan(180^\circ - A)}{\sec(A - 720^\circ) \cdot \sin(A + 540^\circ) \cdot \cot(A - 90^\circ)}$$

$$= \frac{(-\sin A) \cdot (\sec A) \cdot (-\tan A)}{\sec(2 \times 360 - A) \cdot \sin(3 \times 180^\circ + A) \cdot [-\cot(90^\circ - A)]}$$

$$(\because \sec(-\theta) = \sec \theta)$$

$$\text{and } \cot(-\theta) = -\cot \theta$$

$$= \frac{\sin A \cdot \sec A \cdot \tan A}{\sec A (-\sin A) (-\tan A)}$$

$$= \frac{\sin A \cdot \sec A \cdot \tan A}{\sin A \cdot \sec A \cdot \tan A} = 1$$

Ex.7 $\sin 720^\circ - \cot 270^\circ - \sin 150^\circ$
 $\cos 120^\circ$ is equal to:-

$$(a) \frac{1}{2} \quad (b) \frac{1}{3}$$

$$(c) \frac{1}{5} \quad (d) \frac{1}{4}$$

Sol. (d). $\sin 720^\circ - \cot 270^\circ - \sin 150^\circ \cdot \cos 120^\circ$

$$= \sin(2 \times 360^\circ + 0^\circ) - \cot(360^\circ - 90^\circ) - \sin$$

$$(90^\circ + 60^\circ) \cdot \cos(90^\circ + 30^\circ)$$

$$= \sin 0^\circ + \cot 90^\circ + \cos 60^\circ \cdot \sin 30^\circ$$

$$= 0 + 0 + \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Ex. 8. Find the value of :- $\frac{\sin 37^\circ}{\cos 53^\circ}$

- (a) 1 (b) -1 (c) 0 (d) 0

Sol. (a)

$$\frac{\sin 37^\circ}{\cos 53^\circ} = \frac{\sin 37^\circ}{\cos(90^\circ - 37^\circ)} = \frac{\sin 37^\circ}{\sin 37^\circ} = 1$$

Ex. 9. Evaluate :- $\sin^2 60^\circ + \cos^2 30^\circ + \cot^2 45^\circ$

$$+ \sec^2 60^\circ - \operatorname{cosec}^2 30^\circ + \cos^2 0^\circ :-$$

- (a) $\frac{3}{2}$ (b) $\frac{5}{2}$

- (c) $\frac{7}{2}$ (d) 2

Sol. (c) We know that $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}, \cot 45^\circ = 1$$

$$\sec 60^\circ = 2$$

$$\therefore \sin^2 60^\circ + \cos^2 30^\circ + \cot^2 45^\circ + \sec^2 60^\circ$$

$$- \operatorname{cosec}^2 30^\circ + \cos^2 0^\circ$$

$$= \left[\left(\frac{\sqrt{3}}{2} \right)^2 + \left(\frac{\sqrt{3}}{2} \right)^2 + 1^2 + 2^2 - 2^2 + 1^2 \right] = \frac{7}{2}$$

Ex. 10 If $\frac{x \operatorname{cosec}^2 30^\circ \cdot \sec^2 45^\circ}{8 \cos^2 45^\circ \cdot \sin^2 60^\circ}$

= $\tan^2 60^\circ - \tan^2 30^\circ$, then the value of x is :-

- (a) -1 (b) 0 (c) 1 (d) 2

Sol. (c)

$$\frac{x \times (2)^2 \times (\sqrt{2})^2}{8 \times \left(\frac{1}{\sqrt{2}} \right)^2 \times \left(\frac{\sqrt{3}}{2} \right)^2} = (\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}} \right)^2$$

$$\Rightarrow 8x = \left(\frac{8}{3} \times 8 \times \frac{1}{2} \times \frac{3}{4} \right) \Rightarrow x = 1$$

Ex. 11. If $\sec \theta = \frac{13}{5}$ and θ lies in the

fourth quadrant, then the value of $\sin \theta$ is :-

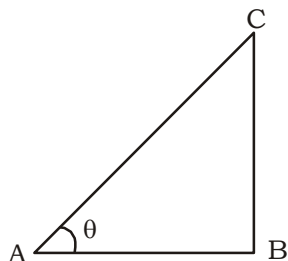
- (a) $\frac{12}{13}$ (b) $-\frac{12}{13}$

- (c) $-\frac{5}{13}$ (d) $\frac{5}{13}$

Sol. (b) $\sec \theta = \frac{13}{5} = \frac{AC}{AB}$

$$\text{and } BC = \sqrt{13^2 - 5^2} = 12$$

$$\therefore \sin \theta = \frac{BC}{AC} = \frac{12}{13}$$



since θ lies in the fourth quadrant

$$\therefore \sin \theta = -\frac{12}{13}$$

Ex. 12. If $3 \tan \theta + 4 = 0$, where

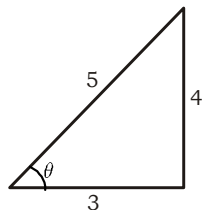
$\frac{\pi}{2} < \theta < \pi$, then the value of

$2 \cot \theta - 5 \cos \theta + \sin \theta$ is :-

- (a) $-\frac{53}{10}$ (b) $\frac{7}{10}$

- (c) $\frac{23}{10}$ (d) $\frac{37}{10}$

Sol. (c) $3 \tan \theta + 4 = 0 \Rightarrow \tan \theta = -\frac{4}{3}$



$$\Rightarrow \sin \theta = \frac{4}{5}, \cos \theta = -\frac{3}{5}, \cot \theta = -\frac{3}{4}$$

[$\because \sin \theta$ is positive and $\cos \theta$ is negative in II quadrant].

Ex. 13 If $\sin 17^\circ = \frac{x}{y}$, then the

value of $\sec 17^\circ - \sin 73^\circ$ is:

- (a) $\frac{y^2 - x^2}{xy}$ (b) $\frac{x^2}{\sqrt{y^2 - x^2}}$

- (c) $\frac{x^2}{y\sqrt{y^2 + x^2}}$ (d) $\frac{x^2}{y\sqrt{y^2 - x^2}}$

Sol. (d) $\sec 17^\circ - \sin 73^\circ$

$$= \sec 17^\circ - \sin (90^\circ - 17^\circ)$$

$$= \sec 17^\circ - \cos 17^\circ$$

$$= \frac{1}{\cos 17^\circ} - \cos 17^\circ$$

$$= \frac{1 - \cos^2 17^\circ}{\cos 17^\circ} = \frac{\sin^2 17^\circ}{\cos 17^\circ} = \frac{\frac{x^2}{y^2}}{\sqrt{1 - \frac{x^2}{y^2}}}$$

$$= \frac{\frac{x^2}{y^2}}{\frac{\sqrt{y^2 - x^2}}{y}} = \frac{x^2}{y\sqrt{y^2 - x^2}}$$

Ex. 14. If $\operatorname{cosec} 39^\circ = x$, the value of

$$\frac{1}{\operatorname{cosec}^2 51^\circ} + \sin^2 39^\circ +$$

$$\tan^2 51^\circ - \frac{1}{\sin^2 51^\circ \sec^2 39^\circ}$$
 is:

- (a) $\sqrt{x^2 - 1}$ (b) $\sqrt{1 - x^2}$

- (c) $x^2 - 1$ (d) $1 - x^2$

Sol. (c) $\frac{1}{\operatorname{cosec}^2 51^\circ} + \sin^2 39^\circ + \tan^2 51^\circ$

$$= \frac{1}{\sin^2 51^\circ \cdot \sec^2 39^\circ}$$

$$= \sin^2 51^\circ + \sin^2 39^\circ + \tan^2 (90^\circ - 39^\circ)$$

$$= \frac{1}{\sin^2 (90^\circ - 39^\circ) \cdot \sec^2 39^\circ}$$

$$= \cos^2 39^\circ + \sin^2 39^\circ + \cot^2 39^\circ$$

$$= \frac{1}{\cos^2 39^\circ \cdot \sec^2 39^\circ}$$

$$[\because \sin (90^\circ - \theta) = \cos \theta],$$

$$\tan (90^\circ - \theta) = \cot \theta]$$

$$= 1 + \cot^2 39^\circ - 1 = \operatorname{cosec}^2 39^\circ - 1$$

$$= x^2 - 1$$

Ex. 15 Find the value of $\cos (180^\circ + A) + \cos (180^\circ + B) + \cos (180^\circ + C) + \cos (180^\circ + D)$ Where A, B, C and D are the vertices of a cyclic quadrilateral?

- (a) 0 (b) 1
(c) 2 (d) $2 \cos A$

Sol. (a)

$$\cos (180^\circ + A) + \cos (180^\circ + B) + \cos$$

$$(180^\circ + C) + \cos (180^\circ + D)$$

$$= -\cos A - \cos B - \cos C - \cos D$$

$$= -\cos(180^\circ - C) - \cos(180^\circ - D)$$

$$= -\cos C - \cos D$$

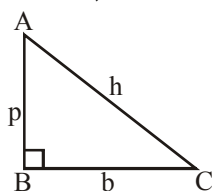
[$\because A + C = B + D = 180^\circ$ cyclic quadrilateral]

$$= \cos C + \cos D - \cos C - \cos D = 0$$

Some Useful formula

- (i) $\sin^2 \theta + \cos^2 \theta = 1$
 or $\sin^2 \theta = 1 - \cos^2 \theta$
 or $\cos^2 \theta = 1 - \sin^2 \theta$
- (ii) $1 + \tan^2 \theta = \sec^2 \theta$
 or $\sec^2 \theta - 1 = \tan^2 \theta$
 or $\sec^2 \theta - \tan^2 \theta = 1$
- (iii) $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$
 or $\operatorname{cosec}^2 \theta - 1 = \cot^2 \theta$
 or $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

Proof we know,



$$\sin \theta = \frac{p}{h} \quad \cos \theta = \frac{b}{h}$$

Now,

$$\sin^2 \theta + \cos^2 \theta$$

$$= \left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2 = \frac{p^2}{h^2} + \frac{b^2}{h^2} =$$

$$\frac{p^2 + b^2}{h^2}$$

$\because$ In right angle $\triangle ABC$

$$p^2 + b^2 = h^2$$

then $\sin^2 \theta + \cos^2 \theta$

$$= \frac{h^2}{h^2} = 1$$

★ Same as we can proof all remaining results same this process

TYPE - III

If $A + B = 90^\circ$,

Results

- (i) $\sin A \cdot \sec B = 1$
 or $\sin A = \cos B$
 (ii) $\cos A \cdot \operatorname{cosec} B = 1$
 or $\sec A = \operatorname{cosec} B$
 (iii) $\tan A \cdot \tan B = 1$
 or $\tan A = \cot B$
 (iv) $\cot A \cdot \cot B = 1$
 (v) $\sin^2 A + \sin^2 B = 1$
 (vi) $\cos^2 A + \cos^2 B = 1$

Proof

(i) $\sin A \cdot \sec B = 1$

$A + B = 90^\circ$ (given)

$B = 90^\circ - A$

Then, $\sin A \cdot \sec(90^\circ - A)$

Now, $\sin A \cdot \operatorname{cosec} A$

$$\Rightarrow \sin A \times \frac{1}{\sin A} = 1$$

★ Same as we can proof all remaining results same this process

★ **And their vice-versa are also true.**

when $\sin A \cdot \sec B = 1$,

then we can say $A + B = 90^\circ$

Ex. 16 The value of $(\sin 25^\circ \cdot \sec 65^\circ)$ is equal to:-

Sol. $25^\circ + 65^\circ = 90^\circ$

$$\left(\begin{array}{l} \text{If } A + B = 90^\circ \\ \sin A \cdot \sec B = 1 \end{array} \right)$$

So, $\sin 25^\circ \cdot \sec 65^\circ = 1$

Ex. 17 The value of $(\tan 23^\circ \times \tan 67^\circ)$ is equal to :-

Sol. $23^\circ + 67^\circ = 90^\circ$

$$\left(\begin{array}{l} \text{If } A + B = 90^\circ \\ \tan A \cdot \tan B = 1 \end{array} \right)$$

So, $\tan 23^\circ \cdot \tan 67^\circ = 1$

Ex. 18 The value of $\tan 10^\circ \cdot \tan 25^\circ \cdot \tan 65^\circ \cdot \tan 80^\circ$ is

Sol.

$$\tan 10^\circ \tan 25^\circ \tan 65^\circ \tan 80^\circ = 1$$

Ex. 19 If $\sin(3x - 6) = \cos(6x - 3)$ find the value of x .

Sol. $3x - 6 + 6x - 3 = 90^\circ$

$$9x = 99^\circ$$

$$x = 11$$

$$\left(\begin{array}{l} \text{If } A + B = 90, \\ \text{then } \sin A = \cos B \end{array} \right)$$

Ex. 20 The value of $\cos 40^\circ \cdot \operatorname{cosec} 50^\circ$

Sol. $40^\circ + 50^\circ = 90^\circ$

$$\left(\begin{array}{l} \text{If } A + B = 90^\circ \\ \cos A \cdot \operatorname{cosec} B = 1 \end{array} \right)$$

So, $\cos 40^\circ \cdot \operatorname{cosec} 50^\circ = 1$

Ex. 21 If $\tan 2\theta \cdot \tan 3\theta = 1$

find the value of θ

Sol. $2\theta + 3\theta = 90^\circ$

$$5\theta = 90^\circ$$

$$\theta = 18^\circ$$

$$\left(\begin{array}{l} \text{If } A + B = 90^\circ \\ \tan A \cdot \tan B = 1 \end{array} \right)$$

Ex. 22 If $\cot 2\theta \cdot \cot 3\theta = 1$ find the value of

$$\sin \frac{5\theta}{2} \cdot \cos \frac{5\theta}{2}$$

Sol. $2\theta + 3\theta = 90^\circ$

$$5\theta = 90^\circ$$

$$\left(\begin{array}{l} \text{If } A + B = 90^\circ \\ \tan A \cdot \tan B = 1 \end{array} \right)$$

$$\text{Now, } \sin \frac{5\theta}{2} \cdot \cos \frac{5\theta}{2}$$

put the value of 5θ

$$= \sin \frac{90}{2} \cdot \cos \frac{90}{2}$$

$$= \sin 45^\circ \cdot \cos 45^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

Ex. 23 If $\sin(x + 4)^\circ \sec(x - 4)^\circ = 1$

find the value of $\tan \frac{2x}{3}$

Sol. $\sin(x + 4)^\circ \sec(x - 4)^\circ = 1$

$$x + 4 + x - 4 = 90^\circ$$

$$2x = 90^\circ$$

$$x = 45$$

Now,

$$\tan \frac{2x}{3}$$

put of value of x

$$= \tan \frac{2 \times 45}{3} = \tan \frac{90^\circ}{3}$$

$$= \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Ex. 24 If $\cos(90 - \theta) = \sin(3\theta - 5)$ find the value of θ

Sol. $\cos(90 - \theta) = \sin(3\theta - 50)$

$$\cos(90 - \theta) \operatorname{cosec}(3\theta - 50) = 1$$

$$90 - \theta + 3\theta - 50 = 90^\circ$$

$$\left(\begin{array}{l} \text{if } \cos A \cdot \operatorname{cosec} B = 1 \\ A + B = 90^\circ \end{array} \right)$$

$$2\theta = 50^\circ$$

$$\theta = 25^\circ$$

Ex.25 If $\cot(x - 50) = \tan(80 - 2x)$
find the value of $\tan x + \sin x$
Sol. $\cot(x - 50) = \tan(80 - 2x)$
 $\cot(x - 50) \cot(80 - 2x) = 1$

$$\left(\begin{array}{l} \text{if } \cot A \cdot \cot B = 1 \\ \text{then } A + B = 90^\circ \end{array} \right)$$

$$x - 50 + 80 - 2x = 90$$

$$-x + 30 = 90$$

$$x = -60^\circ$$

Now, $\tan x + \sin x$
 $= \tan(-60^\circ) + \sin(-60^\circ)$
 $= -\tan 60^\circ - \sin 60^\circ$

$$\left(\begin{array}{l} \tan(-\theta) = -\tan \theta \\ \sin(-\theta) = -\sin \theta \end{array} \right)$$

$$= -(\tan 60^\circ + \sin 60^\circ)$$

$$= -\left(\sqrt{3} + \frac{\sqrt{3}}{2} \right) = \frac{-3\sqrt{3}}{2}$$

TYPE-IV

Sum and Difference Formula

(i) $\sin(A+B) = \sin A \cdot \cos B + \cos A \sin B$

(ii) $\sin(A-B) = \sin A \cdot \cos B - \cos A \sin B$

(iii) $\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$

(iv) $\cos(A-B) = \cos A \cdot \cos B + \sin A \sin B$

(v) $2 \sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$

(vi) $2 \cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$

(vii) $2 \sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

(viii) $2 \cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$

(ix) $\sin^2 A - \sin^2 B = \sin(A+B) \cdot \sin(A-B)$

(x) $\cos^2 A - \cos^2 B = \cos(A+B) \cdot \cos(A-B)$

Tangent Formulae

(i) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

(ii) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

(iii) $\cot(A+B) = \frac{\cot A \cdot \cot B - 1}{\cot A + \cot B}$

(iv) $\cot(A-B) = \frac{\cot B \cdot \cot A + 1}{\cot B - \cot A}$

(v) $\tan(45 + \theta) = \frac{1 + \tan \theta}{1 - \tan \theta}$
 $= \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$

(vi) $\tan(45 - \theta) = \frac{1 - \tan \theta}{1 + \tan \theta}$

$$= \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}$$

Ex.26 Find the value of the following

(i) $\sin 75^\circ$ (ii) $\cos 75^\circ$ (iii)

$\tan 15^\circ$ (iv) $\tan 75^\circ$

Sol.(i) $\sin 75^\circ$
 $\sin(45^\circ + 30^\circ)$
 $= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$
 $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$

★ $\sin 75^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}} = \cos 15^\circ$

(ii) $\cos 75^\circ$
 $\cos(45^\circ + 30^\circ)$
 $= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$
 $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$

★ $\cos 75^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}} = \sin 15^\circ$

(iii) $\tan 15^\circ$
 $\tan(45 - 30)$
 $= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ}$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

★ $\tan 15^\circ = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \cot 75^\circ$

(iv) $\tan 75^\circ$
 $\tan(45 + 30)$
 $= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

★ $\tan 75^\circ = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \cot 15^\circ$

Trigonometric Ratios of Specific Angles

(i) $\sin 18^\circ = \left(\frac{\sqrt{5} - 1}{4} \right)$

(ii) $\cos 18^\circ = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$

(iii) $\cos 36^\circ = \left(\frac{\sqrt{5} + 1}{4} \right)$

(iv) $\sin 36^\circ = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

(vii) $\sin 22 \frac{1}{2}^\circ = \frac{\sqrt{2 - \sqrt{2}}}{2}$

(viii) $\cos 22 \frac{1}{2}^\circ = \frac{\sqrt{2 + \sqrt{2}}}{2}$

Ex.27 The value of $\frac{\cos 15^\circ - \sin 15^\circ}{\cos 15^\circ + \sin 15^\circ}$ is

Sol. $\frac{\cos 15^\circ - \sin 15^\circ}{\cos 15^\circ + \sin 15^\circ}$
 $= \tan(45 - 15)$
 $= \tan 30^\circ = \frac{1}{\sqrt{3}}$

Ex.28 The value of $\tan 40^\circ + 2 \tan 10^\circ$ is equal to
(a) $\tan 40^\circ$ (b) $\cot 40^\circ$
(c) $\sin 40^\circ$ (d) $\cos 40^\circ$

Sol. We know,
 $40^\circ + 10^\circ = 50^\circ$
both sides take $\tan$
 $\tan(40^\circ + 10^\circ) = \tan 50^\circ$
 $\Rightarrow \frac{\tan 40^\circ + \tan 10^\circ}{1 - \tan 40^\circ \cdot \tan 10^\circ} = \tan 50^\circ$
 $\Rightarrow \tan 40^\circ + \tan 10^\circ = \tan 50^\circ - \tan 50^\circ \cdot \tan 40^\circ \cdot \tan 10^\circ$

$$\Leftrightarrow 1 \left(\begin{array}{l} \therefore \tan A \cdot \tan B = 1 \\ \text{if } A + B = 90 \end{array} \right)$$

$\Rightarrow \tan 40^\circ + \tan 10^\circ = \tan 50^\circ - \tan 10^\circ$
 $\Rightarrow \tan 40^\circ + 2 \tan 10^\circ = \tan 50^\circ$
Now, $\tan 50^\circ = \tan(90^\circ - 40^\circ) = \cot 40^\circ$

Ex.29 The value of $\frac{\tan 57^\circ + \cot 37^\circ}{\tan 33^\circ + \cot 53^\circ}$ is equal to
(a) $\tan 33^\circ \cdot \cot 53^\circ$
(b) $\tan 53^\circ \cdot \cot 37^\circ$
(c) $\tan 33^\circ \cdot \cot 57^\circ$
(d) $\tan 57^\circ \cdot \cot 37^\circ$
Sol. $\frac{\tan 57^\circ + \cot 37^\circ}{\tan 33^\circ + \cot 53^\circ}$
 $\Rightarrow \frac{\tan 57^\circ + \cot 37^\circ}{\tan(90 - 57) + \cot 53^\circ}$

$$\Rightarrow \frac{\tan 57^\circ + \frac{1}{\tan 37^\circ}}{\cot 57^\circ + \cot(90 - 37)^\circ}$$

$$\Rightarrow \frac{\tan 57^\circ + \frac{1}{\tan 37^\circ}}{\frac{1}{\tan 57^\circ} + \tan 37^\circ}$$

$$\Rightarrow \frac{(\tan 57^\circ \tan 37^\circ + 1) \tan 37^\circ}{(\tan 57^\circ \tan 37^\circ + 1) \tan 57^\circ}$$

$$\Rightarrow \frac{1}{\tan 37^\circ} \times \tan 57^\circ$$

$$\Rightarrow \tan 57^\circ \cdot \cot 37^\circ$$

TYPE-V

Use of componendo and dividendo-

If $\frac{x}{y} = \frac{a}{b}$, Then $\frac{x}{y} = \frac{a}{b}$

$$\frac{x+y}{x-y} = \frac{a+b}{a-b}$$

Proof

$$\frac{x}{y} = \frac{a}{b}$$

Add 1 in both side.

$$\frac{x}{y} + 1 = \frac{a}{b} + 1$$

$$\frac{x+y}{y} = \frac{a+b}{b} \quad \dots\dots\dots(i)$$

subtract 1 in both side.

$$\frac{x}{y} - 1 = \frac{a}{b} - 1$$

$$\frac{x-y}{y} = \frac{a-b}{b} \quad \dots\dots\dots(ii)$$

(i) / (ii)

$$\frac{x+y}{x-y} = \frac{a+b}{a-b}$$

Ex.30 If $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = 9$ find the value $\tan \theta$ and $\cos \theta$

$$\text{Sol. } \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = \frac{9}{1}$$

Apply C & D

$$\Rightarrow \frac{(\sin \theta + \cos \theta) + (\sin \theta - \cos \theta)}{(\sin \theta + \cos \theta) - (\sin \theta - \cos \theta)}$$

$$= \frac{9+1}{9-1}$$

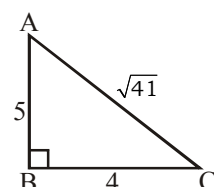
$$\Rightarrow \frac{2 \sin \theta}{2 \cos \theta} = \frac{10}{8}$$

$$\tan \theta = \frac{5}{4}$$

Now, $\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$

$$\text{Hypotenuse} = \sqrt{(5)^2 + (4)^2}$$

$$= \sqrt{41}$$



$$\text{Then, } \cos \theta = \frac{b}{h} = \frac{4}{\sqrt{41}}$$

Ex.31 If $\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} = \frac{5}{3}$, then find

The value of $\sin \theta$

$$\text{Sol. } \frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} = \frac{5}{3}$$

Apply C & D

$$\Rightarrow \frac{(\sec \theta + \tan \theta) + (\sec \theta - \tan \theta)}{(\sec \theta + \tan \theta) - (\sec \theta - \tan \theta)}$$

$$= \frac{5+3}{5-3}$$

$$\Rightarrow \frac{2 \sec \theta}{2 \tan \theta} = \frac{8}{2}$$

$$\Rightarrow \frac{\frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} = 4 \Rightarrow \frac{1}{\sin \theta} = 4$$

$$\text{So, } \sin \theta = \frac{1}{4}$$

Ex. 32 If $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = \frac{5}{4}$, find the

value of $\frac{\tan^2 \theta + 1}{\tan^2 \theta - 1}$

$$\text{Sol. } \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = \frac{5}{4},$$

Apply C and D

$$\tan \theta = 9$$

$$\text{Now, } \frac{\tan^2 \theta + 1}{\tan^2 \theta - 1} = \frac{(9)^2 + 1}{(9)^2 - 1}$$

$$= \frac{82}{80} = \frac{41}{40}$$

Ex. 33 If $\tan \theta = \frac{4}{3}$, then the value of

$$\frac{3 \sin \theta + 2 \cos \theta}{3 \sin \theta - 2 \cos \theta} \text{ is}$$

$$\text{Sol. } \tan \theta = \frac{4}{3}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{4}{3}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{2 \times 2}{3}$$

$$\frac{3 \sin \theta}{2 \cos \theta} = \frac{2}{1}$$

Apply C and D

$$\frac{3 \sin \theta + 2 \cos \theta}{3 \sin \theta - 2 \cos \theta} = \frac{2+1}{2-1}$$

$$\boxed{\frac{3 \sin \theta + 2 \cos \theta}{3 \sin \theta - 2 \cos \theta} = 3}$$

Alternate:-

$$\frac{3 \sin \theta + 2 \cos \theta}{3 \sin \theta - 2 \cos \theta}$$

divide all terms by $\cos \theta$

$$\frac{3 \tan \theta + 2}{3 \tan \theta - 2}$$

$$\therefore \tan \theta = \frac{4}{3} \text{ (given)}$$

$$= \frac{3 \times \frac{4}{3} + 2}{3 \times \frac{4}{3} - 2} = \frac{4+2}{4-2} = \frac{6}{2} = 3$$

Ex.34 If $2 \cot \theta = 3$, Then find the value of $\frac{2 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta}$

$$\text{Sol. } 2 \cot \theta = 3$$

$$\cot \theta = \frac{3}{2}$$

$$\frac{\cos \theta}{\sin \theta} = \frac{3}{2}$$

$$\frac{2 \cos \theta}{\sin \theta} = \frac{3}{1}$$

Apply C and D

$$\frac{2\cos\theta + \sin\theta}{2\cos\theta - \sin\theta} = \frac{3+1}{3-1}$$

$$\frac{2\cos\theta + \sin\theta}{2\cos\theta - \sin\theta} = \frac{4}{2} = 2$$

So,

$$\frac{2\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = \frac{1}{2}$$

Alternate:-

$$\frac{2\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$$

Divide all terms (in numerator and in denominator) by $\sin\theta$

$$\frac{2\cot\theta - 1}{2\cot\theta + 1}$$

$$\therefore \cot\theta = \frac{3}{2} \text{ (given)}$$

$$\frac{2 \times \frac{3}{2} - 1}{2 \times \frac{3}{2} + 1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

TYPE - VI

Some pythagorean natural number will help in solving the problem on trigonometric ratio angle.

pythagorean theorem

$$(\text{Base})^2 + (\text{Perpendicular})^2 = (\text{Hypotenuse})^2$$

$$\begin{aligned} 3^2 + 4^2 &= 5^2, & 6^2 + 8^2 &= 10^2, \\ 5^2 + 12^2 &= 13^2, & 10^2 + 24^2 &= 26^2, \\ 8^2 + 15^2 &= 17^2, & 7^2 + 24^2 &= 25^2, \\ 20^2 + 21^2 &= 29^2, & 9^2 + 40^2 &= 41^2, \text{ etc.} \end{aligned}$$

Ex.35 If $\sin\theta + \cos\theta = \frac{17}{13}$ find the value of $\sin\theta \cdot \cos\theta$

Sol. $\sin\theta + \cos\theta = \frac{17}{13}$
squaring of both side

$$\Rightarrow (\sin\theta + \cos\theta)^2 = \left(\frac{17}{13}\right)^2$$

$$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta \cos\theta = \frac{289}{169}$$

$$\Rightarrow 1 + 2\sin\theta \cos\theta = \frac{289}{169}$$

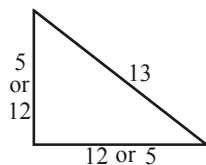
$$\Rightarrow 2\sin\theta \cos\theta = \frac{289}{169} - 1$$

$$\Rightarrow 2\sin\theta \cos\theta = \frac{289-169}{169}$$

$$\Rightarrow 2\sin\theta \cos\theta = \frac{120}{169}$$

$$\Rightarrow \sin\theta \cos\theta = \frac{60}{169}$$

Alternate:-



$$\begin{array}{ccc} \sin\theta + \cos\theta = \frac{17}{13} & \rightarrow & \frac{p+b}{h} \\ \downarrow & & \downarrow \\ \frac{p}{h} & & \frac{b}{h} \end{array}$$

Apply pythagorean here hypotenuse is 13, Then other sides of right angle triangle will be 5 and 12.

Now,

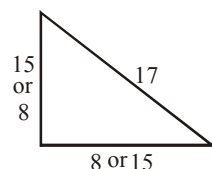
$$\frac{5}{13} + \frac{12}{13} = \frac{17}{13}$$

But we cannot find exact value of base and perpendicular, here no affect of value of $\sin\theta$ and $\cos\theta$. This question because both are product.

$$\text{Hence, } \sin\theta \cos\theta = \frac{5}{13} \times \frac{12}{13} = \frac{60}{169}$$

Ex.36 If $\sin\theta + \cos\theta = \frac{23}{17}$, find the value of $\sin\theta \cdot \cos\theta$

Sol.



$$\sin\theta + \cos\theta = \frac{23}{17}$$

$\downarrow \quad \downarrow$

$$\frac{p}{h} + \frac{b}{h} = \frac{p+b}{h}$$

so, $h \rightarrow 17$

Apply pythagorean sides here hypotenuse is 17, then other sides 8 and 15

$$\text{Now, Check } \frac{8}{17} + \frac{15}{17} = \frac{23}{17}$$

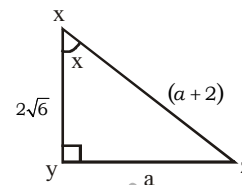
$$\text{Hence, } \sin\theta \cdot \cos\theta = \frac{8}{17} \times \frac{15}{17} = \frac{120}{289}$$

Ex. 37 In a Δxyz , $\angle y = 90^\circ$

$$xy = 2\sqrt{6} \text{ and } xz - yz = 2$$

Find the value of $\sec x + \tan x = ?$

Sol.



Let $yz = a$,

Then $xz = 2 + yz = 2 + a$

Apply pythagorean theorem

$$\Rightarrow (2\sqrt{6})^2 + (a)^2 = (a+2)^2$$

$$\Rightarrow (2\sqrt{6})^2 = (a+2)^2 - (a)^2$$

$$\Rightarrow 24 = (a+2-a)(a+2+a)$$

$$\Rightarrow 24 = 2(2a+2)$$

$$\Rightarrow 24 = 4(a+1)$$

$$\Rightarrow 6 = a+1$$

$$\Rightarrow a = 5$$

$$\text{Hence, } xz = (5+2) = 7$$

$$yz = 5$$

$$\Rightarrow \sec x + \tan x = \frac{7}{2\sqrt{6}} + \frac{5}{2\sqrt{6}}$$

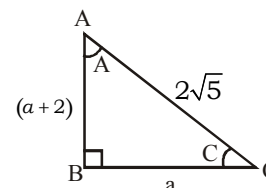
$$= \frac{12}{2\sqrt{6}} = \frac{6}{\sqrt{6}} = \sqrt{6}$$

Ex. 38 In a ΔABC , $\angle B = 90^\circ$

$$AB - BC = 2, \text{ and } AC = 2\sqrt{5}$$

find the value of $\cos^2 A - \cos^2 C = ?$

Sol.



Let the triangle of side $BC = a$

Then $AB = BC + 2 = a + 2$

Now,

$$\Rightarrow (a+2)^2 + (a)^2 = (2\sqrt{5})^2$$

$$\Rightarrow a^2 + 4 + 4a + a^2 = 20$$

$$\Rightarrow 2a^2 + 4a = 16$$

$$\Rightarrow a^2 + 2a = 8$$

$$\Rightarrow a^2 + 2a - 8 = 0$$

$$\Rightarrow a^2 + 4a - 2a - 8 = 0$$

$$\Rightarrow (a+4)(a-2) = 0$$

$$a = -4, a = 2$$

side of Δ is always positive hence,

We take $a = 2$

Now,
 $\Rightarrow AB = 4, BC = 2$
 $\Rightarrow \cos^2 A - \cos^2 C$
 $\Rightarrow \left(\frac{4}{2\sqrt{5}}\right)^2 - \left(\frac{2}{2\sqrt{5}}\right)^2$
 $\Rightarrow \frac{16}{20} - \frac{4}{20} = \frac{12}{20} = \frac{3}{5}$

Ex. 39 If $2\sin\alpha + 15\cos^2\alpha = 7$,
 $(0^\circ < \alpha < 90^\circ)$ find the value of $\cot\alpha$

(a) $\frac{3}{4}$ (b) $\frac{5}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Sol. $2\sin\alpha + 15\cos^2\alpha = 7$,
 $\Rightarrow 2\sin\alpha + 15(1 - \sin^2\alpha) = 7$
 $\Rightarrow 2\sin\alpha + 15 - 15\sin^2\alpha = 7$
 $\Rightarrow 15\sin^2\alpha - 2\sin\alpha - 8 = 0$
 $\Rightarrow 15\sin^2\alpha - 12\sin\alpha + 10\sin\alpha - 8 = 0$
 $\Rightarrow (3\sin\alpha + 2)(5\sin\alpha - 4) = 0$
 $\Rightarrow 3\sin\alpha + 2 = 0$ or $5\sin\alpha - 4 = 0$
 $\Rightarrow \sin\alpha = -\frac{2}{3}$ or $\sin\alpha = \frac{4}{5}$

Value of α between 0° and 90°
so $\sin\alpha$ is positive. Then we

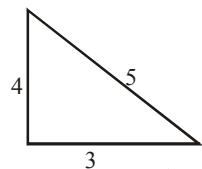
take $\sin\alpha = \frac{4}{5}$

$\sin\alpha = \frac{4}{5} = \frac{p}{h}$

$b = \sqrt{(5)^2 - (4)^2} = 3$

Hence, $\cot\alpha = \frac{b}{p} = \frac{3}{4}$

Alternate:-



We take option (a)

$\cot\alpha = \frac{3}{4} = \frac{b}{p}$

$h = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

Now, check

$2\sin\alpha + 15\cos^2\alpha = 7$

$2 \times \frac{4}{5} + 15 \times \left(\frac{3}{5}\right)^2$

$= \frac{8}{5} + 15 \times \frac{9}{25}$

$= \frac{8}{5} + \frac{27}{5} = \frac{35}{5} = 7$

L.H.S = R.H.S

So, $\cot\alpha = \frac{3}{4}$

Ex. 40 If $2 - \cos^2\theta = 3\sin\theta \cos\theta$,
find the value of $\tan\theta$

(a) $\frac{1}{2}$ (b) 0 (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

Sol. $2 - \cos^2\theta = 3\sin\theta \cos\theta$
divide by $\cos^2\theta$ both sides

$\Rightarrow \frac{2 - \cos^2\theta}{\cos^2\theta} = \frac{3\sin\theta \cos\theta}{\cos^2\theta}$

$\Rightarrow 2\sec^2\theta - 1 = 3\tan\theta$

$\Rightarrow 2(1 + \tan^2\theta) - 1 = 3\tan\theta$

$\Rightarrow 2 + 2\tan^2\theta - 1 = 3\tan\theta$

$\Rightarrow 2\tan^2\theta - 3\tan\theta + 1 = 0$

$\Rightarrow (2\tan\theta - 1)(\tan\theta - 1) = 0$

$\Rightarrow 2\tan\theta - 1 = 0$, $\tan\theta - 1 = 0$

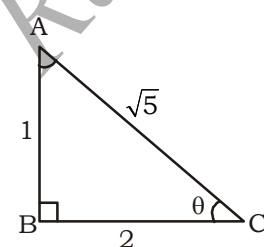
$\Rightarrow \tan\theta = \frac{1}{2}$ or 1

So, option (a) is correct

$\tan\theta = \frac{1}{2}$

Alternate:-

Take options (a)



So, $\tan\theta = \frac{1}{2} = \frac{p}{b}$

then, $h = \sqrt{(2)^2 + (1)^2}$

$h = \sqrt{5}$

Now,

$2 - \cos^2\theta = 3\sin\theta \cos\theta$

$2 - \left(\frac{2}{\sqrt{5}}\right)^2 = 3 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$

$\frac{6}{5} = \frac{6}{5}$

L.H.S = R.H.S

$\therefore \tan\theta = \frac{1}{2}$

TYPE -VII

Function and Inverse function $\rightarrow$

(a) If $\sin\theta + \operatorname{cosec}\theta = 2$
then $\sin\theta = \operatorname{cosec}\theta = 1$
 $\therefore \sin^n\theta + \operatorname{cosec}^n\theta = 2$
 $n \rightarrow$ natural no.

Ex.41 If $\sin\theta + \operatorname{cosec}\theta = 2$ find the
value of $\sin^{100}\theta + \operatorname{cosec}^{100}\theta$

Sol. $\sin\theta + \operatorname{cosec}\theta = 2$
Then, $\sin\theta = \operatorname{cosec}\theta = 1$
so, $\sin^{100}\theta + \operatorname{cosec}^{100}\theta = 1$
 $= (1)^{100} + (1)^{100} = 2$

(b) If $\cos\theta + \sec\theta = 2$
then $\cos\theta = \sec\theta = 1$
 $\therefore \cos^n\theta + \sec^n\theta = 2$

Ex.42 If $\cos\theta + \sec\theta = 2$, find the
value of $\cos^{10}\theta + \sec^{10}\theta = ?$

Sol. $\cos\theta + \sec\theta = 2$
 $\cos\theta = \sec\theta = 1$
Then, $\cos^{10}\theta + \sec^{10}\theta$
 $= (1)^{10} + (1)^{10} = 1 + 1 = 2$

(c) If $\tan\theta + \cot\theta = 2$
so $\tan\theta = \cot\theta = 1$
 $\tan^n\theta + \cot^n\theta = 2$

Ex.43 If $\tan\theta + \cot\theta = 2$ find the
value of $\tan^{50}\theta + \cot^{60}\theta$

Sol. $\tan\theta + \cot\theta = 2$
 $\tan\theta = \cot\theta = 1$

$\tan^{50}\theta + \cot^{60}\theta = (1)^{50} + (1)^{60} = 1 + 1 = 2$

(d) If $\sin A + \cos B = 2$
Then $A = 90^\circ$
 $B = 0^\circ$

Ex.44 If $\sin A + \cos B = 2$, then find the
value of $\tan \frac{(A+B)}{2}$

Sol. $\sin A + \cos B = 2$
 $\downarrow \quad \quad \downarrow$
 $1 \quad \quad 1$
 $A = 90^\circ$
 $B = 0^\circ$

Now, $\tan \frac{(A+B)}{2}$
 $= \tan \frac{(90+0)}{2} = \tan 45^\circ = 1$

(e) If $\sin A + \cos B = 0$
then $A = 0^\circ$, $B = 90^\circ$

Ex.45 If $\sin A + \cos B + \sin C = 3$, then
find the value of $\cot \frac{(A+B+C)}{3}$

Sol. $\sin A + \cos B + \sin C = 3$
 $\downarrow \quad \downarrow \quad \downarrow$
 $1 \quad 1 \quad 1$
 $\therefore A = 90^\circ$
 $B = 0^\circ$
 $C = 90^\circ$
 $\cot \frac{(90+0+90)}{3} = \cot 60^\circ = \frac{1}{\sqrt{3}}$

Ex. 46 If θ is acute and $\tan \theta + \cot \theta = 2$, then $\tan^7 \theta + \cot^9 \theta$ is equal to:
 (a) $\sqrt{3}$ (b) 3 (c) 2 (d) 4

Sol. $\tan \theta + \cot \theta = 2$
 $\tan \theta = \cot \theta = 1$
 So, $\therefore \tan^7 \theta + \cot^9 \theta = 1 + 1 = 2$

Ex. 47 If $\tan(x+y) = \sqrt{3}$ and $\cot(x-y) = \sqrt{3}$, then what are the smallest positive value of x and y respectively?
 (a) $45^\circ, 30^\circ$
 (b) $15^\circ, 60^\circ$
 (c) $45^\circ, 15^\circ$
 (d) $30^\circ, 45^\circ$

Sol. $\tan(x+y) = \sqrt{3}$
 then $x+y = 60^\circ$ (i)
 $\cot(x-y) = \sqrt{3}$
 Then $x-y = 30^\circ$ (ii)
 from (i) and (ii)
 $x = 45^\circ$ & $y = 15^\circ$

Ex. 48 If $2\cos 3\theta_1 = 1$ and $2\sin 2\theta_2 = \sqrt{3}$, then what will be the value of θ_1 and θ_2
 (a) $30^\circ, 20^\circ$ (b) $60^\circ, 40^\circ$
 (c) $20^\circ, 30^\circ$ (d) $45^\circ, 45^\circ$

Sol. $2\cos 3\theta_1 = 1 \Rightarrow \cos 3\theta_1 = \frac{1}{2} = \cos 60^\circ$
 $\Rightarrow 3\theta_1 = 60^\circ \Rightarrow \theta_1 = 20^\circ$
 $2\sin 2\theta_2 = \sqrt{3} \Rightarrow \sin 2\theta_2 = \frac{\sqrt{3}}{2} = \sin 60^\circ$
 $\Rightarrow 2\theta_2 = 60^\circ \Rightarrow \theta_2 = 30^\circ$

Ex. 49 If $\sin \alpha + \cos \beta = 2$ ($0^\circ \leq \beta \leq \alpha \leq 90^\circ$), then $\sin \left(\frac{2\alpha + 2\beta}{3} \right) =$
 (a) $\sin \frac{\alpha}{2}$ (b) $\cos \frac{\alpha}{3}$
 (c) $\sin \frac{\alpha}{3}$ (d) $\cos \frac{2\alpha}{3}$

Sol. $\sin \alpha + \cos \beta = 2$ $\sin \alpha \leq 1$;
 $\cos \beta \leq 1$
 $\Rightarrow \alpha = 90^\circ$; $\beta = 0^\circ$
 $\therefore \sin \left(\frac{2\alpha + \beta}{3} \right) = \sin \left(\frac{180^\circ}{3} \right)$
 $= \sin 60^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$
 $\cos \frac{\alpha}{3} = \cos 30^\circ = \frac{\sqrt{3}}{2}$

Ex. 50 If $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$, then $\cos \theta_1 + \cos \theta_2 + \cos \theta_3 = ?$
 (a) 0 (b) 1 (c) 2 (d) 3

Sol. $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3$
 $\Rightarrow \sin \theta_1 = \sin \theta_2 = \sin \theta_3$
 $\Rightarrow \theta_1 = \theta_2 = \theta_3 = \frac{\pi}{2}$
 $\therefore \cos \theta_1 + \cos \theta_2 + \cos \theta_3 = 0$

Ex. 51 If $\cos^2 \alpha + \cos^2 \beta = 2$, then the value of $\tan^3 \alpha + \sin^5 \beta$ is :
 (a) -1 (b) 0 (c) 1 (d) $\frac{1}{\sqrt{3}}$

Sol. $\cos^2 \alpha + \cos^2 \beta = 2$
 $\Rightarrow 1 - \sin^2 \alpha + 1 - \sin^2 \beta = 2$
 $\Rightarrow \sin^2 \alpha + \sin^2 \beta = 0$
 $\Rightarrow \sin \alpha = \sin \beta = 0$
 $\Rightarrow \alpha = \beta = 0$
 $\therefore \tan^3 \alpha + \sin^5 \beta = 0$

TYPE - VIII

Series Base $\longrightarrow$

Ex. 52 The value of $\cos 1^\circ, \cos 2^\circ, \cos 3^\circ, \dots, \cos 179^\circ$ is:-

(a) 1 (b) -1 (c) 2 (d) 0

Sol. $\therefore \cos 90^\circ = 0$
 $\therefore \cos 1^\circ \cdot \cos 2^\circ \dots \cos 179^\circ = 0$
Ex. 53 The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$ is :

(a) 1 (b) 0 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

Sol. $\tan 1^\circ \cdot \tan 2^\circ \cdot \tan 3^\circ \dots \tan 45^\circ \dots \tan 88^\circ \tan 89^\circ$
 $= (\tan 1^\circ \cdot \tan 89^\circ)$
 $(\tan 2^\circ \cdot \tan 88^\circ) \dots \tan 45^\circ$
 $= (\tan 1^\circ \cdot \cot 1^\circ) \cdot (\tan 2^\circ \cdot \cot 2^\circ) \dots \tan 45^\circ = 1$
 $[\because \tan(90^\circ - \theta) = \cot \theta, \tan \theta \cdot \cot \theta = 1]$

Ex. 54 The value of : $\cos 20^\circ + \cos 40^\circ + \cos 60^\circ + \dots + \cos 160^\circ + \cos 180^\circ$ is:-
 (a) 1 (b) -1
 (c) 0 (d) $\frac{1}{2}$

Sol. $\cos(180^\circ - \theta) = -\cos \theta$
 $\therefore \cos 160^\circ = \cos(180^\circ - 20^\circ) = -\cos 20^\circ$
 similarly
 $\Rightarrow \cos 140^\circ = -\cos 40^\circ$,
 $\cos 120^\circ = -\cos 60^\circ$
 $\cos 100^\circ = -\cos 80^\circ$
 Now,
 $\Rightarrow (\cos 20^\circ + \cos 160^\circ) + (\cos 40^\circ + \cos 140^\circ) + (\cos 60^\circ + \cos 120^\circ) + (\cos 80^\circ + \cos 100^\circ) + \cos 180^\circ$
 $= (\cos 20^\circ - \cos 20^\circ) + (\cos 40^\circ - \cos 40^\circ) + (\cos 60^\circ - \cos 60^\circ) + (\cos 80^\circ - \cos 80^\circ) + \cos 180^\circ$
 $\Rightarrow \cos 180^\circ = -1$

Ex. 55 $\sin^2 5^\circ + \sin^2 6^\circ + \dots + \sin^2 84^\circ + \sin^2 85^\circ = ?$
 (a) $39\frac{1}{2}$ (b) $40\frac{1}{2}$
 (c) 40 (d) $39\frac{1}{\sqrt{2}}$

Sol. Let the number of terms be n , then By $t_n = a + (n-1)d$
 Here,
 $\Rightarrow a = 5, d = 1$

$$\begin{aligned} \Rightarrow 85 &= 5 + (n-1)1 \\ \Rightarrow n-1 &= 85-5=80 \\ \Rightarrow n &= 81 \\ \therefore \sin^2 5^\circ + \sin^2 6^\circ + \dots + \sin^2 45^\circ \\ &+ \dots + \sin^2 84^\circ + \sin^2 85^\circ \\ &= (\sin^2 5^\circ + \sin^2 85^\circ) + (\sin^2 6^\circ + \sin^2 84^\circ) + \dots + \text{to 40 terms} + \sin^2 45^\circ \\ &= (\sin^2 5^\circ + \cos^2 5^\circ) + (\sin^2 6^\circ + \cos^2 6^\circ) + \dots + \text{to 40 terms} + \sin^2 45^\circ \\ &= 40 + \frac{1}{2} = 40\frac{1}{2} \end{aligned}$$

$$\left[\begin{array}{l} \sin(90^\circ - \theta) = \cos \theta \\ \sin^2 \theta + \cos^2 \theta = 1 \end{array} \right]$$

Ex.56 The value of $\sin 10^\circ + \sin 20^\circ + \dots + \sin 340^\circ + \sin 350^\circ$

Sol. $\sin 10^\circ + \sin 20^\circ + \dots + \sin 340^\circ + \sin 350^\circ$
 $\Rightarrow \sin(360^\circ - 350^\circ) + \sin(360^\circ - 340^\circ) + \dots + \sin 180^\circ + \dots + \sin 340^\circ + \sin 350^\circ$
 $\Rightarrow -\sin 350^\circ - \sin 340^\circ + \dots + \sin 180^\circ + \dots + \sin 340^\circ + \sin 350^\circ = 0$
 $[\sin(360^\circ - \theta) = -\sin \theta, \sin 180^\circ = 0]$

Ex.57 The value of $\cos^2 1^\circ + \cos^2 3^\circ + \dots + \cos^2 89^\circ + \cos^2 90^\circ$

Sol. $\cos^2 1^\circ + \cos^2 3^\circ + \cos^2 5^\circ + \dots + \cos^2 89^\circ + \cos^2 90^\circ$
 $n = \frac{89-1}{2} + 1 = 45$
 $\text{sum} = \frac{45}{2}$
 $\therefore \frac{45}{2} + \underbrace{\cos^2 90^\circ}_0 = \frac{45}{2}$

Ex.58 The value of

$$\sin^2 \frac{\pi}{40} + \sin^2 \frac{2\pi}{40} + \sin^2 \frac{3\pi}{40} + \dots + \sin^2 \frac{20\pi}{40}$$

Sol. $\sin^2 \frac{\pi}{40} + \sin^2 \frac{2\pi}{40} + \sin^2 \frac{3\pi}{40} + \dots + \sin^2 \frac{19\pi}{40} + \sin^2 \frac{20\pi}{40}$

$$\begin{aligned} \Rightarrow & \sin^2 \frac{\pi}{40} + \sin^2 \frac{2\pi}{40} + \dots + \sin^2 \frac{18\pi}{40} + \sin^2 \frac{19\pi}{40} + \sin^2 \frac{20\pi}{40} \\ \Rightarrow & \left(\sin^2 \frac{\pi}{40} + \sin^2 \frac{19\pi}{40} \right) + \left(\sin^2 \frac{2\pi}{40} + \sin^2 \frac{18\pi}{40} \right) + \dots + \sin^2 \frac{20\pi}{40} \end{aligned}$$

Here,

$$\frac{\pi}{40} + \frac{19\pi}{40} = \frac{20\pi}{40} = \frac{\pi}{2}$$

$$\text{So, sum of 2 terms} = \sin^2 \frac{\pi}{2} = 1$$

$$n = 19$$

$$\text{sum} = \frac{19}{2} + \sin^2 \frac{20\pi}{40}$$

$$= \frac{19}{2} + \sin^2 90^\circ$$

$$\Rightarrow \frac{19}{2} + 1 = \frac{21}{2}$$

Ex.59 If $1 + \sin x + \sin^2 x + \sin^3 x + \dots \infty = 4 + 2\sqrt{3}$, find the value of x

Sol. It is a G.P Series

$$\text{then, } S_\infty = \frac{a}{1-r}$$

$$a = 1, r = \sin x$$

$$\frac{1}{1-\sin x} = 4 + 2\sqrt{3} \quad \times$$

$$\frac{(4-2\sqrt{3})}{4-2\sqrt{3}}$$

$$\Rightarrow \frac{1}{1-\sin x} = \frac{4}{4-2\sqrt{3}}$$

$$\Rightarrow \frac{1}{1-\sin x} = \frac{\frac{4}{4}}{\frac{4}{4} - \frac{2\sqrt{3}}{4}}$$

(divide by 4 all terms)

$$\frac{1}{1-\sin x} = \frac{1}{1-\frac{\sqrt{3}}{2}}$$

comparing both equation

$$\sin x = \frac{\sqrt{3}}{2}$$

$$\sin x = \sin 60^\circ$$

$$x = 60^\circ$$

TYPE- IX

(A)

$$\boxed{\begin{array}{l} \sin^2 \theta + \cos^2 \theta = 1 \\ \text{or} \\ \sin^2 \theta = 1 - \cos^2 \theta \\ \text{or} \\ \cos^2 \theta = 1 - \sin^2 \theta \end{array}}$$

Ex. 60 What is the value of $\sin^2 1000^\circ + \cos^2 1000^\circ$?

- (a) 1000 (b) 100 (c) 10 (d) 1

Sol.(d) $\sin^2 1000^\circ + \cos^2 1000^\circ = 1$
 for every value of θ in $\sin^2 \theta + \cos^2 \theta$ will be 1

Ex. 61 If $\sin^2 60^\circ + \cos^2(3x - 9^\circ) = 1$
 Then value of x is

Sol. $\sin^2 60^\circ + \cos^2(3x - 9^\circ) = 1$
 This is similar to $\sin^2 \theta + \cos^2 \theta$

$$\begin{aligned} \Rightarrow \text{So,} \\ 60^\circ &= 3x - 9 \\ 69^\circ &= 3x \\ x &= 23^\circ \end{aligned}$$

Ex.62 If $3\sin^2 \alpha + 7\cos^2 \alpha = 4$, then the value of $\tan \alpha$ is (where $0 < \alpha < 90^\circ$):

- (a) $\sqrt{2}$ (b) $\sqrt{5}$ (c)

- (d) $\sqrt{6}$

Sol.(a) $3\sin^2 \alpha + 7(1 - \sin^2 \alpha) = 4$

$$\Rightarrow 3\sin^2 \alpha + 7 - 7\sin^2 \alpha = 4$$

$$\Rightarrow 7 - 4\sin^2 \alpha = 4$$

$$\Rightarrow 4\sin^2 \alpha = 3 \Rightarrow \sin \alpha = \frac{\sqrt{3}}{2}$$

$$\text{So, } \alpha = 60^\circ$$

$$\tan \alpha = \tan 60^\circ = \sqrt{3}$$

Ex.63 If $\cos^4 \theta - \sin^4 \theta = \frac{2}{3}$, then the value of $2\cos^2 \theta - 1$ is :

- (a) 0 (b) 1 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

Sol.(c) $\cos^4 \theta - \sin^4 \theta = \frac{2}{3}$

$$\Rightarrow (\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta) = \frac{2}{3}$$

$$\Rightarrow \cos^2 \theta - \sin^2 \theta = \frac{2}{3}$$

$$\Rightarrow \cos^2 \theta - (1 - \cos^2 \theta) = \frac{2}{3}$$

$$\Rightarrow 2\cos^2 \theta - 1 = \frac{2}{3}$$

Ex.64 $\sin \theta + \sin^2 \theta = 1$

Find the value of $\cos^2 \theta + \cos^4 \theta$

Sol. $\sin \theta + \sin^2 \theta = 1$

$$\sin \theta = 1 - \sin^2 \theta$$

$$\sin \theta = \cos^2 \theta$$

Now, $\cos^2 \theta + \cos^4 \theta$

Put the value $\cos^2 \theta$

$$\cos^2 \theta + (\cos^2 \theta)^2$$

$$\sin \theta + \sin^2 \theta = 1 \text{ (given)}$$

Ex.65 If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$ is equal to :

- (a) 2 (b) 1 (c) 0 (d) -1

Sol. (c) $\sin x + \sin^2 x = 1$

$$\Rightarrow \sin x = 1 - \sin^2 x = \cos^2 x$$

$$\therefore \sin^2 x = \cos^4 x$$

$$\text{Now, } \cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 1$$

$$= (\cos^4 x + \cos^2 x)^3 - 1 = (\sin^2 x + \sin x)^3 - 1$$

$$= 1^3 - 1 = 0$$

Ex.66 If $\sin \theta + \sin^2 \theta + \sin^3 \theta = 1$, Find the value of $\cos^6 \theta - 4\cos^4 \theta + 8\cos^2 \theta = ?$

Sol. $\sin \theta + \sin^2 \theta + \sin^3 \theta = 1$

$$\Rightarrow \sin \theta + \sin^3 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \sin \theta (1 + \sin^2 \theta) = \cos^2 \theta$$

$$\Rightarrow \sin \theta (1 + 1 - \cos^2 \theta) = \cos^2 \theta$$

$$\Rightarrow \sin \theta (2 - \cos^2 \theta) = \cos^2 \theta$$

Squaring. (for making $\sin \theta$ into $\cos \theta$)

$$\Rightarrow \sin^2 \theta (2 - \cos^2 \theta)^2 = \cos^4 \theta$$

$$\Rightarrow (1 - \cos^2 \theta) [4 + \cos^4 \theta - 4\cos^2 \theta] = \cos^4 \theta$$

$$\Rightarrow 4 + \cos^4 \theta - 4\cos^2 \theta - 4\cos^2 \theta - \cos^6 \theta + 4\cos^4 \theta = \cos^4 \theta$$

$$\Rightarrow -\cos^6 \theta + 4\cos^4 \theta - 8\cos^2 \theta = -4$$

Hence,

$$\cos^6 \theta - 4\cos^4 \theta + 8\cos^2 \theta = 4$$

Ex.67 If $\cos \theta + \cos^2 \theta = 1$, Find the value of $\sin^8 \theta + 2\sin^6 \theta + \sin^4 \theta$

Sol. $\cos \theta = 1 - \cos^2 \theta$

$$\cos \theta = \sin^2 \theta$$

squaring both side

$$\cos^2 \theta = \sin^4 \theta \quad \dots\dots(i)$$

Now, $\sin^8 \theta + 2\sin^6 \theta + \sin^4 \theta$

$$\sin^8 \theta + 2\sin^6 \theta + \sin^4 \theta$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$a^2 \quad 2ab \quad b^2$$

$$(\sin^4 \theta + \sin^2 \theta)^2$$

From equation (i),

$$(\cos^2 \theta + \sin^2 \theta)^2 = 1$$

Ex.68 If $x = (1 + \sin \theta) (1 + \sin \alpha) (1 + \sin \beta) = (1 - \sin \theta) (1 - \sin \alpha) (1 - \sin \beta)$ Then, Find the value of x

$$(a) \pm \cos \theta \cdot \cos \alpha \cdot \cos \beta$$

$$(b) \pm \cos^2 \theta \cdot \cos^2 \alpha \cdot \cos^2 \beta$$

$$(c) \pm \sec \theta \cdot \sec \alpha \cdot \sec \beta$$

$$(d) \pm \sin \theta \cdot \sin \alpha \cdot \sin \beta$$

Sol. $x = (1 + \sin \theta) (1 + \sin \alpha) (1 + \sin \beta)$ (i)

$$x = (1 - \sin \theta) (1 - \sin \alpha) (1 - \sin \beta) \quad \dots\dots(ii)$$

$$(i) \times (ii)$$

$$x^2 = \cos^2 \theta \cdot \cos^2 \alpha \cdot \cos^2 \beta$$

$$x = \pm \cos \theta \cos \alpha \cos \beta$$

Ex. 69 If $\frac{\cos \alpha}{\cos \beta} = m$ and $\frac{\cos \alpha}{\sin \beta} = n$,

Find the value of $(m^2 + n^2) \cos^2 \beta$

Sol. $\frac{\cos \alpha}{\cos \beta} = m$

$$\cos \alpha = m \cos \beta \quad \dots\dots(i)$$

$$\frac{\cos \alpha}{\sin \beta} = n$$

$$\cos \alpha = n \sin \beta \quad \dots\dots(ii)$$

from (i) and (ii) squaring both sides

$$\Rightarrow m^2 \cos^2 \beta = n^2 \sin^2 \beta$$

$$\Rightarrow m^2 \cos^2 \beta = n^2 (1 - \cos^2 \beta)$$

$$\Rightarrow m^2 \cos^2 \beta = n^2 + n^2 \cos^2 \beta$$

$$\Rightarrow m^2 \cos^2 \beta + n^2 \cos^2 \beta = n^2$$

$$\Rightarrow (m^2 + n^2) \cos^2 \beta = n^2$$

Ex. 70 If $r \sin \theta = 1$, and $r \cos \theta = \sqrt{3}$

Then the value of

$$(\sqrt{3} \tan \theta + 1) \text{ And } r$$

Sol. $r \sin \theta = 1 \quad \dots\dots(i)$

$$r \cos \theta = \sqrt{3} \quad \dots\dots(ii)$$

$$(i)^2 + (ii)^2$$

$$(r \sin \theta)^2 + (r \cos \theta)^2 = (1)^2 + (\sqrt{3})^2$$

$$r^2 \sin^2 \theta + r^2 \cos^2 \theta = 1 + 3$$

$$r^2 (\sin^2 \theta + \cos^2 \theta) = 4$$

$$r^2 = 4$$

$$r = 2$$

$$(i)/(ii)$$

$$\frac{r \sin \theta}{r \cos \theta} = \frac{1}{\sqrt{3}}$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

Now, $(\sqrt{3} \tan \theta + 1)$

Put the value of $\tan \theta$

$$= \sqrt{3} \times \frac{1}{\sqrt{3}} + 1 = 2$$

Ex.71 $\frac{\sin \theta}{x} = \frac{\cos \theta}{y}$, then $\sin \theta - \cos \theta$ is equal to

$$(a) \frac{x-y}{x+y}$$

$$(b) \frac{x-y}{x+y}$$

$$(c) \frac{x-y}{\sqrt{x^2+y^2}}$$

$$(d) \frac{y-x}{\sqrt{x^2+y^2}}$$

Sol. $\frac{\sin \theta}{x} = \frac{\cos \theta}{y} = k$ (Let)

$$\sin \theta = kx \quad \dots\dots(i)$$

$$\cos \theta = ky \quad \dots\dots(ii)$$

$$(i)^2 + (ii)^2$$

$$\sin^2 \theta + \cos^2 \theta = (kx)^2 + (ky)^2$$

$$1 = k^2 (x^2 + y^2)$$

$$k = \frac{1}{\sqrt{x^2+y^2}}$$

Now, $\sin \theta - \cos \theta = kx - ky = k(x - y)$
Put the value of k

$$\therefore \sin \theta - \cos \theta = \frac{x-y}{\sqrt{x^2+y^2}}$$

Ex.72 If $3\sin \theta + 4\cos \theta = 5$ find the value of $\tan \theta$.

Sol. $3\sin \theta + 4\cos \theta = 5$

$$\frac{3}{5} \sin \theta + \frac{4}{5} \cos \theta = 1 \quad \dots\dots(i)$$

We know, $\sin^2 \theta + \cos^2 \theta = 1$
or

$$\sin \theta \cdot \sin \theta + \cos \theta \cdot \cos \theta = 1 \quad \dots\dots(ii)$$

comparison of (i) and (ii)

$$\text{Then, } \sin \theta = \frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{3/5}{4/5} = \frac{3}{4}$$

Alternative:-

$$3 \sin \theta + 4 \cos \theta = 5$$

$$\downarrow \quad \downarrow$$

$$p \quad b$$

$$\tan \theta = \frac{p}{b} = \frac{3}{4}$$

Ex.73 If $(a^2 - b^2) \sin \theta + 2ab \cos \theta = a^2 + b^2$ then find the value of $\tan \theta$

Sol. $(a^2 - b^2) \sin \theta + 2ab \cos \theta = a^2 + b^2$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ p & & b \end{array}$$

$$\tan \theta = \frac{p}{b} = \frac{a^2 - b^2}{2ab}$$

Ex.74 If $x \cos \theta - y \sin \theta = \sqrt{x^2 + y^2}$,

$$\text{and } \frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} = \frac{1}{x^2 + y^2}$$

which statement of true

(i) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(ii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(iii) $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

(iv) $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$

Sol. $x \cos \theta - y \sin \theta = \sqrt{x^2 + y^2}$

$$\frac{x \cos \theta}{\sqrt{x^2 + y^2}} + \frac{(-y) \sin \theta}{\sqrt{x^2 + y^2}} = 1 \dots (i)$$

We know,

$$\cos^2 \theta + \sin^2 \theta = 1 \dots (ii)$$

comparison of (i) and (ii)

$$\cos \theta = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\sin \theta = \frac{-y}{\sqrt{x^2 + y^2}}$$

$$\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} = \frac{1}{x^2 + y^2} \dots (iii)$$

Put the value of $\cos \theta$ & $\sin \theta$ in (iii)

$$\left(\frac{x}{\sqrt{x^2 + y^2}} \right)^2 \times \frac{1}{a^2} +$$

$$\left(\frac{-y}{\sqrt{x^2 + y^2}} \right)^2 \times \frac{1}{b^2} = \frac{1}{x^2 + y^2}$$

$$\frac{x^2}{(x^2 + y^2)a^2} + \frac{y^2}{(x^2 + y^2)b^2}$$

$$= \frac{1}{x^2 + y^2}$$

Hence, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Ex.75 If $29 \tan \theta = 31$, find the value

$$\text{of } \frac{1 + 2 \sin \theta \cos \theta}{1 - 2 \sin \theta \cos \theta}$$

Sol. $(\sin \theta + \cos \theta)^2 = \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta$

$$= 1 + 2 \sin \theta \cos \theta \text{ and } (\sin \theta - \cos \theta)^2 = 1 - 2 \sin \theta \cos \theta.$$

$$\text{Now, } = \frac{1 + 2 \sin \theta \cos \theta}{1 - 2 \sin \theta \cos \theta}$$

$$= \frac{(\sin \theta + \cos \theta)^2}{(\sin \theta - \cos \theta)^2}$$

Divide $\cos \theta$ all terms

$$\left(\frac{\tan \theta + 1}{\tan \theta - 1} \right)^2$$

$$\therefore \tan \theta = \frac{31}{29} \text{ (given)}$$

Put the value of $\tan \theta$

$$= \left(\frac{\frac{31}{29} + 1}{\frac{31}{29} - 1} \right)^2 = \left(\frac{60/29}{2/29} \right)^2$$

$$= (30)^2 = 900$$

Ex.76 If $\tan \alpha = n \tan \beta$ and $\sin \alpha =$

$$m \sin \beta$$

Find the value of $\cos^2 \alpha$

Sol. $\tan \alpha = n \tan \beta$

$$\frac{\sin \alpha}{\cos \alpha} = \frac{n \sin \beta}{\cos \beta}$$

$$\text{put } (\sin \alpha = m \sin \beta)$$

$$m \cos \beta = n \cos \alpha \dots (i)$$

$$m \sin \beta = \sin \alpha \dots (ii)$$

adding after squaring (i) and (ii)

$$m^2 \cos^2 \beta + m^2 \sin^2 \beta = n^2$$

$$\cos^2 \alpha + \sin^2 \alpha$$

$$m^2 (\cos^2 \beta + \sin^2 \beta) = n^2 \cos^2 \alpha$$

$$+ (1 - \cos^2 \alpha)$$

$$m^2 = n^2 \cos^2 \alpha + 1 - \cos^2 \alpha$$

$$m^2 - 1 = \cos^2 \alpha (n^2 - 1)$$

$$\cos^2 \alpha = \frac{m^2 - 1}{n^2 - 1}$$

Ex.77 If $x \sin^3 \theta + y \cos^3 \theta$

$$= \sin \theta \cdot \cos \theta \text{ and } x \sin \theta - y$$

$$\cos \theta = 0 \text{ find the value of } x^2 + y^2$$

Sol. $x \sin \theta = y \cos \theta \dots (i) \text{ (given)}$

$$\Rightarrow x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cdot \cos \theta.$$

$$\Rightarrow x \sin \theta \cdot \sin^2 \theta + y \cos^3 \theta$$

$$= \sin \theta \cdot \cos \theta$$

$$\Rightarrow y \cos \theta \cdot \sin^2 \theta + y \cos^3 \theta$$

$$= \sin \theta \cdot \cos \theta$$

$$\Rightarrow y \cos \theta (\sin^2 \theta + \cos^2 \theta)$$

$$= \sin \theta \cdot \cos \theta$$

$$\Rightarrow y \cos \theta = \sin \theta \cdot \cos \theta$$

$$\Rightarrow y = \sin \theta$$

$$\text{Thus } x = \cos \theta$$

$$\text{Now, } x^2 + y^2 = \cos^2 \theta + \sin^2 \theta = 1$$

Ex.78 $10 \sin^4 \alpha + 15 \cos^4 \alpha = 6$, find the value of $27 \operatorname{cosec}^6 \alpha + 8 \sec^6 \alpha$.

Sol. $10 \sin^4 \alpha + 15 \cos^4 \alpha = 6 \times 1$

$$10 \sin^4 \alpha + 15 \cos^4 \alpha = 6 \times$$

$$(\sin^2 \theta + \cos^2 \theta)^2$$

$$10 \sin^4 \alpha + 15 \cos^4 \alpha = 6 (\sin^4 \alpha + \cos^4 \alpha + 2 \sin^2 \alpha \cos^2 \alpha)$$

divide by $\cos^4 \alpha$ both sides

$$10 \tan^4 \alpha + 15 = 6 (\tan^4 \alpha + 1 + 2 \tan^2 \alpha)$$

$$\Rightarrow 10 \tan^4 \alpha + 15 = 6 \tan^4 \alpha + 6 + 12 \tan^2 \alpha$$

$$\Rightarrow 4 \tan^4 \alpha - 12 \tan^2 \alpha + 9 = 0$$

$$\Rightarrow (2 \tan^2 \alpha - 3) = 0$$

$$\Rightarrow 2 \tan^2 \alpha = 3$$

$$\Rightarrow \tan^2 \alpha = \frac{3}{2}$$

$$\text{Now, } 27 \operatorname{cosec}^6 \alpha + 8 \sec^6 \alpha = 27 (\operatorname{cosec}^2 \alpha)^3 + 8 (\sec^2 \alpha)^3$$

$$= 27 (1 + \cot^2 \alpha)^3 + 8 (1 + \tan^2 \alpha)^3$$

$$\text{Put } \tan^2 \alpha = \frac{3}{2} \text{ then } \cot^2 \alpha = \frac{2}{3}$$

$$= 27 \left(1 + \frac{2}{3} \right)^3 + 8 \left(1 + \frac{3}{2} \right)^3 = 27$$

$$\left(\frac{5}{3} \right)^3 + 8 \left(\frac{5}{2} \right)^3$$

$$= 27 \times \frac{125}{27} + 8 \times \frac{125}{8} = 125 + 125 = 250$$

Alternate:-

$$10 \sin^4 \alpha + 15 \cos^4 \alpha = 6$$

$$\frac{10}{6} \sin^2 \alpha \cdot \sin^2 \alpha + \frac{15}{6} \cos^2 \alpha \cdot \cos^2 \alpha$$

$$\cos^2 \alpha = 1 \dots (i)$$

$$\sin^2 \alpha + \cos^2 \alpha = 1 \dots (ii)$$

comparison (i) and (ii)

$$\frac{10}{6} \sin^2 \alpha = 1 \text{ and } \frac{15}{6} \cos^2 \alpha = 1$$

$$\sin^2 \alpha = \frac{3}{5} \text{ and } \cos^2 \alpha = \frac{2}{5}$$

$$\begin{aligned} \text{Now, } 27 (\operatorname{cosec}^2 \alpha)^3 + 8 (\sec^2 \alpha)^3 \\ = 27 \left(\frac{5}{3}\right)^3 + 8 \left(\frac{5}{2}\right)^3 = 27 \times \\ \frac{125}{27} + 8 \times \frac{125}{8} \\ = 125 + 125 = 250 \end{aligned}$$

Ex.79 If $T_n = \sin^n \theta + \cos^n \theta$, then

$$\frac{T_3 - T_5}{T_1} = ?$$

(a) $\sin \theta \cdot \cos \theta$

(b) $\sin^2 \theta \cdot \cos \theta$

(c) $\sin^2 \theta \cdot \cos^2 \theta$

(d) $\sin \theta \cdot \cos^2 \theta$

Sol.(c) $\frac{T_3 - T_5}{T_1}$

$$= \frac{(\sin^3 \theta + \cos^3 \theta) - (\sin^5 \theta + \cos^5 \theta)}{\sin \theta + \cos \theta}$$

$$= \frac{(\sin^3 \theta - \sin^5 \theta) + (\cos^3 \theta - \cos^5 \theta)}{\sin \theta + \cos \theta}$$

$$= \frac{\sin^3 \theta (1 - \sin^2 \theta) + \cos^3 \theta (1 - \cos^2 \theta)}{\sin \theta + \cos \theta}$$

$$= \frac{\sin^3 \theta \cdot \cos^2 \theta + \cos^3 \theta \cdot \sin^2 \theta}{\sin \theta + \cos \theta}$$

$$= \frac{\sin^2 \theta \cdot \cos^2 \theta (\sin \theta + \cos \theta)}{(\sin \theta + \cos \theta)}$$

$$= \sin^2 \theta \cdot \cos^2 \theta$$

Alternative:-

Let $\theta = 45^\circ$, then

$$T_n = \left(\frac{1}{\sqrt{2}}\right)^n + \left(\frac{1}{\sqrt{2}}\right)^n = 2 \left(\frac{1}{\sqrt{2}}\right)^n = 2^{\left(\frac{2-n}{2}\right)}$$

$$\therefore \frac{T_3 - T_5}{T_1} = \frac{2^{-1/2} - 2^{-3/2}}{2^{1/2}} = \frac{\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}}}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} = \frac{1}{4}$$

$$\text{i.e. } \frac{T_3 - T_5}{T_1} = \sin^2 \theta \cdot \cos^2 \theta$$

Ex.80 If $u_n = \cos^n \alpha + \sin^n \alpha$, then the value of $2u_6 - 3u_4 + 1$ is :

(a) 1 (b) 4 (c) 6 (d) 0

Sol. $u_n = \cos^n \alpha + \sin^n \alpha$

$$\therefore u_6 = \cos^6 \alpha + \sin^6 \alpha$$

$$(\cos^2 \alpha)^3 + (\sin^2 \alpha)^3$$

$$\Rightarrow (\cos^2 \alpha + \sin^2 \alpha)^3 - 3 \cos^2 \alpha \cdot \sin^2 \alpha (\sin^2 \alpha + \cos^2 \alpha)$$

$$[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)]$$

$$\Rightarrow 1 - 3 \cos^2 \alpha \cdot \sin^2 \alpha$$

$$u_6 = 1 - 3 \cos^2 \alpha \cdot \sin^2 \alpha$$

$$u_4 = \cos^4 \alpha + \sin^4 \alpha$$

$$\Rightarrow (\cos^2 \alpha + \sin^2 \alpha)^2 - 2 \cos^2 \alpha \cdot \sin^2 \alpha$$

$$\Rightarrow 1 - 2 \cos^2 \alpha \cdot \sin^2 \alpha$$

$$\therefore 2u_6 - 3u_4 + 1$$

$$\Rightarrow 2(1 - 3 \sin^2 \alpha \cos^2 \alpha) - 3(1 - 2 \sin^2 \alpha \cos^2 \alpha) + 1$$

$$\Rightarrow 2 - 3 + 1 = 0$$

Alternate:-

$$u_n = \cos^n \alpha + \sin^n \alpha$$

$$u_6 = \cos^6 \alpha + \sin^6 \alpha$$

$$u_4 = \cos^4 \alpha + \sin^4 \alpha$$

$$\text{Let } \alpha = 0^\circ$$

$$\text{Then, } u_6 = 1 \text{ \& } u_4 = 1$$

$$\text{Now, } 2u_6 - 3u_4 + 1$$

$$\Rightarrow 2 \times 1 - 3 \times 1 + 1 = 0$$

(B). If $a \sin \theta + b \cos \theta = m \dots (i)$

$b \sin \theta - a \cos \theta = n \dots (ii)$

Then $a^2 + b^2 = m^2 + n^2$

Proof (i)² + (ii)²

$$\Rightarrow (a \sin \theta + b \cos \theta)^2 + (b \sin \theta - a \cos \theta)^2 = m^2 + n^2$$

$$\Rightarrow (a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta) + (b^2 \sin^2 \theta + a^2 \cos^2 \theta - 2ab \sin \theta \cos \theta) = m^2 + n^2$$

$$\Rightarrow a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 (\cos^2 \theta + \sin^2 \theta) = m^2 + n^2$$

$$a^2 + b^2 = m^2 + n^2$$

Ex.81 If $\sin \theta + \cos \theta = x$, find the value of $\sin \theta - \cos \theta$

Sol. $1 \times \sin \theta + 1 \times \cos \theta = x$

$$\text{Let } 1 \times \sin \theta - 1 \times \cos \theta = n$$

$$\text{use property } a^2 + b^2 = m^2 + n^2$$

$$\text{Here, } a = b = 1, m = x, n(\text{let})$$

$$(1)^2 + (1)^2 = (x)^2 + (n)^2$$

$$2 - x^2 = n^2$$

$$n = \pm \sqrt{2 - x^2}$$

$$\text{So, } \sin \theta - \cos \theta = \pm \sqrt{2 - x^2}$$

Ex.82 If $a \cos \theta + b \sin \theta = c$, Then find $(a \sin \theta - b \cos \theta) = ?$

Sol. $a \cos \theta + b \sin \theta = c \dots (i)$

$$a \sin \theta - b \cos \theta = x \text{ (let)} \dots (ii)$$

$$(i)^2 + (ii)^2$$

$$a^2 + b^2 = c^2 + x^2$$

$$x^2 = a^2 + b^2 - c^2$$

$$\text{So, } (a \sin \theta - b \cos \theta)^2 = a^2 + b^2 - c^2$$

Ex.83 If $\sin \theta + \cos \theta = \frac{7}{5}$, find the value of $\cos \theta - \sin \theta = ?$

Sol. $\sin \theta + \cos \theta = \frac{7}{5}$

$$\cos \theta - \sin \theta = x \text{ (let)}$$

$$\Rightarrow (1)^2 + (1)^2 = \left(\frac{7}{5}\right)^2 + (x)^2$$

$$\Rightarrow 2 - \frac{49}{25} = x^2$$

$$\Rightarrow x^2 = 2 - \frac{49}{25} = \frac{1}{25}$$

$$\text{So, } x = \pm \frac{1}{5}$$

Ex.84 If $\sin \theta + \cos \theta = \sqrt{2} \cos \theta$, then find the value of $\sin \theta - \cos \theta$.

Sol. $\sin \theta + \cos \theta = \sqrt{2} \cos \theta$

$$\sin \theta - \cos \theta = x \text{ (let)}$$

$$\Rightarrow (1)^2 + (1)^2 = (\sqrt{2} \cos \theta)^2 + (x)^2$$

$$\Rightarrow 2 = 2 \cos^2 \theta + x^2$$

$$\Rightarrow x^2 = 2 - 2 \cos^2 \theta$$

$$\Rightarrow x^2 = 2(1 - \cos^2 \theta)$$

$$\Rightarrow x^2 = 2 \sin^2 \theta$$

$$x = \pm \sqrt{2} \sin \theta$$

Ex.85 If $\frac{x}{a} \sin \theta + \frac{y}{b} \cos \theta = 1$, then.

Find the value of $\frac{y}{b} \sin \theta - \frac{x}{a} \cos \theta = ?$

Sol. $\frac{x}{a} \sin \theta + \frac{y}{b} \cos \theta = 1$

$$\frac{y}{b} \sin \theta - \frac{x}{a} \cos \theta = P \text{ (let)}$$

$$\Rightarrow \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = (1)^2 + (P)^2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + P^2$$

$$P = \pm \sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1}$$

Ex.86 If $\sin \theta + \cos \theta = \frac{1}{2}$, Then
find the value of $\sin \theta - \cos \theta$
=?

Sol. $\sin \theta + \cos \theta = \frac{1}{2}$
 $\sin \theta - \cos \theta = x$ (let)
 $(1)^2 + (1)^2 = \left(\frac{1}{2}\right)^2 + (x)^2$
 $x^2 = 2 - \frac{1}{4}$
 $x^2 = \frac{7}{4}$
 $x = \pm \frac{\sqrt{7}}{2}$

Ex.87 If $3 \sin \theta + 4 \cos \theta = 5$, find the
value of $4 \sin \theta - 3 \cos \theta$?

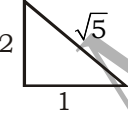
Sol. $3 \sin \theta + 4 \cos \theta = 5$
 $4 \sin \theta - 3 \cos \theta = x$ (let)
 $(3)^2 + (4)^2 = (5)^2 + x^2$
 $x = 0$
 $\therefore$ So, $4 \sin \theta - 3 \cos \theta = 0$

TYPE - X

(A) $1 + \tan^2 \theta = \sec^2 \theta$
 $\sec^2 \theta - 1 = \tan^2 \theta$
 $\sec^2 \theta - \tan^2 \theta = 1$

Ex.88 If $\sec^2 \theta + \tan^2 \theta = 9$ find the
value of $\sin \theta$ ($0^\circ < \theta < 90^\circ$)

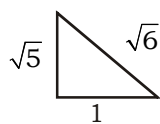
Sol. $\sec^2 \theta + \tan^2 \theta = 9$
 $1 + \tan^2 \theta + \tan^2 \theta = 9$
 $2 \tan^2 \theta = 8$
 $\tan^2 \theta = 4$
 $\tan \theta = 2, = \frac{p}{b}$

Now,  $\sin \theta = \frac{p}{h} = \frac{2}{\sqrt{5}}$

Ex.89 If $\sec^2 \theta + \tan^2 \theta = 11$, find
the value of $\csc \theta$
($\therefore 0^\circ < \theta < 90^\circ$)

Sol. $\sec^2 \theta + \tan^2 \theta = 11$
 $1 + \tan^2 \theta + \tan^2 \theta = 11$
 $2 \tan^2 \theta = 10$
 $\tan^2 \theta = 5$
 $\tan \theta = \sqrt{5}$

Now,



$$\csc \theta = \frac{\sqrt{6}}{\sqrt{5}}$$

Ex.90 If $\sec^2 \theta + \tan^2 \theta = \frac{5}{3}$ find the
value of θ . while
($0^\circ \leq \theta \leq 90^\circ$)

Sol. $\sec^2 \theta + \tan^2 \theta = \frac{5}{3}$
 $1 + \tan^2 \theta + \tan^2 \theta = \frac{5}{3}$

$$2 \tan^2 \theta = \frac{5}{3} - 1$$

$$2 \tan^2 \theta = \frac{2}{3}$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\text{so, } \theta = 30^\circ$$

Ex.91 If $\tan^2 \alpha = 1 + 2 \tan^2 \beta$, find
the value of $\sqrt{2} \cos \alpha - \cos \beta$
=?

Sol. $\tan^2 \alpha = 1 + 2 \tan^2 \beta$ (Using
identity)
 $\Rightarrow \sec^2 \alpha - 1 = 1 + 2(\sec^2 \beta - 1)$
 $\Rightarrow \sec^2 \alpha - 1 = 1 + 2 \sec^2 \beta - 2$
 $\Rightarrow \sec^2 \alpha - 1 = 2 \sec^2 \beta - 1$
 $\Rightarrow \sec^2 \alpha = 2 \sec^2 \beta$
 $\Rightarrow \sec \alpha = \sqrt{2} \sec \beta$

$$\Rightarrow \frac{1}{\cos \alpha} = \sqrt{2} \left(\frac{1}{\cos \beta} \right)$$

$$\Rightarrow \cos \beta = \sqrt{2} \cos \alpha$$

$$\Rightarrow \sqrt{2} \cos \alpha - \cos \beta = 0$$

Alternative:-

$$\alpha = 45^\circ \text{ \& \; } \beta = 0^\circ \text{ satisfies}$$

$$\tan^2 \alpha = 1 + 2 \tan^2 \beta$$

$$\text{put } \alpha = 45^\circ \text{ \& \; } \beta = 0^\circ \text{ in } \sqrt{2}$$

$$\cos \alpha - \cos \beta$$

$$= \sqrt{2} \cos 45^\circ - \cos 0^\circ = 1 - 1 = 0$$

(B). $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$

$$\sec \theta - \tan \theta = \frac{1}{\sec \theta + \tan \theta}$$

$$\sec \theta + \tan \theta = \frac{1}{\sec \theta - \tan \theta}$$

if $\sec \theta + \tan \theta = x$

$$\text{then, } \sec \theta - \tan \theta = \frac{1}{x}$$

Ex.92 If $\sec \theta + \tan \theta = 3$, find the
value of $\cos \theta$

Sol. $\sec \theta + \tan \theta = 3$ (i)
then $\sec \theta - \tan \theta = \frac{1}{3}$ (ii)

adding (i) + (ii)

$$2 \sec \theta = 3 + \frac{1}{3}$$

$$2 \sec \theta = \frac{10}{3}$$

$$\sec \theta = \frac{5}{3}$$

$$\text{Hence, } \cos \theta = \frac{3}{5}$$

Ex.93 If $\sec \theta + \tan \theta = x$, find the
value of $\tan \theta$

Sol. $\sec \theta + \tan \theta = x$ (i)
 $\sec \theta - \tan \theta = \frac{1}{x}$ (ii)
(i) and (ii)

$$2 \tan \theta = x - \frac{1}{x} = \frac{x^2 - 1}{x}$$

$$\tan \theta = \frac{x^2 - 1}{2x}$$

Ex.94 If $\sec \theta + \tan \theta = 5$, find the
value of $\sin \theta$.

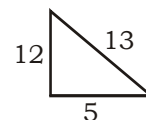
Sol. $\sec \theta + \tan \theta = 5$ (i)
then $\sec \theta - \tan \theta = \frac{1}{5}$ (ii)

Adding (i) and (ii)

$$2 \sec \theta = 5 + \frac{1}{5} = \frac{26}{5}$$

$$\sec \theta = \frac{13}{5} = \frac{h}{b}$$

Now,



$$\sin \theta = \frac{12}{13}$$

Ex.95 If $\sec \theta + \tan \theta = 5$, Then the

value of $\frac{\tan \theta + 1}{\tan \theta - 1}$ is

Sol. $\sec \theta + \tan \theta = 5$ (i)

$\sec \theta - \tan \theta = \frac{1}{5}$ (ii)

(i) - (ii)

$2 \tan \theta = 5 - \frac{1}{5}$

$2 \tan \theta = \frac{24}{5}$

$\tan \theta = \frac{12}{5}$

Now, $\frac{\tan \theta + 1}{\tan \theta - 1}$

Put the value $\tan \theta$

$\frac{\frac{12}{5} + 1}{\frac{12}{5} - 1} = \frac{\frac{17}{5}}{\frac{7}{5}} = \frac{17}{7}$

Ex.96 If $\sec \theta = x + \frac{1}{4x}$ ($0^\circ < \theta < 90^\circ$), then $\sec \theta + \tan \theta$ is equal to

(a) $\frac{x}{2}$ (b) $2x$ (c) x (d) $\frac{1}{2x}$

Sol. $\sec \theta = x + \frac{1}{4x}$ (given)(i)

Let $\sec \theta + \tan \theta = a$ (ii)

Then $\sec \theta - \tan \theta = \frac{1}{a}$ (iii)

(ii) + (iii)

$2 \sec \theta = a + \frac{1}{a}$

$\sec \theta = \frac{a + \frac{1}{a}}{2}$ (iv)

Compare (i) and (iv)

Hence, $x = \frac{a}{2}$, $a = 2x$

so, $\sec \theta + \tan \theta = 2x$

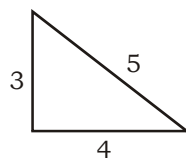
Alternate:-

$\sec \theta = x + \frac{1}{4x}$

Let $x = 1$

Then, $\sec \theta = 1 + \frac{1}{4 \times 1} = \frac{5}{4}$

Now,



$\sec \theta + \tan \theta$

$\frac{5}{4} + \frac{3}{4} = \frac{8}{4} = 2$

Take option (b) $2x = 2 \times 1 = 2$

Ex.97 If $\tan^2 \theta = 1 - e^2$, then $\sec \theta + \tan^3 \theta \cdot \operatorname{cosec} \theta = ?$

(a) $(2 - e^2)^{3/2}$

(b) $(2 - e^2)^{2/3}$

(c) $(2 - e^2)^{1/2}$

(d) None of these

Sol. We have, $\sec \theta + \tan^3 \theta \cdot \operatorname{cosec} \theta$

$= \sec \theta (1 + \tan^3 \theta \frac{\operatorname{cosec} \theta}{\sec \theta})$

$= \sec \theta (1 + \tan^2 \theta)$

$= \sec \theta \cdot \sec^2 \theta$

$= \sec^3 \theta = (\sec^2 \theta)^{3/2} = (1 + \tan^2 \theta)^{3/2}$

$= (1 + 1 - e^2)^{3/2}$

$\therefore \tan^2 \theta = 1 - e^2 = (2 - e^2)^{3/2}$

Alternate:-

Let $\theta = 45^\circ$

Then, $\tan^2 \theta = 1 - e^2$

$\tan^2 45 = 1 - e^2$

$1 = 1 - e^2$

$e^2 = 0$

Now, $\sec \theta + \tan^3 \theta \cdot \operatorname{cosec} \theta$

Put $\theta = 45^\circ$

$= \sec 45^\circ + \tan^2 45^\circ \cdot \operatorname{cosec} 45^\circ$

$= \sqrt{2} + 1 \times \sqrt{2}$

$= 2\sqrt{2} = (2)^{3/2}$

Now, we take option (a)

$(2 - e^2)^{3/2}$

Put $e^2 = 0$

Then $= (2)^{3/2}$

So, (a) option is correct

Ex.98 Find the value of

$\frac{\sec \theta + \tan \theta - 1}{\tan \theta - \sec \theta + 1}$

Sol.

$\frac{\sec \theta + \tan \theta - 1}{\tan \theta - \sec \theta + 1}$

$\Rightarrow \frac{(\sec \theta + \tan \theta) - (\sec^2 \theta - \tan^2 \theta)}{\tan \theta - \sec \theta + 1}$

$(\because \sec^2 \theta - \tan^2 \theta = 1)$

$\Rightarrow \frac{(\sec \theta + \tan \theta) - (\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}{\tan \theta - \sec \theta + 1}$

$\Rightarrow \frac{(\sec \theta + \tan \theta)(1 - \sec \theta + \tan \theta)}{(\tan \theta - \sec \theta + 1)}$

$\Rightarrow \sec \theta + \tan \theta$

$\Rightarrow \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = \frac{1 + \sin \theta}{\cos \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta}$

$\Rightarrow \frac{1 - \sin^2 \theta}{\cos \theta (1 - \sin \theta)} = \frac{\cos^2 \theta}{\cos \theta (1 - \sin \theta)}$

$\Rightarrow \text{So, } \frac{\cos \theta}{1 - \sin \theta}$

(C). $a \sec \theta - b \tan \theta = c$ (i)

$b \sec \theta - a \tan \theta = d$ (ii)

or

$a \tan \theta - b \sec \theta = d$

Then, $(a^2 - b^2 = c^2 - d^2)$

or $a \sec \theta + b \tan \theta = c$ (i)

$b \sec \theta + a \tan \theta = d$ (ii)

then, $(a^2 - b^2 = c^2 - d^2)$

Proof (i)² - (ii)²

$\Rightarrow (a \sec \theta - b \tan \theta)^2 - (b \sec \theta - a \tan \theta)^2 = c^2 - d^2$

$\Rightarrow (a^2 \sec^2 \theta + b^2 \tan^2 \theta - 2ab \sec \theta \tan \theta)$

$-(b^2 \sec^2 \theta + a^2 \tan^2 \theta - 2ab \sec \theta \tan \theta) = c^2 - d^2$

$\Rightarrow a^2 \sec^2 \theta + b^2 \tan^2 \theta - b^2 \sec^2 \theta - a^2 \tan^2 \theta = c^2 - d^2$

$\Rightarrow a^2 (\sec^2 \theta - \tan^2 \theta) - b^2 (\sec^2 \theta - \tan^2 \theta) = c^2 - d^2$

$a^2 - b^2 = c^2 - d^2$

Ex.99 If $a \sec \theta - b \tan \theta = 10$

and $b \sec \theta - a \tan \theta = 5$

find the value of $a^2 - b^2$

Sol. $a \sec \theta - b \tan \theta = 10$

$b \sec \theta - a \tan \theta = 5$

(use property)

$a^2 - b^2 = (10)^2 - (5)^2$

$a^2 - b^2 = 100 - 25 = 75$

Ex.100 If $5 \sec \theta - 3 \tan \theta = 7$ find the value of

$3 \sec \theta - 5 \tan \theta = ?$

Sol. $5 \sec \theta - 3 \tan \theta = 7$
 $3 \sec \theta - 5 \tan \theta = m$ (let)
 $(5)^2 - (3)^2 = (7)^2 - (m)^2$
 $25 - 9 = 49 - m^2$
 $m^2 = 33$
 $m = \pm \sqrt{33}$

TYPE - XI

(A) $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$
 $\operatorname{cosec}^2 \theta - 1 = \cot^2 \theta$
 $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

Ex. 101 $\operatorname{cosec}^2 \theta + 2 \cot^2 \theta = 10$, then find the value of $\sin \theta + \cos \theta$ when $0^\circ < \theta < 90^\circ$

Sol. $\operatorname{cosec}^2 \theta + 2 \cot^2 \theta = 10$
 $1 + \cot^2 \theta + 2 \cot^2 \theta = 10$
 $3 \cot^2 \theta = 9$
 $\cot \theta = \sqrt{3}$

So, $\theta = 30^\circ$

Now, $\sin \theta + \cos \theta$
 $= \sin 30^\circ + \cos 30^\circ$
 $= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$

Ex. 102 If $\operatorname{cosec}^2 \theta + \cot^2 \theta = 3$, find the value of $\cos \theta$ when $(0^\circ < \theta < 90^\circ)$

Sol. $\operatorname{cosec}^2 \theta + \cot^2 \theta = 3$
 $1 + \cot^2 \theta + \cot^2 \theta = 3$
 $2 \cot^2 \theta = 2$
 $\cot^2 \theta = 1$
 $\cot \theta = 1$

So, $\theta = 45^\circ$

Now, $\cos \theta$
 $= \cos 45^\circ = \frac{1}{\sqrt{2}}$

(B) $\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$
 $(\operatorname{cosec} \theta - \cot \theta)(\operatorname{cosec} \theta + \cot \theta) = 1$
 $\operatorname{cosec} \theta - \cot \theta = \frac{1}{\operatorname{cosec} \theta + \cot \theta}$

If $\operatorname{cosec} \theta - \cot \theta = x$,

then, $\operatorname{cosec} \theta + \cot \theta = \frac{1}{x}$

Ex. 103 If $\operatorname{cosec} \theta - \cot \theta = 4$, find the value of $\cos \theta$

Sol. $\operatorname{cosec} \theta - \cot \theta = 4$ (i)

then, $\operatorname{cosec} \theta + \cot \theta = \frac{1}{4}$ (ii)

(i) and (ii)

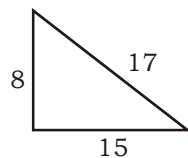
$$2 \operatorname{cosec} \theta = 4 + \frac{1}{4}$$

$$2 \operatorname{cosec} \theta = \frac{17}{4}$$

$$\operatorname{cosec} \theta = \frac{17}{8} = \frac{h}{p}$$

$$b = \sqrt{(17)^2 - (8)^2} = 15$$

Now,



$$\cos \theta = \frac{b}{h} = \frac{15}{17}$$

Ex. 104 If $\operatorname{cosec} \theta + \cot \theta = \sqrt{5} + 2$, then find the value of $\sin \theta$.

Sol. $\operatorname{cosec} \theta + \cot \theta = \sqrt{5} + 2$ (i)

Then, $\operatorname{cosec} \theta - \cot \theta = \frac{1}{\sqrt{5} + 2} = \sqrt{5} - 2$ (ii)

(i) + (ii)
 $2 \operatorname{cosec} \theta = 2\sqrt{5}$
 $\operatorname{cosec} \theta = \sqrt{5}$

So, $\sin \theta = \frac{1}{\sqrt{5}}$

(c) $a \operatorname{cosec} \theta - b \cot \theta = c$
 $b \operatorname{cosec} \theta - a \cot \theta = d$
or

$a \cot \theta - b \operatorname{cosec} \theta = d$
 $(i)^2 - (ii)^2$

Then, $(a^2 - b^2 = c^2 - d^2)$
or

$a \operatorname{cosec} \theta + b \cot \theta = c$ (i)

$b \operatorname{cosec} \theta + a \cot \theta = d$ (ii)

$(i)^2 - (ii)^2$
 $(a^2 - b^2 = c^2 - d^2)$

Ex. 105 If $4 \operatorname{cosec} \theta + 5 \cot \theta = 7$, then find the value of $5 \operatorname{cosec} \theta + 4 \cot \theta$ = ?

Sol. $4 \operatorname{cosec} \theta + 5 \cot \theta = 7$ (given)

$5 \operatorname{cosec} \theta + 4 \cot \theta = m$ (let)

Using identity
 $(4)^2 - (5)^2 = (7)^2 - (m)^2$

$$16 - 25 = 49 - m^2$$

$$m^2 = 49 + 9$$

$$m = \pm \sqrt{58}$$

TYPE - XII

(A) If $A+B = 45^\circ$ or 225° then,

(i) $(1 + \tan A)(1 + \tan B) = 2$ and

(ii) $(1 - \cot A)(1 - \cot B) = 2$

Proof

(i) $A+B = 45^\circ$

Both side take tan.

$\tan (A+B) = \tan 45^\circ$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$\Rightarrow \tan A + \tan B = 1 - \tan A \tan B$

$\Rightarrow \tan A + \tan B + \tan A \tan B = 1$

Adding 1 both side.

$1 + \tan A + \tan B + \tan A \tan B = 1 + 1$

$1(1 + \tan A) + \tan B(1 + \tan A) = 2$

Hence, $(1 + \tan B)(1 + \tan A) = 2$

$A+B = 45^\circ$

Both side take cot.

$\cot (A+B) = \cot 45^\circ$

$$\frac{\cot A \cdot \cot B - 1}{\cot A + \cot B} = 1$$

$\Rightarrow \cot A \cot B - 1 = \cot A + \cot B$

$\Rightarrow \cot A [\cot B - 1] - 1 - \cot B + 1 = 0$

$\Rightarrow \cot A [\cot B - 1] - 1 [\cot B - 1] = 2$

$\Rightarrow (\cot A - 1)(\cot B - 1) = 2$

Ex. 106 Find the value of $(1 + \tan 5^\circ)(1 + \tan 40^\circ)$

Sol. $A+B = 5^\circ + 40^\circ = 45^\circ$

then, $(1 + \tan 5^\circ)(1 + \tan 40^\circ) = 2$

Ex. 107 Find the value of

$$(1 + \tan 1^\circ)(1 + \tan 2^\circ)(1 + \tan 3^\circ) \dots (1 + \tan 44^\circ)(1 + \tan 43^\circ)(1 + \tan 42^\circ)$$

Sol. $(1 + \tan 1^\circ)(1 + \tan 44^\circ)$

$(1 + \tan 2^\circ)(1 + \tan 43^\circ)$

$(1 + \tan 3^\circ)(1 + \tan 42^\circ)$

$1^\circ + 44^\circ = 2^\circ + 43^\circ$

$= 3^\circ + 42^\circ = 45^\circ$

so, 3 pair of such term

$= 2 \times 2 \times 2 = 8$

Ex. 108 If $A+B = 45^\circ$, then find

$$\frac{\tan A}{1 - \tan A} \cdot \frac{\tan B}{1 - \tan B} = ?$$

Sol. $\frac{\tan A}{1 - \tan A} \cdot \frac{\tan B}{1 - \tan B}$

$$= \frac{\frac{1}{\cot A}}{1 - \frac{1}{\cot A}} \cdot \frac{\frac{1}{\cot B}}{1 - \frac{1}{\cot B}}$$

$$= \frac{1}{\cot A - 1} \cdot \frac{1}{\cot B - 1} =$$

$$\frac{1}{(\cot A - 1)(\cot B - 1)} = \frac{1}{2}$$

(B) If A+B+C = 180°

then,

(i) tan A + tan B + tan C = tan A . tan B . tan C

(ii) cot A . cot B + cot B . cot C + cot C . cot A = 1

(iii) sin 2A + sin 2B + sin 2C = 4 sin A . sin B . sin C

Proof,

$$A+B+C = 180^\circ$$

$$A+B = 180^\circ - C$$

Both side take tan

$$\tan(A+B) = \tan(180^\circ - C)$$

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{-\tan C}{1}$$

$$\Rightarrow \tan A + \tan B = -\tan C + \tan A \tan B \tan C$$

$$\Rightarrow \text{so, } \tan A + \tan B + \tan C = \tan A . \tan B . \tan C$$

(tan A . tan B . tan C) divided by both side

then,

$$\frac{1}{\tan B \tan C} + \frac{1}{\tan A \tan C} + \frac{1}{\tan A \tan B} = 1$$

$$\Rightarrow \cot A \cot B + \cot B \cot C + \cot C \cot A = 1$$

Ex.109 The value of $\tan 37^\circ + \tan 73^\circ + \tan 70^\circ$ is equal to

Sol. Sum of angle $37^\circ + 73^\circ + 70^\circ = 180^\circ$

$$\text{so, } \tan 37^\circ + \tan 73^\circ + \tan 70^\circ = \tan 37^\circ . \tan 73^\circ . \tan 70^\circ$$

TYPE - XIII

Morri's law

If $4\theta < 60^\circ$

$$\text{(i) } \sin \theta . \sin 2\theta . \sin 4\theta = \frac{1}{4} \sin 3\theta$$

$$\text{(ii) } \cos \theta . \cos 2\theta . \cos 4\theta = \frac{1}{4} \cos 3\theta$$

$$\text{(iii) } \tan \theta . \tan 2\theta . \tan 4\theta = \tan 3\theta$$

For all value of θ

$$\text{(i) } \sin(60-\theta) \sin \theta . \sin(60+\theta)$$

$$= \frac{1}{4} \sin 3\theta$$

$$\text{(ii) } \cos(60-\theta) \cos \theta . \cos(60+\theta)$$

$$= \frac{1}{4} \cos 3\theta$$

$$\text{(iii) } \tan(60-\theta) \tan \theta . \tan(60+\theta) = \tan 3\theta$$

Ex.110 The value of $\tan 10^\circ \tan 20^\circ \tan 40^\circ = ?$

Sol. Here $\theta = 10^\circ$

$$\therefore \tan(10^\circ) \tan(2 \times 10^\circ) \tan(4 \times 10^\circ) = \tan(3 \times 10^\circ)$$

$$\Rightarrow \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Ex.111 The value of $\sin 20^\circ . \sin 40^\circ . \sin 80^\circ = ?$

Sol. $\sin 20^\circ . \sin 40^\circ . \sin 80^\circ$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\theta \quad 60-\theta \quad 60+\theta$$

Here, $\theta = 20^\circ$

$$= \frac{1}{4} \sin 3\theta$$

$$= \frac{1}{4} \sin(3 \times 20^\circ)$$

$$= \frac{1}{4} \sin 60^\circ$$

$$= \frac{1}{4} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$$

Ex.112 The value of

$$\sin \frac{\pi}{9} . \sin \frac{5\pi}{9} . \sin \frac{7\pi}{9} . \sin \frac{3\pi}{9} . \text{ is}$$

equal to

$$\text{Sol. } \sin \frac{\pi}{9} . \sin \frac{5\pi}{9} . \sin \frac{7\pi}{9} . \sin \frac{3\pi}{9}$$

Put value of $\pi = 180^\circ$

$$\Rightarrow \sin 20^\circ . \sin 100^\circ . \sin 140^\circ . \sin 60^\circ$$

$$\Rightarrow \sin 20^\circ . \sin(180^\circ - 80^\circ) .$$

$$\sin(180^\circ - 40^\circ) . \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin 20^\circ . \sin 40^\circ . \sin 80^\circ . \frac{\sqrt{3}}{2}$$

$$[\therefore \sin(180^\circ - \theta) = \sin \theta]$$

$$\Rightarrow \frac{1}{4} \sin 60^\circ \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{1}{4} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \Rightarrow \frac{3}{16}$$

Ex.113 The value of $\sin 12^\circ . \sin 48^\circ . \sin 54^\circ = ?$

Sol. $\sin 12^\circ . \sin 48^\circ . \sin 54^\circ$

($\sin 72^\circ$ divide and multiply)

$$\sin 12^\circ . \sin 48^\circ . \sin 72^\circ \times$$

$$\frac{1}{\sin 72^\circ} . \sin 54^\circ$$

here $\theta = 12^\circ$

$$\Rightarrow \frac{1}{4} \sin 3 \times 12^\circ \times \frac{1}{\sin 72^\circ} . \sin(90^\circ - 36^\circ)$$

$$\Rightarrow \frac{1}{4} \sin 36^\circ . \cos 36^\circ \times \frac{1}{\sin 72^\circ}$$

$$\Rightarrow \frac{1}{4 \times 2} . 2 \sin 36^\circ . \cos 36^\circ$$

$$\times \frac{1}{\sin 72^\circ}$$

$$\Rightarrow \frac{1}{8} \sin 2 \times 36^\circ \times \frac{1}{\sin 72^\circ}$$

$$\Rightarrow \frac{1}{8} \sin 72^\circ \times \frac{1}{\sin 72^\circ} = \frac{1}{8}$$

Ex.114 The value of $1 - \sin 10^\circ \sin 50^\circ \sin 70^\circ = ?$

Sol. $1 - \sin 10^\circ \sin 50^\circ \sin 70^\circ$

here $\theta = 10^\circ$

$$= 1 - \left(\frac{1}{4} \sin 3 \times 10^\circ \right)$$

$$= 1 - \frac{1}{4} \times \sin 30^\circ = 1 - \frac{1}{4} \times \frac{1}{2} = \frac{7}{8}$$

TYPE - XIV

T-radius of Multiple Angles :-

$$\text{(i) } \sin 2A = 2 \sin A . \cos A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$\text{(ii) } \cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\text{(iii) } \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\text{(iv) } \sin 3A = 3 \sin A - 4 \sin^3 A$$

$$\text{(v) } \cos 3A = 4 \cos^3 A - 3 \cos A$$

$$\text{(vi) } \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

$$\text{(vii) } \sin C + \sin D = 2$$

$$\sin \left(\frac{C+D}{2} \right) . \cos \left(\frac{C-D}{2} \right)$$

(viii) $\sin C - \sin D$

$$= 2 \cos \left(\frac{C+D}{2} \right) \cdot \sin \left(\frac{C-D}{2} \right)$$

(ix) $\cos C + \cos D = 2 \cos \left(\frac{C+D}{2} \right) \cdot \cos \left(\frac{C-D}{2} \right)$

(x) $\cos C - \cos D = 2 \sin \left(\frac{C+D}{2} \right) \cdot \sin \left(\frac{C-D}{2} \right)$

Ex.115. If $\sin 2x = \frac{1}{5}$, the value of

$(\sin x + \cos x)$ is :-

(a) $\sqrt{\frac{7}{5}}$ (b) $\sqrt{\frac{4}{5}}$

(c) $\sqrt{\frac{6}{5}}$ (d) $\sqrt{\frac{2}{5}}$

Sol. $\sin 2x = \frac{1}{5}$

add 1 both side

$$1 + \sin 2x = 1 + \frac{1}{5} = \frac{6}{5}$$

$$\therefore \sin^2 x + \cos^2 x + 2 \sin x \cdot \cos x = \frac{6}{5}$$

$[\because \sin^2 x + \cos^2 x = 1 \text{ and } \sin 2x = 2 \sin x \cdot \cos x]$

$$\Rightarrow (\sin x + \cos x)^2 = \frac{6}{5}$$

$$\Rightarrow \sin x + \cos x = \sqrt{\frac{6}{5}}$$

Ex.116 The value of

$$\cos 15^\circ \cdot \cos 7\frac{1}{2}^\circ \cdot \sin 7\frac{1}{2}^\circ = ?$$

Sol. $\cos 15^\circ \cdot \cos 7\frac{1}{2}^\circ \cdot \sin 7\frac{1}{2}^\circ$

Multiply and divide by 2

$$\Rightarrow \cos 15^\circ \cdot \frac{1}{2}$$

$$\left[2 \cos 7\frac{1}{2}^\circ \cdot \sin 7\frac{1}{2}^\circ \right]$$

$$\Rightarrow \frac{1}{2} \cos 15^\circ \times \sin 2 \cdot \frac{15}{2}$$

$$\Rightarrow \frac{1}{2} \cos 15^\circ \cdot \sin 15^\circ$$

Again multiply and divide by 2

$$\Rightarrow \frac{1}{2 \times 2} \times 2 \sin 15^\circ \cdot \cos 15^\circ$$

$$\Rightarrow \frac{1}{4} \cdot \sin 30^\circ \Rightarrow \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

Ex.117. The value of

$$\frac{\sin 2x}{\sin \frac{x}{4}}$$

Sol. $\frac{\sin 2x}{\sin \frac{x}{4}}$

First we solve $\sin 2x$

$$\sin 2x = 2 \sin x \cdot \cos x$$

$$\Rightarrow 2 \sin 2 \left(\frac{x}{2} \right) \cdot \cos x$$

$$= 4 \sin \frac{x}{2} \cdot \cos \frac{x}{2} \cdot \cos x$$

$$= 4 \sin 2 \left(\frac{x}{2 \times 2} \right) \cdot \cos \frac{x}{2} \cdot \cos x$$

$$= 4 \times 2 \sin \frac{x}{4} \cdot \cos \frac{x}{4} \cdot \cos \frac{x}{2} \cdot \cos x$$

now,

$$\Rightarrow \frac{4 \times 2 \sin \frac{x}{4} \cdot \cos \frac{x}{4} \cdot \cos \frac{x}{2} \cdot \cos x}{\sin \frac{x}{4}}$$

$$\Rightarrow 8 \cos \frac{x}{4} \cdot \cos \frac{x}{2} \cdot \cos x$$

Ex.118. Find the value of $\frac{1 - \tan^2 22\frac{1}{2}^\circ}{1 + \tan^2 22\frac{1}{2}^\circ}$

Sol. $\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$

$$\therefore \cos 2 \times 22\frac{1}{2}^\circ = \cos 45^\circ$$

$$= \frac{1}{\sqrt{2}}$$

Ex.119 Find the value $\sin 22^\circ \frac{1}{2} = ?$

Sol. $[\cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A]$

$$22\frac{1}{2}^\circ = A$$

$$45^\circ = 2A$$

$$\Rightarrow \cos 45^\circ = 1 - 2 \sin^2 22\frac{1}{2}^\circ$$

$$\left(\sin^2 22\frac{1}{2}^\circ = x \text{ (let)} \right)$$

$$\Rightarrow \frac{1}{\sqrt{2}} = 1 - 2x^2$$

$$\Rightarrow 2x^2 = 1 - \frac{1}{\sqrt{2}}$$

$$\Rightarrow x^2 = \frac{\sqrt{2} - 1}{2\sqrt{2}}$$

$$x = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

Ex.120 The value of

$$3 \sin 20^\circ - 4 \sin^3 20^\circ$$

Sol. $3 \sin 20^\circ - 4 \sin^3 20^\circ = \sin 3 \times 20^\circ$

$$= \sin 60^\circ$$

Ex.121 Find the value of

$$\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$$

Sol. $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$

$$= \frac{\cos 10^\circ - \sqrt{3} \sin 10^\circ}{\sin 10^\circ \cos 10^\circ}$$

$$= \frac{2 \left(\frac{1}{2} \cos 10^\circ - \frac{\sqrt{3}}{2} \sin 10^\circ \right)}{\sin 10^\circ \cos 10^\circ}$$

We can write $\frac{1}{2} = \sin 30^\circ$ &

$$\frac{\sqrt{3}}{2} = \cos 30^\circ$$

$$= \frac{2 [\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ]}{\sin 10^\circ \cos 10^\circ}$$

multiply and divide by 2

$$= \frac{2.2 \sin (30^\circ - 10^\circ)}{2 \sin 10^\circ \cos 10^\circ}$$

$$= \frac{4 \sin 20^\circ}{\sin 2 \times 10^\circ} = 4$$

Ex.122. If $\cos x = \frac{2\cos y - 1}{2 - \cos y}$ then find the value of

$$\tan \frac{x}{2} \cot \frac{y}{2}$$

Sol. $\frac{\cos x}{1} = \frac{2\cos y - 1}{2 - \cos y}$

$\Rightarrow$ Applying Componendo-Dividendo rule

$\Rightarrow \frac{\cos x + 1}{\cos x - 1}$

$$= \frac{2\cos y - 1 + 2 - \cos y}{2\cos y - 1 - 2 + \cos y}$$

$\Rightarrow \frac{\cos x + 1}{\cos x - 1} = \frac{1 + \cos y}{3(\cos y - 1)}$

$\Rightarrow \frac{2\cos^2 \frac{x}{2} - 1 + 1}{1 - 2\sin^2 \frac{x}{2} - 1}$

$$= \frac{1 + 2\cos^2 \frac{y}{2} - 1}{3\left(1 - 2\sin^2 \frac{y}{2} - 1\right)}$$

$\Rightarrow \frac{2\cos^2 \frac{x}{2}}{-2\sin^2 \frac{x}{2}} = \frac{2\cos^2 \frac{y}{2}}{3\left(-2\sin^2 \frac{y}{2}\right)}$

$\Rightarrow \cot^2 \frac{x}{2} = \frac{1}{3} \cot^2 \frac{y}{2}$

$\Rightarrow 3 = \tan^2 \frac{x}{2} \cdot \cot^2 \frac{y}{2}$

Hence, $\tan \frac{x}{2} \cdot \cot \frac{y}{2} = \sqrt{3}$

Ex.123 Find the value of $\frac{1}{2} \operatorname{cosec} 10^\circ - 2\sin 70^\circ$

Sol. $\frac{1}{2} \operatorname{cosec} 10^\circ - 2\sin 70^\circ$
 $= \frac{1}{2\sin 10^\circ} - 2\sin 70^\circ$
 $= \frac{1 - 2 \cdot 2\sin 70^\circ \sin 10^\circ}{2\sin 10^\circ}$
 $= \frac{1 - 2[\cos 60^\circ - \cos 80^\circ]}{2\sin 10^\circ}$

(Using formula; $2\sin A \cdot \sin B = \cos(A - B) - \cos(A + B)$)

$$\begin{aligned} &= \frac{1 - 2\left(\frac{1}{2} - \cos 80^\circ\right)}{2\sin(90^\circ - 80^\circ)} \\ &= \frac{1 - 1 + 2\cos 80^\circ}{2\cos 80^\circ} \\ &= \frac{2\cos 80^\circ}{2\cos 80^\circ} = 1 \end{aligned}$$

TYPE- XV

Trigonometry expression is independent of angle so we can put any value of θ except result should not indetermine

$$\left(\infty, \frac{\infty}{\infty}, \frac{0}{0}, \frac{1}{0}\right)$$

Note:- $\frac{1}{\infty} = 0, \frac{1}{\infty + 1} = \frac{1}{\infty} = 0$

$\Rightarrow$ It is better to put $\theta = 0^\circ$ if expression does not contain cosec θ or cot θ otherwise $\theta = 45^\circ$

(i) If $\sin \theta, \cos \theta$ in equation, Try to put $\theta = 0^\circ$ or 90°

(ii) If $\sin \theta, \cos \theta, \tan \theta, \sec \theta, \operatorname{cosec} \theta, \cot \theta$ try to put $\theta = 45^\circ$

Ex.124 If $\tan \theta + \sin \theta = m$ and $\tan \theta - \sin \theta = n$, then find the value of $m^2 - n^2 = ?$

(a) $4\sqrt{mn}$ (b) mn

(c) m^2n^2 (d) m^3n^3

Sol. $m^2 - n^2 = (\tan \theta + \sin \theta)^2 - (\tan \theta - \sin \theta)^2$

$$= 4 \tan \theta \cdot \sin \theta$$

$$[\because (a + b)^2 - (a - b)^2 = 4ab]$$

$$= 4 \tan \theta \cdot \sin \theta$$

$$= 4\sqrt{\tan^2 \theta \cdot \sin^2 \theta}$$

$$= 4\sqrt{\tan^2 \theta (1 - \cos^2 \theta)}$$

$$= 4\sqrt{\tan^2 \theta - \tan^2 \theta \cdot \cos^2 \theta}$$

$$= 4\sqrt{\tan^2 \theta - \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \cos^2 \theta}$$

$$= 4\sqrt{\tan^2 \theta - \sin^2 \theta}$$

Now, $mn = (\tan \theta + \sin \theta)(\tan \theta - \sin \theta)$

$$mn = \tan^2 \theta - \sin^2 \theta \quad \dots (i)$$

From equation (i)

$$= 4\sqrt{mn}$$

Alternative:-

Let $\theta = 45^\circ$

$$m = \tan \theta + \sin \theta = 1 + \frac{1}{\sqrt{2}}$$

Now, $n = \tan \theta - \sin \theta = 1 - \frac{1}{\sqrt{2}}$
 $m^2 - n^2 = (m + n)(m - n)$

$$= (2) \times \left(2 \times \frac{1}{\sqrt{2}}\right) = 4 \times \frac{1}{\sqrt{2}}$$

take option (a) $4\sqrt{mn}$

$$= mn = \left(1 + \frac{1}{\sqrt{2}}\right)\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$= (1)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 = 1 - \frac{1}{2} = \frac{1}{2}$$

So, $4\sqrt{mn} = 4 \times \sqrt{\frac{1}{2}}$

So option (a) is correct.

Ex.125 If $\sin \theta + \cos \theta = p$ and

$\sec \theta + \operatorname{cosec} \theta = q$, then what is the value of $2p$:

(a) $p(q^2 - 1)$

(b) $p(1 - q^2)$

(c) $q(1 - p^2)$

(d) $q(p^2 - 1)$

Sol. Let $\theta = 45^\circ$

then $p = \sin 45^\circ + \cos 45^\circ = \sqrt{2}$ and

$$q = \sec 45^\circ + \operatorname{cosec} 45^\circ = 2\sqrt{2}$$

Now, $2p = 2\sqrt{2}$

Take option (d)

$$\therefore q(p^2 - 1) = 2\sqrt{2} \left[(\sqrt{2})^2 - 1 \right] = 2\sqrt{2} = 2p$$

Hence, option (d) is correct.

Ex.126 Find the value of :

$$1 + 2 \sec^2 A \cdot \tan^2 A - \sec^4 A - \tan^4 A$$

(a) 0 (b) 1

(c) $\sec^2 A \cdot \tan^2 A$

(d) None of these

Sol. $1 - (\sec^4 A + \tan^4 A - 2 \sec^2 A \cdot \tan^2 A)$

$$= 1 - (\sec^2 A - \tan^2 A)^2$$

$$= 1 - 1 = 0$$

Alternative:-

Take $A = 45^\circ$, then

Given Exp.

$$= 1 + 2 \sec^2 45^\circ \cdot \tan^2 45^\circ - \sec^4 45^\circ - \tan^4 45^\circ$$

$$= 1 + 2(\sqrt{2})^2 \times 1 - (\sqrt{2})^4 - 1$$

$$= 0$$

Ex.127 If $x = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$, then $\frac{2x}{1-x^2} = ?$

- (a) $\sec\theta$ (b) $\tan\theta$ (c) $\cot\theta$ (d) $\cos\theta$

Sol. Let $\theta = 60^\circ$

$$\therefore x = \sqrt{\frac{1 - \frac{1}{2}}{1 + \frac{1}{2}}} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{2x}{1-x^2} = \frac{2/\sqrt{3}}{1-\frac{1}{3}} = \sqrt{3}$$

and option (B). $\tan\theta = \tan 60^\circ$

$$= \sqrt{3}$$

& option (B) is correct.

Note:- at $\theta = 0^\circ$ $\cot\theta = \infty$

$\therefore \theta$ can't be 0° & at $\theta = 45^\circ$ option (B) and (C) contradicts.

Ex.128 If $x = \operatorname{cosec}\theta - \sin\theta$ and $y = \sec\theta - \cos\theta$, then the value of $x^2y^2(x^2 + y^2 + 3)$ is :

- (a) 0 (b) 1 (c) 2 (d) 3

Sol. take $\theta = 45^\circ$

$$\therefore x = \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \text{ \& } y = \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\therefore x^2y^2(x^2 + y^2 + 3) = \frac{1}{2} \times \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} + 3 \right) = \frac{1}{4}(4) = 1$$

Ex.129 If $m = \operatorname{cosec}\theta - \sin\theta$ and $n = \sec\theta - \cos\theta$, then find the value of

$$m^{2/3} + n^{2/3} = ?$$

- (a) $(mn)^{2/3}$ (b) $(mn)^{-2/3}$ (c) $(mn)^{-1/3}$ (d) $(mn)^{1/3}$

Sol. $m = \operatorname{cosec}\theta - \sin\theta = \frac{1 - \sin^2\theta}{\sin\theta}$

$$= \frac{\cos^2\theta}{\sin\theta} \text{ and } n = \sec\theta - \cos\theta$$

$$= \frac{1 - \cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$$

$$\therefore mn = \frac{\cos^2\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos\theta} = \sin\theta \cdot \cos\theta$$

and $m^{2/3} + n^{2/3} = \left(\frac{\cos^2\theta}{\sin\theta} \right)^{2/3} + \left(\frac{\sin^2\theta}{\cos\theta} \right)^{2/3}$

$$= \frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} + \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{(\sin\theta \cdot \cos\theta)^{2/3}} = \frac{1}{(mn)^{2/3}}$$

$$= (mn)^{-2/3}$$

Alternative:-

Let $\theta = 45^\circ$, then

$$m = \frac{1}{\sqrt{2}} \text{ and } n = \frac{1}{\sqrt{2}}$$

$$\therefore m^{2/3} + n^{2/3} = \left(\frac{1}{\sqrt{2}} \right)^{2/3} + \left(\frac{1}{\sqrt{2}} \right)^{2/3}$$

$$\Rightarrow 2 \cdot \left(\frac{1}{\sqrt{2}} \right)^{2/3} = 2 \cdot \left(\frac{1}{2} \right)^{1/3}$$

$$\Rightarrow 2 \cdot 2^{-1/3} = 2^{1-1/3} = 2^{2/3}$$

$$\& (mn)^{-2/3} = \left(\frac{1}{2} \right)^{-2/3} = 2^{2/3}$$

$$\Rightarrow m^{2/3} + n^{2/3} = (mn)^{-2/3}$$

Ex.130 If $\operatorname{cosec}\theta - \sin\theta = l^3$ and $\sec\theta - \cos\theta = m^3$, then the value of $l^2m^2(l^2 + m^2)$ is :

- (a) -1 (b) 0 (c) 1 (d) 2

Sol. take $\theta = 45^\circ$

$$l^3 = \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$m^3 = \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow l^3 = m^3$$

So, $l = m$

$$\Rightarrow l^2m^2(l^2 + m^2)$$

Put $(l = m)$

$$\Rightarrow l^2 \times l^2 (l^2 + l^2)$$

$$\Rightarrow 2l^6 = 2(l^3)^2 = 2 \times \left(\frac{1}{\sqrt{2}} \right)^2 = 1$$

Ex.131 If $a \cos^3\theta + 3a \cos\theta \cdot \sin^2\theta = m$ and $a \sin^3\theta + 3a \sin\theta \cdot \cos^2\theta = n$, then the find value of

$$(m+n)^{2/3} + (m-n)^{2/3} = ?$$

$$(a) 2a^{1/3} \quad (b) 2a^{2/3}$$

$$(c) a^{2/3} \quad (d) a^{1/3}$$

Sol. $m + n = a \cos^3\theta + 3a \cos\theta \sin^2\theta + a \sin^3\theta + 3a \sin\theta \cos^2\theta$

$$= a(\cos\theta + \sin\theta)^3$$

Similarly,

$$m - n = a(\cos\theta - \sin\theta)^3$$

$$\therefore \cos\theta + \sin\theta = \left(\frac{m+n}{a} \right)^{1/3} \text{ and}$$

$$\cos\theta - \sin\theta = \left(\frac{m-n}{a} \right)^{1/3}$$

$$\therefore \left(\frac{m+n}{a} \right)^{2/3} + \left(\frac{m-n}{a} \right)^{2/3}$$

$$\Rightarrow (\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2$$

$$\Rightarrow 2(\cos^2\theta + \sin^2\theta) = 2$$

$$\Rightarrow (m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$$

Alternate:-

Let $\theta = 0^\circ$, then

$$m = a \text{ and } n = 0$$

$$(m+n)^{2/3} + (m-n)^{2/3} = a^{2/3} + a^{2/3} = 2a^{2/3}$$

EXERCISE

1. $\frac{\tan \theta + \cot \theta}{\tan \theta - \cot \theta} = 2, (0 \leq \theta \leq 90^\circ)$, then the value of $\sin \theta$ is
(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) 1
2. The value of $\cot 10^\circ \cdot \cot 20^\circ \cdot \cot 60^\circ \cdot \cot 70^\circ \cdot \cot 80^\circ$ is
(a) 1 (b) -1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$
3. The value of $\cot 18^\circ \left(\cot 72^\circ \cos^2 22^\circ + \frac{1}{\tan 72^\circ \sec^2 68^\circ} \right)$ is
(a) 1 (b) $\sqrt{2}$ (c) 3 (d) $\frac{1}{\sqrt{3}}$
4. If $\tan 15^\circ = 2 - \sqrt{3}$, the value of $\tan 15^\circ \cot 75^\circ + \tan 75^\circ \cot 15^\circ$ is
(a) 14 (b) 12 (c) 10 (d) 8
5. If x, y are acute angles, $0 < x + y < 90^\circ$ and $\sin(2x - 20^\circ) = \cos(2y + 20^\circ)$, then the value of $\tan(x + y)$ is :
(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) 1
6. $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \dots + \sin^2 85^\circ + \sin^2 90^\circ$ is equal to
(a) $7\frac{1}{2}$ (b) $8\frac{1}{2}$ (c) 9 (d) $9\frac{1}{2}$
7. The value of $\frac{\sin 39^\circ}{\cos 51^\circ} + 2 \tan 11^\circ \tan 31^\circ \tan 45^\circ \tan 59^\circ \tan 79^\circ - 3(\sin^2 21^\circ + \sin^2 69^\circ)$ is ;
(a) 2 (b) -1 (c) 1 (d) 0
8. If $\frac{\cos^2 \theta}{\cot^2 \theta - \cos^2 \theta} = 3$ and $0^\circ < \theta < 90^\circ$, then value of θ is:
(a) 30° (b) 45°
(c) 60° (d) None of these
9. If $A = \tan 11^\circ \tan 29^\circ$, $B = 2 \cot 61^\circ \cot 79^\circ$, then ;
(a) $A = 2B$ (b) $A = -2B$
(c) $2A = B$ (d) $2A = -B$
10. If $\sin \alpha + \cos \beta = 2; (0^\circ \leq \beta < \alpha \leq 90^\circ)$, then $\sin\left(\frac{2\alpha + \beta}{3}\right) =$
(a) $\sin \frac{\alpha}{2}$ (b) $\cos \frac{\alpha}{3}$
(c) $\sin \frac{\alpha}{3}$ (d) $\cos \frac{2\alpha}{3}$
11. If $\sin \alpha \sec(30^\circ + \alpha) = 1, (0 < \alpha < 60^\circ)$, then the value of $\sin \alpha + \cos 2\alpha$ is
(a) 1 (b) $\frac{2 + \sqrt{3}}{2\sqrt{3}}$
(c) 0 (d) $\sqrt{2}$
12. If $\tan \theta = 1$, then the value of $\frac{8 \sin \theta + 5 \cos \theta}{\sin^3 \theta - 2 \cos^3 \theta + 7 \cos \theta}$ is
(a) 2 (b) $2\frac{1}{2}$ (c) 3 (d) $\frac{4}{5}$
13. If ' θ ' be a positive acute angle satisfying $\cos^2 \theta + \cos^4 \theta = 1$, then the value of $\tan^2 \theta + \tan^4 \theta$ is
(a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) 0
14. If $\tan \theta = \frac{4}{3}$, then the value of $\frac{3 \sin \theta + 2 \cos \theta}{3 \sin \theta - 2 \cos \theta}$ is
(a) 0.5 (b) -0.5 (c) 3.0 (d) -3.0
15. The simplified value of $(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$ is
(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2
16. If ' θ ' be an acute angle and $\tan \theta + \cot \theta = 2$, then the value of $\tan^5 \theta + \cot^{10} \theta$ is
(a) 1 (b) 2 (c) 3 (d) 4
17. If $\sin \theta - \cos \theta = \frac{7}{13}$ and $0^\circ < \theta < 90^\circ$, then the value of $\sin \theta + \cos \theta$ is
(a) $\frac{17}{13}$ (b) $\frac{13}{17}$ (c) $\frac{1}{13}$ (d) $\frac{1}{17}$
18. If $2 \cos \theta - \sin \theta = \frac{1}{\sqrt{2}}, (0^\circ < \theta < 90^\circ)$ the value of $2 \sin \theta + \cos \theta$ is
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{3}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$
19. If $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = 3$ then the value of $\sin^4 \theta - \cos^4 \theta$ is
(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{4}{5}$
20. If $\sec^2 \theta + \tan^2 \theta = 7$, then the value of θ when $0^\circ \leq \theta \leq 90^\circ$, is
(a) 60° (b) 30° (c) 0° (d) 90°
21. The simplified value of $(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2$ is :
(a) -1 (b) 0 (c) $\sec^2 x$ (d) 1
22. If $A = \sin^2 \theta + \cos^4 \theta$ for any value of θ , then the value of A is
(a) $1 \leq A \leq 1$ (b) $\frac{3}{4} \leq A \leq 1$
(c) $\frac{13}{16} \leq A \leq 1$ (d) $\frac{3}{4} \leq A \leq \frac{13}{16}$
23. If $\sin \theta + \operatorname{cosec} \theta = 2$, then the value of $\sin^5 \theta + \operatorname{cosec}^5 \theta$, when $0^\circ \leq \theta \leq 90^\circ$ is
(a) 0 (b) 1 (c) 10 (d) 2
24. If $\tan 2\theta \cdot \tan 4\theta = 1$, then the value of $\tan 3\theta$ is
(a) $\sqrt{3}$ (b) 10 (c) 1 (d) $\frac{1}{\sqrt{3}}$
25. If $\cos^2 \alpha + \cos^2 \beta = 2$, then the value of $\tan^3 \alpha + \sin^5 \beta$ is ;
(a) -1 (b) 0 (c) 1 (d) $\frac{1}{\sqrt{3}}$

26. In a triangle ABC, $\angle ABC = 75^\circ$ and $\angle ACB = \frac{\pi^c}{4}$, The circular measure of $\angle BAC$ is
 (a) $\frac{5\pi}{12}$ radian (b) $\frac{\pi}{3}$ radian
 (c) $\frac{\pi}{6}$ radian (d) $\frac{\pi}{2}$ radian
27. The angles of a triangle are $(x + 5)^\circ$, $(2x - 3)^\circ$ and $(3x + 4)^\circ$. The value of x is
 (a) 30° (b) 31° (c) 29° (d) 28°
28. If $\angle A$ and $\angle B$ are complementary to each other, then the value of $\sec^2 A + \sec^2 B - \sec^2 A \cdot \sec^2 B$ is
 (a) 1 (b) -1 (c) 2 (d) 0
29. If $\sin 17^\circ = \frac{x}{y}$, then the value of $(\sec 17^\circ - \sin 73^\circ)$ is
 (a) $\frac{y^2}{x\sqrt{y^2 - x^2}}$ (b) $\frac{x^2}{y\sqrt{y^2 - x^2}}$
 (c) $\frac{x^2}{y\sqrt{x^2 - y^2}}$ (d) $\frac{y^2}{x\sqrt{x^2 - y^2}}$
30. The value of $\frac{\cot 30^\circ - \cot 75^\circ}{\tan 15^\circ - \tan 60^\circ}$ is
 (a) 0 (b) 1
 (c) $\sqrt{3} - 1$ (d) -1
31. The value of $\cot \theta \cdot \tan(90^\circ - \theta) - \sec(90^\circ - \theta) \cdot \operatorname{cosec} \theta + (\sin^2 25^\circ + \sin^2 65^\circ) + \sqrt{3}(\tan 5^\circ \tan 15^\circ \tan 30^\circ \tan 75^\circ \tan 85^\circ)$ is
 (a) 1 (b) -1 (c) 2 (d) 0
32. If $\sin(3x - 20^\circ) = \cos(3y + 20^\circ)$ then the value of $(x + y)$ is
 (a) 20° (b) 30° (c) 40° (d) 45°
33. If $\cos \theta \operatorname{cosec} 23^\circ = 1$, the value of θ is
 (a) 23° (b) 37° (c) 63° (d) 67°
34. If $2(\cos^2 \theta - \sin^2 \theta) = 1$, θ is a positive acute angle, then the value of θ is
 (a) 60° (b) 30°
 (c) 45° (d) $22\frac{1}{2}^\circ$
35. If $\sec(7\theta + 28^\circ) = \operatorname{cosec}(30^\circ - 3\theta)$, then the value of θ is
 (a) 8° (b) 5° (c) 60° (d) 9°
36. If $\tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \sqrt{3}$, the value of $\cos \theta$ is:
 (a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) 1
37. The value of $\cos 25^\circ - \sin 25^\circ$
 (a) positive but less than 1
 (b) positive but greater than 1
 (c) negative
 (d) 0
38. In a right angled ΔABC , right angle at B, if $\cos A = \frac{4}{5}$, then what is $\sin C$ is equal to?
 (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{2}{5}$
39. If α and β are complementary angles, then what is $\sqrt{\cos \alpha \operatorname{cosec} \beta - \cos \alpha \sin \beta}$ equal to?
 (a) $\sec \beta$ (b) $\cos \alpha$
 (c) $\sin \alpha$ (d) $-\tan \beta$
40. If $2 \cot \theta = 3$, then what is $\frac{2 \cos \theta - \sin \theta}{2 \cos \theta + \sin \theta}$ equal to?
 (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$
41. If $\sin \theta \cos \theta = \frac{1}{2}$, then what is $\sin^6 \theta + \cos^6 \theta$ equal to?
 (a) 1 (b) 2 (c) 3 (d) $\frac{1}{4}$
42. If $\sec \theta + \tan \theta = 2$, then what is the value of $\sec \theta$?
 (a) $\frac{3}{2}$ (b) $\sqrt{2}$ (c) $\frac{5}{2}$ (d) $\frac{5}{4}$
43. What is $\operatorname{cosec}(75^\circ + \theta) - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot(35^\circ - \theta)$ equal to?
 (a) -1 (b) 0 (c) 1 (d) $\frac{3}{2}$
44. If $\sin \theta + 2 \cos \theta = 1$, where $0 < \theta < \pi/2$, what is $2 \sin \theta - \cos \theta$ equal to?
 (a) -1 (b) $1/2$ (c) 2 (d) 1
45. If $\tan 8\theta = \cot 2\theta$, where $0 < 8\theta < \frac{\pi}{2}$, then what is the value of $\tan 5\theta$?
 (a) $\frac{1}{\sqrt{3}}$ (b) 1 (c) $\sqrt{3}$ (d) 0
46. If $\sin(A + B) = 1$, where $0 < B < 45^\circ$, then what is $\cos(A - B)$ equal to?
 (a) $\sin 2B$ (b) $\sin B$
 (c) $\cos 2B$ (d) $\cos B$
47. If $5 \sin \theta + 12 \cos \theta = 13$, then what is $5 \cos \theta - 12 \sin \theta$ equal to?
 (a) -2 (b) -1 (c) 0 (d) 1
48. If $4 \tan \theta = 3$, then what is $\frac{4 \sin \theta - \cos \theta}{4 \sin \theta + 9 \cos \theta}$ equal to?
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$
49. If $\sin \theta - \cos \theta = 0$, then what is $\sin^4 \theta + \cos^4 \theta$ equal to?
 (a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
50. What is $\frac{(\sin \theta + \cos \theta)(\tan \theta + \cot \theta)}{\sec \theta + \operatorname{cosec} \theta}$ equal to
 (a) 1 (b) 2
 (c) $\sin \theta$ (d) $\cos \theta$
51. What is $\sin^6 \theta + \cos^6 \theta + 3 \sin^2 \theta \cos^2 \theta$ equal to?
 (a) 0 (b) 1 (c) 2 (d) 4
52. What is $\frac{(1 + \sec \theta - \tan \theta) \cos \theta}{(1 + \sec \theta + \tan \theta)(1 - \sin \theta)}$ equal to?
 (a) 1 (b) 2
 (c) $\tan \theta$ (d) $\cot \theta$
53. If $\sin \theta + \cos \theta = \sqrt{3}$, then what is $\tan \theta + \cot \theta$ equal to?
 (a) 1 (b) $\sqrt{2}$ (c) 2 (d) $\sqrt{3}$
54. If $\tan \theta + \sec \theta = m$, then what is $\sec \theta$ equal to?
 (a) $\frac{m^2 - 1}{2m}$ (b) $\frac{m^2 + 1}{2m}$
 (c) $\frac{m + 1}{m}$ (d) $\frac{m^2 + 1}{m}$
55. What is $\operatorname{cosec}(75^\circ + \theta) - \sec(15^\circ - \theta)$ equal to?
 (a) 0 (b) 1
 (c) $2 \sin \theta$ (d) $2 \cos \theta$

56. If $\triangle ABC$ is right angled at C, then what is $\cos(A + B) + \sin(A + B)$ equal to?
 (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2
57. If α , β and γ are acute angles such that $\sin \alpha = \frac{\sqrt{3}}{2}$, $\cos \beta = \frac{\sqrt{3}}{2}$ and $\tan \gamma = 1$, then what is $\alpha + \beta + \gamma$ equal to?
 (a) 105° (b) 120° (c) 135° (d) 150°
58. If $\cos A + \cos^2 A = 1$, then what is the value of $2(\sin^2 A + \sin^4 A)$?
 (a) 4 (b) 2 (c) 1 (d) $\frac{1}{2}$
59. $(1 - \tan A)^2 + (1 + \tan A)^2 + (1 - \cot A)^2 + (1 + \cot A)^2$ is equal to
 (a) $\sin^2 A \cdot \cos^2 A$
 (b) $\sec^2 A \cdot \csc^2 A$
 (c) $2\sec^2 A \cdot \csc^2 A$
 (d) None of these
60. What is the value of $\frac{\tan A - \sin A}{\sin^3 A}$?
 (a) $\frac{\sec A}{1 - \cos A}$ (b) $\frac{\sec A}{1 + \cos^2 A}$
 (c) $\frac{\sec A}{1 + \cos A}$ (d) None of these
61. If $\sin \theta = \frac{x^2 - y^2}{x^2 + y^2}$, then which one of the following is correct?
 (a) $\cos \theta = \frac{2xy}{x^2 - y^2}$
 (b) $\cos \theta = \frac{2xy}{x^2 + y^2}$
 (c) $\cos \theta = \frac{x - y}{x^2 + y^2}$
 (d) $\cos \theta = \frac{xy(x - y)}{x^2 + y^2}$
62. If $a^2 = \frac{1 + 2\sin \theta \cos \theta}{1 - 2\sin \theta \cos \theta}$, then what is the value of $\frac{a+1}{a-1}$?
 (a) $\sec \theta$ (b) 1
 (c) 0 (d) $\tan \theta$
63. If the angle θ is in the first quadrant and $\tan \theta = 3$, then what is the value of $(\sin \theta + \cos \theta)$?
 (a) $\frac{1}{\sqrt{10}}$ (b) $\frac{2}{\sqrt{10}}$
 (c) $\frac{3}{\sqrt{10}}$ (d) $\frac{4}{\sqrt{10}}$
64. If $0^\circ < \theta < 90^\circ$, then all the trigonometric ratios can be obtained when
 (a) Only $\sin \theta$ is given
 (b) Only $\cos \theta$ is given
 (c) Only $\tan \theta$ is given
 (d) any one of the six ratios is given
65. What is the value of $\sin A \cos A \tan A + \cos A \sin A \cot A$?
 (a) $\sin^2 A + \cos A$
 (b) $\sin^2 A + \tan^2 A$
 (c) $\sin^2 A + \cot^2 A$
 (d) $\csc^2 A - \cot^2 A$
66. What is the value of $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta}$?
 (a) $2\csc \theta$ (b) $2\sec \theta$
 (c) $\sec \theta$ (d) $\csc \theta$
67. If $\sin \theta \cos \theta = \sqrt{3}/4$, then $\sin^4 \theta + \cos^4 \theta$ is equal to
 (a) $7/8$ (b) $5/8$
 (c) $3/8$ (d) $1/8$
68. Consider the following
 (i) $\sin^2 1^\circ + \cos^2 1^\circ = 1$
 (ii) $\sec^2 33^\circ - \cot^2 57^\circ = \csc^2 37^\circ - \tan^2 53^\circ$
 Which of the above statement is/are correct?
 (a) Only I (b) Only II
 (c) Both I and II (d) Neither I nor II
69. If $p = a \sin x + b \cos x$ and $q = a \cos x - b \sin x$, then what is the value of $p^2 + q^2$?
 (a) $a + b$ (b) ab
 (c) $a^2 + b^2$ (d) $a^2 - b^2$
70. The expression $\sin^2 x + \cos^2 x - 1 = 0$ is satisfied by how many values of x ?
 (a) Only one value of x
 (b) Two values of x
 (c) Infinite values of x
 (d) No value of x
71. Consider the following statements
 I. There is only one value of x in the first quadrant that satisfies $\sin x + \cos x = 2$
 II. There is only one values of x in the first quadrant that satisfies $\sin x - \cos x = 0$.
 Which of the statements above is/are correct?
 (a) Only I
 (b) Only II
 (c) Both I and II
 (d) Neither I nor II
72. If $7 \sin^2 \theta + 3 \cos^2 \theta = 4$, ($0^\circ \leq \theta \leq 90^\circ$), then value of θ is
 (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$
73. If $\tan(2\theta + 45^\circ) = \cot 3\theta$, where $(2\theta + 45^\circ)$ and 3θ are acute angles, then the value of θ is
 (a) 5° (b) 9° (c) 12° (d) 15°
74. If θ be acute angle and $\cos \theta = \frac{15}{17}$, then the value of $\cot(90^\circ - \theta)$ is
 (a) $\frac{2\sqrt{8}}{15}$ (b) $\frac{8}{15}$
 (c) $\frac{\sqrt{2}}{17}$ (d) $\frac{8\sqrt{2}}{17}$
75. If $\sec^2 \theta + \tan^2 \theta = \frac{7}{12}$, then $\sec^4 \theta - \tan^4 \theta =$
 (a) $\frac{7}{12}$ (b) $\frac{1}{2}$ (c) $\frac{5}{12}$ (d) 1
76. If $0 < x < \frac{\pi}{2}$ and $\sec x = \csc y$ then the value of $\sin(x+y)$ is;
 (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{3}}$
77. If A, B and C be the angles of a triangle, the incorrect relation is;
 (a) $\sin\left(\frac{A+B}{2}\right) = \cos \frac{C}{2}$
 (b) $\cos\left(\frac{A+B}{2}\right) = \sin \frac{C}{2}$
 (c) $\tan\left(\frac{A+B}{2}\right) = \sec \frac{C}{2}$
 (d) $\cot\left(\frac{A+B}{2}\right) = \tan \frac{C}{2}$

78. If θ is a positive acute angle and $\tan 2\theta \cdot \tan 3\theta = 1$, then the value of $\left(2\cos^2 \frac{5\theta}{2} - 1\right)$ is
(a) $-\frac{1}{2}$ (b) 1 (c) 0 (d) $\frac{1}{2}$
79. If $2\sin\left(\frac{\pi x}{2}\right) = x^2 + \frac{1}{x^2}$, then the value of $\left(x - \frac{1}{x}\right)$ is
(a) -1 (b) 2 (c) 1 (d) 0
80. If $\cos \theta + \sec \theta = 2$, the value of $\cos^6 \theta + \sec^6 \theta$ is
(a) 4 (b) 8 (c) 1 (d) 2
81. The numerical value of $\frac{5}{\sec^2 \theta} + \frac{2}{1 + \cot^2 \theta} + 3\sin^2 \theta$ is;
(a) 5 (b) 2 (c) 3 (d) 4
82. The numerical value of $\left(\frac{1}{\cos \theta} + \frac{1}{\cot \theta}\right) \left(\frac{1}{\cos \theta} - \frac{1}{\cot \theta}\right)$ is
(a) 0 (b) -1 (c) +1 (d) 2
83. If $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = \frac{5}{4}$, the value of $\frac{\tan^2 \theta + 1}{\tan^2 \theta - 1}$ is
(a) $\frac{25}{16}$ (b) $\frac{41}{9}$ (c) $\frac{41}{40}$ (d) $\frac{40}{41}$
84. If $\tan 7\theta \tan 2\theta = 1$, then the value of $\tan 3\theta$ is
(a) $\sqrt{3}$ (b) $-\frac{1}{\sqrt{3}}$
(c) $\frac{1}{\sqrt{3}}$ (d) $-\sqrt{3}$
85. The value of $(2\cos^2 \theta - 1) \left(\frac{1 + \tan \theta}{1 - \tan \theta} + \frac{1 - \tan \theta}{1 + \tan \theta}\right)$ is
(a) 4 (b) 1 (c) 3 (d) 2
86. If $\sec \theta + \tan \theta = 2$, then the value of $\sec \theta$ is
(a) $\frac{4}{5}$ (b) 5 (c) $\frac{5}{4}$ (d) $\sqrt{2}$
87. If $\sec \theta = x + \frac{1}{4x}$, ($0^\circ < \theta < 90^\circ$) then $\sec \theta + \tan \theta$ is equal to
(a) $\frac{x}{2}$ (b) $2x$ (c) x (d) $\frac{1}{2x}$
88. The value of $(\sin^2 25^\circ + \sin^2 65^\circ)$ is
(a) $\frac{\sqrt{3}}{2}$ (b) 1 (c) 0 (d) $\frac{2}{\sqrt{3}}$
89. If $\sec \theta + \tan \theta = \sqrt{3}$ ($0^\circ \leq \theta \leq 90^\circ$), then the value of $\tan 3\theta = ?$
(a) undefined (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{3}$
90. If $\sin(60^\circ - \theta) = \cos(\psi - 30^\circ)$, then the value of $\tan(\psi - \theta)$ is (assume that θ and ψ are both positive acute angles with $\theta < 60^\circ$ and $\psi > 30^\circ$).
(a) $\frac{1}{\sqrt{3}}$ (b) 0 (c) $\sqrt{3}$ (d) 1
91. If $a \sin \theta + b \cos \theta = c$ then the value of $a \cos \theta - b \sin \theta$ is;
(a) $\pm \sqrt{-a^2 + b^2 + c^2}$
(b) $\pm \sqrt{a^2 + b^2 - c^2}$
(c) $\pm \sqrt{a^2 - b^2 - c^2}$
(d) $\pm \sqrt{a^2 - b^2 + c^2}$
92. If $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$ where $A > B > 0$ and $A + B$ is an acute angle, then the value B is
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$
93. The value of $152 \left(\sin 30^\circ + 2\cos^2 45^\circ + 3\sin 30^\circ + 4\cos^2 45^\circ + \dots + 17\sin 30^\circ + 18\cos^2 45^\circ \right)$ is
(a) an integer but not perfect square
(b) a rational number but not an integer
(c) a perfect square of an integer
(d) irrational
94. Evaluate : $3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$
(a) 1 (b) 3 (c) 2 (d) 5
95. $\sin^2 \theta - 3\sin \theta + 2 = 0$ will be true if
(a) $0 \leq \theta < 90$ (b) $0 < \theta < 90$
(c) $\theta = 0^\circ$ (d) $\theta = 90^\circ$
96. If $\tan \alpha = n \tan \beta$ and $\sin \alpha = m \sin \beta$, then $\cos^2 \alpha$ is
(a) $\frac{m^2}{n^2 + 1}$ (b) $\frac{m^2}{n^2}$
(c) $\frac{m^2 - 1}{n^2 - 1}$ (d) $\frac{m^2 + 1}{n^2 + 1}$
97. If $\operatorname{cosec} \theta - \cot \theta = \frac{7}{2}$, then value of $\operatorname{cosec} \theta$ is;
(a) $\frac{47}{28}$ (b) $\frac{51}{28}$ (c) $\frac{53}{28}$ (d) $\frac{49}{28}$
98. If $x \sin 45^\circ = y \operatorname{cosec} 30^\circ$, then $\frac{x^4}{y^4}$ is equal to
(a) 4^3 (b) 6^3 (c) 2^3 (d) 8^3
99. If $5 \tan \theta = 4$, then $\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 2 \cos \theta}$ is equal to
(a) $\frac{2}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$
100. $2 \operatorname{cosec}^2 23^\circ \cot^2 67^\circ - \sin^2 23^\circ - \sin^2 67^\circ - \cot^2 67^\circ$ is equal to
(a) 1 (b) $\sec^2 23^\circ$
(c) $\tan^2 23^\circ$ (d) 0
101. The equation $\cos^2 \theta = \frac{(x+y)^2}{4xy}$ is only possible when
(a) $x = -y$ (b) $x > y$
(c) $x = y$ (d) $x < y$
102. If $\alpha + \beta = 90^\circ$, then the value of $(1 - \sin^2 \alpha)(1 - \cos^2 \alpha) \times (1 + \cot^2 \beta)(1 + \tan^2 \beta)$ is
(a) 1 (b) -1 (c) 0 (d) 2

103. $\frac{2\sin 68^\circ}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\tan 75^\circ} - \frac{3\tan 45^\circ \cdot \tan 20^\circ \cdot \tan 40^\circ \cdot \tan 50^\circ \cdot \tan 70^\circ}{5}$ is equal to
 (a) -1 (b) 0 (c) 1 (d) 2
104. If $\sin 7x = \cos 11x$, then the value of $\tan 9x + \cot 9x$ is
 (a) 1 (b) 2 (c) 3 (d) 4
105. If $\tan^2 \alpha = 1 + 2\tan^2 \beta$ (here, α, β are positive acute angles), then $\sqrt{2} \cos \alpha - \cos \beta$ is equal to
 (a) 0 (b) $\sqrt{2}$ (c) 1 (d) -1
106. If $\tan \theta + \cot \theta = 2$, then the value of $\tan^{100} \theta + \cot^{100} \theta$ is
 (a) 2 (b) 0 (c) 1 (d) $\sqrt{3}$
107. $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta}$ is equal to
 (a) $1 - \tan \theta - \cot \theta$
 (b) $1 + \tan \theta - \cot \theta$
 (c) $1 - \tan \theta + \cot \theta$
 (d) $1 + \tan \theta + \cot \theta$
108. If $\sec \theta + \tan \theta = 2 + \sqrt{5}$, then the value of $\sin \theta + \cos \theta$ is;
 (a) $\frac{3}{\sqrt{5}}$ (b) $\sqrt{5}$ (c) $\frac{7}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{5}}$
109. The value of $(1 + \cot \theta - \operatorname{cosec} \theta)(1 + \tan \theta + \sec \theta)$ is equal to
 (a) 1 (b) 2 (c) 0 (d) -1
110. If $x = a \sec \theta \cos \phi$, $y = b \sec \theta \sin \phi$, $z = c \tan \theta$, then the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$ is;
 (a) 1 (b) 4 (c) 9 (d) 0
111. If $\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} = \frac{5}{3}$, then $\sin \theta$ is equal to;
 (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$
112. If $(1 + \sin \alpha)(1 + \sin \beta)(1 + \sin \gamma) = (1 - \sin \alpha)(1 - \sin \beta)(1 - \sin \gamma)$, then third side is equal to
 (a) $\pm \cos \alpha \cos \beta \cos \gamma$
 (b) $\pm \sin \alpha \sin \beta \sin \gamma$
 (c) $\pm \sin \alpha \cos \beta \sec \gamma$
 (d) $\pm \sin \alpha \sin \beta \cos \gamma$
113. The numerical value of $\frac{1}{1 + \cot^2 \theta} + \frac{3}{1 + \tan^2 \theta} + 2\sin^2 \theta$ will be
 (a) 2 (b) 5 (c) 6 (d) 3
114. The value of $\frac{4}{1 + \tan^2 \alpha} + \frac{1}{1 + \cot^2 \alpha} + 3\sin^2 \alpha$ is
 (a) 4 (b) -1 (c) 2 (d) 3
115. The value of $3(\sin x + \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$ is
 (a) 14 (b) 11 (c) 12 (d) 13
116. The value of $\sec \theta \left(\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} \right) - 2\tan^2 \theta$ is
 (a) 4 (b) 1 (c) 2 (d) 0
117. The elimination of θ from $x \cos \theta - y \sin \theta = 2$ and $x \sin \theta + y \cos \theta = 4$ will give
 (a) $x^2 + y^2 = 20$ (b) $3x^2 + y^2 = 20$
 (c) $x^2 - y^2 = 20$ (d) $3x^2 - y^2 = 10$
118. The value of $\left[\frac{\cos^2 A (\sin A + \cos A)}{\operatorname{cosec}^2 A (\sin A - \cos A)} + \frac{\sin^2 A (\sin A - \cos A)}{\sec^2 A (\sin A + \cos A)} \right] \times (\sec^2 A - \operatorname{cosec}^2 A)$
 (a) 1 (b) 3 (c) 2 (d) 4
119. The value of $\frac{1}{\operatorname{cosec} \theta - \cot \theta} - \frac{1}{\sin \theta}$ is
 (a) 1 (b) $\cot \theta$
 (c) $\operatorname{cosec} \theta$ (d) $\tan \theta$
120. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, then $\cos \theta - \sin \theta$ is
 (a) $\sqrt{2} \tan \theta$ (b) $-\sqrt{2} \cos \theta$
 (c) $-\sqrt{2} \sin \theta$ (d) $\sqrt{2} \sin \theta$
121. If $\sin \theta - \cos \theta = \frac{1}{2}$ then value of $\sin \theta + \cos \theta$ is;
 (a) -2 (b) ± 2
 (c) $\frac{\sqrt{7}}{2}$ (d) 2
122. The value of $\frac{\sin A}{1 + \cos A} + \frac{\sin A}{1 - \cos A}$ is ($0^\circ < A < 90^\circ$)
 (a) $2 \operatorname{cosec} A$ (b) $2 \sec A$
 (c) $2 \sin A$ (d) $2 \cos A$
123. If $\tan \theta - \cot \theta = 0$, find the value of $\sin \theta + \cos \theta$,
 (a) 0 (b) 1 (c) $\sqrt{2}$ (d) 2
124. If $3\sin \theta + 5\cos \theta = 5$, then $5\sin \theta - 3\cos \theta$ is equal to
 (a) ± 3 (b) ± 5 (c) 1 (d) 0
125. If $x \sin 60^\circ \cdot \tan 30^\circ = \sec 60^\circ \cdot \cot 45^\circ$, then the value of x is
 (a) 2 (b) $2\sqrt{3}$
 (c) 4 (d) $4\sqrt{3}$
126. If $\theta = 60^\circ$, then $\frac{1}{2}\sqrt{1 + \sin \theta} + \frac{1}{2}\sqrt{1 - \sin \theta}$ is equal to
 (a) $\cot \frac{\theta}{2}$ (b) $\sec \frac{\theta}{2}$
 (c) $\sin \frac{\theta}{2}$ (d) $\cos \frac{\theta}{2}$
127. If $\frac{2\tan^2 30^\circ}{1 - \tan^2 30^\circ} + \sec^2 45^\circ - \sec^2 0^\circ = x \sec 60^\circ$, then the value of x is
 (a) 2 (b) 1 (c) 0 (d) -1
128. If $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$, then $\sin \alpha + \cos \alpha$ is
 (a) $\pm \sqrt{2} \sin \theta$ (b) $\pm \sqrt{2} \cos \theta$
 (c) $\pm \frac{1}{\sqrt{2}} \sin \theta$ (d) $\pm \frac{1}{\sqrt{2}} \cos \theta$
129. If $7\sin^2 \theta + 3\cos^2 \theta = 4$, ($0^\circ < \theta < 90^\circ$), then the value of $\tan \theta$ is
 (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{3}$

130. If $\tan 9^\circ = \frac{p}{q}$, then the value of

$$\frac{\sec^2 81^\circ}{1 + \cot^2 81^\circ} \text{ is}$$

- (a) $\frac{q}{p}$ (b) 1 (c) $\frac{p^2}{q^2}$ (d) $\frac{q^2}{p^2}$

131. If $\sec \theta + \tan \theta = 5$ then the value of $\frac{\tan \theta + 1}{\tan \theta - 1}$ is

- (a) $\frac{11}{7}$ (d) $\frac{13}{7}$ (c) $\frac{15}{7}$ (d) $\frac{17}{7}$

132. If $\frac{\cos \alpha}{\cos \beta} = a$ and $\frac{\sin \alpha}{\sin \beta} = b$, then

the value of $\sin^2 \beta$ in terms of a and b is

- (a) $\frac{a^2 + 1}{a^2 - b^2}$ (b) $\frac{a^2 - b^2}{a^2 + b^2}$
(c) $\frac{a^2 - 1}{a^2 - b^2}$ (d) $\frac{a^2 - 1}{a^2 + b^2}$

133. The value of

$$\frac{\cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ} \text{ is}$$

- (a) $\frac{64}{\sqrt{3}}$ (b) $\frac{55}{12}$ (c) $\frac{67}{12}$ (d) $\frac{67}{10}$

134. If $\cos \pi x = x^2 - x + \frac{5}{4}$, the value of x will be

- (a) 0 (b) 1 (d) -1
(d) None of the above

135. The numerical value of

$$1 + \frac{1}{\cot^2 63^\circ} - \sec^2 27^\circ + \frac{1}{\sin^2 63^\circ} - \operatorname{cosec}^2 27^\circ \text{ is}$$

- (a) 1 (b) 2 (c) -1 (d) 0

136. If $x = \frac{\cos \theta}{1 - \sin \theta}$, then $\frac{\cos \theta}{1 + \sin \theta}$ is equal to

- (a) $x - 1$ (b) $\frac{1}{x}$
(b) $\frac{1}{x+1}$ (d) $\frac{1}{1-x}$

137. In $\triangle ABC$, $\angle B = 90^\circ$ and $AB : BC = 2 : 1$, Then value of $(\sin A + \cot C)$

- (a) $3 + \sqrt{5}$ (b) $\frac{2 + \sqrt{5}}{2\sqrt{5}}$
(c) $2 + \sqrt{5}$ (d) $3\sqrt{5}$

138. If $\sin \frac{\pi x}{2} = x^2 - 2x + 2$, then the value of x is

- (a) 0 (b) 1
(c) -1 (d) None of these

139. The value of

$$\frac{\sin 43^\circ}{\cos 47^\circ} + \frac{\cos 19^\circ}{\sin 71^\circ} - 8 \cos^2 60^\circ$$

- (a) 0 (b) 1 (c) 2 (d) -1

140. The value of

$$\left(\sin^2 7 \frac{1^\circ}{2} + \sin^2 82 \frac{1^\circ}{2} \right) \text{ is}$$

- (a) 1 (b) 2 (c) 0 (d) 4

141. If $3 \sin x + 5 \cos x = 5$, then what is the value of $(3 \cos x - 5 \sin x)$?

- (a) 0 (b) 2 (c) 3 (d) 5

142. If α and β are complementary angles, then what is

$$\sqrt{(\operatorname{cosec} \alpha \cdot \operatorname{cosec} \beta)} \left(\frac{\sin \alpha}{\sin \beta} + \frac{\cos \alpha}{\cos \beta} \right)^{\frac{1}{2}}$$

equal to?

- (a) 0 (b) 1
(c) 2 (d) None of these

143. If A, B, C and D are the successive angles of a cyclic quadrilateral, then what is

$\cos A + \cos B + \cos C + \cos D$ are equal to?

- (a) 4 (b) 2 (c) 1 (d) 0

144. How many degrees are there in an angle which equals two-third of its complement?

- (a) 36° (b) 45° (c) 48° (d) 60°

145. If $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 2$ with $0 < \theta$

$< 90^\circ$, then what is θ equal to?

- (a) 30° (b) 45° (c) 60° (d) 75°

146. If $\sin 3\theta = \cos(\theta - 2^\circ)$, where 3θ and $(\theta - 2^\circ)$ are acute angles, what is the value of θ ?

- (a) 22° (b) 23° (c) 24° (d) 25°

147. What is $\frac{\sin^6 \theta - \cos^6 \theta}{\sin^2 \theta - \cos^2 \theta}$ equal to?

- (a) $\sin^4 \theta - \cos^4 \theta$
(b) $1 - \sin^2 \theta \cos^2 \theta$
(c) $1 + \sin^2 \theta \cos^2 \theta$
(d) $1 - 3 \sin^2 \theta \cos^2 \theta$

148. If $\tan A = \frac{1 - \cos B}{\sin B}$, then what is

$$\frac{2 \tan A}{1 - \tan^2 A} \text{ equal to?}$$

- (a) $\frac{\tan B}{2}$ (b) $2 \tan B$
(c) $\tan B$ (d) $4 \tan B$

149. Assume the Earth to be a sphere of radius R. What is the radius of the circle of latitude $40^\circ S$?

- (a) $R \cos 40^\circ$ (b) $R \sin 80^\circ$
(c) $R \sin 40^\circ$ (d) $R \tan 40^\circ$

150. If $\cos \theta \geq \frac{1}{2}$ in the first quadrant, then which one of the following is correct?

- (a) $\theta \leq \frac{\pi}{3}$ (b) $\theta \geq \frac{\pi}{3}$
(c) $\theta \leq \frac{\pi}{6}$ (d) $\theta \geq \frac{\pi}{6}$

151. If $\sin \theta + \cos \theta = 1$, then what is the value of $\sin \theta \cdot \cos \theta$?

- (a) 2 (b) 0
(c) 1 (d) $\frac{1}{2}$

152. What is $\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}}$ equal to?

- (a) $\sec \theta - \tan \theta$
(b) $\sec \theta + \tan \theta$
(c) $\operatorname{cosec} \theta + \cot \theta$
(d) $\operatorname{cosec} \theta - \cot \theta$

153. Two sides of an acute angle triangle are 6 cm and 2 cm, respectively. Which one of the following represents the correct range of the third side in cm?

- (a) (4, 8) (b) $(4, 2\sqrt{10})$
(c) $(4\sqrt{2}, 8)$ (d) $(4\sqrt{2}, 2\sqrt{10})$

154. If $\cos 1^\circ = p$ and $\cos 89^\circ = q$, then which one of the following is correct?

- (a) p is close to 0 and q is close to 1

- (b) $p < q$
 (c) $p = q$
 (d) p is close to 1 and q is close to 0
155. If $7 \cos^2 \theta + 3 \sin^2 \theta = 4$ and $0 < \theta < \frac{\pi}{2}$, then what is the value of $\tan \theta$?
 (a) $\sqrt{7}$ (b) $\frac{7}{3}$ (c) 3 (d) $\sqrt{3}$
156. What is the value of $[(1 - \sin^2 \theta) \sec^2 \theta + \tan^2 \theta] (\cos^2 \theta + 1)$ when $0^\circ < \theta < 90^\circ$?
 (a) 2 (b) > 2 (c) ≥ 2 (d) < 2
157. If $0 \leq \theta < \frac{\pi}{2}$ and $p = \sec^2 \theta$, then which one of the following is correct?
 (a) $p < 1$ (b) $p = 1$
 (c) $p > 1$ (d) $p \geq 1$
158. In a $\triangle ABC$, $\angle ABC = 90^\circ$, $\angle ACB = 30^\circ$, $AB = 5$ cm. What is the length of AC ?
 (a) 10 cm (b) 5 cm
 (c) $5\sqrt{2}$ cm (d) $5\sqrt{3}$ cm
159. If $0 \leq \theta \leq \frac{\pi}{2}$ and $\cos \theta + \sqrt{3} \sin \theta = 2$, then what is the value of θ ?
 (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$
160. If ABC is a right angled triangle at C and having u units, v units and w units as the length of its sides opposite to be vertices A, B and C respectively, then what is $\tan A + \tan B$ equal to?
 (a) $\frac{u^2}{vw}$ (b) 1
 (c) $u + v$ (d) $\frac{w^2}{uv}$
161. ABC is a right triangle with right angle at A . If the value of $\tan B = \frac{1}{\sqrt{3}}$, then for any real k the length of the hypotenuse is of the form.
 (a) $3k$ (b) $2k$ (c) $5k$ (d) $9k$
162. If α is an acute angle and $\sin \alpha = \sqrt{\frac{x-1}{2x}}$, then what is $\tan \alpha$ equal to?
 (a) $\sqrt{\frac{x-1}{x+1}}$ (b) $\sqrt{\frac{x+1}{x-1}}$
 (c) $\sqrt{x^2-1}$ (d) $\sqrt{x^2+1}$
163. $\frac{\cos \theta}{1 - \sin \theta} - \frac{\cos \theta}{1 + \sin \theta} = 2$ is satisfied by which one of the following values of θ ?
 (a) $\pi/2$ (b) $\pi/3$
 (c) $\pi/4$ (d) $\pi/6$
164. If $0^\circ < x < 45^\circ$ and $45^\circ < y < 90^\circ$, then which one of the following is correct?
 (a) $\sin x = \sin y$
 (b) $\sin x < \sin y$
 (c) $\sin x > \sin y$
 (d) $\sin x \leq \sin y$
165. What is the value of $\sin^3 60^\circ \cot 30^\circ - 2 \sec^2 45^\circ + 3 \cos 60^\circ \tan^2 45^\circ - \tan^2 60^\circ$?
 (a) $35/8$ (b) $-35/8$
 (c) $-11/8$ (d) $11/8$
166. If $\tan \theta = \frac{p}{q}$, then what is $\frac{p \sec \theta - q \operatorname{cosec} \theta}{p \sec \theta + q \operatorname{cosec} \theta}$ equal to?
 (a) $\frac{p-q}{p+q}$ (b) $\frac{q^2-p^2}{q^2+p^2}$
 (c) $\frac{p^2-q^2}{q^2+p^2}$ (d) 1
167. The value of $\operatorname{cosec}^2 \theta - 2 + \sin^2 \theta$ is always
 (a) less than zero
 (b) non-negative
 (c) zero
 (d) 1
168. Find the value of $1 - 2 \sin^2 \theta + \sin^4 \theta$,
 (a) $\sin^4 \theta$ (b) $\cos^4 \theta$
 (c) $\operatorname{cosec}^4 \theta$ (d) $\sec^4 \theta$
169. $\sin \theta = 0.7$, then $\cos \theta$, $0^\circ \leq \theta < 90^\circ$ is
 (a) 0.3 (b) $\sqrt{0.49}$
 (c) $\sqrt{0.51}$ (d) $\sqrt{0.9}$
170. The value of $\sin^2 65^\circ + \sin^2 25^\circ + \cos^2 35^\circ + \cos^2 55^\circ$ is
 (a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$
171. If $\cos^2 \theta - \sin^2 \theta = \frac{1}{3}$, where $0 \leq \theta < \frac{\pi}{2}$, then the value of $\cos^4 \theta - \sin^4 \theta$ is
 (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$
172. If $\tan \theta = \frac{1}{\sqrt{11}}$ and $0^\circ < \theta < \frac{\pi}{2}$, then the value of $\frac{\cos^2 \theta - \sec^2 \theta}{\cos^2 \theta + \sec^2 \theta}$ is
 (a) $\frac{3}{4}$ (b) $\frac{4}{5}$ (c) $\frac{5}{6}$ (d) $\frac{6}{7}$
173. If $\sin \theta = \frac{3}{5}$, then the value of $\frac{\tan \theta + \cos \theta}{\cot \theta + \operatorname{cosec} \theta}$ is equal to
 (a) $\frac{29}{60}$ (b) $\frac{31}{60}$ (c) $\frac{34}{60}$ (d) $\frac{37}{60}$
174. If $(\sin \alpha + \operatorname{cosec} \alpha)^2 + (\cos \alpha + \sec \alpha)^2 = k + \tan^2 \alpha + \cot^2 \alpha$, then the value of k is
 (a) 1 (b) 7 (c) 3 (d) 5
175. If $\sin 21^\circ = \frac{x}{y}$, then $\sec 21^\circ - \sin 69^\circ$ is equal to
 (a) $\frac{x^2}{y\sqrt{y^2-x^2}}$ (b) $\frac{y^2}{x\sqrt{y^2-x^2}}$
 (c) $\frac{x^2}{y\sqrt{x^2-y^2}}$ (d) $\frac{y^2}{x\sqrt{x^2-y^2}}$
176. If $\sec \alpha + \tan \alpha = 2$, then the value of $\sin \alpha$ is (assume that $0^\circ < \alpha < 90^\circ$)
 (a) 0.4 (b) 0.5
 (c) 0.6 (d) 0.8
177. If $7 \sin \alpha = 24 \cos \alpha$; $0 < \alpha < \frac{\pi}{2}$, then the value of $14 \tan \alpha - 75 \cos \alpha - 7 \sec \alpha$ is equal to
 (a) 3 (b) 4 (c) 1 (d) 2

178. ABCD is a rectangle of which AC is a diagonal. The value of $(\tan^2 \angle CAD + 1) \sin^2 \angle BAC$ is

- (a) 2 (b) $\frac{1}{4}$ (c) 1 (d) 0

179. For any real value of θ , $\sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}}$ = ?

- (a) $\cot \theta - \operatorname{cosec} \theta$
(b) $\sec \theta - \tan \theta$
(c) $\operatorname{cosec} \theta - \cot \theta$
(d) $\tan \theta - \sec \theta$

180. In a $\triangle ABC$, $\angle B = \frac{\pi}{3}$, $\angle C = \frac{\pi}{4}$ and D divides BC internally in the ratio 1 : 3 then, $\frac{\sin \angle BAD}{\sin \angle CAD}$ is equal to

- (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{6}}$ (d) $\sqrt{6}$

181. If $\sin 3A = \cos(A - 26^\circ)$, where $3A$ is an acute angle then the value of A is

- (a) 29° (b) 26° (c) 23° (d) 28°

182. The value of $\sec^2 \theta -$

$$\frac{\sin^2 \theta - 2 \sin^4 \theta}{2 \cos^4 \theta - \cos^2 \theta}$$
 is

- (a) 1 (b) 2 (c) -1 (d) 0

183. If $x = a(\sin \theta + \cos \theta)$, $y = b(\sin \theta - \cos \theta)$ then the value of

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}$$
 is

- (a) 0 (b) 1 (c) 2 (d) -2

184. If $\sin 5\theta = \cos 20^\circ$ ($0^\circ < \theta < 90^\circ$) then the value of θ is

- (a) 4° (b) 22° (c) 10° (d) 14°

185. If $0^\circ < \theta < 90^\circ$ and $2 \sec \theta = 3 \operatorname{cosec}^2 \theta$, then θ is

- (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{5}$

186. $\sqrt{\frac{1+\sin \theta}{1-\sin \theta}} + \sqrt{\frac{1-\sin \theta}{1+\sin \theta}}$ is equal to

- (a) $2 \cos \theta$ (b) $2 \sin \theta$
(c) $2 \cot \theta$ (d) $2 \sec \theta$

187. If α and β are positive acute angles, $\sin(4\alpha - \beta) = 1$ and

$\cos(2\alpha + \beta) = \frac{1}{2}$, then the value of $\sin(\alpha + 2\beta)$ is

- (a) 0 (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$

188. If θ is a positive acute angle and $\operatorname{cosec} \theta = \sqrt{3}$, then the value of $\cot \theta - \operatorname{cosec} \theta$ is

- (a) $\frac{3\sqrt{2} - \sqrt{3}}{3}$ (b) $\frac{\sqrt{2}(3 + \sqrt{3})}{3}$

- (c) $(\sqrt{2} - \sqrt{3})$ (d) $\frac{3\sqrt{2} + \sqrt{3}}{3}$

189. If $(r \cos \theta - \sqrt{3})^2 + (r \sin \theta - 1)^2 = 0$, then the value of

$$\frac{r \tan \theta + \sec \theta}{r \sec \theta + \tan \theta}$$
 is equal to

- (a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{\sqrt{5}}{4}$

190. If $\frac{\cos \alpha}{\cos \beta} = a$, $\frac{\sin \alpha}{\sin \beta} = b$, then $\sin^2 \beta$ is equal to

- (a) $\frac{a^2 - 1}{a^2 + b^2}$ (b) $\frac{a^2 + 1}{a^2 - b^2}$

- (c) $\frac{a^2 - 1}{a^2 - b^2}$ (d) $\frac{a^2 + 1}{a^2 + b^2}$

191. If $\sqrt{3} \tan \theta = 3 \sin \theta$, then the value of $(\sin^2 \theta - \cos^2 \theta)$ is

- (a) 1 (b) 3

- (c) $\frac{1}{3}$ (d) None of these

192. ABC is a right angle triangle and right angle at B and $\angle A = 60^\circ$ and AB = 20cm, then the ratio of sides BC and CA is

- (a) $\sqrt{3} : 1$ (b) $1 : \sqrt{3}$
(c) $\sqrt{3} : \sqrt{2}$ (d) $\sqrt{3} : 2$

193. If $\tan(A + B) = \sqrt{3}$ and $\tan$

$$(A - B) = \frac{1}{\sqrt{3}}, (\angle A + \angle B) < 90^\circ,$$

$A \geq B$, then $\angle A$ is

- (a) 90° (b) 30° (c) 45° (d) 60°

194. The value of $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$ is equal to

- (a) $\sin \theta$ (b) $\cos \theta$
(c) $\tan \theta$ (d) $\cot \theta$

195. If $\cot \theta = \frac{2xy}{x^2 - y^2}$, then what is $\cos \theta$ equal to?

- (a) $\frac{x^2 - y^2}{x^2 + y^2}$ (b) $\frac{x^2 + y^2}{x^2 - y^2}$

- (c) $\frac{2xy}{x^2 + y^2}$ (d) $\frac{2xy}{\sqrt{x^2 + y^2}}$

196. For what value of θ is $(\sin \theta + \operatorname{cosec} \theta) = 2.5$, where $0^\circ < \theta < 90^\circ$?

- (a) 30° (b) 45° (c) 60° (d) 90°

197. If $x \cos 60^\circ + y \cos 0^\circ = 3$ and $4x \sin 30^\circ - y \cot 45^\circ = 2$, then what is the value of x ?

- (a) -1 (b) 0 (c) 1 (d) 2

198. What is $\log(\tan 1^\circ) + \log(\tan 2^\circ) + \log(\tan 3^\circ) + \dots + \log(\tan 89^\circ)$ equal to?

- (a) 0 (b) 1 (c) 2 (d) -1

199. If $\sin x \cos x = 1/2$, then what is the value of $\sin x - \cos x$?

- (a) 2 (b) 1 (c) 0 (d) -1

200. If $\tan^2 y \operatorname{cosec}^2 x - 1 = \tan^2 y$, then which one of the following is correct?

- (a) $x - y = 0$ (b) $x = 2y$
(c) $y = 2x$ (d) $x - y = 1$

201. If $\frac{\cos x}{1 + \operatorname{cosec} x} + \frac{\cos x}{\operatorname{cosec} x - 1} = 2$, then which one of the following is one of the values of x ?

- (a) $\pi/2$ (b) $\pi/3$
(c) $\pi/4$ (d) $\pi/6$

202. If $x + y = 90^\circ$ and $\sin x : \sin y = \sqrt{3} : 1$, then what is $x : y$ equal to?

- (a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 3 : 2

203. If $\frac{\cos x}{\cos y} = n$ and $\frac{\sin x}{\sin y} = m$, then $(m^2 - n^2) \sin^2 y$ is equal to

- (a) $1 - n^2$ (b) $1 + n^2$
(c) m^2 (d) n^2

204. If $p = \tan^2 x + \cot^2 x$, then which one of the following is correct?

- (a) $p \leq 2$ (b) $p \geq 2$
(c) $p < 2$ (d) $p > 2$

205. What is the value of

$$\frac{5 \sin 75^\circ \sin 77^\circ + 2 \cos 13^\circ \cos 15^\circ}{\cos 15^\circ \sin 77^\circ}$$

$$- \frac{7 \sin 81^\circ}{\cos 9^\circ}$$

- (a) -1 (b) 0 (c) 1 (d) 2

206. If $\sin x + \sin y = a$ and $\cos x + \cos y = b$, what is $\sin x \cdot \sin y + \cos x \cdot \cos y$ equal to?

- (a) $a + b - ab$ (b) $a + b + ab$

- (c) $a^2 + b^2 - 2$ (d) $\left(\frac{a^2 + b^2 - 2}{2}\right)$

207. If α is the angle of first quadrant such that $\operatorname{cosec}^4 \alpha = 17 + \cot^4 \alpha$, then what is the value of $\sin \alpha$?

- (a) $1/3$ (b) $1/4$
(c) $1/9$ (d) $1/16$

208. If $x + \left(\frac{1}{x}\right) = 2 \cos \alpha$, then what

is the value $x^2 + \left(\frac{1}{x^2}\right)$?

- (a) $4 \cos^2 \alpha$
(b) $4 \cos^2 \alpha - 1$
(c) $2 \cos^2 \alpha - 2 \sin^2 \alpha$
(d) $\cos^2 \alpha - \sin^2 \alpha$

209. What is the value

$$\cot^2 \theta - \frac{1}{\sin^2 \theta} ?$$

- (a) $1/2$ (b) -1
(c) $-1/2$ (d) $3/2$

210. If $\sin x = \cos y$ and angle x and angle y are acute then what is the relation between x and y ?

- (a) $x - y = \pi/2$
(b) $x + y = 3\pi/2$
(c) $x + y = \pi/2$
(d) $x + y = \pi/4$

211. If $\sin \theta = \frac{m^2 - n^2}{m^2 + n^2}$, then what is the value of $\tan \theta$?

- (a) $\frac{m^2 + n^2}{m^2 - n^2}$ (b) $\frac{2mn}{m^2 + n^2}$
(c) $\frac{m^2 - n^2}{2mn}$ (d) $\frac{m^2 + n^2}{2mn}$

212. If $\sin(x - y) = 1/2$ and $\cos(x + y) = 1/2$, then what is the value of x ?

- (a) 15° (b) 30° (c) 45° (d) 60°

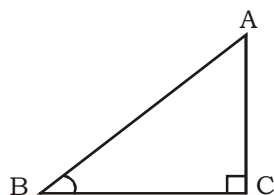
213. If $1 + \tan \theta = \sqrt{2}$, then what is the value of $\cot \theta - 1$?

- (a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 0 (d) 1

214. If $\sin(x + 54^\circ) = \cos x$, where $0 < x, x + 54^\circ < 90^\circ$, then what is the value of x ?

- (a) 54° (b) 36° (c) 27° (d) 18°

215. In the given figure, $BC = 15$ cm and $\sin B = 4/5$. What is the value of AB ?



- (a) 25 cm (b) 20 cm
(c) 5 cm (d) 4 cm

216. The smallest side of a right angled triangle has length 2 cm. The tangent of one acute angle

is $\frac{3}{4}$. What is the hypotenuse of the triangle?

- (a) 5 cm (b) 2.5 cm
(c) 1.25 cm (d) $\frac{10}{3}$ cm

217. If $\sin x - \cos x = 0$, then what is the value of $\sin^4 x + \cos^4 x$?

- (a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

218. What is the expression $\frac{\tan x}{1 + \sec x} -$

$\frac{\tan x}{1 - \sec x}$ equal to?

- (a) $\operatorname{cosec} x$ (b) $2 \operatorname{cosec} x$
(c) $2 \sin x$ (d) $2 \cos x$

219. What is the expression $(\sin^4 x - \cos^4 x + 1) \operatorname{cosec}^2 x$ equal to?

- (a) 1 (b) 2 (c) 0 (d) -1

220. If $x + y = 90^\circ$, then what is

$\sqrt{\cos x \cdot \operatorname{cosec} y} - \cos x \cdot \sin y$ equal to?

- (a) $\cos x$ (b) $\sin x$
(c) $\sqrt{\cos x}$ (d) $\sqrt{\sin x}$

221. If $p = \sin^{10} x$, then which one of the following correct for any value x ?

- (a) $p \geq 1$ (b) $0 \leq p \leq 1$
(c) $1 \leq p \leq 2$ (d) None of these

222. What is the value of the expression $\cos^2 \frac{\pi}{8} + 4 \cos^2 \frac{\pi}{4} - \sec \frac{\pi}{3} + 5$

$\tan^2 \frac{\pi}{3} + \sin^2 \frac{\pi}{8}$?

- (a) 8 (b) 10 (c) 16 (d) 18

223. If $\operatorname{cosec} \theta = p/q$ and θ is acute, then what is the value of

$$\left(\sqrt{p^2 - q^2}\right) \tan \theta ?$$

- (a) p (b) q
(c) pq (d) $\sqrt{p^2 + q^2}$

224. If $2x^2 \cos 60^\circ - 4 \cot^2 45^\circ - 2 \tan 60^\circ = 0$, then what is the value of x ?

- (a) 2 (b) 3
(c) $\sqrt{3} - 1$ (d) $\sqrt{3} + 1$

225. Which one of the following statements is true in respect of the expression $\sin 31^\circ + \sin 32^\circ$?

- (a) Its value is 0
(b) Its value is 1
(c) Its value is less than 1
(d) Its value is greater than 1

226. Which one of the following is correct?

- (a) $\sin 35^\circ > \cos 55^\circ$

- (b) $\cos 61^\circ > \frac{1}{2}$

- (c) $\sin 32^\circ > \frac{1}{2}$

- (d) $\tan 44^\circ > 1$

227. If $\sin \theta + \operatorname{cosec} \theta = 2$, then what is the value of $\sin^4 \theta + \cos^4 \theta$?

- (a) 2 (b) 2^2 (c) 2^3 (d) 1

228. If $\theta \in \mathbb{R}$ be such that $\sec \theta > 0$ and $2 \sec^2 \theta + \sec \theta - 6 = 0$

Then, what is the value of $\operatorname{cosec} \theta$?

- (a) $\sqrt{5}$ (b) $\sqrt{3}/2$

- (c) $3/\sqrt{5}$ (d) $2/\sqrt{3}$

229. Under which one of the following conditions is the trigonometrical identity.

$\sin x / (1 + \cos x) = (1 - \cos x) / \sin x$ true?

- (a) x is not a multiple of 360°
(b) x is not an odd multiple of 180°
(c) x is not a multiple of 180°
(d) None of the above

230. If $3 \sin \theta + 4 \cos \theta = 5$, then what is $3 \cos \theta - 4 \sin \theta$ equal to?

- (a) 0 (b) 3 (c) 4 (d) 5

231. If $\sec \theta = 13/5$, then what is value

$$\text{of } \frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta} ?$$

- (a) 1 (b) 2 (c) 3 (d) 4

232. If $r \sin \theta = \frac{7}{2}$ and $r \cos \theta = \frac{7\sqrt{3}}{2}$ then value of r is
(a) 4 (b) 3 (c) 5 (d) 7
233. If $\theta + \phi = \frac{\pi}{2}$ and $\sin \theta = \frac{1}{2}$, then the value of $\sin \phi$ is
(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$
234. If $\sec \theta + \tan \theta = P$, ($P \neq 0$) the $\sec \theta$ is equal to:
(a) $\frac{1}{3} \left(P + \frac{1}{P} \right)$, $P \neq 0$
(b) $\frac{1}{2} \left(P + \frac{1}{P} \right)$, $P \neq 0$
(c) $2 \left(P + \frac{1}{P} \right)$, $P \neq 0$
(d) $\left(P - \frac{1}{P} \right)$, $P \neq 0$
235. The value of $\sin^2 22^\circ + \sin^2 68^\circ + \cot^2 30^\circ$ is:
(a) $5/4$ (b) $3/4$
(c) 3 (d) 4
236. If $\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} = 2 \frac{51}{79}$ then the value of $\sin \theta$ is
(a) $\frac{91}{144}$ (b) $\frac{39}{72}$ (c) $\frac{65}{144}$ (d) $\frac{35}{72}$
237. If $1 + \cos^2 \theta = 3 \sin \theta \cos \theta$, then the integral value of $\cot \theta$ is
 $\left(0 < \theta < \frac{\pi}{2} \right)$
(a) 2 (b) 1 (c) 3 (d) 0
238. The value of following is $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ$
(a) $-1/2$ (b) $1/2$
(c) 2 (d) 1
239. If $\sin A + \operatorname{cosec} A = 3$, then find the value of $\frac{\sin^4 A + 1}{\sin^2 A}$.
(a) 1 (b) 0 (c) 7 (d) 0
240. $\cos 7^\circ \cos 23^\circ \cos 45^\circ \operatorname{cosec} 83^\circ \operatorname{cosec} 67^\circ = ?$
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$
241. If $\frac{\tan \theta + \cot \theta}{\tan \theta - \cot \theta} = 2$, ($0^\circ \leq \theta \leq 90^\circ$), then the value of $\sin \theta$ is
(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) 1
242. If $\sin x + \cos x = c$, then $\sin^6 x + \cos^6 x$ is equal to.
(a) $\frac{1+6c^2-3c^4}{16}$
(b) $\frac{1+6c^2-3c^4}{4}$
(c) $\frac{1+6c^2+3c^4}{16}$
(d) $\frac{1+6c^2+3c^4}{4}$
243. $\frac{1 - \sin A \cos A}{\cos A (\sec A - \operatorname{cosec} A)} \cdot \frac{\sin^2 A - \cos^2 A}{\sin^3 A + \cos^3 A} = ?$
(a) $\sin A$ (b) $\cos A$
(c) $\tan A$ (d) $\operatorname{cosec} A$
244. If $\cot \theta + \cos \theta = m$ and $\cot \theta - \cos \theta = n$, then find the value of $m^2 - n^2$.
(a) $\sqrt{mn}$ (b) $2\sqrt{mn}$
(c) $3\sqrt{mn}$ (d) $4\sqrt{mn}$
245. If $\tan \theta + \sin \theta = m$ and $\tan \theta - \sin \theta = n$, then find the value of $\sqrt{mn}$.
(a) $\frac{1}{2} (m^2 - n^2)$ (b) $2(m^2 - n^2)$
(c) $\frac{1}{4} (m^2 + n^2)$ (d) $\frac{1}{4} (m^2 - n^2)$
246. If $\cot \theta + \tan \theta = x$ and $\sec \theta - \cos \theta = y$, then $(x^2 y)^{2/3} - (xy^2)^{2/3} = ?$
(a) 4 (b) 3 (c) 2 (d) 1
247. $\frac{\sin^8 \theta - \cos^8 \theta}{\cos 2\theta (1 + \cos^2 2\theta)} = ?$
(a) 1 (b) $-\frac{1}{2}$ (c) -1 (d) 2
248. If $(\sec \alpha + \tan \alpha) (\sec \beta + \tan \beta) (\sec \gamma + \tan \gamma) = (\sec \alpha - \tan \alpha) (\sec \beta - \tan \beta) (\sec \gamma - \tan \gamma)$, then each of the side is equal to
(a) ± 1 (b) -1 (c) +1 (d) 4
249. If $a \sec \theta + b \tan \theta + c = 0$ and $p \sec \theta + q \tan \theta + r = 0$, then $(br - qc)^2 - (pc - ar)^2 = ?$
(a) $(aq - bp)^2$ (b) $(ap - bq)^2$
(c) $(aq + bp)^2$ (d) $(aq - bp)^3$
250. If $P = a \cos^3 x + 3a \cos x \cdot \sin^2 x$ and $Q = a \sin^3 x + 3a \cos^2 x \cdot \sin x$, then $(P+Q)^{2/3} + (P-Q)^{2/3} = ?$
(a) $2a^{2/3}$ (b) $a^{1/3}$
(c) $2a^{1/3}$ (d) $a^{1/3}$
251. If $8 \cos^2 \theta + 8 \sec^2 \theta = 65$ and $0^\circ < \theta < \frac{\pi}{2}$, then $4 \cos 2\theta$ is equal to
(a) $-\frac{23}{8}$ (b) $-\frac{31}{8}$
(c) $-\frac{31}{32}$ (d) $-\frac{33}{32}$
252. If $\cos(\theta - A) = a$, $\cos(\theta - B) = b$, then $\sin^2(A-B) + 2ab \cos(A-B)$ is equal to
(a) $a^2 - b^2$ (b) $a^2 + b^2$
(c) $b^2 - a^2$ (d) $2ab$
253. $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} = ?$
(a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) 0
254. If $\sin \theta + \cos \theta = a$ and $\sec \theta + \operatorname{cosec} \theta = b$, then the value of $b(a^2 - 1)$ is equal to
(a) $2a$ (b) $3a$ (c) 0 (d) $2ab$
255. $\cos 15^\circ \cos 7\frac{1}{2}^\circ \cos 82\frac{1}{2}^\circ = ?$
(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{16}$
256. $3 \tan \theta \tan \phi = 1$, then $\frac{\cos(\theta - \phi)}{\cos(\theta + \phi)} = ?$
(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) 3
257. $\tan 20^\circ + \tan 40^\circ + \sqrt{3} \tan 20^\circ \cdot \tan 40^\circ = ?$
(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) 1
258. If $x \cos q - \sin q = 1$, then $x^2 + (1 + x^2) \sin q$ equals
(a) 1 (b) -1 (c) 0 (d) 2

259. The simplified form of the given expression $\sin A \cos A (\tan A - \cot A)$ is (where $0^\circ < A < 90^\circ$):

- (a) 1 (b) $1 - \cos^2 A$
(c) $1 - 2 \sin^2 A$ (d) $2 \sin^2 A - 1$

260. $\frac{\cos \alpha}{\sin \beta} = n$ and $\frac{\cos \alpha}{\cos \beta} = m$, then the value of $\cos^2 \beta$ is:

- (a) $\frac{m^2-1}{n^2-1}$ (b) $\frac{m^2-3}{n^2-4}$
(c) $\frac{m^2+3}{n^2+3}$ (d) $\frac{n^2}{m^2+n^2}$

261. Provided, $\sin(A - B) = \sin A \cos B - \cos A \sin B$, then $\sin 15^\circ$ will be

- (a) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2\sqrt{2}}$
(c) $\frac{\sqrt{3}-1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

262. If α is an acute angle and $2\sin \alpha + 15 \cos^2 \alpha = 7$, then the value of $\cot \alpha$ is

- (a) $4/3$ (b) $5/4$
(c) $4/5$ (d) $3/4$

263. If $\sin^4 \theta + \cos^4 \theta = 2\sin^2 \theta \cos^2 \theta$, θ is an acute angle, then value of $\tan \theta$ is

- (a) $\sqrt{2}$ (b) 1 (c) $3/5$ (d) 0

264. If A is an acute angle and $\cot A + \operatorname{cosec} A = 3$, then the value of $\sin A$ is

- (a) 1 (b) $4/5$
(c) $3/5$ (d) 0

265. The value of $\cot 41^\circ \cdot \cot 42^\circ \cdot \cot 43^\circ \cdot \cot 44^\circ \cdot \cot 45^\circ \cdot \cot 46^\circ \cdot \cot 47^\circ \cdot \cot 48^\circ \cdot \cot 49^\circ$.

- (a) $\frac{\sqrt{3}}{2}$ (b) 1 (c) 0 (d) $\frac{1}{\sqrt{2}}$

266. If $5 \cos \theta + 12 \sin \theta = 13$, $\theta < 90^\circ$, then the value of $\sin \theta$ is

- (a) $\frac{6}{13}$ (b) $-\frac{12}{13}$
(c) $\frac{5}{13}$ (d) $\frac{12}{13}$

267. If $\tan \theta - \cot \theta = 0$ and θ is positive acute angle, then the value of $\frac{\tan(\theta+15^\circ)}{\tan(\theta-15^\circ)}$ is

- (a) 3 (b) $\frac{1}{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

268. If $\sec \theta - \tan \theta = \frac{1}{\sqrt{3}}$, the value

of $\sec \theta \cdot \tan \theta$ is:

- (a) $\frac{4}{\sqrt{3}}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{2}{3}$ (d) $\frac{1}{\sqrt{3}}$

269. The value of $(\operatorname{cosec} a - \sin a)$

$(\sec a - \cos a)(\tan a + \cot a)$

- (a) 4 (b) 6 (c) 2 (d) 1

270. If $\tan A = n \tan B$ and $\sin A = m \sin B$, then the value of $\cos^2 A$ is

- (a) $\frac{m^2+1}{n^2+1}$ (b) $\frac{m^2+1}{n^2-1}$
(c) $\frac{m^2-1}{n^2-1}$ (d) $\frac{m^2-1}{n^2+1}$

271. If $x \cos^2 30^\circ \cdot \sin 60^\circ = \frac{\tan^2 45^\circ \cdot \sec 60^\circ}{\operatorname{cosec} 60^\circ}$ then the value of x

- (a) $\frac{1}{\sqrt{3}}$ (b) $2\frac{2}{3}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

272. If $\sin(\theta+30^\circ) = \frac{3}{\sqrt{12}}$, then find $\cos^2 \theta$

- (a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

273. If $0 \leq \theta \leq 90^\circ$ and

$$4\cos^2 \theta - 4\sqrt{3}\cos \theta + 3 = 0,$$

then the value of θ is

- (a) 30° (b) 90° (c) 45° (d) 60°

274. The value of $\sec^4 A (1 - \sin^4 A) - 2\tan^2 A$ is

- (a) $\frac{1}{2}$ (b) 0 (c) 2 (d) 1

275. If A, B, C are the angles of a ΔABC then following is equal to:

$$\sin\left(\frac{B+C}{2}\right)$$

- (a) $\sec \frac{B}{2}$ (b) $\sec \frac{A}{2}$
(c) $\operatorname{cosec} \frac{A}{2}$ (d) $\cos \frac{A}{2}$

276. The numerical value of

$$\frac{9}{\operatorname{cosec}^2 \theta} + 4 \cos^2 \theta + \frac{5}{1+\tan^2 \theta} :$$

- (a) 7 (b) 9 (c) 4 (d) 5

277. If $\cos \theta = \frac{p}{\sqrt{p^2+q^2}}$, then the value of $\tan \theta$ is:

- (a) $\frac{q}{p}$ (b) $\frac{p}{p^2+q^2}$
(c) $\frac{q}{\sqrt{p^2+q^2}}$ (d) $\frac{q}{\sqrt{p^2-q^2}}$

278. If $\sec \theta + \tan \theta = 3$, θ being acute, the value of $5 \sin \theta$ is:

- (a) $\sqrt{3/5}$ (b) $\sqrt{5/3}$
(c) 4 (d) $\frac{5}{2}$

279. If $\frac{x - x \tan^2 30^\circ}{1 + \tan^2 30^\circ} = \sin^2 30^\circ + 4$

$\cot^2 45^\circ - \sec^2 60^\circ$ Then value of x is:

- (a) $\frac{1}{4}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$

280. If $\cos \theta + \sin \theta = m$ and $\sec \theta + \operatorname{cosec} \theta = n$ then the value of $n(m^2-1)$ is equal to:

- (a) mn (b) $4mn$
(c) $2n$ (d) $2m$

281. If $\alpha + \beta = 90^\circ$ then the expression

$$\frac{\tan \alpha}{\tan \beta} + \sin^2 \alpha + \sin^2 \beta$$

is equal to :

- (a) $\tan^2 \alpha$ (b) $\tan^2 \beta$
(c) $\sin^2 \beta$ (d) $\sec^2 \alpha$

282. The value of x in the equation

$$\tan^2 \frac{\pi}{4} - \cos^2 \frac{\pi}{3} = x \sin \frac{\pi}{4} \cos$$

$$\frac{\pi}{4} \tan \frac{\pi}{3}$$
 is :

- (a) $\frac{3\sqrt{3}}{4}$ (b) $\frac{2}{\sqrt{3}}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

283. If $\sin 2\theta = \frac{\sqrt{3}}{2}$ then the value of $\sin 3\theta$ is equal to : (Take $0^\circ \leq \theta \leq 90^\circ$)

- (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 0 (d) 1

284. If $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = 3$ then the value of $\sin^4 \theta$ is :

- (a) $\frac{16}{25}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{3}{5}$

285. If $\cos 20^\circ = m$ and $\cos 70^\circ = n$, then the value of $m^2 + n^2$ is

- (a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) $\frac{1}{\sqrt{2}}$

286. If $\sin A - \cos A = \frac{\sqrt{3}-1}{2}$ then the value of $\sin A \cdot \cos A$ is

- (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{4}$ (d) $\frac{\sqrt{3}}{4}$

287. If $\sin \theta = \frac{a}{b}$, then the value of $\sec \theta - \cos \theta$ is (where $0^\circ < \theta < 90^\circ$)

- (a) $\frac{a}{b\sqrt{b^2-a^2}}$ (b) $\frac{b^2}{a\sqrt{b^2-a^2}}$
(c) $\frac{a^2}{b\sqrt{b^2-a^2}}$ (d) $\frac{\sqrt{b^2+a^2}}{\sqrt{b^2-a^2}}$

288. The expression

$\frac{\sqrt{1+\sin \theta} + \sqrt{1-\sin \theta}}{\sqrt{1-\sin \theta} + \sqrt{1+\sin \theta}}$ is equal to:

(a) $2 \sin \theta$ (b) $2 \tan \theta$
(c) $2 \sec \theta$ (d) $2 \operatorname{cosec} \theta$

289. If $\frac{\sec^2 70^\circ - \cot^2 20^\circ}{2(\operatorname{cosec}^2 59^\circ - \tan^2 31^\circ)} = \frac{2}{m}$,

then m is equal to:

- (a) 2 (b) 3 (c) 4 (d) 1

290. If $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin$

$(\alpha - \beta) = \frac{5}{13}$ α, β lie between 0

and $\frac{\pi}{4}$, then $\tan 2\alpha = ?$

- (a) $\frac{56}{33}$ (b) $\frac{36}{33}$ (c) $\frac{33}{56}$ (d) $\frac{49}{36}$

291. $\cos \frac{\pi}{7} + \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} + \dots$

$+ \dots \cos \frac{6\pi}{7} = ?$

- (a) 0 (b) 1 (c) -1 (d) 2

292. $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ = ?$

- (a) 1 (b) 2 (c) 3 (d) 4

293. If $2 \cos x + \sin x = 1$, then find $7 \cos x + 6 \sin x$.

- (a) 6 (b) 2 (c) 7 (d) 1

294. $\cos^2 48^\circ - \sin^2 12^\circ = ?$

- (a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}+1}{8}$

- (c) $\frac{\sqrt{3}-1}{4}$ (d) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

295. $\sin\left(\frac{\pi}{10}\right) \sin\left(\frac{3\pi}{10}\right) = ?$

- (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 1

296. $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = ?$

- (a) $2 \tan 2\theta$ (b) $2 \cot 2\theta$
(c) $\tan 2\theta$ (d) $\cot 2\theta$

297. If $\sin(\alpha - \beta) = \frac{1}{2}$ and $\cos(\alpha + \beta) =$

$\frac{1}{2}$, where α, β are positive acute

angles, then α & β are

- (a) 45° and 15° (b) 60° and 15°
(c) 15° and 45° (d) 45° and 60°

298. If $\alpha + \beta - \gamma = \pi$, then $\sin^2 \alpha + \sin^2 \beta - \sin^2 \gamma = ?$

- (a) $2 \sin \alpha \sin \beta \cos \gamma$
(b) $2 \cos \alpha \cos \beta \cos \gamma$
(c) $2 \sin \alpha \sin \beta \sin \gamma$
(d) $2 \cos \alpha \sin \beta \sin \gamma$

299. $1 + \cos 2x + \cos 4x + \cos 6x = ?$

- (a) $2 \cos x \cos 2x \cos 3x$
(b) $4 \sin x \cos 2x \cos 3x$
(c) $4 \cos x \cos 2x \cos 3x$
(d) $\cos x \cos 2x \cos 3x$

300. $\sin 12^\circ \sin 48^\circ \sin 54^\circ = ?$

- (a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{32}$

301. $\frac{1}{4} [\sqrt{3} \cos 23^\circ - \sin 23^\circ] = ?$

- (a) $\cos 43^\circ$ (b) $\cos 70^\circ$
(c) $\cos 53^\circ$ (d) $\frac{1}{2} \cos 53^\circ$

302. $2 \cos x - \cos 3x - \cos 5x = ?$

- (a) $16 \cos^3 x \sin^2 x$
(b) $\sin^3 x \cos^2 x$
(c) $4 \cos^3 x \sin^2 x$
(d) $4 \sin^3 x \cos^2 x$

303. $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ = ?$

- (a) 2 (b) $\frac{2 \sin 20^\circ}{\sin 40^\circ}$

- (c) 4 (d) $\frac{4 \sin 20^\circ}{\sin 40^\circ}$

304. $\frac{\cos 12^\circ - \sin 12^\circ}{\cos 12^\circ + \sin 12^\circ} + \frac{\sin 147^\circ}{\cos 147^\circ} = ?$

- (a) 1 (b) -1 (c) 0 (d) $\frac{1}{2}$

305. $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ = ?$

- (a) 2 (b) $-\frac{1}{2}$
(c) 0 (d) $\frac{1}{2}$

306. If $m \tan(\theta - 30^\circ) = n \tan(\theta + 120^\circ)$, then $\frac{m+n}{m-n} = ?$

- (a) $2 \cos 2\theta$ (b) $\cos 2\theta$
(c) $2 \sin 2\theta$ (d) $\sin 2\theta$

307. If $\tan \theta = \frac{x \sin \phi}{1 - x \cos \phi}$ & $\tan \phi =$

$\frac{y \sin \theta}{1 - y \cos \theta}$, then $\frac{x}{y} = ?$

- (a) $\frac{\sin \theta}{\sin \phi}$ (b) $\frac{\sin \phi}{\sin \theta}$
(c) $\frac{\sin \phi}{1 - \cos \theta}$ (d) $\frac{\sin \theta}{1 - \cos \phi}$

308. If $\tan x = \frac{b}{a}$, then $\sqrt{\frac{a+b}{a-b}} +$

$\sqrt{\frac{a-b}{a+b}} = ?$

- (a) $\frac{2 \sin x}{\sqrt{\sin 2x}}$ (b) $\frac{2 \cos x}{\sqrt{\cos 4x}}$
(c) $\frac{2 \cos x}{\sqrt{\sin 2x}}$ (d) $\frac{2 \cos x}{\sqrt{\cos 2x}}$

309. $\tan \alpha + 2 \tan 2\alpha + 4 \tan 4\alpha + 8 \cot 8\alpha = ?$
 (a) $\tan \alpha$ (b) $\tan 2\alpha$
 (c) $\cot \alpha$ (d) $\cot 2\alpha$

310. If $\sin A = n \sin B$, then $\left(\frac{n-1}{n+1}\right)$

$\tan \left(\frac{A+B}{2}\right) = ?$

(a) $\sin \left(\frac{A-B}{2}\right)$ (b) $\tan \left(\frac{A-B}{2}\right)$

(c) $\cot \left(\frac{A-B}{2}\right)$ (d) $\tan \left(\frac{A+B}{2}\right)$

311. $2 \sin A \cos^3 A - 2 \sin^3 A \cos A = ?$

(a) $\sin 4A$ (b) $\frac{1}{2} \sin 4A$

(c) $\frac{1}{4} \sin 4A$ (d) $\frac{1}{8} \sin 4A$

312. $\tan A + \tan (180^\circ + A) + \cot (90^\circ + A) + \cot (360^\circ - A) = ?$

(a) 0 (b) $2 \tan A$
 (c) $2 \cot A$ (d) $\tan A - \cot A$

313. If $\frac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha} = y$, then

$\frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha} = ?$

(a) y (b) $1/y$

(c) $1+y$ (d) $\frac{1}{1+y}$

314. $\frac{\sin 70^\circ + \cos 40^\circ}{\cos 70^\circ + \sin 40^\circ} = ?$

(a) 1 (b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $-\sqrt{3}$

315. If $\frac{\sin^4 A}{a} + \frac{\cos^4 A}{b} = \frac{1}{a+b}$, then

$\frac{\sin^8 A}{a^3} + \frac{\cos^8 A}{b^3} = ?$

(a) $\frac{1}{(a+b)^3}$ (b) $\frac{a^2 b^2}{(a+b)^2}$

(c) $\frac{a^3 b^3}{(a+b)^2}$ (d) $\frac{ab}{a+b}$

316. If $2y \cos \theta = x \sin \theta$ and $2x \sec \theta - y \operatorname{cosec} \theta = 3$, then $x^2 + 4y^2 = ?$

(a) 4 (b) -4 (c) ± 4 (d) 0

317. If $\tan \theta - \cot \theta = a$ and $\cos \theta + \sin \theta = b$, then $(b^2 - 1)^2 (a^2 + 4) = ?$

(a) 2 (b) -4 (c) ± 4 (d) 4

318. If $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$, then $\sin \alpha + \cos \alpha$ and $\sin \alpha - \cos \alpha$ are equal to

(a) $\sqrt{2} \cos \theta, \sqrt{2} \sin \theta$

(b) $\sqrt{2} \sin \theta, \sqrt{2} \cos \theta$

(c) $\sqrt{2} \sin \theta, \sqrt{2} \sin \theta$

(d) $\sqrt{2} \cos \theta, \sqrt{2} \cos \theta$

319. If $\sin \theta + \sin \phi = a$ and $\cos \theta + \cos \phi = b$, then $\tan \left(\frac{\theta - \phi}{2}\right) = ?$

(a) $\sqrt{\frac{a^2 + b^2}{4 - a^2 - b^2}}$ (b) $\sqrt{\frac{4 - a^2 - b^2}{a^2 + b^2}}$

(c) $\sqrt{\frac{a^2 + b^2}{4 + a^2 + b^2}}$ (d) $\sqrt{\frac{4 + a^2 + b^2}{a^2 + b^2}}$

320. $\cos^2 \alpha + \cos^2 (\alpha + 120^\circ) + \cos^2 (\alpha - 120^\circ) = ?$

(a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) 0

321. If $\tan A = \frac{1 - \cos B}{\sin B}$, express $\tan 2A$ in terms of $\tan B$

(a) $\tan 2A = \tan B$

(b) $\tan 2A = \tan^2 B$

(c) $\tan 2A = \tan^2 A + \tan^2 B$

(d) $\tan 2A = \tan^2 A - \tan^2 B$

322. If $\tan (A + B) = p$ and $\tan (A - B) = q$, then the value of $\tan 2A$ is

(a) $\frac{p+q}{p-q}$ (b) $\frac{p-q}{1+pq}$

(c) $\frac{p+q}{1-pq}$ (d) $\frac{1+pq}{p-q}$

323. $\frac{\sec 8A - 1}{\sec 4A - 1} = ?$

(a) $\frac{\tan 2A}{\tan 8A}$ (b) $\frac{\tan 8A}{\tan 2A}$

(c) $\frac{\cot 8A}{\cot 2A}$ (d) $\frac{\cot 2A}{\cot 8A}$

324. In a $\triangle ABC$, $\angle C = 90^\circ$, then the equation whose roots are $\tan A$ & $\tan B$ is

(a) $abx^2 + c^2 + ab = 0$

(b) $abx^2 + c^2x - ab = 0$

(c) $abx^2 + c^2x - ab = 0$

(d) $abx^2 - c^2x + ab = 0$

325. If $\cos(A - B) = \frac{3}{5}$ and $\tan A \tan B = 2$, then

(a) $\cos A \cos B = \frac{2}{5}$

(b) $\sin A \sin B = \frac{2}{5}$

(c) $\cos A \cos B = \frac{1}{5}$

(d) Both (b) and (c)

326. $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = ?$

(a) $\tan 54^\circ$ (b) $\tan 36^\circ$

(c) $\tan 18^\circ$ (d) $\cot 18^\circ$

327. If $\tan \alpha = \frac{1}{7}$ and $\tan \beta = \frac{1}{3}$, the

$\cos 2\alpha = ?$

(a) $\sin 2\beta$ (b) $\sin 4\beta$

(c) $\sin 3\beta$ (d) $\cos 3\beta$

328. If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, then $\cos 2A = ?$

(a) $\sin B$ (b) $\sin 2B$

(c) $\sin 3B$ (d) $\cos 3B$

329. $2 \sin^2 \beta + 4 \cos(\alpha + \beta) \sin \alpha \sin \beta + \cos^2(\alpha + \beta) = ?$

(a) $\sin^2 \alpha$ (b) $\cos^2 \beta$

(c) $\cos^2 \alpha$ (d) $\sin^2 \beta$

330. $\sin 12^\circ \sin 24^\circ \sin 48^\circ \sin 84^\circ = ?$

(a) $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$

(b) $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$

(c) $\frac{3}{16}$ (d) $\frac{1}{16}$

331. $\tan 5x \tan 3x \tan 2x = ?$

(a) $\tan 5x - \tan 3x - \tan 2x$

(b) 0

(c) $\frac{\sin 6x - \sin 3x - \sin 2x}{\cos 5x - \cos 3x - \cos 2x}$

(d) $\tan 9x$

332. If $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$, then $\cos 2\alpha + \cos 2\beta = ?$

(a) $-2 \sin(\alpha + \beta)$

(b) $2 \cos(\alpha + \beta)$

(c) $2 \sin(\alpha + \beta)$

(d) $-2 \cos(\alpha + \beta)$

333. If $\cos A = a \cos B$ and $\sin A = b \sin B$, then $(b^2 - a^2) \sin^2 B = ?$

(a) $1 + a^2$ (b) $2 + a^2$

(c) $1 - a^2$ (d) $2 - a^2$

334. If $A + B + C = \pi$, then $\cos 2A + \cos 2B + \cos 2C = ?$

(a) $1 + 4 \cos A \cos B \cos C$

(b) $-1 + 4 \sin A \sin B \sin C$

(c) $-1 - 4 \cos A \cos B \cos C$

(d) $1 + 4 \sin A \sin B \sin C$

335. If A, B, C are angles of a triangle, then $\sin^2 A + \sin^2 B + \sin^2 C - 2 \cos A \cos B \cos C = ?$

(a) 1 (b) 2 (c) 3 (d) 4

336. If $A + B + C = 180^\circ$, then $\sin 2A + \sin 2B + \sin 2C = ?$

- (a) $4 \sin A \sin B \cos C$
(b) $4 \cos A \cos B \cos C$
(c) $4 \sin A \sin B \sin C$
(d) $8 \sin A \sin B \sin C$

337. $\tan 10^\circ - \tan 50^\circ + \tan 70^\circ = ?$

- (a) 0 (b) 1 (c) $\sqrt{3}$ (d) 3

338. For what value of α does the equation $4\cos \alpha + 3\cos 2\alpha - 2\sin 3\alpha + \cos 4\alpha = 2\sqrt{3} - 1$?

- (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

339. If $(\sin A + \sin B + \sin C)^2 = \sin^2 A + \sin^2 B + \sin^2 C$, then which one is true?

- (a) $\sin A + \sin B + \sin C = 0$
(b) $\cos A + \cos B + \cos C = 0$

(c) $\frac{1}{\sin A} + \frac{1}{\sin B} + \frac{1}{\sin C} = 0$

- (d) None of these

340. If $\frac{\sin x}{\sin y} = p$ and $\frac{\cos x}{\cos y} = q$, then $\tan x = ?$

- (a) $\frac{p}{q} \sqrt{\frac{q^2 - 2}{1 - p^2}}$ (b) $\frac{p}{q} \sqrt{\frac{q^2 - 1}{1 - p^2}}$

- (c) $\frac{p}{q} \sqrt{\frac{1 - q^2}{1 - p^2}}$ (d) $\frac{q}{p} \sqrt{\frac{q^2 - 1}{1 - p^2}}$

341. If $y = a \cos^2 x + 2b \sin x \cos x + c \sin^2 x$ and $z = a \sin^2 x - 2b \sin x \cos x + c \cos^2 x$, then

- (a) $y + z = a + c$
(b) $y + z = a + b$
(c) $y + a = x + b$
(d) None of these

342. If $A + B + C = \pi$, then

$$\frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B} = ?$$

- (a) 0 (b) 1 (c) 2 (d) 3

343. If $\sin A$, $\cos A$ and $\tan A$ are in GP, then $\cos^3 A + \cos^2 A = ?$

- (a) 1 (b) 2 (c) 3 (d) 4

344. If $A + B = C$ and $\tan A = k \tan B$, and $A - B = \phi$, then $\sin C = ?$

- (a) 0 (b) 1
(c) $\frac{k+1}{k-1}$ (d) $\frac{k+1}{k-1} \sin \phi$

345. If $\tan \alpha$, $\tan \beta$ are the roots of $x^2 + px + q = 0$

($p \neq q$) then $\tan(\alpha + \beta) = ?$

- (a) $p - 1$ (b) $\frac{p}{q - 1}$
(c) $2q + p$ (d) None of these

346. If $A + B + C = \frac{3\pi}{2}$, then $\cos 2A + \cos 2B + \cos 2C = ?$

- (a) $1 - 4 \sin A \sin B \sin C$
(b) $1 - \sin A \sin B \sin C$
(c) $1 - 2 \sin A \sin B \sin C$
(d) $1 - 3 \sin A \sin B \sin C$

347. $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then $\frac{\tan x}{\tan y} = ?$

- (a) a (b) b
(c) a/b (d) b/a

348. $\tan \frac{2\pi}{5} - \tan \frac{\pi}{15} - \sqrt{3} \tan \frac{2\pi}{5} \tan \frac{\pi}{15} = ?$

$\frac{\pi}{15} = ?$

- (a) 0 (b) 1 (c) $\sqrt{2}$ (d) $\sqrt{3}$

349. If $A + B + C = 180^\circ$, then $\tan^2$

$$\frac{A}{2} + \tan^2 \frac{B}{2} + \tan^2 \frac{C}{2} = ?$$

- (a) 1 (b) 3 (c) 2 (d) 0

350. $\frac{\cot^2 15^\circ - 1}{\cot^2 15^\circ + 1} = ?$

- (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$
(c) $-\frac{2}{\sqrt{3}}$ (d) $\sqrt{\frac{2}{3}}$

ANSWER KEY

1. (b)	36. (c)	71. (b)	106. (a)	141. (c)	176. (c)	211. (c)	246. (d)	281. (d)	316. (a)
2. (d)	37. (a)	72. (c)	107. (d)	142. (b)	177. (d)	212. (c)	247. (b)	282. (d)	317. (d)
3. (a)	38. (b)	73. (b)	108. (a)	143. (d)	178. (c)	213. (b)	248. (a)	283. (d)	318. (a)
4. (a)	39. (c)	74. (b)	109. (b)	144. (a)	179. (c)	214. (d)	249. (a)	284. (a)	319. (b)
5. (d)	40. (c)	75. (a)	110. (a)	145. (b)	180. (c)	215. (a)	250. (a)	285. (b)	320. (a)
6. (d)	41. (d)	76. (b)	111. (a)	146. (b)	181. (a)	216. (d)	251. (b)	286. (d)	321. (a)
7. (d)	42. (d)	77. (c)	112. (a)	147. (b)	182. (a)	217. (c)	252. (b)	287. (c)	322. (c)
8. (c)	43. (b)	78. (c)	113. (d)	148. (c)	183. (c)	218. (b)	253. (a)	288. (c)	323. (b)
9. (c)	44. (a)	79. (d)	114. (a)	149. (a)	184. (d)	219. (b)	254. (a)	289. (a)	324. (d)
10. (b)	45. (b)	80. (d)	115. (d)	150. (a)	185. (c)	220. (b)	255. (b)	290. (a)	325. (d)
11. (a)	46. (a)	81. (a)	116. (c)	151. (b)	186. (d)	221. (b)	256. (b)	291. (a)	326. (a)
12. (a)	47. (c)	82. (c)	117. (a)	152. (b)	187. (d)	222. (c)	257. (c)	292. (d)	327. (b)
13. (b)	48. (d)	83. (c)	118. (c)	153. (b)	188. (c)	223. (b)	258. (a)	293. (a)	328. (b)
14. (c)	49. (c)	84. (c)	119. (b)	154. (d)	189. (a)	224. (d)	259. (d)	294. (b)	329. (c)
15. (c)	50. (a)	85. (d)	120. (d)	155. (d)	190. (c)	225. (d)	260. (d)	295. (c)	330. (d)
16. (b)	51. (b)	86. (c)	121. (c)	156. (b)	191. (c)	226. (c)	261. (a)	296. (a)	331. (a)
17. (a)	52. (a)	87. (b)	122. (a)	157. (d)	192. (d)	227. (d)	262. (d)	297. (a)	332. (d)
18. (c)	53. (a)	88. (b)	123. (c)	158. (a)	193. (c)	228. (c)	263. (b)	298. (a)	333. (c)
19. (b)	54. (b)	89. (a)	124. (a)	159. (a)	194. (c)	229. (c)	264. (c)	299. (c)	334. (c)
20. (a)	55. (a)	90. (c)	125. (c)	160. (d)	195. (c)	230. (a)	265. (b)	300. (b)	335. (b)
21. (d)	56. (c)	91. (b)	126. (d)	161. (b)	196. (a)	231. (c)	266. (d)	301. (d)	336. (c)
22. (b)	57. (c)	92. (b)	127. (b)	162. (a)	197. (d)	232. (d)	267. (a)	302. (a)	337. (c)
23. (d)	58. (b)	93. (c)	128. (b)	163. (c)	198. (a)	233. (d)	268. (c)	303. (c)	338. (a)
24. (c)	59. (c)	94. (d)	129. (a)	164. (b)	199. (c)	234. (b)	269. (d)	304. (c)	339. (a)
25. (b)	60. (c)	95. (d)	130. (d)	165. (b)	200. (a)	235. (d)	270. (c)	305. (d)	340. (b)
26. (b)	61. (b)	96. (c)	131. (d)	166. (c)	201. (c)	236. (c)	271. (b)	306. (a)	341. (a)
27. (c)	62. (d)	97. (c)	132. (c)	167. (b)	202. (c)	237. (b)	272. (b)	307. (a)	342. (c)
28. (d)	63. (d)	98. (a)	133. (b)	168. (b)	203. (a)	238. (b)	273. (a)	308. (d)	343. (a)
29. (b)	64. (d)	99. (c)	134. (d)	169. (c)	204. (b)	239. (c)	274. (d)	309. (c)	344. (d)
30. (d)	65. (d)	100. (b)	135. (d)	170. (c)	205. (b)	240. (d)	275. (d)	310. (b)	345. (b)
31. (a)	66. (a)	101. (c)	136. (b)	171. (a)	206. (d)	241. (b)	276. (b)	311. (b)	346. (a)
32. (b)	67. (b)	102. (a)	137. (b)	172. (c)	207. (a)	242. (b)	277. (a)	312. (d)	347. (c)
33. (d)	68. (a)	103. (c)	138. (b)	173. (b)	208. (c)	243. (a)	278. (c)	313. (a)	348. (d)
34. (b)	69. (c)	104. (b)	139. (a)	174. (b)	209. (b)	244. (d)	279. (c)	314. (c)	349. (a)
35. (a)	70. (c)	105. (a)	140. (a)	175. (a)	210. (c)	245. (d)	280. (d)	315. (a)	350. (a)

1. (b) $\frac{\tan \theta + \cot \theta}{\tan \theta - \cot \theta} = 2$

By componendo and dividendo rule,

$$\frac{\tan \theta}{\cot \theta} = \frac{3}{1}$$

$$\frac{\sin \theta}{\cos \theta} \times \frac{\sin \theta}{\cos \theta} = 3$$

$$\sin^2 \theta = 3 \cos^2 \theta$$

$$\sin^2 \theta = 3(1 - \sin^2 \theta)$$

$$4 \sin^2 \theta = 3$$

$$\sin^2 \theta = \frac{3}{4} \Rightarrow \sin \theta = \frac{\sqrt{3}}{2}$$

Alternate:-

$$\frac{\tan \theta + \cot \theta}{\tan \theta - \cot \theta} = 2$$

By C and D rule,

$$\frac{\tan \theta}{\cot \theta} = \frac{3}{1}$$

$$\tan^2 \theta = 3$$

$$\tan \theta = \sqrt{3} = \tan 60^\circ$$

$$\theta = 60^\circ$$

$$\therefore \sin \theta = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

2. (d) $\cot 10^\circ \cdot \cot 20^\circ \cdot \cot 60^\circ \cdot \cot 70^\circ \cdot \cot 80^\circ$

[In $\cot A \cdot \cot B$ If $A + B = 90^\circ$ then, $\cot A \cdot \cot B = 1$]

$$\cot 10^\circ \cdot \cot 80^\circ \cdot \cot 20^\circ \cdot \cot 70^\circ \cdot \cot 60^\circ$$

$$= 1 \times 1 \times \frac{1}{\sqrt{3}}$$

3. (a)

$$\cot 18^\circ \left[\cot 72^\circ \cdot \cos^2 22^\circ + \frac{1}{\tan 72^\circ \cdot \sec^2 68^\circ} \right]$$

$$\cot 18^\circ \cdot \cot 72^\circ \cdot \cos^2 22^\circ + \frac{\cot 18^\circ}{\tan 72^\circ \cdot \sec^2 68^\circ}$$

(If $A + B = 90^\circ$, then $\cot A \cdot \cot B = 1$)

$$= \cos^2 22^\circ + \frac{\cot 18^\circ}{\tan 72^\circ \cdot \sec^2 68^\circ}$$

$$(\because \tan 72^\circ = \cot 18^\circ)$$

$$= \cos^2 22^\circ + \cos^2 68^\circ$$

$$= \cos^2 22^\circ + \sin^2 22^\circ = 1$$

$$(\because \sin^2 \theta + \cos^2 \theta = 1)$$

4. (a) $\tan 15^\circ \cot 75^\circ + \tan 75^\circ \cot 15^\circ$
 $= \tan 15^\circ \cdot \cot(90^\circ - 15^\circ) + \tan(90^\circ - 15^\circ) \cdot \cot 15^\circ$

$$= \tan 15^\circ \cdot \tan 15^\circ + \cot 15^\circ \cdot \cot 15^\circ$$

$$= \tan^2 15^\circ + \cot^2 15^\circ \quad \dots(i)$$

[Formula]

$$\cot(90^\circ - \theta) = \tan \theta$$

$$\tan(90^\circ - \theta) = \cot \theta$$

Put value of $\tan 15^\circ$,

$$\left[\begin{aligned} \cot 15^\circ &= \frac{1}{\tan 15^\circ} \\ \cot 15^\circ &= \frac{1}{(2 - \sqrt{3})} \Rightarrow \frac{1}{(2 - \sqrt{3})} \times \frac{(2 + \sqrt{3})}{(2 + \sqrt{3})} \end{aligned} \right]$$

$$\cot 15^\circ = 2 + \sqrt{3}$$

Now put value in eq (i)

$$\tan^2 15^\circ + \cot^2 15^\circ$$

$$= (2 - \sqrt{3})^2 + (2 + \sqrt{3})^2$$

$$= 4 + 3 - 4\sqrt{3} + 4 + 3 + 4\sqrt{3}$$

$$= 14$$

5. (d) $\sin(2x - 20^\circ) = \cos(2y + 20^\circ)$
 $(2x - 20^\circ) + (2y + 20^\circ) = 90^\circ$

$$\left[\begin{aligned} \text{if } \sin A &= \cos B \\ \text{then} \\ A + B &= 90^\circ \end{aligned} \right]$$

$$2(x + y) = 90^\circ$$

$$x + y = 45^\circ$$

$$\tan(x + y) = \tan 45^\circ = 1$$

Alternate:-

$$\sin(2x - 20^\circ) = \cos(2y + 20^\circ)$$

$$\Rightarrow \sin(2x - 20^\circ) = \sin[90^\circ - (2y + 20^\circ)]$$

$$\Rightarrow 2x - 20^\circ = 90^\circ - (2y + 20^\circ)$$

$$\Rightarrow 2x - 20^\circ = 90^\circ - 2y - 20^\circ$$

$$\Rightarrow 2x + 2y = 90^\circ$$

$$\Rightarrow 2(x + y) = 90^\circ$$

$$\Rightarrow x + y = 45^\circ$$

$$\therefore \tan(x + y) = \tan 45^\circ = 1$$

6. (d)

$$(\sin^2 5^\circ + \sin^2 10^\circ + \dots + \sin^2 85^\circ) + \sin^2 90^\circ$$

$$\text{No. of terms} = \left\{ \left(\frac{85 - 5}{5} \right) + 1 \right\} = 17$$

$$\text{Sum of series} = \frac{17}{2} + 1 = 9 \frac{1}{2}$$

[use this approach in exam]

7. (d) $\frac{\sin 39^\circ}{\cos 51^\circ} + 2 \tan 11^\circ \tan 31^\circ$

$$\frac{\tan 45^\circ \tan 59^\circ \tan 79^\circ}{3(\sin^2 21^\circ + \sin^2 69^\circ)}$$

$$= \frac{\sin 39^\circ}{\cos(90^\circ - 39^\circ)} + 2 \tan 11^\circ \cdot \tan(90^\circ - 11^\circ) \cdot \tan 31^\circ$$

$$\cdot \tan(90^\circ - 31^\circ) \cdot 1 - 3(\sin^2 21^\circ + \sin^2(90^\circ - 21^\circ))$$

$$= \frac{\sin 39^\circ}{\sin 39^\circ} + 2 \tan 11^\circ \cdot \cot 11^\circ \cdot \tan 31^\circ$$

$$\cot 31^\circ - 3(\sin^2 21^\circ + \cos^2 21^\circ)$$

$$= 1 + 2 - 3 = 0$$

8. (c) $\frac{\cos^2 \theta}{\cot^2 \theta - \cos^2 \theta} = 3$

$$\Rightarrow \cos^2 \theta = 3(\cot^2 \theta - \cos^2 \theta)$$

$$\Rightarrow \cos^2 \theta = 3 \cot^2 \theta - 3 \cos^2 \theta$$

$$\Rightarrow 4 \cos^2 \theta = 3 \cot^2 \theta$$

$$\Rightarrow 4 \cos^2 \theta = 3 \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$\Rightarrow 4 \sin^2 \theta = 3$$

$$\sin \theta = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

$$\therefore \theta = 60^\circ$$

Alternate method:-

Put $\theta = 60^\circ$

$$\frac{\cos^2 60^\circ}{\cot^2 60^\circ - \cos^2 60^\circ} = 3$$

$$\Rightarrow \text{LHS} = \frac{\frac{1}{4}}{\frac{1}{3} - \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{12}} = 3$$

$$\therefore \text{LHS} = \text{RHS} = 3$$

Here,

$$3 = 3$$

So, $\theta = 60^\circ$

9. (c) $\frac{A}{B} = \frac{\tan 11^\circ \tan 29^\circ}{2 \cot 61^\circ \cot 79^\circ}$

$$\frac{A}{B} = \frac{\tan 11^\circ \tan 29^\circ}{2[\cot(90^\circ - 29^\circ) \cot(90^\circ - 11^\circ)]}$$

$$\frac{A}{B} = \frac{\tan 11^\circ \tan 29^\circ}{2 \tan 11^\circ \tan 29^\circ}$$

$$\frac{A}{B} = \frac{1}{2}$$

$$2A = B$$

10. (b) $\sin \alpha + \cos \beta = 2$

put, $\alpha = 90^\circ$, $\beta = 0^\circ$

$$\Rightarrow \sin 90^\circ + \cos 0^\circ = 2$$

$$\Rightarrow 1 + 1 = 2$$

$$2 = 2 \text{ matched}$$

So, $\alpha = 90^\circ$, $\beta = 0^\circ$

$$\Rightarrow \sin\left(\frac{2\alpha + \beta}{3}\right)^\circ$$

$$= \sin\left(\frac{2 \times 90 + 0}{3}\right)^\circ = \sin\left(\frac{180}{3}\right)^\circ$$

$$= \sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Take option 'b'

$$\cos \frac{\alpha}{3} = \cos \frac{90^\circ}{3} = \cos 30^\circ$$

So, this is answer.

11. (a) $\sin \alpha \sec(30^\circ + \alpha) = 1$

put value of α between 0° to 60° ,

if $\alpha = 30^\circ$

$$\sin 30^\circ \sec(30^\circ + 30^\circ) = 1$$

$$\sin 30^\circ \sec 60^\circ = 1$$

$$\Rightarrow \frac{1}{2} \times 2 = 1$$

$$\Rightarrow 1 = 1 \text{ (satisfy)}$$

So, $\alpha = 30^\circ$

$$\sin \alpha + \cos 2\alpha$$

$$= \sin 30^\circ + \cos 2 \times 30^\circ$$

$$= \sin 30^\circ + \cos 60^\circ = \frac{1}{2} + \frac{1}{2} = 1$$

Alternate:-

If, $\sin \alpha \sec \beta = 1$

then, $\alpha + \beta = 90^\circ$

$$\sin \alpha \sec(30^\circ + \alpha) = 1$$

$$\alpha + 30^\circ + \alpha = 90^\circ$$

$$2\alpha = 60^\circ$$

$$\alpha = 30^\circ$$

$$\sin \alpha + \cos 2\alpha$$

$$\sin 30^\circ + \cos 2 \times 30^\circ = 1$$

12. (a) If, $\tan \theta = 1$

It means, $\theta = 45^\circ$

$$\frac{8 \sin \theta + 5 \cos \theta}{\sin^3 \theta - 2 \cos^3 \theta + 7 \cos \theta}$$

$$= \frac{8 \sin 45^\circ + 5 \cos 45^\circ}{\sin^3 45^\circ - 2 \cos^3 45^\circ + 7 \cos 45^\circ}$$

$$= \frac{8 \times \frac{1}{\sqrt{2}} + 5 \times \frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^3 - 2\left(\frac{1}{\sqrt{2}}\right)^3 + 7\left(\frac{1}{\sqrt{2}}\right)} = 2$$

13. (b) $\cos^2 \theta + \cos^4 \theta = 1$

$$\cos^4 \theta = 1 - \cos^2 \theta$$

$$\cos^4 \theta = \sin^2 \theta$$

$$\cos^2 \theta \cdot \cos^2 \theta = \sin^2 \theta$$

$$\cos^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\cos^2 \theta = \tan^2 \theta$$

$$\cos^2 \theta + \cos^4 \theta = \tan^2 \theta + \tan^4 \theta$$

$$\therefore \cos^2 \theta + \cos^4 \theta = 1$$

Here, $\tan^2 \theta + \tan^4 \theta = 1$

14. (c) $\frac{3 \sin \theta + 2 \cos \theta}{3 \sin \theta - 2 \cos \theta}$

divide numerator & denominator by $\cos \theta$

$$= \frac{3 \frac{\sin \theta}{\cos \theta} + \frac{2 \cos \theta}{\cos \theta}}{3 \frac{\sin \theta}{\cos \theta} - \frac{2 \cos \theta}{\cos \theta}} \left[\because \frac{\sin \theta}{\cos \theta} = \tan \theta \right]$$

$$= \frac{3 \tan \theta + 2}{3 \tan \theta - 2}$$

put value of $\tan \theta$

$$\frac{3 \times \frac{4}{3} + 2}{3 \times \frac{4}{3} - 2} = \frac{6}{2} = 3$$

15. (c) $(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$
 $= (\sec^2 A + \cos^2 A - 2 \sec A \cos A) +$
 $(\operatorname{cosec}^2 A + \sin^2 A - 2 \operatorname{cosec} A \sin A) - (\cot^2 A + \tan^2 A - 2 \cot A \tan A)$
 $= \sec^2 A - \tan^2 A + \cos^2 A + \sin^2 A + \operatorname{cosec}^2 A - \cot^2 A - 2$
 $= 3 - 2 = 1$

Alternate:-

$$(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 - (\cot A - \tan A)^2$$

put $\theta = 45^\circ$,

$$= (\sec 45^\circ - \cos 45^\circ)^2 + (\operatorname{cosec} 45^\circ - \sin 45^\circ)^2 - (\cot 45^\circ - \tan 45^\circ)^2$$

$$= \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 - (1 - 1)^2$$

$$= \frac{1}{2} + \frac{1}{2} - 0 = 1$$

16. (b) $\tan \theta + \cot \theta = 2$

If we put $\theta = 45^\circ$,

$$\tan 45^\circ + \cot 45^\circ = 2$$

$$1 + 1 = 2 \Rightarrow 2 = 2$$

So, $\theta = 45^\circ$

$$\tan^5 \theta + \cot^{10} \theta$$

$$= \tan^5 45^\circ + \cot^{10} 45^\circ$$

$$= (1)^5 + (1)^{10} = 1 + 1 = 2$$

17. (a) $\sin \theta - \cos \theta = \frac{7}{13}$

$$\sin \theta + \cos \theta = \sqrt{2 - x^2}$$

$$= \sqrt{2 - \left(\frac{7}{13}\right)^2} = \sqrt{2 - \left(\frac{49}{169}\right)}$$

$$= \sqrt{\frac{289}{169}} = \frac{17}{13}$$

18. (c) $2 \cos \theta - \sin \theta = \frac{1}{\sqrt{2}}$

put value of $\theta = 45^\circ$,

$$2 \cos 45^\circ - \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$2 \times \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

So, it satisfies with question.

Now,

$$2 \sin \theta + \cos \theta = 2 \sin 45^\circ + \cos 45^\circ$$

$$= 2 \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

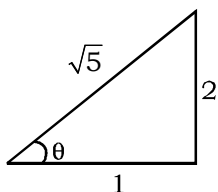
19. (b) $\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = 3$

$$\sin \theta + \cos \theta = 3 \sin \theta - 3 \cos \theta$$

$$2 \sin \theta = 4 \cos \theta \Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{2}{1}$$

$$\Rightarrow \tan \theta = \frac{2}{1} = \frac{P}{B}$$

$$\left[\therefore \sin \theta = \frac{P}{H} = \frac{2}{\sqrt{5}} \text{ and } \cos \theta = \frac{B}{H} = \frac{1}{\sqrt{5}} \right]$$



$$\begin{aligned} \Rightarrow \sin^4 \theta - \cos^4 \theta \\ \Rightarrow (\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta) \\ \Rightarrow 1(\sin^2 \theta - \cos^2 \theta) \\ \Rightarrow \left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2 = \frac{4}{5} - \frac{1}{5} = \frac{3}{5} \end{aligned}$$

$$\begin{aligned} \text{20. (a) } \sec^2 \theta + \tan^2 \theta &= 7 \\ \Rightarrow 1 + \tan^2 \theta + \tan^2 \theta &= 7 \\ \Rightarrow 2\tan^2 \theta &= 6 \\ \Rightarrow \tan^2 \theta &= 3 \\ \Rightarrow \tan \theta &= \sqrt{3} = \tan 60^\circ \\ \Rightarrow \theta &= 60^\circ \end{aligned}$$

Alternate:-

$$\begin{aligned} \text{take help from option} \\ \text{put } \theta &= 60^\circ, \\ \sec^2 60^\circ + \tan^2 60^\circ &= 7 \\ (2)^2 + (\sqrt{3})^2 &= 7 \\ 7 &= 7 \text{ (matched)} \\ \text{So, } \theta &= 60^\circ \end{aligned}$$

$$\begin{aligned} \text{21. (d) } (\sec x \cdot \sec y + \tan x \cdot \tan y)^2 - (\sec x \cdot \tan y + \tan x \cdot \sec y)^2 \\ \text{put value of } x=y=45^\circ, \\ = (\sec 45^\circ \sec 45^\circ + \tan 45^\circ \tan 45^\circ)^2 \\ - (\sec 45^\circ \tan 45^\circ + \tan 45^\circ \sec 45^\circ)^2 \\ = (\sqrt{2} \times \sqrt{2} + 1 \times 1)^2 - (\sqrt{2} \times 1 + 1 \times \sqrt{2})^2 \\ = (2+1)^2 - (2\sqrt{2})^2 = 9 - 8 = 1 \end{aligned}$$

$$\begin{aligned} \text{22. (b) According to question,} \\ A &= \sin^2 \theta + \cos^4 \theta \\ \text{Put } \theta &= 90^\circ \text{ for maximum value of } A \\ A &= \sin^2 90^\circ + \cos^4 90^\circ \\ A &= 1 + 0 \\ A &= 1 \\ \text{Put } \theta &= 45^\circ \\ \text{for minimum value of } A \\ A &= \sin^2 45^\circ + \cos^4 45^\circ \\ A &= \frac{1}{2} + \frac{1}{4} \\ A &= \frac{3}{4} \therefore A \text{ lies in } \frac{3}{4} \leq A \leq 1 \end{aligned}$$

$$\begin{aligned} \text{23. (d) } \sin \theta + \operatorname{cosec} \theta &= 2 \\ \text{put } \theta &= 90^\circ \end{aligned}$$

$$\begin{aligned} \Rightarrow \sin 90^\circ + \operatorname{cosec} 90^\circ &= 2 \\ \Rightarrow 1 + 1 &= 2 \\ \Rightarrow 2 &= 2 \end{aligned}$$

It, satisfies the question

$$\begin{aligned} \sin^5 \theta + \operatorname{cosec}^5 \theta \\ = \sin^5 90^\circ + \operatorname{cosec}^5 90^\circ \\ = (1)^5 + (1)^5 = 1 + 1 = 2 \end{aligned}$$

$$\begin{aligned} \text{24. (c) } \tan 2\theta \cdot \tan 4\theta &= 1 \\ [\text{If } \tan A \cdot \tan B = 1, \text{ then } A + B = 90^\circ] \\ 2\theta + 4\theta &= 90^\circ \\ \Rightarrow 6\theta &= 90^\circ \\ \Rightarrow 3\theta &= 45^\circ \\ \therefore \tan 3\theta &= \tan 45^\circ = 1 \end{aligned}$$

$$\begin{aligned} \text{25. (b) } \cos^2 \alpha + \cos^2 \beta &= 2 \\ \text{put value of } \alpha &= \beta = 0^\circ \\ \Rightarrow \cos^2 0^\circ + \cos^2 0^\circ &= 2 \\ \Rightarrow (1)^2 + (1)^2 &= 2 \\ \Rightarrow 2 &= 2 \\ [\text{If satisfies the question}] \\ \tan^3 \alpha + \sin^5 \beta \\ = \tan^3 0^\circ + \sin^5 0^\circ &= 0 + 0 = 0 \end{aligned}$$

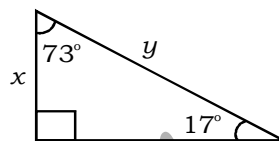
$$\begin{aligned} \text{26. (b)} \\ \text{Diagram: A triangle ABC with interior angles } \angle B = 75^\circ \text{ and } \angle C = \frac{\pi}{4}. \\ \frac{\pi^c}{4} = \frac{180^\circ}{4} = 45^\circ \\ \angle BAC = 180^\circ - 75^\circ - 45^\circ = 60^\circ \\ 180^\circ \rightarrow \pi^c \\ 1^\circ \rightarrow \frac{\pi^c}{180^\circ} \\ 60^\circ \rightarrow \frac{\pi^c}{180^\circ} \times 60^\circ = \frac{\pi}{3} \text{ radian} \end{aligned}$$

$$\begin{aligned} \text{27. (c) } (x+5)^\circ + (2x-3)^\circ + (3x+4)^\circ &= 180^\circ \\ (\text{Sum of all angles in triangle is } 180^\circ) \\ 6x + 6^\circ &= 180^\circ \\ (x+1) &= 30^\circ \\ \therefore x &= 29^\circ \end{aligned}$$

$$\begin{aligned} \text{28. (d) } A + B &= 90^\circ \\ (\text{Complementary angle}) \\ \text{We can put } A &= B = 45^\circ \\ \text{Or } A &= 30^\circ, B = 60^\circ \\ &= \sec^2 A + \sec^2 B - \sec^2 A \cdot \sec^2 B \\ &= \sec^2 45^\circ + \sec^2 45^\circ - \sec^2 45^\circ \cdot \sec^2 45^\circ \end{aligned}$$

$$\begin{aligned} &= (\sqrt{2})^2 + (\sqrt{2})^2 - (\sqrt{2})^2 \cdot (\sqrt{2})^2 \\ &= 4 - 4 = 0 \end{aligned}$$

$$\text{29. (b) } \sin 17^\circ = \frac{x}{y} \Rightarrow \frac{P}{H}$$



$$\begin{aligned} \Rightarrow \frac{\sqrt{y^2 - x^2}}{y} - \sin 73^\circ \\ = \frac{\sqrt{y^2 - x^2}}{y} - \frac{\sqrt{y^2 - x^2}}{y} \\ = \frac{y^2 - (y^2 - x^2)}{(y)(\sqrt{y^2 - x^2})} \\ = \frac{y^2 - y^2 + x^2}{y\sqrt{y^2 - x^2}} = \frac{x^2}{y\sqrt{y^2 - x^2}} \end{aligned}$$

$$\begin{aligned} \text{30. (d) } \frac{\cot 30^\circ - \cot 75^\circ}{\tan 15^\circ - \tan 60^\circ} \\ = \frac{\tan 60^\circ - \tan 15^\circ}{\tan 15^\circ - \tan 60^\circ} \\ = \frac{-(\tan 15^\circ - \tan 60^\circ)}{\tan 15^\circ - \tan 60^\circ} = -1 \end{aligned}$$

Alternate:-

$$\begin{aligned} \frac{\cot 30^\circ - \cot 75^\circ}{\tan 15^\circ - \tan 60^\circ} &= \frac{\cot 30^\circ - \cot 75^\circ}{\cot 75^\circ - \cot 30^\circ} \\ &= \frac{\cot 30^\circ - \cot 75^\circ}{-(\cot 30^\circ - \cot 75^\circ)} = -1 \end{aligned}$$

$$\begin{aligned} \text{31. (a) } \cot \theta \cdot \tan (90^\circ - \theta) - \sec (90^\circ - \theta) \operatorname{cosec} \theta + (\sin^2 25^\circ + \sin^2 65^\circ) \\ + \sqrt{3} (\tan 5^\circ \cdot \tan 15^\circ \cdot \tan 30^\circ \cdot \tan 75^\circ \cdot \tan 85^\circ) \\ = \cot \theta \cdot \cot \theta - \operatorname{cosec} \theta \cdot \operatorname{cosec} \theta \\ + (\sin^2 25^\circ + \cos^2 25^\circ) + \sqrt{3} [(\tan 5^\circ \cdot \tan 85^\circ) \cdot (\tan 15^\circ \cdot \tan 75^\circ) \cdot \tan 30^\circ] \\ = (\cot^2 \theta - \operatorname{cosec}^2 \theta) + 1 + \sqrt{3} (1 \cdot 1 \cdot \frac{1}{\sqrt{3}}) \\ \left[\tan A \cdot \tan B = 1 \right] \\ \text{If } A+B=90^\circ \\ = (-1) + 1 + \sqrt{3} \times \frac{1}{\sqrt{3}} \\ = -1 + 1 + 1 = 1 \end{aligned}$$

32. (b) $\sin(3x - 20^\circ) = \cos(3y + 20^\circ)$
 [If, $\sin A = \cos B$ then, $A + B = 90^\circ$]
 $\Rightarrow (3x - 20^\circ) + (3y + 20^\circ) = 90^\circ$
 $\Rightarrow 3x + 3y = 90^\circ$
 $\Rightarrow x + y = 30^\circ$

Alternate:-

$\sin(3x - 20^\circ) = \cos(3y + 20^\circ)$
 $\sin(3x - 20^\circ) = \sin[90^\circ - (3y + 20^\circ)]$
 $3x - 20^\circ = 90^\circ - 3y - 20^\circ$
 $(\because \sin(90^\circ - \theta) = \cos \theta)$
 $3x + 3y = 90^\circ$
 $\Rightarrow x + y = 30^\circ$

33. (d) $\cos \theta \cdot \operatorname{cosec} 23^\circ = 1$

$$\cos \theta \cdot \frac{1}{\sin 23^\circ} = 1$$

$$\Rightarrow \cos \theta = \sin 23^\circ$$

(If $\sin A = \cos B$ then $A + B = 90^\circ$)

$$\frac{1}{\operatorname{cosec} A} = \cos B$$

$$\sin A = \cos B$$

$$A + B = 90^\circ$$

$$23^\circ + B = 90^\circ$$

$$B = 90^\circ - 23^\circ = 67^\circ$$

$$\cos B = \cos \theta$$

$$\Rightarrow B = \theta = 67^\circ$$

$$\therefore \theta = 67^\circ$$

Alternate:-

$$\cos \theta \cdot \operatorname{cosec} 23^\circ = 1$$

$$\cos \theta \cdot \frac{1}{\sin 23^\circ} = 1$$

$$\cos \theta = \sin 23^\circ$$

$$\cos \theta = \cos(90^\circ - 23^\circ)$$

$$\theta = 90^\circ - 23^\circ$$

$$\therefore \theta = 67^\circ$$

34. (b) $2(\cos^2 \theta - \sin^2 \theta) = 1$

$$\Rightarrow \cos^2 \theta - \sin^2 \theta = \frac{1}{2}$$

$$\Rightarrow 2\sin^2 \theta = 1 - \frac{1}{2}$$

$$\Rightarrow \sin^2 \theta = \frac{1}{4}$$

$$\Rightarrow \sin \theta = \frac{1}{2} = \sin 30^\circ$$

$$\theta = 30^\circ$$

35. (a) $\sec(7\theta + 28^\circ) = \operatorname{cosec}(30^\circ - 3\theta)$

[If $\sec A \cdot \sin B = 1$, then $A + B = 90^\circ$]

$$\Rightarrow (7\theta + 28^\circ) + (30^\circ - 3\theta) = 90^\circ$$

$$\Rightarrow 4\theta + 58^\circ = 90^\circ$$

$$\Rightarrow 4\theta = 32^\circ \Rightarrow \theta = 8^\circ$$

36. (c) $\tan\left[\frac{\pi}{2} - \frac{\theta}{2}\right] = \sqrt{3}$

$$\Rightarrow \tan\left[90^\circ - \frac{\theta}{2}\right] = \sqrt{3} [\pi = 180^\circ]$$

$$\Rightarrow \cot \frac{\theta}{2} = \sqrt{3} = \cot 30^\circ$$

$$\Rightarrow \frac{\theta}{2} = 30^\circ$$

$$\Rightarrow \theta = 60^\circ$$

$$\therefore \cos 60^\circ = \frac{1}{2}$$

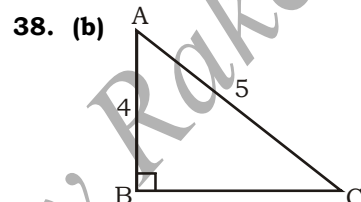
37. (a) Since, value of $\cos \theta$ decreases, from 0° to 90° at 45° it is equal to the value of $\sin \theta$.

Similarly,

Value of $\sin \theta$ increases from 0 to 90° and at 45° it is equal to the value of $\cos \theta$

For $0^\circ < \theta < 45^\circ$, $\cos \theta > \sin \theta$

So, value of $\cos 25^\circ - \sin 25^\circ$ is always positive but less than 1.



$\triangle ABC$,

$$\cos A = \frac{4}{5} \text{ i.e.,}$$

$$AB = 4 \text{ and } AC = 5$$

$$\sin C = \frac{AB}{AC} = \frac{4}{5}$$

39. (c) Since, α and β are complementary angle.

$$\therefore \alpha = 90^\circ - \beta$$

Now,

$$\sqrt{\cos \alpha \operatorname{cosec} \beta - \cos \alpha \sin \beta}$$

$$= \sqrt{\frac{\cos \alpha}{\sin \beta} - \cos \alpha \sin \beta}$$

$$= \sqrt{\frac{\cos \alpha}{\cos(90^\circ - \beta)} - \cos \alpha \cdot \cos(90^\circ - \beta)}$$

$$= \sqrt{\frac{\cos \alpha}{\cos \alpha} - \cos \alpha \cdot \cos \alpha}$$

$$= \sqrt{1 - \cos^2 \alpha} = \sqrt{\sin^2 \alpha} = \sin \alpha$$

40. (c) $\therefore 2\cot \theta = 3$

$$\Rightarrow \cot \theta = \frac{3}{2}$$

divide numerator and denominator by $\sin \theta$,

$$\therefore \frac{2\cos \theta - \sin \theta}{2\cos \theta + \sin \theta} = \frac{2\cot \theta - 1}{2\cot \theta + 1}$$

$$= \frac{2 \times \frac{3}{2} - 1}{2 \times \frac{3}{2} + 1} = \frac{3 - 1}{3 + 1} = \frac{2}{4} = \frac{1}{2}$$

41. (d) $\sin^6 \theta + \cos^6 \theta$

$$= (\sin^2 \theta)^3 + (\cos^2 \theta)^3$$

$$= (\sin^2 \theta + \cos^2 \theta)(\sin^4 \theta + \cos^4 \theta - \sin^2 \theta \cos^2 \theta)$$

$$= (\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta$$

$$(\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$= (1 - 3\sin^2 \theta \cos^2 \theta)$$

$$= 1 - 3 \times \frac{1}{4} = 1 - \frac{3}{4} = \frac{1}{4}$$

42. (d) By trigonometric identity,

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$$

$$\Rightarrow \sec \theta - \tan \theta = \frac{1}{2} \quad \dots (i)$$

and given, $\sec \theta + \tan \theta = 2$

On adding Eqs. (i) and (ii) we get,

$$2\sec \theta = \frac{1}{2} + 2 \quad \dots (ii)$$

$$\sec \theta = \frac{5}{4}$$

Alternate:-

$$\sec \theta + \tan \theta = 2$$

$$\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = 2$$

$$\frac{1 + \sin \theta}{\cos \theta} = 2$$

Squaring both sides,

$$\frac{(1 + \sin \theta)^2}{\cos^2 \theta} = 4$$

$$\frac{(1 + \sin \theta)^2}{1 - \sin^2 \theta} = 4$$

$$\frac{(1 + \sin \theta)^2}{(1 - \sin \theta)(1 + \sin \theta)} = 4$$

$$\frac{1 + \sin \theta}{1 - \sin \theta} = 4$$

$$1 + \sin \theta = 4 - 4\sin \theta$$

$$5\sin\theta = 3$$

$$\sin\theta = \frac{3}{5}$$

$$\cos\theta = \sqrt{1 - \sin^2\theta}$$

$$\cos\theta = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$\cos\theta = \sqrt{1 - \frac{9}{25}}$$

$$\cos\theta = \frac{4}{5}$$

$$\therefore \sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$$

$$43. (b) \operatorname{cosec}(75^\circ + \theta) - \sec(15^\circ - \theta) - \tan(55^\circ + \theta) + \cot(35^\circ - \theta)$$

$$\Rightarrow \operatorname{cosec}(75^\circ + \theta) - \operatorname{cosec}[90^\circ - (15^\circ - \theta)]$$

$$- \tan(55^\circ + \theta) + \tan[90^\circ - (35^\circ - \theta)]$$

$$\Rightarrow \operatorname{cosec}(75^\circ + \theta) - \operatorname{cosec}(75^\circ + \theta) - \tan(55^\circ + \theta) + \tan(55^\circ + \theta) = 0$$

$$44. (c) \sin\theta + 2\cos\theta = 1$$

On squaring both sides, we get

$$(\sin\theta + 2\cos\theta)^2 = 1$$

$$\Rightarrow \sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = 1$$

$$\Rightarrow (1 - \cos^2\theta) + 4(1 - \sin^2\theta) + 4\sin\theta\cos\theta = 1$$

$$\Rightarrow -(\cos^2\theta + 4\sin^2\theta) + 4\sin\theta\cos\theta = 1 - 5$$

$$\Rightarrow \cos^2\theta + 4\sin^2\theta - 4\sin\theta\cos\theta = 4$$

$$\Rightarrow (2\sin\theta - \cos\theta)^2 = 4$$

$$\Rightarrow 2\sin\theta - \cos\theta = 2$$

$$45. (b) \tan 8\theta = \cot 2\theta$$

$$\Rightarrow \tan 8\theta = \tan(90^\circ - 2\theta)$$

$$\Rightarrow 8\theta = 90^\circ - 2\theta$$

$$\Rightarrow \theta = 9^\circ$$

$$\therefore \tan 5\theta$$

$$\Rightarrow \tan 45^\circ = 1$$

$$46. (a) \sin(A + B) = 1$$

$$A + B = \sin^{-1} 1$$

$$\Rightarrow (A + B) = 90^\circ$$

$$(\because \sin^{-1} 1 = 90^\circ)$$

$$\therefore B = 90^\circ - A \Rightarrow A = 90^\circ - B$$

$$\text{Now,}$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$= \cos(90^\circ - B) \cos B + \sin(90^\circ - B) \sin B$$

$$= \sin B \cos B + \cos B \sin B$$

$$= 2\sin B \cos B = \sin 2B$$

$$47. (c) \therefore 5\sin\theta + 12\cos\theta = 13$$

On squaring both sides, we get

$$25\sin^2\theta + 144\cos^2\theta + 120\sin\theta\cos\theta = 169$$

$$\Rightarrow 25(1 - \cos^2\theta) + 144(1 - \sin^2\theta) + 120\sin\theta\cos\theta = 169$$

$$\Rightarrow 25 - 25\cos^2\theta + 144 - 144\sin^2\theta + 120\sin\theta\cos\theta = 169$$

$$\Rightarrow 25\cos^2\theta + 144\sin^2\theta - 120\sin\theta\cos\theta = 169 - 169$$

$$\Rightarrow (5\cos\theta - 12\sin\theta)^2 = 0$$

$$\therefore 5\cos\theta - 12\sin\theta = 0$$

$$48. (d) \frac{4\sin\theta - \cos\theta}{4\sin\theta + 9\cos\theta}$$

On dividing both numerator and denominator by $\cos\theta$, we get

$$\frac{4\sin\theta - \cos\theta}{4\sin\theta + 9\cos\theta} = \frac{\frac{4\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}{\frac{4\sin\theta}{\cos\theta} + \frac{9\cos\theta}{\cos\theta}} = \frac{4\tan\theta - 1}{4\tan\theta + 9}$$

$$= \frac{3 - 1}{3 + 9} = \frac{2}{12} = \frac{1}{6}$$

$$49. (c) \therefore \sin\theta - \cos\theta = 0$$

$$\therefore \sin\theta = \cos\theta$$

Since, $\sin\theta$ and $\cos\theta$ are equal for $\theta = 45^\circ$

$$\therefore \sin^4\theta + \cos^4\theta = (\sin 45^\circ)^4 + (\cos 45^\circ)^4$$

$$\left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$50. (a) \frac{(\sin\theta + \cos\theta)(\tan\theta + \cot\theta)}{\sec\theta + \operatorname{cosec}\theta}$$

$$= \frac{(\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)}{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}$$

$$= \frac{(\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)}{\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}}$$

$$(\because \sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{(\sin\theta + \cos\theta)\left(\frac{1}{\sin\theta\cos\theta}\right)}{\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}}$$

$$\frac{\sin\theta + \cos\theta}{\frac{\sin\theta\cos\theta}{\sin\theta + \cos\theta}} = 1$$

Alternate:-

$$\frac{(\sin\theta + \cos\theta)(\tan\theta + \cot\theta)}{\sec\theta + \operatorname{cosec}\theta}$$

$$\text{Put } \theta = 45^\circ$$

$$\Rightarrow \frac{\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)(1+1)}{\sqrt{2} + \sqrt{2}}$$

$$\Rightarrow \frac{\frac{2}{\sqrt{2}} \times 2}{2\sqrt{2}} = \frac{4}{\sqrt{2}} \times \frac{1}{2\sqrt{2}} = 1$$

$$51. (b) \sin^6\theta + \cos^6\theta + 3\sin^2\theta \cos^2\theta$$

$$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta \cos^2\theta$$

$$\cos^2\theta (\sin^2\theta + \cos^2\theta)$$

$$[\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)]$$

Here,

$$a = \sin^2\theta \text{ and } b = \cos^2\theta$$

$$= (\sin^2\theta + \cos^2\theta)^3$$

$$= (\sin^2\theta + \cos^2\theta)^3$$

$$= (1)^3 = 1 [\because \sin^2\theta + \cos^2\theta = 1]$$

$$52. (a) \frac{(1 + \sec\theta - \tan\theta)\cos\theta}{(1 + \sec\theta + \tan\theta)(1 - \sin\theta)}$$

$$= \frac{\left(1 + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)\cos\theta}{\left(1 + \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta)}$$

$$= \frac{\left(\frac{\cos\theta + 1 - \sin\theta}{\cos\theta}\right)\cos\theta}{\frac{(\cos\theta + 1 + \sin\theta)(1 - \sin\theta)}{\cos\theta}}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\cos\theta + 1 + \sin\theta - \sin\theta\cos\theta - \sin\theta - \sin^2\theta}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\cos\theta + 1 - \sin^2\theta - \sin\theta\cos\theta}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\cos\theta + \cos^2\theta - \sin\theta\cos\theta}$$

$$[\because 1 - \sin^2\theta = \cos^2\theta]$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\cos\theta(\cos\theta + 1 - \sin\theta)} = 1$$

53. (a) $\sin \theta + \cos \theta = \sqrt{3}$

On squaring both sides, we get,

$$(\sin \theta + \cos \theta)^2 = (\sqrt{3})^2$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 3$$

$$\Rightarrow 1 + 2 \sin \theta \cos \theta = 3$$

$$(\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$\Rightarrow \sin \theta \cos \theta = \frac{3-1}{2} = \frac{2}{2} = 1$$

Now,

$$\tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta}$$

$$\therefore \tan \theta + \cot \theta = \frac{1}{1} = 1$$

54. (b) $\tan \theta + \sec \theta = m$

$$\Rightarrow \sec \theta = m - \tan \theta$$

On squaring both sides, we get

$$(\sec \theta)^2 = (m - \tan \theta)^2$$

$$\Rightarrow \sec^2 \theta = m^2 + \tan^2 \theta - 2m \tan \theta$$

$$\Rightarrow \sec^2 \theta - \tan^2 \theta = m^2 - 2m \tan \theta$$

$$\Rightarrow 1 = m^2 - 2m \tan \theta$$

$$(\because \sec^2 \theta - \tan^2 \theta = 1)$$

$$\Rightarrow \tan \theta = \frac{m^2 - 1}{2m}$$

On putting the value of $\tan \theta$ in initial equation, we get

$$\frac{m^2 - 1}{2m} + \sec \theta = m$$

$$\Rightarrow \sec \theta = m - \frac{m^2 - 1}{2m}$$

$$\therefore \sec \theta = \frac{2m^2 - m^2 + 1}{2m} = \frac{m^2 + 1}{2m}$$

Alternate:-

$$\tan \theta + \sec \theta = m$$

$$\text{Put } \theta = 0^\circ$$

$$\Rightarrow \tan 0^\circ + \sec 0^\circ = m$$

$$\Rightarrow 0 + 1 = m$$

$$\Rightarrow m = 1$$

$$\therefore \sec \theta = \sec 0^\circ = 1$$

Now check from option

Option:- (b) : $\frac{m^2 + 1}{2m} = \frac{1+1}{2 \times 1}$
 $= \frac{2}{2} = 1$ (Satisfy)

55. (a) $\operatorname{cosec}(75^\circ + \theta) - \sec(15^\circ - \theta)$
 $= \operatorname{cosec}(75^\circ + \theta) - \sec[90^\circ - (75^\circ + \theta)]$
 $= \operatorname{cosec}(75^\circ + \theta) - \operatorname{cosec}(75^\circ + \theta)$
 $= 0$

56. (c) In $\triangle ABC$, if $\angle C$ is 90° , then
 $\angle A + \angle B = 180^\circ - 90^\circ = 90^\circ$
 Now,
 $\cos(A + B) + \sin(A + B)$
 $= 0 + 1 = 1$

57. (c) $\sin \alpha = \frac{\sqrt{3}}{2}$

$$\Rightarrow \alpha = 60^\circ \left(\because \sin 60^\circ = \frac{\sqrt{3}}{2} \right)$$

Now,

$$\cos \beta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \beta = 30^\circ \left(\because \sin 30^\circ = \frac{\sqrt{3}}{2} \right)$$

$$\text{and } \tan \gamma = 1$$

$$\Rightarrow \gamma = 45^\circ \left(\because \tan 45^\circ = 1 \right)$$

$$\therefore \alpha + \beta + \gamma = 60^\circ + 30^\circ + 45^\circ = 135^\circ$$

58. (b) Given that, $\cos A + \cos^2 A = 1$
 $\Rightarrow \cos A = 1 - \cos^2 A = \sin^2 A$ (i)

Now,

$$2(\sin^2 A + \sin^4 A)$$

$$= 2(\sin^2 A + \cos^2 A) \text{ [from Eq. (i)]}$$

$$= 2 \cdot (1) \left[\because \sin^2 \theta + \cos^2 \theta = 1 \right] = 2$$

59. (c) $(1 - \tan A)^2 + (1 + \tan A)^2 + (1 - \cot A)^2 + (1 + \cot A)^2$
 $= 1 + \tan^2 A - 2 \tan A + 1 + \tan^2 A$
 $+ 2 \tan A + 1 + \cot^2 A - 2 \cot A + 1$
 $+ \cot^2 A + 2 \cot A$
 $= 4 + 2(\tan^2 A + \cot^2 A)$
 $= (2 + 2 \tan^2 A) + (2 + 2 \cot^2 A)$
 $= 2 \sec^2 A + 2 \operatorname{cosec}^2 A$

$$= 2 \left(\frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} \right)$$

$$= 2 \left(\frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A} \right)$$

$$= \frac{2 \cdot (1)}{\sin^2 A \cos^2 A}$$

$$= 2 \sec^2 A \operatorname{cosec}^2 A$$

Alternate:-

$$(1 - \tan A)^2 + (1 + \tan A)^2 + (1 - \cot A)^2 + (1 + \cot A)^2$$

$$\text{Put } A = 45^\circ$$

$$(1-1)^2 + (1+1)^2 + (1-1)^2 + (1+1)^2$$

$$4 + 4 = 8$$

Now check from option

Option:- (c)

$$2 \sec^2 A \operatorname{cosec}^2 A$$

$$= 2 \times \sec^2 45^\circ \operatorname{cosec}^2 45^\circ$$

$$= 2 \times 2 \times 2 = 8 \text{ (satisfy)}$$

60. (c) $\frac{\tan A - \sin A}{\sin^3 A} = \frac{\frac{\sin A}{\cos A} - \sin A}{\sin^3 A}$

$$= \frac{\sin A \left(\frac{1}{\cos A} - 1 \right)}{\sin^3 A} = \frac{1 - \cos A}{\cos A \sin^2 A}$$

Rationalising above equation

$$= \frac{(1 - \cos A)}{\cos A \sin^2 A} \times \frac{(1 + \cos A)}{(1 + \cos A)}$$

$$= \frac{(1 - \cos^2 A)}{\cos A \sin^2 A (1 + \cos A)}$$

$$= \frac{\sin^2 A}{\cos A \sin^2 A (1 + \cos A)}$$

$$= \frac{1}{\cos A} \cdot \frac{1}{1 + \cos A} = \frac{\sec A}{1 + \cos A}$$

Alternate:-

$$\frac{\tan A - \sin A}{\sin^3 A}$$

$$\text{Put } A = 45^\circ$$

$$= \frac{\tan 45^\circ - \sin 45^\circ}{\sin^3 45^\circ} = \frac{1 - \frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}} \right)^3}$$

$$= \frac{\frac{\sqrt{2}-1}{\sqrt{2}}}{\frac{1}{2\sqrt{2}}} = \frac{\sqrt{2}-1}{\sqrt{2}} \times 2\sqrt{2}$$

$$= 2(\sqrt{2}-1)$$

Now check from options

Option:- $\frac{\sec A}{1 + \cos A}$

$$= \frac{\sec 45^\circ}{1 + \cos 45^\circ} = \frac{\sqrt{2}}{1 + \frac{1}{\sqrt{2}}}$$

$$= \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2} + 1} = \frac{2}{\sqrt{2} + 1}$$

Rationalise above equation

$$= \frac{2}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1}$$

$$= \frac{2(\sqrt{2}-1)}{2-1} = 2(\sqrt{2}-1) \text{ (Satisfy)}$$

61. (b) Given that, $\sin \theta = \frac{x^2 - y^2}{x^2 + y^2}$

$$\therefore \cos^2 \theta = 1 - \sin^2 \theta$$

$$= 1 - \left(\frac{x^2 - y^2}{x^2 + y^2} \right)^2$$

$$= \frac{(x^2 + y^2)^2 - (x^2 - y^2)^2}{(x^2 + y^2)^2}$$

$$= \frac{2x^2 \cdot 2y^2}{(x^2 + y^2)^2}$$

$$= \frac{4x^2 \cdot y^2}{(x^2 + y^2)^2} = \left(\frac{2xy}{x^2 + y^2} \right)^2$$

$$\therefore \cos \theta = 2xy / x^2 + y^2$$

Alternate:-

$$\sin \theta = \frac{x^2 - y^2}{x^2 + y^2}$$

$$\text{Put } \theta = 30^\circ$$

$$x = \sqrt{3}$$

$$y = 1$$

$$\sin 30^\circ = \frac{(\sqrt{3})^2 - (1)^2}{(\sqrt{3})^2 + (1)^2}$$

$$\frac{1}{2} = \frac{3-1}{3+1}$$

$$\frac{1}{2} = \frac{2}{4}$$

$$\frac{1}{2} = \frac{1}{2}$$

62. (d) Given that,

$$\Rightarrow a^2 = \frac{1+2\sin\theta \cdot \cos\theta}{1-2\sin\theta \cdot \cos\theta}$$

$$\Rightarrow a^2 = \frac{(\sin^2\theta + \cos^2\theta) + 2\sin\theta \cdot \cos\theta}{(\sin^2\theta + \cos^2\theta) - 2\sin\theta \cdot \cos\theta}$$

$$\Rightarrow a^2 = \frac{(\sin\theta + \cos\theta)^2}{(\sin\theta - \cos\theta)^2}$$

$$\Rightarrow \frac{a}{1} = \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

(applying componendo dividendo formula)

$$\Rightarrow \frac{a+1}{a-1} = \frac{(\sin\theta + \cos\theta) + (\sin\theta - \cos\theta)}{(\sin\theta + \cos\theta) - (\sin\theta - \cos\theta)}$$

$$\Rightarrow \frac{a+1}{a-1} = \frac{2\sin\theta}{2\cos\theta} = \tan\theta$$

63. (d) Given that, θ lies in first quadrant and $\tan \theta = 3$

$$\therefore \tan^2 \theta = 9 \Rightarrow 1 + \tan^2 \theta = 10$$

$$\Rightarrow \sec^2 \theta = 10 \Rightarrow \sec \theta = \sqrt{10}$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{10}} \quad \dots\dots(i)$$

$$\therefore \sin^2 \theta = 1 - \cos^2 \theta$$

$$= 1 - \frac{1}{10} = \frac{9}{10}$$

$$\Rightarrow \sin \theta = \frac{3}{\sqrt{10}} \quad \dots\dots(ii)$$

Now,

$$\sin \theta + \cos \theta = \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \frac{4}{\sqrt{10}}$$

(since, θ lies in first quadrant)

64. (d) If $0^\circ < \theta < 90^\circ$, then all the trigonometric ratios can be obtained when any one of the six ratios is given.

Since, We use any of the following identity to get any trigonometric ratios

$$\sin^2 \theta + \cos^2 \theta = 1, 1 + \tan^2 \theta$$

$$= \sec^2 \theta \text{ and } 1 + \cot^2 \theta$$

$$= \operatorname{cosec}^2 \theta$$

65. (d) $\sin A \cdot \cos A \cdot \tan A + \cos A \cdot \sin A \cdot \cot A$

$$= \sin A \cdot \cos A \cdot \frac{\sin A}{\cos A} + \cos A \cdot \sin A \cdot \frac{\cos A}{\sin A}$$

$$\sin A \cdot \frac{\cos A}{\sin A} = \sin^2 A + \cos^2 A = 1$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \operatorname{cosec}^2 A - \cot^2 A$$

$$[\because \operatorname{cosec}^2 A - \cot^2 A = 1]$$

Alternate:-

$$\sin A \cos A \tan A + \cos A \sin A \cot A$$

$$\text{Put } A = 45^\circ$$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 1 + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} = 1$$

Now check from option,

$$\operatorname{cosec}^2 A - \cot^2 A$$

$$\text{Put } A = 45^\circ$$

$$(\sqrt{2})^2 - 1$$

$$2 - 1 = 1$$

(Satisfy)

66. (a) Let

$$f(\theta) = \frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta}$$

$$= \frac{2\sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}{1 + 2\cos^2 \frac{\theta}{2} - 1} + \frac{1 + 2\cos^2 \frac{\theta}{2} - 1}{2\sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}$$

$$= \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} + \frac{\cos \frac{\theta}{2}}{\sin \frac{\theta}{2}}$$

$$= \frac{2}{2} \cdot \frac{(\sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2})}{\sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}$$

$$= \frac{2}{\sin \theta} = 2\operatorname{cosec} \theta$$

Alternate:-

$$\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta}$$

$$\text{Put } \theta = 90^\circ$$

$$\frac{\sin 90^\circ}{1 + \cos 90^\circ} + \frac{1 + \cos 90^\circ}{\sin 90^\circ}$$

$$\frac{1}{1} + \frac{1}{1} = 2$$

Now check the option,

Option: (a)

$$2\operatorname{cosec} \theta$$

$$= 2\operatorname{cosec} 90^\circ$$

$$= 2 \times 1 = 2 \text{ (Satisfy)}$$

67. (b) Given that, $\sin \theta \cdot \cos \theta = \frac{\sqrt{3}}{4}$

$$\therefore \sin^4 \theta + \cos^4 \theta = (\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta \cos^2 \theta$$

$$= (1)^2 - 2(\sin \theta \cdot \cos \theta)^2$$

$$= 1 - 2\left(\frac{\sqrt{3}}{4}\right)^2 = 1 - 2 \cdot \frac{3}{16} = 1 - \frac{3}{8} = \frac{5}{8}$$

Alternate:-

$$\sin \theta \cos \theta = \frac{\sqrt{3}}{4}$$

$$\text{Put } \theta = 30^\circ$$

$$\sin 30^\circ \cos 30^\circ = \frac{\sqrt{3}}{4}$$

$$\Rightarrow \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

$$\Rightarrow \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4} \quad \text{(Satisfy)}$$

$$\therefore \sin^4 \theta + \cos^4 \theta$$

$$\Rightarrow \sin^4 30^\circ + \cos^4 30^\circ$$

$$\Rightarrow \left(\frac{1}{2}\right)^4 + \left(\frac{\sqrt{3}}{2}\right)^4$$

$$\Rightarrow \frac{1}{16} + \frac{9}{16}$$

$$\Rightarrow \frac{1+9}{16} = \frac{10}{16}$$

$$\Rightarrow \frac{5}{8}$$

68. (a) We know that, $\sin^2 \theta + \cos^2 \theta = 1$ is true

I. $\sin^2 1^\circ + \cos^2 1^\circ = 1$ which is also true.

II. $\sec^2 33^\circ - \cot^2 57^\circ = \operatorname{cosec}^2 37^\circ - \tan^2 53^\circ$

Now,

$$\sec^2(90 - 57)^\circ = \operatorname{cosec}^2 57^\circ \text{ and } \cot^2 57^\circ = \cot^2(90 - 33)^\circ = \tan^2 33^\circ$$

$$\therefore \sec^2 33^\circ - \cot^2 57^\circ = \operatorname{cosec}^2 57^\circ - \tan^2 33^\circ$$

Hence,

Statement II is not true.

69. (c) Given,

$$p = a \sin x + b \cos x$$

$$q = a \cos x - b \sin x$$

$$\Rightarrow p^2 = a^2 \sin^2 x + b^2 \cos^2 x + 2ab \sin x \cos x$$

$$\text{and } q^2 = a^2 \cos^2 x + b^2 \sin^2 x - 2ab \sin x \cos x$$

$$\therefore p^2 + q^2 = a^2 (\sin^2 x + \cos^2 x) + b^2 (\cos^2 x + \sin^2 x) = a^2 + b^2$$

70. (c) Given that, $\sin^2 x + \cos^2 x - 1 = 0$
 $\sin^2 x + \cos^2 x = 1$

which is an identity of trigonometric ratio and always true for every real value of x .

So, the equation have an infinite solution.

71. (b) I. Given that, $\sin x + \cos x = 2$
 On Squaring both sides, we get

$$\Rightarrow (\sin x + \cos x)^2 = 4$$

$$\Rightarrow (\sin^2 x + \cos^2 x) + 2 \sin x \cdot \cos x = 4$$

$$\Rightarrow 1 + \sin 2x = 4$$

$$\Rightarrow \sin 2x = 3 \Rightarrow \sin 2x \neq 3$$

Hence, there is no value of x in the first quadrant that satisfies $\sin x + \cos x = 2$

II. $\sin x - \cos x = 0$

$$\Rightarrow \tan x = 1 = \tan \frac{\pi}{4} \Rightarrow x = \frac{\pi}{4}$$

Also, there is only one value of x in the first quadrant that satisfies in $\sin x - \cos x = 0$

72. (c) $7 \sin^2 \theta + 3 \cos^2 \theta = 4$

$$\Rightarrow 7 \sin^2 \theta + 3(1 - \sin^2 \theta) = 4$$

$$\Rightarrow 7 \sin^2 \theta + 3 - 3 \sin^2 \theta = 4$$

$$\Rightarrow 4 \sin^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = \frac{1}{4}$$

$$\Rightarrow \sin \theta = \frac{1}{2} = \sin 30^\circ$$

$$\theta = 30^\circ = \frac{\pi}{6} \quad [\because \pi^\circ = 180^\circ]$$

73. (b) $\tan(2\theta + 45^\circ) = \cot 3\theta$

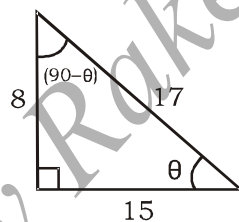
[if $\tan A = \cot B$ then $A + B = 90^\circ$]

$$(2\theta + 45^\circ) + 3\theta = 90^\circ$$

$$5\theta + 45^\circ = 90^\circ$$

$$\theta = \frac{45}{5} = 9^\circ$$

74. (b) $\cos \theta = \frac{15 \rightarrow \text{Base}}{17 \rightarrow \text{Hypo.}}$



$$\text{perpendicular} = 8$$

$$= \cot(90^\circ - \theta) = \tan \theta$$

$$= \frac{8}{15} \quad [\because \tan \theta = \frac{P}{B}]$$

75. (a) $(\sec^4 \theta - \tan^4 \theta)$

$$\Rightarrow (\sec^2 \theta - \tan^2 \theta) (\sec^2 \theta + \tan^2 \theta)$$

$$\Rightarrow 1 \times (\sec^2 \theta + \tan^2 \theta)$$

$$(\because \sec^2 \theta - \tan^2 \theta = 1)$$

$$\Rightarrow 1 \times \frac{7}{12} = \frac{7}{12}$$

76. (b) $\sec x = \operatorname{cosec} y$

$$\left[\begin{array}{l} \text{if } \sec A = \operatorname{cosec} B \\ \text{then } A + B = 90^\circ \end{array} \right]$$

$$x + y = 90^\circ$$

$$\sin(x + y) = \sin 90^\circ = 1$$

77. (c) $A + B + C = \pi = 180^\circ$

$$\Rightarrow \frac{A+B}{2} = \frac{180}{2} - \frac{C}{2}$$

$$\Rightarrow \sin\left(\frac{A+B}{2}\right)$$

$$= \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cos \frac{C}{2}$$

Similarly,

$$\cos\left(\frac{A+B}{2}\right) = \sin \frac{C}{2}$$

$$\cot\left(\frac{A+B}{2}\right) = \tan \frac{C}{2}$$

$$\tan\left(\frac{A+B}{2}\right) = \cot \frac{C}{2}$$

So, option (C) is incorrect

78. (c) $\tan 2\theta \tan 3\theta = 1$

$$(2\theta + 3\theta) = 90^\circ$$

$$5\theta = 90^\circ$$

(If $\tan A \cdot \tan B = 1$ then $A + B = 90^\circ$)

$$= 2 \cos^2 \frac{5\theta}{2} - 1 = 2 \cos^2 45^\circ - 1$$

$$= 2 \times \frac{1}{2} - 1 = 0$$

79. (d) $2 \sin \left[\frac{\pi x}{2} \right] = x^2 + \frac{1}{x^2}$

$$\text{Let } x = 1$$

$$2 \sin 90 \cdot 1 = 1^2 + \frac{1}{1^2}$$

$$2 \times 1 = 1 + 1$$

$$2 = 2 \text{ matched, so } x = 1$$

$$\text{so, } x - \frac{1}{x}$$

$$\Rightarrow 1 - \frac{1}{1} = 0$$

80. (d) $\cos \theta + \sec \theta = 2$

$$\text{Put } \theta = 0^\circ$$

$$\cos 0^\circ + \sec 0^\circ = 2$$

$$\Rightarrow 1 + 1 = 2 \Rightarrow 2 = 2$$

$$(\text{matched, so } \theta = 0^\circ)$$

$$= \cos^6 \theta + \sec^6 \theta$$

$$= \cos^6 0^\circ + \sec^6 0^\circ$$

$$= (1)^6 + (1)^6 = 1 + 1 = 2$$

$$\begin{aligned}
 81. (a) \quad & \frac{5}{\sec^2 \theta} + \frac{2}{1 + \cot^2 \theta} + 3\sin^2 \theta = \left(-\frac{1}{2}\right) \left[\frac{(1+3+2\sqrt{3}) + (1+3-2\sqrt{3})}{1-3} \right] \\
 & = 5\cos^2 \theta + \frac{2}{\operatorname{cosec}^2 \theta} + 3\sin^2 \theta = \left(-\frac{1}{2}\right) \left[\frac{4+2\sqrt{3}+4-2\sqrt{3}}{-2} \right] \\
 & = 5\cos^2 \theta + 2\sin^2 \theta + 3\sin^2 \theta = -\frac{1}{2} \left[-\frac{8}{2} \right] = \frac{1}{2} \times 4 = 2 \\
 & = 5(\cos^2 \theta + \sin^2 \theta) \\
 & (\because \sin^2 \theta + \cos^2 \theta = 1) \\
 & = 5
 \end{aligned}$$

$$\begin{aligned}
 82. (c) \quad & \left[\frac{1}{\cos \theta} + \frac{1}{\cot \theta} \right] \left[\frac{1}{\cos \theta} - \frac{1}{\cot \theta} \right] \\
 & = (\sec \theta + \tan \theta) (\sec \theta - \tan \theta) \\
 & = \sec^2 \theta - \tan^2 \theta \\
 & [\because 1 + \tan^2 \theta = \sec^2 \theta] = 1
 \end{aligned}$$

$$\begin{aligned}
 83. (c) \quad & \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} \times \frac{5}{4} \\
 \Rightarrow & 4\sin \theta + 4\cos \theta = 5\sin \theta - 5\cos \theta \\
 \Rightarrow & \sin \theta = 9\cos \theta \\
 \Rightarrow & \frac{\sin \theta}{\cos \theta} = 9 \\
 \Rightarrow & \tan \theta = 9 \\
 \frac{\tan^2 \theta + 1}{\tan^2 \theta - 1} &= \frac{9^2 + 1}{9^2 - 1} = \frac{82}{80} = \frac{41}{40}
 \end{aligned}$$

$$\begin{aligned}
 84. (c) \quad & \tan 7\theta \cdot \tan 2\theta = 1 \\
 [\text{If, } \tan A \cdot \tan B = 1 \text{ then, } A + B = 90^\circ] \\
 \Rightarrow & 7\theta + 2\theta = 90^\circ \\
 \Rightarrow & 9\theta = 90^\circ \\
 \Rightarrow & \theta = 10^\circ \\
 \therefore \tan 3\theta &= \tan 30^\circ = \frac{1}{\sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 85. (d) \quad & (2\cos^2 \theta - 1) \left[\frac{1 + \tan \theta}{1 - \tan \theta} + \frac{1 - \tan \theta}{1 + \tan \theta} \right] \\
 \text{Put } \theta &= 60^\circ \\
 \Rightarrow & (2\cos^2 60^\circ - 1) \left[\frac{1 + \tan 60^\circ}{1 - \tan 60^\circ} + \frac{1 - \tan 60^\circ}{1 + \tan 60^\circ} \right] \\
 & = \left[2\left(\frac{1}{2}\right)^2 - 1 \right] \left[\frac{1 + \sqrt{3}}{1 - \sqrt{3}} + \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \right] \\
 & = \left[\frac{1}{2} - 1 \right] \left[\frac{(1 + \sqrt{3})^2 + (1 - \sqrt{3})^2}{(1 - \sqrt{3})(1 + \sqrt{3})} \right]
 \end{aligned}$$

Alternate (I):-

$$\begin{aligned}
 & (2\cos^2 \theta - 1) \times \left(\frac{1 + \tan \theta}{1 - \tan \theta} + \frac{1 - \tan \theta}{1 + \tan \theta} \right) \\
 & = (2\cos^2 \theta - (\sin^2 \theta + \cos^2 \theta)) \\
 & \times \left(\frac{(1 + \tan \theta)^2 + (1 - \tan \theta)^2}{1 - \tan^2 \theta} \right) \\
 & = (\cos^2 \theta - \sin^2 \theta) \\
 & \times \left(\frac{1 + \tan^2 \theta + 2\tan \theta + 1 + \tan^2 \theta - 2\tan \theta}{1 - \frac{\sin^2 \theta}{\cos^2 \theta}} \right) \\
 & = (\cos^2 \theta - \sin^2 \theta) \\
 & \left(\frac{2 + 2\tan^2 \theta}{\cos^2 \theta - \sin^2 \theta} \right) \times \cos^2 \theta \\
 & = 2(1 + \tan^2 \theta) \times \cos^2 \theta \\
 & = 2\sec^2 \theta \cdot \cos^2 \theta = 2
 \end{aligned}$$

Alternate(II):-

$$\begin{aligned}
 & (2\cos^2 \theta - 1) \left(\frac{1 + \tan \theta}{1 - \tan \theta} + \frac{1 - \tan \theta}{1 + \tan \theta} \right) \\
 \text{put } \theta &= 0^\circ \\
 & = (2\cos^2 0^\circ - 1) \times \left(\frac{1 + \tan 0^\circ}{1 - \tan 0^\circ} + \frac{1 - \tan 0^\circ}{1 + \tan 0^\circ} \right) \\
 & = (2 \times 1 - 1) \left(\frac{1+0}{1-0} + \frac{1-0}{1+0} \right) \\
 & = (2-1)(1+1) = 2 \\
 86. (c) \quad & \sec \theta + \tan \theta = 2 \dots\dots(i) \\
 \therefore \sec^2 \theta - \tan^2 \theta &= 1 \\
 \Rightarrow (\sec \theta - \tan \theta) (\sec \theta + \tan \theta) &= 1 \\
 \Rightarrow (\sec \theta - \tan \theta) &= \frac{1}{\sec \theta + \tan \theta} \\
 (\sec \theta - \tan \theta) &= \frac{1}{2} \dots\dots(ii) \\
 \text{adding equation (i) and (ii)} \\
 \sec \theta + \tan \theta + \sec \theta - \tan \theta &= 2 + \frac{1}{2}
 \end{aligned}$$

$$\Rightarrow 2\sec \theta = \frac{5}{2} \Rightarrow \sec \theta = \frac{5}{4}$$

$$\begin{aligned}
 87. (b) \quad & \sec \theta = \frac{4x^2 + 1}{4x} \\
 \tan \theta &= \sqrt{\sec^2 \theta - 1} \\
 &= \sqrt{\left[\frac{4x^2 + 1}{4x} \right]^2 - 1} \\
 &= \sqrt{\frac{(4x^2 + 1)^2 - (4x)^2}{(4x)^2}} \\
 &= \sqrt{\frac{16x^4 + 1 + 8x^2 - 16x^2}{(4x)^2}} \\
 &= \sqrt{\frac{16x^4 + 1 - 8x^2}{(4x)^2}} \\
 &= \sqrt{\frac{(4x^2 - 1)^2}{(4x)^2}} \\
 &= \frac{4x^2 - 1}{4x} \\
 \therefore \sec \theta + \tan \theta &= \frac{4x^2 + 1}{4x} + \frac{4x^2 - 1}{4x} \\
 &= \frac{4x^2 + 1 + 4x^2 - 1}{4x} \\
 &= \frac{8x^2}{4x} = 2x
 \end{aligned}$$

Alternate:-

$$\begin{aligned}
 \sec \theta &= x + \frac{1}{4x} \\
 \text{put } x &= 1 \\
 \sec \theta &= 1 + \frac{1}{4} = \frac{5}{4} = \frac{H}{B} \\
 \tan \theta &= \frac{P}{B} = \frac{3}{4} \\
 \text{Now,} \\
 \sec \theta + \tan \theta &= \frac{5}{4} + \frac{3}{4} = \frac{5+3}{4} = \frac{8}{4} = 2 \\
 \text{by option (b),} \\
 2x &= 2 \times 1 = 2
 \end{aligned}$$

$$\begin{aligned}
 88. (b) \sin^2 25^\circ + \sin^2 65^\circ \\
 &= \sin^2 25^\circ + \sin^2 (90^\circ - 25^\circ) \\
 &= \sin^2 25^\circ + \cos^2 25^\circ = 1
 \end{aligned}$$

Alternate:-

$$\begin{aligned}
 \text{If } A + B &= 90^\circ \\
 \text{then } \sin^2 A + \sin^2 B &= 1
 \end{aligned}$$

$$\text{So, } \sin^2 25^\circ + \sin^2 65^\circ = 1$$

$$\begin{aligned}
 99. (a) \sec \theta + \tan \theta &= \sqrt{3} \dots (i) \\
 \therefore \sec^2 \theta - \tan^2 \theta &= 1 \\
 \Rightarrow (\sec \theta - \tan \theta)(\sec \theta + \tan \theta) &= 1
 \end{aligned}$$

$$\Rightarrow \sec \theta - \tan \theta = \frac{1}{\sqrt{3}} \dots (ii)$$

Subtract equation (i) from (ii)

$$\Rightarrow 2 \tan \theta = \sqrt{3} - \frac{1}{\sqrt{3}}$$

$$\Rightarrow 2 \tan \theta = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow \theta = 30^\circ \left[\because \tan 30^\circ = \frac{1}{\sqrt{3}} \right]$$

$$\Rightarrow \tan 3\theta = \tan 90^\circ = \text{undefined}$$

$$\begin{aligned}
 90. (c) \sin(60^\circ - \theta) &= \cos(\psi - 30^\circ) \\
 (60^\circ - \theta) + (\psi - 30^\circ) &= 90^\circ
 \end{aligned}$$

$$\begin{aligned}
 [\text{if } \sin A &= \cos B \\
 \text{then } A + B &= 90^\circ]
 \end{aligned}$$

$$(\psi - \theta) = 90^\circ - 30^\circ$$

$$(\psi - \theta) = 60^\circ$$

$$\tan(\psi - \theta) = \tan 60^\circ$$

$$\tan 60^\circ = \sqrt{3}$$

$$91. (b) a \sin \theta + b \cos \theta = c \dots (i)$$

let

$$a \cos \theta - b \sin \theta = x \dots (ii)$$

Squaring and adding equation (i) and (ii)

$$= a^2 + b^2 = c^2 + x^2$$

$$= a^2 + b^2 - c^2 = x^2$$

$$= x = \pm \sqrt{a^2 + b^2 - c^2}$$

$$92. (b) \sin(A - B) = \frac{1}{2} = \sin 30^\circ$$

$$\therefore A - B = 30^\circ$$

$$\cos(A + B) = \frac{1}{2} = \cos 60^\circ$$

$$\therefore A + B = 60^\circ$$

Adding both side

$$\Rightarrow (A - B) + (A + B) = 30^\circ + 60^\circ$$

$$\Rightarrow 2A = 90^\circ$$

$$\Rightarrow A = 45^\circ$$

$$\therefore A - B = 30^\circ$$

$$B = A - 30^\circ = 45^\circ - 30^\circ = 15^\circ$$

$$\Rightarrow \frac{15^\circ \times \pi}{180^\circ} = \frac{\pi}{12} \text{ radian}$$

$$\begin{aligned}
 93. (c) 152 (\sin 30^\circ + 2 \cos^2 45^\circ \\
 + 3 \sin 30^\circ \dots \dots \dots + 17 \sin 30^\circ + \\
 18 \cos^2 45^\circ) \\
 \Rightarrow 152
 \end{aligned}$$

$$\left(\frac{1}{2} + 2 \left(\frac{1}{\sqrt{2}} \right)^2 + 3 \times \frac{1}{2} \dots \dots \dots + 17 \times \frac{1}{2} + 18 \left(\frac{1}{\sqrt{2}} \right)^2 \right)$$

$$\Rightarrow 152 \left[\frac{1}{2} + 1 + 1 \frac{1}{2} + \dots \dots \dots + 8 \frac{1}{2} + 9 \right]$$

$\Rightarrow$ This is in A.P. where,

$$a = \frac{1}{2}, d = \frac{1}{2}, n = 18$$

$$\Rightarrow S_{152} = 152 \left[\frac{18}{2} \left(2 \times \frac{1}{2} + (18-1) \frac{1}{2} \right) \right]$$

$$\Rightarrow 152 \left[\frac{18}{2} \left(1 + \frac{17}{2} \right) \right]$$

$$\Rightarrow 152 \times 9 \times \frac{19}{2} \Rightarrow 12996 \Rightarrow$$

$$\sqrt{12996} = 114$$

$$94. (d) 3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$$

$$\Rightarrow 3 \cos 80^\circ \frac{1}{\sin 10^\circ} + 2 \cos 59^\circ \frac{1}{\sin 31^\circ}$$

$$\Rightarrow 3 \cos 80^\circ \frac{1}{\sin(90^\circ - 80^\circ)} + 2$$

$$\cos 59^\circ \frac{1}{\sin(90^\circ - 59^\circ)}$$

$$\Rightarrow 3 + 2 = 5$$

$$95. (d) \sin^2 \theta - 3 \sin \theta + 2 = 0$$

$$\Rightarrow \sin^2 \theta - 2 \sin \theta - \sin \theta + 2 = 0$$

$$\Rightarrow \sin \theta (\sin \theta - 2) - 1 (\sin \theta - 2) = 0$$

$$\Rightarrow (\sin \theta - 1) (\sin \theta - 2) = 0$$

$$[\because \sin \theta \neq 2]$$

$$\therefore \sin \theta = 1 = \sin 90^\circ$$

$$\Rightarrow \theta = 90^\circ$$

Alternate:-

Put value of $\theta = 90^\circ$ [take help from options]

$$\sin^2 \theta - 3 \sin \theta + 2 = 0$$

$$\sin^2 90^\circ - 3 \sin 90^\circ + 2 = 0$$

$$1 - 3 \times 1 + 2 = 0$$

$$[\because \sin 90^\circ = 1]$$

$$0 = 0$$

[matched]

So, this is answer.

$$96. (c) \sin \alpha = m \sin \beta$$

Squaring both sides

$$\sin^2 \alpha = m^2 \sin^2 \beta \dots (i)$$

$$\tan \alpha = n \tan \beta$$

Squaring both sides

$$\tan^2 \alpha = n^2 \tan^2 \beta$$

$$\frac{\sin^2 \alpha}{\cos^2 \alpha} = n^2 \frac{\sin^2 \beta}{\cos^2 \beta}$$

(value put in $\sin^2 \beta$)

$$\frac{\sin^2 \alpha}{\cos^2 \alpha} = \frac{n^2 \sin^2 \alpha}{m^2 (1 - \sin^2 \beta)}$$

(from equation (i))

$$\frac{1}{\cos^2 \alpha} = \frac{n^2}{m^2 \left(1 - \frac{\sin^2 \alpha}{m^2} \right)}$$

$$(m^2 - \sin^2 \alpha) = n^2 \cos^2 \alpha$$

$$m^2 - (1 - \cos^2 \alpha) = n^2 \cos^2 \alpha$$

$$m^2 - 1 + \cos^2 \alpha = n^2 \cos^2 \alpha$$

$$m^2 - 1 = \cos^2 \alpha (n^2 - 1)$$

$$\cos^2 \alpha = \frac{m^2 - 1}{n^2 - 1}$$

Alternate:-

According to question,

$$\tan \alpha = n \tan \beta$$

$$\sin \alpha = m \sin \beta$$

$$n = \frac{\tan \alpha}{\tan \beta}, m = \frac{\sin \alpha}{\sin \beta}$$

Put $\alpha = 30^\circ$ and $\beta = 60^\circ$

$$n = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

$$m = \frac{\sin 30^\circ}{\sin 60^\circ} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$\text{Then, } \cos^2 \alpha = \cos^2 30^\circ = \frac{3}{4}$$

by option (c),

$$\frac{m^2 - 1}{n^2 - 1}$$

Put the value of m and n,

$$\frac{\left(\frac{1}{\sqrt{3}}\right)^2 - 1}{\left(\frac{1}{3}\right)^2 - 1} = \frac{\frac{1}{3} - 1}{\frac{1}{9} - 1}$$

$$= \frac{3}{4} \text{ (satisfied)}$$

97. (c) $\operatorname{cosec} \theta - \cot \theta = \frac{7}{2} \dots (i)$

$$\therefore \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$\Rightarrow [\operatorname{cosec} \theta - \cot \theta](\operatorname{cosec} \theta + \cot \theta) = 1$$

$$\Rightarrow (\operatorname{cosec} \theta + \cot \theta)$$

$$= \frac{1}{(\operatorname{cosec} \theta - \cot \theta)}$$

$$\Rightarrow \operatorname{cosec} \theta + \cot \theta = \frac{2}{7} \dots (ii)$$

Adding both equations,

$$\Rightarrow 2 \operatorname{cosec} \theta = \frac{7}{2} + \frac{2}{7}$$

$$\Rightarrow \frac{49 + 4}{14} = \frac{53}{14}$$

$$\Rightarrow \operatorname{cosec} \theta = \frac{53}{28}$$

98. (a) $x \sin 45^\circ = y \operatorname{cosec} 30^\circ$

$$\Rightarrow \frac{x}{y} = \frac{\operatorname{cosec} 30^\circ}{\sin 45^\circ}$$

$$\Rightarrow \frac{x}{y} = \frac{2}{1/\sqrt{2}} = \frac{2\sqrt{2}}{1}$$

$$\Rightarrow \frac{x^4}{y^4} = \left(\frac{2\sqrt{2}}{1}\right)^4 = \frac{64}{1}$$

$$= 4^3$$

99. (c) $\therefore 5 \tan \theta = 4$

$$\Rightarrow \tan \theta = \frac{4}{5}$$

$$\frac{5 \sin \theta - 3 \cos \theta}{5 \sin \theta + 2 \cos \theta}$$

Divide numerator and denominator by $\cos \theta$

$$= \frac{5 \frac{\sin \theta}{\cos \theta} - 3 \frac{\cos \theta}{\cos \theta}}{5 \frac{\sin \theta}{\cos \theta} + 2 \frac{\cos \theta}{\cos \theta}} = \frac{5 \tan \theta - 3}{5 \tan \theta + 2}$$

$$= \frac{\left(5 \times \frac{4}{5}\right) - 3}{\left(5 \times \frac{4}{5}\right) + 2} = \frac{1}{6}$$

100. (b) According to question,

$$2 \operatorname{cosec}^2 23^\circ \cot^2 67^\circ - \sin^2 23^\circ - \sin^2 67^\circ - \cot^2 67^\circ$$

$$= 2 \operatorname{cosec}^2 23^\circ \cot^2 (90^\circ - 23^\circ) - \sin^2 23^\circ - \sin^2 (90^\circ - 23^\circ) - \cot^2 67^\circ$$

$$= 2 \operatorname{cosec}^2 23^\circ \tan^2 23^\circ - (\sin^2 23^\circ + \cos^2 23^\circ) - \cot^2 67^\circ$$

$$= \frac{2}{\cos^2 23^\circ} - 1 - \cot^2 67^\circ$$

$$= 2 \sec^2 23^\circ - 1 - \cot^2 (90^\circ - 23^\circ)$$

$$= 2 \sec^2 23^\circ - 1 - \tan^2 23^\circ$$

$$= 2 \sec^2 23^\circ - (1 + \tan^2 23^\circ)$$

$$= 2 \sec^2 23^\circ - \sec^2 23^\circ = \sec^2 23^\circ$$

101. (c) $\cos^2 \theta = \frac{(x+y)^2}{4xy}$

$$\therefore \text{max. value of } \cos^2 \theta = 1$$

$$\therefore 1 = \frac{(x+y)^2}{4xy}$$

$$\Rightarrow 4xy = (x+y)^2$$

$$\Rightarrow 4xy = x^2 + y^2 + 2xy$$

$$\Rightarrow 0 = x^2 + y^2 - 2xy$$

$$\Rightarrow 0 = (x-y)^2$$

$$\Rightarrow 0 = x - y \Rightarrow x = y$$

102. (a) $(1 - \sin^2 \alpha)(1 - \cos^2 \alpha) \times (1 + \cot^2 \beta)(1 + \tan^2 \beta)$

$$= (\cos^2 \alpha)(\sin^2 \alpha)(\operatorname{cosec}^2 \beta)(\sec^2 \beta)$$

$$\text{put } \alpha = \beta = 45^\circ,$$

$$= \cos^2 45^\circ \cdot \sin^2 45^\circ \cdot \operatorname{cosec}^2 45^\circ \cdot \sec^2 45^\circ$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot 2 \cdot 2 = 1$$

Alternate:-

$$\alpha + \beta = 90^\circ \text{ or } \alpha = 90^\circ - \beta$$

$$(1 - \sin^2 \alpha)(1 - \cos^2 \alpha) \times$$

$$(1 + \cot^2 \beta)(1 + \tan^2 \beta)$$

$$= (\cos^2 \alpha)(\sin^2 \alpha)(\operatorname{cosec}^2 \beta)(\sec^2 \beta)$$

$$= \cos^2 (90^\circ - \beta) \cdot \sin^2 \alpha \cdot$$

$$\operatorname{cosec}^2 \beta \cdot \sec^2 (90^\circ - \alpha)$$

$$= \sin^2 \beta \cdot \operatorname{cosec}^2 \beta \cdot \sin^2 \alpha \cdot$$

$$\operatorname{cosec}^2 \alpha = 1$$

103. (c) $\frac{2 \sin 68^\circ}{\cos 22^\circ} - \frac{2 \cot 15^\circ}{5 \tan 75^\circ} -$

$$\frac{3 \tan 45^\circ \cdot \tan 20^\circ \cdot \tan 40^\circ \cdot \tan 50^\circ \cdot \tan 70^\circ}{5}$$

$$= \frac{2 \sin 68^\circ}{\cos (90^\circ - 68^\circ)} - \frac{2 \cot 15^\circ}{5 \tan (90^\circ - 15^\circ)} -$$

$$\frac{3 \times 1 \cdot (\tan 20^\circ \cdot \tan 70^\circ)(\tan 40^\circ \cdot \tan 50^\circ)}{5}$$

$$= \frac{2 \sin 68^\circ}{\sin 68^\circ} - \frac{2 \cot 15^\circ}{5 \cot 15^\circ}$$

$$- \frac{3 \times 1 \times 1 \times 1}{5}$$

$$(\tan A \cdot \tan B = 1 \text{ if } A + B = 90^\circ)$$

$$= 2 - \frac{2}{5} - \frac{3}{5} = 1$$

104. (b) $\sin 7x = \cos 11x$

$$\Rightarrow \sin 7x = \sin (90^\circ - 11x)$$

$$\Rightarrow 7x = 90^\circ - 11x$$

$$\Rightarrow 7x + 11x = 90^\circ$$

$$\Rightarrow 18x = 90^\circ$$

$$\Rightarrow x = 5^\circ$$

$$= \tan 9x + \cot 9x$$

$$= \tan 45^\circ + \cot 45^\circ$$

$$= 1 + 1 = 2$$

105. (a) $\tan^2 \alpha = 1 + 2 \tan^2 \beta$

$$\Rightarrow \sec^2 \alpha - 1 = 1 + 2(\sec^2 \beta - 1)$$

$$\Rightarrow \sec^2 \alpha - 1 = 2 \sec^2 \beta - 1$$

$$\Rightarrow \frac{1}{\cos^2 \alpha} = \frac{2}{\cos^2 \beta}$$

$$\Rightarrow 2 \cos^2 \alpha = \cos^2 \beta$$

$$\Rightarrow \sqrt{2} \cos \alpha = \cos \beta$$

$$\therefore \sqrt{2} \cos \alpha - \cos \beta = 0$$

Alternate:-

$$\tan^2 \alpha = 1 + 2 \tan^2 \beta$$

$$\text{Put } \beta = 45^\circ$$

$$\Rightarrow \tan^2 \alpha = 1 + 2 \cdot \tan^2 45^\circ$$

$$\Rightarrow \tan^2 \alpha = 3$$

$$\Rightarrow \tan \alpha = \sqrt{3}$$

$$\Rightarrow \alpha = 60^\circ$$

$$\text{Put } \alpha = 60^\circ, \beta = 45^\circ$$

$$\sqrt{2} \cos \alpha - \cos \beta$$

$$= \sqrt{2} \cos 60^\circ - \cos 45^\circ$$

$$= \sqrt{2} \times \frac{1}{2} - \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$$

$$106. (a) \tan \theta + \cot \theta = 2$$

$$\text{Put } \theta = 45^\circ$$

$$1 + 1 = 2 \text{ (matched)}$$

$$\text{So, } \theta = 45^\circ$$

$$\Rightarrow \tan^{100} 45^\circ + \cot^{100} 45^\circ$$

$$\Rightarrow 1^{100} + 1^{100} = 2$$

$$107. (d) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta}$$

$$= \frac{\tan \theta}{1 - \frac{1}{\tan \theta}} + \frac{\frac{1}{\tan \theta}}{1 - \tan \theta}$$

$$= \frac{\tan^2 \theta}{\tan \theta - 1} + \frac{1}{\tan \theta (1 - \tan \theta)}$$

$$= \frac{\tan^2 \theta}{\tan \theta - 1} - \frac{1}{\tan \theta (\tan \theta - 1)}$$

$$= \frac{\tan^3 \theta - 1}{\tan \theta (\tan \theta - 1)}$$

$$= \frac{(\tan \theta - 1)(\tan^2 \theta + \tan \theta + 1)}{\tan \theta (\tan \theta - 1)}$$

$$[\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]$$

$$= \frac{\tan^2 \theta + \tan \theta + 1}{\tan \theta}$$

$$= \tan \theta + \cot \theta + 1$$

$$108. (a)$$

$$\sec \theta + \tan \theta = 2 + \sqrt{5} \quad \dots(i)$$

$$\therefore \sec^2 \theta - \tan^2 \theta = 1$$

$$\Rightarrow (\sec \theta - \tan \theta)(\sec \theta + \tan \theta) = 1$$

$$\Rightarrow \sec \theta - \tan \theta = \frac{1}{2 + \sqrt{5}}$$

$$\Rightarrow \sec \theta - \tan \theta = \frac{1}{\sqrt{5} + 2}$$

$$\Rightarrow \sec \theta - \tan \theta = \sqrt{5} - 2 \quad \dots(ii)$$

$$\left[\because \frac{1}{\sqrt{5} + 2} \times \frac{\sqrt{5} - 2}{\sqrt{5} - 2} = \frac{\sqrt{5} - 2}{5 - 4} = \sqrt{5} - 2 \right]$$

$$\text{add eq (i) + (ii)}$$

$$2 \sec \theta = 2 + \sqrt{5} + \sqrt{5} - 2$$

$$\Rightarrow 2 \sec \theta = 2\sqrt{5}$$

$$\Rightarrow \sec \theta = \sqrt{5}$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{5}}$$

$$\therefore \sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \sin^2 \theta = 1 - \left(\frac{1}{\sqrt{5}} \right)^2$$

$$\Rightarrow \sin^2 \theta = \frac{4}{5}$$

$$\Rightarrow \sin \theta = \frac{2}{\sqrt{5}}$$

$$\therefore \sin \theta + \cos \theta$$

$$= \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} = \frac{3}{\sqrt{5}}$$

$$109. (b) (1 + \cot \theta - \operatorname{cosec} \theta)$$

$$(1 + \tan \theta + \sec \theta)$$

$$\text{Put, } \theta = 45^\circ$$

$$= (1 + \cot 45^\circ - \operatorname{cosec} 45^\circ)$$

$$(1 + \tan 45^\circ + \sec 45^\circ)$$

$$= (1 + 1 - \sqrt{2})(1 + 1 + \sqrt{2})$$

$$= (2 - \sqrt{2})(2 + \sqrt{2})$$

$$= [2^2 - (\sqrt{2})^2]$$

$$[\because (a - b)(a + b) = a^2 - b^2]$$

$$= 4 - 2 = 2$$

$$110. (a) x = a \sec \theta \cos \phi$$

$$y = b \sec \theta \sin \phi$$

$$z = c \tan \theta$$

$$\Rightarrow \frac{x}{a} = \sec \theta \cos \phi$$

$$\Rightarrow \frac{y}{b} = \sec \theta \sin \phi$$

$$\Rightarrow \frac{z}{c} = \tan \theta$$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$$

$$\sec^2 \theta \cdot \cos^2 \phi + \sec^2 \theta \cdot \sin^2 \phi - \tan^2 \theta$$

$$= \sec^2 \theta (\cos^2 \phi + \sin^2 \phi) - \tan^2 \theta$$

$$(\because \sin^2 \phi + \cos^2 \phi = 1)$$

$$= \sec^2 \theta - \tan^2 \theta = 1$$

$$111. (a) \frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} = \frac{5}{3}$$

$$\Rightarrow 3 \sec \theta + 3 \tan \theta = 5 \sec \theta - 5 \tan \theta$$

$$\Rightarrow 8 \tan \theta = 2 \sec \theta \Rightarrow \frac{\sec \theta}{\tan \theta} = 4$$

$$\Rightarrow \frac{1}{\cos \theta} \times \frac{\cos \theta}{\sin \theta} = 4 \Rightarrow \sin \theta = \frac{1}{4}$$

$$112. (a) \text{ Let,}$$

$$(1 + \sin \alpha)(1 + \sin \beta)(1 + \sin \gamma) = (1 -$$

$$\sin \alpha)(1 - \sin \beta)(1 - \sin \gamma) = x$$

$$\therefore x \cdot x = (1 + \sin \alpha)(1 - \sin \alpha)(1 + \sin \beta)$$

$$(1 - \sin \beta)(1 + \sin \gamma)(1 - \sin \gamma)$$

$$x^2 = (1 - \sin^2 \alpha)(1 - \sin^2 \beta)(1 - \sin^2 \gamma)$$

$$x^2 = \cos^2 \alpha \cdot \cos^2 \beta \cdot \cos^2 \gamma$$

$$x = \pm \cos \alpha \cdot \cos \beta \cdot \cos \gamma$$

$$113. (d)$$

$$\frac{1}{1 + \cot^2 \theta} + \frac{3}{1 + \tan^2 \theta} + 2 \sin^2 \theta$$

$$= \frac{1}{\operatorname{cosec}^2 \theta} + \frac{3}{\sec^2 \theta} + 2 \sin^2 \theta$$

$$= \sin^2 \theta + 3 \cos^2 \theta + 2 \sin^2 \theta$$

$$= 3(\sin^2 \theta + \cos^2 \theta)$$

$$= 3 \times (1) = 3$$

$$(\because \sin^2 \theta + \cos^2 \theta = 1)$$

Alternate:-

$$\frac{1}{1 + \cot^2 \theta} + \frac{3}{1 + \tan^2 \theta} + 2 \sin^2 \theta$$

$$\text{Put } \theta = 45^\circ$$

$$\Rightarrow \frac{1}{1+1} + \frac{3}{1+1} + 2 \times \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} + \frac{3}{2} + 1$$

$$\Rightarrow \frac{1+3+2}{2} = \frac{6}{2} = 3$$

$$114. (a)$$

$$\frac{4}{1 + \tan^2 \alpha} + \frac{1}{1 + \cot^2 \alpha} + 3 \sin^2 \alpha$$

$$\begin{aligned}
&= \frac{4}{\sec^2 \alpha} + \frac{1}{\operatorname{cosec}^2 \alpha} + 3 \sin^2 \alpha \\
&= 4 \cos^2 \alpha + \sin^2 \alpha + 3 \sin^2 \alpha \\
&= 4(\cos^2 \alpha + \sin^2 \alpha) \\
&= 4 \times (1) = 4 \\
&(\because \sin^2 \alpha + \cos^2 \alpha = 1)
\end{aligned}$$

Alternate:-

$$\begin{aligned}
&\frac{4}{1 + \tan^2 \alpha} + \frac{1}{1 + \cot^2 \alpha} + 3 \sin^2 \alpha \\
&\text{Put } \alpha = 45^\circ
\end{aligned}$$

$$\Rightarrow \frac{4}{1+1} + \frac{1}{1+1} + 3 \times \frac{1}{2}$$

$$\Rightarrow 2 + \frac{1}{2} + \frac{3}{2}$$

$$\Rightarrow \frac{4+1+3}{2} = \frac{8}{2} = 4$$

115. (d) $3(\sin x + \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$
Put, $x = 90^\circ$
 $3(\sin 90^\circ + \cos 90^\circ)^4 + 6(\sin 90^\circ + \cos 90^\circ)^2 + 4(\sin^6 90^\circ + \cos^6 90^\circ)$
 $= 3(1+0)^4 + 6(1+0)^2 + 4(1^6+0)$
 $= 3 + 6 + 4 = 13$

116. (c)

$$\sec \theta \left(\frac{1+\sin \theta}{\cos \theta} + \frac{\cos \theta}{1+\sin \theta} \right) - 2 \tan^2 \theta$$

$$\text{Take, } \theta = 0^\circ$$

$$= \sec 0^\circ \times \left(\frac{1+\sin 0^\circ}{\cos 0^\circ} + \frac{\cos 0^\circ}{1+\sin 0^\circ} \right) - 2 \tan^2 0^\circ$$

$$= 1 \times \left(\frac{1+0}{1} + \frac{1}{1+0} \right) - 2 \times 0 = 2$$

117. (a) $x \sin \theta + y \cos \theta = 4$

Squaring both sides,

$$x^2 \sin^2 \theta + y^2 \cos^2 \theta + 2xy \sin \theta \cos \theta = 16 \dots (i)$$

$$x \cos \theta - y \sin \theta = 2$$

$$\text{again squaring both sides,}$$

$$x^2 \cos^2 \theta + y^2 \sin^2 \theta - 2xy \sin \theta \cos \theta = 4 \dots (ii)$$

$$\text{on adding eqn. (i) and (ii),}$$

$$(x^2 + y^2)(\sin^2 \theta + \cos^2 \theta)$$

$$= 16 + 4$$

$$x^2 + y^2 = 20$$

118. (c) $\left[\frac{\cos^2 A \cdot \sin^2 A (\sin A + \cos A)}{(\sin A - \cos A)} + \right.$

$$\left. \frac{\sin^2 A \cdot \cos^2 A (\sin A - \cos A)}{(\sin A + \cos A)} \right] \times \left[\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A} \right]$$

$$= \left[\frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)} \right]$$

$$\times [\sin^2 A - \cos^2 A]$$

$$= 2(\sin^2 A + \cos^2 A) = 2$$

119. (b) $\frac{1}{\operatorname{cosec} \theta - \cot \theta} - \frac{1}{\sin \theta}$

$$= \frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\operatorname{cosec} \theta - \cot \theta} - \operatorname{cosec} \theta$$

$$[\because \operatorname{cosec}^2 \theta - \cot^2 \theta = 1]$$

$$= \operatorname{cosec} \theta + \cot \theta - \operatorname{cosec} \theta$$

$$= \cot \theta$$

120. (d) $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$

Squaring both sides,

$$\Rightarrow \cos^2 \theta + \sin^2 \theta + 2 \cos \theta \sin \theta$$

$$= 2 \cos^2 \theta$$

$$\Rightarrow 2 \cos^2 \theta - \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos \theta \sin \theta$$

$$\Rightarrow \cos^2 \theta - \sin^2 \theta$$

$$= 2 \sin \theta \cos \theta$$

$$\Rightarrow (\cos \theta - \sin \theta)(\cos \theta + \sin \theta)$$

$$= 2 \sin \theta \cos \theta$$

$$\Rightarrow (\cos \theta - \sin \theta)(\sqrt{2} \cos \theta)$$

$$= 2 \sin \theta \cos \theta$$

$$\Rightarrow \cos \theta - \sin \theta = \frac{2 \sin \theta \cos \theta}{\sqrt{2} \cos \theta}$$

$$= \sqrt{2} \sin \theta$$

Alternate:-

$$\cos \theta + \sin \theta = \sqrt{2} \cos \theta$$

$$\cos \theta - \sin \theta = x$$

$$2(\sin^2 \theta + \cos^2 \theta) = (\sqrt{2})^2 \cos^2 \theta + x^2$$

$$2 - 2 \cos^2 \theta = x^2$$

$$2(1 - \cos^2 \theta) = x^2$$

$$2 \sin^2 \theta = x^2$$

$$x = \sqrt{2} \sin \theta$$

121. (c) $\sin \theta - \cos \theta = \frac{1}{2} \dots (i)$

$$\sin \theta + \cos \theta = m \dots (ii)$$

on squaring and adding both sides,

$$2(\sin^2 \theta + \cos^2 \theta) = \left(\frac{1}{2} \right)^2 + m^2$$

$$\Rightarrow 2 = \frac{1}{4} + m^2$$

$$\Rightarrow m^2 = 2 - \frac{1}{4} = \frac{7}{4}$$

$$\Rightarrow m = \frac{\sqrt{7}}{2}$$

$$\therefore \sin \theta + \cos \theta = \frac{\sqrt{7}}{2}$$

122. (a) $\frac{\sin A}{1 + \cos A} + \frac{\sin A}{1 - \cos A}$

$$= \frac{\sin A(1 - \cos A) + \sin A(1 + \cos A)}{(1 + \cos A)(1 - \cos A)}$$

$$= \frac{\sin A - \sin A \cos A + \sin A + \sin A \cos A}{1 - \cos^2 A}$$

$$= \frac{2 \sin A}{\sin^2 A} = 2 \operatorname{cosec} A$$

123. (c) $\tan \theta - \cot \theta = 0$

$$\text{Put } \theta = 45^\circ$$

$$\tan 45^\circ - \cot 45^\circ = 0$$

$$1 - 1 = 0$$

$$0 = 0 \text{ (matched)}$$

$$\theta = 45^\circ$$

So,

$$\sin \theta + \cos \theta$$

$$= \sin 45^\circ + \cos 45^\circ$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

124. (a) $3 \sin \theta + 5 \cos \theta = 5$

$$5 \sin \theta - 3 \cos \theta = x$$

and squaring and adding both sides,

$$5^2(\sin^2 \theta + \cos^2 \theta) + 3^2$$

$$(\sin^2 \theta + \cos^2 \theta) = x^2 + 5^2$$

$$\Rightarrow 5^2 + 3^2 = x^2 + 5^2$$

$$\Rightarrow x^2 = 9$$

$$\therefore x = \pm 3$$

125. (c) $x \sin 60^\circ \cdot \tan 30^\circ$
 $= \sec 60^\circ \cdot \cot 45^\circ$

$$\Rightarrow x \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{3}} = 2 \cdot 1$$

$$\Rightarrow \frac{x}{2} = 2$$

$$\Rightarrow x = 4$$

126. (d) $\theta = 60^\circ$

$$= \frac{1}{2}\sqrt{1+\sin\theta} + \frac{1}{2}\sqrt{1-\sin\theta}$$

$$= \frac{1}{2}\sqrt{1+\sin 60^\circ} + \frac{1}{2}\sqrt{1-\sin 60^\circ}$$

$$= \frac{1}{2}\sqrt{1+\left(\frac{\sqrt{3}}{2}\right)} + \frac{1}{2}\sqrt{1-\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{1}{2\sqrt{2}}(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}})$$

$$= \frac{1}{2\sqrt{2}} \times \frac{1}{\sqrt{2}}(\sqrt{4+2\sqrt{3}} + \sqrt{4-2\sqrt{3}})$$

$$= \frac{1}{4}(\sqrt{(\sqrt{3}+1)^2} + \sqrt{(\sqrt{3}-1)^2})$$

$$= \frac{1}{4}(\sqrt{3}+1 + \sqrt{3}-1)$$

$$= \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} = \cos 30^\circ = \cos \frac{\theta}{2}$$

127. (b) $\frac{2\tan^2 30^\circ}{1-\tan^2 30^\circ} + \sec^2 45^\circ - \sec^2 0^\circ$
 $= x \sec 60^\circ$

$$\Rightarrow \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)^2}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} + (\sqrt{2})^2 - 1$$

$$= x \times 2$$

$$\Rightarrow \frac{2 \times \frac{1}{3}}{1 - \frac{1}{3}} + 2 - 1 = 2x$$

$$\Rightarrow \left(\frac{2}{3} \times \frac{3}{2}\right) + 2 - 1 = 2x$$

$$\Rightarrow 2 = x \times 2$$

$$\therefore x = 1$$

128. (b) $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$

Squaring both sides and after that adding '1' both sides,

$$\Rightarrow 1 + \tan^2 \theta = 1 + \frac{(\sin \alpha - \cos \alpha)^2}{(\sin \alpha + \cos \alpha)^2}$$

$$\Rightarrow \sec^2 \theta$$

$$= \frac{(\sin \alpha + \cos \alpha)^2 + (\sin \alpha - \cos \alpha)^2}{(\sin \alpha + \cos \alpha)^2}$$

$$(\because 1 + \tan^2 \theta = \sec^2 \theta)$$

$$\Rightarrow \sec^2 \theta = \frac{2(\sin^2 \alpha + \cos^2 \alpha)}{(\sin \alpha + \cos \alpha)^2}$$

$$\Rightarrow \frac{1}{\cos^2 \theta} = \frac{2}{(\sin \alpha + \cos \alpha)^2}$$

$$\Rightarrow \frac{1}{\cos \theta} = \frac{\pm \sqrt{2}}{\sin \alpha + \cos \alpha}$$

$$\Rightarrow \sin \alpha + \cos \alpha = \pm \sqrt{2} \cos \theta$$

129. (a) $7\sin^2 \theta + 3\cos^2 \theta = 4$

$$\Rightarrow 7\sin^2 \theta + 3(1 - \sin^2 \theta) = 4$$

$$\Rightarrow 7\sin^2 \theta + 3 - 3\sin^2 \theta = 4$$

$$\Rightarrow 4\sin^2 \theta = 4 - 3$$

$$\Rightarrow \sin^2 \theta = \frac{1}{4}$$

$$\Rightarrow \sin \theta = \frac{1}{2}$$

$$\Rightarrow \theta = 30^\circ$$

$$\therefore \tan \theta = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

130. (d) $\tan 9^\circ = \frac{p}{q}$

$$\frac{\sec^2 81^\circ}{1 + \cot^2 81^\circ} = \frac{\sec^2 81^\circ}{\operatorname{cosec}^2 81^\circ}$$

$$(\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta)$$

$$= \frac{1}{\cos^2 81^\circ} \times \sin^2 81^\circ$$

$$= \tan^2 81^\circ = \tan^2 (90^\circ - 9^\circ)$$

$$= \cot^2 9^\circ = \frac{1}{\tan^2 9^\circ} = \frac{q^2}{p^2}$$

131. (d) If $\sec \theta + \tan \theta = 5$ (i)

$$\therefore \sec^2 \theta - \tan^2 \theta = 1$$

$$\Rightarrow (\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$$

$$(\sec \theta - \tan \theta) = \frac{1}{5} \quad \text{....(ii)}$$

subtracting eq. (ii) from (i)

$$\Rightarrow (\sec \theta + \tan \theta) - (\sec \theta - \tan \theta)$$

$$= 5 - \frac{1}{5}$$

$$\Rightarrow 2 \tan \theta = \frac{25-1}{5} = \frac{24}{5}$$

$$\tan \theta = \frac{12}{5}$$

$$\Rightarrow \frac{\tan \theta + 1}{\tan \theta - 1} = \frac{\frac{12}{5} + 1}{\frac{12}{5} - 1} = \frac{12+5}{12-5} = \frac{17}{7}$$

132. (c) $\frac{\cos \alpha}{\cos \beta} = a$

$$= \cos \alpha = a \cos \beta$$

On squaring both sides,

$$\Rightarrow \cos^2 \alpha = a^2 \cos^2 \beta$$

$$\Rightarrow 1 - \sin^2 \alpha = a^2 (1 - \sin^2 \beta) \quad \text{....(i)}$$

$$\text{Again, } \sin \alpha = b \sin \beta$$

Squaring both sides

$$\Rightarrow \sin^2 \alpha = b^2 \sin^2 \beta$$

put the value of $\sin^2 \alpha$ in equation (i)

$$\Rightarrow 1 - b^2 \sin^2 \beta = a^2 (1 - \sin^2 \beta)$$

$$\Rightarrow 1 - b^2 \sin^2 \beta = a^2 - a^2 \sin^2 \beta$$

$$\Rightarrow a^2 \sin^2 \beta - b^2 \sin^2 \beta = a^2 - 1$$

$$\Rightarrow \sin^2 \beta (a^2 - b^2) = a^2 - 1$$

$$\Rightarrow \sin^2 \beta = \frac{a^2 - 1}{a^2 - b^2}$$

133. (b)

$$\frac{\cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$= \frac{\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - 1^2}{1}$$

$$(\because \sin^2 A + \cos^2 A = 1)$$

$$= \frac{1}{4} + \frac{4 \times 4}{3} - 1 = \frac{1}{4} + \frac{16}{3} - 1$$

$$= \frac{3+64-12}{12} = \frac{55}{12}$$

134. (d) $\cos \pi x = x^2 - x + \frac{5}{4}$

$$= x^2 - 2 \times x \times \frac{1}{2} + \frac{1}{4} - \frac{1}{4} + \frac{5}{4}$$

$$= \left(x - \frac{1}{2}\right)^2 + 1 > 1$$

$$= -1 \leq \cos x \leq 1$$

$\therefore$ so value of x is none of the above

135. (d) $1 + \frac{1}{\cot^2 63^\circ} - \sec^2 27^\circ$

$$+ \frac{1}{\sin^2 63^\circ} - \operatorname{cosec}^2 27^\circ$$

$$= 1 + \tan^2 63^\circ - \sec^2 27^\circ + \operatorname{cosec}^2 63^\circ - \operatorname{cosec}^2 27^\circ$$

$$= 1 + \cot^2 27^\circ - \sec^2 27^\circ + \sec^2 27^\circ - \operatorname{cosec}^2 27^\circ$$

$$= 1 + \cot^2 27^\circ - \operatorname{cosec}^2 27^\circ$$

$$= 1 - 1 = 0$$

136. (b) $x = \frac{\cos \theta}{1 - \sin \theta}$

$$= \frac{\cos \theta (1 + \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)}$$

$$= \frac{\cos \theta (1 + \sin \theta)}{1 - \sin^2 \theta}$$

$$= \frac{\cos \theta (1 + \sin \theta)}{\cos^2 \theta}$$

$$= \frac{1 + \sin \theta}{\cos \theta}$$

$$\frac{\cos \theta}{1 + \sin \theta} = \frac{1}{x}$$

Alternate:-

$$\therefore x = \frac{\cos \theta}{1 - \sin \theta}$$

put $\theta = 0^\circ$

$$x = \frac{\cos 0^\circ}{1 - \sin 0^\circ} = \frac{1}{1 - 0}$$

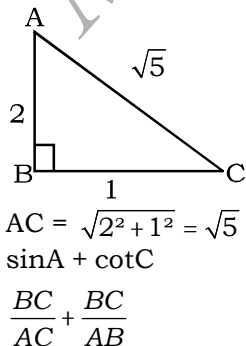
$$\Rightarrow x = 1$$

$$= \frac{\cos \theta}{1 + \sin \theta} = \frac{\cos 0^\circ}{1 + \sin 0^\circ} = \frac{1}{1 + 0}$$

$$= 1$$

now check option by putting $x = 1$ only option (b) satisfying.

137. (b)



$$\frac{1}{\sqrt{5}} + \frac{1}{2} \Rightarrow \frac{2 + \sqrt{5}}{2\sqrt{5}}$$

138. (b) $\sin \frac{\pi x}{2} = x^2 - 2x + 2$

put value of x from options

$$x = 1$$

$$\sin \frac{\pi}{2} \times 1 = 1^2 - 2 \times 1 + 2$$

$$\sin 90^\circ = 1 - 2 + 2$$

$$1 = 1 \text{ (satisfied)}$$

139. (a) $\frac{\sin 43^\circ}{\cos 47^\circ} + \frac{\cos 19^\circ}{\sin 71^\circ} - 8 \cos^2 60^\circ$

$$= 1 + 1 - \left(8 \times \left(\frac{1}{2} \right)^2 \right)$$

(If $A + B = 90^\circ$, then $\sin A = \cos B$)

$$\frac{\sin A}{\cos B} = 1 \text{ or } \frac{\cos B}{\sin A} = 1$$

$$= 2 - 2 = 0$$

140. (a) $\sin^2 7\frac{1}{2}^\circ + \sin^2 82\frac{1}{2}^\circ$

$$= \sin^2 7\frac{1}{2}^\circ + \sin^2 \left(90^\circ - 7\frac{1}{2}^\circ \right)$$

$$= \sin^2 7\frac{1}{2}^\circ + \cos^2 7\frac{1}{2}^\circ$$

$$= 1 \text{ (} \because \sin^2 \theta + \cos^2 \theta = 1 \text{)}$$

141. (c) Given that, $3 \sin x + 5 \cos x = 5$

On squaring both sides, we get

$$9 \sin^2 x + 25 \cos^2 x + 30 \sin x \cos x = 25$$

$$\Rightarrow 9(1 - \cos^2 x) + 25(1 - \sin^2 x) + 30 \sin x \cos x = 25$$

$$\Rightarrow 9 + 25 - (9 \cos^2 x + 25 \sin^2 x - 30 \sin x \cos x) = 25$$

$$\Rightarrow 9 = (3 \cos x - 5 \sin x)^2$$

$$\Rightarrow 3 \cos x - 5 \sin x = 3$$

142. (a) If $\alpha + \beta = 90^\circ$

Put $\alpha = 45^\circ$

$$\beta = 45^\circ$$

$$\sqrt{(\operatorname{cosec} \alpha \operatorname{cosec} \beta)} \times \left(\frac{\sin \alpha}{\sin \beta} + \frac{\cos \alpha}{\cos \beta} \right)^{-\frac{1}{2}}$$

$$\Rightarrow \sqrt{2 \times 2} \times \left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} + \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} \right)^{-\frac{1}{2}}$$

$$\Rightarrow \sqrt{2} \times (2)^{-\frac{1}{2}}$$

$$\Rightarrow \sqrt{2} \times \frac{1}{\sqrt{2}} = 1$$

143. (d) We know that, in a cyclic quadrilateral sum of opposite angle is 180°

$$\therefore A + C = 180^\circ \text{(i)}$$

and $B + D = 180^\circ \text{(ii)}$

$$\therefore \cos A + \cos B + \cos C + \cos D$$

$$= \cos A + \cos B + \cos(180^\circ - A) + \cos(180^\circ - B)$$

From Eqs. (i) and (ii),

$$= \cos A + \cos B - \cos A - \cos B = 0$$

144. (a) Given, $\alpha + \beta = 90^\circ \text{(i)}$

By given condition,

$$\beta = \frac{2}{3} \alpha$$

$$\therefore \beta = \frac{2}{3} \alpha = \frac{2}{3} (90^\circ - \beta)$$

{from Eq. (i)}

$$\Rightarrow \beta = 60^\circ - \frac{2}{3} \beta \Rightarrow \beta = 36^\circ$$

145. (b) Given, $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 2$

$$\therefore \sin^2 \theta + \cos^2 \theta = 2 \sin \theta \cos \theta$$

$$\Rightarrow \sin 2\theta = 1 = \sin 90^\circ$$

$$\Rightarrow 2\theta = 90^\circ \Rightarrow \theta = 45^\circ$$

146. (b) Given, $\sin 3\theta = \cos(\theta - 2^\circ)$

$$\Rightarrow \sin 3\theta = \sin[90^\circ - (\theta - 2^\circ)]$$

$$\Rightarrow 3\theta = 90^\circ - \theta + 2^\circ$$

$$\Rightarrow 4\theta = 92^\circ \Rightarrow \theta = \frac{92^\circ}{4} = 23^\circ$$

147. (b) $\frac{\sin^6 \theta - \cos^6 \theta}{\sin^2 \theta - \cos^2 \theta}$

$$= \frac{(\sin^2 \theta)^3 - (\cos^2 \theta)^3}{\sin^2 \theta - \cos^2 \theta}$$

$$= \frac{(\sin^2 \theta - \cos^2 \theta)(\sin^4 \theta + \cos^4 \theta + \sin^2 \theta \cos^2 \theta)}{\sin^2 \theta - \cos^2 \theta}$$

$$= \sin^4 \theta + \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta$$

$$= (\sin^2 \theta + \cos^2 \theta)^2 - \sin^2 \theta \cos^2 \theta$$

$$= 1 - \sin^2 \theta \cos^2 \theta$$

148. (c) $\tan A = \frac{1 - \cos B}{\sin B}$

Put $A = 30^\circ$, $B = 60^\circ$

$$\tan 30^\circ = \frac{1 - \cos 60^\circ}{\sin 60^\circ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1 - \frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ (Satisfy)}$$

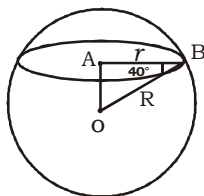
$$\therefore \frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$$

Now check the option,
Option (c):- $\tan B$

$$\tan 60^\circ = \sqrt{3}$$

149. (a)



In $\triangle OAB$,

$$\cos 40^\circ = \frac{AB}{OB} \Rightarrow \cos 40^\circ = \frac{r}{R}$$

$$\Rightarrow r = R \cos 40^\circ$$

So, the radius of the circle of latitude 40° S is $R \cos 40^\circ$.

150. (a) We know that, If value of $\cos \theta$ increases, then the value of θ decreases.

$$\therefore \cos \theta \geq \frac{1}{2}$$

$$\therefore \cos \theta \geq \cos 60^\circ \Rightarrow \theta \leq \frac{\pi}{3}$$

151. (b) Given, $\sin \theta + \cos \theta = 1$

On squaring both sides, we get

$$(\sin^2 \theta + \cos^2 \theta) + 2 \sin \theta \cos \theta = 1$$

$$\Rightarrow 1 + 2 \sin \theta \cos \theta = 1$$

$$\Rightarrow \sin \theta \cdot \cos \theta = 0$$

$$\mathbf{152. (b)} \quad \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} = \sqrt{\frac{(1 + \sin \theta)(1 + \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)}}$$

$$= \sqrt{\frac{(1 + \sin \theta)^2}{(1 - \sin^2 \theta)}} = \sqrt{\frac{(1 + \sin \theta)^2}{\cos^2 \theta}}$$

$$= \frac{1 + \sin \theta}{\cos \theta} = \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$$

$$= \sec \theta + \tan \theta$$

$$\mathbf{153. (b)} \quad \therefore \cos \theta = \frac{a^2 + b^2 - C^2}{2ab}$$

(By cosine rule)

$$= \frac{6^2 + 2^2 - C^2}{2 \times 6 \times 2} = \frac{40 - C^2}{24}$$

For acute angle,

$$\cos \theta > 0 \Rightarrow \frac{40 - C^2}{24} > 0 \Rightarrow C^2 < 40$$

$$\Rightarrow 0 < C < 2\sqrt{10}$$

(since, C cannot be negative)(i)

$$\text{Also, } b + c > a \Rightarrow 6 - 2 > c > 4$$

$$\text{From Eqs. (i) and (ii), } C \in (4, 2\sqrt{10})$$

154. (d) We know that, the value of $\cos \theta$ is decreasing from 0° to 90° .

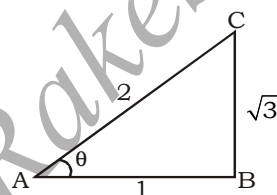
$$\therefore \cos 1^\circ > \cos 89^\circ$$

$$\Rightarrow p > q$$

Also, $\cos 1^\circ$ is close to 1 and $\cos 89^\circ$ is close to 0.

Hence, option (d) is correct.

155. (d)



$$\text{Given, } 7 \cos^2 \theta + 3 \sin^2 \theta = 4$$

$$\Rightarrow 7(1 - \sin^2 \theta) + 3(\sin^2 \theta) = 4$$

$$\Rightarrow 7 - 4 \sin^2 \theta = 4$$

$$\Rightarrow 4 \sin^2 \theta = 3$$

$$\Rightarrow \sin \theta = \pm \frac{\sqrt{3}}{2}$$

$$\text{For, } 0 < \theta < \frac{\pi}{2}$$

$$\sin \theta = \frac{\sqrt{3}}{2}, \theta = 60^\circ$$

$$\therefore \tan \theta = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\mathbf{156. (b)} \quad [(1 - \sin^2 \theta) \sec^2 \theta + \tan^2 \theta]$$

$$(\cos^2 \theta + 1)$$

$$(\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$= [\cos^2 \theta \cdot \sec^2 \theta + \tan^2 \theta](\cos^2 \theta + 1)$$

$$(\because \cos^2 \theta \cdot \sec^2 \theta = 1)$$

$$= (1 + \tan^2 \theta)(\cos^2 \theta + 1)$$

$$(\because \sec^2 \theta - \tan^2 \theta = 1)$$

$$= \sec^2 \theta (\cos^2 \theta + 1)$$

$$= \sec^2 \theta \cdot \cos^2 \theta + \sec^2 \theta$$

$$= 1 + \sec^2 \theta > 1 + 1 > 2$$

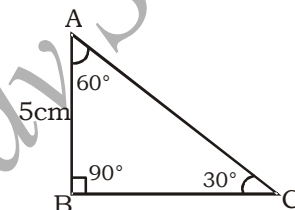
$$(\because \sec^2 \theta > 1 \text{ for } 0 < \theta < 90^\circ)$$

157. (d) We know in the interval $\theta \in$

$$\left[0, \frac{\pi}{2}\right], \sec^2 \theta \text{ is increasing from}$$

$$1 \text{ to } \infty \therefore p \geq 1$$

158. (a) In $\triangle ABC$,



$$\cos 60^\circ = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{5}{AC}$$

$$\therefore AC = 10 \text{ cm}$$

159. (a) Given, $\cos \theta + \sqrt{3} \sin \theta = 2$

$$\Rightarrow \frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta = 1$$

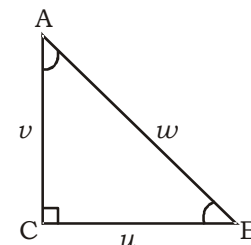
$$\Rightarrow \sin 30^\circ \cos \theta + \cos 30^\circ \sin \theta = 1$$

$$\Rightarrow \sin(30^\circ + \theta) = \sin 90^\circ$$

$$\Rightarrow 30^\circ + \theta = 90^\circ$$

$$\therefore \theta = 60^\circ = \frac{\pi}{3}$$

160. (d) In $\triangle ABC$,



$$\tan A = \frac{BC}{AC} = \frac{u}{v}$$

$$\text{and } \tan B = \frac{v}{u}$$

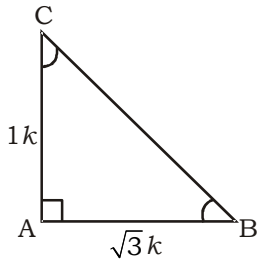
$$\text{Also, } u^2 + v^2 = w^2 \quad \dots (i)$$

(by Pythagoras theorem)

$$\therefore \tan A + \tan B = \frac{u}{v} + \frac{v}{u} = \frac{u^2 + v^2}{uv}$$

$$= \frac{w^2}{uv} \quad [\text{from Eq. (i)}]$$

161. (b) Given,



$$\tan B = \frac{k}{\sqrt{3}k}$$

In $\triangle ABC$,
 $AB^2 + AC^2 = BC^2$ (i)
 (by Pythagoras theorem)

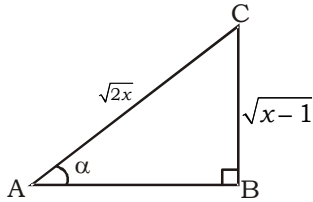
$$\Rightarrow (\sqrt{3}k)^2 + (1k)^2 = BC^2$$

$$\Rightarrow BC^2 = 4k^2 \Rightarrow BC = 2k$$

162. (a) Given,

$$\sin \alpha = \sqrt{\frac{x-1}{2x}}$$

In $\triangle ABC$, using Pythagoras theorem,
 $AC^2 = AB^2 + BC^2$



$$\Rightarrow 2x = AB^2 + (x-1)$$

$$\Rightarrow AB^2 = x+1 \Rightarrow AB = \sqrt{x+1}$$

$$\therefore \tan \alpha = \frac{BC}{AB} = \frac{\sqrt{x-1}}{\sqrt{x+1}} = \sqrt{\frac{x-1}{x+1}}$$

163. (c) Given,

$$\frac{\cos \theta}{1 - \sin \theta} - \frac{\cos \theta}{1 + \sin \theta} = 2$$

$$\Rightarrow \frac{\cos \theta + \sin \theta \cos \theta - \cos \theta + \cos \theta \sin \theta}{1 - \sin^2 \theta} = 2$$

$$(\because 1 - \sin^2 \theta = \cos^2 \theta)$$

$$\Rightarrow 2 \sin \theta \cos \theta = 2 \cos^2 \theta$$

$$\Rightarrow \sin \theta = \cos \theta$$

$$\Rightarrow \tan \theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

164. (b) As we know that, $\sin x$ is increasing from 0° to 90° .

$\therefore \sin y > \sin x$.

165. (b) $\sin^3 60^\circ \cdot \cot 30^\circ - 2 \sec^2 45^\circ + 3 \cos 60^\circ \cdot \tan^2 45^\circ - \tan^2 60^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)^3 \times \sqrt{3} - 2(\sqrt{2})^2 + 3$$

$$\times \left(\frac{1}{2}\right) \times (1)^2 - (\sqrt{3})^2$$

$$= \frac{9}{8} - 4 + \frac{3}{2} - 3 = \frac{-35}{8}$$

$$= \frac{1}{3} \times 1$$

$$\cos^4 \theta - \sin^4 \theta = \frac{1}{3}$$

$$(\because (a^2 - b^2)(a^2 + b^2) = a^4 - b^4)$$

172. (c) $\tan \theta = \frac{1}{\sqrt{11}}$

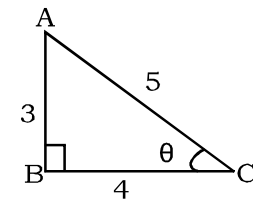
$$\frac{\cos \sec^2 \theta - \sec^2 \theta}{\cos \sec^2 \theta + \sec^2 \theta}$$

$$= \frac{\frac{1}{\sin^2 \theta} - \frac{1}{\cos^2 \theta}}{\frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta}} \Rightarrow \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta} = \frac{\sin^2 \theta \cdot \cos^2 \theta}{\sin^2 \theta \cdot \cos^2 \theta}$$

$$\Rightarrow \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta} = \frac{\cos^2 \theta \left(1 - \frac{\sin^2 \theta}{\cos^2 \theta}\right)}{\cos^2 \theta \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta}\right)}$$

$$\Rightarrow \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1 - \frac{1}{11}}{1 + \frac{1}{11}} = \frac{5}{6}$$

173. (b) $\sin \theta = \frac{3}{5}$



$$\sin \theta = \frac{3}{5} = \frac{P}{H}$$

So, $B = 4$

$P = 3$

$H = 5$

$$\Rightarrow \frac{\tan \theta + \cos \theta}{\cot \theta + \sec \theta} = \frac{\frac{P}{B} + \frac{B}{H}}{\frac{B}{P} + \frac{H}{P}}$$

$$\Rightarrow \frac{\frac{3}{4} + \frac{4}{5}}{\frac{4}{3} + \frac{5}{3}} = \frac{\frac{15+16}{20}}{\frac{4+5}{3}} = \frac{\frac{31}{20}}{\frac{9}{3}}$$

$$= \frac{31}{60}$$

174. (b) $(\sin \alpha + \csc \alpha)^2 + (\cos \alpha + \sec \alpha)^2 = k + \tan^2 \alpha + \cot^2 \alpha$
 put $\alpha = 45^\circ$,

166. (c) $\tan \theta = \frac{p}{q}$

$$= \frac{p \sec \theta - q \csc \theta}{p \sec \theta + q \csc \theta}$$

$$= \frac{\csc \theta \left(\frac{p \sec \theta}{\csc \theta} - q \right)}{\csc \theta \left(\frac{p \sec \theta}{\csc \theta} + q \right)}$$

$$= \frac{p \tan \theta - q}{p \tan \theta + q}$$

$$= \frac{p \times \frac{p}{q} - q}{p \times \frac{p}{q} + q}$$

$$= \frac{p^2 - q^2}{p^2 + q^2}$$

167. (b) $\csc^2 \theta - 2 + \sin^2 \theta$

$$= (\sin \theta - \csc \theta)^2$$

Hence, it is always non-negative.

168. (b) $1 - 2 \sin^2 \theta + \sin^4 \theta$

$$\Rightarrow (1 - \sin^2 \theta)^2$$

$$\Rightarrow (\cos^2 \theta)^2$$

$$\Rightarrow \cos^4 \theta$$

169. (c) $\sin \theta = 0.7$

$$\therefore \sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow (0.7)^2 + \cos^2 \theta = 1$$

$$\Rightarrow 0.49 + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - 0.49$$

$$\Rightarrow \cos \theta = \sqrt{0.51}$$

170. (c) $\sin^2 65^\circ + \sin^2 25^\circ + \cos^2 35^\circ + \cos^2 55^\circ$

$$= \sin^2 65^\circ + \sin^2 (90^\circ - 65^\circ) +$$

$$[\cos^2 35^\circ + \cos^2 (90^\circ - 35^\circ)]$$

$$= (\sin^2 65^\circ + \cos^2 65^\circ)$$

$$+ (\cos^2 35^\circ + \sin^2 35^\circ)$$

$$= 1 + 1 = 2$$

171. (a) $\cos^2 \theta - \sin^2 \theta = \frac{1}{3}$ (given)

$$\cos^2 \theta + \sin^2 \theta = 1 \text{ (property)}$$

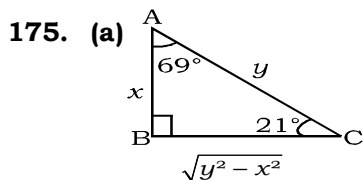
$$(\cos^2 \theta - \sin^2 \theta) (\cos^2 \theta + \sin^2 \theta)$$

$$\Rightarrow (\sin 45^\circ + \operatorname{cosec} 45^\circ)^2 + (\cos 45^\circ + \sec 45^\circ)^2 = K + \tan^2 45^\circ + \cot^2 45^\circ$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}} + \sqrt{2} \right)^2 + \left(\frac{1}{\sqrt{2}} + \sqrt{2} \right)^2 = k + 1 + 1$$

$$\Rightarrow \frac{1}{2} + 2 + \left(2\sqrt{2} \times \frac{1}{\sqrt{2}} \right) + \frac{1}{2} + 2 + \left(2\sqrt{2} \times \frac{1}{\sqrt{2}} \right) = k + 2$$

$$\Rightarrow 4\frac{1}{2} + 4\frac{1}{2} = k + 2 \Rightarrow k = 7$$



In $\triangle ABC$, $\sin 21^\circ = \frac{x}{y}$

$$AB = x$$

$$AC = y$$

$$BC = \sqrt{y^2 - x^2}$$

$$\Rightarrow \sec 21^\circ - \sin 69^\circ$$

$$\Rightarrow \frac{AC}{BC} - \frac{BC}{AC}$$

$$\Rightarrow \frac{(AC)^2 - (BC)^2}{(BC)(AC)} = \frac{y^2 - (\sqrt{y^2 - x^2})^2}{y\sqrt{y^2 - x^2}}$$

$$\Rightarrow \frac{y^2 - y^2 + x^2}{y\sqrt{y^2 - x^2}} = \frac{x^2}{y\sqrt{y^2 - x^2}}$$

176. (c) $\sec \alpha + \tan \alpha = 2$ (i)

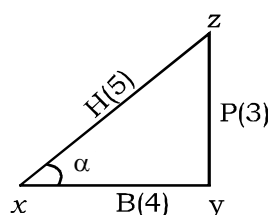
$$\sec \alpha - \tan \alpha = \frac{1}{2}$$
(ii)

On adding equation (i) and (ii)

$$2\sec \alpha = 2 + \frac{1}{2}$$

$$\sec \alpha = \frac{5}{4} \rightarrow H$$

$$\frac{5}{4} \rightarrow B$$

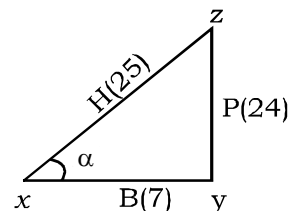


$$\sin \alpha = \frac{3}{5} = 0.6$$

177. (d) $7 \sin \alpha = 24 \cos \alpha$

$$\frac{\sin \alpha}{\cos \alpha} = \frac{24}{7}$$

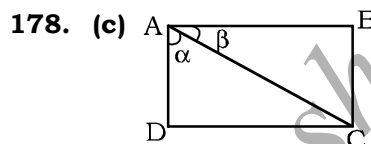
$$\Rightarrow \tan \alpha = \frac{24 \rightarrow P}{7 \rightarrow B}$$



$$14 \tan \alpha - 75 \cos \alpha - 7 \sec \alpha$$

$$= 14 \times \frac{24}{7} - 75 \times \frac{7}{25} - 7 \times \frac{25}{7}$$

$$= 48 - 21 - 25 = 2$$



$$= (\tan^2 \alpha + 1) \sin^2 \beta$$

$$= (\tan^2 45^\circ + 1) \sin^2 45^\circ$$

$$= (1 + 1) \left(\frac{1}{\sqrt{2}} \right)^2 = 2 \times \frac{1}{2} = 1$$

179. (c) $\sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} = \sqrt{\frac{1}{\cos \theta} - 1}$

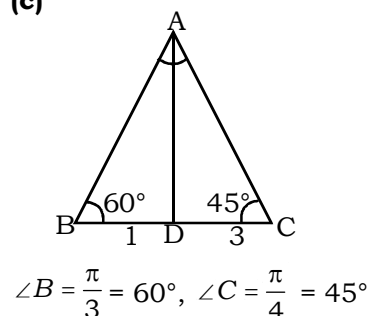
$$= \sqrt{\frac{1}{\cos \theta} + 1}$$

$$= \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \sqrt{\frac{(1 - \cos \theta)^2}{\sin^2 \theta}}$$

$$= \frac{1 - \cos \theta}{\sin \theta} = \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}$$

$$= \operatorname{cosec} \theta - \cot \theta$$

180. (c)



In $\triangle ABD$,

$$\Rightarrow \frac{BD}{\sin \angle BAD} = \frac{AD}{\sin 60^\circ} \text{ (By sin rule)}$$

$$\Rightarrow \frac{BD}{\sin \angle BAD} = \frac{AD}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow AD = \frac{\sqrt{3}}{2} \cdot \frac{BD}{\sin \angle BAD} \text{(i)}$$

from $\triangle ADC$

$$\frac{CD}{\sin \angle DAC} = \frac{AD}{\sin 45^\circ} \Rightarrow AD$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{CD}{\sin \angle DAC} \text{(ii)}$$

from equation (i) and (ii)

$$\frac{\sqrt{3}}{2} \cdot \frac{BD}{\sin \angle BAD} = \frac{1}{\sqrt{2}} \cdot \frac{CD}{\sin \angle DAC}$$

$$\Rightarrow \frac{\sin \angle BAD}{\sin \angle DAC} = \frac{\sqrt{3}}{2} \times \sqrt{2} \times \frac{1}{3}$$

$$= \frac{1}{\sqrt{3} \times \sqrt{2}} = \frac{1}{\sqrt{6}}$$

181. (a) $\sin 3A = \cos(A - 26^\circ)$

$$\Rightarrow \sin 3A = \sin[90^\circ - (A - 26^\circ)]$$

$$\Rightarrow 3A = 90^\circ - A + 26^\circ$$

$$\Rightarrow 4A = 116^\circ$$

$$\Rightarrow A = 29^\circ$$

182. (a) $\sec^2 \theta - \frac{\sin^2 \theta - 2 \sin^4 \theta}{2 \cos^4 \theta - \cos^2 \theta}$

$$= \sec^2 \theta - \frac{\sin^2 \theta (1 - 2 \sin^2 \theta)}{\cos^2 \theta (2 \cos^2 \theta - 1)}$$

$$[\because \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta]$$

$$= \sec^2 \theta - \tan^2 \theta = 1$$

Alternate:-

$$\sec^2 \theta - \frac{\sin^2 \theta - 2 \sin^4 \theta}{2 \cos^4 \theta - \cos^2 \theta}$$

$$\text{Put } \theta = 0^\circ$$

$$\sec^2 0^\circ - \frac{\sin^2 0^\circ - 2 \sin^4 0^\circ}{2 \cos^4 0^\circ - \cos^2 0^\circ}$$

$$= 1 - \frac{0 - 0}{2 - 1}$$

$$= 1 - 0 = 1$$

183. (c) $x = a(\sin\theta + \cos\theta)$
 $y = b(\sin\theta - \cos\theta)$

$$\Rightarrow \frac{x}{a} = (\sin\theta + \cos\theta)$$

$$\Rightarrow \frac{x^2}{a^2}$$

$$= (\sin\theta + \cos\theta)^2 \quad \dots(i)$$

$$\Rightarrow \frac{y}{b} = (\sin\theta - \cos\theta)$$

$$\Rightarrow \frac{y^2}{b^2} = (\sin\theta - \cos\theta)^2 \quad \dots(ii)$$

On adding equation (i) and (ii)

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$= (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2(\sin^2\theta + \cos^2\theta)$$

$$= 2 \times 1 = 2$$

184. (d) $\sin 5\theta = \cos 20^\circ$

$$\Rightarrow 5\theta + 20^\circ = 90^\circ$$

(If $\sin A = \cos B$ then $A + B = 90^\circ$)

$$\Rightarrow 5\theta = 70^\circ$$

$$\Rightarrow \theta = 14^\circ$$

185. (c) $2 \sec\theta = 3 \operatorname{cosec} 2\theta$

$$\frac{2}{\cos\theta} = \frac{3}{\sin^2\theta} = \frac{3}{1 - \cos^2\theta}$$

$$\Rightarrow 2 - 2\cos^2\theta = 3\cos\theta$$

$$\Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$$

$$\Rightarrow 2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$$

$$\Rightarrow 2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$$

$$\Rightarrow (2\cos\theta - 1)(\cos\theta + 2) = 0$$

$$\therefore 2\cos\theta - 1 = 0 \text{ or } \cos\theta + 2 \neq 0$$

$$\Rightarrow \cos\theta = \frac{1}{2} = \cos 60^\circ \text{ or } \cos \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

186. (d) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$$= \frac{(\sqrt{1+\sin\theta})^2 + (\sqrt{1-\sin\theta})^2}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$$

$$= \frac{2}{\cos\theta} = 2\sec\theta$$

Alternate:-

$$\text{put } \theta = 30^\circ,$$

$$= \sqrt{\frac{1+\sin 30^\circ}{1-\sin 30^\circ}} + \sqrt{\frac{1-\sin 30^\circ}{1+\sin 30^\circ}}$$

$$= \sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}} + \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}}$$

$$= \sqrt{\frac{3}{1}} + \sqrt{\frac{1}{3}} = \frac{4}{\sqrt{3}}$$

Now check with option by putting $\theta = 30^\circ$,

$$2 \sec 30^\circ = \frac{2 \times 2}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

187. (d) $\sin(4\alpha - \beta) = 1 = \sin 90^\circ$

$$\cos(2\alpha + \beta) = \frac{1}{2} = \cos 60^\circ$$

$$4\alpha - \beta = 90^\circ$$

$$2\alpha + \beta = 60^\circ$$

$$\text{adding } 6\alpha = 150^\circ$$

$$\alpha = 25^\circ$$

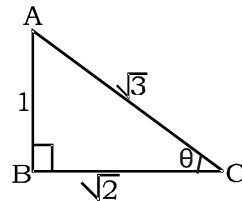
$$\Rightarrow \beta = 10^\circ$$

$$\Rightarrow \sin(\alpha + 2\beta)$$

$$\Rightarrow \sin(25^\circ + 2 \times 10^\circ)$$

$$\Rightarrow \sin 45^\circ = \frac{1}{\sqrt{2}}$$

188. (c)



$$\operatorname{cosec}\theta = \sqrt{3}$$

$$\sin\theta = \frac{1}{\sqrt{3}} \rightarrow P$$

$$\Rightarrow \cot\theta - \operatorname{cosec}\theta$$

$$\Rightarrow \frac{\sqrt{2}}{1} - \frac{\sqrt{3}}{1}$$

$$\Rightarrow \sqrt{2} - \sqrt{3}$$

189. (a) $(r \cos\theta - \sqrt{3})^2 + (r \sin\theta - 1)^2 = 0$

$$(r \cos\theta - \sqrt{3})^2 = 0, (r \sin\theta - 1)^2 = 0$$

$$\text{Or } \cos\theta = \frac{\sqrt{3}}{r} \quad \dots(i)$$

$$r \sin\theta = 1 \quad \dots(ii)$$

squaring and adding equation (i) and (ii)

$$r^2 \cos^2\theta + r^2 \sin^2\theta = 3 + 1$$

$$r^2(\cos^2\theta + \sin^2\theta) = 4$$

$$r^2 = 4$$

$$r = 2$$

$$\tan\theta = \frac{r \sin\theta}{r \cos\theta} = \frac{1}{\sqrt{3}} \text{ and } r \cos\theta$$

$$= \sqrt{3}$$

$$\cos\theta = \frac{\sqrt{3}}{r} \sec\theta = \frac{r}{\sqrt{3}}$$

$$\frac{r \tan\theta + \sec\theta}{r \sec\theta + \tan\theta} = \frac{\frac{r}{\sqrt{3}} + \frac{r}{\sqrt{3}}}{\frac{r^2}{\sqrt{3}} + \frac{1}{\sqrt{3}}}$$

$$= \frac{r \left(\frac{2}{\sqrt{3}} \right)}{\frac{r^2 + 1}{\sqrt{3}}} = \frac{2r}{r^2 + 1} = \frac{2 \times 2}{2^2 + 1} = \frac{4}{5}$$

Alternate:-

$$r = 2$$

$$\tan\theta = \frac{r \sin\theta}{r \cos\theta} = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

$$= \frac{2 \tan 30^\circ + \sec 30^\circ}{2 \sec 30^\circ + \tan 30^\circ}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}}{2 \times \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{4}{5}$$

190. (c) $\frac{\sin\alpha}{\sin\beta} = b \Rightarrow \sin\alpha = b \sin\beta$

$$\frac{\cos\alpha}{\cos\beta} = a \Rightarrow \frac{\cos^2\alpha}{\cos^2\beta} = a^2$$

$$\Rightarrow \frac{1 - \sin^2\alpha}{1 - \sin^2\beta} = a^2$$

$$\Rightarrow 1 - \sin^2\alpha = a^2 (1 - \sin^2\beta)$$

$$\Rightarrow 1 - b^2 \sin^2\beta = a^2 - a^2 \sin^2\beta$$

[value put in $\sin\alpha$]

$$\Rightarrow 1 - a^2 = b^2 \sin^2 \beta - a^2 \sin^2 \beta$$

$$\Rightarrow 1 - a^2 = (b^2 - a^2) \sin^2 \beta$$

$$\Rightarrow \sin^2 \beta = \frac{1 - a^2}{b^2 - a^2} = \frac{a^2 - 1}{a^2 - b^2}$$

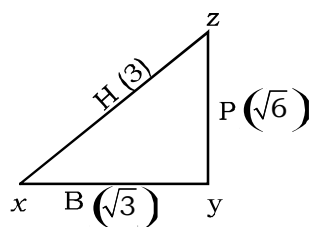
191. (c) $\sqrt{3} \tan \theta = 3 \sin \theta$

$$\sqrt{3} \frac{\sin \theta}{\cos \theta} = 3 \sin \theta$$

$$\frac{\sqrt{3}}{\cos \theta} = 3$$

$$\cos \theta = \frac{\sqrt{3}}{3}$$

then perpendicular = $\sqrt{6}$



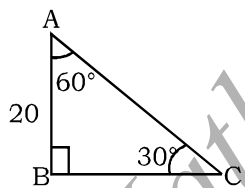
$$(\sin^2 \theta - \cos^2 \theta)$$

$$\Rightarrow \left(\frac{P}{H}\right)^2 - \left(\frac{B}{H}\right)^2$$

$$\Rightarrow \left(\frac{\sqrt{6}}{3}\right)^2 - \left(\frac{\sqrt{3}}{3}\right)^2$$

$$\Rightarrow \frac{6}{9} - \frac{3}{9} = \frac{1}{3}$$

192. (d)



AB = 20 cm

BC : CA = ?

$$\Rightarrow \frac{BC}{CA} = \cos C$$

$$\Rightarrow \frac{BC}{CA} = \cos 30^\circ$$

$$(\angle C = 180^\circ - 90^\circ - 60^\circ \Rightarrow 30^\circ)$$

$$\Rightarrow \frac{BC}{CA} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sqrt{3} : 2$$

193. (c) $\tan(A + B) = \sqrt{3} = \tan 60^\circ$

$$\tan(A - B) = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\Rightarrow (A + B) = 60^\circ \dots\dots(i)$$

$$(A - B) = 30^\circ \dots\dots(ii)$$

Adding both equation

$$2A = 90^\circ$$

$$A = \frac{90^\circ}{2}$$

$$A = 45^\circ$$

$$B = 15^\circ$$

194. (c) $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$

$$\Rightarrow \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} \times \frac{(1 - 2(1 - \cos^2 \theta))}{2 \cos^2 \theta - 1}$$

$$\Rightarrow \frac{\tan \theta (1 + 2 \cos^2 \theta - 2)}{2 \cos^2 \theta - 1}$$

$$\Rightarrow \frac{\tan \theta (2 \cos^2 \theta - 1)}{(2 \cos^2 \theta - 1)}$$

$$\Rightarrow \tan \theta$$

Alternate:-

$$\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$$

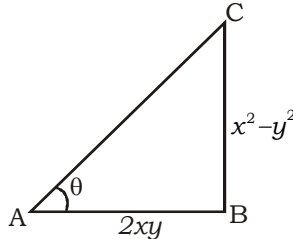
$$\Rightarrow \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

$$(1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1 = \cos 2\theta)$$

$$= \cos^2 \theta - \sin^2 \theta$$

$$\Rightarrow \tan \theta$$

195. (c) Given, $\cot \theta = \frac{2xy}{x^2 - y^2}$



In ΔABC ,

$$AC^2 = (x^2 - y^2)^2 + (2xy)^2$$

$$\Rightarrow AC^2 = (x^2 + y^2)^2 \Rightarrow AC = x^2 + y^2$$

$$\therefore \cos \theta = \frac{AB}{AC} = \frac{2xy}{x^2 + y^2}$$

196. (a) Given, $(\sin \theta + \operatorname{cosec} \theta) = 2.5$

$$\Rightarrow \left(\sin \theta + \frac{1}{\sin \theta}\right) = \frac{5}{2}$$

$$\Rightarrow 2 \sin^2 \theta - 5 \sin \theta + 2 = 0$$

$$\Rightarrow 2 \sin^2 \theta - 4 \sin \theta - \sin \theta + 2 = 0$$

$$\Rightarrow 2 \sin \theta (\sin \theta - 2) - 1 (\sin \theta - 2) = 0$$

$$\Rightarrow (2 \sin \theta - 1) (\sin \theta - 2) = 0$$

$$\Rightarrow \sin \theta = \frac{1}{2} \quad (\because \sin \theta \neq 2)$$

$$\therefore \theta = 30^\circ$$

Alternate:-

$$\sin \theta + \operatorname{cosec} \theta = 2.5$$

$$\text{Put } \theta = 30^\circ$$

$$\sin 30^\circ + \operatorname{cosec} 30^\circ = 2.5$$

$$\Rightarrow \frac{1}{2} + 2 = 2.5$$

$$\Rightarrow 2.5 = 2.5 \quad (\text{Satisfy})$$

$$\therefore \theta = 30^\circ$$

197. (d) Given, $x \cos 60^\circ + y \cos 0^\circ = 3$

$$\Rightarrow \frac{x}{2} + y = 3$$

$$\Rightarrow x + 2y = 6 \quad \dots\dots(i)$$

$$\Rightarrow \text{and } 4x \sin 30^\circ - y \cot 45^\circ = 2$$

$$\Rightarrow 4x \times \frac{1}{2} - y \cdot 1 = 2$$

$$\Rightarrow 2x - y = 2 \quad \dots\dots(ii)$$

On solving Eqs. (i) and (ii),

we get $x = y = 2$

198. (a) $\log(\tan 1^\circ) + \log(\tan 2^\circ) + \dots$

$$\dots + \log(\tan 89^\circ)$$

$$= \log(\tan 1^\circ \tan 2^\circ \dots \tan 45^\circ$$

$$\dots \tan 88^\circ \tan 89^\circ)$$

$$[\because \tan 89^\circ = \tan(90^\circ - 1^\circ) = \cot 1^\circ]$$

$$= \log[\tan 1^\circ \cot 1^\circ (\tan 2^\circ \cot 2^\circ)$$

$$\dots \tan \dots \tan 45^\circ]$$

$$= \log(1^\circ \cdot 1^\circ \dots 1^\circ) = 0$$

199. (c) Now, $(\sin x - \cos x)^2 = (\sin^2 x + \cos^2 x) - 2 \sin x \cos x$

$$= 1 - 2 \left(\frac{1}{2}\right)$$

$$\left(\because \sin x \cos x = \frac{1}{2}, \text{ given}\right) = 0$$

Alternate:-

$$\sin x \cos x = \frac{1}{2}$$

$$\text{Put } x = 45^\circ$$

$$\sin 45^\circ \cos 45^\circ = \frac{1}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{2} \text{ (Satisfy)}$$

$$\therefore \frac{\sin x - \cos x}{\sin 45^\circ - \cos 45^\circ}$$

$$\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$$

200. (a) $\tan^2 y \operatorname{cosec}^2 x - 1 = \tan^2 y$

Put $x = y = 45^\circ$

$$\tan^2 45^\circ \operatorname{cosec}^2 45^\circ - 1 = \tan^2 45^\circ$$

$$\Rightarrow 1 \times (\sqrt{2})^2 - 1 = (1)^2$$

$$\Rightarrow 2 - 1 = 1$$

$$\Rightarrow 1 = 1 \text{ (Satisfy)}$$

$$\therefore x = y$$

$$x - y = 0$$

201. (c) Given,

$$\frac{\cos x}{1 + \operatorname{cosec} x} + \frac{\cos x}{\operatorname{cosec} x - 1} = 2$$

$$\Rightarrow \frac{2 \cos x \operatorname{cosec} x}{\operatorname{cosec}^2 x - 1} = 2$$

$$\Rightarrow \frac{\cos x \operatorname{cosec} x}{\cot^2 x} = 1$$

$$\Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$$

202. (c) Given, $\sin x : \sin y = \sqrt{3} : 1$

$$= \frac{\sqrt{3}}{2} : \frac{1}{2} = \sin 60^\circ : \sin 30^\circ$$

$$\therefore x : y = 60 : 30$$

$$\Rightarrow x : y = 2 : 1$$

203. (a) Given, $\frac{\cos x}{\cos y} = n \dots (i)$

$$\frac{\sin x}{\sin y} = m \dots (ii)$$

Now, $(m^2 - n^2) \sin^2 y$

$$= \left(\frac{\sin^2 x}{\sin^2 y} - \frac{\cos^2 x}{\cos^2 y} \right) \sin^2 y$$

$$= \frac{(1 - \cos^2 x) \cos^2 y - \cos^2 x (1 - \cos^2 y)}{\cos^2 y}$$

$$= \frac{\cos^2 y - \cos^2 x}{\cos^2 y} = 1 - n^2 \text{ [from Eq. (i)]}$$

Alternate:-

$$\frac{\cos x}{\cos y} = n \text{ and } \frac{\sin x}{\sin y} = m$$

Put $x = y = 45^\circ$

$$\frac{\cos 45^\circ}{\cos 45^\circ} = n \text{ and } \frac{\sin 45^\circ}{\sin 45^\circ} = m$$

$$n = 1 \quad m = 1$$

$$\therefore (m^2 - n^2) \sin^2 y = (1^2 - 1^2) \sin^2 45^\circ = 0$$

Now check from options

Option (a):- $1 - n^2$

$$\Rightarrow 1 - (1)^2$$

$$\Rightarrow 0$$

(Satisfy)

204. (b) Given, $p = \tan^2 x + \cot^2 x$
 $= (\tan x + \cot x)^2 - 2$

$$= \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right)^2 - 2$$

$$= \left(\frac{2}{\sin 2x} \right)^2 - 2 = \frac{4}{\sin^2 2x} - 2$$

Since, the maximum value of $\sin 2x$ is 1.

$$\therefore p_{\min} = \frac{4}{1} - 2 = 2$$

$$\therefore p \geq 2$$

Hence, $p \geq 2$

Alternate:-

$$P = \tan^2 x + \cot^2 x$$

Put $x = 45^\circ$

$$P = \tan^2 45^\circ + \cot^2 45^\circ$$

$$P = 1 + 1$$

$$P = 2$$

Put $x = 30^\circ$

$$P = \tan^2 30^\circ + \cot^2 30^\circ$$

$$P = \left(\frac{1}{\sqrt{3}} \right)^2 + (\sqrt{3})^2$$

$$P = 0.33 + 2.99$$

$$P = 3.32$$

$$\therefore P \geq 2$$

205. (b)

$$\frac{5 \sin 75^\circ \sin 77^\circ + 2 \cos 13^\circ \cos 15^\circ}{\cos 15^\circ \sin 77^\circ}$$

$$- \frac{7 \sin 81^\circ}{\cos 9^\circ}$$

$$= \frac{5 \cos 15^\circ \sin 77^\circ + 2 \sin 77^\circ \cos 15^\circ}{\cos 15^\circ \sin 77^\circ}$$

$$- \frac{7 \cos 9^\circ}{\cos 9^\circ}$$

$$= \frac{7 \cos 15^\circ \cdot \sin 77^\circ}{\cos 15^\circ \cdot \sin 77^\circ} - \frac{7 \cos 9^\circ}{\cos 9^\circ}$$

$$= 7 - 7 = 0$$

206. (d) Given, $\sin x + \sin y = a$ and $\cos x + \cos y = b$

on squaring both sides, we get

$$\sin^2 x + \sin^2 y + 2 \sin x \sin y = a^2 \dots (i)$$

$$\cos^2 x + \cos^2 y + 2 \cos x \cos y = b^2 \dots (ii)$$

On adding Eqs. (i) and (ii), we get

$$(\sin^2 x + \cos^2 x) + (\sin^2 y + \cos^2 y) + 2(\sin x \sin y + \cos x \cos y) = a^2 + b^2$$

$$\Rightarrow (\sin x \sin y + \cos x \cos y)$$

$$= \frac{a^2 + b^2 - 2}{2}$$

Alternate:-

$$\sin x + \sin y = a$$

$$\cos x + \cos y = b$$

Put $x = y = 45^\circ$

$$\sin 45^\circ + \sin 45^\circ = a$$

$$a = \sqrt{2}$$

$$b = \sqrt{2}$$

$$\therefore \sin x \sin y + \cos x \cos y = \frac{1}{2} + \frac{1}{2} = 1$$

Now check from option,

Option: (d) $\frac{a^2 + b^2 - 2}{2}$

$$\Rightarrow \frac{(\sqrt{2})^2 + (\sqrt{2})^2 - 2}{2}$$

$$\Rightarrow \frac{2 + 2 - 2}{2}$$

$$\Rightarrow \frac{2}{2} = 1 \text{ (Satisfy)}$$

207. (a) Given, $\operatorname{cosec}^4 \alpha - \cot^4 \alpha = 17$
 $\Rightarrow (\operatorname{cosec}^2 \alpha - \cot^2 \alpha) (\operatorname{cosec}^2 \alpha + \cot^2 \alpha) = 17$

$$\Rightarrow 1 \left(\frac{1 + \cos^2 \alpha}{\sin^2 \alpha} \right) = 17$$

$$(\because \operatorname{cosec}^2 \alpha - \cot^2 \alpha = 1)$$

$$\Rightarrow 2 - \sin^2 \alpha = 17 \sin^2 \alpha$$

$$\Rightarrow 18 \sin^2 \alpha = 2 \Rightarrow \sin^2 \alpha = \frac{1}{9}$$

$$\therefore \sin \alpha = \frac{1}{3}$$

(since, α lie in first quadrant)

208. (c) Given, $x + \left(\frac{1}{x} \right) = 2 \cos \alpha$

On squaring both sides, we get

$$x^2 + \frac{1}{x^2} + 2 = 4 \cos^2 \alpha$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 2(2\cos^2 \alpha - 1)$$

$$= 2(2\cos^2 \alpha - \sin^2 \alpha - \cos^2 \alpha)$$

$$= 2\cos^2 \alpha - 2\sin^2 \alpha$$

209. (b) $\cot^2 \theta - \frac{1}{\sin^2 \theta}$

$$= \frac{\cos^2 \theta}{\sin^2 \theta} - \frac{1}{\sin^2 \theta} = \frac{\cos^2 \theta - 1}{\sin^2 \theta}$$

$$= \frac{-(1 - \cos^2 \theta)}{\sin^2 \theta} = \frac{-\sin^2 \theta}{\sin^2 \theta} = -1$$

$$(\because 1 - \cos^2 \theta = \sin^2 \theta)$$

Alternate:-

$$\cot^2 \theta - \frac{1}{\sin^2 \theta}$$

Put $\theta = 45^\circ$

$$\cot^2 45^\circ - \frac{1}{\sin^2 45^\circ}$$

$$\Rightarrow 1 - \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}$$

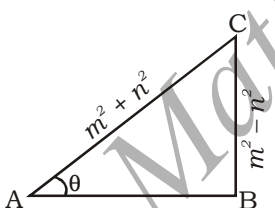
$$\Rightarrow 1 - 2 = -1$$

210. (c) Since, $\sin x = \cos y$
As x and y are acute angles,
then

$$x = y = \frac{\pi}{4}$$

$$\therefore x + y = \frac{\pi}{2}$$

211. (c) Given, $\sin \theta = \frac{m^2 - n^2}{m^2 + n^2}$



In ΔABC ,

$$AB = \sqrt{(AC)^2 - (BC)^2}$$

$$= \sqrt{m^4 + n^4 + 2m^2n^2 - (m^4 + n^4 - 2m^2n^2)}$$

$$= \sqrt{4m^2n^2} = 2mn$$

$$\therefore \tan \theta = \frac{m^2 - n^2}{2mn}$$

212. (c) Given, $\sin(x - y) = \frac{1}{2}$ and

$$\cos(x + y) = \frac{1}{2}$$

$$\Rightarrow \sin(x - y) = \sin 30^\circ$$

$$\text{and } \cos(x + y) = \cos 60^\circ$$

$$\Rightarrow x - y = 30^\circ \text{ and } x + y = 60^\circ$$

$$\therefore x = 45^\circ \text{ and } y = 15^\circ$$

213. (b) Given, $1 + \tan \theta = \sqrt{2}$

$$\Rightarrow \tan \theta = \sqrt{2} - 1$$

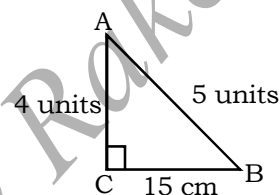
$$\therefore \cot \theta - 1 = \frac{1}{\sqrt{2} - 1} - 1$$

$$= \frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} - 1$$

$$= \frac{\sqrt{2} + 1}{2 - 1} - 1 = \sqrt{2}$$

214. (d) Given, $\sin(x + 54^\circ) = \cos x$
 $\Rightarrow \sin(x + 54^\circ) = \sin(90^\circ - x)$
 $(\because 0^\circ < x < 90^\circ)$
 $\Rightarrow x + 54^\circ = 90^\circ - x$
 $\Rightarrow 2x = 36^\circ \Rightarrow x = 18^\circ$

215. (a) By using Pythagoras theorem



$$AB^2 = AC^2 + BC^2$$

$$(5)^2 = (4)^2 + BC^2$$

$$BC = 3 \text{ units}$$

$$3 \text{ units} = 15 \text{ cm}$$

$$1 \text{ unit} = \frac{15}{3}$$

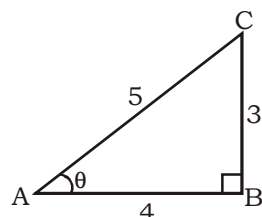
$$5 \text{ units} = \frac{15}{3} \times 5 = 25 \text{ cm}$$

$$\therefore AB = 25 \text{ cm}$$

216. (d) Since, $\tan \theta = \frac{3}{4} = \frac{P}{B}$

$$H = \sqrt{P^2 + B^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Let the length of hypotenuse = x cm



$$\therefore \sin \theta = \frac{2}{x} = \frac{3}{5}$$

$$\Rightarrow \frac{2 \times 5}{3} = \frac{10}{3} \text{ cm}$$

217. (c) Given, $\sin x - \cos x = 0$
 $\Rightarrow \sin x = \cos x \Rightarrow \tan x = 1$

$$\Rightarrow \tan x = \tan \frac{\pi}{4} \Rightarrow x = \frac{\pi}{4}$$

$$\therefore \sin^4 x + \cos^4 x = \sin^4 \frac{\pi}{4} + \cos^4 \frac{\pi}{4}$$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

218. (b) $= \frac{\tan x}{1 + \sec x} - \frac{\tan x}{1 - \sec x}$

$$= \frac{\tan x(1 - \sec x - 1 - \sec x)}{1 - \sec^2 x}$$

$$= \frac{-2 \tan x \sec x}{-\tan^2 x}$$

$$= \frac{2}{\frac{\cos x}{\sin x}} = 2 \operatorname{cosec} x$$

Alternate:-

$$\frac{\tan x}{1 + \sec x} - \frac{\tan x}{1 - \sec x}$$

Put $x = 45^\circ$

$$\frac{\tan 45^\circ}{1 + \sec 45^\circ} - \frac{\tan 45^\circ}{1 - \sec 45^\circ}$$

$$\Rightarrow \frac{1}{1 + \sqrt{2}} - \frac{1}{1 - \sqrt{2}}$$

$$\Rightarrow \frac{1 - \sqrt{2} - 1 - \sqrt{2}}{1 - (\sqrt{2})^2}$$

$$\Rightarrow \frac{-2\sqrt{2}}{-1}$$

$$2\sqrt{2}$$

Now check from option.

Option: (b)

$$2 \operatorname{cosec} x = 2 \times \operatorname{cosec} 45^\circ$$

$$= 2 \times \sqrt{2} \text{ (Satisfy)}$$

219. (b) $(\sin^4 x - \cos^4 x + 1) \operatorname{cosec}^2 x$
 $= \{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) + 1\} \operatorname{cosec}^2 x$
 $[\because a^2 - b^2 = (a+b)(a-b)]$

$$\begin{aligned}
 & (\because 1 - \cos^2 x = \sin^2 x) \\
 & = \{\sin^2 x - \cos^2 x + 1\} \operatorname{cosec}^2 x \\
 & (\because 1 - \cos^2 x = \sin^2 x) \\
 & = 2\sin^2 x \cdot \frac{1}{\sin^2 x} = 2
 \end{aligned}$$

Alternate:-

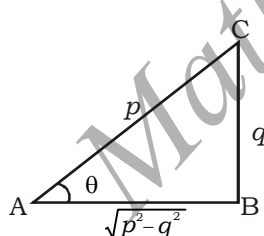
$$\begin{aligned}
 & (\sin^4 x - \cos^4 x + 1) \operatorname{cosec}^2 x \\
 & \text{Put } x = 90^\circ \\
 & (\sin^4 90^\circ - \cos^4 90^\circ + 1) \\
 & \operatorname{cosec}^2 90^\circ \\
 & \Rightarrow (1 - 0 + 1) \times 1 \\
 & \Rightarrow 2
 \end{aligned}$$

220. (b) $\sqrt{\cos x \operatorname{cosec} y - \cos x \sin y}$
 $(\because x + y = 90^\circ, \text{ given})$
 $= \sqrt{\cos x \operatorname{cosec}(90^\circ - x) - \cos x \sin(90^\circ - x)}$
 $= \sqrt{\cos x \sec x - \cos^2 x}$
 $= \sqrt{1 - \cos^2 x} = \sqrt{\sin^2 x} = \sin x$

221. (b) We know that, $0 \leq \sin^2 x \leq 1$
 $\Rightarrow 0 \leq \sin^{10} x \leq 1$
 $\Rightarrow 0 \leq p \leq 1$
 $(\because p = \sin^{10} x)$

222. (c) $\left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8}\right) + 4\cos^2 \frac{\pi}{4}$
 $- \sec \frac{\pi}{3} + 5 \tan^2 \frac{\pi}{3}$
 $= 1 + 4 \left(\frac{1}{\sqrt{2}}\right)^2 - 2 + 5(\sqrt{3})^2$
 $(\because \sin^2 \theta + \cos^2 \theta = 1)$
 $= 1 + 2 - 2 + 15 = 16$

223. (b) Given, $\operatorname{cosec} \theta = \frac{p}{q}$



In ΔABC , $\tan \theta = \frac{q}{\sqrt{p^2 - q^2}}$
 $\Rightarrow \sqrt{p^2 - q^2} \cdot \tan \theta = q$

Alternate:-

$$\begin{aligned}
 \operatorname{cosec} \theta &= \frac{p}{q} \\
 \theta &= 30^\circ
 \end{aligned}$$

$$\operatorname{cosec} 30^\circ = \frac{p}{q}$$

$$2 = \frac{p}{q}$$

$$p = 2q$$

$$\begin{aligned}
 \therefore \sqrt{p^2 - q^2} \times \tan \theta & \\
 &= \sqrt{4q^2 - q^2} \times \tan 30^\circ \\
 &= \sqrt{3q^2} \times \frac{1}{\sqrt{3}} = q
 \end{aligned}$$

224. (d) Given, $2x^2 \cos 60^\circ - 4 \cot^2 45^\circ - 2 \tan 60^\circ = 0$

$$\begin{aligned}
 \Rightarrow 2x^2 \times \frac{1}{2} - 4(1)^2 - 2 \times \sqrt{3} &= 0 \\
 \Rightarrow x^2 - 4 - 2\sqrt{3} &= 0 \\
 \Rightarrow x^2 &= 4 + 2\sqrt{3} \\
 \Rightarrow x^2 &= 3 + 1 + 2\sqrt{3} \\
 \Rightarrow x^2 &= (\sqrt{3})^2 + (1)^2 + 2\sqrt{3} \cdot 1 \\
 \Rightarrow x^2 &= (\sqrt{3} + 1)^2 \\
 \Rightarrow x &= \sqrt{3} + 1
 \end{aligned}$$

225. (d) We know that, $\sin 30^\circ = \frac{1}{2}$

Value of $\sin$ increases 0° to 90°
 $\therefore \sin 31^\circ > \sin 30^\circ$ and $\sin 32^\circ > \sin 30^\circ$

$$\Rightarrow \sin 31^\circ > \frac{1}{2} \text{ and } \sin 32^\circ > \frac{1}{2}$$

On adding both sides, we get

$$\begin{aligned}
 \sin 31^\circ + \sin 32^\circ &> \frac{1}{2} + \frac{1}{2} \\
 \Rightarrow \sin 31^\circ + \sin 32^\circ &> 1
 \end{aligned}$$

226. (c) We know that, $\sin \theta$ is increasing in 0° to 90° .

$$\therefore \sin 30^\circ = \frac{1}{2}$$

$$\therefore \sin 32^\circ > \frac{1}{2}$$

227. (d) Given, $\sin \theta + \operatorname{cosec} \theta = 2$

$$\Rightarrow \sin \theta + \frac{1}{\sin \theta} = 2$$

$$\Rightarrow \sin^2 \theta - 2 \sin \theta + 1 = 0$$

$$\Rightarrow (\sin \theta - 1)^2 = 0 \Rightarrow \sin \theta = 1$$

$$\Rightarrow \sin \theta = \sin 90^\circ \Rightarrow \theta = 90^\circ$$

$$\begin{aligned}
 \Rightarrow \sin^4 \theta + \cos^4 \theta &= \sin^4 90^\circ + \cos^4 90^\circ \\
 \therefore &= 1 + 0 = 1
 \end{aligned}$$

228. (c) Given, $2 \sec^2 \theta + \sec \theta - 6 = 0$

$$\Rightarrow 2 \sec^2 \theta + 4 \sec \theta - 3 \sec \theta - 6 = 0$$

$$\Rightarrow 2 \sec \theta (\sec \theta + 2) - 3(\sec \theta + 2) = 0$$

$$\Rightarrow (2 \sec \theta - 3)(\sec \theta + 2) = 0$$

$$(\sec \theta \neq -2, \sec \theta > 0)$$

$$\Rightarrow \sec \theta = \frac{3}{2}$$

$$\Rightarrow \cos \theta = \frac{2}{3}$$

$$\Rightarrow \sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

$$\therefore \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}}$$

229. (c) $\frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$

$$\Rightarrow \sin^2 x = (1 - \cos x)(1 + \cos x)$$

$$\Rightarrow (1 - \cos^2 x) = (1 - \cos^2 x)$$

which is possible for all values of x except multiples of 180° .
 Since for $x = 180^\circ$, $\sin x = 0$ and $1 + \cos x = 0$

230. (a) $3 \sin \theta + 4 \cos \theta = 5$

$$3 \cos \theta - 4 \sin \theta = x \text{ (Let)}$$

$$\text{Using identity,}$$

$$3^2 + 4^2 = 5^2 + x^2$$

$$x = 0$$

So,

$$3 \cos \theta - 4 \sin \theta = 0$$

231. (c) Given, $\sec \theta = \frac{13}{5}$

$$\Rightarrow \sec^2 \theta = \frac{169}{25}$$

$$\Rightarrow 1 + \tan^2 \theta = \frac{169}{25}$$

$$\Rightarrow \tan^2 \theta = \frac{169}{25} - 1$$

$$\Rightarrow \tan^2 \theta = \frac{144}{25} \Rightarrow \tan \theta = \frac{12}{5}$$

$$\begin{aligned}
 \therefore \frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta} &= \frac{2 \frac{\sin \theta}{\cos \theta} - 3}{4 \frac{\sin \theta}{\cos \theta} - 9} \\
 &= \frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta}
 \end{aligned}$$

$$= \frac{2 \tan \theta - 3}{4 \tan \theta - 9} = \frac{2\left(\frac{12}{5}\right) - 3}{4\left(\frac{12}{5}\right) - 9}$$

[From Eq. (i)]

$$= \frac{24 - 15}{48 - 45} = \frac{9}{3} = 3$$

232. (d) $r \sin \theta = \frac{7}{2}$ (i)

$$r \cos \theta = \frac{7\sqrt{3}}{2}$$
(ii)

On squaring and adding both equation

$$r^2 \sin^2 \theta + r^2 \cos^2 \theta = \left(\frac{7}{2}\right)^2 + \left(\frac{7\sqrt{3}}{2}\right)^2$$

$$r^2 (\sin^2 \theta + \cos^2 \theta) = \frac{49}{4} + \frac{147}{4}$$

$$r^2 = \frac{196}{4} = 49$$

$$r = \sqrt{49} = 7$$

233. (d) $\theta + \phi = \frac{\pi}{2}$

$$\theta + \phi = 90^\circ \text{(i)}$$

$$\sin \theta = \frac{1}{2}$$

$$\sin \theta = \sin 30^\circ = \frac{1}{2} \text{(ii)}$$

put $\theta = 30^\circ$ in equation (i)

$$30^\circ + \phi = 90^\circ$$

$$\phi = 60^\circ$$

$$\sin \phi = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

234. (b) Given

$$\Rightarrow \sec \theta + \tan \theta = p \text{(i)}$$

Then,

$$\sec \theta - \tan \theta = \frac{1}{p} \text{(ii)}$$

From equation (i) + (ii)

$$\Rightarrow 2 \sec \theta = p + \frac{1}{p}$$

$$\sec \theta = \frac{1}{2} \left(p + \frac{1}{p} \right)$$

235. (d) Given

$$\Rightarrow \sin^2 22^\circ + \sin^2 68^\circ + \cot^2 30^\circ$$

$$\Rightarrow \cos^2 68^\circ + \sin^2 68^\circ + \cot^2 30^\circ$$

$$\begin{cases} \sin(90 - \theta) = \cos \theta \\ \cos(90 - \theta) = \sin \theta \end{cases}$$

$$\Rightarrow 1 + \cot^2 30^\circ$$

$$\Rightarrow 1 + (\sqrt{3})^2$$

$$[\because \cot 30^\circ = \sqrt{3}]$$

$$\Rightarrow 4$$

236. (c) Given,

$$\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} = 2 \frac{51}{79}$$

$$\Rightarrow \frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} = \frac{209}{79}$$

$$\left[\begin{array}{l} \text{by componendo-dividendo} \\ \frac{a}{b} = \frac{c}{d}, \frac{a+b}{a-b} = \frac{c+d}{c-d} \end{array} \right]$$

$$\Rightarrow \frac{\sec \theta}{\tan \theta} = \frac{288}{130}$$

$$\frac{1}{\sin \theta} = \frac{288}{130}$$

$$\Rightarrow \frac{1}{\sin \theta} = \frac{288}{130}$$

$$\Rightarrow \text{Therefore, } \sin \theta = \frac{130}{288}$$

$$\Rightarrow \sin \theta = \frac{65}{144}$$

237. (b) Given, $1 + \cos^2 \theta = 3 \sin \theta$

$$\cos \theta \quad [0 < \theta < \pi/2]$$

$$1 + \cos^2 \theta = 3 \sin \theta \cdot \cos \theta$$

$$\text{Find } \cot \theta = ?$$

By dividing $\sin^2 \theta$ both sides

$$\Rightarrow \frac{1 + \cos^2 \theta}{\sin^2 \theta} = \frac{3 \sin \theta \cos \theta}{\sin^2 \theta}$$

$$\Rightarrow \csc^2 \theta + \cot^2 \theta = 3 \cot \theta$$

$$\Rightarrow 1 + \cot^2 \theta + \cot^2 \theta = 3 \cot \theta$$

$$[1 + \cot^2 \theta = \csc^2 \theta]$$

$$\Rightarrow 1 + 2 \cot^2 \theta = 3 \cot \theta$$

$$\Rightarrow 2 \cot^2 \theta = 3 \cot \theta - 1$$

$$\text{Let } \theta = 45^\circ$$

$$\therefore \cot 45^\circ = 1$$

$$2 \cot^2 45^\circ - 3 \cot 45^\circ + 1 = 0$$

$$2 - 3 + 1 = 0$$

$$0 = 0$$

$$\text{Therefore } \cot \theta = \cot 45^\circ = 1$$

238. (b) The value of $\cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ$

$$\text{We know that, } \cos(180^\circ \pm \theta) = -\cos \theta$$

$$\Rightarrow \cos 24^\circ + \cos 55^\circ + \cos(180^\circ - 55^\circ) + \cos(180^\circ + 24^\circ) + \cos(360^\circ - 60^\circ)$$

$$\Rightarrow \cos 24^\circ + \cos 55^\circ - \cos 55^\circ - \cos 24^\circ + \cos 60^\circ$$

$$= \cos 60^\circ = \frac{1}{2}$$

239. (c) $\sin A + \operatorname{cosec} A = 3$

$$\sin A + \frac{1}{\sin A} = 3$$

Squaring both sides

$$\sin^2 A + \frac{1}{\sin^2 A} + 2 = 9$$

$$\frac{\sin^4 A + 1}{\sin^2 A} = 9 - 2 = 7$$

240. (d) $\cos \alpha \operatorname{cosec} \beta = 1$ if $\alpha + \beta = 90^\circ$

$$\cos 7^\circ \cos 23^\circ \cos 45^\circ \operatorname{cosec} 83^\circ \operatorname{cosec} 67^\circ$$

$$= (\cos 7^\circ \operatorname{cosec} 83^\circ) (\cos 23^\circ \operatorname{cosec} 67^\circ) \cos 45^\circ$$

$$= 1 \times 1 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

241. (b) $\frac{\tan \theta + \cot \theta}{\tan \theta - \cot \theta} = 2$

$$\tan \theta + \cot \theta = 2 \tan \theta - 2 \cot \theta$$

$$3 \cot \theta = \tan \theta$$

$$\frac{3}{\tan \theta} = \tan \theta$$

$$\tan^2 \theta = 3$$

$$\Rightarrow \tan \theta = \sqrt{3} \text{ or } \theta = 60^\circ$$

$$\sin \theta = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

242. (b) $\sin x + \cos x = c$

squaring both sides

$$\sin^2 x + \cos^2 x + 2 \sin x \cos x = c^2$$

$$\sin x \cos x = \frac{c^2 - 1}{2}$$

we know that,

$$\sin^6 x + \cos^6 x = 1 - 3 \sin^2 x \cos^2 x$$

$$= 1 - 3 \left(\frac{c^2 - 1}{2} \right)^2$$

$$= 1 - 3 \left(\frac{c^4 + 1 - 2c^2}{4} \right)$$

$$= \frac{1 + 6c^2 - 3c^4}{4}$$

Alternate:-

Put, $x = 0$

then $c = 1$

put, $c = 1$ in all option

option (a) = $\frac{1}{4}$, option (b) = 1

option (c) = $\frac{5}{8}$, option (d) = $\frac{5}{2}$

Hence, $\sin^6 x + \cos^6 x = 1 = \text{option (b)}$

243. (a)

$$\frac{1 - \sin A \cdot \cos A}{\cos A (\sec A - \csc A)} \cdot \frac{\sin^2 A - \cos^2 A}{\sin^3 A + \cos^3 A}$$

$$= \frac{(1 - \sin A \cos A)}{\cos A \left(\frac{\sin A - \cos A}{\sin A \cos A} \right)}$$

$$= \frac{(\sin A + \cos A)(\sin A - \cos A)}{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A \cos A)}$$

$$= \frac{\sin A(1 - \sin A \cdot \cos A)}{(1 - \sin A \cdot \cos A)} = \sin A$$

244. (d) $mn = \cot^2 \theta - \cos^2 \theta$

$$= \frac{\cos^2 \theta (1 - \sin^2 \theta)}{\sin^2 \theta}$$

$$mn = \cot^2 \theta \cdot \cos^2 \theta$$

$$\Rightarrow \sqrt{mn} = \cot \theta \cdot \cos \theta$$

$$m^2 - n^2 = (m + n)(m - n) = 2 \cot \theta \cdot 2 \cos \theta$$

$$= 4 \cot \theta \cos \theta = 4 \sqrt{mn}$$

Alternate:-

$$\cot \theta + \cos \theta = m$$

$$\cot \theta - \cos \theta = n$$

$$\text{Put } \theta = 45^\circ$$

$$\cot 45^\circ + \cos 45^\circ = m$$

$$\Rightarrow 1 + \frac{1}{\sqrt{2}} = m$$

$$\Rightarrow m = 1 + \frac{1}{\sqrt{2}}$$

$$\Rightarrow n = 1 - \frac{1}{\sqrt{2}}$$

$$m^2 - n^2 = \left(1 + \frac{1}{\sqrt{2}} \right)^2 - \left(1 - \frac{1}{\sqrt{2}} \right)^2$$

$$= 1 + \frac{1}{2} + \frac{2}{\sqrt{2}} - 1 - \frac{1}{2} + \frac{2}{\sqrt{2}}$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Now check from option

Option:- (d) $4\sqrt{mn}$

$$\Rightarrow \Rightarrow 4 \sqrt{\left(1 + \frac{1}{\sqrt{2}} \right) \left(1 - \frac{1}{\sqrt{2}} \right)}$$

$$\Rightarrow 4 \sqrt{1^2 - \frac{1}{(\sqrt{2})^2}}$$

$$\Rightarrow 4 \sqrt{1 - \frac{1}{2}}$$

$$\Rightarrow 4 \sqrt{\frac{1}{2}} \Rightarrow 2\sqrt{4}$$

$$= 2\sqrt{2} \text{ (satisfy)}$$

245. (d) $\tan \theta + \sin \theta = m$

$$\tan \theta - \sin \theta = n$$

$$\text{Put } \theta = 45^\circ$$

$$\tan 45^\circ + \sin 45^\circ = m$$

$$1 + \frac{1}{\sqrt{2}} = m$$

$$1 - \frac{1}{\sqrt{2}} = n$$

$$\therefore \sqrt{mn} = \sqrt{\left(1 - \frac{1}{\sqrt{2}} \right) \left(1 + \frac{1}{\sqrt{2}} \right)}$$

$$= \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Now check from option,

$$\text{Option (d):- } \frac{1}{4}(m^2 - n^2)$$

$$= \frac{1}{4} \left[\left(1 + \frac{1}{\sqrt{2}} \right)^2 - \left(1 - \frac{1}{\sqrt{2}} \right)^2 \right]$$

$$= \frac{1}{4} \left[1 + \frac{1}{2} + \frac{2}{\sqrt{2}} - 1 - \frac{1}{2} + \frac{2}{\sqrt{2}} \right]$$

$$= \frac{1}{4} \left[\frac{4}{\sqrt{2}} \right] = \frac{1}{\sqrt{2}} \text{ (Satisfy)}$$

246. (d) $x = \cot \theta + \tan \theta$

$$= \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta}$$

$$y = \sec \theta - \cos \theta = \frac{1 - \cos^2 \theta}{\cos \theta} = \frac{\sin^2 \theta}{\cos \theta}$$

$$(x^2 y)^{2/3} = \left(\frac{1}{\sin^2 \theta \cdot \cos^2 \theta} \cdot \frac{\sin^2 \theta}{\cos \theta} \right)^{2/3}$$

$$= \frac{1}{\cos^2 \theta}$$

$$(xy)^{2/3} = \left(\frac{1}{\sin \theta \cdot \cos \theta} \cdot \frac{\sin^4 \theta}{\cos^2 \theta} \right)^{2/3}$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$(x^2 y)^{2/3} - (xy)^{2/3} = \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{\cos^2 \theta}{\cos^2 \theta} = 1$$

Alternate:-

$$\text{Put } \theta = 45^\circ$$

$$x = \cot 45^\circ + \tan 45^\circ = 2$$

$$y = \sec 45^\circ - \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$(x^2 y)^{2/3} - (xy)^{2/3} = \left(\frac{4}{\sqrt{2}} \right)^{2/3} - \left(\frac{2}{2} \right)^{2/3}$$

$$= 2 - 1 = 1$$

247. (b) $\frac{\sin^8 \theta - \cos^8 \theta}{\cos 2\theta(1 + \cos^2 \theta)}$

$$= \frac{(\sin^4 \theta + \cos^4 \theta)(\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta)}{(\cos^2 \theta - \sin^2 \theta)(1 + (\cos^2 \theta - \sin^2 \theta)^2)}$$

$$= - \frac{(\sin^4 \theta + \cos^4 \theta)}{(1 + \cos^4 \theta + \sin^4 \theta - 2 \sin^2 \theta \cdot \cos^2 \theta)}$$

We can say

$$\sin^4 \theta + \cos^4 \theta = (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta$$

$$= - \frac{(1 - 2 \sin^2 \theta \cdot \cos^2 \theta)}{(1 + 1 - 2 \sin^2 \theta \cdot \cos^2 \theta - 2 \sin^2 \theta \cdot \cos^2 \theta)}$$

$$= - \frac{1}{2}$$

Alternate:-

Answer is independent of angle θ , so put $\theta = 90^\circ$

$$\frac{\sin^8 90^\circ - \cos^8 90^\circ}{\cos 180^\circ(1 + \cos^2 180^\circ)}$$

$$= \frac{1 - 0}{-1(1 + 1)} = - \frac{1}{2}$$

248. (a) $k = (\sec \alpha + \tan \alpha)(\sec \beta + \tan \beta)(\sec \gamma + \tan \gamma) \dots (i)$

$k = (\sec \alpha - \tan \alpha)(\sec \beta - \tan \beta)(\sec \gamma - \tan \gamma) \dots (ii)$

Multiplying equation (i) & (ii) $k^2 = (\sec^2 \alpha - \tan^2 \alpha)(\sec^2 \beta - \tan^2 \beta)(\sec^2 \gamma - \tan^2 \gamma)$
 $k^2 = 1 \Rightarrow k = \pm 1$

249. (a) $a \sec \theta + b \tan \theta + c = 0$

$p \sec \theta + q \tan \theta + r = 0$

Apply cross multiplication method to solve these eqⁿ

$$\frac{\sec \theta}{br - qc} = \frac{\tan \theta}{pc - ar} = \frac{1}{aq - bp}$$

$$\sec \theta = \frac{br - qc}{aq - bp} \text{ \& } \tan \theta = \frac{pc - ar}{aq - bp}$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\left(\frac{br - qc}{aq - bp} \right)^2 - \left(\frac{pc - ar}{aq - bp} \right)^2 = 1$$

Then, $(br - qc)^2 - (pc - ar)^2 = (aq - bp)^2$

250. (a) $P = a \cos^3 x + 3a \cos x \sin^2 x$
 $Q = a \sin^3 x + 3a \cos^2 x \sin x$
 Put $x = 45^\circ$

$$P = \frac{a}{2\sqrt{2}} + \frac{3a}{2\sqrt{2}},$$

$$Q = \frac{a}{2\sqrt{2}} + \frac{3a}{2\sqrt{2}}$$

$$P = \frac{4a}{2\sqrt{2}} = \sqrt{2}a, Q = \sqrt{2}a$$

$$(P + Q)^{2/3} + (P - Q)^{2/3}$$

$$= (\sqrt{2}a + \sqrt{2}a)^{2/3} + 0$$

$$= (2\sqrt{2}a)^{2/3} = 2a^{2/3}$$

251. (b) $8 \cos^2 \theta + 8 \sec^2 \theta = 65$

$$0^\circ < \theta < \frac{\pi}{2}$$

$$\Rightarrow 8 \cos^2 \theta + \frac{8}{\cos^2 \theta} = 65$$

$$\Rightarrow 8 \cos^4 \theta + 8 = 65 \cos^2 \theta$$

$$\Rightarrow 8 \cos^4 \theta - 64 \cos^2 \theta - \cos^2 \theta + 8 = 0$$

$$\Rightarrow 8 \cos^2 \theta [\cos^2 \theta - 8] - 1 (\cos^2 \theta - 8) = 0$$

$$\Rightarrow \cos^2 \theta = \frac{1}{8},$$

$$\Rightarrow \cos^2 \theta = 8 \text{ (not possible)}$$

$$(\because \cos 2\theta = 2\cos^2 \theta - 1)$$

$$\Rightarrow \cos 2\theta = 2 \times \frac{1}{64} - 1$$

$$= \frac{2 - 64}{64} = \frac{-62}{64} = -\frac{31}{32}$$

$$\Rightarrow 4 \cos 2\theta = -\frac{31}{8}$$

252. (b) $\cos(\theta - A) = a, \cos(\theta - B) = b$

Let $\theta = 90^\circ$

$$a = \cos(90^\circ - A) = \sin A,$$

$$b = \cos(90^\circ - B) = \sin B$$

$$\cos A = \sqrt{1 - a^2}, \cos B = \sqrt{1 - b^2}$$

$$\Rightarrow \sin^2(A - B) + 2ab \cos(A - B)$$

$$= (\sin A \cos B - \cos A \sin B)^2 + 2ab (\cos A \cos B + \sin A \sin B)$$

$$= (a\sqrt{1 - b^2} - b\sqrt{1 - a^2})^2$$

$$+ 2ab (\sqrt{1 - a^2} \cdot \sqrt{1 - b^2} + ab)$$

$$= a^2(1 - b^2) + b^2(1 - a^2) - 2ab$$

$$\sqrt{1 - a^2} \cdot \sqrt{1 - b^2}$$

$$+ 2ab \sqrt{1 - a^2} \cdot \sqrt{1 - b^2} + 2a^2b^2$$

$$= a^2 - a^2b^2 + b^2 - a^2b^2 + 2a^2b^2$$

$$= a^2 + b^2$$

Alternate:-

$$\cos(\theta - A) = a$$

$$\cos(\theta - B) = b$$

Put $\theta = 90^\circ$

$$\cos(90^\circ - A) = a$$

$$a = \sin A$$

$$b = \sin B$$

Put $A = 60^\circ$

$$B = 30^\circ$$

$$a = \frac{\sqrt{3}}{2}$$

$$b = \frac{1}{2}$$

$$\therefore \sin^2(A - B) + 2ab \cos(A - B)$$

$$\sin^2(60^\circ - 30^\circ) + 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} \cos(60^\circ - 30^\circ)$$

$$\Rightarrow \frac{1}{4} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{1}{4} + \frac{3}{4} = 1$$

Now check the option

Option (b):- $a^2 + b^2$

$$\Rightarrow \left(\frac{\sqrt{3}}{2} \right)^2 + \left(\frac{1}{2} \right)^2$$

$$\Rightarrow \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \text{ (Satisfy)}$$

253. (a) $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8}$

$$= \sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \left(\pi - \frac{3\pi}{8} \right)$$

$$+ \sin^4 \left(\pi - \frac{\pi}{8} \right)$$

$$= 2 \left(\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} \right)$$

$$= 2 \left(\sin^4 \frac{\pi}{8} + \sin^4 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) \right)$$

$$= 2 \left(\sin^4 \frac{\pi}{8} + \cos^4 \frac{\pi}{8} \right)$$

$$\left(\because \sin \frac{3\pi}{8} = \sin \left(\frac{\pi}{2} - \frac{\pi}{8} \right) = \cos \frac{\pi}{8} \right)$$

$$= 2 \left(\left(\sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} \right)^2 - 2 \sin^2 \frac{\pi}{8} \cdot \cos^2 \frac{\pi}{8} \right)$$

$$= 2 \left(1 - \frac{1}{2} \left(\sin \frac{\pi}{4} \right)^2 \right) = 2 \left(1 - \frac{1}{4} \right) = \frac{3}{2}$$

254. (a) $\sin \theta + \cos \theta = a,$
 $\sec \theta + \operatorname{cosec} \theta = b$

$$b(a^2 - 1) = \left(\frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta} \right) (\sin^2$$

$$+ \cos^2 \theta + 2 \sin \theta \cos \theta - 1)$$

$$= \frac{(\sin \theta + \cos \theta)}{\sin \theta \cos \theta} \times 2 \sin \theta \cos \theta$$

$$= 2a$$

255. (b) $\frac{1}{2} \left(\cos 15^\circ \cdot \cos 7\frac{1}{2}^\circ \cdot \cos 82\frac{1}{2}^\circ \right)$
 $\times 2$

$$\Rightarrow \frac{1}{2} \left(\cos 15^\circ \cdot 2 \times \cos 7\frac{1}{2}^\circ \cdot \sin 7\frac{1}{2}^\circ \right)$$

$$\Rightarrow \frac{1}{2} \cos 15^\circ \cdot \sin 15^\circ$$

Multiply and divide by 2

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times 2 \cos 15^\circ \sin 15^\circ$$

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \sin 2 \times 15^\circ$$

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \sin 30^\circ$$

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

256. (b) $3 \tan \theta \tan \phi = 1$

$\theta = \phi = 30^\circ$

$$\frac{\cos(30^\circ - 30^\circ)}{\cos(30^\circ + 30^\circ)} = \frac{\cos 0^\circ}{\cos 60^\circ} = 2$$

257. (c) $\tan 60^\circ$

$$= \frac{\tan 20^\circ + \tan 40^\circ}{1 - \tan 20^\circ \tan 40^\circ}$$

$$\tan 20^\circ + \tan 40^\circ = \sqrt{3} - \sqrt{3} \tan 20^\circ \tan 40^\circ$$

$$\tan 20^\circ + \tan 40^\circ + \sqrt{3} \tan 20^\circ \tan 40^\circ$$

$$\tan 40^\circ = \sqrt{3}$$

258. (a) Given let $x = 1$ and $\theta = 0^\circ$

$$\Rightarrow x \cos \theta - \sin \theta = 1 \quad \dots (i)$$

$$\Rightarrow 1 \times 1 - (0) = 1$$

$$\Rightarrow 1 = 1$$

putting value $x = 1$ and $\theta = 0^\circ$ in equation (i)

$$\Rightarrow x^2 + (1 + x^2) \sin \theta$$

$$\Rightarrow 1^2 + (1 + 1^2) \times \sin 0^\circ = 1$$

259. (d) Simplest form of

$$\Rightarrow \sin A \cdot \cos A (\tan A - \cot A)$$

$$\Rightarrow \sin A \cdot \cos A \left(\frac{\sin A}{\cos A} - \frac{\cos A}{\sin A} \right)$$

$$\Rightarrow \sin^2 A - \cos^2 A$$

$$\{\because \cos^2 A = 1 - \sin^2 A\}$$

$$\Rightarrow 2 \sin^2 A - 1$$

260. (d) Given:-

$$n = \frac{\cos \alpha}{\sin \beta}, \quad m = \frac{\cos \alpha}{\cos \beta}$$

$$\Rightarrow \cos \alpha = n \sin \beta, \quad \text{and}$$

$$\cos \alpha = m \cos \beta$$

$$\cos^2 \alpha = n^2 \sin^2 \beta \quad \dots (i)$$

$$\cos^2 \alpha = m^2 \cos^2 \beta \quad \dots (ii)$$

$$\text{equation (i) = (ii)}$$

$$\Rightarrow n^2 \sin^2 \beta = m^2 \cos^2 \beta$$

$$\Rightarrow n^2 (1 - \cos^2 \beta) = m^2 \cos^2 \beta$$

$$\Rightarrow \cos^2 \beta = \frac{n^2}{m^2 + n^2}$$

261. (a) Given, $\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

Let $A = 45^\circ, B = 30^\circ$

$$\sin(45-30) = \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

262. (d) α is an acute angle

$$\Rightarrow 2 \sin \alpha + 15 \cos^2 \alpha = 7$$

$$\Rightarrow 2 \sin \alpha + 15 (1 - \sin^2 \alpha) = 7$$

$$\Rightarrow 2 \sin \alpha + 15 - 15 \sin^2 \alpha = 7$$

$$\Rightarrow 15 \sin^2 \alpha - 2 \sin \alpha - 8 = 0$$

$$\text{Let } \sin \alpha = x$$

$$\Rightarrow 15x^2 - 2x - 8 = 0$$

$$\Rightarrow 15x^2 - (12 - 10)x - 8 = 0$$

$$\Rightarrow 15x^2 - 12x + 10x - 8 = 0$$

$$\Rightarrow 3x(5x - 4) + 2(5x - 4) = 0$$

$$\Rightarrow (3x + 2)(5x - 4) = 0$$

$$\Rightarrow x = -\frac{2}{3} \text{ (Rejected)}$$

$$\Rightarrow x = \frac{4}{5}$$

$$\Rightarrow \sin \theta = \frac{4}{5} = \frac{\text{Perp.}}{\text{Hypo.}}$$

$$\text{Base} = 3$$

(Using triplets 3, 4, 5)

$$\Rightarrow \text{Therefore, } \cot \alpha = \frac{\text{Base}}{\text{Perp.}}$$

$$\cot \alpha = \frac{3}{4}$$

263. (b) According to the question,

$$\sin^4 \theta + \cos^4 \theta = 2 \sin^2 \cos^2 \theta$$

$$\text{Put } \theta = 45^\circ$$

$$\sin^4 45^\circ + \cos^4 45^\circ = 2 \sin^2 \cos^2 \theta$$

$$\frac{1}{4} + \frac{1}{4} = 2 \times \frac{1}{2} \times \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} \text{ [Satisfy]}$$

$$\therefore \tan 45^\circ = 1$$

264. (c) According to the question,

$$\operatorname{cosec} A + \cot A = 3$$

$$\frac{\operatorname{cosec} A - \cot A}{2 \operatorname{cosec} A} = \frac{1}{3}$$

$$\operatorname{cosec} A = \frac{10}{6}$$

$$\sin A = \frac{6}{10} = \frac{3}{5}$$

265. (b) $\cot 41^\circ \cdot \cot 42^\circ \cdot \cot 43^\circ \cdot \cot 44^\circ \cdot \cot 45^\circ \cdot \cot 46^\circ \cdot \cot 47^\circ \cdot \cot 48^\circ \cdot \cot 49^\circ$

$$\Rightarrow (\cot 41^\circ \cdot \cot 49^\circ) \cdot (\cot 42^\circ \cdot \cot 48^\circ) \cdot (\cot 43^\circ \cdot \cot 47^\circ) \cdot (\cot 44^\circ \cdot \cot 46^\circ) \cdot \cot 45^\circ$$

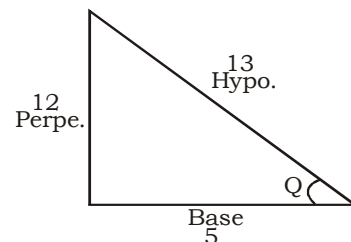
$$\cot B = 1 \quad [A + B = 90^\circ] = 1$$

266. (d) $5 \cos \theta + 12 \sin \theta = 13$

$$\frac{5}{13} \cos \theta + \frac{12}{13} \sin \theta = \frac{13}{13}$$

[Divide whole question by R.H.S. value]

$$\frac{5}{13} \frac{\text{Base}}{\text{Hypo}} + \frac{12}{13} \frac{\text{Perpendicular}}{\text{Hypo}} = 1$$



$$\sin \theta = \frac{P}{H} = \frac{12}{13}$$

267. (a) $\tan \theta - \cot \theta = 0$

$$\text{Put } \theta = 45^\circ$$

$$\tan 45^\circ - \cot 45^\circ = 0$$

$$1 - 1 = 0$$

$$0 = 0 \text{ (Satisfied)}$$

$$\text{So, } \theta = 45^\circ$$

$$\text{Now, } \frac{\tan(\theta + 15^\circ)}{\tan(\theta - 15^\circ)}$$

$$\Rightarrow \frac{\tan(45^\circ + 15^\circ)}{\tan(45^\circ - 15^\circ)}$$

$$\Rightarrow \frac{\tan 60^\circ}{\tan 30^\circ} \Rightarrow \frac{\sqrt{3}}{1} = \sqrt{3} \times \sqrt{3} = 3$$

268. (c) SHORTCUT METHOD

$$\text{Put } \theta = 30^\circ$$

$$\sec \theta - \tan \theta = \frac{1}{\sqrt{3}}$$

$$\sec 30^\circ - \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ (satisfy)}$$

$$\sec \theta = 30^\circ$$

$$\Rightarrow \sec \theta \cdot \tan \theta$$

$$\Rightarrow \sec 30^\circ \cdot \tan 30^\circ$$

$$\Rightarrow \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{2}{3}$$

269. (d) $(\operatorname{cosec} a - \sin a)$

$$(\sec a - \cos a) (\tan a + \cot a)$$

$$\text{Put } a = 45^\circ$$

$$(\operatorname{cosec} 45^\circ - \sin 45^\circ) (\sec 45^\circ - \cos 45^\circ) (\tan 45^\circ + \cot 45^\circ)$$

$$\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right) \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right) (1+1)$$

$$\frac{1}{2} \times 2 = 1$$

270. (c) According to the question
 $\tan A = n \tan B$ and $\sin A = m \sin B$

$$n = \frac{\tan A}{\tan B} \quad m = \frac{\sin A}{\sin B}$$

$$\text{Put } A = 30^\circ \text{ and } B = 60^\circ$$

$$n = \frac{1}{\sqrt{3}} \quad m = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$n = \frac{1}{3} \quad m = \frac{1}{\sqrt{3}}$$

$$\therefore \cos^2 A = \cos^2 30^\circ = \frac{3}{4}$$

Now check from option to save your valuable time

Option (c): $\frac{m^2-1}{n^2-1}$

$$= \frac{\left(\frac{1}{\sqrt{3}}\right)^2 - 1}{\left(\frac{1}{3}\right)^2 - 1} = \frac{\frac{1}{3} - 1}{\frac{1}{9} - 1} = \frac{-\frac{2}{3}}{-\frac{8}{9}}$$

$$= \frac{3}{4} \text{ (satisfy)}$$

271.. (b) $x \cos^2 30^\circ \cdot \sin 60^\circ$

$$= \frac{\tan^2 45^\circ \cdot \sec 60^\circ}{\operatorname{cosec} 60^\circ}$$

$$x \left(\frac{\sqrt{3}}{2}\right)^2 \left(\frac{\sqrt{3}}{2}\right) = \frac{(1)^2 (2)}{\left(\frac{2}{\sqrt{3}}\right)}$$

$$x = 2 \frac{2}{3}$$

272.(b) $\sin(\theta + 30^\circ) = \frac{3}{\sqrt{12}}$

$$= \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\sin(\theta + 30^\circ) = \sin 60^\circ$$

$$\therefore \theta = 30^\circ$$

$$\cos^2 \theta = \cos^2 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

273. (a) $4 \cos^2 \theta - 4\sqrt{3} \cos \theta + 3 = 0$

Hit & trial method

$$\text{Put } \theta = 30^\circ \text{ option (a)}$$

$$4 \cos^2 30^\circ - 4\sqrt{3} \cos 30^\circ + 3 = 0$$

$$4\left(\frac{3}{4}\right) - 4\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) + 3 = 0$$

$$0 = 0$$

274. (d) According to the question

$$\Rightarrow \sec^4 A (1 - \sin^4 A) - 2 \tan^2 A$$

$$\text{Put } A = 45^\circ$$

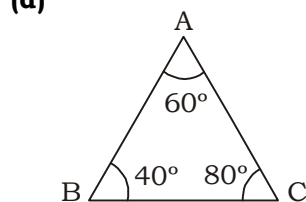
$$\Rightarrow \sec^4 45^\circ (1 - \sin^4 45^\circ) - 2 \tan^2 45^\circ$$

$$\Rightarrow 4 \left(1 - \frac{1}{4}\right) - 2$$

$$\Rightarrow 4 \times \frac{3}{4} - 2$$

$$\Rightarrow 3 - 2 = 1$$

275. (d)



$\Rightarrow$ As we know

$$\Rightarrow \sin(90^\circ - \theta) = \cos \theta$$

$$\Rightarrow \sin \left[\frac{(B+C)}{2} \right] = \cos \frac{A}{2}$$

Note:- Let $\angle A = 60^\circ$, $\angle B = 40^\circ$,
 $\angle C = 80^\circ$

$$\Rightarrow \sin \left[\frac{B+C}{2} \right] = \cos \frac{A}{2}$$

$$\Rightarrow \sin \frac{(40^\circ + 80^\circ)}{2}$$

$$= \cos \frac{60^\circ}{2}$$

$$\Rightarrow \sin \left(\frac{120^\circ}{2} \right) = \cos \frac{60^\circ}{2}$$

$$\Rightarrow \sin 60^\circ = \cos 30^\circ$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$\Rightarrow$ Therefore

$$\sin \left(\frac{B+C}{2} \right) = \cos \left(\frac{A}{2} \right)$$

276. (b)

$$= \frac{9}{\operatorname{cosec}^2 \theta} + 4 \cos^2 \theta + \frac{5}{1 + \tan^2 \theta}$$

$$= 9 \sin^2 \theta + 4 \cos^2 \theta + \frac{5}{1 + \tan^2 \theta}$$

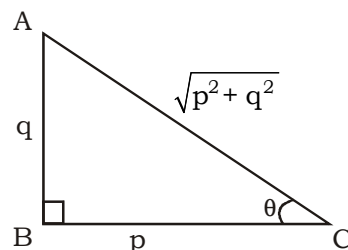
$$\therefore (1 + \tan^2 \theta = \sec^2 \theta)$$

$$= 9 \sin^2 \theta + 4 \cos^2 \theta + 5 \cos^2 \theta$$

$$= 9 (\sin^2 \theta + \cos^2 \theta) = 9$$

277. (a)

$$\cos \theta = \frac{\text{Base}}{\text{Hyp}} = \frac{p}{\sqrt{p^2 + q^2}}$$



$$(\text{Hyp})^2 = \text{Base}^2 + \text{perp}^2.$$

Perpendicular = q

$$\tan \theta = \frac{\text{Perp.}}{\text{Base}} = \frac{q}{p}$$

278.(c) $\tan \theta + \sec \theta = 3$

$$\sec \theta - \tan \theta = \frac{1}{3}$$

$$2 \sec \theta = \frac{10}{3}$$

$$\cos \theta = \frac{3}{5}; \sin \theta = \frac{4}{5}$$

$$5 \sin \theta = 4$$

$$279. (c) \frac{x - x \tan^2 30^\circ}{1 + \tan^2 30^\circ}$$

$$= \sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ$$

$$\frac{x(1 - \tan^2 30^\circ)}{1 + \tan^2 30^\circ} = \left(\frac{1}{2}\right)^2 + 4 \times 1 - (2)^2$$

$$\frac{x \left[1 - \left(\frac{1}{\sqrt{3}}\right)^2 \right]}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1}{4} + 4 - 4$$

$$\frac{x \left[1 - \frac{1}{3} \right]}{1 + \frac{1}{3}} = \frac{1}{4}$$

$$x \times \frac{2}{3} = \frac{1}{4} \times \frac{4}{3}$$

$$x = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$$

280.(d) According to question

$$\cos \theta + \sin \theta = m$$

$$\sec \theta + \operatorname{cosec} \theta = n$$

$$\text{put } \theta = 45^\circ$$

$$\cos 45^\circ + \sin 45^\circ = m,$$

$$\sec 45^\circ + \operatorname{cosec} 45^\circ = n$$

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = m, \sqrt{2} + \sqrt{2} = n$$

$$m = \sqrt{2}, n = 2\sqrt{2}$$

$$\therefore n(m^2 - 1) = 2\sqrt{2} \left((\sqrt{2})^2 - 1 \right)$$

$$= 2\sqrt{2} (2 - 1) = 2\sqrt{2}$$

Now check from option only one option satisfy

$$2m = 2 \times \sqrt{2} = 2\sqrt{2}$$

281. (d) Given:-

$$\alpha + \beta = 90^\circ$$

$$\therefore \alpha + \beta = 90^\circ$$

$$\Rightarrow \alpha = 90 - \beta$$

$\Rightarrow \alpha, \beta$ are complementary angles

$$\Rightarrow \frac{\tan \alpha}{\tan(90 - \alpha)} + \sin^2 \alpha + \sin^2(90 - \alpha)$$

$$\Rightarrow \frac{\tan \alpha}{\cot \alpha} + \sin^2 \alpha + \cos^2 \alpha$$

$$\Rightarrow \tan^2 \alpha + 1$$

$$(\because \sin^2 \alpha + \cos^2 \alpha = 1)$$

$$\Rightarrow \sec^2 \alpha$$

$$(\because 1 + \tan^2 \alpha = \sec^2 \alpha)$$

282. (d)

$$\tan^2 \frac{\pi}{4} - \cos^2 \frac{\pi}{3} = x \sin \frac{\pi}{4} \cos \frac{\pi}{4} \tan \frac{\pi}{3}$$

$$\Rightarrow \tan^2 45^\circ - \cos^2 60^\circ = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$$

$$\Rightarrow 1 - \frac{1}{4} = x \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \sqrt{3}$$

$$\Rightarrow \frac{3}{4} = \frac{x \times \sqrt{3}}{2}$$

$$\Rightarrow x = \frac{\sqrt{3}}{2}$$

$$283. (d) \sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin 2\theta = \sin 60^\circ$$

$$\Rightarrow 2\theta = 60^\circ$$

$$\Rightarrow \theta = \frac{60}{2} = 30^\circ$$

$$\Rightarrow \text{So } \sin 3\theta = \sin 3 \times 30^\circ$$

$$= \sin 90^\circ = 1$$

$$284. (a) \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = \frac{3}{1}$$

$$\text{to find } \sin^4 \theta = ?$$

$$\frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = \frac{3}{1} \text{ (by componendo dividendo)}$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{3+1}{3-1}$$

$$\Rightarrow \tan \theta = 2$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{2}{1}$$

$$\Rightarrow \sin^4 \theta = \left(\frac{2}{\sqrt{5}} \right)^4 = \frac{16}{25}$$

$$285. (b) \cos 20^\circ = m$$

$$\cos 70^\circ = n$$

$$\text{So, } m^2 + n^2 = \cos^2 20^\circ + \cos^2 70^\circ$$

$$= 1$$

$$\left[\begin{array}{l} \cos^2 A + \cos^2 B = 1 \\ \text{if } A + B = 90^\circ \end{array} \right]$$

$$286. (d) \sin A - \cos A = \frac{\sqrt{3} - 1}{2}$$

Shortcut Method:-

$$\text{Put } \theta = 60^\circ$$

$$\Rightarrow \sin 60^\circ - \cos 60^\circ = \frac{\sqrt{3} - 1}{2}$$

$$\Rightarrow \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3} - 1}{2}$$

$$\Rightarrow \frac{\sqrt{3} - 1}{2} = \frac{\sqrt{3} - 1}{2}$$

(Matched)

$$\text{Hence, } \sin A \cdot \cos A$$

$$\Rightarrow \sin 60^\circ \cdot \cos 60^\circ$$

$$\Rightarrow \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4}$$

Alternate:-

$$\sin A - \cos A = \frac{\sqrt{3} - 1}{2}$$

Squaring both side,

$$\Rightarrow \sin^2 A + \cos^2 A - 2 \sin A \cos A$$

$$= \left(\frac{\sqrt{3} - 1}{2} \right)^2$$

$$\Rightarrow 1 - 2 \sin A \cos A$$

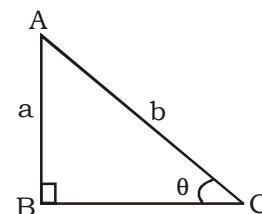
$$= \frac{3 + 1 - 2\sqrt{3}}{4}$$

$$\Rightarrow 2 \sin A \cos A = 1 - 2 \frac{(2 - \sqrt{3})}{4}$$

$$\Rightarrow 2 \sin A \cdot \cos A = \frac{2 - 2 + \sqrt{3}}{2}$$

$$\sin A \cdot \cos A = \frac{\sqrt{3}}{4}$$

$$287. (c) \sin \theta = \frac{a}{b} = \frac{p}{h}$$



$$BC = \sqrt{b^2 - a^2}$$

[using pythagoras theorem]

$$\therefore \sec \theta - \cos \theta$$

$$\begin{aligned}
 &= \frac{H}{B} - \frac{B}{H} = \frac{AC}{BC} - \frac{BC}{AC} \\
 &= \frac{b}{\sqrt{b^2 - a^2}} - \frac{\sqrt{b^2 - a^2}}{b} \\
 &= \frac{b^2 - (\sqrt{b^2 - a^2})^2}{b\sqrt{b^2 - a^2}} \\
 &= \frac{b^2 - b^2 + a^2}{b\sqrt{b^2 - a^2}} = \frac{a^2}{b\sqrt{b^2 - a^2}}
 \end{aligned}$$

288. (c) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$$\begin{aligned}
 &= \frac{(\sqrt{1+\sin\theta})^2 + (\sqrt{1-\sin\theta})^2}{\sqrt{1-\sin^2\theta}} \\
 &= \frac{1+\sin\theta + 1-\sin\theta}{\sqrt{\cos^2\theta}} \\
 &= \frac{2}{\cos\theta} = 2\sec\theta
 \end{aligned}$$

Alternate:-

Put $\theta = 30^\circ$

$$\begin{aligned}
 &\Rightarrow \sqrt{\frac{1+\sin 30^\circ}{1-\sin 30^\circ}} + \sqrt{\frac{1-\sin 30^\circ}{1+\sin 30^\circ}} \\
 &\Rightarrow \sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}} + \sqrt{\frac{1-\frac{1}{2}}{1+\frac{1}{2}}} \\
 &\Rightarrow \sqrt{\frac{\frac{3}{2}}{\frac{1}{2}}} + \sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}} \\
 &\Rightarrow \sqrt{3} + \frac{1}{\sqrt{3}} \\
 &\Rightarrow \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}
 \end{aligned}$$

Now check from option to save your valuable time, option 'c'

$$2\sec\theta = 2\sec 30^\circ = 2 \times \frac{2}{\sqrt{3}}$$

$$= \frac{4}{\sqrt{3}} \text{ (Satisfied)}$$

289. (c) $\frac{\sec^2 70^\circ - \cot^2 20^\circ}{2(\cos^2 59^\circ - \tan^2 31^\circ)}$

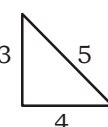
$$= \frac{2}{m}$$

$$\begin{aligned}
 &\frac{\sec^2 70^\circ - \cot^2 (90^\circ - 70^\circ)}{2(\cos^2 59^\circ - \tan^2 (90^\circ - 59^\circ))} \\
 &= \frac{2}{m}
 \end{aligned}$$

$$\frac{\sec^2 70^\circ - \tan^2 70^\circ}{2(\cos^2 59^\circ - \cot^2 59^\circ)} = \frac{2}{m}$$

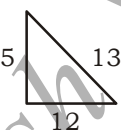
$$\begin{aligned}
 \frac{1}{2} &= \frac{2}{m} \left[\begin{array}{l} \sec^2 \theta - \tan^2 \theta = 1 \\ \cos^2 \theta - \cot^2 \theta = 1 \end{array} \right] \\
 m &= 2 \times 2 = 4
 \end{aligned}$$

290. (a) $\cos(\alpha + \beta) = \frac{4}{5}$



$$\Rightarrow \tan(\alpha + \beta) = \frac{3}{4}$$

$\sin(\alpha - \beta) = \frac{5}{13}$



$$\Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$$

$$\Rightarrow \tan 2\alpha = \tan(\alpha + \beta + \alpha - \beta)$$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta)\tan(\alpha - \beta)}$$

$$\begin{aligned}
 &= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{14}{12} \times \frac{48}{48-15} = \frac{56}{33}
 \end{aligned}$$

291. (a) $\cos \frac{\pi}{7} + \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} +$

$$\cos \frac{4\pi}{7} + \cos \frac{5\pi}{7} + \cos \frac{6\pi}{7}$$

$$= \cos \frac{\pi}{7} + \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} + \cos \left(\pi - \frac{3\pi}{7} \right)$$

$$+ \cos \left(\pi - \frac{2\pi}{7} \right) + \cos \left(\pi - \frac{\pi}{7} \right)$$

$$= \cos \frac{\pi}{7} + \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} - \cos \frac{2\pi}{7}$$

$$- \cos \frac{\pi}{7} - \cos \frac{3\pi}{7} = 0$$

292. (d) $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

$$\tan 9^\circ + \cot 9^\circ - (\tan 27^\circ + \cot 27^\circ)$$

$$= \frac{\sin 9^\circ}{\cos 9^\circ} + \frac{\cos 9^\circ}{\sin 9^\circ} - \left(\frac{\sin 27^\circ}{\cos 27^\circ} + \frac{\cos 27^\circ}{\sin 27^\circ} \right)$$

$$= \frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\sin 27^\circ \cos 27^\circ}$$

$$= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} = \frac{2(\sin 54^\circ - \sin 18^\circ)}{\sin 18^\circ \sin 54^\circ}$$

Using SinC and sinD formula

$$= \frac{2(2\sin 18^\circ \cos 36^\circ)}{\sin 18^\circ \cos 36^\circ} = 4$$

293. (a) $\sin x + 2\cos x = 1 \quad \dots(i)$

Put $x = 90^\circ$

$$\sin 90^\circ + 2\cos 90^\circ = 1$$

$$1 + 0 = 1$$

$$1 = 1 \text{ (Satisfy)}$$

Now,

$$7\cos x + 6\sin x$$

$$= 7\cos 90^\circ + 6\sin 90^\circ = 6$$

294. (b) $\cos^2 48^\circ - \sin^2 12^\circ$

$$[\because \cos^2 A - \sin^2 B = \cos(A+B) \cdot \cos(A-B)]$$

$$\cos 60^\circ \cdot \cos 36^\circ = \frac{1}{2} \left(\frac{\sqrt{5}+1}{4} \right) = \frac{\sqrt{5}+1}{8}$$

$$[\text{Using } \cos 36^\circ = \frac{\sqrt{5}+1}{4}]$$

295. (c) $\sin\left(\frac{\pi}{10}\right) \sin\left(\frac{3\pi}{10}\right)$

$$\frac{1}{2} \left[2\sin\left(\frac{\pi}{10}\right) \sin\frac{3\pi}{10} \right]$$

$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$= \frac{1}{2} \left(\cos \frac{2\pi}{10} - \cos \frac{4\pi}{10} \right)$$

$$= \frac{1}{2} (\cos 36^\circ - \cos 72^\circ)$$

$$= \frac{1}{2} (\cos 36^\circ - \sin 18^\circ)$$

(By putting the value of $\cos 36^\circ$ and $\sin 18^\circ$)

$$= \frac{1}{2} \left(\frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4} \right) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

296. (a) $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right)$

$$= \frac{1+\tan\theta}{1-\tan\theta} - \frac{1-\tan\theta}{1+\tan\theta}$$

$$= \frac{1 + \tan^2 \theta + 2 \tan \theta - 1 - \tan^2 \theta + 2 \tan \theta}{1 - \tan^2 \theta}$$

$$= \frac{2.2 \tan \theta}{1 - \tan^2 \theta} = 2 \tan 2\theta$$

Alternate:-

$$\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right)$$

Put $\theta = 15^\circ$

$$\tan(60^\circ) - \tan(30^\circ)$$

$$\Rightarrow \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \frac{2}{\sqrt{3}}$$

Now check from option,

Option (a):-

$$2 \tan 2\theta$$

$$\Rightarrow 2 \tan 2 \times 15^\circ$$

$$\Rightarrow 2 \tan 30^\circ = \frac{2}{\sqrt{3}} \text{ (Satisfy)}$$

297. (a) $\sin(\alpha - \beta) = \cos(\alpha + \beta) = \sin(90^\circ - \alpha - \beta)$

$$\alpha - \beta = 90 - \alpha - \beta$$

$$\alpha = 45^\circ, \beta = 15^\circ$$

Alternate:-

$$\sin(\alpha - \beta) = \frac{1}{2}$$

$$\Rightarrow \sin(\alpha - \beta) = \sin 30^\circ$$

$$\alpha - \beta = 30^\circ$$

$$\Rightarrow \cos(\alpha + \beta) = \frac{1}{2}$$

$$\Rightarrow \cos(\alpha + \beta) = \cos 60^\circ$$

$$\alpha + \beta = 60^\circ$$

Solve eq(i) and (ii) we get

$$\alpha = 45^\circ \text{ and } \beta = 15^\circ$$

298. (a) $\alpha + \beta = \pi + \gamma$

$$\Rightarrow \sin(\alpha + \beta) = \sin(\pi + \gamma)$$

$$\Rightarrow \sin \alpha \cos \beta + \cos \alpha \sin \beta = -\sin \gamma$$

squaring both the sides

$$\Rightarrow \sin^2 \alpha (1 - \sin^2 \beta) + \sin^2 \beta (1 - \sin^2 \alpha) + 2 \sin \alpha \cos \beta \cos \alpha \sin \beta = \sin^2 \gamma$$

$$\cos \alpha \sin \beta = \sin^2 \gamma$$

$$\Rightarrow \sin^2 \alpha - \sin^2 \alpha \sin^2 \beta + \sin^2 \beta - \sin^2 \alpha \sin^2 \beta + 2 \sin \alpha \cos \beta \cos \alpha \sin \beta = \sin^2 \gamma$$

$$\cos \alpha \sin \beta = \sin^2 \gamma$$

$$\Rightarrow \sin^2 \alpha + \sin^2 \beta - \sin^2 \gamma$$

$$= 2 \sin \alpha \sin \beta$$

$$(\sin \alpha \sin \beta - \cos \alpha \cos \beta)$$

$$= -2 \sin \alpha \sin \beta \cos(\alpha + \beta)$$

$$= -2 \sin \alpha \sin \beta \cos(180^\circ + \gamma)$$

$$= 2 \sin \alpha \sin \beta \cos \gamma$$

Alternate:-

$$\alpha + \beta - \gamma = \pi$$

$$\therefore \sin^2 \alpha + \sin^2 \beta - \sin^2 \gamma$$

$$\text{Put } \alpha = 90^\circ$$

$$\beta = 90^\circ$$

$$\gamma = 0^\circ$$

$$\therefore \sin^2 90^\circ + \sin^2 90^\circ - \sin^2 0^\circ$$

$$\Rightarrow 1 + 1 - 0$$

$$\Rightarrow 2$$

Now check from option,

Option (a):- $2 \sin \alpha \sin \beta \cos \gamma$

$$\Rightarrow 2 \sin 90^\circ \sin 90^\circ \cos 0^\circ$$

$$\Rightarrow 2 \times 1 \times 1 \times 1$$

$$\Rightarrow 2 \text{ (Satisfy)}$$

299. (c) $1 + \cos 2x + \cos 4x + \cos 6x$
 $1 + 2 \cos^2 x - 1 + 2 \cos$

$$\left(\frac{4x+6x}{2}\right) \cos\left(\frac{6x-4x}{2}\right)$$

$$= 2 \cos^2 x + 2 \cos 5x \cos x$$

$$= 2 \cos x (\cos x + \cos 5x)$$

$$\dots(i) = 2 \cos x \left[2 \cos\left(\frac{5x+x}{2}\right) \cos\left(\frac{5x-x}{2}\right) \right]$$

$$= 2 \cos x \cdot 2 \cos 3x \cos 2x$$

$$= 4 \cos x \cos 2x \cos 3x$$

Alternate :

We can put $x = 0^\circ$ in question and all option

$$1 + \cos 2x + \cos 4x + \cos 6x$$

$$= 1 + \cos 0^\circ + \cos 0^\circ + \cos 0^\circ = 4$$

Option

(a) $2 \cos x \cos 2x \cos 3x = 2$

(b) $4 \sin x \cos 2x \cos 3x = 0$

(c) $4 \cos x \cos 2x \cos 3x = 4$

(d) $\cos x \cos 2x \cos 3x = 1$

Hence, option (c) is correct

300. (b)

divide and multiply by $\sin 72^\circ$,

$$\frac{\sin 12^\circ \cdot \sin 48^\circ \cdot \sin 72^\circ \cdot \sin 54^\circ}{\sin 72^\circ}$$

$$\text{Use } \frac{1}{4} \sin 3\theta$$

$$= \sin(60^\circ - \theta) \cdot \sin \theta \cdot \sin(60^\circ + \theta)$$

Here, $\theta = 12^\circ$

$$= \frac{1}{4} \cdot \frac{\sin 36^\circ \cdot \sin 54^\circ}{2 \sin 36^\circ \cos 36^\circ}$$

$$= \frac{1}{8} \cdot \frac{\sin 54^\circ}{\sin 54^\circ} = \frac{1}{8}$$

301. (d) $\frac{1}{4}(\sqrt{3} \cos 23^\circ - \sin 23^\circ)$

$$\frac{1}{2} \left(\frac{\sqrt{3}}{2} \cos 23^\circ - \frac{1}{2} \sin 23^\circ \right)$$

$$= \frac{1}{2} (\cos 30^\circ \cos 23^\circ - \sin 30^\circ \sin 23^\circ)$$

$$[\cos A \cos B - \sin A \sin B = \cos(A + B)]$$

$$= \frac{1}{2} (\cos 53^\circ)$$

302. (a) $2 \cos x - \cos 3x - \cos 5x$
 $= 2 \cos x - (\cos 3x + \cos 5x)$

$$= 2 \cos x - \left[2 \cos\left(\frac{3x+5x}{2}\right) \cos\left(\frac{5x-3x}{2}\right) \right]$$

$$= 2 \cos x - 2 \cos 4x \cos x$$

$$= 2 \cos x (1 - \cos 4x)$$

$$= 2 \cos x (1 - 1 + 2 \sin^2 2x)$$

$$= 4 \cos x \sin^2 2x$$

$$= 4 \cos x (2 \sin x \cos x)^2$$

$$= 16 \cos^3 x \cdot \sin^2 x$$

Alternate:-

Put $x = 45^\circ$ in question and all option

$$2 \cos 45^\circ - \cos 135^\circ - \cos 225^\circ$$

$$= \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}}$$

When $x = 45^\circ$, $(\sin x = \cos x)$

$$\text{Option (a)} \quad 16 \cos^5 x = 16 \left(\frac{1}{\sqrt{2}} \right)^5$$

$$= \frac{16}{4\sqrt{2}} = \frac{4}{\sqrt{2}}$$

$$\text{Option (b)} \quad \sin^5 x = \left(\frac{1}{\sqrt{2}} \right)^5 = \frac{1}{4\sqrt{2}}$$

$$\text{Option (c)} \quad 4 \cos^5 x = 4 \left(\frac{1}{\sqrt{2}} \right)^5 = \frac{4}{4\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\text{Option (d)} \quad 4 \cos^5 x = 4 \left(\frac{1}{\sqrt{2}} \right)^5 = \frac{4}{4\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

Hence, Option (a) is correct.

$$\begin{aligned}
 303. (c) \quad & \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} \\
 &= \frac{\sqrt{3} \cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cdot \cos 20^\circ} \\
 &= \frac{2 \left[\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ \right]}{\sin 20^\circ \cdot \cos 20^\circ} \\
 &= \frac{2.2[\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ]}{2 \cdot \sin 20^\circ \cdot \cos 20^\circ} \\
 &= \frac{4 \sin 40^\circ}{\sin 40^\circ} = 4
 \end{aligned}$$

$$\begin{aligned}
 304. (c) \quad & \frac{\cos 12^\circ - \sin 12^\circ}{\cos 12^\circ + \sin 12^\circ} + \frac{\sin 147^\circ}{\cos 147^\circ} \\
 &= \frac{\sin 78^\circ - \sin 12^\circ}{\sin 78^\circ + \sin 12^\circ} \\
 &+ \frac{\sin 147^\circ}{\cos 147^\circ}
 \end{aligned}$$

$$\begin{aligned}
 & \left[\begin{array}{l} \sin(180^\circ - \theta) = \sin \theta \\ \cos(180^\circ - \theta) = -\cos \theta \end{array} \right] \\
 &= \frac{2 \cos 45^\circ \cdot \sin 33^\circ}{2 \sin 45^\circ \cdot \cos 33^\circ} - \frac{\sin 33^\circ}{\cos 33^\circ} = 0
 \end{aligned}$$

$$\begin{aligned}
 305. (d) \quad & \cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ \\
 &= \cos 24^\circ + \cos(90^\circ - 35^\circ) + \cos(90^\circ + 35^\circ) + \cos(180^\circ + 24^\circ) + \cos(360^\circ - 60^\circ) \\
 &= \cos 24^\circ + \sin 35^\circ - \sin 35^\circ - \cos 24^\circ + \cos 60^\circ = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 306. (a) \quad & m \tan(\theta - 30^\circ) = n \tan(\theta + 120^\circ) \\
 & m \tan(\theta - 30^\circ) = -n \cot(\theta + 30^\circ)
 \end{aligned}$$

$$\tan(\theta - 30^\circ) \cdot \tan(\theta + 30^\circ) = \frac{-n}{m}$$

$$\frac{\sqrt{3} \tan \theta - 1}{\sqrt{3} + \tan \theta} \cdot \frac{\sqrt{3} \tan \theta + 1}{\sqrt{3} - \tan \theta} = \frac{-n}{m}$$

$$\left[\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \right]$$

$$\frac{3 \tan^2 \theta - 1}{3 - \tan^2 \theta} = \frac{-n}{m}$$

$$\frac{m}{n} = \frac{3 - \tan^2 \theta}{1 - 3 \tan^2 \theta}$$

Use Componendo and Dividendo

$$\frac{m+n}{m-n} = \frac{4 - 4 \tan^2 \theta}{2 + 2 \tan^2 \theta}$$

$$\begin{aligned}
 &= \frac{4(1 - \tan^2 \theta)}{2(1 + \tan^2 \theta)} = 2 \cos 2\theta
 \end{aligned}$$

Alternate:-

$$\frac{n}{m} = \frac{\tan(\theta - 30^\circ)}{\tan(\theta + 120^\circ)}$$

$$\text{Put } \theta = 90^\circ$$

$$\Rightarrow \frac{n}{m} = \frac{\tan(90^\circ - 30^\circ)}{\tan(90^\circ + 120^\circ)}$$

$$\Rightarrow \frac{n}{m} = \frac{\tan 60^\circ}{\tan 210^\circ}$$

$$\Rightarrow \frac{n}{m} = \frac{\tan 60^\circ}{\tan(180^\circ + 30^\circ)}$$

$$\Rightarrow \frac{n}{m} = \frac{\tan 60^\circ}{\tan 30^\circ}$$

$$\Rightarrow \frac{n}{m} = \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}}$$

$$\Rightarrow \frac{n}{m} = 3$$

$$\Rightarrow \frac{m}{n} = \frac{1}{3}$$

$$\therefore \frac{m+n}{m-n} = \frac{1+3}{1-3} = \frac{4}{-2} = -2$$

Now check from option,

Option (a):- $2 \cos 2\theta$

$$\begin{aligned}
 \text{Put } \theta &= 90^\circ \\
 &= 2 \cos 180^\circ \\
 &= 2 \cos(90^\circ + 90^\circ) \\
 &= -2 \sin 90^\circ \\
 &= -2 \text{ (Satisfy)}
 \end{aligned}$$

$$307. (a) \quad \tan \theta = \frac{x \sin \phi}{1 - x \cos \phi}$$

$$\Rightarrow \frac{1}{\tan \theta} = \frac{1}{x \sin \phi} - \cot \phi$$

$$\frac{1}{\tan \theta} + \cot \phi = \frac{1}{x \sin \phi}$$

$$\Rightarrow \cot \theta + \cot \phi = \frac{1}{x \sin \phi} \dots (i)$$

$$\tan \phi = \frac{y \sin \theta}{1 - y \cos \theta}$$

$$\Rightarrow \frac{1}{\tan \phi} = \frac{1}{y \sin \theta} - \cot \theta$$

$$\frac{1}{\tan \phi} + \cot \theta = \frac{1}{y \sin \theta}$$

$$\Rightarrow \cot \phi + \cot \theta = \frac{1}{y \sin \theta} \dots (ii)$$

from equation (i) and (ii)

$$\frac{1}{x \sin \phi} = \frac{1}{y \sin \theta}$$

$$\Rightarrow \frac{x}{y} = \frac{\sin \theta}{\sin \phi}$$

$$308. (d) \quad \tan x = \frac{b}{a}$$

$$= \sqrt{1 + \frac{b}{a}} + \sqrt{1 - \frac{b}{a}}$$

$$= \sqrt{\frac{1 + \tan x}{1 - \tan x}} + \sqrt{\frac{1 - \tan x}{1 + \tan x}}$$

Rationalising above equation

$$= \sqrt{\frac{1 + \tan x}{1 - \tan x}} \times \frac{1 + \tan x}{1 + \tan x}$$

$$+ \sqrt{\frac{1 - \tan x}{1 + \tan x}} \times \frac{1 - \tan x}{1 - \tan x}$$

$$= \sqrt{\frac{(1 + \tan x)^2}{1 - \tan^2 x}} + \sqrt{\frac{(1 - \tan x)^2}{1 - \tan^2 x}}$$

$$= \frac{1 + \tan x}{\sqrt{1 - \tan^2 x}} + \frac{1 - \tan x}{\sqrt{1 - \tan^2 x}}$$

$$= \frac{(1 + \tan x) + (1 - \tan x)}{\sqrt{1 - \tan^2 x}} = \frac{2}{\sqrt{1 - \frac{\sin^2 x}{\cos^2 x}}}$$

$$= \frac{2}{\sqrt{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}} = \frac{2 \cos x}{\sqrt{\cos 2x}}$$

$$\begin{aligned}
 309. (c) \quad & \tan \alpha + 2 \tan 2\alpha + 4 \tan 4\alpha + \frac{8(1 - \tan^2 4\alpha)}{2 \tan 4\alpha} \\
 & \left(\because \cot 2\theta = \frac{1 - \tan^2 \theta}{2 \tan \theta} \right)
 \end{aligned}$$

$$= \tan \alpha + 2 \tan 2\alpha + \frac{8 \tan^2 4\alpha + 8 - 8 \tan^2 4\alpha}{2 \tan 4\alpha}$$

$$= \tan \alpha + 2 \tan 2\alpha + \frac{4}{\tan 4\alpha}$$

$$\begin{aligned}
 &= \tan \alpha + 2 \tan 2\alpha + \frac{4(1 - \tan^2 2\alpha)}{2 \tan 2\alpha} \\
 &= \tan \alpha + \frac{4 \tan^2 2\alpha + 4 - 4 \tan^2 2\alpha}{2 \tan 2\alpha} \\
 &= \tan \alpha + \frac{2}{\tan 2\alpha} \\
 &= \tan \alpha + \frac{2(1 - \tan^2 \alpha)}{2 \tan \alpha} \\
 &= \frac{2 \tan^2 \alpha + 2 - 2 \tan^2 \alpha}{2 \tan \alpha} \\
 &= \frac{1}{\tan \alpha} = \cot \alpha
 \end{aligned}$$

Alternate:-

$$\begin{aligned}
 &\tan \alpha + 2 \tan 2\alpha + 4 \tan 4\alpha + 8 \cot 8\alpha \\
 &\text{Put } \alpha = 30^\circ \\
 &= \tan 30^\circ + 2 \tan 60^\circ + 4 \tan 120^\circ + 8 \cot 240^\circ \\
 &= \tan 30^\circ + 2 \tan 60^\circ - 4 \cot 30^\circ + 8 \cot 60^\circ
 \end{aligned}$$

$$= \frac{1}{\sqrt{3}} + 2\sqrt{3} - 4\sqrt{3} + \frac{8}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} - 2\sqrt{3} + \frac{8}{\sqrt{3}}$$

$$= \frac{9}{\sqrt{3}} - 2\sqrt{3}$$

$$= 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$$

Now check from options

Option (c):- $\cot \alpha = \cot 30^\circ$

$$= \sqrt{3} \text{ (Satisfy)}$$

310. (b) $\sin A = n \sin B$

$$\frac{\sin A}{\sin B} = \frac{n}{1}$$

applying componendo dividendo rule

$$\Rightarrow \frac{\sin A + \sin B}{\sin A - \sin B} = \frac{n+1}{n-1}$$

$$\Rightarrow \frac{2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)}{2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)}$$

$$= \frac{n+1}{n-1}$$

$$\Rightarrow \frac{\tan \left(\frac{A+B}{2} \right)}{\tan \left(\frac{A-B}{2} \right)} = \frac{n+1}{n-1}$$

$$\left(\frac{n-1}{n+1} \right) \tan \left(\frac{A+B}{2} \right) = \tan \left(\frac{A-B}{2} \right)$$

Alternate:-

$$\sin A = n \sin B$$

$$\text{Put, } A = 90^\circ \text{ \& } B = 30^\circ$$

$$\text{then, } n = 2$$

$$\left(\frac{n-1}{n+1} \right) \tan 60^\circ = \frac{1}{3} \times \sqrt{3} = \frac{1}{\sqrt{3}}$$

Put these value in options

$$\text{Option (a) } \sin \left(\frac{A-B}{2} \right) = \sin 30^\circ = \frac{1}{2}$$

$$\text{Option (b) } \tan \left(\frac{A-B}{2} \right) = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\text{Option (c) } \cot \left(\frac{A-B}{2} \right) = \cot 30^\circ = \sqrt{3}$$

$$\text{Option (d) } \tan \left(\frac{A+B}{2} \right) = \tan 60^\circ = \sqrt{3}$$

Hence, option (b) is correct.

311. (b) $2 \sin A \cos^3 A - 2 \sin^3 A \cos A$

$$2 \sin A \cos A (\cos^2 A - \sin^2 A)$$

$$\frac{1}{2} (2 \sin 2A \cdot \cos 2A) = \frac{1}{2} \sin 4A$$

Alternate:-

$$2 \sin A \cos^3 A - 2 \sin^3 A \cos A$$

$$\text{Put } A = 30^\circ$$

$$2 \sin 30^\circ \cos^3 30^\circ - 2 \sin^3 30^\circ \cos 30^\circ$$

$$\frac{3\sqrt{3}}{8} - 2 \times \frac{1}{8} \times \frac{\sqrt{3}}{2}$$

$$\frac{3\sqrt{3}}{8} - \frac{\sqrt{3}}{8} = \frac{\sqrt{3}}{4}$$

Now check from options

$$\text{Option (b):- } \frac{1}{2} \sin 4A$$

$$\text{Put } A = 30^\circ$$

$$\Rightarrow \frac{1}{2} \times \sin 120^\circ$$

$$\Rightarrow \frac{1}{2} \times \sin(90 + 30^\circ)$$

$$\Rightarrow \frac{1}{2} \times \cos 30^\circ$$

$$\Rightarrow \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} \text{ (Satisfy)}$$

312. (d) $\tan A + \tan A - \tan A - \cot A$
 $= \tan A - \cot A$

$$\text{313. (a)} \frac{2 \sin \alpha}{1 + \cos \alpha + \sin \alpha} = y$$

$$\text{Put } \alpha = 45^\circ$$

$$= \frac{\frac{2}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = y$$

$$= y = \frac{2}{2 + \sqrt{2}} = 2 - \sqrt{2}$$

$$= \frac{1 - \cos \alpha + \sin \alpha}{1 + \sin \alpha}$$

$$= \frac{1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{\sqrt{2} + 1}$$

$$= \frac{\sqrt{2}(\sqrt{2} - 1)}{1} = 2 - \sqrt{2} = y$$

$$\text{314. (c)} \frac{\sin 70^\circ + \cos 40^\circ}{\cos 70^\circ + \sin 40^\circ}$$

$$= \frac{\cos 20^\circ + \cos 40^\circ}{\sin 20^\circ + \sin 40^\circ}$$

$$= \frac{2 \cos 30^\circ \cdot \cos 10^\circ}{2 \sin 30^\circ \cdot \cos 10^\circ}$$

$$= \cot 30^\circ = \sqrt{3}$$

$$\text{315. (a)} \frac{\sin^4 A}{a} + \frac{\cos^4 A}{b} = \frac{1}{a+b}$$

$$\Rightarrow \left(\frac{a+b}{a} \right) \sin^4 A + \left(\frac{a+b}{b} \right) \cos^4 A = 1$$

$$\Rightarrow \left(\frac{a+b}{a} \sin^2 A \right) \sin^2 A +$$

$$\left(\frac{a+b}{b} \cos^2 A \right) \cos^2 A = 1$$

$$\left(\because \frac{a+b}{a} \sin^2 A = 1 \text{ \& } \frac{a+b}{b} \cos^2 A = 1 \right)$$

$$\Rightarrow \sin^2 A = \frac{a}{a+b} \text{ \& } \cos^2 A = \frac{b}{a+b}$$

Now,

$$\Rightarrow \frac{\sin^8 A}{a^3} + \frac{\cos^8 A}{b^3}$$

$$= \frac{a^4}{(a+b)^4 a^3} + \frac{b^4}{(a+b)^4 b^3}$$

$$= \frac{a+b}{(a+b)^4} = \frac{1}{(a+b)^3}$$

Alternate:-

Put $A = 45^\circ$ and $a = 1, b = 1$

$$\Rightarrow \frac{\sin^4 45^\circ}{1} + \frac{\cos^4 45^\circ}{1} = \frac{1}{1+1}$$

$$\Rightarrow \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{2} \text{ (Satisfy)}$$

$$\therefore \frac{\sin^8 A}{a^3} + \frac{\cos^8 A}{b^3}$$

$$\frac{\sin^8 45^\circ}{1^3} + \frac{\cos^8 45^\circ}{1^3}$$

$$\frac{1}{16} + \frac{1}{16} = \frac{1}{8}$$

Now check from option

Option (a):- $\frac{1}{(a+b)^3}$

$$\Rightarrow \frac{1}{(1+1)^3} \Rightarrow \frac{1}{8} \text{ (Satisfy)}$$

316. (a) $2y \cos \theta = x \sin \theta$

$$= 2y \operatorname{cosec} \theta = x \sec \theta$$

Put value of $x \sec \theta$ as following

$$2x \sec \theta - y \operatorname{cosec} \theta = 3$$

$$2.2y \operatorname{cosec} \theta - y \operatorname{cosec} \theta = 3$$

$$4y \operatorname{cosec} \theta - y \operatorname{cosec} \theta = 3$$

$$3y \operatorname{cosec} \theta = 3$$

$$= y \operatorname{cosec} \theta = 1$$

$$\text{or } \sin \theta = y$$

$$\text{then, } 2y \cos \theta = xy$$

$$\Rightarrow 2 \cos \theta = x$$

$$\text{Hence,}$$

$$x^2 + 4y^2 = 4 \cos^2 \theta + 4 \sin^2 \theta = 4$$

Alternate:-

$$\text{Put } \theta = 45^\circ$$

$$2y \cos 45^\circ = x \sin 45^\circ \Rightarrow x = 2y$$

$$2x \sec 45^\circ - y \operatorname{cosec} 45^\circ = 3$$

$$\sqrt{2} (2x - y) = 3 \Rightarrow \sqrt{2} (4y - y) = 3$$

$$y = \frac{1}{\sqrt{2}}$$

$$x = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$x^2 + 4y^2 = 2 + 4 \times \frac{1}{2} = 4$$

317. (d) $\tan \theta - \cot \theta$

$$= a \& \cos \theta + \sin \theta = b$$

$$\text{Put, } \theta = 45^\circ$$

$$\tan 45^\circ - \cot 45^\circ = a,$$

$$\cos 45^\circ + \sin 45^\circ = b,$$

$$a = 0, \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = b$$

$$\frac{1+1}{\sqrt{2}} = b \Rightarrow b = \sqrt{2}$$

$$(b^2 - 1)^2 (a^2 + 4)$$

$$= \left[(\sqrt{2})^2 - 1 \right]^2 (0 + 4)$$

$$= (2-1)^2 (4) = 4$$

318. (a) $\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$

$$\frac{\sin \theta}{\cos \theta} = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$$

$$\text{let, } \sin \alpha - \cos \alpha = k \sin \theta$$

$$\sin \alpha + \cos \alpha = k \cos \theta$$

$$\text{Adding after squaring,}$$

$$\sin^2 \alpha + \cos^2 \alpha - 2 \sin \alpha \cos \alpha +$$

$$\sin^2 \alpha + \cos^2 \alpha + 2 \sin \alpha \cos \alpha$$

$$= k^2 (\sin^2 \theta + \cos^2 \theta)$$

$$2 = k^2$$

$$k = \sqrt{2}$$

$$\text{Hence, } \sin \alpha - \cos \alpha = \sqrt{2} \sin \theta$$

$$\sin \alpha + \cos \alpha = \sqrt{2} \cos \theta$$

319. (b) $\sin \theta + \sin \phi = a,$

$$\cos \theta + \cos \phi = b$$

$$\text{Put, } \theta = 90^\circ \& \phi = 30^\circ$$

$$a = 1 + \frac{1}{2} = \frac{3}{2}, b = \frac{\sqrt{3}}{2}$$

$$\tan \left(\frac{\theta - \phi}{2} \right) = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\text{option (b)} \Rightarrow \sqrt{\frac{4 - a^2 - b^2}{a^2 + b^2}}$$

$$= \sqrt{\frac{4 - \frac{9}{4} - \frac{3}{4}}{\frac{9}{4} + \frac{3}{4}}} = \frac{1}{\sqrt{3}}$$

Hence, option (b) is correct

320. (a) Answer is independent of α so

$$\text{put } \alpha = 0^\circ$$

$$\cos^2 \alpha + \cos^2 (\alpha + 120^\circ) + \cos^2 (\alpha - 120^\circ)$$

$$= \cos^2 0^\circ + \cos^2 120^\circ + \cos^2 (-120^\circ)$$

$$= 1 + \frac{1}{4} + \frac{1}{4} = \frac{3}{2}$$

321. (a) $\tan A = \frac{1 - \cos B}{\sin B}$

$$= \frac{1 - \cos 2 \times \frac{B}{2}}{\sin 2 \times \frac{B}{2}} = \frac{1 - 1 + 2 \sin^2 \frac{B}{2}}{2 \sin \frac{B}{2} \cos \frac{B}{2}}$$

$$\tan A = \frac{\sin \frac{B}{2}}{\cos \frac{B}{2}} = \tan \frac{B}{2}$$

$$A = \frac{B}{2} \Rightarrow B = 2A$$

$$\tan 2A = \tan B$$

322. (c) $\tan 2A = \tan (A + B + A - B)$

$$= \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B) \cdot \tan(A-B)} = \frac{p+q}{1-pq}$$

$$\text{323. (b)} \frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\frac{1}{\cos 8A} - 1}{\frac{1}{\cos 4A} - 1}$$

$$= \frac{1 - \cos 8A}{1 - \cos 4A} \times \frac{\cos 4A}{\cos 8A}$$

$$= \frac{2 \sin^2 4A}{2 \sin^2 2A} \times \frac{\cos 4A}{\cos 8A}$$

$$= \frac{2 \sin 4A \cos 4A \cdot \sin 4A}{\cos 8A \times 2 \sin^2 2A}$$

$$= \tan 8A \cdot \frac{\cos 2A}{\sin 2A} = \frac{\tan 8A}{\tan 2A}$$

Alternate:-

$$\frac{\sec 8A - 1}{\sec 4A - 1}$$

$$\text{Put } A = 15^\circ$$

$$\Rightarrow \frac{\sec 120^\circ - 1}{\sec 60^\circ - 1}$$

$$\Rightarrow \frac{\frac{1}{\cos 120^\circ} - 1}{\frac{1}{\cos 60^\circ} - 1}$$

$$\Rightarrow \frac{-\frac{1}{\sin 30^\circ} - 1}{\frac{1}{\cos 60^\circ} - 1} = \frac{-2-1}{2-1} = -3$$

Now check from options

Option (b):- $\frac{\tan 8A}{\tan 2A}$

Put $A = 15^\circ$

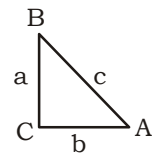
$$\frac{\tan 120^\circ}{\tan 30^\circ} = \frac{-\cot 30^\circ}{\tan 30^\circ}$$

$$= -\sqrt{3} \times \sqrt{3} = -3 \text{ (Satisfy)}$$

324. (d) $\alpha + \beta = \tan A + \tan B = \frac{a}{b} + \frac{b}{a}$

$$= \frac{a^2 + b^2}{ab}$$

$$\alpha \beta = \tan A \cdot \tan B = \frac{a}{b} \cdot \frac{b}{a} = 1$$



then equation

$$x^2 - (\alpha + \beta)x + \alpha \beta = 0$$

$$x^2 - \left(\frac{a^2 + b^2}{ab} \right)x + 1 = 0$$

$$abx^2 - (a^2 + b^2)x + ab = 0$$

$$abx^2 - c^2x + ab = 0$$

$$(\because a^2 + b^2 = c^2)$$

325. (d) $\cos(A - B) = \frac{3}{5}$ and

$$\tan A \tan B = \frac{\sin A \sin B}{\cos A \cos B} = 2$$

$$\sin A \sin B = 2 \cos A \cos B \quad (i)$$

$$\cos A \cos B + \sin A \sin B = \frac{3}{5}$$

From eq (i)

$$3 \cos A \cos B = \frac{3}{5}$$

$$\cos A \cos B = \frac{1}{5}$$

$$\sin A \sin B = \frac{2}{5}$$

326. (a)

$$\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \frac{\sin 81^\circ + \sin 9^\circ}{\sin 81^\circ - \sin 9^\circ}$$

$$= \frac{2 \sin 45^\circ \cdot \cos 36^\circ}{2 \cos 45^\circ \cdot \sin 36^\circ} = \cot 36^\circ$$

$$= \tan 54^\circ$$

327. (b) $\tan \alpha = \frac{1}{7}$, $\tan \beta = \frac{1}{3}$

$$\cos 2\alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$$

$$\frac{1 - \frac{1}{49}}{1 + \frac{1}{49}} = \frac{48}{50} = \frac{24}{25}$$

$$\sin 2\beta = \frac{2 \tan \beta}{1 + \tan^2 \beta} = \frac{\frac{2}{3}}{1 + \frac{1}{9}}$$

$$= \frac{2}{3} \times \frac{9}{10} = \frac{3}{5}$$

$$\cos 2\beta = \frac{4}{5}$$

$$\sin 4\beta = 2 \sin 2\beta \cos 2\beta$$

$$= 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$$

$$\text{hence, } \cos 2\alpha = \sin 4\beta$$

328. (b) $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$

$$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \frac{1}{4}}{1 + \frac{1}{4}} = \frac{3}{5}$$

$$\sin 2B = \frac{2 \tan B}{1 + \tan^2 B} = \frac{\frac{2}{3}}{1 + \frac{1}{9}}$$

$$= \frac{2}{3} \times \frac{9}{10} = \frac{3}{5}$$

$$\text{Hence, } \cos 2A = \sin 2B$$

329. (c) Put $\beta = 0^\circ$ such that no option are same

$$2 \sin^2 \beta + 4 \cos(\alpha + \beta) \cdot$$

$$\sin \alpha \cdot \sin \beta + \cos^2(\alpha + \beta)$$

$$= 0 + 0 + \cos^2 \alpha$$

$$= \cos^2 \alpha \text{ option (c) (We can also put } \alpha = 0^\circ)$$

330. (d)

$$\frac{(\sin 12^\circ \cdot \sin 48^\circ \cdot \sin 72^\circ) \cdot (\sin 24^\circ \cdot \sin 36^\circ \cdot \sin 84^\circ)}{\sin 72^\circ \cdot \sin 36^\circ}$$

Multiply and divide by $\sin 36^\circ$ & $\sin 72^\circ$

$$\sin \theta \cdot \sin(60 - \theta) \cdot \sin(60 + \theta) = \frac{1}{4} \sin 3\theta \quad (a)$$

$$= \frac{1}{4} \sin 36^\circ \times \frac{1}{4} \sin 72^\circ \times \quad (b)$$

$$\frac{1}{\sin 72^\circ \sin 36^\circ} = \frac{1}{16}$$

331. (a) $\tan 5x = \frac{\tan 3x + \tan 2x}{1 - \tan 3x \tan 2x}$

$$\Rightarrow \tan 5x - \tan 3x \tan 2x = \tan 3x + \tan 2x$$

$$\Rightarrow \tan 5x \tan 3x \tan 2x = \tan 5x - \tan 3x - \tan 2x$$

332. (d) $\cos \alpha + \cos \beta = \sin \alpha + \sin \beta$

Squaring both sides

$$\Rightarrow \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta$$

$$= \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta$$

$$\Rightarrow (\cos^2 \alpha - \sin^2 \alpha) + (\cos^2 \beta - \sin^2 \beta)$$

$$+ 2(\cos \alpha \cos \beta - \sin \alpha \sin \beta) = 0$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta + 2 \cos(\alpha + \beta) = 0$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta = -2 \cos(\alpha + \beta)$$

333. (c) $\cos A = a \cos B \Rightarrow \cos^2 A = a^2$

$$\cos^2 B = a^2 (1 - \sin^2 B) \quad \dots (i)$$

$$\sin A = b \sin B \Rightarrow \sin^2 A = b \sin^2 B \quad \dots (ii)$$

Adding both equation

$$\cos^2 A + \sin^2 A = a^2 - a^2 \sin^2 B + b^2 \sin^2 B$$

$$1 = a^2 + \sin^2 B (b^2 - a^2)$$

$$(b^2 - a^2) \sin^2 B = 1 - a^2$$

Alternate:-

$$a = \frac{\cos A}{\cos B}, \quad b = \frac{\sin A}{\sin B}$$

$$\text{Put } A = 0^\circ, \quad B = 60^\circ$$

$$a = 2, \quad b = 0$$

$$(b^2 - a^2) \sin^2 B = (0^2 - 2^2) \times \sin^2 60^\circ$$

$$= -4 \times \frac{3}{4} = -3$$

Now check from option

Option (c):- $1 - a^2$

$$\text{Put } a = 2$$

$$1 - (2)^2 = 1 - 4 = -3 \text{ (Satisfy)}$$

334. (c) Put $A = 90^\circ$, $B = 60^\circ$, $C = 30^\circ$

$$(\because A + B + C = \pi)$$

$$\text{then, } \cos 2A + \cos 2B + \cos 2C$$

$$= \cos 180^\circ + \cos 120^\circ + \cos 60^\circ$$

$$= -1 - \frac{1}{2} + \frac{1}{2} = -1$$

Put these value in all option

$$= 1 + 4 \cos A \cdot \cos B \cdot \cos C$$

$$= 1$$

$$= -1 + 4 \sin A \sin B \cdot \cos C$$

$$= -1 + 4 \times 1 \times \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= -1 + 3 = 2$$

$$\begin{aligned} \text{(c)} &= -1 - 4 \cos A \cos B \cdot \cos C \\ &= -1 - 0 = -1 \\ \text{(d)} &= 1 + 4 \sin A \sin B \cdot \sin C \end{aligned}$$

$$= 1 + 4 \times 1 \times \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = 1 + \sqrt{3}$$

Hence, option (c) is correct

$$\begin{aligned} \text{335. (b)} \quad &\text{Let } A = B = 45^\circ \text{ and } C = 90^\circ \\ &\sin^2 A + \sin^2 B + \sin^2 C - 2 \cos A \cdot \cos B \cdot \cos C \end{aligned}$$

$$= \frac{1}{2} + \frac{1}{2} + 1 - 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 0 = 2$$

$$\begin{aligned} \text{336. (c)} \quad &\sin 2A + \sin 2B + \sin 2C \\ &= 2 \sin(A+B) \cdot \cos(A-B) + 2 \sin C \cdot \cos C \\ &= 2 \sin(180^\circ - C) \cdot \cos(A-B) + 2 \sin C \cdot \cos C \\ &= 2 \sin C [\cos(A-B) + \cos C] \\ &= 2 \sin C [\cos(A-B) + \cos(180^\circ - (A+B))] \\ &= 2 \sin C [\cos(A-B) - \cos(A+B)] \\ &= 2 \sin C (2 \sin A \cdot \sin B) = 4 \sin A \sin B \sin C \end{aligned}$$

Alternate:-

$$A+B+C=180^\circ$$

Put $A = B = 45^\circ$ and $C = 90^\circ$ in equation and option too.

$$\begin{aligned} \sin 2A + \sin 2B + \sin 2C &= \sin 90^\circ + \sin 90^\circ + \sin 180^\circ \\ &= 1 + 1 + 0 = 2 \end{aligned}$$

Now check for option

as

$$\text{(a)} = 0 \quad \text{(b)} = 0$$

$$\text{(c)} = 4 \times \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot 1 = 2$$

$$\text{(d)} = 8 \times \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot 1 = 4$$

Option (c) is correct.

$$\begin{aligned} \text{337. (c)} \quad &\tan 10^\circ - \tan 50^\circ + \tan 70^\circ \\ \Rightarrow &\tan 10^\circ + \tan(60^\circ + 10^\circ) - \tan(60^\circ - 10^\circ) \end{aligned}$$

$$\begin{aligned} \Rightarrow &\tan 10^\circ + \frac{\tan 60^\circ + \tan 10^\circ}{1 - \tan 60^\circ \cdot \tan 10^\circ} \\ &- \frac{\tan 60^\circ - \tan 10^\circ}{1 + \tan 60^\circ \cdot \tan 10^\circ} \end{aligned}$$

$$\Rightarrow \tan 10^\circ + \frac{\sqrt{3} + \tan 10^\circ}{1 - \sqrt{3} \tan 10^\circ}$$

$$- \frac{\sqrt{3} - \tan 10^\circ}{1 + \sqrt{3} \tan 10^\circ}$$

$$\Rightarrow \tan 10^\circ + \frac{8 \tan 10^\circ}{1 - 3 \tan^2 10^\circ}$$

$$\Rightarrow \frac{9 \tan 10^\circ - 3 \tan^3 10^\circ}{1 - 3 \tan^2 10^\circ}$$

$$= \frac{3(3 \tan 10^\circ - \tan^3 10^\circ)}{1 - 3 \tan^2 10^\circ}$$

$$\left[\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right]$$

$$= 3 \tan 30^\circ = \sqrt{3}$$

338. (a) Solve it by option

$$\text{put, } \alpha = \frac{\pi}{6}$$

$$4 \cos \frac{\pi}{6} + 3 \cos 2 \frac{\pi}{6} - 2 \sin 3 \frac{\pi}{6} +$$

$$\cos 4 \frac{2\pi}{6}$$

$$= 4 \cdot \frac{\sqrt{3}}{2} + \frac{3}{2} - 2 - \frac{1}{2}$$

$$= \frac{4\sqrt{3}}{2} - 1 = 2\sqrt{3} - 1$$

hence, option (a) is correct.

$$\text{339. (a)} \quad (\sin A + \sin B + \sin C)^2 = \sin^2 A + \sin^2 B + \sin^2 C$$

As we know that

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$$

$$\therefore 2(\sin A \sin B + \sin B \sin C + \sin C \sin A) = 0$$

$$\Rightarrow \sin A \sin B + \sin B \sin C + \sin C \sin A = 0$$

above condition will be true only when

$$A = B = C = 0$$

$$\therefore \sin A + \sin B + \sin C = 0$$

$$\text{340. (b)} \quad \frac{\sin x}{\sin y} = p \text{ \& } \frac{\cos x}{\cos y} = q$$

$$\frac{\sin x}{p} = \sin y$$

$$\Rightarrow \frac{\sin^2 x}{p^2} = \sin^2 y$$

$$\frac{\cos x}{q} = \cos y \Rightarrow \frac{\cos^2 x}{q^2} = \cos^2 y$$

Adding both equation

$$\frac{\sin^2 x}{p^2} + \frac{\cos^2 x}{q^2} = \sin^2 y + \cos^2 y = 1$$

divide by $\cos^2 x$

$$\frac{\tan^2 x}{p^2} + \frac{1}{q^2} = \frac{1}{\cos^2 x}$$

$$= \sec^2 x = 1 + \tan^2 x$$

$$\tan^2 x \left(\frac{1}{p^2} - 1 \right) = 1 - \frac{1}{q^2}$$

$$\tan^2 x = \frac{\frac{q^2-1}{q^2}}{\frac{1-p^2}{p^2}}$$

$$\tan x = \frac{p}{q} \sqrt{\frac{q^2-1}{1-p^2}}$$

Alternate:-

$$p = \frac{\sin x}{\sin y}$$

$$q = \frac{\cos x}{\cos y}$$

Put $x = 30^\circ$ and $y = 60^\circ$

$$p = \frac{\sin 30^\circ}{\sin 60^\circ} = \frac{1}{\sqrt{3}}$$

$$q = \frac{\cos 30^\circ}{\cos 60^\circ} = \sqrt{3}$$

$$\therefore \tan x = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

Now check from options

$$\text{Option (b):- } \frac{p}{q} \sqrt{\frac{q^2-1}{1-p^2}}$$

$$\Rightarrow \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} \sqrt{\frac{(\sqrt{3})^2-1}{1-\left(\frac{1}{\sqrt{3}}\right)^2}}$$

$$\Rightarrow \frac{1}{3} \sqrt{\frac{3-1}{3-1}}$$

$$\Rightarrow \frac{1}{3} \sqrt{\frac{2}{2}} \times 3$$

$$\Rightarrow \frac{1}{\sqrt{3}} \quad (\text{Satisfy})$$

$$\text{341. (a)} \quad y = a \cos^2 x + 2b \sin x \cos x + c \sin^2 x$$

$$z = a \sin^2 x - 2b \sin x \cos x + c \cos^2 x$$

$$y + z = a (\sin^2 x + \cos^2 x) + c (\sin^2 x + \cos^2 x)$$

$$y + z = a + c$$

$$\text{342. (c)} \quad A + B + C = \pi$$

answer is independent of A, B & C

So put $A = B = C = 60^\circ$

$$= \frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B}$$

$$= \frac{\cos 60^\circ}{\sin 60^\circ \cdot \sin 60^\circ} + \frac{\cos 60^\circ}{\sin 60^\circ \cdot \sin 60^\circ} + \frac{\cos 60^\circ}{\sin 60^\circ \cdot \cos 60^\circ}$$

$$= 3 \times \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}} = 3 \cdot \frac{\frac{1}{2}}{\frac{3}{4}} = 2$$

343. (a) $\sin A$, $\cos A$ & $\tan A$ are in G.P.
then, $\cos^2 A = \sin A \cdot \tan A$

$$\cos^2 A = \frac{\sin A \cdot \sin A}{\cos A}$$

$$\cos^3 A = \sin^2 A = 1 - \cos^2 A$$

$$\cos^3 A + \cos^2 A = 1$$

344. (d) $A + B = C$, $\tan A = k \tan B$
and $A - B = \phi$

$$\frac{\sin A}{\cos A} = \frac{k \sin B}{\cos B}$$

$$\frac{\sin A \cos B}{\cos A \sin B} = \frac{k}{1}$$

Using componendo & dividendo rule

$$\frac{\sin A \cos B + \cos A \sin B}{\cos B \sin A - \cos A \sin B} = \frac{k+1}{k-1}$$

$$\frac{\sin(A+B)}{\sin(A-B)} = \frac{k+1}{k-1}$$

$$\frac{\sin C}{\sin \phi} = \frac{k+1}{k-1}$$

$$\sin C = \frac{k+1}{k-1} \sin \phi$$

345. (b) $\tan \alpha$, $\tan \beta$ are roots of $x^2 + px + q = 0$

$$\tan \alpha + \tan \beta = -p$$

$$\tan \alpha \cdot \tan \beta = q$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

$$= \frac{-p}{1-q} = \frac{p}{q-1}$$

Alternate:-

$$x^2 + px + q = 0$$

$$\text{Put } \alpha = 45^\circ$$

$$\beta = 0^\circ$$

$$\tan 45^\circ = 1$$

$$\tan 0^\circ = 0$$

$$\therefore 1^2 + p \times 1 + q = 0$$

$$p + q = -1$$

$$0^2 + p \times 0 + q = 0$$

$$q = 0$$

From (i) and (ii)

$$p = -1 \text{ and } q = 0$$

$$\therefore \tan(\alpha + \beta) = \tan 45^\circ = 1$$

Now check from options,

$$\text{Option (b):- } \frac{p}{q-1} = \frac{-1}{0-1}$$

$$= 1$$

(Satisfy)

346. (a) If $A + B + C = \frac{3\pi}{2}$

$$\text{Put } A = B = C = \frac{\pi}{2} \text{ in question}$$

and all options.

$$\cos 2A + \cos 2B + \cos 2C = \cos \pi +$$

$$\cos \pi + \cos \pi = -3$$

option (a)

$$= 1 - 4 \sin \frac{\pi}{2} \cdot \sin \frac{\pi}{2} \cdot \sin \frac{\pi}{2} = 1 - 4$$

$$= -3$$

347. (c) $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$

$$\frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y} = \frac{a+b}{a-b}$$

Using componendo & dividendo rule

$$\frac{2 \sin x \cdot \cos y}{2 \cos x \cdot \sin y} = \frac{2a}{2b}$$

$$= \frac{\tan x}{\tan y} = \frac{a}{b}$$

$$\dots(i) \quad \text{348. (d)} \quad \tan \frac{\pi}{3} = \tan \left(\frac{2\pi}{5} - \frac{\pi}{15} \right)$$

\dots(ii)

$$\tan \frac{\pi}{3} = \frac{\tan \frac{2\pi}{5} - \tan \frac{\pi}{15}}{1 + \tan \frac{2\pi}{5} \tan \frac{\pi}{15}}$$

$$\Rightarrow \tan \frac{\pi}{3} + \tan \frac{\pi}{3} \tan \frac{2\pi}{5} \tan \frac{\pi}{15}$$

$$= \tan \frac{2\pi}{5} - \tan \frac{\pi}{15}$$

$$\Rightarrow \tan \frac{2\pi}{5} - \tan \frac{\pi}{15} - \sqrt{3} \tan \frac{2\pi}{5}$$

$$\tan \frac{\pi}{15} = \sqrt{3}$$

349. (a) Answer is independent of A, B, C and $A + B + C = 180^\circ$
So put, $A = B = C = 60^\circ$

$$\tan^2 \frac{A}{2} + \tan^2 \frac{B}{2} + \tan^2 \frac{C}{2} = \tan^2 30^\circ +$$

$$\tan^2 30^\circ + \tan^2 30^\circ$$

$$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

$$\text{350. (a)} \quad \frac{\cot^2 15^\circ - 1}{\cot^2 15^\circ + 1}$$

$$= \frac{\cos^2 15^\circ - \sin^2 15^\circ}{\cos^2 15^\circ + \sin^2 15^\circ}$$

divide by $\cos^2 15^\circ$ all terms

$$= \frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ} = \cos 2 \times 15^\circ$$

$$= \cos 30^\circ = \frac{\sqrt{3}}{2}$$

MAXIMUM AND MINIMUM VALUE OF TRIGONOMETRIC FUNCTIONS

Limit of the values of Trigonometric Functions:

	Min	Max
$\sin \theta$ & $\cos \theta$ (odd power)	-1	+1
$\sin^2 \theta$ & $\cos^2 \theta$ (even power)	0	+1
$\tan \theta$ & $\cot \theta$ (odd power)	$-\infty$	$+\infty$
$\tan^2 \theta$ & $\cot^2 \theta$ (even power)	0	∞
$\sec \theta$ & $\operatorname{cosec} \theta$ (odd power)	$-\infty$	$+\infty$
$\sec^2 \theta$ & $\operatorname{cosec}^2 \theta$ (even power)	1	∞

Note:- The value of $\sec \theta$ & $\operatorname{cosec} \theta$ can be anything $-\infty$ to $+\infty$ but value of $\sec \theta$ & $\operatorname{cosec} \theta$ can't be between -1 & 1 but it can be -1 & 1.

Increasing and Decreasing functions :

- (i) The value of $\sin \theta$ increases from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$. As θ increases in this interval, then $\sin \theta$ also increases.
- (ii) **1st quadrant :** $\sin \theta$ increases from 0 to 1; $\cos \theta$ decreases from 1 to 0 and $\tan \theta$ increases from 0 to ∞ .
- (iii) **2nd quadrant :** $\sin \theta$ decreases from 1 to 0; $\cos \theta$ decreases from 0 to -1 and $\tan \theta$ decreases from ∞ to 0.
- (iii) **3rd quadrant :** $\sin \theta$ decreases from 0 to -1; $\cos \theta$ increases from -1 to 0; $\tan \theta$ increases from 0 to ∞ .
- (iv) **4th quadrant :** $\sin \theta$ increases from -1 to 0; $\cos \theta$ increases from 0 to 1; $\tan \theta$ decreases from ∞ to 0.

TYPE-I

Ex.1 Choose the correct statements among following.

- (a) $\cos 40^\circ > \cos 70^\circ$
 (b) $\sin 35^\circ > \sin 65^\circ$
 (c) $\tan 45^\circ < \tan 46^\circ$
 (d) $\cot 40^\circ < \cot 39^\circ$
 (e) $\sec 20^\circ > \sec 40^\circ$
 (f) $\operatorname{cosec} 20^\circ < \operatorname{cosec} 30^\circ$

Sol. When $0 < \theta < 90^\circ$,

(a) Value of $\cos \theta$ decreases as θ increases, hence, $40^\circ < 70^\circ \Rightarrow \cos 40^\circ > \cos 70^\circ$. The statement is true.

(b) As θ increases, value of $\sin \theta$ also increase, hence $65^\circ > 35^\circ \Rightarrow \sin 65^\circ > \sin 35^\circ$. Hence, the statement is false.

(c) As θ increases value of $\tan \theta$ increases, hence $45^\circ < 46^\circ \Rightarrow \tan 45^\circ < \tan 46^\circ$. Hence, given statement is true.

(d) As θ increases, value of $\cot \theta$ decreases, hence $40^\circ > 39^\circ \Rightarrow \cot 40^\circ < \cot 39^\circ$. Hence, the statement is true.

(e) As θ increases, value of $\sec \theta$ increases hence $20^\circ < 40^\circ \Rightarrow \sec 20^\circ < \sec 40^\circ$. Hence, the statement is false.

(f) As θ increases, value of $\operatorname{cosec} \theta$ decreases, Hence $20^\circ < 30^\circ \Rightarrow \operatorname{cosec} 20^\circ > \operatorname{cosec} 30^\circ$. Hence, given statement is false.

Ex.2 Which statement is incorrect

- (a) $\sin \theta = \frac{3}{4}$ (b) $\cos \theta = \frac{1}{3}$
 (c) $\sec \theta = 4$ (d) $\operatorname{cosec} \theta = \frac{2}{3}$

Sol. option (d) is incorrect because

$$\operatorname{cosec} \theta = \frac{2}{3},$$

it means $\sin \theta = \frac{3}{2}$, it is not possible
 therefore $\sin \theta$ maximum value is 1

Ex.3 Find minimum & maximum value of $15 + \sin^2 \theta$

Sol.

$$\begin{array}{c} \sin^2 \theta \\ \swarrow \quad \searrow \\ \text{min.} \quad \text{max.} \\ 0 \quad \quad +1 \end{array}$$

$$\begin{aligned} \text{min. value} &= 15 + 0 = 15 \\ \text{max. value} &= 15 + 1 = 16 \end{aligned}$$

Ex.4 Find minimum & maximum value of $15 - \sin^2 \theta$

Sol.

$$\begin{array}{c} \sin^2 \theta \\ \swarrow \quad \searrow \\ \text{min.} \quad \text{max.} \\ 0 \quad \quad +1 \\ \text{max. value} = 15 - 0 = 15 \\ \text{min. value} = 15 - 1 = 14 \end{array}$$

Ex.5 Find min & max value of $10 + \sec^2 \theta$

Sol.

$$\begin{array}{c} \sec^2 \theta \\ \swarrow \quad \searrow \\ \text{min.} \quad \text{max.} \\ +1 \quad \quad \infty \\ \text{min value} = 10 + 1 = 11 \\ \text{max. value} = 10 + \infty = \infty \end{array}$$

* maximum value will not be asked & equal to ∞

Ex.6 Find minimum & maximum value of $10 \sin \theta - 1$

Sol.

$$\begin{array}{c} \sin \theta \\ \swarrow \quad \searrow \\ \text{min.} \quad \text{max.} \\ -1 \quad \quad +1 \\ \text{max. value} = 10 \times (1) - 1 = 9 \\ \text{min. value} = 10 \times (-1) - 1 = -11 \end{array}$$

Ex.7 Find minimum & maximum value of

$$12\sin^2\theta - 3$$

Sol. $12\sin^2\theta - 3$

$$\text{max. value} = 12(1) - 3 = 9$$

$$\text{min value} = 12 \times (0) - 3 = -3$$

Ex.8 Find minimum & maximum value of $11 + \cos^2\theta$

Sol.

$$\begin{array}{cc} & \cos^2\theta \\ & \swarrow \quad \searrow \\ \text{max.} & \text{min.} \\ +1 & 0 \\ \text{max.} = 11+1 = 12 & \\ \text{min.} = 11+0 = 11 & \end{array}$$

TYPE - II

(i) In the expression of $a \sin^2\theta + b \cos^2\theta$.

$$\begin{array}{l|l} \text{if } a > b & \text{if } b > a \\ \hline \text{max} = a & \text{max} = b \\ \text{min} = b & \text{min} = a \end{array}$$

Ex.9 find the maximum and minimum value of $15 \sin^2\theta + 10 \cos^2\theta$.

Sol. $15 \sin^2\theta + 10 \cos^2\theta$

$$= 15 \sin^2\theta + 10 (1 - \sin^2\theta)$$

$$= 15 \sin^2\theta + 10 - 10 \sin^2\theta$$

$$= 5 \sin^2\theta + 10$$

$$= 10 + 5 \sin^2\theta$$

$$(0 < \sin^2\theta < 1)$$

$$\text{max.} = 10 + 5 \times 1 = 15$$

$$\text{min.} = 10 + 5 \times 0 = 10$$

Alternate:-

$$15 \sin^2\theta + 10 \cos^2\theta$$

$$\text{max.} = 15 \text{ (use above identity)}$$

$$\text{min.} = 10$$

Ex.10 find the maximum and minimum value of $4 \sin^2\theta + 7 \cos^2\theta + 5$

Sol. $4 \sin^2\theta + 7 \cos^2\theta + 5$

$$\text{min} = 4 + 5 = 9$$

$$\text{max} = 7 + 5 = 12$$

Ex.11 Find the maximum and minimum value of

$$12 \sin^2\theta - 17 \cos^2\theta$$

Sol. $12 \sin^2\theta + (-17) \cos^2\theta$

$$\text{max.} = 12$$

$$\text{min} = -17$$

Ex.12 Find the maximum and minimum value of

$$-14 \sin^2\theta - 21 \cos^2\theta$$

Sol. $\text{max} = -14$

$$\text{min} = -21$$

(ii) when angle is different & Independent then.

$\sin^2\alpha$	$\cos^2\beta$
$\text{max} = +1$	$\text{max} = +1$
$\text{min} = 0$	$\text{min} = 0$

Ex.13 Find the maximum and minimum value of $7 \sin^2\alpha + 20 \cos^2\beta$

Sol. $7 \sin^2\alpha + 20 \cos^2\beta$

Now,

$$\text{max.} = 7 \times 1 + 20 \times 1 = 27$$

$$\text{min.} = 7 \times 0 + 20 \times 0 = 0$$

Ex.14 Find the maximum and minimum value of $10 \tan^2\theta + 15 \sec^2\theta$

Sol. $10 \tan^2\theta + 15 \sec^2\theta$

$$= 10 \tan^2\theta + 15 (1 + \tan^2\theta)$$

$$= 10 \tan^2\theta + 15 + 15 \tan^2\theta$$

$$= 25 \tan^2\theta + 15$$

$$\text{min. value} = 25 \times 0 + 15$$

$$= 15 (\because \tan^2\theta \text{ min} = 0)$$

* When a $\sec^2\theta + b \tan^2\theta$ is given then coefficient of $\sec^2\theta$ is value minimum is equal to a.

(iii) If expression with a $\sin\theta \pm b \cos\theta$

$$\text{max. value} = \sqrt{a^2 + b^2}$$

$$\text{min. value} = -\sqrt{a^2 + b^2}$$

Ex.15 Find the maximum and minimum value of $5 \sin\theta + 12 \cos\theta$

Sol. $5 \sin\theta + 12 \cos\theta$

$$\text{maxi. value} = \sqrt{(5)^2 + (12)^2} =$$

$$\sqrt{25 + 144} = 13$$

$$\text{mini. value} = -\sqrt{(5)^2 + (12)^2} = -13$$

Ex.16 Find the maximum and minimum value of $5 \cos\theta + 3 \cos(\theta + 60^\circ) + 5$

Sol. $5 \cos\theta + 3 \cos(\theta + 60^\circ) + 5$

$$\Rightarrow 5 \cos\theta + 3 (\cos\theta \cos 60^\circ - \sin\theta \sin 60^\circ) + 5$$

$$\Rightarrow 5 \cos\theta + 3 \left(\cos\theta \times \frac{1}{2} - \sin\theta \right)$$

$$\frac{\sqrt{3}}{2} + 5$$

$$\Rightarrow 5 \cos\theta + \frac{3}{2} \cos\theta - \frac{3\sqrt{3}}{2} \sin\theta + 5$$

$$\Rightarrow \frac{13}{2} \cos\theta - \frac{3\sqrt{3}}{2} \sin\theta + 5$$

max. value

$$= \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} + 5$$

$$= \sqrt{\frac{169}{4} + \frac{27}{4}} + 5 = \frac{14}{2} + 5 = 12$$

$$\text{min. value} = -\sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}$$

$$+ 5 = -7 + 5 = -2$$

Ex.17 Find the maximum and minimum value of

$$16^{\sin\theta} \cdot 4^{\sin\theta} \cdot 2^{\cos\theta} \cdot 128^{\cos\theta}$$

Sol. $16^{\sin\theta} \cdot 4^{\sin\theta} \cdot 2^{\cos\theta} \cdot 128^{\cos\theta}$

$$\Rightarrow (2^4)^{\sin\theta} \cdot (2^2)^{\sin\theta} \cdot (2)^{\cos\theta} \cdot (2^7)^{\cos\theta}$$

$$\Rightarrow 2^{4\sin\theta} \cdot 2^{2\sin\theta} \cdot 2^{\cos\theta} \cdot 2^{7\cos\theta}$$

$$\Rightarrow 2^{6\sin\theta + 8\cos\theta}$$

Max. Value $6 \sin\theta + 8 \cos\theta$.

$$= \sqrt{6^2 + 8^2} = 10$$

Now,

$$\text{max. value} = 2^{10}$$

$$\text{min. value} = 2^{-10}$$

Ex.18 Find the maximum and minimum value of $27^{\sin x} \times$

$$81^{\cos x}$$

Sol. $27^{\sin x} \times 81^{\cos x}$

$$\Rightarrow (3^3)^{\sin x} \times (3^4)^{\cos x}$$

$$\Rightarrow 3^{3\sin x} \times 3^{4\cos x}$$

$$\Rightarrow 3^{3\sin x + 4\cos x}$$

max. value = $3 \sin x + 4 \cos x$

$$= \sqrt{(3)^2 + (4)^2} = 5$$

so, max. value = 3^5

$$\text{min. value} = 3^{-5}$$

Ex.19 find maximum & minimum value of $5 \sin^2 x - 12 \sin x \cos x + 10 \cos^2 x$

Sol. $5 \sin^2 x - 12 \sin x \cos x + 10 \cos^2 x$

$$\Rightarrow (4 \sin^2 x - 12 \sin x \cos x + 9 \cos^2 x) + \cos^2 x + \sin^2 x$$

$$\Rightarrow (2 \sin x - 3 \cos x)^2 + 1$$

for minimum value of $2 \sin x - 3 \cos x = 0$

then,

minimum value = 1

for maximum value

$$2 \sin x - 3 \cos x = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$

$$\begin{aligned}\text{maximum value} &= 1 + (\sqrt{13})^2 \\ &= 1 + 13 = 14\end{aligned}$$

Ex.20 Find the minimum & maximum value of $10 \sin \theta \cdot \cos \theta + 1 - 2 \sin^2 \theta$

Sol. $10 \sin \theta \cdot \cos \theta + 1 - 2 \sin^2 \theta$
 $\Rightarrow 5 \times 2 \sin \theta \cos \theta + 1 - 2 \sin^2 \theta$
 $\Rightarrow 5 \sin 2\theta + 1 - 2 \sin^2 \theta$
 $\text{max.} = +\sqrt{5^2 + 1^2} = +\sqrt{26}$
 $\text{min.} = -\sqrt{26}$

TYPE-III

In expression of $\sin^{2n} \theta + \cos^{2n} \theta$.
maximum value = 1
minimum value = put $\theta = 45^\circ$

Ex.21 Find the minimum & maximum value of $\sin^4 \theta + \cos^4 \theta$

Sol. $\sin^4 \theta + \cos^4 \theta$
 $= (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta$
 $= 1 - 2 \sin^2 \theta \cos^2 \theta$
 multiply & divide by 2
 $= 1 - \frac{2 \times 2}{2} \sin^2 \theta \cos^2 \theta$
 $= 1 - \frac{1}{2} (2 \sin \theta \cos \theta)^2$
 $= 1 - \frac{1}{2} \sin^2 2\theta$

$$\text{maxi. value} = 1 - \frac{1}{2} \times (0) = 1$$

$$\text{mini. value} = 1 - \frac{1}{2} (1) = \frac{1}{2}$$

Alternate:-

$$\begin{aligned}\sin^4 \theta + \cos^4 \theta \\ \text{maxi. value} &= 1 \\ \text{for minimum value } \theta &= 45^\circ \\ \text{mini. value} &= \sin^4 45^\circ + \cos^4 45^\circ \\ &= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}\end{aligned}$$

Ex.22 Find the minimum & maximum value of $\sin^6 \theta + \cos^6 \theta$.

Sol. $\sin^6 \theta + \cos^6 \theta$
 $= (\sin^2 \theta)^3 + (\cos^2 \theta)^3$
 $= (\sin^2 \theta + \cos^2 \theta)^3 - 3 \sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta)$
 $= 1 - 3 \sin^2 \theta \cos^2 \theta$
 multiply & divide by 4

$$= 1 - \frac{3}{4} \times 4 \sin^2 \theta \cos^2 \theta$$

$$= 1 - \frac{3}{4} (2 \sin \theta \cos \theta)^2$$

$$= 1 - \frac{3}{4} \sin^2 2\theta$$

$$\text{max. value} = 1 - \frac{3}{4} \times (0) = 1$$

$$\text{min. value} = 1 - \frac{3}{4} \times (1) = \frac{1}{4}$$

Alternate:-

$$\begin{aligned}\sin^6 \theta + \cos^6 \theta \\ \text{maxi. value} &= 1\end{aligned}$$

$$\text{mini. value} = \left(\frac{1}{\sqrt{2}}\right)^6 + \left(\frac{1}{\sqrt{2}}\right)^6$$

$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

Ex.23 Find the minimum & maximum value of $\sin^2 \theta + \cos^4 \theta$.

Sol. $\sin^2 \theta + \cos^4 \theta$
 $\text{max. value} = 1$
 $\text{min. value put } \theta = 45^\circ$
 $= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^4$
 $= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

TYPE-IV

In the expression of $\sin^n \theta \cdot \cos^n \theta$.

$$\text{maxi. value} = \frac{1}{2^n}$$

min. value =

(i) When n is even then min. value 0

(ii) When n is odd

$$\text{then min value} = \left(\frac{-1}{2^n}\right)$$

Ex.24 Find the minimum & maximum value of $\sin^4 \theta \cdot \cos^4 \theta$

Sol. $\sin^4 \theta \cdot \cos^4 \theta$
 $= \frac{1 \times 2^4}{2^4} \sin^4 \theta \cdot \cos^4 \theta$
 $= \frac{1}{2^4} (2 \sin \theta \cdot \cos \theta)^4$
 $= \frac{1}{2^4} \sin^4 2\theta$

$$\text{maxi.} = \frac{1}{2^4} \times (1) = \frac{1}{2^4}$$

$$\text{mini.} = \frac{1}{2^4} \times 0 = 0$$

($\sin^n \theta$ when n even max.=1, min.=0)

Ex.25 Find the minimum & maximum value of $\sin^3 \theta \cos^3 \theta$

Sol. $\text{max} = \frac{1}{2^3} = \frac{1}{8}$
 $\text{min.} = \frac{-1}{2^3} = \frac{-1}{8}$ (3 - odd power)

Ex.26 Find the minimum & maximum value of $\sin^{110} \theta \cdot \cos^{110} \theta$.

Sol. $\text{max} = \frac{1}{2^{110}}$
 $\text{min} = 0$ (even power)

TYPE-V

(A) $a \tan^2 \theta + b \cot^2 \theta$ or

$$a \tan^2 \theta + \frac{b}{\tan^2 \theta}$$

$$\text{min. value} = 2\sqrt{ab}$$

$$\text{max. value} = \infty$$

Proof

$$\begin{aligned}a \tan^2 \theta + b \cot^2 \theta \\ &= (\sqrt{a} \tan \theta - \sqrt{b} \cot \theta)^2 + 2\sqrt{a} \sqrt{b} \tan \theta \cot \theta \\ &= (\sqrt{a} \tan \theta - \sqrt{b} \cot \theta)^2 + 2\sqrt{ab}\end{aligned}$$

But $(\sqrt{a} \tan \theta - \sqrt{b} \cot \theta)^2$ is either 0 or greater than zero.

$$\therefore a \tan^2 \theta + b \cot^2 \theta \geq 0 + 2\sqrt{ab}$$

$$\text{or, } a \tan^2 \theta + b \cot^2 \theta \geq 2\sqrt{ab}$$

Since value of $a \tan^2 \theta + b \cot^2 \theta$ is greater than or equal to $2\sqrt{ab}$,

its minimum value is $2\sqrt{ab}$

Note- Do not write the given expression as $(\sqrt{a} \tan \theta + \sqrt{b} \cot \theta)^2 - 2\sqrt{ab} \tan \theta \cot \theta$.

In this situation minimum value of, $\sqrt{a} \tan \theta + \sqrt{b} \cot \theta$ cannot be zero.

Ex.27 Find the minimum value of $16 \tan^2 \theta + 9 \cot^2 \theta$.

Sol. $16 \tan^2 \theta + 9 \cot^2 \theta$

$$\begin{aligned}\text{min. value} &= 2\sqrt{ab} = 2\sqrt{16 \times 9} \\ &= 2 \times 12 = 24\end{aligned}$$

Ex.28 Find the minimum value of $4\tan^2\theta + 25\cot^2\theta$

Sol. $4\tan^2\theta + 25\cot^2\theta$
 min. value = $2\sqrt{4 \times 25}$
 = $2 \times 10 = 20$

Ex.29 Find the minimum value of $4\sec^2\theta + 25\operatorname{cosec}^2\theta$

Sol. $4\sec^2\theta + 25\operatorname{cosec}^2\theta$
 = $4(1+\tan^2\theta) + 25(1+\cot^2\theta)$
 = $4 + 4\tan^2\theta + 25 + 25\cot^2\theta$
 = $29 + 4\tan^2\theta + 25\cot^2\theta$
 min. value = $4\tan^2\theta + 25\cot^2\theta$
 = $2\sqrt{4 \times 25} = 20$
 min. value = $29 + 20 = 49$

Ex.30 Find the minimum value of $\tan^2\theta + \cot^2\theta$.

Sol. $\tan^2\theta + \cot^2\theta$
 min. value = $2\sqrt{ab}$
 = $2\sqrt{1 \times 1} = 2$

(B) in expression of $a\sin^2\theta + b\operatorname{cosec}^2\theta$

(i) if $a \leq b$

min. value = $a+b$

(ii) if $a \geq b$

min. value = $2\sqrt{ab}$

Ex.31 Find the minimum value of $4\sin^2\theta + 25\operatorname{cosec}^2\theta$

Sol. minimum value = $4+25 = 29$
 ($\because a < b$ min value = $a+b$)

Ex.32 Find the minimum value of $16\sin^2\theta + 9\operatorname{cosec}^2\theta$

Sol. $16\sin^2\theta + 9\operatorname{cosec}^2\theta$
 min. value = $2\sqrt{16 \times 9}$
 = $2 \times 12 = 24$

($\because a > b$ min value = $2\sqrt{ab}$)

Ex.33 Find the minimum value of $25\operatorname{cosec}^2\theta + 25\sin^2\theta$.

Sol. $25\operatorname{cosec}^2\theta + 25\sin^2\theta$
 mini. value = $2\sqrt{ab}$
 $2\sqrt{25 \times 25}$
 $2 \times 25 = 50$

or

mini. value = $a+b = 25+25 = 50$

Ex.34 Find the minimum value of $\sin^2\theta + \operatorname{cosec}^2\theta$

Sol. $\sin^2\theta + \operatorname{cosec}^2\theta$
 min. value = $2\sqrt{1 \times 1} = 2$
or
 min. value $1+1 = 2$

(C) In expression of

$a\cos^2\theta + b\sec^2\theta$

(i) If $a \leq b$

minimum value = $a+b$

(ii) If $a \geq b$

minimum value = $2\sqrt{ab}$

Ex.35 Find the minimum value of $10\cos^2\theta + 15\sec^2\theta$

Sol. $10\cos^2\theta + 15\sec^2\theta$
 minimum value = $10+15 = 25$
 ($\because a < b$ min. value = $a+b$)

Ex.36 Find the minimum value of $\cos^2\theta + \sec^2\theta$

Sol. $\cos^2\theta + \sec^2\theta$
 min. value = $1+1 = 2$

(D) In the expression of $a\sec^2\theta + b\operatorname{cosec}^2\theta$

min. value = $(\sqrt{a} + \sqrt{b})^2$

maxi. value = ∞

Ex.37 Find the value of $4\sec^2\theta + 9\operatorname{cosec}^2\theta$.

Sol. mini. value = $(\sqrt{a} + \sqrt{b})^2 =$
 $(\sqrt{4} + \sqrt{9})^2$
 = $(2+3)^2 = 25$

Ex.38 Find the minimum value of $\sin^2\theta + \operatorname{cosec}^2\theta + \cos^2\theta + \sec^2\theta + \tan^2\theta + \cot^2\theta$.

Sol. $\sin^2\theta + \operatorname{cosec}^2\theta + \cos^2\theta + \sec^2\theta + \tan^2\theta + \cot^2\theta$
 = $\sin^2\theta + 1 + \cot^2\theta + \cos^2\theta + 1 + \tan^2\theta + \tan^2\theta + \cot^2\theta$
 $\Rightarrow 2 + \sin^2\theta + \cos^2\theta + 2\tan^2\theta + 2\cot^2\theta$
 = $2 + 1 + 2\tan^2\theta + 2\cot^2\theta$
 = $3 + 2\tan^2\theta + 2\cot^2\theta$
 min. value of $2\tan^2\theta + 2\cot^2\theta$
 = $2\sqrt{ab}$
 = $2\sqrt{2 \times 2} = 4$
 min. value = $3 + 4 = 7$

EXERCISE

- Find the maximum value of $\sin x + \cos x$.
 (a) $\sqrt{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 2 (d) 3
- For $0 < \theta < \frac{\pi}{2}$ the inequality which holds is :
 (a) $\theta < \sin\theta < \tan\theta$
 (b) $\sin\theta < \theta < \tan\theta$
 (c) $\theta < \tan\theta < \sin\theta$
 (d) $\tan\theta < \theta < \sin\theta$
- Which is smaller ?
 (a) $\sin 76^\circ$ (b) $\cos 76^\circ$
 (c) both are equal
 (d) None of these
- If the sine of an angle is $1/3$, then cosine of that angle is :

- (a) equal to $1/3$
 (b) less than $1/3$
 (c) greater than $1/3$
 (d) not known
- The maximum value of $24\sin\theta + 7\cos\theta$ is
 (a) 7 (b) 17 (c) 24 (d) 25
- Which of the following is correct?
 (a) $\sin 1^\circ < \sin 1$
 (B) $\sin 1^\circ = \sin 1$
 (c) $\sin 1^\circ > \sin 1$
 (d) $\sin 1^\circ = \frac{\pi}{180} \sin 1$
- The greatest value of $\cos(xe^{|x|} + 7x^2 - 3x)$, $x \in [-1, \infty)$ is
 (a) 0 (b) 1 (c) -1
 (d) None of these

- The greatest value of $\sin^4\theta + \cos^4\theta$ is :
 (a) 1 (b) $\frac{1}{2}$ (c) 0 (d) $\frac{\sqrt{3}}{2}$
- The least value of $2\sin^2\theta + 3\cos^2\theta$ is:
 (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3
- What is the minimum value of $\sin^2\theta + \cos^4\theta$:
 (a) 0 (b) $\frac{3}{4}$ (c) 2 (d) $\frac{1}{4}$
- Find the maximum value of $\sin^{113}\theta \cdot \cos^{113}\theta$:

- (a) $\left(\frac{3}{2}\right)^{113}$ (b) 1
(c) $\left(\frac{1}{4}\right)^{113}$ (d) $\left(\frac{1}{2}\right)^{113}$
12. Find the minimum value of $16\operatorname{cosec}^2 \theta + 25\sec^2 \theta$:
(a) 81 (b) 41 (c) 82 (d) 90
13. The least value of $\sin^2 \theta + \operatorname{cosec}^2 \theta + \cos^2 \theta + \sec^2 \theta$ is:
(a) 3 (b) 4 (c) 5 (d) 6
14. If $0 \leq \theta < \frac{\pi}{2}$, then which of the following trigonometric ratios can have the value 1.1?
(a) $\sin \theta$ (b) $\cos \theta$
(c) $2\sec \theta$ (d) $2\tan \theta$
15. $\cos 3\theta + \sin 3\theta$ is maximum when θ is:
(a) 15° (b) 30° (c) 45° (d) 60°
16. The value of x , for maximum value of $(\sin x + \cos x)$ is:
(a) 30° (b) 45° (c) 60° (d) 90°
17. The maximum value of $\sin x \cdot \cos x$ is:
(a) 2 (b) $\sqrt{2}$ (c) $\frac{1}{2}$ (d) 1
18. The maximum and minimum value of $(1 + \cos 2x)$ are:
(a) -1 and 1 (b) 1 and 2
(c) $-\frac{1}{2}$ and $\frac{1}{2}$ (d) 0 and 2
19. If $P = \sin^2 20^\circ + \sin^2 40^\circ + \sin^2 50^\circ + \sin^2 70^\circ$, then.
(a) $0 < P < 1$ (b) $1 < P < 1.5$
(c) $1.5 < P < 2$ (d) $P = 2$
20. If $0 \leq \theta \leq \frac{\pi}{2}$, then which of the following is true?
(a) $(\tan^2 \theta + \cot^2 \theta) \geq 2$
(b) $(\tan^2 \theta + \cot^2 \theta) \leq 2$
(c) $(\tan^2 \theta + \cot^2 \theta) \leq 1$
(d) None of these
21. If $0 < \theta < \frac{\pi}{2}$, which of the following is true?
(a) $\sin^2 \theta + \frac{1}{\sin^2 \theta} < 2$
(b) $\sin^2 \theta + \frac{1}{\sin^2 \theta} = 2$
(c) $\sin^2 \theta + \frac{1}{\sin^2 \theta} > 2$
(d) None of these
22. The least value of $(4\sec^2 \theta + 9\operatorname{cosec}^2 \theta)$ is
(a) 1 (b) 19 (c) 25 (d) 7
23. If $\alpha + \beta = 90^\circ$, then the maximum value of $\sin \alpha \cdot \sin \beta$ is:
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$
(d) None of these
24. $\sin x + \sqrt{3} \cos x$ is maximum when
(a) $x = 30^\circ$ (b) $x = 0^\circ$
(c) $x = 45^\circ$ (d) $x = 60^\circ$
25. The least value of $\tan^2 \theta + \cot^2 \theta$ is:
(a) 2 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$
26. What is the minimum value of $\sin^6 \theta + \cos^6 \theta$?
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1
(d) None of these
27. The greatest value of $81^{\sin x} \cdot 27^{\cos x}$ is:
(a) 3^5 (b) 3^4 (c) 3 (d) 3^3
28. If $0^\circ < A < 90^\circ$ and $\cos A - \sin A > 0$ then $\cos A + \sin A$ can not be greater than:
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$
29. In a $\triangle ABC$, if angle C is obtuse, then
(a) $\tan A \cdot \tan B < 1$
(b) $\tan A \cdot \tan B \leq 1$
(c) $\tan A \cdot \tan B > 1$
(d) None of these
30. The ratio of the greatest value of $2 - \cos x + \sin^2 x$ to its least value is:
(a) $\frac{1}{4}$ (b) $\frac{9}{4}$ (c) $\frac{13}{4}$ (d) $\frac{7}{4}$

ANSWER KEY

1.(a)	4.(c)	7.(b)	10.(b)	13.(c)	16.(b)	19.(d)	22.(c)	25.(a)	28.(d)
2.(b)	5.(d)	8.(a)	11.(d)	14.(d)	17.(c)	20.(a)	23.(b)	26.(b)	29.(a)
3.(b)	6.(a)	9.(c)	12.(a)	15.(a)	18.(d)	21.(c)	24.(a)	27.(a)	30.(c)

1.(a) Maximum Value of :

$$a \sin x + b \cos x = \sqrt{a^2 + b^2}$$

here $a = b = 1$

Maximum Value of :

$$\sin x + \cos x = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Alternate

$$\sin x + \cos x = \left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x \right) \sqrt{2}$$

$$= (\sin x \cos 45^\circ + \cos x \sin 45^\circ) \sqrt{2}$$

$$= \sqrt{2} \sin(x + 45^\circ)$$

$$\text{Now, } -1 \leq \sin(x + 45^\circ) \leq 1$$

$$-1 \times \sqrt{2} \leq \sqrt{2} \sin(x + 45^\circ) \leq 1 \times \sqrt{2}$$

$$\Rightarrow -\sqrt{2} \leq (\sin x + \cos x) \leq \sqrt{2}$$

2.(b) In Ist quadrant, $\sin \theta < \theta < \tan \theta$ is true.

3.(b) In Ist quadrant as θ increases, the value of $\sin \theta$ increases.

$$\text{Now, } \cos 76^\circ = \cos(90^\circ - 14^\circ) = \sin 14^\circ$$

out of $\sin 76^\circ$ and $\sin 14^\circ$, the smaller one is $\sin 14^\circ$

Hence, $\cos 76^\circ$ is smaller.

4.(c)

$$\cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} > \sqrt{\frac{1}{9}}$$

$$\text{So, } \cos \theta > \frac{1}{3}$$

5.(d) Max. value of $a \sin \theta + b \cos \theta$

$$= \sqrt{a^2 + b^2}$$

$$\text{Max. value of } 24 \sin \theta + 7 \cos \theta$$

$$= \sqrt{(24)^2 + (7)^2} = 25$$

6.(a) Since 1 radian = $57^\circ 16' 22''$ approx,

and $\sin 57^\circ 16' 22'' > \sin 1^\circ$,
 $\sin 1^\circ < \sin 1$

7.(b) since $\cos \theta \leq 1$ for all real θ .
the given expression ≤ 1 for all x

Hence, the greatest value = 1

8.(a)

$$\sin^4 \theta + \cos^4 \theta = (\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 2 \sin^2 \theta \cos^2 \theta$$

$$= (\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta$$

$$= 1 - 2 \sin^2 \theta \cos^2 \theta \leq 1$$

Greatest value of $\sin^4 \theta + \cos^4 \theta = 1$

$$9.(c) 2 \sin^2 \theta + 3 \cos^2 \theta =$$

$$2(\sin^2 \theta + \cos^2 \theta) + \cos^2 \theta$$

$$= 2 + \cos^2 \theta \geq 2$$

$$\text{Least value of } 2 \sin^2 \theta + 3 \cos^2 \theta = 2$$

$$10.(b) x = \sin^2 \theta + \cos^4 \theta$$

$$= 1 - \cos^2 \theta + \cos^4 \theta$$

$$x = 1 - \cos^2 \theta (1 - \cos^2 \theta)$$

$$x = 1 - \cos^2 \theta \sin^2 \theta$$

$$x = 1 - \frac{1}{4} (2 \sin \theta \cos \theta)^2$$

$$x = 1 - \frac{1}{4} \sin^2 2\theta \quad 0 \leq \sin^2 2\theta \leq 1$$

$$\text{when } \sin^2 2\theta = 0 \Rightarrow x = 1 - \frac{1}{4} (0) = 1$$

and when

$$\sin^2 2\theta = 1, \Rightarrow x = 1 - \frac{1}{4} (1) = \frac{3}{4}$$

$$\text{i.e. } \frac{3}{4} \leq x \leq 1$$

i.e. the least value of

$$\sin^2 \theta + \cos^4 \theta = \frac{3}{4}$$

$$11.(d) \sin^{113} \theta \cdot \cos^{113} \theta$$

$$= \frac{1}{2^{113}} (2 \sin \theta \cos \theta)^{113}$$

$$= \left(\frac{1}{2} \right)^{113} (\sin 2\theta)^{113} \leq \left(\frac{1}{2} \right)^{113}$$

$$(\because -1 \leq \sin 2\theta \leq 1)$$

Hence, the greatest value of

$$\sin^{113} \theta \cdot \cos^{113} \theta = \left(\frac{1}{2} \right)^{113}$$

Short Cut :

The maximum value of

$$\sin^n \theta \cdot \cos^n \theta = \left(\frac{1}{2} \right)^n$$

The maximum value of

$$\sin^{113} \theta \cdot \cos^{113} \theta = \left(\frac{1}{2} \right)^{113}$$

$$12.(a) 16 \operatorname{cosec}^2 \theta + 25 \sec^2 \theta$$

$$= 16(1 + \cot^2 \theta) + 25(1 + \tan^2 \theta)$$

$$= 41 + 16 \cot^2 \theta + 25 \tan^2 \theta$$

(i)

Now,

$\therefore$ Minimum value of

$$ax^2 + \frac{b}{x^2} = 2\sqrt{ab}$$

Minimum value of :

$$16 \cot^2 \theta + 25 \tan^2 \theta = 2\sqrt{16 \times 25} = 40$$

From (i)

Minimum value of

$$16 \operatorname{cosec}^2 \theta + 25 \sec^2 \theta = 41 + 40 = 81$$

Short Cut :

Minimum value of

$$(a \operatorname{cosec}^2 \theta + b \sec^2 \theta)$$

$$= (\sqrt{a} + \sqrt{b})^2$$

$\therefore$ Minimum value of

$$(16 \operatorname{cosec}^2 \theta + 25 \sec^2 \theta)$$

$$= (\sqrt{16} + \sqrt{25})^2 = (9)^2 = 81$$

13.(c)

$$(\sin^2 \theta + \cos^2 \theta) + (\operatorname{cosec}^2 \theta + \sec^2 \theta)$$

$$= 1 + (\operatorname{cosec}^2 \theta + \sec^2 \theta)$$

$\therefore$ Minimum value of

$$(a \operatorname{cosec}^2 \theta + b \sec^2 \theta) = (\sqrt{a} + \sqrt{b})^2$$

$\therefore$ Minimum value of

$$(\operatorname{cosec}^2 \theta + \sec^2 \theta) = (\sqrt{1} + \sqrt{1})^2 = 4$$

$\therefore$ Minimum value of the given

$$\text{Exp.} = 1 + 4 = 5$$

14.(d) Clearly, $0 \leq \sin \theta < 1$ and

$$0 \leq \cos \theta < 1$$

$$\text{for } 0 \leq \theta < \frac{\pi}{2}$$

Also $\sec \theta \geq 1$ so $2 \sec \theta \geq 2$

Now, $\tan \theta$ may have the value 0.55

$\therefore$ $2 \tan \theta$ may have the value 1.1.

$$15.(a) \cos 3\theta + \sin 3\theta$$

$$= \sqrt{2} \left(\frac{1}{\sqrt{2}} \cos 3\theta + \frac{1}{\sqrt{2}} \sin 3\theta \right)$$

$$= \sqrt{2}(\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ)$$

$$= \sqrt{2} \sin(45^\circ + 30^\circ)$$

The maximum value occurs when

$$\sin(45^\circ + 30^\circ) = 1$$

$$\text{i.e. } 30^\circ = 45^\circ \Rightarrow \theta = 15^\circ$$

$$16.(b) \sin x + \cos x$$

$$= \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x \right)$$

$$= \sqrt{2} (\sin x \cos 45^\circ + \cos x \sin 45^\circ)$$

$$= \sqrt{2} \sin(x + 45^\circ)$$

$$= \sqrt{2}, \text{ when } x = 45^\circ$$

$\therefore$ max.

$$(\sin x + \cos x) = \sqrt{2}$$

$$17.(c) \sin x \cdot \cos x = \frac{1}{2} (2 \sin x \cdot \cos x)$$

$$= \frac{1}{2} \sin 2x \leq \frac{1}{2}$$

$$[\because -1 \leq \sin 2x \leq 1]$$

Hence max. value of $\sin x \cdot \cos x$ is $\frac{1}{2}$.

$$18.(d) \text{ We know that}$$

$$-1 \leq \cos \theta \leq 1$$

$$\therefore -1 \leq \cos 2x \leq 1 \text{ OR}$$

$$-1 + 1 \leq 1 + \cos 2x \leq 1 + 1$$

$$\Rightarrow 0 \leq 1 + \cos 2x \leq 2$$

$$19.(d) P = (\sin^2 20^\circ + \sin^2 70^\circ) +$$

$$(\sin^2 40^\circ + \sin^2 50^\circ)$$

$$= (\sin^2 20^\circ + \cos^2 20^\circ) +$$

$$(\sin^2 40^\circ + \cos^2 40^\circ)$$

$$= 1 + 1 = 2$$

$$20.(a) \tan^2 \theta + \cot^2 \theta =$$

$$\tan^2 \theta + \cot^2 \theta -$$

$$2 \tan \theta \cdot \cot \theta + 2 \tan \theta \cdot \cot \theta$$

$$= (\tan \theta - \cot \theta)^2 + 2 \geq 2$$

$$21.(c) \sin^2 \theta + \frac{1}{\sin^2 \theta}$$

$$= \sin^2 \theta + \frac{1}{\sin^2 \theta} - 2 + 2$$

$$= \left(\sin \theta - \frac{1}{\sin \theta} \right)^2 + 2 > 2$$

$$\left[\because 0 < \theta < \frac{\pi}{2} \Rightarrow \sin \theta \neq 1 \right]$$

$$22.(c) \text{ least value of}$$

$$(a \sec^2 \theta + b \operatorname{cosec}^2 \theta) = (\sqrt{a} + \sqrt{b})^2$$

$$\therefore \text{ least value of}$$

$$(4 \sec^2 \theta + 9 \operatorname{cosec}^2 \theta)$$

$$= (\sqrt{4} + \sqrt{9})^2 = (5)^2 = 25$$

$$23.(b) \text{ Let } y = \sin \alpha \cdot \sin \beta$$

$$= \frac{1}{2} (2 \sin \alpha \cdot \sin \beta)$$

$$= \frac{1}{2} [2 \sin \alpha \cdot \sin(90^\circ - \alpha)]$$

$$= \frac{1}{2} (2 \sin \alpha \cdot \cos \alpha)$$

OR

$$y = \frac{1}{2} (\sin 2\alpha)$$

Since, Maximum value of $\sin 2\alpha$ is 1.

$$\therefore \text{ Maximum value of } Y = \frac{1}{2}$$

$$24.(a) \text{ Let } y = \sin x + \sqrt{3} \cos x$$

$$= 2 \left(\frac{1}{2} \sin x + \frac{\sqrt{3}}{2} \cos x \right)$$

$$= 2 \sin(x + 60^\circ)$$

y will be maximum, when $\sin(x + 60^\circ)$ maximum

$$\text{i.e. } \sin(x + 60^\circ) = 1 = \sin$$

$$90^\circ \Rightarrow x + 60^\circ = 90^\circ$$

$$\Rightarrow x = 30^\circ$$

$$25.(a) \tan^2 \theta + \cot^2 \theta$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$= \frac{\sin^4 \theta + \cos^4 \theta}{\sin^2 \theta \cdot \cos^2 \theta}$$

$$= \frac{(\sin^2 \theta - \cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta}{\sin^2 \theta \cdot \cos^2 \theta}$$

$$= \left(\frac{(\sin^2 \theta - \cos^2 \theta)}{\sin \theta \cdot \cos \theta} \right)^2 + 2 \geq 2$$

$$\text{Least value of } \tan^2 \theta + \cot^2 \theta = 2$$

Alternate:-

$$\tan^2 \theta + \cot^2 \theta = \tan^2 \theta + \frac{1}{\tan^2 \theta}$$

$$\text{It is in the form of } ax^2 + \frac{b}{x^2}$$

$$\text{Where } a = b = 1$$

$$\therefore \text{ minimum value}$$

$$= 2\sqrt{ab} = 2\sqrt{1 \times 1} = 2$$

$$26.(b) x = \sin^6 \theta + \cos^6 \theta$$

$$\Rightarrow x = (\sin^2 \theta)^3 + (\cos^2 \theta)^3$$

$$\Rightarrow x = (\sin^2 \theta + \cos^2 \theta)$$

$$(\sin^4 \theta + \cos^4 \theta - \sin^2 \theta \cdot \cos^2 \theta)$$

$$x = 1 \times [(\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cdot \cos^2 \theta]$$

$$x = 1 - 3 \sin^2 \theta \cdot \cos^2 \theta$$

$$x = 1 - \frac{3}{4} (2 \sin \theta \cdot \cos \theta)^2$$

$$x = 1 - \frac{3}{4} (\sin 2\theta)^2$$

$$= 1 - \frac{3}{4} \sin^2 2\theta$$

$$\therefore 0 \leq \sin^2 2\theta \leq 1$$

$$\therefore \text{ at } \sin^2 2\theta = 0$$

$$x = 1 - \frac{3}{4} (0) = 1$$

$$\text{and at } \sin^2 2\theta = 1$$

$$x = 1 - \frac{3}{4} (1) = \frac{1}{4}$$

$$\text{i.e. } \frac{1}{4} \leq x \leq 1$$

i.e. least value of

$$\sin^6 \theta + \cos^6 \theta = \frac{1}{4}$$

$$27.(a)$$

$$81^{\sin x} \cdot 27^{\cos x} = 3^{4 \sin x} \cdot 3^{3 \cos x}$$

$$= 3^{4 \sin x + 3 \cos x}$$

For maximum value,

$4 \sin x + 3 \cos x$ must be maximum and maximum value of :

$$4 \sin x + 3 \cos x$$

$$= \sqrt{4^2 + 3^2} = 5$$

$\therefore$ Greatest value of

$$81^{\sin x} \cdot 27^{\cos x} = 3^5$$

$$28.(d) \quad \cos A - \sin A > 0$$

$$\sin A < \cos A$$

$$\tan A < 1 \Rightarrow A < 45^\circ$$

$$\therefore \sin A + \cos A < \sin 45^\circ + \cos 45^\circ$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\text{i.e. } \sin A + \cos A = \sqrt{2}$$

$$29.(a) \quad \text{since } A+B+C=\pi$$

$$\therefore A+B = \pi - C$$

$$\Rightarrow \tan(A+B) = \tan(\pi - C)$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$$

$\therefore$ angle C is obtuse

$$\therefore \tan C < 0$$

$$\therefore \frac{\tan A + \tan B}{1 - \tan A \tan B} > 0 \quad \text{---(i)}$$

But since C is obtuse angle, so A and B will

both be less than $\frac{\pi}{2}$

$\therefore$ Both $\tan A$ and $\tan B$ are positive.

Hence, from

$$(i) \quad 1 - \tan A \cdot \tan B > 0$$

$$\Rightarrow \tan A \cdot \tan B < 1$$

$$30.(c) \quad 2 - \cos x + \sin^2 x$$

$$= 2 - \cos x + 1 - \cos^2 x$$

$$= -(\cos^2 x + \cos x) + 3$$

$$= -\left[\left(\cos x + \frac{1}{2}\right)^2 - \frac{1}{4}\right] + 3$$

$$= \frac{13}{4} - \left(\cos x + \frac{1}{2}\right)^2$$

$\therefore$ Max. value occurs at

$$\cos x = -\frac{1}{2}$$

$$\text{and it is } \frac{13}{4}$$

and Min. value occurs at $\cos x = 1$ and it is 1

$\therefore$ The required ratio is $\frac{13}{4}$.



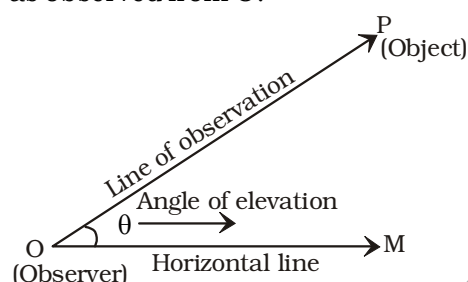
HEIGHT & DISTANCE

INTRODUCTION

One of the important application of trigonometry is in finding the height and distance of the point which are not directly measurable. This is done with the help of trigonometric ratios.

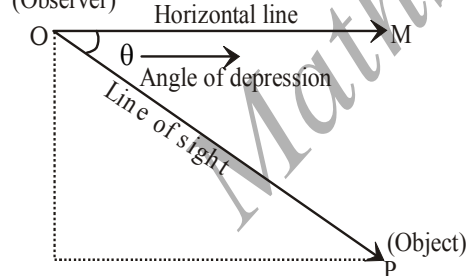
Angle of elevation

Let O and P be two point where P is at a higher level than O. Let O be at the position of the observer and P be the position of the object. Draw a horizontal line OM through the point O. OP is called the line of observation or line of sight, Then $\angle POM = \theta$ is called of elevation of P as observed from O.



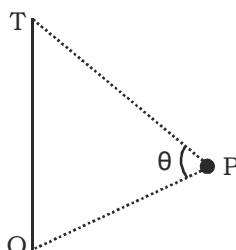
Angle of Depression

In the above figure, if P be at a lower level than O, then $\angle MOP = \theta$ is called the angle of depression.



Angle subtended by a line at a point

In the adjacent figure suppose OT is a tower, where O is the foot and T is the top of a tower. Suppose P is a point any where in the space (including ground). Join O - P and T - P, then $\angle OPT = \theta$ is the angle subtended by tower OT at point P.

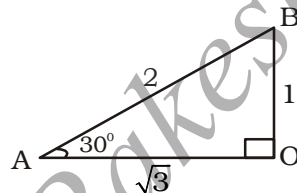


Some useful point

In this chapter we solve all the questions with the help of ratio. Some important ratios are as following :-

TYPE - I

(A)



Angle Ratio of $30^\circ =$

Base : Height : Hypotenuse
 $\sqrt{3} : 1 : 2$

Proof:

In $\triangle OBA$

$$\tan 30^\circ = \frac{OB}{OA}$$

$$\frac{1}{\sqrt{3}} = \frac{OB}{OA}$$

$$OB : OA$$

$$1 : \sqrt{3}$$

$$AB^2 = (OB)^2 + (OA)^2 = (\sqrt{3})^2 + (1)^2$$

$$AB = \sqrt{3+1} = 2.$$

OB : OA : AB

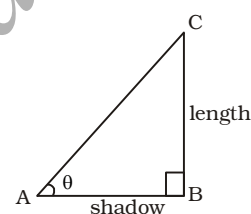
$$1 : \sqrt{3} : 2$$

Ex.1 If the ratio of the length of a pen to its shadow is $1 : \sqrt{3}$,

the angle of elevation of the source of light is :-

- (a) 40° (b) 30° (c) 60° (d) 90°

Sol. $\tan \theta = \frac{BC}{AB} = \frac{1}{\sqrt{3}} = \tan 30^\circ$
 $\Rightarrow \theta = 30^\circ$



Ex.2 The angle of elevation of the top of a tower at a distance of 500 m from its foot is 30° . The height of the tower is :-

(a) $\frac{500(\sqrt{3} - 1)}{3}$ m

(b) 500 m

(c) $\frac{500\sqrt{3}}{3}$ m

(d) $\frac{500(\sqrt{3} + 1)}{3}$

Sol. $\tan 30^\circ = \frac{BC}{AB} = \frac{h}{500}$

$$\Rightarrow h = 500 \times \frac{1}{\sqrt{3}} = \frac{500\sqrt{3}}{3}$$

Alternative

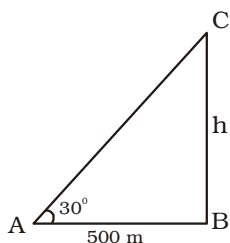
In 30° :- height : Base

$$1 : \sqrt{3}$$

but Base = 500 (given)

$$\therefore \sqrt{3} \rightarrow 500$$

$$\therefore 1 \rightarrow \frac{500}{\sqrt{3}} = \frac{500\sqrt{3}}{3}$$



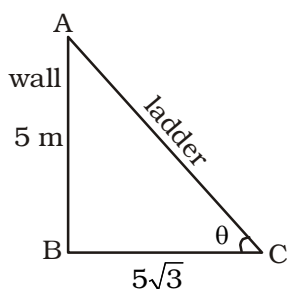
Ex.3 The foot of a ladder leaning against a wall of length 5 metre rest on a level ground $5\sqrt{3}$ metre from the base of the wall. The angle of inclination of the ladder with the ground is :-

- (a) 60° (b) 50° (c) 40° (d) 30°

Sol. In right angled $\triangle ABC$,

$$\tan \theta = \frac{AB}{BC} = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

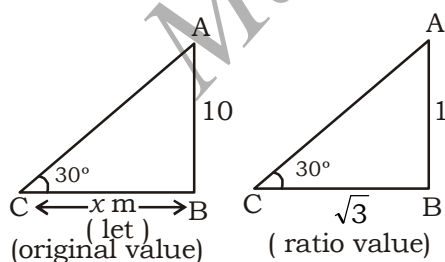
$$\Rightarrow \theta = 30^\circ$$



Ex.4 A ladder is resting against a wall at a height of 10m. If the ladder is inclined at an angle of 30° with the ground the distance of the ladder from the wall is:

- (a) $\frac{10}{\sqrt{3}}$ m (b) $\frac{20}{\sqrt{3}}$ m
(c) $10\sqrt{3}$ m (d) $20\sqrt{3}$ m

Sol.



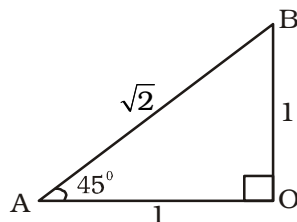
Ratio value
Original value

$$\begin{array}{lcl} AB \rightarrow 1 & \rightarrow & 10 \text{ m} \\ \therefore BC \rightarrow \sqrt{3} & \rightarrow & 10\sqrt{3} \text{ m} \end{array}$$

i.e. Required distance (BC ratio value = $\sqrt{3}$)

$$= x = 10\sqrt{3} \text{ m}$$

(B)



Angle Ratio of 45°

$$= \text{Base} : \text{Height} : \text{Hypotenuse} \\ 1 : 1 : \sqrt{2}$$

Proof:

In $\triangle OBA$

$$\tan 45^\circ = \frac{OB}{OA}$$

$$1 = \frac{OB}{OA}$$

$$\Rightarrow OB : OA$$

$$1 : 1$$

$$\Rightarrow AB^2 = (1)^2 + (1)^2$$

$$\Rightarrow AB = \sqrt{2}$$

$$\boxed{OB : OA : AB}$$

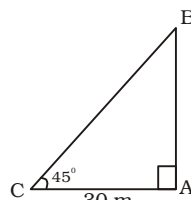
$$\boxed{1 : 1 : \sqrt{2}}$$

Ex.5 The angle of elevation of the top of a tower at a distance of 30 m from its foot is 45° . The height of the tower is:-

- (a) 20 m (b) 30 m (c)

$$15\sqrt{2} \text{ m} \quad (d) \frac{15}{\sqrt{2}} \text{ m}$$

Sol.



$$\text{Base} : \text{height} = 1 : 1$$

$$\therefore \text{height} = \text{Base} = 30 \text{ m}$$

Alternate:-

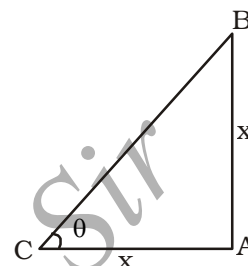
$$\text{In } \triangle ABC, \tan 45^\circ = \frac{AB}{AC}$$

$$\Rightarrow 1 = \frac{AB}{30} \Rightarrow AB = 30 \text{ m}$$

Ex.6 The angle of elevation of a moon when the length of the shadow of a pole is equal to its height is :-

- (a) 30° (b) 45° (c) 60° (d) 90°

Sol.



Let $AB = x$, then $AC = x$

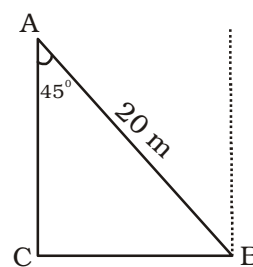
$$\therefore \tan \theta = \frac{AB}{AC} = \frac{x}{x} = 1$$

$$\Rightarrow \theta = 45^\circ$$

Ex.7 The banks of a river are parallel. A swimmer starts from a point on one of the banks and swims in a straight line inclined to the bank at 45° and reaches the opposite bank at a point 20 m from the point opposite to the starting point. The breadth of the river is :-

- (a) 20 m (b) 28.28m (c) 14.14 m (d) 40 m

Sol.

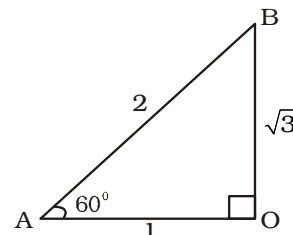


Let A be the starting point and B, the end point of the swimmer. Then $AB = 20 \text{ m}$ & $\angle BAC = 45^\circ$

$$\text{Now, } \sin 45^\circ = \frac{BC}{AB}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{20} \Rightarrow BC = 10\sqrt{2} = 14.14 \text{ m}$$

(C)



Angle Ratio of 60° =

Base : Height : Hypotenuse

$$1 : \sqrt{3} : 2$$

Proof:

In $\triangle OBA$

$$\tan 60^\circ = \frac{OB}{OA}$$

$$\sqrt{3} = \frac{OB}{OA}$$

$$\Rightarrow OB : OA$$

$$\sqrt{3} : 1$$

$$\Rightarrow AB^2 = (OB)^2 + (OA)^2$$

$$= (\sqrt{3})^2 + (1)^2$$

$$\Rightarrow AB = \sqrt{3+1} = 2.$$

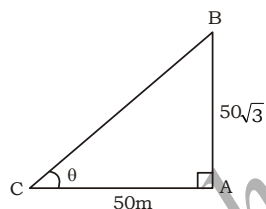
OB : OA : AB

$$\sqrt{3} : 1 : 2$$

Ex.8 A tower is $50\sqrt{3}$ meters high. Find the angle of elevation of its top from a point 50 meters away from its foot:-

- (a) $\theta = 60^\circ$ (b) $\theta = 45^\circ$
(c) $\theta = 30^\circ$ (d) $\theta = 22\frac{1}{2}^\circ$

Sol.



Height of tower (AB) = $50\sqrt{3}$

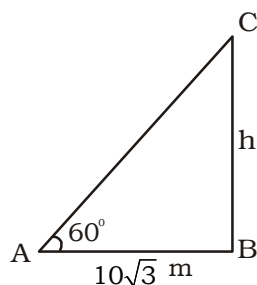
$$\therefore \tan \theta = \frac{AB}{AC} = \frac{50\sqrt{3}}{50} = \sqrt{3} = \tan 60^\circ$$

$$\Rightarrow \theta = 60^\circ$$

Ex.9 If the angle of elevation of the top of a building from a point $10\sqrt{3}$ m away from its base is 60° , the height of the building is:-

- (a) 10 m (b) 20 m
(c) $\frac{10}{\sqrt{3}}$ m (d) 30 m

Sol.



$$\tan 60^\circ = \frac{h}{10\sqrt{3}}$$

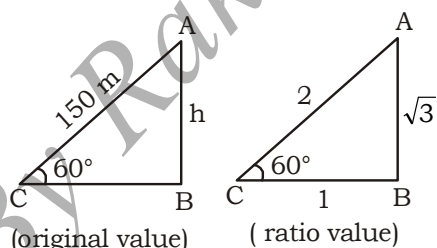
$$\sqrt{3} = \frac{h}{10\sqrt{3}}$$

$$h = 30 \text{ m.}$$

Ex.10 One flies a kite with a thread 150 metre long. If the thread of the kite makes an angle of 60° with the horizontal line, then the height of the kite from the ground (assuming the thread to be in a straight line) is:-

- (a) 50 metre
(b) $75\sqrt{3}$ m
(c) $25\sqrt{3}$ metre
(d) 80 metre

Sol.



(original value) (ratio value)
AC = length of thread, let height = h m

Ratio value Original value

$$AC \rightarrow 2 \longrightarrow 150$$

$$\therefore 1 \longrightarrow \frac{150}{2} = 75$$

$$\therefore \sqrt{3} \longrightarrow 75\sqrt{3}$$

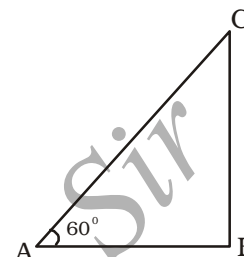
i.e. the height of the kite

$$= AB = h = 75\sqrt{3} \text{ m}$$

Ex.11 The length of a string between a kite and a point on the ground is 50 m. The string makes an angle of 60° with the level ground. If there is no slack in the string, the height of the kite is:-

- (a) $50\sqrt{3}$ m
(b) $25\sqrt{3}$ m
(c) 25 m
(d) $\frac{25}{\sqrt{3}}$ m

Sol.



Let C be the position of the kite and AC be the string.

$$\therefore AC = 50 \text{ m and}$$

$$\angle BAC = 60^\circ$$

$$\therefore \frac{BC}{AC} = \sin 60^\circ \Rightarrow \frac{BC}{50} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow BC = 25\sqrt{3} \text{ m}$$

Hence, Height of the kite

$$= 25\sqrt{3} \text{ m}$$

Alternate :-

In 60° AC : BC

$$= 2 : \sqrt{3}$$

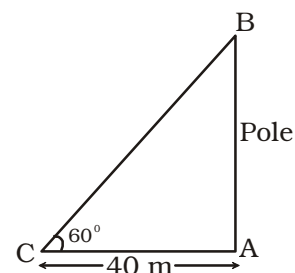
$$\times 25 \quad \times 25$$

$$50 \quad 25\sqrt{3}$$

Ex.12 The angle of the sun at any distance is 60° . The height of the vertical pole that will cast a shadow of 40 m is :-

- (a) 20 m (b) $\frac{40}{\sqrt{3}}$ m
(c) $40\sqrt{3}$ m (d) $20\sqrt{3}$ m

Sol.



Let height of pole AB = h m

$$\therefore \tan 60^\circ = \frac{h}{40}$$

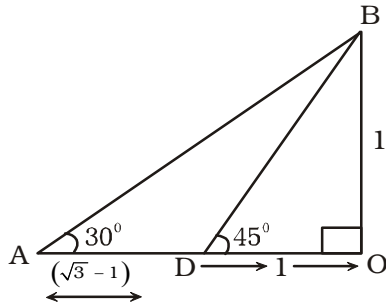
$$\Rightarrow h = 40\sqrt{3} \text{ m}$$

Alternative :

In $60^\circ = \text{Base} : \text{height}$

$$\begin{array}{ccc} 1 & : & \sqrt{3} \\ \times 40 & & \times 40 \\ \hline 40 & & 40\sqrt{3} \end{array}$$

$$\therefore \text{height} = 40\sqrt{3}$$

TYPE-II**(A)****Proof: In $\triangle AOB$**

$$\tan 30^\circ = \frac{OB}{OA}$$

$$\frac{1}{\sqrt{3}} = \frac{OB}{OA}$$

$$\Rightarrow \text{OB} : \text{OA}$$

$$1 : \sqrt{3}$$

$\triangle OBD$

$$\tan 45^\circ = \frac{OB}{OD}$$

$$1 = \frac{OB}{OD}$$

OB : OD

$$1 : 1$$

From (i) and (ii)

To make equal ratio

OB : OA

OB : OD

$$1 : \sqrt{3} \quad 1 : 1$$

$$OA : OB : OD$$

$$\sqrt{3} : 1 : 1$$

$$\sqrt{3} : 1 : 1$$

$$\text{OB} : \text{OA} : \text{OD}$$

$$1 : \sqrt{3} : 1$$

then,

$$\text{AD} = \text{OA} - \text{OD}$$

$$\text{AD} = \sqrt{3} - 1$$

Ex.13 The shadow of an electric pole standing on a ground is 40 m less when the angle of elevation changes from 30° to 45° . The length of the pole is:-

(a) $20(\sqrt{3} + 1)$ m

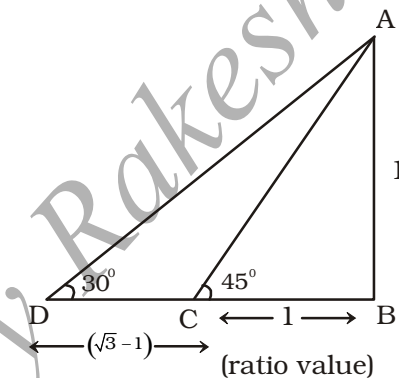
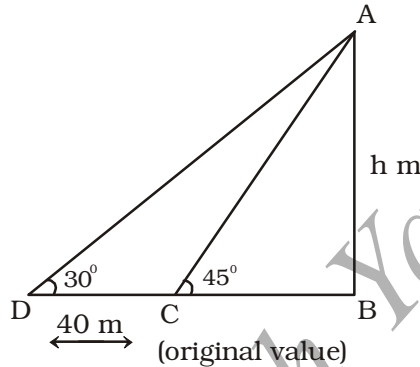
(b) $20(\sqrt{3} - 1)$ m

(c) 20 m

(d) $20\sqrt{3}$ m

Sol.

(By ratio)



$$\therefore (\sqrt{3} - 1) \rightarrow 40 \text{ m}$$

$$\therefore 1 \rightarrow \frac{40}{\sqrt{3} - 1} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)} = 20(\sqrt{3} + 1)$$

$$\therefore \text{length of pole} = 20(\sqrt{3} + 1)$$

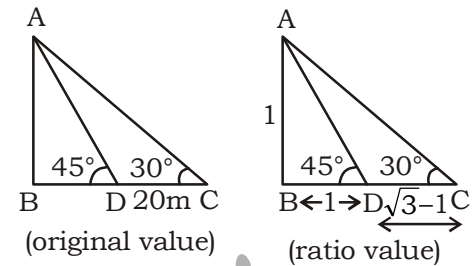
Ex.14 If the angle of elevation of the Sun changes from 30° to 45° , the length of the shadow of a pillar decreases by 20 metres. The height of the pillar is:

(a) $20(\sqrt{3} - 1)$ m

(b) $20(\sqrt{3} + 1)$ m

(c) $10(\sqrt{3} - 1)$ m

(d) $10(\sqrt{3} + 1)$ m

Sol.

Let AB be a pillar of height h metre.

Ratio value original value

$$CD \rightarrow \sqrt{3} - 1 \rightarrow 20$$

$$\therefore 1 \rightarrow \frac{20}{\sqrt{3} - 1} \times$$

$$\frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

height of the pillar = 10

$$(\sqrt{3} + 1) \text{ metre}$$

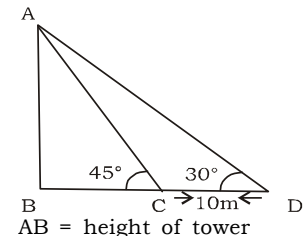
Ex.15 The length of the shadow of a vertical tower on level ground increases by 10 metres when the altitude of the sun changes from 45° to 30° . Then the height of the tower is

(a) $5(\sqrt{3} + 1)$ metres

(b) $5(\sqrt{3} - 1)$ metres

(c) $5\sqrt{3}$ metres

(d) $\frac{5}{\sqrt{3}}$ metres

Sol.

AB = height of tower

In $\triangle ABC$

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{AB}{BC} = AB : BC = 1 : 1$$

.....(i)

In $\triangle ABD$

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BD}$$

$$\Rightarrow AB : BD = 1 : \sqrt{3} \quad \text{.....(ii)}$$

Now,

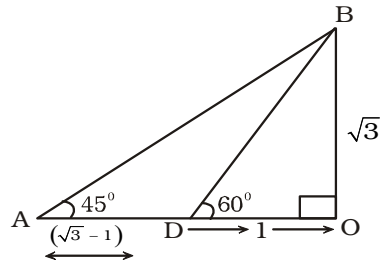
$$\begin{array}{lcl} BC & : & AB \\ 1 & : & 1 \end{array} \quad \begin{array}{lcl} & : & BD \\ & : & \sqrt{3} \end{array}$$

$$CD = BD - BC = (\sqrt{3} - 1) \text{ units} = 10 \text{ m}$$

$$= 1 \text{ unit} = \frac{10}{\sqrt{3} - 1}$$

$$AB = 1 \text{ unit} = 5(\sqrt{3} + 1) \text{ metres}$$

(B)



$\triangle OAB$

$$\tan 45^\circ = \frac{OB}{OA} = 1$$

$$OB : OA$$

$$1 : 1$$

$\triangle OBD$

$$\tan 60^\circ = \frac{OB}{OD}$$

$$\sqrt{3} = \frac{OB}{OD}$$

$$OB : OD$$

$$\sqrt{3} : 1$$

From (i) and (ii)

To make equal ratio

$$\frac{OB}{OA} = \frac{OB}{OD} \Rightarrow \frac{1}{1} = \frac{\sqrt{3}}{1}$$

$$OB : OA : OD$$

$$\sqrt{3} : \sqrt{3} : 1$$

$$AD = OA - OD = \sqrt{3} - 1$$

Ex. 16 The angle of elevation of the top of a tower at a point on level ground is 45° . When moved 20 m towards the tower, the angle of elevation becomes 60° . What is the height of the tower?

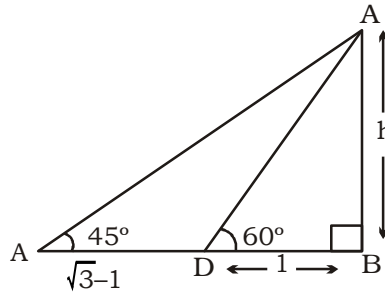
$$(a) 10(\sqrt{3} - 1) \text{ m}$$

$$(b) 10(\sqrt{3} + 1) \text{ m}$$

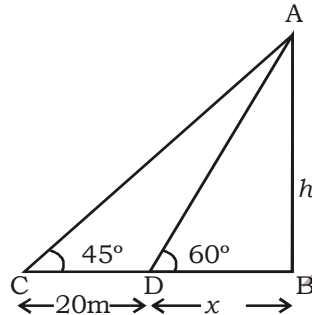
$$(c) 10(3 - \sqrt{3}) \text{ m}$$

$$(d) 10(3 + \sqrt{3}) \text{ m}$$

Sol. (d) Let the height of tower be h m and $BD = xm$



Ratio Figure



Both of the figures are in comparison ratio

$$(\sqrt{3} - 1)R = 20$$

$$R = \frac{20}{\sqrt{3} - 1}$$

$$x = 1 \quad R = \frac{20}{\sqrt{3} - 1}$$

$$= \frac{20}{\sqrt{3} - 1} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)} = \frac{20(\sqrt{3} + 1)}{2}$$

$$= 10(\sqrt{3} + 1) \text{ m.}$$

Height of tower

$$= AB = \sqrt{3} \times 10(\sqrt{3} + 1)$$

$$= 10(3 + \sqrt{3}) \text{ m.}$$

Ex. 17 The elevation of the top of a tower from a point on the ground is 45° . On travelling 60 m from the point towards the tower the elevation of the top becomes 60° . The height of the tower (in metres) is

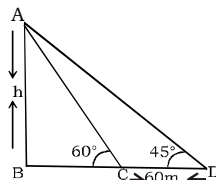
$$(a) 30$$

$$(b) 30(3 - \sqrt{3})$$

$$(c) 30(3 + \sqrt{3})$$

$$(d) 30\sqrt{3}$$

Sol.



AB height of tower

In $\triangle ABC$

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{BC} \Rightarrow AB : BC = \sqrt{3} : 1 \dots (i)$$

In $\triangle ABD$

$$\tan 45^\circ = \frac{AB}{BD}$$

$$1 = \frac{AB}{BD}$$

$$\Rightarrow AB : BD = 1 : 1 \dots (ii)$$

Now

$$\frac{AB}{BD} = \frac{AB}{BC} \Rightarrow \frac{1}{1} = \frac{\sqrt{3}}{1}$$

$$\sqrt{3} : \sqrt{3} : 1$$

$$CD = BD - BC = (\sqrt{3} - 1)$$

$$(\sqrt{3} - 1) = 60 \text{ metre}$$

$$1 \text{ unit} = \frac{60}{\sqrt{3} - 1}$$

$$AB = \sqrt{3} \text{ units} = \frac{60}{\sqrt{3} - 1} \times \sqrt{3}$$

$$= 30(3 + \sqrt{3}) \text{ m}$$

Ex. 18 The angle of elevation of the tip of a tower from a point on the ground is 45° . Moving 21 m directly towards the base of the tower, the angle of elevation changes to 60° . What is the height of the tower, to the nearest meter?

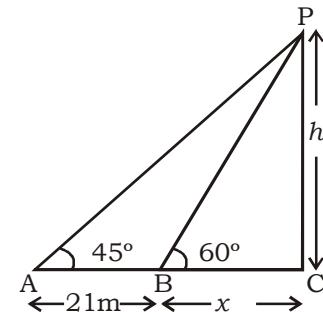
$$(a) 48 \text{ m}$$

$$(b) 49 \text{ m}$$

$$(c) 50 \text{ m}$$

$$(d) 51 \text{ m}$$

Sol. In $\triangle PBC$,



$$\tan 60^\circ = \frac{h}{x} \Rightarrow \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \dots (i)$$

$$\text{In } \triangle PAC, \tan 45^\circ = \frac{h}{21 + x} = 1$$

$$\Rightarrow h = 21 + x$$

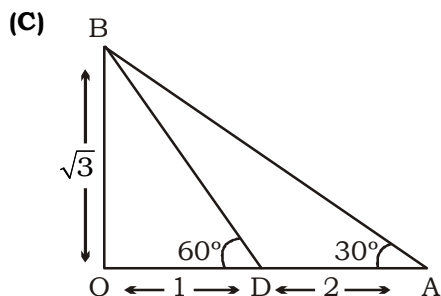
$$\Rightarrow h = 21 + \frac{h}{\sqrt{3}} \text{ [from Eq. (i)]}$$

$$\Rightarrow h \left(1 - \frac{1}{\sqrt{3}} \right) = 21$$

$$\therefore h = \frac{21\sqrt{3} \cdot (\sqrt{3} + 1)}{(\sqrt{3} - 1) \cdot (\sqrt{3} + 1)}$$

$$= \frac{21\sqrt{3}(\sqrt{3} + 1)}{2}$$

$$= 49.68 \approx 50 \text{ m}$$



Proof

$\triangle OAB$

$$\tan 30^\circ = \frac{OB}{OA}$$

$$\frac{1}{\sqrt{3}} = \frac{OB}{OA}$$

$$OB : OA$$

$$1 : \sqrt{3}$$

$\triangle OBD$

$$\tan 60^\circ = \frac{OB}{OD}$$

$$\sqrt{3} = \frac{OB}{OD}$$

$$OB : OD$$

$$\sqrt{3} : 1$$

From (i) & (ii)

to make equal ratio

$$OB : OA \quad OB : OD$$

$$1 : \sqrt{3} \quad \sqrt{3} : 1$$

$$OB : OA : OD$$

$$\sqrt{3} : 3 : 1$$

$$AD = OA - OD = 3 - 1 = 2$$

Ex. 19 A man from the top a 50 m high tower, sees a car moving towards the tower at an angle of depression of 30° . After some time, the angle of

depression becomes 60° . The distance (in m) travelled by the car during this time is:-

(a) $50\sqrt{3}$ (b) $\frac{50\sqrt{3}}{3}$

(c) $\frac{100\sqrt{3}}{3}$ (d) $100\sqrt{3}$

Sol. In $\triangle ADC$

$$\Rightarrow \tan 30^\circ = \frac{DC}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{AC}$$

$$AC = 50\sqrt{3}$$

In $\triangle DBC$

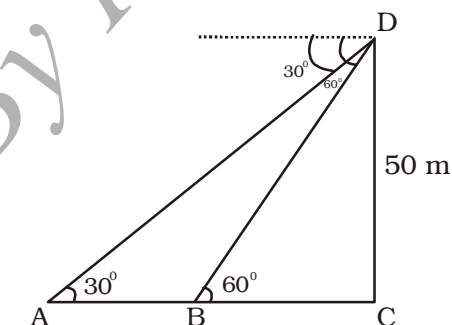
$$\Rightarrow \tan 60^\circ = \frac{50}{BC}$$

$$BC = \frac{50}{\sqrt{3}}$$

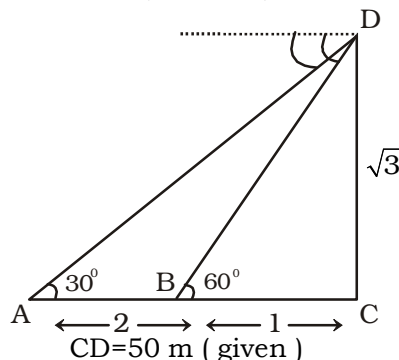
(i) - (ii)

$$AB = AC - BC = 50 \left(\sqrt{3} - \frac{1}{\sqrt{3}} \right)$$

$$= \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$$



Alternate :- (by ratio)



but $CD = \sqrt{3}$ (according to ratio)

Ratio value original value

$$CD \sqrt{3} \longrightarrow 50$$

$$\therefore 1 \longrightarrow \frac{50}{\sqrt{3}}$$

$$\therefore 2 \longrightarrow \frac{100}{\sqrt{3}}$$

$$\therefore AB (\text{ratio value}) = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$$

Ex. 20 The angle of elevation of top of a tree on the bank of a river from its other bank is 60° and from a point 20 m further away from this is 30° . The width of the river is:-

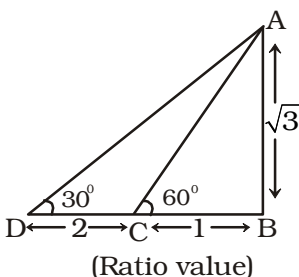
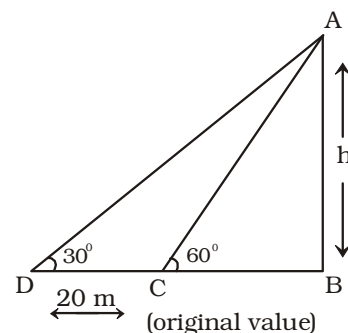
(a) $10\sqrt{3}$ m (b) 10 m

(c) 20 m (d) $20\sqrt{3}$ m

Sol.

(By ratio)

BC = width of river



Ratio value original value

$$2 \longrightarrow 20 \text{ m}$$

$$\therefore 1 \longrightarrow 10 \text{ m}$$

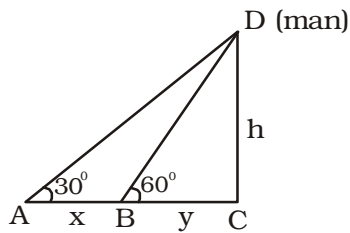
Hence, width of the river = BC = 10 m

Ex. 21 A man on the top of a rock rising on a sea-shore observes a boat coming towards it. If it takes 20 minute for the angle of depression to change from 30° to 60° , how soon will the boat reach the shore?

(a) 20 minute

- (b) 30 minute
(c) 10 minute
(d) 15 minute

Sol.



From figure, $\tan 60^\circ = \frac{h}{y}$

$$\Rightarrow h = y\sqrt{3} \dots\dots\dots(i)$$

$$\text{and } \tan 30^\circ = \frac{h}{x+y}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y\sqrt{3}}{x+y}$$

[using (i)]

$$\therefore 3y = x + y$$

$$\Rightarrow 2y = x \Rightarrow y = \frac{x}{2}$$

$\therefore$ Time taken to cover a distance from A to B = 20 minute

$\therefore$ time taken to cover a unit distance

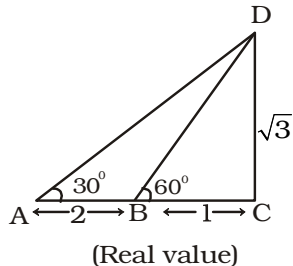
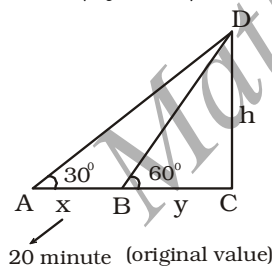
$$= \frac{20}{x} \text{ minute}$$

$\therefore$ For distance 'y' time taken

$$= \frac{20}{x} \times y$$

$$= \frac{20}{x} \times \frac{x}{2} = 10 \text{ minute}$$

Alternative (By ratio)



Ratio value original value

$$2 \longrightarrow 20 \text{ minute}$$

$$\therefore 1 \longrightarrow 10 \text{ minute}$$

$\therefore$ For distance y (or ratio value = 1) time taken = 10 minute

Ex.22 From the top of a 60 m high tower the angle of depression of the top and bottom of a building are observed to be 30° and 60° respectively. The height of the building is:-

(a) $60\sqrt{3}$ m (b) $40\sqrt{3}$ m

(c) 40 m (d) 20 m

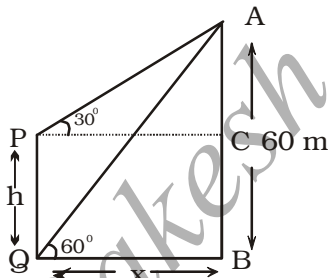
Sol. Let AB be the tower and PQ the building.

From right angled $\triangle ABQ$,

$$\tan 60^\circ = \frac{AB}{BQ} \Rightarrow \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow x = 20\sqrt{3}$$

$\dots\dots(i)$



and From right angled $\triangle APC$,

$$\tan 30^\circ = \frac{AC}{PC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AC}{x}$$

$$AC = \frac{x}{\sqrt{3}}$$

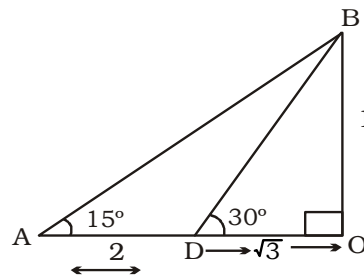
put the value of x in equation

$$(i) \quad AC = \frac{20\sqrt{3}}{\sqrt{3}} = 20$$

$$\therefore PQ = BC = AB - AC = 60 - 20 = 40$$

$$\therefore \text{height of the building} = h = 40 \text{ m}$$

(D)



In $\triangle OAB$

$$\tan 15^\circ = \frac{OB}{OA}$$

$$(2 - \sqrt{3}) = \frac{OB}{OA}$$

$$\left(\tan 15^\circ = 2 - \sqrt{3} = \frac{1}{2 + \sqrt{3}} \right)$$

$$\frac{1}{2 + \sqrt{3}} = \frac{OB}{OA}$$

$$OB : OA$$

$\dots\dots(i)$

$$1 : 2 + \sqrt{3}$$

In $\triangle BOD$

$$\tan 30^\circ = \frac{OB}{OD}$$

$$\frac{1}{\sqrt{3}} = \frac{OB}{OD}$$

$$OB : OD$$

$\dots\dots(ii)$

$$1 : \sqrt{3}$$

From (i) & (ii)

to make same ratio

$$OB : OA \quad OB : OD$$

$$1 : 2 + \sqrt{3} \quad 1 : \sqrt{3}$$

$$OB : OA : OD$$

$$1 : 2 + \sqrt{3} : \sqrt{3}$$

then,

$$AD = OA - OD$$

$$= (2 + \sqrt{3}) - (\sqrt{3}) = 2$$

Ex.23 The angles of elevation of the top of a tower from two points A and B lying on the horizontal through the foot of the tower are respectively 15° and 30° . If A and B are on the same side of the tower and $AB = 48$ metre, then the height of the tower is:

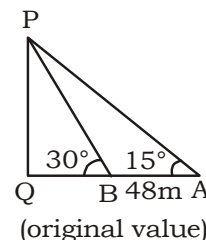
(a) $24\sqrt{3}$ metre

(b) 24 metre

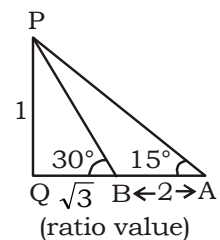
(c) $24\sqrt{2}$ metre

(d) 96 metre

Sol.



(original value)



(ratio value)

$$PQ = \text{Tower} = h \text{ metre}$$

Ratio value	Original value
AB $\rightarrow 2$	$\rightarrow 48$
$\therefore 1$	$\rightarrow \frac{48}{2}$
	$= 24 \text{ metre}$

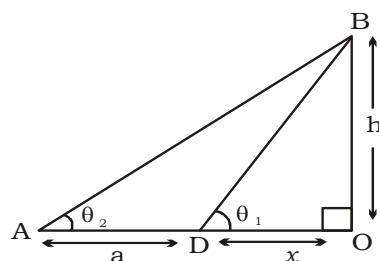
i.e. height of the building
 $= PQ$ (ratio value $= 1$) $= h$
 $= 24 \text{ metre}$.

Note:- you also can make other ratio for different angle.

Alternate:-

We can use formula for this type

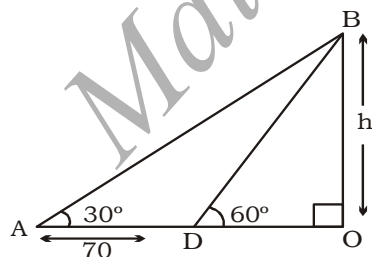
- * The angle of elevation of the top of a tower from a point on the horizontal is θ_2 and moving 'a' distance towards the tower it becomes θ_1 , the height of the tower is-



$$h = \frac{a}{\cot \theta_2 - \cot \theta_1}$$

- Ex.24** The angle of elevation of the top of a tower from a point on the ground is 30° and moving 70 metres towards the tower it becomes 60° . The height of the tower is:

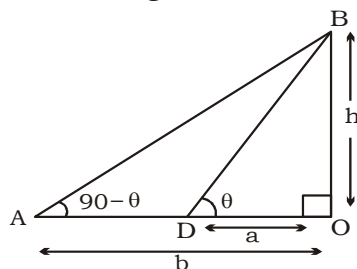
Sol.



$$h = \frac{70}{\cot 30^\circ - \cot 60^\circ} = \frac{70}{\sqrt{3} - \frac{1}{\sqrt{3}}} = \frac{70\sqrt{3}}{2} = 35\sqrt{3}$$

Type - III

- * If the angles of a tower from two points distant a and b from its foot and in the same straight line from it is complementary each other then height of tower is.



$$h = \sqrt{ab}$$

- Ex.25** The angle of elevation of the top of a tower at two points which are at a distance a and b from the foot in the same horizontal line and on the same side of the tower, are complementary. The height of the tower is :-

- (a) ab (b) $\sqrt{ab}$
 (c) $\sqrt{\frac{a}{b}}$ (d) $\sqrt{\frac{b}{a}}$

- Sol.** Let PQ be the given tower of height h. If A, B be given points then suppose.

$$\angle PAQ = \alpha \text{ and } \angle PBQ = \beta$$

$$\therefore \alpha + \beta = 90^\circ$$

Now in ΔPAQ ,

$$\tan \alpha = \frac{h}{a} \dots \dots \dots (i)$$

in ΔPBQ ,

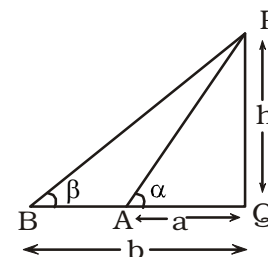
$$\tan \beta = \frac{h}{b} \Rightarrow \tan(90^\circ - \alpha) = \frac{h}{b}$$

$$\Rightarrow \cot \alpha = \frac{h}{b} \Rightarrow \tan \alpha = \frac{b}{h}$$

$$\Rightarrow \frac{h}{a} = \frac{b}{h}$$

$$[\text{from (i)- } \tan \alpha = \frac{h}{a}]$$

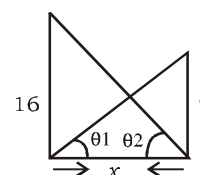
$$\Rightarrow h^2 = ab \Rightarrow h = \sqrt{ab}$$



- Ex.26** The distance between two pillars of length 16 metres and 9 metres is x metres. If two angles of elevation of their respective top from the bottom of the other are complementary to each other then the value of x (in metres) is

- (a) 15 (b) 16 (c) 12 (d) 9

Sol.



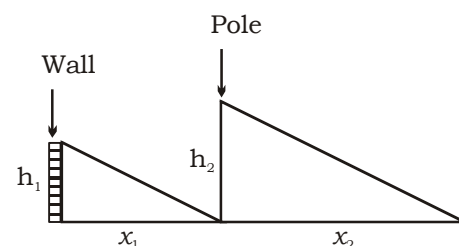
$$\text{If } \theta_1 + \theta_2 = 90^\circ \text{ then } x = \sqrt{h_1 \times h_2}$$

(h = height of towers)

$$x = \sqrt{16 \times 9} = \sqrt{144} = 12 \text{ mtr}$$

TYPE - IV

- * At a particular time for all object ratio of height shadow are same



$$\tan \theta = \frac{h_1}{x_1} = \frac{h_2}{x_2}$$

- Ex.27** A vertical stick 12 cm long casts a shadow 8 cm long on the ground. At the same time, a tower casts a shadow 40 m long on the ground. The height of the tower is

- (a) 72 m (b) 60 m
 (c) 65 m (d) 70 m

$$\text{Sol. } \frac{12}{8} = \frac{h_2}{40}$$

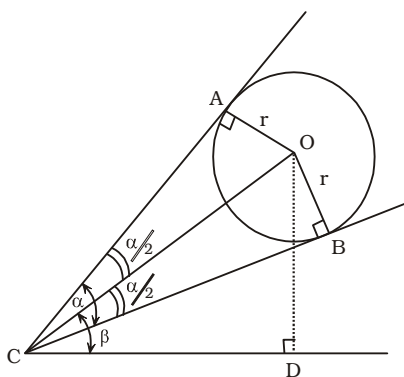
$$h_2 = \frac{40 \times 12}{8}$$

$$h_2 = 60 \text{ m.}$$

TYPE - V

A round balloon of radius r subtends an angle α at the eye of an observer while the angle of elevation of its centre is β . Then the height of its centre from horizontal is

$$h = r \sin \beta \cdot \operatorname{cosec} \frac{\alpha}{2}$$



Proof....

Let O be the centre of balloon of radius r . The observer's eye is at C , $\angle ACB = \alpha$ and $\angle OCD = \beta$ clearly, CA and CB are tangents to the circle.

$$\text{so } \angle ACO = \angle BCO = \frac{\alpha}{2}$$

In right angled $\triangle OBC$,

$$\sin \frac{\alpha}{2} = \frac{OB}{OC} \Rightarrow OC = \frac{OB}{\sin \frac{\alpha}{2}} = r \operatorname{cosec} \frac{\alpha}{2}$$

In right angled $\triangle OCD$,

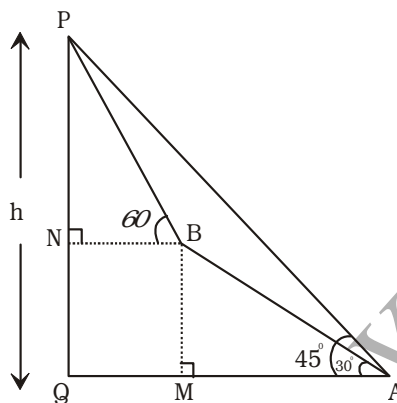
$$\sin \beta = \frac{OD}{OC} \Rightarrow OD = OC \sin \beta = r \operatorname{cosec} \frac{\alpha}{2} \cdot \sin \beta$$

$\therefore$ Height of the centre of the balloon is

$$r \sin \beta \cdot \operatorname{cosec} \frac{\alpha}{2}$$

TYPE - VI

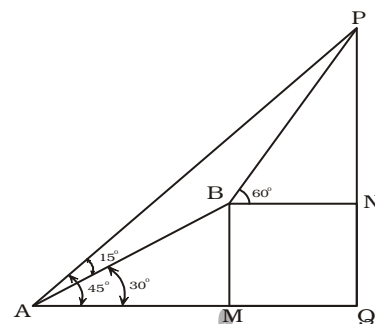
At the foot of the mountain the elevation of its summit is 45° ; after ascending AB km towards the mountain up a slope of 30° inclination, the elevation is found to be 60° . Then the height of the mountain.



$$h = AB \left(\frac{\sqrt{3} + 1}{2} \right)$$

Ex.28 At the foot of the mountain the elevation of its summit is 45° ; after ascending 4 km towards the mountain up a slope of 30° inclination, the elevation is found to be 60° . Then find the height of the mountain:-

Sol. Suppose P be the summit of the mountain and Q be the foot.



Here, $BN \perp PQ$ and $BM \perp AQ$,

$$AB = 4 \text{ km}, \angle MAB = 30^\circ,$$

$$\angle MAP = 45^\circ$$

$$\angle NBP = 60^\circ \therefore \angle BAP = 15^\circ$$

$$\text{and } \angle APQ = 45^\circ$$

$$\text{and } \angle BPN = 30^\circ$$

$$\therefore \angle APB = 15^\circ$$

$$\therefore \triangle ABP \text{ is isosceles and}$$

$$AB = BP = 4 \text{ km}$$

In $\triangle PBN$,

$$PN = BP \sin 60^\circ$$

In $\triangle ABM$,

$$BM = AB \sin 30^\circ$$

$$\therefore PQ = PN + NQ = PN + BM$$

$$= BP \sin 60^\circ + AB \sin 30^\circ$$

$$= 4 \frac{\sqrt{3}}{2} + 4 \frac{1}{2} = 4 \left(\frac{\sqrt{3} + 1}{2} \right)$$

$$= 2(\sqrt{3} + 1) \text{ km}$$

$$\therefore \text{Height of the mountain is}$$

$$= 2(\sqrt{3} + 1) \text{ km}$$

Alternae:-

$$\begin{aligned} PQ &= AB \left(\frac{\sqrt{3} + 1}{2} \right) = 4 \left(\frac{\sqrt{3} + 1}{2} \right) \\ &= 2(\sqrt{3} + 1) \text{ km} \end{aligned}$$

EXERCISE

1. At a point on a horizontal line through the base of a monument the angle of elevation of the top of the monument is found to be such that its tangent is $\frac{1}{5}$. On walking 138 metres towards the monument the secant of the angle of elevation is found to be $\frac{\sqrt{193}}{12}$. The height of the monument (in metre) is
(a) 42 (b) 49 (c) 35 (d) 56
2. The angle of elevation of the top of a building from the top and bottom of a tree are x and y respectively. If the height of the tree is h metre, then (in metre) the height of the building is
(a) $\frac{h \cot x}{\cot x + \cot y}$ (b) $\frac{h \cot y}{\cot x + \cot y}$
(c) $\frac{h \cot x}{\cot x - \cot y}$ (d) $\frac{h \cot y}{\cot x - \cot y}$
3. The angle of elevation of the top of a tower from a point A on the ground is 30° . On moving a distance of 20 metres towards the foot of the tower to a point B, the angle of elevation increases to 60° . The height of the tower is
(a) $\sqrt{3}$ m (b) $5\sqrt{3}$ m
(c) $10\sqrt{3}$ m (d) $20\sqrt{3}$ m
4. Two poles of equal height are standing opposite to each other on either side of a road which is 100m wide. From a point between them on road, angle of elevation of their tops are 30° and 60° . The height of each pole (in meter) is
(a) $25\sqrt{3}$ (b) $20\sqrt{3}$
(c) $28\sqrt{3}$ (d) $30\sqrt{3}$ m
5. The angle of elevation of the top of a chimney and roof of the building from a point on the ground are 45° and x° respectively. The height of building is h metre. Then the height of the chimney, (in metre) is
(a) $h \cot x + h$ (b) $h \cot x - h$
(c) $h \tan x - h$ (d) $h \tan x + h$
6. There are two vertical posts, one on each side of a road, just opposite to each other. One post is 108 metre high. From the top of this post the angle of depression of the top and foot of the other post are 30° and 60° respectively. The height of the other post (in metre) is
(a) 36 (b) 72 (c) 108 (d) 110
7. Two posts are x metres apart and the height of one is double that of the other. If from the mid-point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height (in metres) of the shorter post is
(a) $\frac{x}{2\sqrt{2}}$ (b) $\frac{x}{4}$
(c) $x\sqrt{2}$ (d) $\frac{x}{2}$
8. An aeroplane when flying at a height of 5000m from the ground passes vertically above another aeroplane at an instant, when the angles of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. The vertical distance between the aeroplanes at that instant is
(a) $5000(\sqrt{3} - 1)$ m
(b) $5000(3 - \sqrt{3})$ m
(c) $5000\left(1 - \frac{1}{\sqrt{3}}\right)$ m
(d) 4500 m
9. A man standing at a point P is watching the top of a tower, which makes an angle of elevation of 30° . The man walks some distance towards the tower and then his angle of elevation of the top of the tower is 60° . If the height of tower is 30m, then the distance he moves is
(a) 22 m (b) $22\sqrt{3}$ m
(c) 20 m (d) $20\sqrt{3}$ m
10. An aeroplane when flying at a height of 3125m from the ground passes vertically below another plane at an instant when the angle of elevation of the two planes from the same point on the ground are 30° and 60° respectively. The distance between the two planes at that instant is
(a) 6520 m (b) 6000 m
(c) 5000 m (d) 6250 m
11. The shadow of the tower becomes 60 meters longer when the altitude of the sun changes from 45° to 30° . Then the height of the tower is
(a) $20(\sqrt{3} + 1)$ m (b) $24(\sqrt{3} + 1)$ m
(c) $30(\sqrt{3} + 1)$ m (d) $30(\sqrt{3} - 1)$ m
12. A vertical post 15 ft. high is broken at a certain height and its upper part, not completely separated meets the ground at an angle of 30° . Find the height at which the post is broken
(a) 10ft (b) 5ft
(c) $15\sqrt{3}(2 - \sqrt{3})$ ft
(d) $5\sqrt{3}$ ft
13. The shadow of a tower is $\sqrt{3}$ times its height. Then the angle of elevation of the top of the tower is
(a) 45° (b) 30° (c) 60° (d) 90°
14. A man 6ft tall casts a shadow 4ft long. At the same time when a flag pole casts a shadow 50 ft long. The height of the flag pole is
(a) 80ft (b) 75ft (c) 60ft (d) 70ft
15. The angle of elevation of an aeroplane from a point on the ground is 60° . After 15 seconds flight, the elevation changes to 30° . If the aeroplane is flying at a height of $1500\sqrt{3}$ m, find the speed of the plane.
(a) 300 m/sec (b) 200m/sec
(c) 100m/sec (d) 150m/sec
16. There are two temples, one on each bank of a river just opposite to each other. One temple is 54m high. From the top of this temple, the angles of

- depression of the top and the foot of the other temple are 30° and 60° respectively. The length of the temple is;
 (a) 18 m (b) 36 m
 (c) $36\sqrt{3}$ m (d) $18\sqrt{3}$ m
17. The angle of elevation of the top of a tower from the point P and Q at distance of 'a' and 'b' respectively from the base of the tower and in the same straight line with it are complementary. The height of the tower is
 (a) $\sqrt{ab}$ (b) $\frac{a}{b}$
 (c) ab (d) a^2b^2
18. The angle of elevation of a tower from a distance 100 m from its foot is 30° . Height of the tower is
 (a) $\frac{100}{\sqrt{3}}$ m (b) $50\sqrt{3}$ m
 (c) $\frac{200}{\sqrt{3}}$ m (d) $100\sqrt{3}$ m
19. A pole stands vertically inside a scalene triangular park ABC. If the angle of elevation of the top of the pole from each corner of the park is same, then in $\triangle ABC$, the foot of the pole is at the
 (a) centroid
 (b) circumcentre
 (c) incentre
 (d) orthocentre
20. If the angle of elevation of a balloon from two consecutive kilometre-stones along a road are 30° and 60° respectively, then the height of the balloon above the ground will be
 (a) $\frac{\sqrt{3}}{2}$ km (b) $\frac{1}{2}$ km
 (c) $\frac{2}{\sqrt{3}}$ km (d) $3\sqrt{3}$ km
21. A tower standing on a horizontal plane subtends a certain angle at a point 160 m apart from the foot of the tower. On advancing 100 m towards it, the tower is found to subtend an angle twice as before. The height of the tower is
 (a) 80 m (b) 100 m
 (c) 160 m (d) 200 m
22. The angle of elevation of a tower from a distance 50 m from its foot is 30° . The height of the tower is
 (a) $50\sqrt{3}$ m (b) $\frac{50}{\sqrt{3}}$ m
 (c) $75\sqrt{3}$ m (d) $\frac{75}{\sqrt{3}}$ m
23. From two points on the ground lying on a straight line through the foot of a pillar, the two angles of elevation of the top of the pillar are complementary to each other. If the distance of the two points from the foot of the pillar are 9 metres and 16 metres and the two points lie on the same side of the pillar. Then the height of the pillar is
 (a) 5m (b) 10m
 (c) 9m (d) 12m
24. The top of two poles of height 24m and 36 m are connected by a wire. If the wire makes an angle of 60° with the horizontal, then the length of the wire is
 (a) 6m (b) $8\sqrt{3}$ m
 (c) 8 m (d) $6\sqrt{3}$ m
25. From the top of a hill 200 m high the angle of depression of the top and the bottom of a tower are observed to be 30° and 60° . The height of the tower is (in m);
 (a) $\frac{400\sqrt{3}}{3}$ m (b) $166\frac{2}{3}$ m
 (c) $133\frac{1}{3}$ m (d) $200\sqrt{3}$ m
26. From a tower 125 metres high the angle of depression of two objects, which are in horizontal line through the base of the tower are 45° and 30° and they are on the same side of the tower. The distance (in metres) between the objects is
 (a) $125\sqrt{3}$ m (b) $125(\sqrt{3}-1)$ m
 (c) $125/(\sqrt{3}-1)$ m (d) $125(\sqrt{3}+1)$ m
27. From a point P on the ground the angle of elevation of the top of a 10m tall building is 30° . A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45° . Find the length of the flagstaff (Take $\sqrt{3} = 1.732$)
 (a) $10(\sqrt{30}+2)$ m (b) $10(\sqrt{30}+1)$ m
 (c) $10\sqrt{3}$ m (d) 7.32 m
28. The angle of elevation of the top of a vertical tower situated perpendicularly on a plane is observed as 60° from a point P on the same plane. From another point Q, 10m vertically above the point P, the angle of depression of the foot of the tower is 30° . The height of the tower is
 (a) 15 m (b) 30 m
 (c) 20 m (d) 25 m
29. From a point 20 m away from the foot of a tower, the angle of elevation of the top of the tower is 30° . The height of the tower is
 (a) $10\sqrt{3}$ m (b) $20\sqrt{3}$ m
 (c) $\frac{10}{\sqrt{3}}$ m (d) $\frac{20}{\sqrt{3}}$ m
30. The angle of elevation of ladder leaning against a house is 60° and the foot of the ladder is 6.5 metres from the house. The length of the ladder is
 (a) $\frac{13}{\sqrt{3}}$ (b) 13 meters
 (c) 15 meters (d) 3.25 metres
31. The angle of elevation of sun changes from 30° to 45° , the length of the shadow of a pole decreases by 4 metres, the height of the pole is (Assume $\sqrt{3} = 1.732$)
 (a) 1.464m (b) 9.464 m
 (c) 3.648 cm (d) 5.464 m
32. A vertical pole and a vertical tower are standing on the same level ground. Height of the pole is 10 metres. From the top of the pole the angle of elevation of the top of the tower and angle of depression of the foot of the tower are 60° and 30° respectively. The height of the tower is
 (a) 20 m (b) 30 m
 (c) 40 m (d) 50 m
33. If a pole of 12 m height casts a shadow of $4\sqrt{3}$ m long on the ground then the sun's angle of elevation at that instant is
 (a) 30° (b) 60° (c) 45° (d) 90°

34. The angle of elevation of the top of a tower from a point on the ground is 30° and moving 70 meters towards the tower it becomes 60° . The height of the tower is
(a) 10 meter (b) $\frac{10}{\sqrt{3}}$ metre
(c) $10\sqrt{3}$ metre (d) $35\sqrt{3}$ metre
35. From the top of a tower of height 180m the angles of depression of two objects on both sides of the tower are 30° and 45° . Then the distance between the objects are
(a) $180(3+\sqrt{3})$ m
(b) $180(3-\sqrt{3})$ m
(c) $180(\sqrt{3}-1)$ m
(d) $180(\sqrt{3}+1)$ m
36. From the peak of a hill which is 300m high, the angle of depression of two sides of a bridge lying on a ground are 45° and 30° (both ends of the bridge are on the same side of the hill). Then the length of the bridge is
(a) $300(\sqrt{3}-1)$ m (b) $300(\sqrt{3}+1)$
(c) $300\sqrt{3}$ m (d) $\frac{300}{\sqrt{3}}$ m
37. From an aeroplane just over a river, trees on the opposite bank of the river are found to be at an angle of 60° and 30° respectively. If the breadth of the river is 400 metres, then the height of the aeroplane above the river at that instant is (Assume $\sqrt{3} = 1.732$)
(a) 173.2 metres
(b) 346.4 metres
(c) 519.6 metres
(d) 692.8 metres
38. The shadow of a tower standing on a level plane is found to be 50 m longer when the Sun's elevation is 30° , then when it is 60° . What is the height of the tower?
(a) 25 m (b) $25\sqrt{3}$ m
(c) $\frac{25}{\sqrt{3}}$ m (d) 30 m
39. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30° . The distance between the two towers is m times the height of the shorter tower. What is m equal to?
(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$
40. A spherical balloon of radius r subtends angle 60° at the eye of an observer. If the angle of elevation of its centre is 60° and h is height of the centre of the balloon, then which one of the following is correct?
(a) $h = r$ (b) $h = \sqrt{2}r$
(c) $h = \sqrt{3}r$ (d) $h = 2r$
41. What is the angle of elevation of the Sun, when the shadow of a pole of height x m is $\frac{x}{\sqrt{3}}$ m?
(a) 30° (b) 45° (c) 60° (d) 75°
42. On walking 120 m towards a chimney in a horizontal line through its base the angle of elevation of tip of the chimney changes from 30° to 45° . The height of the chimney is
(a) 120 m (b) $60(\sqrt{3}-1)$ m
(c) $60(\sqrt{3}+1)$ m
(d) None of these
43. A ladder 20 m long is placed against a wall, so that the foot of the ladder is 10 m from the wall. The angle of inclination of the ladder to the horizontal will be
(a) 30° (b) 45° (c) 60° (d) 75°
44. The angles of elevation of the top of a tower from two points which are at distance of 10 m and 5 m from the base of the tower and in the same straight line with it are complementary. The height of the tower is
(a) 5 m (b) 15 m
(c) $\sqrt{50}$ m (d) $\sqrt{75}$ m
45. The angles of elevation of the top of an inaccessible tower from two points on the same straight line from the base of the tower are 30° and 60° , respectively. If the point is separated at a distance of 100 m, then the height of the tower is close to
(a) 86.6 m (b) 84.6 m
(c) 82.6 m (d) 80.6 m
46. Two poles of heights 6m and 11m stand on a plane ground. If the distance between their feet is 12 m, what is the distance their tops?
(a) 13 m (b) 17 m
(c) 18 m (d) 23 m
47. From the top of a cliff 200 m high, the angles of depression of the top and bottom of a tower of observed to be 30° and 45° , respectively. What is the height of the tower?
(a) 400 m (b) $400\sqrt{3}$
(c) $400\sqrt{3}$ m
(d) None of these
48. The angle of elevation the tip of a tower from a point on the ground is 45° . Moving 21 m directly towards the base of the tower, the angle of elevation changes to 60° . What is the height of the tower, to the nearest meter?
(a) 48 m (b) 49 m
(c) 50 m (d) 51 m
49. What is the angle of elevation of the Sun when the shadow of a pole is $\sqrt{3}$ times the length of the pole?
(a) 30° (b) 45° (c) 60°
(d) None of these
50. The angles of elevation of the top of a tower from two points situated at distances 36 m and 64 m from its base and in the same straight line with it are complementary. What is the height of the tower?
(a) 50 m (b) 48 m
(c) 25 m (d) 24 m
51. The angle of elevation of the top of an incomplete vertical pillar at a horizontal distance of 100 m from its base is 45° . If the angle of elevation of the top of complete pillar at the same point is to be 60° , then the height of the incomplete pillar is to be increased by
(a) $50\sqrt{2}$ m (b) 100 m

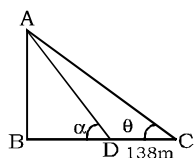
- (c) $100(\sqrt{3}-1)$ m
(d) $100(\sqrt{3}+1)$ m
52. The length of shadow of a tree is 16 m when the angle of elevation of the Sun is 60° . What is the height of the tree?
(a) 8 m (b) 16 m
(c) $16\sqrt{3}$ m (d) $\frac{16}{\sqrt{3}}$ m
53. From a lighthouse the angles of depression of two ships on opposite sides of the lighthouse are observed to 30° and 45° . If the height of lighthouse is h , what is the distance between the ships?
(a) $(\sqrt{3}+1)h$ (b) $(\sqrt{3}-1)h$
(c) $\sqrt{3}h$ (d) $\left(1+\frac{1}{\sqrt{3}}\right)h$
54. A telegraph post gets broken at a point against a storm and its top touches the ground at a distance 20 m from the base of the post making an angle 30° with the ground. What is the height of the post?
(a) $\frac{40}{\sqrt{3}}$ m (b) $20\sqrt{3}$ m
(c) $40\sqrt{3}$ m (d) 30 m
55. The angle of elevation of the top of a tower from the bottom of a building is twice that from its top. What is the height of the building, if the height of the tower is 75 m and the angle of elevation of the top of the tower from the bottom of the building is 60° ?
(a) 25 m (b) 37.5 m
(c) 50 m (d) 60 m
56. The shadow of a tower is 15 m when the Sun's altitude is 30° . What is the length of the shadow when the Sun's altitude is 60° ?
(a) 3 m (b) 4 m (c) 5 m (d) 6 m
57. A man is watching from the top to tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at distance of 60 m from the bottom of tower. After 5 s, the angle of depression becomes 30° . What is the approximate speed of the boat assuming that it is running in still water?
(a) 31.5 km/h (b) 36.5 km/h
(c) 38.5 km/h (d) 40.5 km/h
58. Suppose the angle of elevation of the top of a tree at a point E due East of the tree is 60° and that at a point F due West of the tree is 30° . If the distance between the points E and F is 160 ft, then what is the height of the tree?
(a) $40\sqrt{3}$ ft (b) 60 ft
(c) $\frac{40}{\sqrt{3}}$ ft (d) 23 ft
59. The length of the shadow of a person s cm tall when the angle of elevation of the Sun is α is p cm. It is q cm when the angle of elevation of the Sun is β . Which one of the following is correct when $\beta = 3\alpha$?
(a) $p - q = s \left(\frac{\tan \alpha + \tan 3\alpha}{\tan 3\alpha \tan \alpha} \right)$
(b) $p - q = s \left(\frac{\tan 3\alpha - \tan \alpha}{3 \tan \alpha \tan \alpha} \right)$
(c) $p - q = s \left(\frac{\tan 3\alpha - \tan \alpha}{\tan 3\alpha \tan \alpha} \right)$
(d) $p - q = s \left(\frac{\tan 2\alpha}{\tan 3\alpha \tan \alpha} \right)$
60. A radio transmitter antenna of height 100 m stands at the top of a tall building. At a point on the ground, the angle of elevation of bottom of the antenna is 45° and that of top of antenna is 60° . What is the height of the building?
(a) 100 m (b) 50 m
(c) $50(\sqrt{3}+1)$ m
(d) $50(\sqrt{3}-1)$ m
61. The angle of elevation of the top of an unfinished pillar at a point 150 m from its base is 30° . If the angle of elevation at the same point is to be 45° , then the pillar has to be raised to a height of how many metres?
(a) 59.4 m (b) 61.4 m
(c) 62.4 m (d) 63.4 m
62. A round balloon of unit radius subtends an angle of 90° at the eye of an observer standing at a point, say A. What is the distance of the centre of the balloon from the point A?
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $\frac{1}{2}$
63. Person standing at the end of the shadow is $\frac{1}{\sqrt{3}}$ times the length of the pole. At what angle of elevation will the man see the Sun?
(a) 60° (b) 30° (c) 45° (d) 15°
64. The angle of depression of vertex of a regular hexagon lying in a horizontal plane, from the top of tower of height 75 m located at the centre of the regular hexagon is 60° . What is the length of each side of the hexagon?
(a) $50\sqrt{3}$ m (b) 75 m
(c) $25\sqrt{3}$ m (d) 25 m
65. Two houses are collinear with the base of a tower and are at distance 3 m and 12 m from the base of the tower. The angles of elevation from these two houses of the top of the tower are complementary. What is the height of the tower?
(a) 4 m (b) 6 m
(c) 7.5 m (d) 36 m
66. The angle of elevation from the bank of a river of the top of a tree standing on the opposite bank is 60° . The angle of elevation becomes 30° when observed from a point 40 m backwards in a direction perpendicular to the length of the river. What is the width of the river?
(a) 10 m (b) 20 m
(c) 30 m (d) 40 m

ANSWER KEY

1. (a)	8. (c)	15. (b)	22. (b)	29. (d)	36. (a)	43. (c)	49. (a)	55. (c)	61. (d)
2. (c)	9. (d)	16. (b)	23. (d)	30. (b)	37. (a)	44. (c)	50. (b)	56. (c)	62. (b)
3. (c)	10. (d)	17. (a)	24. (b)	31. (d)	38. (b)	45. (a)	51. (c)	57. (a)	63. (a)
4. (a)	11. (c)	18. (a)	25. (c)	32. (c)	39. (b)	46. (a)	52. (c)	58. (a)	64. (c)
5. (b)	12. (b)	19. (b)	26. (b)	33. (b)	40. (c)	47. (d)	53. (a)	59. (c)	65. (b)
6. (b)	13. (b)	20. (a)	27. (d)	34. (d)	41. (c)	48. (c)	54. (b)	60. (c)	66. (b)
7. (a)	14. (b)	21. (a)	28. (b)	35. (d)	42. (c)				

SOLUTION

1. (a)



Shortcut approach

Ist Case:

$$\tan \theta = \frac{AB}{BC} = \frac{\text{Perpendicular}}{\text{Base}} = \frac{1}{5}$$

IInd Case:

$$\sec \alpha = \frac{AD}{BD} = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\sqrt{193}}{12}$$

In $\triangle ABD$

$$\text{Hypotenuse} = \sqrt{193}$$

$$\text{Base} = 12$$

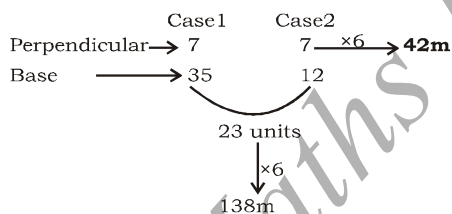
Then perpendicular = 7

(By Pythagoras theorem)

In Case I Perpendicular is 1.

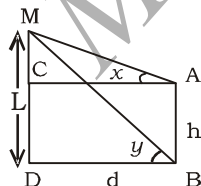
So equal this

$$\tan \theta = \frac{1 \times 7}{5 \times 7} = \frac{7 \leftarrow \text{Perpen.}}{35 \leftarrow \text{Base}}$$



$$AB = 7 \times 6 = 42\text{m}$$

2. (c)



AB = tree 'h'

MD = Building 'L'

DB = CA = 'd'

In $\triangle MCA$,

$$\tan x = \frac{MC}{AC} = \frac{L-h}{d}$$

$$\Rightarrow d = \frac{L-h}{\tan x}$$

$$\Rightarrow d = (L-h) \cot x$$

.....(i)

In $\triangle MDB$,

$$\tan y = \frac{L}{d} = \frac{MD}{DB}$$

$$d = L \cot y$$

.....(ii)

from equation (i) and (ii)

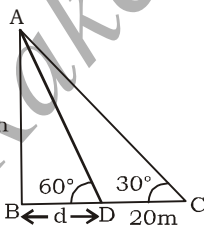
$$(L-h) \cot x = L \cot y$$

$$L \cot x - h \cot x = L \cot y$$

$$h \cot x = L (\cot x - \cot y)$$

$$L = \frac{h \cot x}{\cot x - \cot y}$$

3. (c)



AB = 'h' metre

In $\triangle ABC$

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\Rightarrow \frac{h}{(d+20)}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{(d+20)}$$

$$\sqrt{3}h = d + 20$$

.....(i)

In $\triangle ABD$

$$\tan 60^\circ = \frac{AB}{BD} = \frac{h}{d}$$

$$\sqrt{3} = \frac{h}{d}$$

$$h = \sqrt{3}d$$

$$d = \frac{h}{\sqrt{3}}$$

.....(ii)

Put the value of d in equation (i)

$$\sqrt{3}h = \frac{h}{\sqrt{3}} + 20$$

$$\sqrt{3}h = \frac{h + 20\sqrt{3}}{\sqrt{3}}$$

$$3h = h + 20\sqrt{3}$$

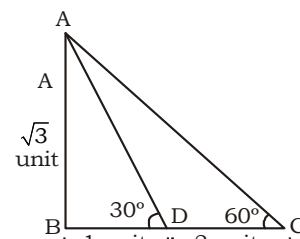
$$2h = 20\sqrt{3}$$

$$h = \frac{20\sqrt{3}}{2}$$

$$h = 10\sqrt{3} \text{ metre}$$

Alternate:-

As we know the ratio of



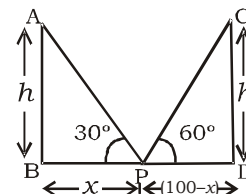
$$2 \text{ unit} = CD = 20 \text{ metres}$$

$$1 \text{ unit} = 10 \text{ metres}$$

$$\sqrt{3} \text{ units} = AB = \sqrt{3} \times 10$$

$$= 10\sqrt{3} \text{ metres}$$

4. (a)



$$BD = 100$$

AB = CD = 'h' metre (Height of pole)

In $\triangle ABP$,

$$\Rightarrow \tan 30^\circ = \frac{h}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \sqrt{3}h = x$$

.....(i)

In $\triangle CDP$

$$\tan 60^\circ = \frac{h}{(100-x)}$$

$$\Rightarrow \sqrt{3}(100-x) = h$$

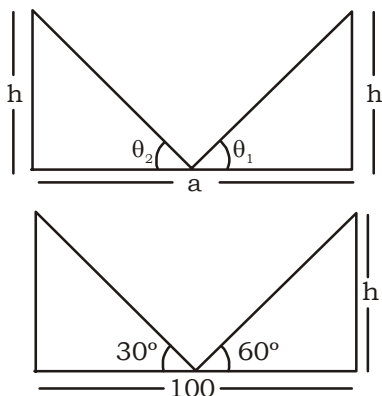
$$\Rightarrow \sqrt{3}(100-\sqrt{3}h) = h$$

(Put the value of x from equation (i))

$$\Rightarrow 100\sqrt{3} - 3h = h \Rightarrow 4h = 100\sqrt{3}$$

$$h = 25\sqrt{3} \text{ metre}$$

Alternate:-

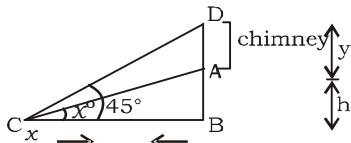


$$h = \frac{a}{\cot \theta_2 + \cot \theta_1}$$

$$h = \frac{100}{\sqrt{3} + \frac{1}{\sqrt{3}}}$$

$$\Rightarrow 25\sqrt{3} \text{ metre}$$

5. (b) $AB = \text{Building} = h \text{ metre}$



$AD = \text{chimney} = 'y' \text{ meter}$

In $\triangle DCB$

$$\tan 45^\circ = \frac{DB}{BC} \Rightarrow 1 = \frac{h+y}{BC} \Rightarrow BC = h+y \quad \dots\dots(i)$$

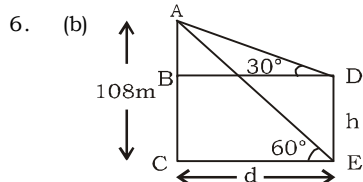
In $\triangle ACB$

$$\tan x^\circ = \frac{AB}{BC} \Rightarrow \tan x = \frac{h}{BC} \Rightarrow BC = h \cot x \dots(ii)$$

from equation (i) and (ii)

$$\Rightarrow h+y = h \cot x$$

$$\Rightarrow y = (h \cot x - h) \text{ metre}$$



In $\triangle ACE$

$$\tan 60^\circ = \frac{AC}{CE}$$

$$\frac{\sqrt{3}}{1} = \frac{AC}{CE} = AC : CE = \sqrt{3} : 1 \dots\dots(i)$$

In $\triangle ABD$

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BD} \Rightarrow AB : BD = 1 : \sqrt{3} \quad \dots(ii)$$

Since $BD = CE$

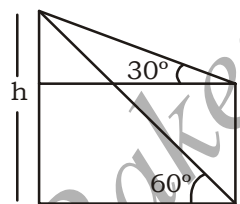
$\therefore AC : CE : AB$

$$\begin{array}{ccc} \text{equation(I)} \rightarrow \sqrt{3} & : & 1 \\ \text{equation(II)} \rightarrow & \sqrt{3} & : 1 \\ \hline & 3 & : 1 \\ & \downarrow \times 36 & \downarrow \times 36 \\ \text{actual height} \rightarrow 108 & & 36 \end{array}$$

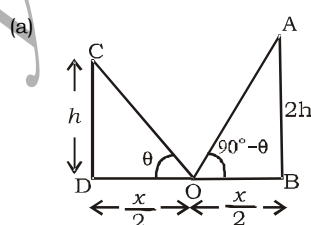
$$\Rightarrow DE = AC - AB = 108 - 36 = 72 \text{ metre}$$

Alternate:

In such question when angles are 30° and 60°



$$x = \frac{2h}{3} = \frac{2 \times 108}{3} = 72 \text{ cm}$$



From figure,

$$OB = OD = \frac{x}{2}$$

In $\triangle OCD$,

$$\tan \theta = \frac{h}{\frac{x}{2}} \Rightarrow \frac{2h}{x} \quad \dots\dots(i)$$

In $\triangle AOB$

$$\tan(90^\circ - \theta) = \frac{AB}{OB}$$

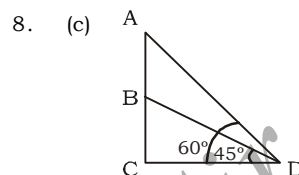
$$\Rightarrow \cot \theta = \frac{2h}{\frac{x}{2}} = \frac{4h}{x} \quad \dots\dots(ii)$$

Multiplying both equations,

$$\tan \theta \cdot \cot \theta = \frac{2h}{x} \times \frac{4h}{x}$$

$$\Rightarrow x^2 = 8h^2$$

$$\Rightarrow h^2 = \frac{x^2}{8} \Rightarrow h = \frac{x}{2\sqrt{2}} \text{ metre}$$



$AC = 5000 \text{ m (given)}$

In $\triangle ACD$,

$$\tan 60^\circ = \frac{AC}{CD}$$

$$\sqrt{3} = \frac{AC}{CD} \Rightarrow AC : CD = \sqrt{3} : 1$$

In $\triangle BCD$,

$$\tan 45^\circ = \frac{BC}{CD}$$

$$1 = \frac{BC}{CD} = BC : CD = 1 : 1 \quad \dots\dots(ii)$$

Now,

$$\begin{array}{ccc} BC & : & CD & : & AC \\ 1 & : & 1 & : & \sqrt{3} \\ & & 1 & : & \sqrt{3} \\ 1 & : & 1 & : & \sqrt{3} \end{array}$$

$AB = AC - BC$

$$= (\sqrt{3} - 1) \text{ units}$$

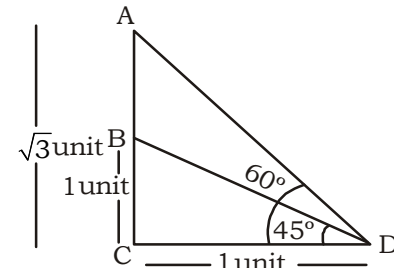
$$\sqrt{3} \text{ units} = 5000 \text{ m}$$

$$1 \text{ unit} = \frac{5000}{\sqrt{3}} \text{ m}$$

$$AB = (\sqrt{3} - 1) \text{ units} = \frac{5000}{\sqrt{3}} (\sqrt{3} - 1)$$

$$= 5000 \left[1 - \frac{1}{\sqrt{3}} \right] \text{ m}$$

Alternate:



$$AC = \sqrt{3} \text{ unit}$$

$$BC = 1 \text{ unit}$$

$$AB = (\sqrt{3} - 1) \text{ unit}$$

According to question,

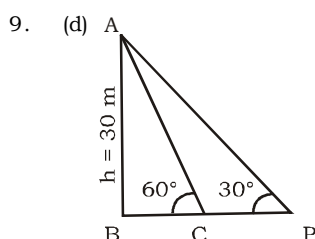
$$AC = \sqrt{3} \text{ unit} = 5000 \text{ m}$$

$$1 \text{ unit} = \frac{5000}{\sqrt{3}} \text{ m}$$

So, the difference between the aeroplane = $AB = (\sqrt{3} - 1) \text{ unit}$

$$(\sqrt{3} - 1) \text{ unit} = \frac{5000}{\sqrt{3}} (\sqrt{3} - 1) \text{ m}$$

$$AB = 5000 \left(1 - \frac{1}{\sqrt{3}} \right) \text{ m}$$



$$h = 30 \text{ m}$$

$$PC = ?$$

In $\triangle ABP$

$$\tan 30^\circ = \frac{AB}{BP}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BP} \Rightarrow AB : BP = 1 : \sqrt{3} \dots (i)$$

In $\triangle ABC$

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\frac{\sqrt{3}}{1} = \frac{AB}{BC} \Rightarrow AB : BC = \sqrt{3} : 1 \dots (ii)$$

$$\begin{array}{ccc} BP & : & AB \\ \sqrt{3} & : & 1 \end{array}$$

$$\begin{array}{ccc} \sqrt{3} & : & 1 \\ 3 & : & \sqrt{3} \end{array}$$

Now

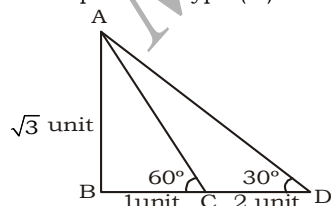
$$AB = \sqrt{3} \text{ units} = 30 \text{ metre}$$

$$1 \text{ unit} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3} \text{ metre}$$

$$PC = 3 - 1 = 2 \text{ units}$$

$$= 10\sqrt{3} \times 2 = 20\sqrt{3} \text{ metre}$$

Alternate:
As we proof in Type (iii)



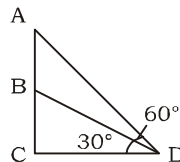
$$AB = 30 \text{ m} = \sqrt{3} \text{ units}$$

$$1 \text{ unit} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3} \text{ metre}$$

$$\text{So, } PC = 2 \text{ units}$$

$$\Rightarrow 2 \times 10\sqrt{3} = 20\sqrt{3} \text{ metre}$$

$$10. (d) \quad BC = 3125 \text{ m}$$



In $\triangle ACD$

$$\tan 60^\circ = \frac{AC}{DC}$$

$$\frac{\sqrt{3}}{1} = \frac{AC}{DC}$$

$$AC : DC = \sqrt{3} : 1 \dots (i)$$

In $\triangle DCB$

$$\tan 30^\circ = \frac{BC}{DC}$$

$$\frac{1}{\sqrt{3}} = \frac{BC}{DC}$$

$$BC : DC = 1 : \sqrt{3} \dots (ii)$$

Now,

$$\begin{array}{ccc} AC & : & DC \\ \sqrt{3} & : & 1 \end{array}$$

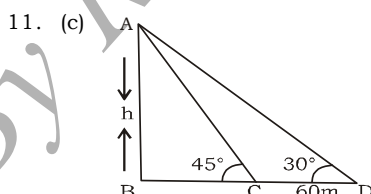
$$\begin{array}{ccc} \sqrt{3} & : & 1 \\ 3 & : & \sqrt{3} \end{array}$$

$$BC = 1 \text{ unit} = 3125 \text{ m}$$

$$AB = AC - BC$$

$$\Rightarrow 3 - 1 = 2 \text{ units}$$

$$AB = 2 \times 3125 = 6250 \text{ m}$$



$$h = \text{height}$$

In $\triangle ABC$

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\frac{1}{1} = \frac{AB}{BC} = AB : BC = 1 : 1 \dots (i)$$

In $\triangle ABD$

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BD}$$

$$\Rightarrow AB : BD = 1 : \sqrt{3} \dots (ii)$$

Now,

$$\begin{array}{ccc} BD & : & AB \\ 1 & : & 1 \end{array}$$

$$\sqrt{3} : 1$$

$$\sqrt{3} : 1 : 1$$

$$CD = BD - BC$$

$$CD = \sqrt{3} - 1$$

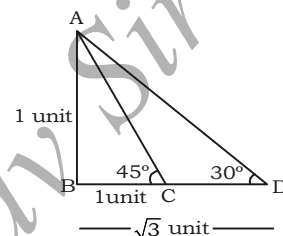
$$(\sqrt{3} - 1) \text{ units} = 60$$

$$1 \text{ unit} = \frac{60}{\sqrt{3} - 1}$$

$$h = \frac{60}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$h = 30(\sqrt{3} + 1) \text{ m}$$

Alternate:



$$AB = 1 \text{ unit}$$

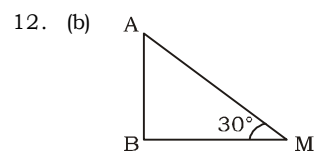
$$BC = 1 \text{ unit}$$

$$CD = (\sqrt{3} - 1) \text{ unit}$$

$$CD = (\sqrt{3} - 1) \text{ unit} = 60 \text{ m}$$

$$\text{Height (AB)} = \frac{60}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= 30(\sqrt{3} + 1) \text{ m}$$



MAB was straight earlier

$$AB + AM = 15 \text{ ft}$$

In $\triangle ABM$

$$\tan 30^\circ = \frac{AB}{BM}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BM}$$

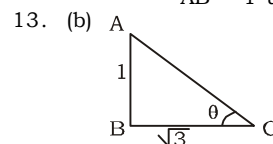
$$\text{If } AB = 1$$

$$\text{and } BM = \sqrt{3}$$

then $AM = 2$ (By pythagoras theorem)

$$AB + AM = 1 + 2 = 3 \text{ units} = 15 \text{ ft}$$

$$AB = 1 \text{ unit} = 5 \text{ ft}$$



In $\triangle ABC$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

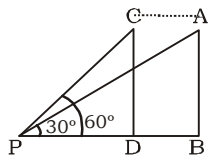
$$\tan \theta = \tan 30^\circ \Rightarrow \theta = 30^\circ$$

14. (b) $\begin{array}{ccc} \text{Height} & & \text{Shadow} \\ 6\text{ft} & & 4\text{ft} \\ 3 & : & 2 \end{array}$

So height of pole will be in same ratio.

$$= 50 \times \frac{3}{2} = 75 \text{ ft}$$

15. (b)



$AB = CD = 1500\sqrt{3}$ m
(height of aeroplane)

In $\triangle PDC$

$$\tan 60^\circ = \frac{CD}{PD}$$

$$\sqrt{3} = \frac{CD}{PD} \Rightarrow CD:PD = \sqrt{3}:1 \dots (i)$$

In $\triangle PBA$

$$\tan 30^\circ = \frac{AB}{PB}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{PB} \Rightarrow AB:PB = 1:\sqrt{3} \dots (ii)$$

$AC = BD$ and $AB = CD$

$PD:AB:PB$

$1:\sqrt{3}$

$1:\sqrt{3}$

$1:\sqrt{3}:3$

$DB = PB - PD$

$= 3 - 1 = 2$ units

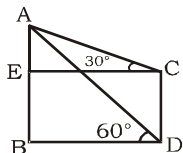
$AB = \sqrt{3}$ units $= 1500\sqrt{3}$ m

$\Rightarrow 1$ unit $= 1500$ m

$CA = DB \Rightarrow 2$ units $= 3000$ metre

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{3000}{15} = 200 \text{ m/s}$$

16. (b)



AB and CD are temples

$BD =$ width of river

$AB = 54$ m

In $\triangle AEC$

$$\tan 30^\circ = \frac{AE}{EC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AE:EC = 1:\sqrt{3}$$

In $\triangle ABD$

$$\tan 60^\circ = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{AB}{BD} \Rightarrow AB:BD = \sqrt{3}:1 \dots (ii)$$

$EB = CD$ and $EC = BD$

Now,

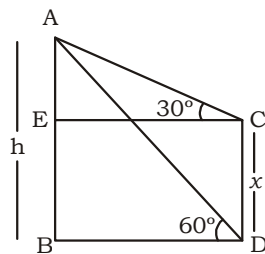
$AB:BD:AE$
 $\sqrt{3}:1:1$

$\sqrt{3}:1$

$3:\sqrt{3}:1$

$$\begin{aligned} CD &= AB - AE \\ &= 3 - 1 = 2 \text{ units} \\ AB &= 3 \text{ units} \times 18 = 54 \text{ m} \\ CD &= 2 \text{ units} \times 18 = 36 \text{ m} \end{aligned}$$

Alternate:



$AB = h = 54$ m

$CD = x$

$$x = \frac{2h}{3} = \frac{2 \times 54}{3} = 36 \text{ m}$$

Alternate:-

$$H = \frac{h \cot \theta}{\cot \theta - \cot \alpha}$$

Where, $\theta =$ smaller angle

$\alpha =$ greater angle

$$54 = \frac{h \cot 30^\circ}{\cot 30^\circ - \cot 60^\circ}$$

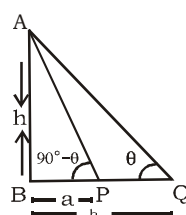
$$54 = \frac{h\sqrt{3}}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

$$54 = \frac{h\sqrt{3}}{\frac{3-1}{\sqrt{3}}}$$

$$54 = \frac{3h}{2}$$

$$h = \frac{54 \times 2}{3} = 36 \text{ m}$$

17. (a)



AB is tower

$\therefore \angle AQB = \theta$ & $\angle APB = 90^\circ - \theta$

$PB = a$, $BQ = b$

In $\triangle AQB$

$$\tan \theta = \frac{AB}{BQ}$$

$$\tan \theta = \frac{h}{b}$$

In $\triangle APB$

$$\tan(90^\circ - \theta) = \frac{h}{PB}$$

.....(i)

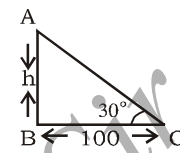
$$\Rightarrow \cot \theta = \frac{h}{a} \dots (ii)$$

By multiplying both equation

$$\tan \theta \cdot \cot \theta = \frac{h}{b} \times \frac{h}{a}$$

$$h^2 = ab \Rightarrow h = \sqrt{ab}$$

18. (a)



In $\triangle ABC$

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC}$$

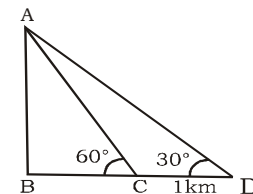
$AB:BC = 1:\sqrt{3}$

$\downarrow \quad \downarrow$
 $\frac{100}{\sqrt{3}} \text{ m} \quad 100 \text{ m}$

$$\therefore \text{Height of tower} = \frac{100}{\sqrt{3}} \text{ m}$$

19. (b) It should be on circumcentre.

20. (a)



$AB =$ height of balloon

In $\triangle ABC$

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{BC} \Rightarrow AB:BC = \sqrt{3}:1$$

In $\triangle ABD$

$$\tan 30^\circ = \frac{AB}{BD} \Rightarrow AB:BD = 1:\sqrt{3}$$

Now,

$BC:AB:BD$
 $1:\sqrt{3}$

$1:\sqrt{3}$

$1:\sqrt{3}:3$

$CD = BD - BC$

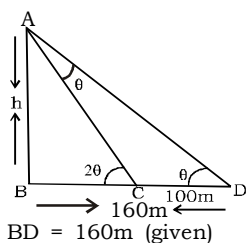
$= 3 - 1 = 2$ units

2 units $= 1$ km

$$1 \text{ unit} = \frac{1}{2}$$

$$AB = \sqrt{3} \text{ unit} = \frac{1}{2} \times \sqrt{3} = \frac{\sqrt{3}}{2} \text{ km}$$

21. (a)



In $\triangle ACD$

exterior $\angle ACB = \angle CAD + \angle ADC$

$$20 = \angle CAD + \theta$$

$$\angle CAD = \theta$$

$$\therefore AC = CD$$

AC = 100 m

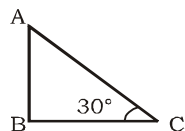
In $\triangle ABC$

$$AC = 100 \text{ m}$$

$$BC = 160 - 100 = 60 \text{ m}$$

Then $AB = 80 \text{ m}$ (By pythagoras theorem)

22. (b)



$AB = \text{Tower}$

$BC = 50 \text{ m}$

(Given)

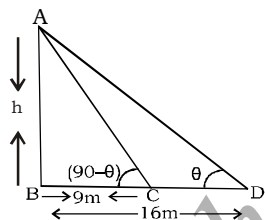
In $\triangle ABC$

$$\tan \theta = \frac{AB}{BC}$$

$$\tan 30^\circ = \frac{AB}{50}$$

$$AB = \frac{50}{\sqrt{3}} \text{ m}$$

23. (d)



$AB = \text{Pillar}$

$BC = 9 \text{ metre}$

$BD = 16 \text{ metre}$

$$\angle ADB = \theta$$

In $\triangle ABC$

$$\tan(90^\circ - \theta) = \frac{AB}{BC}$$

$$\cot \theta = \frac{AB}{BC} = \frac{h}{9} \quad \dots(i)$$

In $\triangle ABD$

$$\tan \theta = \frac{h}{16} \quad \dots(ii)$$

By multiplying equation (i) and (ii)

$$\tan \theta \cdot \cot \theta = \frac{h}{9} \times \frac{h}{16}$$

$$\Rightarrow \frac{h^2}{144} = 1$$

$$\Rightarrow h^2 = 144$$

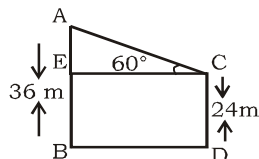
$$h = \sqrt{144}$$

$$h = 12 \text{ metre}$$

Alternate:-

$$h = \sqrt{16 \times 9} = 12 \text{ m.}$$

24. (b)



$AC = \text{wire}$

AB and CD are two poles

In $\triangle AEC$

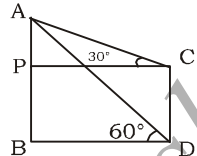
$$\sin 60^\circ = \frac{AE}{AC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{12}{AC}$$

$$(AE = AB - CD = 36 - 24 = 12 \text{ m})$$

$$AC = \frac{24}{\sqrt{3}} = 8\sqrt{3} \text{ m}$$

25. (c)



$AB = \text{hill} = 200 \text{ metre}$

$CD = \text{tower}$

In $\triangle APC$

$$\tan 30^\circ = \frac{AP}{PC}$$

$$\frac{1}{\sqrt{3}} = \frac{AP}{PC} \Rightarrow AP : PC = 1 : \sqrt{3} \quad \dots(i)$$

In $\triangle ABD$

$$\tan 60^\circ = \frac{AB}{BD}$$

$$\sqrt{3} = \frac{AB}{BD} \Rightarrow AB : BD = \sqrt{3} : 1 \quad \dots(ii)$$

$$PB = CD \text{ and } PC = BD$$

Now

$$AB : BD : AP$$

$$\sqrt{3} : 1$$

$$\sqrt{3} : 1$$

$$3 : \sqrt{3} : 1$$

$$CD = PB \Rightarrow AB - AP$$

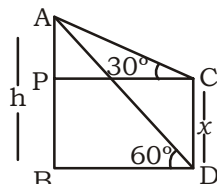
$$CD = 3 - 1 = 2 \text{ units}$$

$$AB = 3 \text{ units} = 200 \text{ metre}$$

$$CD = 2 \text{ units} = \frac{200}{3} \times 2$$

$$= 133 \frac{1}{3} \text{ metre}$$

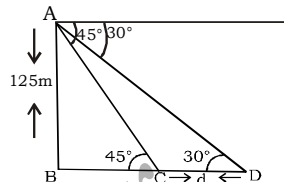
Alternate:-



$$AB = h = 200z$$

$$CD = x = \frac{2h}{3} = \frac{2 \times 200}{3} = 133 \frac{1}{3} \text{ m}$$

26. (b)



$AB = \text{Tower}$

$$\text{In } \triangle ABC \tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{AB}{BC} \Rightarrow AB : BC = 1 : 1 \quad \dots(i)$$

In $\triangle ABD$,

$$\tan 30^\circ = \frac{AB}{BD} \Rightarrow AB : BD = 1 : \sqrt{3} \quad \dots(ii)$$

Now,

$$BC : AB : BD$$

$$1 : 1$$

$$1 : \sqrt{3}$$

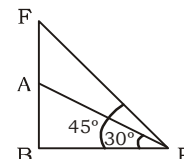
$$1 : 1 : \sqrt{3}$$

$$CD = BD - BC = (\sqrt{3} - 1) \text{ units}$$

$$AB = 1 \text{ unit} = 125 \text{ metre}$$

$$CD = (\sqrt{3} - 1) \text{ units} = 125(\sqrt{3} - 1) \text{ metre}$$

27. (d)



$AB = \text{building} = 10 \text{ m}$

In $\triangle ABP$

$$\tan 30^\circ = \frac{AB}{BP}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BP} \Rightarrow AB : BP$$

$$= 1 : \sqrt{3} \quad \dots(i)$$

In $\triangle FBP$

$$\tan 45^\circ = \frac{FB}{BP}$$

$$1 = \frac{FB}{BP} \Rightarrow FB : BP$$

$$= 1 : 1 \quad \dots(ii)$$

Now,

$$AB : BP : FB$$

$$1 : \sqrt{3}$$

$$1 : \sqrt{3} : \sqrt{3}$$

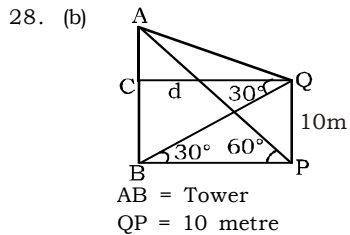
$$\times 10$$

$$10 \text{ m}$$

$$\times 10$$

$$10\sqrt{3} \text{ m}$$

$$\begin{aligned}
 FB &= 17.32 \text{ m} \\
 FA &= FB - AB \\
 &= 17.32 - 10 \\
 &= 7.32 \text{ metre}
 \end{aligned}$$



In $\triangle QBP$, $\tan 30^\circ = \frac{QP}{PB}$

$$\frac{1}{\sqrt{3}} = \frac{QP}{PB} \Rightarrow QP : PB = 1 : \sqrt{3} \dots\dots(i)$$

In $\triangle ABP$

$$\tan 60^\circ = \frac{AB}{BP}$$

$$\sqrt{3} = \frac{AB}{BP} \Rightarrow AB : BP = \sqrt{3} : 1 \dots\dots(ii)$$

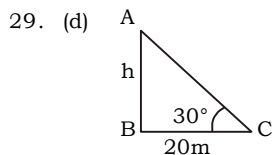
$$CB = QP \text{ and } CQ = BP$$

Now,

$$AB : BP : CB$$

$$\sqrt{3} : 1 : \sqrt{3}$$

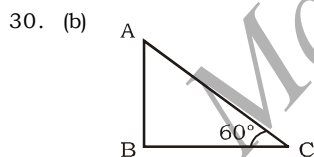
$$\begin{array}{ccc} 3 & : & 1 \\ \downarrow \times 10 & & \downarrow \times 10 \\ 30 \text{ metre} & & 10 \text{ metre} \end{array}$$



In $\triangle ABC$

$$\frac{AB}{BC} = \tan 30^\circ \Rightarrow \frac{h}{20} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{20}{\sqrt{3}} \text{ m}$$



$$AC = \text{Ladder}$$

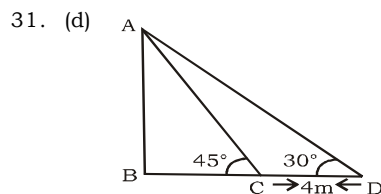
$$BC = 6.5 \text{ metres}$$

In $\triangle ABC$

$$\cos 60^\circ = \frac{BC}{AC}$$

$$\frac{1}{2} = \frac{6.5}{AC} \text{ m}$$

$$AC = 13 \text{ m}$$



AB = pole

In $\triangle ABC$

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{AB}{BC} \Rightarrow AB : BC = 1 : 1 \dots\dots(i)$$

In $\triangle ABD$

$$\tan 30^\circ = \frac{AB}{BD} \Rightarrow AB : BD = 1 : \sqrt{3} \dots\dots(ii)$$

$$\begin{array}{ccc} BC & : & AB : BD \\ 1 & : & 1 : \sqrt{3} \end{array}$$

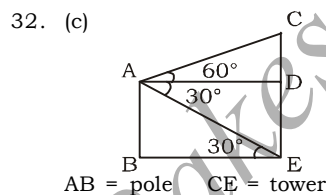
$$\begin{array}{ccc} 1 & : & 1 : \sqrt{3} \\ 1 & : & 1 : \sqrt{3} \end{array}$$

$$CD = BD - BC$$

$$= \sqrt{3} - 1 = \sqrt{3} - 1 \text{ units} = 4 \text{ m}$$

$$AB = 1 \text{ unit} = \frac{4}{\sqrt{3} - 1}$$

$$= 2(\sqrt{3} + 1) = 5.464 \text{ m}$$



In $\triangle ABE$

$$\tan 30^\circ = \frac{AB}{BE}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BE} \Rightarrow AB : BE = 1 : \sqrt{3} \dots\dots(i)$$

In $\triangle ACD$

$$\tan 60^\circ = \frac{CD}{AD}$$

$$\frac{\sqrt{3}}{1} = \frac{CD}{AD} \Rightarrow CD : AD = \sqrt{3} : 1 \dots\dots(ii)$$

$$AD = BE \text{ and } AB = DE$$

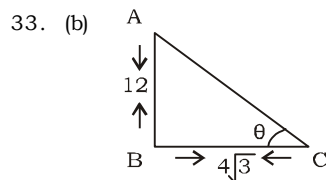
Now,

$$\begin{array}{ccc} AB & : & BE : CD \\ 1 & : & \sqrt{3} \end{array}$$

$$\begin{array}{ccc} 1 & : & \sqrt{3} \\ 1 & : & \sqrt{3} \end{array}$$

$$\begin{array}{ccc} 1 & : & \sqrt{3} : 3 \\ \downarrow \times 10 & & \downarrow \times 10 \\ 10 \text{ metre} & & 30 \text{ metre} \end{array}$$

$$CE = CD + DE = 30 + 10 = 40 \text{ metre}$$



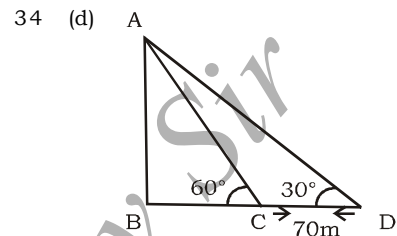
In $\triangle ABC$

$$\tan \theta = \frac{AB}{BC} = \frac{12}{4\sqrt{3}}$$

$$\tan \theta = \frac{3}{\sqrt{3}}$$

$$\tan \theta = \sqrt{3} = \tan 60^\circ$$

$$\theta = 60^\circ$$



In $\triangle ACD$

$$\angle ACB = \angle CAD + \angle ADC$$

$$60^\circ = \angle CAD + 30^\circ$$

$$\angle CAD = 30^\circ$$

So,

$$AC = CD$$

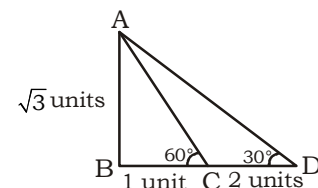
$$AC = 70 \text{ m}$$

$$\operatorname{cosec} 60^\circ = \frac{AC}{AB}$$

$$\frac{2}{\sqrt{3}} = \frac{70}{AB}$$

$$AB = 35\sqrt{3} \text{ m}$$

Alternate:-

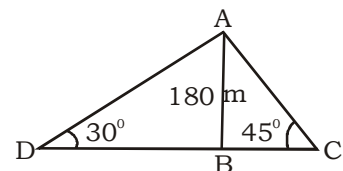


$$CD = 2 \text{ unit} = 70 \text{ m}$$

$$\Rightarrow 1 \text{ unit} = 35 \text{ m}$$

$$AB = 35\sqrt{3} \text{ m}$$

35. (d)



$$AB = 180 \text{ m}$$

In $\triangle ABC$

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{AB}{BC} \Rightarrow AB : BC = 1 : 1 \dots\dots(1)$$

In $\triangle ABD$

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BD}$$

$$AB : BD = 1 : \sqrt{3} \quad \dots(ii)$$

Now,

$$BC : AB : BD$$

$$1 : 1$$

$$1 : \sqrt{3}$$

to make equal ratio

$$BC : AB : BD$$

$$1 : 1 : \sqrt{3}$$

$$CD = BD + BC$$

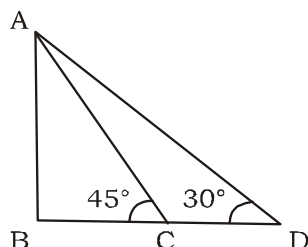
$$CD = (\sqrt{3} + 1) \text{ units}$$

$$AB = 1 \text{ unit} = 180 \text{ m}$$

$$CD = (\sqrt{3} + 1) \text{ units}$$

$$= 180 (\sqrt{3} + 1) \text{ m}$$

36. (a)



AB = height of peak = 300 m

CD = length of Bridge

In $\triangle ABC$

$$\tan 45^\circ = \frac{AB}{BC}$$

$$1 = \frac{AB}{BC} = AB : BC = 1 : 1$$

In $\triangle ABD$

$$\tan 30^\circ = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BD} \Rightarrow AB : BD = 1 : \sqrt{3}$$

Now,

$$BC : AB : BD$$

$$1 : 1 : \sqrt{3}$$

$$1 : 1 : \sqrt{3}$$

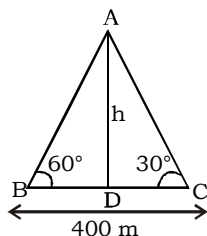
$$CD = BD - BC$$

$$CD = (\sqrt{3} - 1) \text{ unit}$$

$$AB = 1 \text{ unit} = 300 \text{ metre}$$

$$(\sqrt{3} - 1) \text{ units} = 300 (\sqrt{3} - 1) \text{ metre}$$

37. (a)



$$BC = 400 \text{ metres}$$

In $\triangle ABD$

$$\tan 60^\circ = \frac{AD}{BD}$$

$$\frac{\sqrt{3}}{1} = \frac{AD}{BD} \Rightarrow AD : BD = \sqrt{3} : 1 \dots(i)$$

In $\triangle ADC$

$$\tan 30^\circ = \frac{AD}{DC}$$

$$\frac{1}{\sqrt{3}} = \frac{AD}{DC} \Rightarrow AD : DC = 1 : \sqrt{3} \dots(ii)$$

Now,

$$BD : AD : DC$$

$$1 : \sqrt{3}$$

$$1 : \sqrt{3}$$

$$1 : \sqrt{3} : 3$$

$$BC = BD + DC$$

$$= 1 + 3 = 4 \text{ units.}$$

$$4 \text{ units} = 400 \text{ m}$$

$$1 \text{ unit} = 100 \text{ m}$$

$$AD = \sqrt{3} \text{ unit}$$

$$= 100\sqrt{3} = 100 \times 1.732 = 173.2 \text{ m}$$

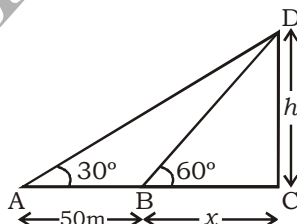
38. (b) Let h be the height of the tower and BC be x m.

$$\text{In } \triangle BCD, \tan 60^\circ = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = x\sqrt{3}$$

... (i)



Now in $\triangle ACD$, $\tan 30^\circ$

$$= \frac{h}{50 + x}$$

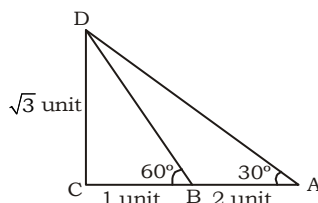
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{50 + x}$$

$$\Rightarrow 50 + x = 3x$$

$$\Rightarrow x = 25 \text{ m}$$

$$\therefore h = 25\sqrt{3} \quad [\text{from eq. (i)}]$$

Alternate:-

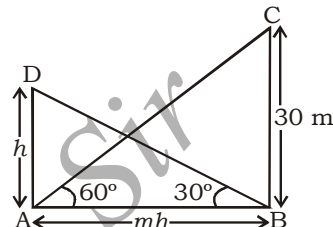


$$2 \text{ units} = 50 \text{ m}$$

$$1 \text{ unit} = 25 \text{ m}$$

$$CD = \sqrt{3} = 25\sqrt{3} \text{ m.}$$

39. (b) Let h be the height of shorter tower, then the distance between the two towers is mh m.

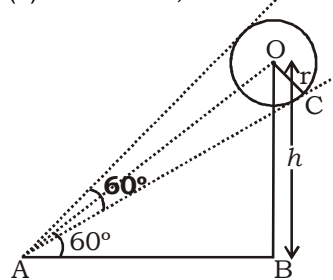


$$\text{In } \triangle ABD, \tan 30^\circ = \frac{h}{mh}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1}{m}$$

$$\therefore m = \sqrt{3}$$

40. (c) In $\triangle ABO$,



$$\sin 60^\circ = \frac{OB}{AO}$$

$$\Rightarrow AO = \frac{OB}{\sin 60^\circ} \quad \dots(i)$$

Now, In $\triangle AOC$,

$$\sin \frac{60^\circ}{2} = \frac{OC}{AO}$$

$$\Rightarrow AO = \frac{OC}{\sin 30^\circ} \quad \dots(ii)$$

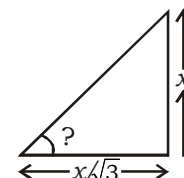
From Eqs. (i) and (ii),

$$\frac{OB}{\sin 60^\circ} = \frac{OC}{\sin 30^\circ}$$

$$\frac{h}{\frac{\sqrt{3}}{2}} = \frac{r}{\frac{1}{2}}$$

$$\therefore h = \sqrt{3}r$$

41. (c) Here, θ is the angle of elevation,



$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{x}{\frac{x}{\sqrt{3}}} = \frac{\sqrt{3}x}{x} = \sqrt{3}$$

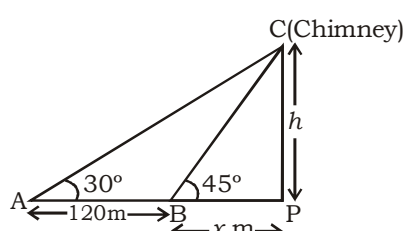
$$\text{Here, } \tan \theta = \sqrt{3}$$

$$\therefore \theta = 60^\circ \quad (\because \tan 60^\circ = \sqrt{3})$$

42. (c) Let h be the height of the chimney.

In $\triangle BPC$,

$$\tan 45^\circ = \frac{h}{x} = 1 \Rightarrow h = x \quad \dots(i)$$



Now, In $\triangle APC$,

$$\tan 30^\circ = \frac{h}{120+x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{h}{120+h} = \frac{1}{\sqrt{3}} \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \sqrt{3}h = 120 + h$$

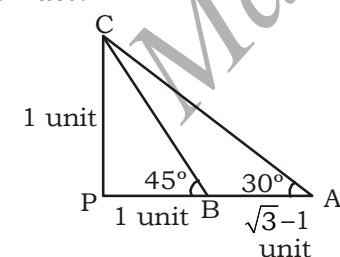
$$\Rightarrow \sqrt{3}h - h = 120$$

$$\Rightarrow h(\sqrt{3} - 1) = 120$$

$$\therefore h = \frac{120}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{120(\sqrt{3}+1)}{2}$$

$\therefore$ Required height of the chimney (h) = $60(\sqrt{3}+1)$ m

Alternate:-



$$AB = 120 \text{ m}$$

$$= (\sqrt{3}-1) \text{ unit} = 120 \text{ m}$$

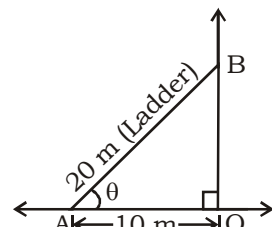
$$1 \text{ unit} = \frac{120}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 60(\sqrt{3}+1) \text{ m}$$

So, the height of PC

$$= 60(\sqrt{3}+1) \text{ metres}$$

43. (c) Let θ be the inclination of the ladder to the horizontal.



Now, In $\triangle AOB$,

$$\cos \theta = \frac{AO}{AB} = \frac{10}{20} = \frac{1}{2}$$

$$\Rightarrow \cos \theta = \cos 60^\circ$$

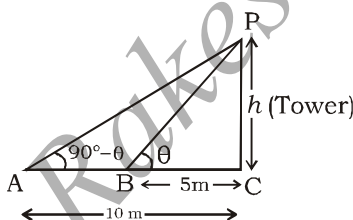
$$\therefore \theta = 60^\circ$$

44. (c) Given that, angles are complementary.

Let $\angle PBC = \theta$

$$\therefore \angle PAC = 90^\circ - \theta$$

Let h be the height of the tower



Now, in $\triangle PBC$

$$\tan \theta = \frac{h}{5} \quad \dots\dots (i)$$

and in $\triangle PAC$

$$\tan(90^\circ - \theta) = \frac{h}{10}$$

$$\Rightarrow \cot \theta = \frac{h}{10} \quad \dots\dots (ii)$$

On multiplying Eqs. (i) and (ii), we get

$$\tan \theta \cdot \cot \theta = \frac{h}{5} \cdot \frac{h}{10}$$

$$\Rightarrow \frac{h^2}{50} = 1 \Rightarrow h = \sqrt{50} \text{ m}$$

Alternate:-

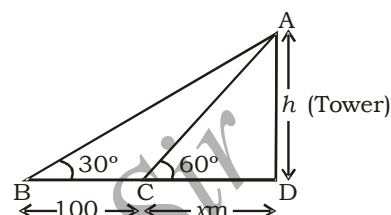
$$h = \sqrt{5 \times 10} = \sqrt{50} \text{ m}$$

45. (a) Let h be the height of inaccessible tower.

Now, in $\triangle ACD$

$$\tan 60^\circ = \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \quad \dots\dots(i)$$



and in $\triangle ABD$

$$\tan 30^\circ = \frac{h}{100+x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}h = 100 + x = 100 + \frac{h}{\sqrt{3}}$$

[From Equation (i)]

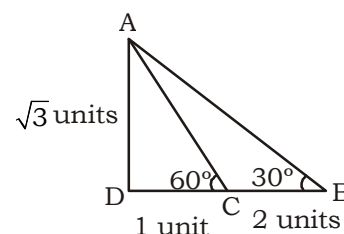
$$\Rightarrow \left(\sqrt{3} - \frac{1}{\sqrt{3}} \right) h = 100$$

$$\Rightarrow \frac{2}{\sqrt{3}} h = 100 \Rightarrow h = 50\sqrt{3}$$

$$\therefore h = 50 \times 1.732 = 86.6 \text{ m}$$

So, the required height is 86.6 m

Alternate:-



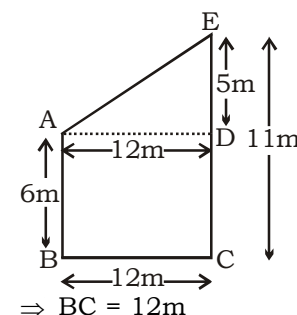
$$BC = 2 \text{ units} = 100 \text{ m}$$

$$1 \text{ unit} = 50 \text{ m}$$

$$\text{Height} = AD = \sqrt{3} \times 50$$

$$= 50\sqrt{3} = 50 \times 1.732 = 86.6 \text{ m}$$

46. (a) Given, $AB = 6 \text{ m}$ and $EC = 11 \text{ m}$



$$\begin{aligned} \therefore BC &= AD = 12\text{m} \\ \text{and } ED &= EC - CD = EC - AB \\ &(\because AB = CD) \end{aligned}$$

$$= 11 - 6 = 5\text{m}$$

In $\triangle AED$,

$$(AE)^2 = (AD)^2 + (ED)^2$$

(by Pythagoras theorem)

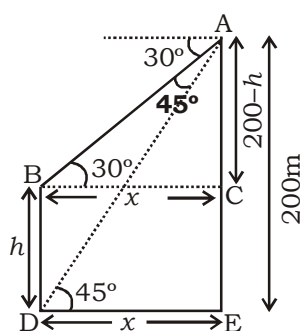
$$= (12)^2 + (5)^2 = 144 + 25$$

$$= 169 = (13)^2$$

$$\therefore AE = 13\text{m}$$

So, the distance between their tops is 13 m.

47. (d) Let $AE = 200\text{ m}$ be the height of the cliff and $BD = h\text{ m}$ be the height of the tower.



In $\triangle ABC$,

$$\tan 30^\circ = \frac{200 - h}{x} = \frac{1}{\sqrt{3}} = \frac{200 - h}{x}$$

$$\Rightarrow x = (200 - h)\sqrt{3}$$

$$\text{and In } \triangle ADE, \tan 45^\circ = \frac{200}{x}$$

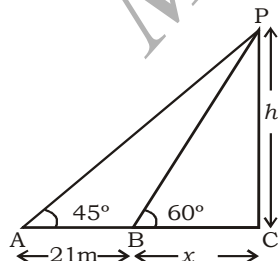
$$\Rightarrow 1 = \frac{200}{x} \Rightarrow x = 200\text{ m}$$

From Eq. (i),

$$200 = (200 - h)\sqrt{3}$$

$$\Rightarrow h = 200\left(\frac{\sqrt{3} - 1}{\sqrt{3}}\right)\text{ m}$$

48. (c) In $\triangle PBC$,



$$\tan 60^\circ = \frac{h}{x} \Rightarrow \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \quad \dots(i)$$

$$\text{In } \triangle PAC, \tan 45^\circ = \frac{h}{21 + x} = 1$$

$$\Rightarrow h = 21 + x$$

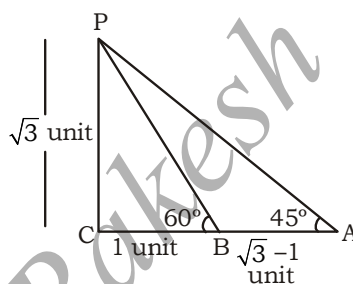
$$\Rightarrow h = 21 + \frac{h}{\sqrt{3}} \quad [\text{from Eq. (i)}]$$

$$\Rightarrow h\left(1 - \frac{1}{\sqrt{3}}\right) = 21$$

$$\therefore h = \frac{21\sqrt{3}(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{21\sqrt{3}(\sqrt{3} + 1)}{2} = 49.68 \approx 50\text{m}$$

Alternate:-



$$AB = (\sqrt{3} - 1)\text{ unit} = 21\text{m}$$

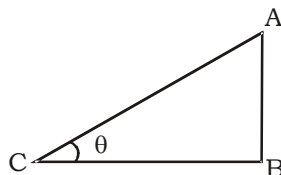
$$1\text{ unit} = \frac{21}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$$

$$1\text{ unit} = \frac{21(\sqrt{3} + 1)}{2}\text{ m}$$

$$PC = \sqrt{3} \times \frac{21(\sqrt{3} + 1)}{2}$$

$$= 49.68 = 50\text{m}$$

$$49. (a) \therefore \tan \theta = \frac{AB}{BC}$$



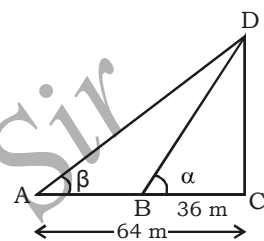
$$BC = \sqrt{3}AB$$

$$\Rightarrow \frac{AB}{BC} = \frac{1}{\sqrt{3}}$$

$$\therefore \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30^\circ \left(\because \tan 30^\circ = \frac{1}{\sqrt{3}} \right)$$

50. (b) Let $CD = h$ be height of the tower.



$$\text{In } \triangle BCD, \tan \alpha = \frac{h}{36}$$

and In $\triangle ACD$,

$$\tan \beta = \frac{h}{64}$$

$$\text{But } \alpha + \beta = \frac{\pi}{2}$$

$$\Rightarrow \tan(\alpha + \beta) = \tan \frac{\pi}{2}$$

$$\Rightarrow \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{1}{0}$$

$$\Rightarrow 1 - \tan \alpha \tan \beta = 0$$

$$\Rightarrow \tan \alpha \tan \beta = 1$$

$$\Rightarrow \frac{h}{36} \times \frac{h}{64} = 1$$

$$\Rightarrow h^2 = 36 \times 64 \Rightarrow h = 6 \times 8$$

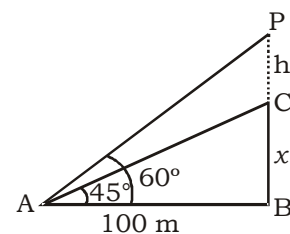
$$\Rightarrow h = 48\text{ m}$$

Alternate:-

$$h = \sqrt{36 \times 64}$$

$$= 6 \times 8 = 48\text{ m.}$$

51. (c) Let the height of the incomplete pillar be $x\text{ m}$ and the increasing height be $PC = h\text{ m}$



In $\triangle ABC$,

$$\tan 45^\circ = \frac{x}{100} \Rightarrow x = 100\text{ m}$$

and In $\triangle APB$,

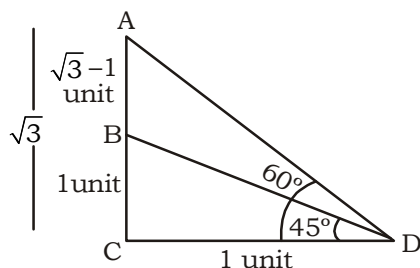
$$\tan 60^\circ = \frac{x+h}{100}$$

$$x+h = 100\sqrt{3}$$

$$\Rightarrow h = 100\sqrt{3} - x = 100\sqrt{3} - 100$$

$$\therefore h = 100(\sqrt{3}-1)\text{m}$$

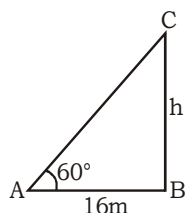
Alternate:-



$$CD = 1 \text{ unit} = 100 \text{ m}$$

$$AB = 100(\sqrt{3}-1)\text{m}$$

52. (c) Let the height of the tree be h m.



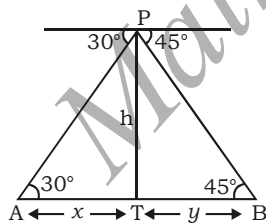
$$\text{In } \triangle ABC, \tan 60^\circ = \frac{h}{16}$$

$$\Rightarrow h = 16\sqrt{3} \text{ m}$$

53. (a) In $\triangle PBT$

$$\tan 45^\circ = \frac{h}{y} = 1$$

$$\therefore y = h$$



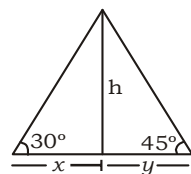
and in $\triangle PTA$

$$\tan 30^\circ = \frac{h}{x} \Rightarrow x = \sqrt{3}h \dots (ii)$$

$\therefore$ Required distance,

$$x+y = \sqrt{3}h + h = h(\sqrt{3}+1)\text{m}$$

Alternate:-

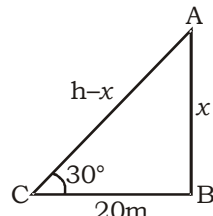


$$x+y = h(\cot 30^\circ + \cot 45^\circ)$$

$$\Rightarrow h(\sqrt{3}+1)$$

$$x+y \Rightarrow (\sqrt{3}+1)h$$

54. (b) Let the height of post be x m.



$$\text{In } \triangle ABC, \tan 30^\circ = \frac{x}{20} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = \frac{20}{\sqrt{3}} \text{ m} \dots (i)$$

$$\text{and } \cos 30^\circ = \frac{20}{h-x}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{20}{h-x}$$

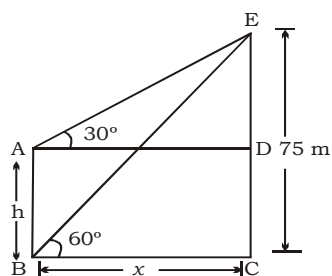
$$\Rightarrow h-x = \frac{40}{\sqrt{3}}$$

From equation (i),

$$h = \frac{40}{\sqrt{3}} + \frac{20}{\sqrt{3}} = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 20\sqrt{3} \text{ m}$$

55. (c) Let height of the building be h m and distance between building and tower be x m.



In $\triangle ADE$,

$$\tan 30^\circ = \frac{ED}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75-h}{x}$$

$$\Rightarrow x = 75\sqrt{3} - h\sqrt{3}$$

and In $\triangle BCE$,

$$\tan 60^\circ = \frac{CE}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{75}{x}$$

$$\Rightarrow x\sqrt{3} = 75 \quad (\text{from eq.(i)})$$

$$\Rightarrow (75\sqrt{3} - h\sqrt{3})\sqrt{3} = 75$$

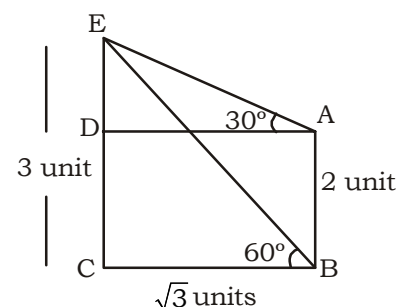
$$\Rightarrow 75 \times 3 - 3h = 75$$

$$\Rightarrow 3h = 75 \times 3 - 75$$

$$\Rightarrow h = \frac{75 \times 2}{3}$$

$$\therefore h = 50 \text{ m}$$

Alternate:-



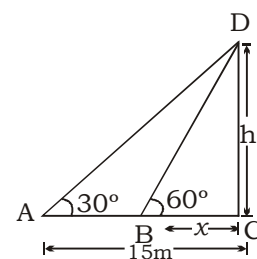
$$AB = \frac{2EC}{3} = \frac{2 \times 75}{3} = 50 \text{ m.}$$

56. (c) In $\triangle ACD$,

$$\tan 30^\circ = \frac{CD}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{15}$$

$$\Rightarrow h = \frac{15}{\sqrt{3}}$$



and in $\triangle BCD$,

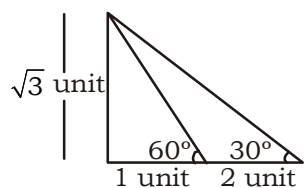
$$\tan 60^\circ = \frac{CD}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \frac{h}{\sqrt{3}} = x$$

$$\therefore x = \frac{15}{3} = 5\text{m} \quad (\text{from Eq. (i)})$$

Alternate:-

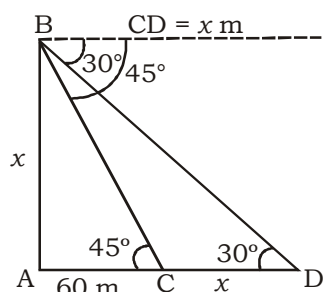


$$3 \text{ units} = 15 \text{ m}$$

$$1 \text{ unit} = \frac{15}{3} = 5\text{m}$$

So, the shadow will be 5m when angle becomes 60°

57. (a) Let AB be a height of tower,



In $\triangle ACB$,

$$\tan 45^\circ = \frac{AB}{AC}$$

$$\Rightarrow 1 = \frac{AB}{60}$$

$$\Rightarrow AB = 60 \text{ m}$$

Now, In $\triangle ADB$,

$$\Rightarrow \tan 30^\circ = \frac{60}{60+x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60}{60+x}$$

$$\Rightarrow 60+x = 60\sqrt{3}$$

$$\Rightarrow x = 60(\sqrt{3}-1) = 60(1.73-1) = 60 \times 0.73 = 43.8 \text{ m}$$

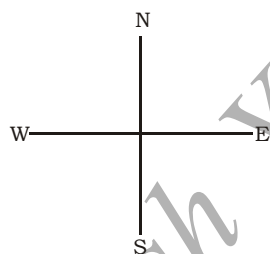
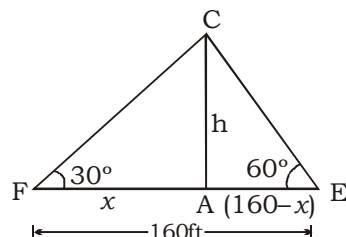
$$\therefore \text{Speed of boat} = \frac{43.8}{5} \times \frac{18}{5} = \frac{788.4}{25} = 31.5 \text{ km/h}$$

58. (a) Let $AC = h$ (Height of a tower)
 x = Distance between A and F
 $\therefore AE = 160 - x$

In $\triangle AFC$,

$$\tan 30^\circ = \frac{h}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = \sqrt{3}h \quad \dots(i)$$



and In $\triangle AEC$,

$$\tan 60^\circ = \frac{h}{160-x}$$

$$\Rightarrow \sqrt{3}(160-x) = h$$

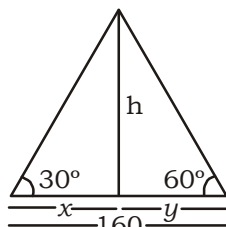
$$\Rightarrow \sqrt{3}(160-\sqrt{3}h) = h \quad [\text{from Eq. (i)}]$$

$$\Rightarrow 160\sqrt{3} - 3h = h$$

$$\Rightarrow 4h = 160\sqrt{3}$$

$$\therefore h = 40\sqrt{3} \text{ ft}$$

Alternate:-



$$x + y = h(\cot 30^\circ + \cot 60^\circ)$$

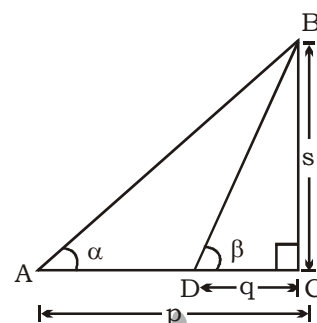
$$160 = h(\cot 30^\circ + \cot 60^\circ)$$

$$160 = h\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) = h\left(\frac{4}{\sqrt{3}}\right)$$

$$h = \frac{160 \times \sqrt{3}}{4} = 40\sqrt{3} \text{ ft.}$$

59. (c) In $\triangle ABC$,

$$\tan \alpha = \frac{s}{p} \Rightarrow p = \frac{s}{\tan \alpha} \quad \dots(i)$$



In $\triangle BDC$,

$$\tan \beta = \frac{s}{q} \Rightarrow q = \frac{s}{\tan 3\alpha}$$

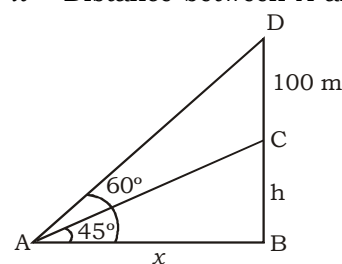
$$(\because \beta = 3\alpha, \text{ given}) \quad \dots(i)$$

On subtracting Eq. (ii) from Eq. (i), we get

$$p - q = \frac{s}{\tan \alpha} - \frac{s}{\tan 3\alpha}$$

$$= s \left(\frac{\tan 3\alpha - \tan \alpha}{\tan \alpha \tan 3\alpha} \right)$$

60. (c) Let BC be a building of height h m and CD = 100 m be a height of antenna.
 x = Distance between A and B



In $\triangle ABC$,

$$\tan 45^\circ = \frac{h}{x} \Rightarrow x = h \quad \dots(i)$$

and in $\triangle ABD$,

$$\tan 60^\circ = \frac{100+h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{100+h}{h} \quad [\text{from Eq. (i)}]$$

$$\Rightarrow \sqrt{3}h = 100 + h$$

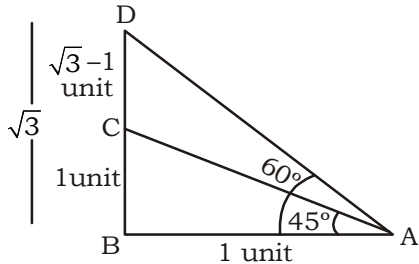
$$\Rightarrow (\sqrt{3}-1)h = 100$$

$$\Rightarrow h = \frac{100}{\sqrt{3}-1}$$

$$\Rightarrow h = \frac{100}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= 50(\sqrt{3}+1)\text{m}$$

Alternate:-



$$CD = DB - BC$$

$$= (\sqrt{3} - 1) \text{ unit}$$

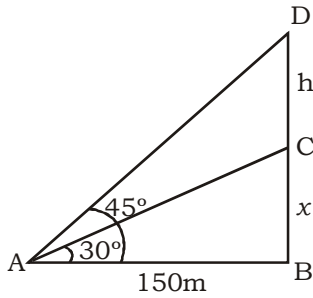
$$(\sqrt{3} - 1) \text{ unit} = 100 \text{ m}$$

$$1 \text{ unit} = \frac{100}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= 50(\sqrt{3} + 1) \text{ metre}$$

$$BC = 1 \text{ unit} = 50(\sqrt{3} + 1) \text{ metres}$$

61. (d) Let $BC = x$ m height of unfinished pillar and $CD = h$ m = Raised height of pillar



In $\triangle ABC$,

$$\tan 30^\circ = \frac{x}{150} \Rightarrow x = \frac{150}{\sqrt{3}} \dots (i)$$

and in $\triangle ABD$,

$$\tan 45^\circ = \frac{h+x}{150} \Rightarrow 1 = \frac{h+x}{150}$$

$$\Rightarrow 150 = h + \frac{150}{\sqrt{3}} \text{ [from Eq. (i)]}$$

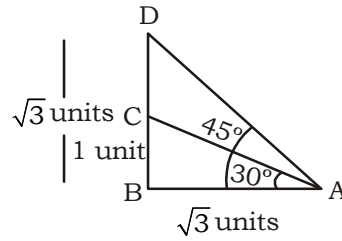
$$\Rightarrow h = \frac{150(\sqrt{3} - 1)}{\sqrt{3}}$$

$$\Rightarrow h = 150 \times \frac{(1.732 - 1)}{1.732}$$

$$= \frac{150 \times 0.732}{1.732} = 63.39$$

$$= 63.4 \text{ m (approx)}$$

Alternate:-



$$AB = \sqrt{3} \text{ units} = 150 \text{ m}$$

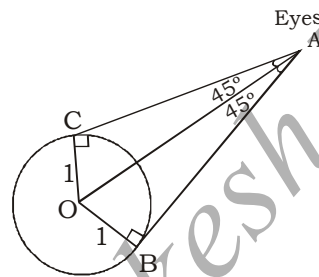
$$1 \text{ unit} = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 50\sqrt{3} \text{ metre}$$

$$\text{So, } CD = 50\sqrt{3}(\sqrt{3} - 1)$$

$$= 150 - 86.60 = 63.4 \text{ metres}$$

62. (b) Let O = Centre of the balloon
 $OB = OC$ = Radii of the balloon

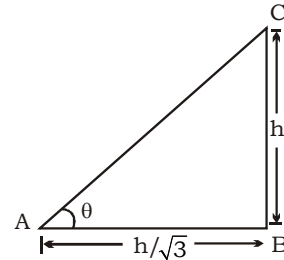


In $\triangle OAB$,

$$\sin 45^\circ = \frac{OB}{OA}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{OA} \Rightarrow OA = \sqrt{2}$$

63. (a) Let h be the height of pole, then its shadow = $h\sqrt{3}$ (given) θ (suppose) be the angle of elevation.



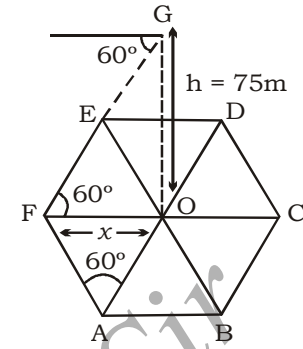
$$\text{Then, } \tan \theta = \frac{h}{h/\sqrt{3}}$$

$$\Rightarrow \tan \theta = \sqrt{3} = \tan 60^\circ$$

$$\therefore \theta = 60^\circ$$

64. (c) Let OG be a height of the tower,
Angle of elevation = Angle of depression

x = Distance between O and F .



In $\triangle FOG$,

$$\tan 60^\circ = \frac{75}{x}$$

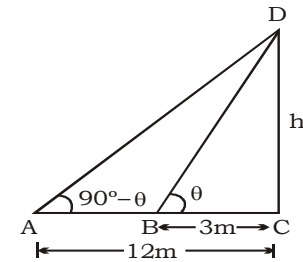
$$\Rightarrow x = \frac{75}{\sqrt{3}} = 25\sqrt{3} \text{ m}$$

In regular hexagon $\triangle OEF$, $\triangle OED$,...are equilateral triangles.

$$\therefore OF = OE = OA = OD = OC = OB = OA = \text{side of hexagon}$$

$$\therefore \text{Length of hexagon} = 25\sqrt{3} \text{ m}$$

65. (b) Let h be the height of the tower and $\angle CBD = \theta$
 $\angle DAC = 90^\circ - \theta$



$\therefore$ In $\triangle BCD$,

$$\tan \theta = \frac{CD}{BC} \Rightarrow \tan \theta = \frac{h}{3} \dots (i)$$

and in $\triangle ACD$

$$\tan (90^\circ - \theta) = \frac{CD}{AC}$$

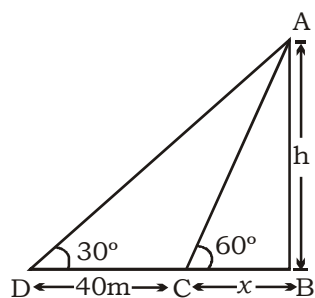
$$\Rightarrow \cot \theta = \frac{h}{12} \dots (ii)$$

On multiplying Eqs. (i) & (ii), we get

$$\tan \theta \cdot \cot \theta = \frac{h}{3} \cdot \frac{h}{12}$$

$$\Rightarrow \frac{h^2}{36} = 1 \Rightarrow h^2 = 36 \Rightarrow h = 6 \text{ m}$$

66. (b) Let $AB = h$ m be the height of the tree and $BC = x$ m be the width of the river



In $\triangle ABC$,

$$\tan 60^\circ = \frac{h}{x} \Rightarrow h = \sqrt{3}x \dots (i)$$

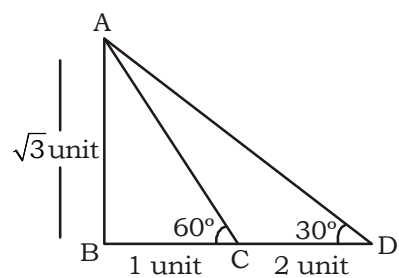
and in $\triangle ADB$, $\tan 30^\circ$

$$= \frac{h}{40+x} \Rightarrow h = \frac{40+x}{\sqrt{3}} \dots (ii)$$

From Eqs. (i) & (ii),

$$\begin{aligned} \sqrt{3}x &= \frac{40+x}{\sqrt{3}} \Rightarrow 2x = 40 \Rightarrow x \\ &= 20 \text{ m} \end{aligned}$$

Alternate:-



$$CD = 40 \text{ m}$$

$$2 \text{ units} = 40 \text{ m}$$

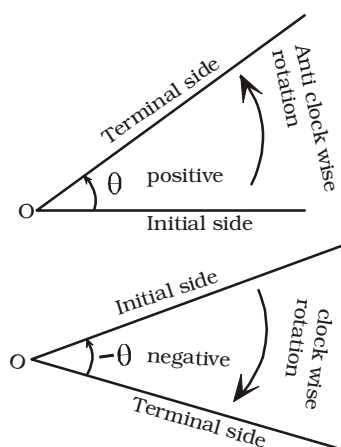
$$1 \text{ unit} = 20 \text{ m}$$

So, the width of river = 20 m



TRIGONOMETRY CIRCULAR MEASURE OF ANGLES

Angles: When two rays (initial and terminal) meet at a point after rotation in a plane then they are said to have described an angle.



Unit of Angle:

1. Degree ($^{\circ}$)
2. Radian (c)
3. Grade (g)

Systems of Measurement of Angles:

1. Hexagonal System

The angle between two perpendicular lines is called a right angle. A right angle is equal to 90 degree, it is written as 90° .

Thus, if a right angle is divided into 90 equal parts then one part is called one degree. It is written as 1° .

If 1° is divided into 60 equal parts, each part is called 1 minute. It is denoted by $1'$

$\frac{1}{60}$ th part of $1'$ is called one second. It is written as $1''$.

$$1 \text{ degree} = 1^{\circ} = \frac{1}{90} \text{ right angle}$$

$$1 \text{ minute} = 1' = \frac{1}{60} \text{ degree}$$

$$1 \text{ second} = 1'' = \frac{1}{60} \text{ minute}$$

In other words:- $90^{\circ} = 1$ right angle

$$60' = 1^{\circ}$$

$$60'' = 1'$$

Hence 1 right angle = 90°

$$= 90 \times 60 = 5400$$

$$= 5400 \text{ minutes}$$

$$= 90 \times 60 \times 60 = 324000''$$

$$= 324000 \text{ seconds}$$

$$\text{Again, } 1^{\circ} = 60' = 60 \times 60'' = 3600''$$

2. Centesimal or French System:-

1 right angle = 100 grades (= 100 g)

1 grade = 60 minutes (= 60')

1 minute = 60 seconds (= 60'')

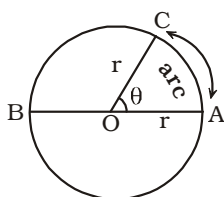
3. Circular System:-

In this system, the unit of measurement is "radian".

Angle (in radian)

$$\theta = \frac{\text{arc AC}}{\text{radius}} = \frac{\widehat{AC}}{r}$$

1 radian or 1^c is the angle subtended by an arc at the centre of a circle whose length is equal to the radius of the circle.



i.e. If arc = radius = r , then

$$\theta = \frac{r}{r} = 1, \text{ radian} = 1^c$$

when arc ACB = πr

4. Relation between degree measure and radian measure:

$$\therefore \pi \text{ rad} = 180^{\circ}$$

$$\therefore x^{\circ} = \frac{\pi x}{180} \text{ rad}$$

$$\text{and } x \text{ rad} = \frac{180}{\pi} x^{\circ}$$

$$1 \text{ rad} = \frac{180}{3.14} = 57^{\circ} 16' 22''$$

Thus to change degree into ra-

dian, multiply by $\frac{\pi}{180}$ and to

change radian into degree mul-

tiple by $\frac{180}{\pi}$.

If mentioned, take $\pi = \frac{22}{7}$ or 3.14.

$$\left\{ \text{and } 1^{\circ} = \frac{\pi}{180} \text{ radian} = \left(\frac{22}{7 \times 180} \right) \text{ rad} \right\}$$

$$\text{or } 1^{\circ} = 0.01746 \text{ radian}$$

For Ex:

$$\begin{array}{cc|cc} \sin 1^{\circ} & \sin 1^{\circ} & \cos 1^{\circ} & \cos 1^{\circ} \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \approx 57^{\circ} & \approx 0^{\circ} & \approx 57^{\circ} & \approx 0^{\circ} \\ \approx \sin 60^{\circ} & \approx \sin 0^{\circ} & \approx \cos 60^{\circ} & \approx \cos 0^{\circ} \\ \text{then } \sin 60^{\circ} > \sin 0^{\circ} & & \text{then } \cos 60^{\circ} < \cos 0^{\circ} & \\ \therefore \boxed{\sin 1^{\circ} > \sin 1^{\circ}} & & \boxed{\cos 1^{\circ} < \cos 1^{\circ}} & \end{array}$$

Degree	Radian	Degree	Radian
30°	$\pi/6$	135°	$3\pi/4$
45°	$\pi/4$	150°	$5\pi/6$
60°	$\pi/3$	180°	π
90°	$\pi/2$	270°	$3\pi/2$
120°	$2\pi/3$	360°	2π

Ex. 1 Convert the following degree measures in the radian measure.

- (i) $42^\circ 30'$ (ii) -520°
 (iii) $72^\circ 45'$ (iv) $115^\circ 40'$
 (v) $15^\circ 12' 30''$

Sol. We know that $x^\circ = \frac{\pi x}{180}$ rad

$$\therefore \text{(i)} \quad 42^\circ 30' = 42 \frac{1^\circ}{2} = \frac{85^\circ}{2}$$

$$= \frac{\pi}{180} \times \frac{85}{2}$$

$$= \frac{\pi \times 17 \times 5}{5 \times 36 \times 2} = \frac{17\pi}{72}$$

$$\text{(ii)} \quad -520^\circ = -520 \times \frac{\pi}{180} = \frac{-26\pi}{9}$$

$$\text{(iii)} \quad 72^\circ 45'$$

$$1^\circ = 60'$$

$$1' = \left(\frac{1}{60}\right)^\circ$$

$$45' = \frac{3^\circ}{4}$$

$$\text{So,} \quad = 72 \frac{3^\circ}{4} = \frac{291^\circ}{4}$$

$$= \frac{291}{4} \times \frac{\pi}{180}$$

$$= \frac{3 \times 97 \times \pi}{4 \times 3 \times 60} = \frac{97\pi}{240}$$

$$\text{(iv)} \quad 115^\circ 40' = 115 \frac{2^\circ}{3}$$

$$\left(\therefore 40' = \frac{2^\circ}{3}\right) = \frac{347^\circ}{3}$$

$$= \frac{347^\circ}{3} \times \frac{\pi}{180} = \frac{347\pi}{540}$$

$$\text{(v)} \quad 15^\circ 12' 30''$$

$$30'' = \left(\frac{30}{60}\right)' = \left(\frac{1}{2}\right)'$$

$$12' 30'' = \left(12 \frac{1}{2}\right)' = \left(\frac{25}{2}\right)'$$

$$= \left(\frac{25}{2} \times \frac{1}{60}\right)^\circ = \left(\frac{5}{24}\right)^\circ$$

$$\therefore 15^\circ 12' 30'' = \left(15 \frac{5}{24}\right)^\circ$$

$$= \left(\frac{365}{24}\right)^\circ = \frac{365}{24} \times \frac{\pi}{180} = \frac{73\pi}{864}$$

Ex. 2 Convert the following radian measure in degree measures

$$\text{(i)} 4^c \quad \text{(ii)} \frac{-5\pi^c}{3} \quad \text{(iii)} \frac{5\pi^c}{24}$$

$$\text{(iv)} \frac{\pi^c}{16} \quad \text{(v)} \frac{5\pi^c}{7}$$

$$\text{Sol.} \quad \pi \text{ radian} = 180^\circ$$

$$1 \text{ radian} = \frac{180}{\pi} = 180^\circ \times \frac{7}{22}$$

$$\text{(i)} \quad 4 \text{ radian} = \frac{180^\circ \times 7 \times 4}{22}$$

$$= \frac{90^\circ \times 7 \times 4}{11} = \frac{2520^\circ}{11}$$

$$= 229 \frac{1}{11} \text{ degree}$$

$$\text{(ii)} \quad -\frac{5\pi}{3} = -\frac{5}{3} \times 180^\circ$$

$$= -5 \times 60^\circ = -300^\circ$$

$$\text{(iii)} \quad \frac{5\pi}{24} = \frac{5\pi}{24} \times \frac{180^\circ}{\pi}$$

$$= \frac{75^\circ}{2} = 37 \frac{1}{2}^\circ$$

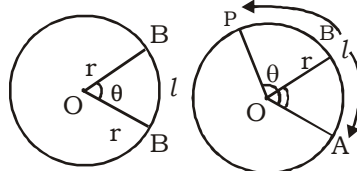
$$\text{(iv)} \quad \frac{\pi}{16} = \frac{\pi}{16} \times \frac{180^\circ}{\pi}$$

$$= \frac{45^\circ}{4} = 11 \frac{1}{4}^\circ$$

$$\text{(v)} \quad \frac{5\pi}{7} \times \frac{180^\circ}{\pi} = \frac{5 \times 180^\circ}{7} = \frac{900^\circ}{7}$$

$$= 128^\circ 57' 14''$$

5. Relation between length of arc (l), radius (r) and angle (θ):



If an arc of length l of a circle subtends θ at its centre and radius of the circle is r then

$$\theta = \frac{l}{r}$$

Hence,

(i) when $\theta = \frac{l}{r}$ and r is constant then $\theta \propto l$

$$\text{i.e., } \theta_1 : \theta_2 = l_1 : l_2$$

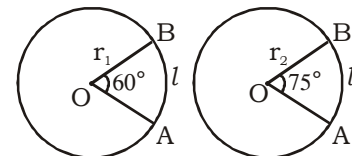
(ii) when $\theta = \frac{l}{r}$ and θ is constant then $l \propto r$ i.e., $l_1 : l_2 = r_1 : r_2$

(iii) when $\theta = \frac{l}{r}$ and l is constant then $\theta \propto \frac{1}{r}$

$$\text{or } r \propto \frac{1}{\theta}$$

i.e., $\theta_1 : \theta_2 = r_2 : r_1$ (reverse order)

Ex. 3 If in two circles, arcs of the same length subtend angle 60° and 75° at the centre, find the ratio of their radii.



Sol. Let the radii of two circles be r_1 and r_2 respectively. According to the question, arc $AB = l$ (say) in the two circle. Given that $\theta_1 = 60^\circ$

$$= 60 \times \frac{\pi}{180} = \frac{60\pi}{180} \text{ radian}$$

$$\text{And } \theta_2 = 75^\circ = \frac{75\pi}{180} \text{ radian}$$

$$\therefore \theta = \frac{l}{r} \therefore \theta_1 = \frac{l}{r_1} \text{ and } \theta_2 = \frac{l}{r_2}$$

$$= \frac{l}{r_2}$$

$$\text{or, } l = r_1 \theta_1 = r_2 \theta_2$$

$$\text{or, } \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\frac{75\pi}{180}}{\frac{60\pi}{180}}$$

$$= \frac{75}{60} = \frac{5}{4}$$

Alternate: -

since l is constant, therefore $r_1 : r_2$

$$= \theta_2 : \theta_1 = 75^\circ : 60^\circ = 5 : 4$$

Ex. 4 If the arc of same length in two circles subtends angle 75° and 120° at their respective centres, then ratio of their diameter is

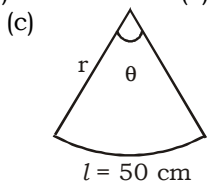
- (a) 8 : 5 (b) 5 : 8
 (c) 3 : 5 (d) 5 : 3

Sol. (a) Since l is constant therefore $r_1 : r_2 = \theta_2 : \theta_1$
 $= 120^\circ : 75^\circ = 8 : 5$
 $\therefore d_1 : d_2 = 8 : 5$

Ex.5 The tip a pendulum swings. It covers an arc of 50 cm and subtends 60° at the fixed point. The length of pendulum is

- (a) 43.72cm (b) 45.72cm
 (c) 47.72cm (d) 45.27 cm

Sol.



$$\theta = 60^\circ = \frac{\pi}{3} \text{ and } l = 50 \text{ cm}$$

$$\therefore \text{ using } \theta = \frac{l}{r}$$

$$r = \frac{l}{\theta} = \frac{50 \text{ cm}}{\frac{\pi}{3}} = \frac{150}{\pi} \text{ cm} = \frac{150}{\frac{22}{7}} = \frac{150 \times 7}{22} = 47.72 \text{ cm}$$

Ex.6 The minutes hand of a watch is 5 cm. How far does the tip move in 20 minutes?

- (a) 10 cm (b) 9.53 cm
 (c) 11 cm (d) 10.47 cm

Sol. (d) In 20 minute, hand covers

$$\frac{20}{60} \times 2\pi = \frac{2\pi}{3} \text{ rad distance.}$$

$$\text{From } \theta = \frac{l}{r}, l = \theta r$$

$$= \frac{2\pi}{3} \times 5 = \frac{10\pi}{3} = \frac{10}{3} \times \frac{22}{7} = \frac{220}{21} = 10.47 \text{ cm}$$

Ex. 7 When a pendulum of length 50 cm oscillates, it produces an arc of 16 cm. The angle so formed in degree measure is (approx) :

- (a) $18^\circ 25'$ (b) $18^\circ 35'$
 (c) $18^\circ 20'$ (d) $18^\circ 08'$

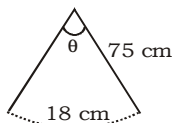
Sol. (c) $l = 16 \text{ cm}$
 $r = 50 \text{ cm}$

$$\therefore \theta = \frac{l}{r} = \frac{16}{50} = \frac{8}{25} \text{ radian}$$

$$= \frac{8}{25} \times \frac{180^\circ}{\pi} = \frac{8}{25} \times \frac{180}{22} \times 7 = \frac{1008}{55}$$

$$= 18 \frac{18^\circ}{55} = 18^\circ \left(\frac{18}{55} \times 60 \right) \approx 18^\circ 20'$$

Ex.8 Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc length 18 cm.



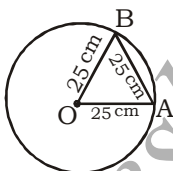
Sol. Suppose the pendulum swings through an angle of

$$\text{radian. then } \theta = \frac{l}{r} = \frac{18}{75} \text{ rad}$$

(see figure)

$$= \frac{6}{25} \text{ rad}$$

Ex.9 In a circle of diameter 50 cm, the length of a chord is 25 cm. Find the length of minor arc and major arc of the chord.



Sol. See the figure
 Given that radius of the circle

$$= \frac{50 \text{ cm}}{2} = 25 \text{ cm and}$$

chord AB of the circle = 25 cm
 Clearly $\triangle OAB$ is an equilateral triangle, therefore $\angle AOB = 60^\circ$

$$= \frac{\pi}{3} = \theta \text{ (say)}$$

In minor arc AB = l then from

$$\theta = \frac{l}{r}$$

$$l = r\theta = \frac{25\pi}{3}$$

Here major angle = $360^\circ - 60^\circ$

$$= 300^\circ = 300 \times \frac{\pi}{180} = \frac{5\pi}{3}$$

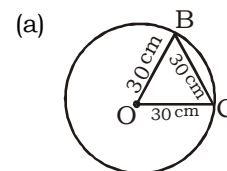
$$\text{Major arc} = 25 \times \frac{5\pi}{3} = \frac{125\pi}{3}$$

Ex.10 The diameter of a circle is 60 cm. The length of minor arc created by a chord of 30 cm is

(a) $31 \frac{3}{7} \text{ cm}$ (b) 34 cm

(c) $32 \frac{2}{7} \text{ cm}$ (d) $32 \frac{4}{7} \text{ cm}$

Sol.



BOC is an equilateral triangle since all sides are equal.

$$\theta = 60^\circ = \frac{\pi}{3} \text{ rad}$$

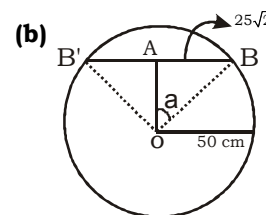
$$\theta = \frac{l}{r} \Rightarrow l = \theta r = \frac{\pi}{3} \times 30$$

$$= 10 \times \frac{22}{7} = 31 \frac{3}{7} \text{ cm}$$

Ex.11 In a circle of radius 50 cm the length of a chord is $50\sqrt{2}$ cm. The length of major arc of the chord is

- (a) 245.5 cm (b) 235.5 cm
 (c) 255.5 cm (d) None of these

Sol.



$$\sin \alpha = \frac{25\sqrt{2}}{50} = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$$

$$\therefore 2\alpha = 90^\circ$$

Hence major arc of chord BB' subtends angle

$$= 360^\circ - 90^\circ = 270^\circ = \frac{3\pi}{2} \text{ at centre.}$$

$$\therefore \text{ using } \theta = \frac{l}{r}$$

$$\text{major arc } l = \theta \times r = \frac{3\pi}{2} \times 50$$

$$= 75\pi = 75 \times 3.14 \text{ cm}$$

$$= 235.50 \text{ cm}$$

6.

If θ is in radian and is very small then $\sin \theta = \theta = \tan \theta$ (approximate)

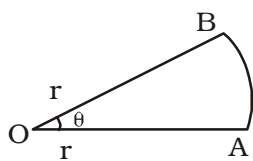
7.

Area of a sector (or Sectorial area) :-

The area of the sector

$$OAB = \frac{1}{2} r^2 \theta$$

Here θ is in radian.



Note :- Radian is a constant angle.

Some Useful Points :-

The angle between two consecutive digits in a clock is

$$30^\circ \left(= \frac{\pi}{6} \text{ radians} \right)$$

□ The hour hand rotates through an angle of 30°

in one hour i.e. $\left(\frac{1}{2}\right)^\circ$ in one minute.

□ The minute hand rotates through an angle of 6° in one minute.

Ex. 12 The angle between the hands of a clock at 4 hour 45 minute is

- (a) $112\frac{1}{2}^\circ$ (b) $122\frac{1}{2}^\circ$
(c) 125° (d) $127\frac{1}{2}^\circ$

Sol. (d) Using $\frac{11}{2} M = 30 H \pm A$

here, $H = 4$, $M = 45$

$$\therefore \frac{11}{2} \times 45 = 30 \times 4 \pm A$$

$$247.5 = 120^\circ \pm A$$

$$\therefore A = 247.5^\circ - 120^\circ = 127.5^\circ$$

Ex. 13 The angle between the hands of a clock at half past one is

- (a) $\frac{3\pi}{4}$ rad (b) $\frac{2\pi}{3}$ rad
(c) $\frac{5\pi}{12}$ rad (d) $\frac{5\pi}{6}$ rad

Sol. (a) Using $\frac{11}{2} M = 30 H \pm A$

$$\frac{11}{2} \times 30 = 30 \times 1 \pm A$$

$$\Rightarrow A = 11 \times 15 - 30 = 135^\circ$$

$$= \frac{135}{180} \times \pi \text{ rad} = \frac{3\pi}{4} \text{ rad}$$

Ex. 14 Find the angle between the minute hand of a clock and the hour hand when the time is 5:20 AM.

- (a) 50° (b) 30° (c) 40° (d) 45°

Sol. (c) $\theta = \left| \frac{11}{2} M - 30 H \right|$

Where θ = angle

M = minute

H = hour

$$\theta = \left| \frac{11}{2} \times 20 - 30 \times 5 \right|$$

$$= |110^\circ - 150^\circ| = 40^\circ$$

Ex. 15 A wheel makes 180 revolutions in one minute. Through how many radians does it turn in one second? Also find its degree measure.

Sol. Wheel makes 180 revolutions in 1 minute.

$$\therefore \text{Wheel makes } \frac{180}{60} = 3 \text{ revolutions in 1 second.}$$

Now, $\therefore$ One complete revolution measure 2π radian.

$\therefore$ Three complete revolutions measure $2\pi \times 3 = 6\pi$ radian

Again π rad = 180°

$$\therefore 6\pi \text{ rad} = 6 \times 180^\circ = 1080^\circ$$

Ex. 16 A wheel makes 240 revolutions per minute. Through how many radians does it turn in 1 second?

- (a) 8π (b) 6π (c) 4π (d) 16π

Sol. (a) Number of revolutions per second = $\frac{240}{60} = 4$

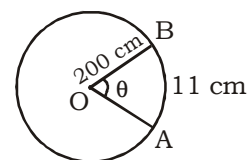
complete circles.

A circle subtends an angle of $2\pi^c$ at its centre it 1 revolution

$$\therefore \text{Number of radians in 4 revolution} = 4 \times 2\pi = 8\pi^c$$

Ex. 17 Find the degree and radian measure of the angle subtended at the centre of a circle of radius 200 cm by an arc of

length 11 cm.



Sol. Given $r = 200$ cm,

$l = \text{Arc AB} = 11$ cm

Suppose angle subtended at the centre of circle be θ radian

$$\text{then, } \theta = \frac{l}{r} = \frac{11}{200} \text{ rad}$$

$$\therefore \pi \text{ rad} = 180^\circ$$

$$\therefore l \text{ rad} = \frac{180^\circ}{\pi}$$

$$\text{or, } l \text{ rad} = \frac{7}{22} \times 180^\circ$$

$$\therefore \frac{11}{200} \text{ rad} = \frac{11}{200} \times \frac{7}{22} \times 180^\circ$$

$$= \frac{7 \times 180^\circ}{200 \times 2} = \frac{7 \times 45^\circ}{100} = \frac{7 \times 9^\circ}{20}$$

$$= \frac{63^\circ}{20} = 3\frac{3}{20} = 3 \text{ degree}$$

$$\frac{3}{20} \times 60 \text{ minutes} = 3^\circ 9'$$

Ex. 18 The moon's distance from the earth is 360000 km and its diameter subtends an angle of $30'$ at the eye of the observer. Find the diameter of the moon.

- (a) 100π km
(b) 1000π km
(c) 1500π km
(d) 2000π km

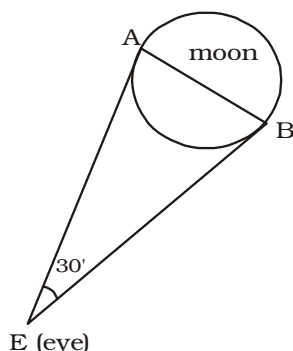
Sol. (b) Diameter $d = \text{Arc AB}$ as the distance between moon and the earth is very large

$$\theta = 30' = \left(\frac{30}{60} \times \frac{\pi}{180} \right)^c$$

$$= \left(\frac{\pi}{360} \right)^c$$

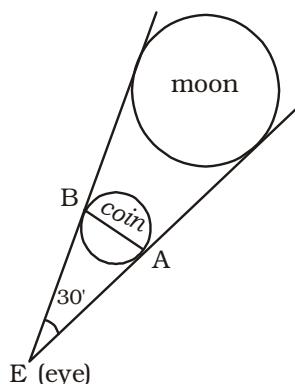
$$\therefore \theta = \frac{\text{arc}}{\text{radius}}$$

$$\Rightarrow \frac{\pi}{360} = \frac{d}{360000} \Rightarrow d = 1000\pi \text{ km}$$



Ex. 19 If the angular diameter of the moon be 30° , how far from the eye a coin of diameter 4.4 cm be kept to hide the moon ?

- (a) 252cm (b) 504cm
(c) 300cm (d) 500 cm
Sol.(b) arc AB = diameter AB = 4.4 cm



$$\theta = 30' = \left(\frac{30}{60}\right)^\circ = \left(\frac{1}{2}\right)^\circ$$

$$= \left(\frac{1}{2} \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{360}\right)^c$$

$$\theta = \frac{\text{arc}}{\text{radius}} \Rightarrow \frac{\pi}{360} = \frac{4.4}{r}$$

$$\Rightarrow r = \frac{4.4 \times 360}{\pi} \text{ cm} = \frac{4.4 \times 360}{22} \times 7$$

$$\Rightarrow r = 504 \text{ cm}$$

EXERCISE

- In radian measure 120° equals
(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{3\pi}{4}$ (d) $\frac{4\pi}{3}$
- $37\frac{1}{2}^\circ$ is equal to which of the following radian measure?
(a) $\frac{5\pi}{12}$ (b) $\frac{7\pi}{12}$ (c) $\frac{5\pi}{24}$ (d) $\frac{7\pi}{24}$
- $11\frac{1}{4}^\circ$ is equivalent to the radian measure
(a) $\frac{\pi}{8}$ rad (b) $\frac{3\pi}{8}$ rad
(c) $\frac{3\pi}{16}$ (d) $\frac{\pi}{16}$ rad
- $\frac{5}{6}$ right angle in radian equals
(a) $\frac{5\pi}{24}$ (b) $\frac{5\pi}{12}$ (c) $\frac{7\pi}{12}$ (d) $\frac{\pi}{12}$
- 1 radian is equal to:-
(a) 100° (b) $\left(\frac{\pi}{180}\right)^\circ$
(c) $\left(\frac{180}{\pi}\right)^\circ$ (d) 90°
- Find the degree measure corresponding to $\left(\frac{1}{6}\right)^c$:-
(a) $9^\circ 32'$ (b) $9^\circ 32' 43.6''$
(c) 10° (d) None of these
- Three interior angles of a quadrilateral are 60° , 120° , 90° . The remaining angle in circular measure is given by :
(a) $\frac{\pi^c}{3}$ (b) $\frac{\pi^c}{2}$ (c) $\frac{\pi^c}{4}$ (d) $\frac{3\pi^c}{4}$
- In $\triangle ABC$, $\angle A = 30^\circ$, $\angle B = 60^\circ$. Find $\angle C$ in circular measure :
(a) $\frac{2\pi^c}{3}$ (b) $\frac{3\pi^c}{4}$ (c) $\frac{\pi^c}{4}$ (d) $\frac{\pi^c}{2}$
- Radian value of $560^\circ 20'$ is
(a) $\frac{1481\pi}{540}$ rad (b) $\frac{1481\pi}{360}$ rad
(c) $\frac{1681\pi}{540}$ rad (d) $\frac{1681\pi}{360}$ rad
- Radian measure of $72^\circ 40'$ is
(a) $\frac{109\pi}{270}$ rad (b) $\frac{109\pi}{180}$ rad
(c) $\frac{219\pi}{540}$ rad (d) $\frac{219\pi}{360}$ rad
- Measure of 6 rad is
(a) $343^\circ 18' 11''$ (b) $341^\circ 18' 11''$
(c) $341^\circ 38' 11''$ (d) $343^\circ 38' 11''$
- If 1 rad = $57^\circ 16' 21''$ then 10 rad equals
(a) $570^\circ 16' 21''$ (b) $573^\circ 43' 10''$
(c) $571^\circ 43' 40''$ (d) $572^\circ 43' 30''$
- If one unit of an angle is $29^\circ 46' 55''$ then five units of the angle equals
(a) $148^\circ 54' 35''$ (b) $146^\circ 54' 35''$
(c) $149^\circ 34' 25''$ (d) $147^\circ 44' 35''$
- If one unit of an angle is $15^\circ 49' 50''$ then measure of 100 units of the angle equals
(a) $1580^\circ 30' 20''$ (b) $1582^\circ 3' 20''$
(c) $1583^\circ 3' 20''$ (d) $1581^\circ 30' 20''$
- A wheel make 90 revolutions in half hour. Through how many degree does it turn in one minute?
(a) 120° (b) 720°
(c) 1080° (d) 540°
- The angle in degree through which a pendulum of length 100 cm swings and the tip describes an arc length of 10 cm is
(a) $5^\circ 43' 38''$ (b) $7^\circ 43' 38''$
(c) $5^\circ 34' 18''$ (d) $7^\circ 34' 18''$
- Find in degrees the angle through which a pendulum swings if its length is 90cm and its tip describes an arc of length 22cm .
(a) 14° (b) $13^\circ 16'$
(c) $14^\circ 8'$ (d) 13°
- A rail road curve is to be laid out on a circle. What radius should be used if the track is to change direction by 25° in a distance of 40 metres ?
(a) 91.64 metres
(b) 90.46 metres
(c) 89.64 metres
(d) 93.64 metres

19. By decreasing 15° of each angle of a triangle, the ratios of their angles are $2 : 3 : 5$. The radian measure of greatest angle is :
- (a) $\frac{11\pi}{24}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{24}$ (d) $\frac{5\pi}{24}$
20. In a triangle ABC, $\angle ABC = 75^\circ$ and $\angle ACB = \frac{\pi^\circ}{4}$. The circular measure of $\angle BAC$ is :
- (a) $\frac{5\pi}{12}$ radian (b) $\frac{\pi}{3}$ radian
- (c) $\frac{\pi}{24}$ (d) $\frac{5\pi}{24}$
21. The minute hand of a big wall-clock is 35 cm long. Taking $\pi = \frac{22}{7}$, length of the arc, its extremity moves in 18 seconds is :
- (a) 11 cm (b) 1.1 cm
(c) 6.6 cm (d) 6 cm
22. Two angles of a triangle are $\frac{3}{2}$ rad and $\frac{4}{3}$ rad. The triangle
- (a) is an acute angled triangle
(b) is an obtuse angled triangle
(c) is a right angled triangle
(d) does not form
23. If two angle of a triangle are 2 rad and $\frac{1}{2}$ rad then its third angle in degree is
- (a) $105\frac{3^\circ}{7}$ (b) $15\frac{3^\circ}{7}$
(c) $105\frac{5^\circ}{7}$ (d) $36\frac{9^\circ}{11}$
24. A wheel revolves 24 times in 10 seconds. How many time does it take in revolving an angle of 110 rad?
- (a) 5 sec (b) 7.3 sec
(c) 10 sec (d) None of these
25. Radian measure of $40^\circ 20' 50''$ is
- (a) $\frac{481}{1196}\pi$ rad (b) $\frac{681}{1296}\pi$ rad
(c) $\frac{581}{2592}\pi$ rad (d) $\frac{581}{1296}\pi$ rad
26. A pendulum of length 60 cm swings and creates an arc of 18 cm. The angle at the fixed point of the pendulum is
- (a) 15° (b) $17\frac{1^\circ}{2}$
(c) 20° (d) $22\frac{1^\circ}{2}$
27. Radius of a circle is 54 cm. If an arc of circle subtends an angle of 20° at centre then length of the arc is
- (a) $19\frac{1}{7}$ (b) $17\frac{4}{7}$ cm
(c) $18\frac{6}{7}$ cm (d) None of these
28. An arc of length 40 cm subtends $22\frac{1^\circ}{2}$ at the centre of the circle. Radius of the circle is
- (a) 92 cm (b) 102 cm
(c) 96 cm (d) 108 cm
29. The minute hand of a watch is 3cm long. How far does its tip move in 50 minute ?
- (a) 10.32 cm (b) 17.67 cm
(c) 15.71 cm (d) 18.23 cm
30. Find the angle between the hour hand and the minute hand at half past four .
- (a) $\frac{\pi}{4}$ radian (b) $\frac{\pi}{6}$ radian
(c) $\frac{2\pi}{3}$ radian (d) $\frac{\pi}{3}$ radian
31. In a circle of diameter 30cm, the length of the chord is 15cm. Find the length of the minor arc corresponding to the chord .
- (a) $\frac{5\pi}{3}$ cm (b) 5π cm
(c) $\frac{5\pi}{2}$ cm (d) None of these
32. The angles of a triangle are in Arithmetic Progression. The ratio of the least angle in degrees to the number of radians in the greatest angle is $60 : \pi$. The angles in degrees are:
- (a) $30^\circ, 60^\circ, 90^\circ$
(b) $35^\circ, 55^\circ, 90^\circ$
(c) $40^\circ, 50^\circ, 90^\circ$
(d) $40^\circ, 55^\circ, 85^\circ$
33. Ananta's (A) and Shailvia's (S) house are situated at a circular road and subtends 90° at a fixed point. If fixed point is at a distance of 100 metre from each house, the distance travelled between the both house on the road is
- (a) 628 metres (b) 314 metres
(c) 157 metres (d) 235.5 metres
34. The angle covered by minute hand of a watch during 1hour 15 minutes noon to half past three noon is
- (a) 4.5π (b) 5π
(c) 4.25π (d) None of these
35. The angle covered by hour hand of a clock from half past six in the morning to three O'clock in the noon is
- (a) 270° (b) 245°
(c) 255° (d) 265°
36. Assuming that the Moon's diameter subtends and angle $\left(\frac{1}{2}\right)^\circ$ at the eye of an observer, find how far from the eye of a coin of 10 cm diameter must be held so as just to hide Moon?
- (a) $112\frac{5}{11}$ cm (b) $110\frac{6}{11}$ cm
(c) $116\frac{5}{11}$ cm (d) $114\frac{6}{11}$ cm
37. The earth revolves in its axis in 24 hours. How much angle does it move in 4 hours and 12 minutes?
- (a) 63° (b) 64°
(c) 65° (d) 70°
38. The angle formed by the hour-hand and the minute-hand of a clock at 2 : 15 p.m. is
- (a) $22\frac{1^\circ}{2}$ (b) 30° (c) $27\frac{1^\circ}{2}$ (d) 45°
39. Two angles of triangle are $\frac{1}{2}$ and $\frac{1}{3}$ radian. The measure of the third angle in degree $\left(\text{taking } \pi = \frac{22}{7}\right)$ is
- (a) $132\frac{1^\circ}{11}$ (b) $132\frac{2^\circ}{11}$
(c) $132\frac{3^\circ}{11}$ (d) 132°

40. A wheel rotates 3.5 times in one second. What time (in second) does the wheel take to rotate 55 radian of angle?
(a) 1.5 (b) 2.5 (c) 3.5 (d) 4.5
41. The radian measure of $63^\circ 14' 51''$ is
(a) $\left(\frac{2811}{8000}\right)^\circ$ (b) $\left(\frac{3811\pi}{8000}\right)^\circ$
(c) $\left(\frac{4811\pi}{8000}\right)^\circ$ (d) $\left(\frac{5811\pi}{8000}\right)^\circ$
42. When a pendulum of length 50 cm oscillates, it produce an arc of 16 cm. The angle so formed in degree measure is (approx)
(a) $22\frac{1}{2}^\circ$ (b) $18^\circ 35'$
(c) $27\frac{1}{2}^\circ$ (d) 45°
43. A rail road curve is to be laid out on a circle. What radius should be used if the track is to change direction by 25° in a distance of 40 metres.
(a) 91.64 m (b) 81.23 m
(c) 95.67 m (d) 98.72 m
44. An arc of a circle of radius 42 cm subtends an angle of 15° at the centre. Taking $\pi = \frac{22}{7}$, the length of the arc is:
(a) $\frac{88}{5}$ cm (b) 11 cm
(c) 12 cm (d) $\frac{14}{5}$ cm
45. If the sum and difference of two angles are 135° and $\frac{\pi}{12}$ respectively, then the value of the angles in degree measure are
(a) $70^\circ, 65^\circ$ (b) $75^\circ, 60^\circ$
(c) $45^\circ, 90^\circ$ (d) $80^\circ, 55^\circ$
46. The degree measure of 1 radian
(taking $\pi = \frac{22}{7}$)
(a) $50^\circ 16' 22''$ (approx.)
(b) $57^\circ 16' 22''$ (approx.)
(c) $57^\circ 22' 16''$ (approx.)
(d) $57^\circ 32' 16''$ (approx.)
47. In a triangle ABC, $\angle ABC = 75^\circ$ and $\angle ACB = \frac{\pi^c}{4}$. The circular measure of $\angle BAC$ is
(a) $\frac{5\pi}{12}$ radian (b) $\frac{\pi}{3}$ radian
(c) $\frac{5\pi}{6}$ radian (d) $\frac{5\pi}{2}$ radian
48. At what point of time after 3 O' clock hour hand and the minute hand of a clock occur at right angles for the first time?
(a) 90' clock
(b) 4th $37\frac{1}{6}$ min
(c) 3h 30 $\frac{8}{11}$ min
(d) 3h 32 $\frac{8}{11}$ min
49. Consider the following
I. The angular measure in radian of a circular arc of fixed length subtending at its centre decreases, if the radius of the arc is increases
II. 1800° is equal to 5 radian. Which of the above statements is/are correct?
(a) Only I (b) Only II
(c) Both I and II
(d) Neither I nor II
50. How many degrees are there in an angles which equals two-third of its complement?
(a) 36° (b) 45° (c) 48° (d) 60°
51. The earth takes 24 h to rotate about its own axis. Through what angle will it turn in 4 h and 12 min?
(a) 64° (b) 63° (c) 65° (d) 70°
52. If $\cos \theta \geq \frac{1}{2}$ in the first quadrant, then which one of the following is correct?
(a) $\theta \leq \frac{\pi}{3}$ (b) $\theta \geq \frac{\pi}{3}$
(c) $\theta \leq \frac{\pi}{6}$ (d) $\theta \geq \frac{\pi}{6}$
53. What is the angle (in radian) included between the hands of a clock, when the time is 10 min past 5?
(a) $17\pi/36$ (b) $19\pi/36$
(c) $5\pi/9$ (d) $7\pi/12$
54. If clock started at noon, then what is the angle turned by hour hands at 3 : 45 pm?
(a) 67.5° (b) 97.5°
(c) 112.5° (d) 142.5°

ANSWER KEY

1. (b)	7. (b)	13. (a)	19. (a)	25. (c)	30. (a)	35. (c)	40. (b)	45. (b)	50. (a)
2. (c)	8. (d)	14. (c)	20. (b)	26. (b)	31. (b)	36. (d)	41. (a)	46. (b)	51. (b)
3. (d)	9. (c)	15. (c)	21. (b)	27. (c)	32. (a)	37. (a)	42. (b)	47. (b)	52. (a)
4. (b)	10. (a)	16. (a)	22. (a)	28. (b)	33. (c)	38. (a)	43. (a)	48. (d)	53. (b)
5. (c)	11. (d)	17. (a)	23. (d)	29. (c)	34. (a)	39. (c)	44. (b)	49. (a)	54. (c)
6. (b)	12. (d)	18. (a)	24. (b)						

1. (b) $120^\circ = \frac{120}{180} \times \pi \text{ rad}$

$$= \frac{2\pi}{3} \text{ rad}$$

2. (c) $37\frac{1}{2}^\circ = \left(\frac{75}{2}\right)^\circ$

$$= \frac{75}{2 \times 180} \times \pi = \frac{5\pi}{2 \times 12} = \frac{5\pi}{24} \text{ rad}$$

3. (d) $11\frac{1}{4}^\circ = \left(\frac{45}{4}\right)^\circ$

$$= \left(\frac{45}{4}\right) \times \pi \text{ rad} = \frac{\pi}{16} \text{ rad}$$

4. (b) $\frac{5}{6}$ right angle

$$= \frac{5}{6} \times \frac{\pi}{2} \text{ rad} = \frac{5\pi}{12}$$

5. (c) $\pi \text{ rad} = 180^\circ$

$$\therefore 1 \text{ rad} = \left(\frac{180}{\pi}\right)^\circ$$

6. (b) $\left(\frac{1}{6}\right)^\circ = \left(\frac{1}{6} \times \frac{180}{\pi}\right)^\circ = \left(\frac{1}{6} \times \frac{180}{22} \times 7\right)^\circ$

$$= \left(\frac{105}{11}\right)^\circ$$

$$= \left(9\frac{6}{11}\right)^\circ = 9^\circ \left(\frac{6}{11} \times 60\right)' = 9^\circ \left(32\frac{8}{11}\right)'$$

$$= 9^\circ 32' \left(\frac{8}{11} \times 60\right)'' = 9^\circ 32' 43.6''$$

7.(b) Fourth angle of quadrilateral

$$= 360^\circ - (60^\circ + 120^\circ + 90^\circ)$$

$$= 90^\circ$$

$$\therefore 180^\circ = \pi^\circ$$

$$90^\circ = \frac{\pi^\circ}{2}$$

8. (d) $\angle C = 180^\circ - 30^\circ - 60^\circ = 90^\circ$

$$\therefore 180^\circ = \pi \text{ radian}$$

$$\therefore 90^\circ = \frac{\pi}{180} \times 90 = \frac{\pi}{2} \text{ radian}$$

9. (c) $560^\circ 20' = \left(560\frac{20}{60}\right)^\circ$

$$= \left(560\frac{1}{3}\right)^\circ = \left(\frac{1681}{3}\right)^\circ$$

$$= \frac{1681}{3} \times \frac{\pi}{180} = \frac{1681}{540} \pi \text{ rad}$$

10. (a) $72^\circ 40' = \left(72\frac{40}{60}\right)^\circ = \left(72\frac{2}{3}\right)^\circ$

$$= \left(\frac{218}{3}\right)^\circ = \left(\frac{218}{3} \times \frac{\pi}{180}\right)$$

$$= \left(\frac{109\pi}{3 \times 90}\right) = \frac{109\pi}{270} \text{ rad}$$

11. (d) $\pi \text{ rad} = 180^\circ$

$$\therefore 1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$6 \text{ rad} = \frac{6 \times 180^\circ}{\pi} = \frac{6 \times 180^\circ \times 7}{22}$$

$$= \frac{21 \times 180^\circ}{11} = \frac{3780^\circ}{11}$$

$$\left(343\frac{7}{11}\right)^\circ = 343^\circ \left(\frac{420}{11}\right)'$$

$$= 343^\circ \left(38\frac{2}{11}\right)'$$

$$= 343^\circ 38' \left(\frac{120}{11}\right)''$$

$$= 343^\circ 38' 11'' \text{ (approximate)}$$

Second Method,

$$\therefore 1 \text{ rad} = 57^\circ 16' 22''$$

$$6 \text{ rad} = 57^\circ \times 6 + 16' \times 6 + 22'' \times 6$$

$$= 342^\circ + 96' + 132''$$

$$= 342^\circ + (1^\circ + 36') + (2' + 12'')$$

$$= (\because 1^\circ = 60' \text{ and } 1' = 60'')$$

$$= 343^\circ 38' 12'' \text{ (approximate)}$$

12. (d) $10 \text{ rad} = (57^\circ 16' 21'') \times 10$

$$= 570^\circ + 160' + 210''$$

$$= 570^\circ + 2^\circ + 40' + 3' + 30''$$

$$= 572^\circ 43' 30''$$

13. (a) $5 \text{ units} = (29^\circ 46' 55'') \times 5$

$$= 145^\circ 230' 275''$$

$$= 145^\circ + 3^\circ + 50' + 4' + 35''$$

$$= 148^\circ 54' 35''$$

14. (c) 100 units

$$= (15^\circ 49' 50'') \times 100$$

$$= 1500^\circ 4900' 5000''$$

$$= 1500^\circ + (81^\circ + 40') + 83' + 20''$$

$$\left(\because \frac{4900}{60} = 81\frac{40}{60}, \frac{5000}{60} = 83\frac{20}{60}\right)$$

$$= 1500^\circ + 81^\circ + 40' + 1^\circ + 23' + 20''$$

$$= 1582^\circ 63' 20'' = 1583^\circ 3' 20''$$

15. (c) Wheel revolves $= \frac{90}{30}$

$$= 3 \text{ turn in one minute}$$

$$\therefore 1 \text{ turn} = 360^\circ$$

$$\therefore 3 \text{ turn} = 1080^\circ$$

16. (a) From $\theta = \frac{l}{r}$, $\theta = \frac{10}{100}$

$$= \frac{1}{10} \text{ rad} = \frac{180^\circ}{\pi \times 10} = \left(\frac{18 \times 7}{22}\right)^\circ$$

$$= \left(\frac{63}{11}\right)^\circ = \left(5\frac{8}{11}\right)^\circ$$

$$= 5^\circ \left(\frac{8}{11} \times 60'\right) = 5^\circ \left(\frac{480}{11}\right)'$$

$$= 5^\circ \left(43\frac{7}{11}\right)' = 5^\circ 43' \left(\frac{7}{11} \times 60''\right)$$

$$= 5^\circ 43' 38'' \text{ (approximate)}$$

17. (a) $r = 90 \text{ cm}$ and

$$\text{arc (s)} = 22 \text{ cm}$$

$$\therefore \theta = \left(\frac{s}{r}\right)^\circ = \left(\frac{22}{90}\right)^\circ = \left(\frac{11}{45}\right)^\circ$$

$$\Rightarrow \theta = \left(\frac{11}{45} \times \frac{180}{\pi}\right)^\circ = \left(\frac{11}{45} \times \frac{180}{22} \times 7\right)^\circ = 14^\circ$$

18. (a) $\theta = 25^\circ$

$$= \frac{25 \times \pi}{180} \text{ radians}$$

$$= \frac{5\pi}{36} \text{ radians}$$

$$\theta = \frac{s}{r}$$

$$\Rightarrow r = \frac{s}{\theta} = \frac{40}{\frac{5\pi}{36}} = \frac{40 \times 36}{5\pi} = \frac{40 \times 36 \times 7}{5 \times 22} \text{ metre}$$

$$= 91.64 \text{ metre}$$

19. (a) $2x + 3x + 5x = 180^\circ - 45^\circ = 135^\circ$
 $\Rightarrow 10x = 135^\circ$

$\Rightarrow x = \frac{135}{10} = \frac{27}{2}$

$\therefore$ Largest angle

$= 5x + 15^\circ = \left(5 \times \frac{27}{2}\right)^\circ + 15^\circ$

$= \frac{135 + 30}{2} = \frac{165^\circ}{2}$

$\therefore 180^\circ = \pi$ radian

$\therefore \frac{165^\circ}{2} = \frac{\pi}{180} \times \frac{165}{2} = \frac{11\pi}{24}$ radian

20. (b) $\angle ABC = 75^\circ$

$\therefore 180^\circ = \pi$ radian

$\therefore 75^\circ = \frac{\pi}{180} \times 75 = \frac{5\pi}{12}$ radian

$\angle BAC = \pi - \frac{\pi}{4} - \frac{5\pi}{12}$

$= \frac{12\pi - 3\pi - 5\pi}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$ radian

21. (b) Traced arc length by minute hand in 60×60 seconds $= 2\pi r$
 $\therefore$ Length of arc made in 18 seconds

$= \frac{2\pi r}{60 \times 60} \times 18$

$= 2 \times \frac{22}{7} \times \frac{35 \times 18}{60 \times 60} = 1.1$ cm

22. (a) 1 right angle $= \frac{\pi}{2}$ rad

$= 1.57$ rad (approximate)

$\frac{3}{2} = 1.5$ rad, which is an acute angle.

$\frac{4}{3} = 1.33$ rad, which is an acute angle

Third angle $= (\pi - 1.5 - 1.33)$
 $\text{and} = (3.14 - 1.5 - 1.33)$
 $= 0.31$ rad which is also an acute angle.

23. (d) Third angle

$= \pi \text{ rad} - \frac{1}{2} \text{ rad} - 2 \text{ rad}$

$= \left(\frac{22}{7} - \frac{5}{2}\right) \text{ rad} = \frac{9}{14}$ rad

$= \frac{9}{14} \times \frac{180^\circ}{\pi} = \frac{9}{14} \times \frac{180}{22} \times 7$

$= \frac{45 \times 9}{11} = \frac{405}{11} = 36 \frac{9}{11}$

24. (b) The wheel makes $\frac{24}{10}$

rotation in one second

It covers $\frac{24}{10} \times 2\pi$ rad angle in 1 second.

Hence in covering 110 rad, wheel takes

$\frac{110}{\frac{24}{10} \times 2\pi} = \frac{110 \times 10 \times 7}{24 \times 2 \times 22}$

$= 7.3$ second

25. (c) $40^\circ 20' 50'' = 40^\circ \left(20 \frac{50}{60}\right)'$

$= 40^\circ \left(\frac{125}{6}\right)' = 40^\circ \left(\frac{125}{6 \times 60}\right)^\circ$

$= \left(40 \frac{25}{72}\right)^\circ = \left(\frac{2905}{72}\right)^\circ$

$= \frac{2905}{72} \times \frac{\pi}{180}$ rad

$= \frac{581\pi}{72 \times 36} = \frac{581\pi}{2592}$ rad

26. (b) $\theta = \frac{l}{r}$, $\theta = \frac{18}{60} = \frac{3}{10}$ rad

$= 0.3$ rad

1 rad $= 57^\circ 16' 22''$ (approximate)

0.3 rad $= 5.7^\circ \times 3 = \text{more than } 17^\circ \text{ and less than } 18^\circ$

$\therefore$ From options $= 17 \frac{1}{2}$

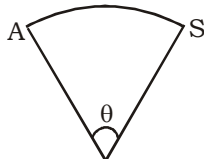
27. (c) $20^\circ = \frac{20^\circ}{180^\circ} \pi = \frac{\pi}{9}$ rad

From, $\theta = \frac{l}{r}$ $l = \theta r = \frac{\pi}{9} \times 54$

$= \frac{22}{7 \times 9} \times 54$ cm

$= \frac{22 \times 6}{7} = \frac{132}{7} = 18 \frac{6}{7}$ cm

28. (b) $l = \text{road}$



22 $\frac{1^\circ}{2} = \left(\frac{45^\circ}{2}\right) \times \pi \text{ rad} = \frac{\pi}{8} \text{ rad}$

$\therefore \theta = \frac{l}{r} \Rightarrow r = \frac{l}{\theta} = \frac{(40)}{\left(\frac{\pi}{8}\right)}$

$= \frac{320}{\pi} = \frac{320 \times 7}{22} = 101.8$ cm

29. (c) The minute hand complete one revolution in 60 minute.

$\therefore$ In 50 minute it will cover

$\frac{50}{60} = \frac{5}{6}$ of the revolution.

$\therefore 1$ revolution $= 2\pi$ radian.

$\therefore \frac{5}{6}$ revolution $= 2\pi \times \frac{5}{6} = \frac{5\pi}{3}$ radian

$\therefore$ Distance moved by tip

$= 3 \times \frac{5\pi}{3} \text{ cm} = 5\pi$ cm

$= 5 \times \frac{22}{7} \text{ cm} = 15.71$ cm

30. (a) Angle traced by the hour hand in 12 hours $= 360^\circ$

$\therefore$ Angle traced by the hour hand in 4 hrs 30 min.

$\left(= \frac{9}{2} \text{ hrs}\right) = \frac{360}{12} \times \frac{9}{2} = 135^\circ$

Angle traced by the minute hand in 60 min. $= 360^\circ$

$\therefore$ Angle traced by the minute hand in 30 min

$= \left(\frac{360}{60} \times 30\right)^\circ = 180^\circ$

Thus, the angle between two hands $= 180^\circ - 135^\circ = 45^\circ$

$= \frac{\pi}{4}$ radian

Alternate:-

$\theta = \left| \frac{11}{2} M - 30H \right|$

Where θ = angle

M = minute

H = hour

$$\theta = \left| \frac{11}{2} \times 30 - 30 \times 4 \right|$$

$$= |165^\circ - 120^\circ|$$

$$= 45^\circ = \frac{\pi}{4} \text{ radian}$$

- 31.(b) OA = OB = 15 cm (radius)
and chord AB = 15 cm

$\therefore \triangle OAB$ is an equilateral triangle.



$$\therefore \angle AOB = 60^\circ = \left(\frac{\pi}{3} \right)^c$$

$$\therefore \theta = \frac{\text{arc}}{\text{radius}} \Rightarrow \text{arc} = \theta \times r = \frac{\pi}{3} \times 15 = 5\pi \text{ cm}$$

- 32.(a) Angles of triangle
(a - d)°, a°, (a + d)°
a - d + a + a + d = 180°
 $\Rightarrow 3a = 180^\circ \Rightarrow a = 60^\circ$

$$\therefore \frac{a-d}{a+d} = \frac{60}{\pi} = \frac{60}{180} = \frac{1}{3}$$

$$\Rightarrow \frac{60-d}{60+d} = \frac{1}{3}$$

$$\Rightarrow 180 - 3d = 60 + d$$

$$\Rightarrow 4d = 120^\circ \Rightarrow d = 30^\circ$$

$\therefore$ Angles of triangle :
a - d = 60° - 30° = 30°
a = 60°
a + d = 60 + 30 = 90°

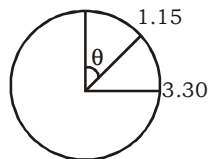
33. (c) $90^\circ = \frac{\pi}{2}$

$$\theta = \frac{l}{r} \text{ or } l = \theta r$$

$$= \frac{\pi}{2} \times 100 \text{ metres}$$

$$= \frac{3.14}{2} \times 100 = 157 \text{ metres}$$

34. (a)



From 1 hour 15 minutes to half past three, minute hand covers 2 hours 15 minutes i.e.,

$2\frac{1}{4}$ rotations.

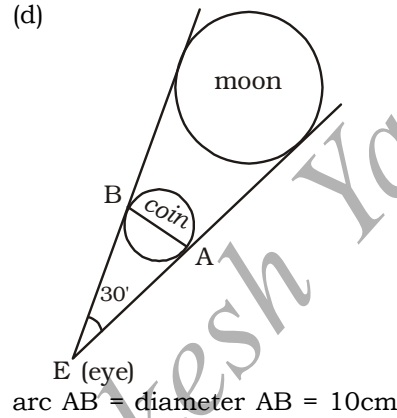
$\therefore$ It covers $2\frac{1}{4} \times 2\pi$

= (4.5) π rad distance.

35. (c) From half past six in the morning to 3 o'clock at noon, time elapsed is 8 hours 30 minutes. Since hour hand covers 30° in 5 minute therefore it covers

$$30^\circ \times 8\frac{1}{2} = 255^\circ \text{ in } 8\frac{1}{2} \text{ hours.}$$

36. (d)



arc AB = diameter AB = 10 cm

$$\theta = \left(\frac{1}{2} \right)^\circ = \frac{1}{2} \times \frac{\pi}{180} = \left(\frac{\pi}{360} \right)^c$$

$$\theta = \frac{l}{r} \Rightarrow \frac{\pi}{360} = \frac{10}{r}$$

$$\Rightarrow r = \frac{360 \times 10 \times 7}{22} = 114\frac{6}{11} \text{ cm}$$

37. (a) Revolution in 24 hours = 360°

$\therefore$ Revolution in 1 hours

$$= \frac{360^\circ}{24} = 15^\circ$$

Revolution in 4 hours

$$= 15^\circ \times 4 = 60^\circ$$

$\therefore$ Revolution in 60 minutes = 15°

Revolution in 12 minutes

$$= \frac{15^\circ \times 12^\circ}{60^\circ} = 3^\circ$$

$\therefore$ Revolution in 4 hours 12 minutes = 60° + 3° = 63°

38. (a) From Trick,

$$\text{using, } = \frac{11}{2} M = 30H \pm A$$

Here, M = 15, H = 2

$$\text{Hence, } \frac{11}{2} \times 15 = 30 \times 2 \pm A$$

$$\text{or, } A = \frac{165}{2} - 60 = 82\frac{1}{2} - 60$$

$$= 22\frac{1}{2}^\circ$$

Therefore, angle will be $22\frac{1}{2}^\circ$

39. (c) Sum of two angle be

$$= \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \text{ rad}$$

$$\therefore \frac{22}{7} \text{ rad} = 180^\circ \left(\because \pi = \frac{22}{7} \right)$$

$$\therefore \frac{5}{6} \text{ rad} = \frac{180^\circ}{22} \times 7 \times \frac{5}{6}$$

$$= \frac{30^\circ \times 35}{22} = \frac{15^\circ \times 35}{11}$$

Remaining angle

$$= 180^\circ - \frac{15^\circ \times 35}{11} = 180^\circ - \frac{525^\circ}{11}$$

$$= 180^\circ - 47\frac{8^\circ}{11} = 132\frac{3^\circ}{11}$$

40. (b) 1 rotation = 2π radian

3.5 rotation = $3.5 \times 2\pi$ radian

$$= 3.5 \times 2 \times \frac{22}{7} = 22 \text{ radian}$$

$\therefore$ Wheel rotation in one second is 22 radian

$\therefore$ Wheel rotation in 55 radian

$$\frac{55}{22} = 2.5 \text{ second.}$$

41. (a) $63^\circ 14' 51'' = 63 \left(14 \frac{51}{60} \right)'$

$$= 63 \left(14 \frac{17}{20} \right)'$$

$$= 63 \left(\frac{297}{20} \right)' = \left(63 \frac{297}{20 \times 60} \right)^\circ$$

$$= \left(63 \frac{99}{20 \times 20} \right)^\circ = \left(63 \frac{99}{400} \right)^\circ$$

$$= \left(\frac{25299}{400} \right)^\circ$$

$$= \left(\frac{25299}{400} \times \frac{\pi}{180} \right) \text{ rad}$$

$$= \left(\frac{2811\pi}{400 \times 20} \right)^\circ = \left(\frac{2811\pi}{8000} \right)^\circ$$

Trick, Value of $63^{\circ}14'51''$ is 60° (approximate)
 $\therefore$ Value of $63^{\circ}14'51''$ should be

more than $\frac{\pi}{3} = 0.33\pi$

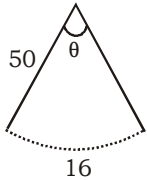
From option (a),

$$\left(\frac{2811\pi}{8000}\right) = \left(\frac{2800\pi}{8000}\right) \text{ (approx)}$$

$$= \left(\frac{28}{80}\pi\right) = 0.35\pi \text{ (approx)}$$

So, option (a) is correct.

42. (b)



$$\theta = \frac{l}{r} \Rightarrow \theta = \frac{16}{50} \text{ rad}$$

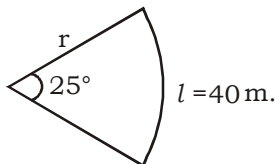
$$= \left(\frac{16}{50} \times \frac{180}{\pi}\right)^{\circ}$$

$$= \left(\frac{16 \times 18}{5} \times \frac{7}{22}\right)^{\circ}$$

$$\left(\frac{16 \times 9 \times 7}{5 \times 11}\right)^{\circ} = \left(\frac{1008}{55}\right)^{\circ}$$

$$\left(18\frac{18}{55}\right)^{\circ} = 18^{\circ}35' \text{ (approx)}$$

43. (a)



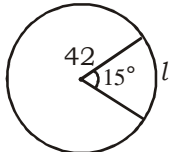
$$25^{\circ} = 25 \times \frac{\pi}{180} \text{ rad} = \frac{5\pi}{36} \text{ rad}$$

$$\text{From } \theta = \frac{l}{r}, \frac{5\pi}{36} = \frac{40}{r}$$

$$\therefore r = \frac{40 \times 36}{5\pi}$$

$$= \frac{1008}{\pi} = 91.64 \text{ meter}$$

44. (b)



$$15^{\circ} = \frac{15}{180} \times \pi = \frac{\pi}{12}$$

$$\text{From } \theta = \frac{l}{r}, l = \theta r$$

$$\text{or, } l = \frac{\pi}{12} \times 42 = \frac{22}{7} \times \frac{1}{12} \times 42$$

$$= 11 \text{ cm}$$

$$45. (b) A + B = 135^{\circ} \dots(i)$$

$$A - B = \frac{\pi}{12} \times \frac{180^{\circ}}{\pi} = 15^{\circ} \dots(ii)$$

$$(i) + (ii) 2A = 150$$

$$A = 75^{\circ}$$

$$B = 135^{\circ} - 75^{\circ} = 60^{\circ}$$

$$46. (b) 1 \text{ radian} = 1 \times \frac{180}{\pi} \text{ degrees}$$

$$= \frac{180}{22} \times 7 = \frac{630^{\circ}}{11} = \left(57\frac{3}{11}\right)^{\circ}$$

$$= 57^{\circ} + \left(\frac{3}{11} \times 60\right)'$$

$$= 57^{\circ} + \left(\frac{180}{11}\right)' = 57^{\circ} + \left(16\frac{4}{11}\right)'$$

$$= 57^{\circ} + 16' + \left(\frac{4}{11} \times 60\right)''$$

$$= 57^{\circ}16'22'' \text{ (approx.)}$$

$$47. (b) \text{ Given, } \angle ABC = 75^{\circ},$$

$$\angle ACB = \frac{\pi}{4} \text{ rad} = \left(\frac{\pi}{4} \times \frac{180}{\pi}\right)^{\circ} = 45^{\circ}$$

$$\angle BAC = 180^{\circ} - (75^{\circ} + 45^{\circ}) = 60^{\circ}$$

$$= \left(60 \times \frac{\pi}{180}\right) \text{ radian}$$

$$= \frac{\pi}{3} \text{ radian}$$

$$48. (d) \text{ In 60 minutes, minute hand rotate } 360^{\circ}$$

In x minutes, minute hand

$$\text{rotate } \frac{360}{60}x = 6x$$

In 60 minutes, hours hand rotate 30°

In x minutes, hour hand rotate

$$\frac{30}{60}x = \frac{x}{2}$$

Initial angle of hour hand at 30' clock is 90° and minutes hand is 0°

Angle after x minutes

$$= 6x - \left(90 + \frac{x}{2}\right) = 90$$

$$\Rightarrow \frac{11x}{2} = 180^{\circ}$$

$$\Rightarrow x = \frac{360}{11} = 3\text{h } 32\frac{8}{11} \text{ min}$$

$$49. (a) \text{ I. Arc length } l = r\theta$$

& for constant l, $\theta = \frac{l}{r}$ so if the radius increases, θ decreases

$$\text{II. } 1800^{\circ} = 1800 \times \frac{\pi}{180} = 10\pi$$

$$50. (a) \text{ Given, } \theta + \phi = 90^{\circ} \text{ (complementary angles)}$$

$$\& \theta = \frac{2}{3}\phi$$

$$\text{or } \theta = \frac{2}{3}(90^{\circ} - \theta)$$

$$\Rightarrow \theta = 60^{\circ} - \frac{2\theta}{3}$$

$$\Rightarrow \frac{5\theta}{3} = 60^{\circ} \Rightarrow \theta = 36^{\circ}$$

$$51. (b) \text{ In 24h, Earth rotate } 360^{\circ}$$

$$\therefore \text{ In 1 hr, Earth rotate } \frac{360^{\circ}}{24} = 15^{\circ}$$

$$\text{In 4h 12 min} = \left(4 + \frac{12}{60}\right) \text{ hr}$$

$$= \frac{21}{5} \text{ hr,}$$

$$\text{Earth rotate} = 15 \times \frac{21}{5} = 63^{\circ}$$

$$52. (a) \text{ If } \theta \text{ decreases then}$$

$\cos\theta$ increases

$$\cos\theta \geq \frac{1}{2}$$

$$\cos\theta \geq \cos\frac{\pi}{3} = \theta \leq \frac{\pi}{3}$$

$$53. (b) \text{ At 5 'o' clock, hour hand ro-}$$

$$\text{tate} = 5 \times \frac{\pi}{6} = \frac{5\pi}{6}$$

In 60 min hour hand rotate

$$= 30^{\circ} = \frac{\pi}{6}$$

10 min hour hand rotate

$$= \frac{10}{60} \times \frac{\pi}{6} = \frac{\pi}{36}$$

Total angle of hour hand

$$= \frac{5\pi}{6} + \frac{\pi}{36} = \frac{30\pi + \pi}{36} = \frac{31\pi}{36}$$

Angle rotate by minute hand in 10 minute

$$= 60^\circ = \frac{\pi}{3}$$

Angle between two hands

$$= \frac{31\pi}{36} - \frac{\pi}{3} = \frac{31\pi - 12\pi}{36} = \frac{19\pi}{36}$$

- 54.** (c) In 12h, hour hand rotate = 360°
 In 1h, hour hand rotate = 30°
 In 3h, hour hand rotate = 90°

In 45 minute, hour hand rotate

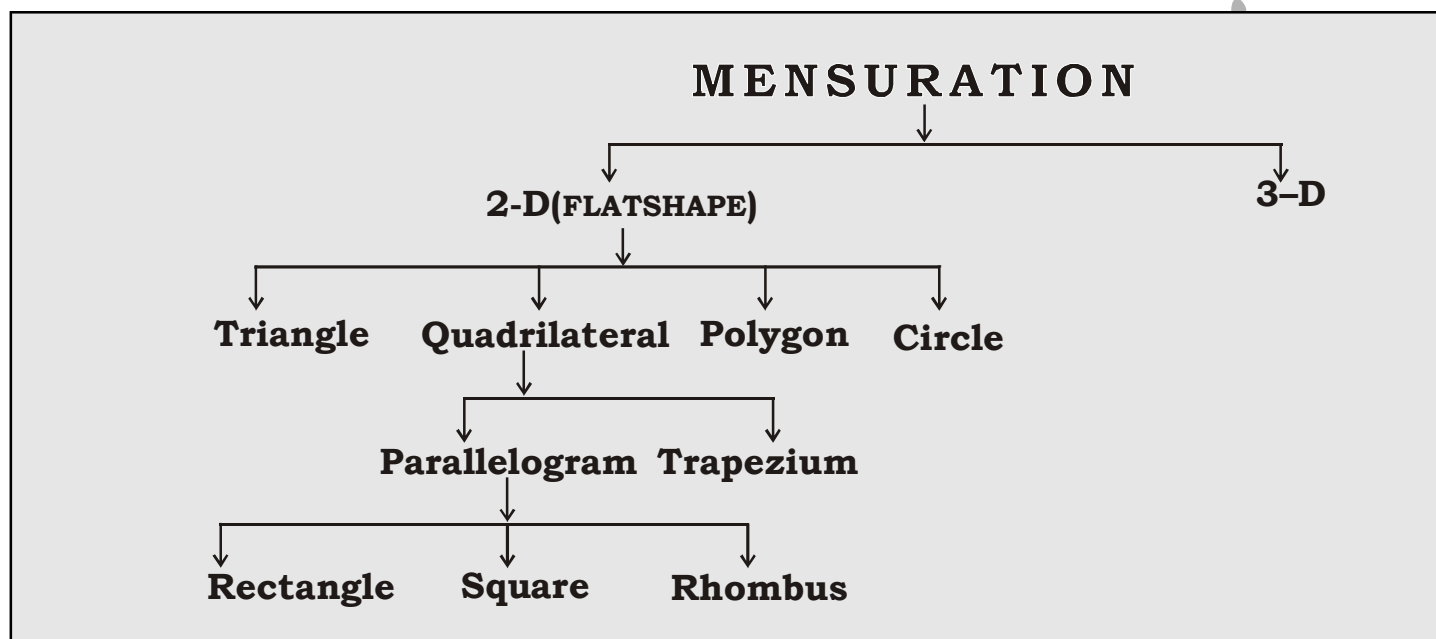
$$= \frac{45}{60} \times 30^\circ = 22.5^\circ$$

Total angle turned by hour hands at 3 : 45 pm = $90^\circ + 22.5^\circ$
 = 112.5°



Maths By Rakesh Yadav Sir

MENSURATION 2-D (TWO DIMENSIONAL)



Area:- The area of any plane figure is the amount of surface enclosed within its bounding lines. It is always expressed in square units e.g. square metres, square inches etc.

□ **Perimeter** : The perimeter of a geometrical figure is the total length of the sides enclosing the figure.

Note:-

1 hectare = 10,000 m²

100 hectare = 1,000,000 m²

Weight (mass) = Volume × density.

What is Quadrilateral?

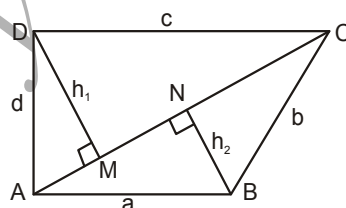
A quadrilateral is a four-sided polygon with four angles. There are many kinds of quadrilaterals. The sum of the angle of quadrilateral is 360°.

Types of quadrilaterals

- | | |
|---------------|-----|
| (1) Rectangle | (2) |
| Square | (3) |
| Parallelogram | (4) |
| Rhombus | (5) |
| Trapezium | |

Quadrilateral : A closed figure bounded by four sides.

(i) $\angle A + \angle B + \angle C + \angle D = 360^\circ$



(ii) area of quadrilateral = $\frac{1}{2} \times$

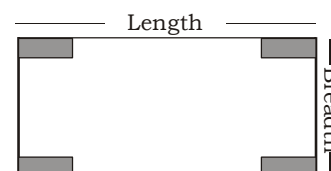
one diagonal × (sum of perpendicular to it from opposite vertices)

$$= \frac{1}{2} (AC)(h_1 + h_2)$$

(iii) $P = a + b + c + d$

Rectangle

A rectangle is a four sided flat shape where every angle is a right angle (90°) also opposite sides are parallel and of equal length. A rectangle has two diagonals, they are equal in length and intersect in the middle.



(1) Its diagonals are equal & bisect each other.

(2) Area = Length × Breadth

(3) Perimeter = 2 (Length + Breadth)

(4) Diagonal (d) = $\sqrt{l^2 + b^2}$

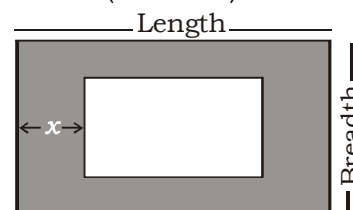
(5) (i) Area of a path inside a rectangular field:-

Area of path = $2x(l + b - 2x)$

(ii) Perimeter (P) = inner P + Outer P

$$= 2(l + b) + 2(l + b - 4x)$$

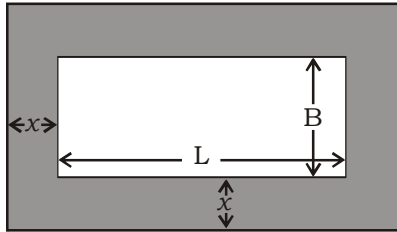
$$= 4(l + b - 2x)$$



(6) (i) Area of path outside a rectangular field:-

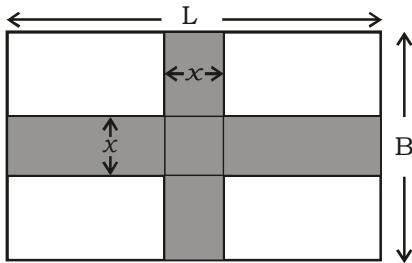
Area of path outside = $2x(l + b + 2x)$

(ii) Perimeter (P) = inner Perimeter + outer Perimeter
 $= 2(l + b) + 2(l + b + 4x)$
 $= 4(l + b + 2x)$



(7) (i) Area of path midway = $x(l + b - x)$

(ii) Perimeter of Path (P) = $2(l + b) - 4x = 2(l + b - 2x)$



(8) Room as a Rectangular figure:-
 Area of four walls of a room = Perimeter $\times$ Height = $2 \times (L + B) \times H$

(9) Area of Roof and 4 walls
 $= 2H(L+B) + LB$
 (this formula can be use when we have to paint a whole room.)

EXAMPLES

1. Area of a rectangular field of breadth 15 cm is 180 sq. cm. Find the length and perimeter of a rectangle

Sol. Area = Length $\times$ Breadth
 $180 = x \times 15$

$x = \frac{180}{15} = 12 \text{ cm} = \text{length}$

Perimeter $\Rightarrow 2(\text{length} + \text{Breadth})$
 $\Rightarrow 2(15 + 12) = 54 \text{ cm}$

2. Area of a rectangular field is 560 sq. metre. Ratio of their length & Breadth is 5:7. Find the diagonal of a rectangle?

Sol. Area = Length $\times$ Breadth
 $560 = 5x \times 7x$
 $560 = 35x^2$
 $x^2 = 16$
 $x = 4$

Length = $5x = 5 \times 4 = 20$,
 Breadth = $7x = 7 \times 4 = 28$

Diagonal = $\sqrt{20^2 + 28^2}$
 $= \sqrt{400 + 784} = \sqrt{1184} \text{ m.}$

3. The ratio between the length and width of the rectangular field is 3 : 2. If only length is increased by 5m. The new area of the field is 2600m². What is the width of the rectangular field?

(a) 60 (b) 50 (c) 40 (d) 65

Sol. Let length = $3x$, then width = $2x$
 $\therefore (3x + 5)2x = 2600$
 $\Rightarrow (3x + 5)x = 1300$
 we, go through the option
 option (c) $2x = 40 \Rightarrow x = 20$
 which satisfy the above equation

$\therefore \text{width} = 2x = 40 \text{m}$

Note : you can also solve the above equation.

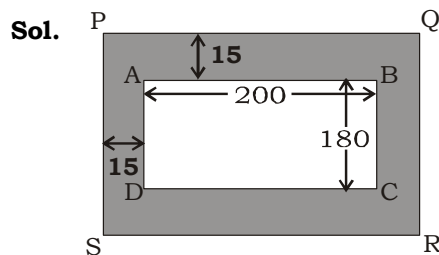
4. The length of rectangle, which is 24cm is equal to the length of a square and the area of the rectangle is 176cm less than the area of the square. What is the breadth of the rectangle?

Sol. Area of square = (side)² = $(24)^2$
 $= 576 \text{cm}^2$

Area of rectangle = length $\times$ breadth = $576 - 176 = 24 \times x$

$\Rightarrow \text{Breadth of rectangle}$
 $= \frac{400}{24} = \frac{50}{3} = 16\frac{2}{3} \text{ cm}$

5. A street of width 15 metres surrounds from outside a rectangular garden whose measurement is 200 m $\times$ 180 m. The area of the path?



Area of Rectangle ABCD = $L \times B$
 $\Rightarrow 200 \times 180 = 36,000 \text{ m}^2$
 Area of Rectangle PQRS
 $= (200 + 30) \times (180 + 30)$

$\Rightarrow 230 \times 210 = 48,300 \text{ m}^2$

Area of Path outside ABCD
 $= \text{Area of PQRS} - \text{Area of ABCD}$
 $\Rightarrow 48,300 - 36,000$
 $\Rightarrow 12,300 \text{ m}^2$

Alternate:-

Area of Path outside
 $= 2x(l + b + 2x)$
 $\Rightarrow 2 \times 15(200 + 180 + 30)$
 $\Rightarrow 30 \times 410 \Rightarrow 12,300 \text{ m}^2$

6. A path of uniform width runs mid-way of the Rectangle field having length 100m & Breadth 50m. If the path occupies 1400m². then the width of the path is?

(a) 5 (b) 10 (c) 12 (d) 8

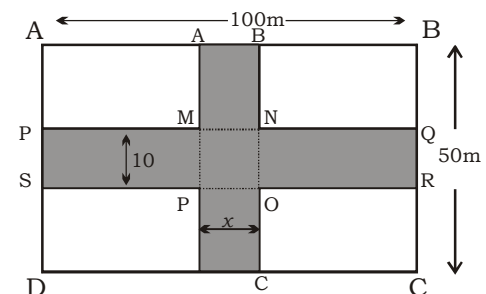
Sol. Area of path midway = $x(l + b - x)$
 $1400 = x(100 + 50 - x)$
 $1400 = 150x - x^2$
 $x^2 - 150x + 1400 = 0$
 $\Rightarrow x^2 - (140 + 10)x + 1400 = 0$
 $\Rightarrow x^2 - 140x - 10x + 1400$
 $\Rightarrow x(x - 140) - 10(x - 140)$
 $= (x - 140)(x - 10)$
 $\Rightarrow x \neq 140$ (is not possible because breadth is less than width)
 $x = 10$

Alternate:-

In these question we take option (b)
 $= 10(100 + 50 - 10)$
 $= 10 \times 140 = 1400$
 So, (b) is correct.

7. Find the perimeter of a path in the sixth question?

Sol.



$L = 100 \text{m}$

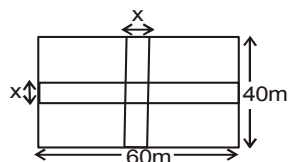
$B = 50 \text{ m}$

$x = \text{width of path} = 10 \text{m.}$

Perimeter of path = perimeter of ABCD + Perimeter of PQRS - Perimeter of MNOP
 $= 2(B + x) + 2(L + x) - 2(x + x)$
 $= 2L + 2B + 4x - 4x$
 $= 2(L + B) = 2(100 + 50) = 300$

8. A rectangular park $60 \times 40 \text{ m}^2$ has two cross roads running in the middle of the park and the rest park has been lawn. If the area of the lawn is 2109 m^2 . What is the width of the road?

Sol.



Total area of park = $60 \times 40 = 2400 \text{ m}^2$
and area of lawn = 2109 m^2 (given)
area of the cross roads = $2400 - 2109 = 291 \text{ m}^2$

$$\Rightarrow x(60 + 40 - x) = 291$$

$$\Rightarrow x^2 - 100x + 291 = 0$$

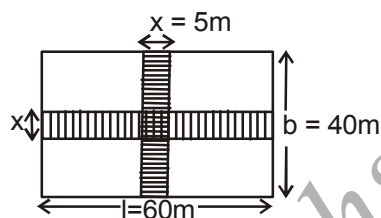
$$\Rightarrow (x - 97)(x - 3) = 0$$

$$\Rightarrow x = 3 \text{ or } 97$$

$$\Rightarrow x = 3$$

[$\therefore x = 97$ is not possible]

9. A rectangular lawn $60 \times 40 \text{ m}^2$ has two roads each 5m wide running between the park. One is parallel to length and other is parallel to width. Cost of gravelling is 60 paise/ m^2 . Find the total cost of gravelling the path?



Sol. Area of path = $x(l + b - x)$
= $5(60 + 40 - 5)$
= $5 \times 95 = 475 \text{ m}^2$
 $\therefore$ Total cost = $475 \times \frac{60}{100}$
= ₹ 285

10. A room 8 meters long, 6 metres broad and 3 metres high. Find the area of Room?

Sol. As room consist of floor, roof & 4 walls, then the area of room will be = Total surface area of room

$$\begin{aligned} &\Rightarrow 2(l + b) \times h + 2lb \\ &\Rightarrow 2[(8 + 6) \times 3 + 48] \\ &\Rightarrow 2[3(14) + 48] \\ &\Rightarrow 2[42 + 48] = 180 \text{ m}^2 \end{aligned}$$

11. A room 8m long, 6m broad and 3m high has two windows $1\frac{1}{2} \text{ m} \times 1 \text{ m}$ and a door

$2 \text{ m} \times 1\frac{1}{2} \text{ m}$. Find the cost of papering the walls with paper 50 cm wide at 25 paise per meter:

Sol. Area of walls = $2(\text{length} + \text{breadth}) \times \text{height} = 2(8 + 6) \times 3 = 84 \text{ m}^2$
Area of two windows and a door

$$= 2\left(1\frac{1}{2} \times 1\right) + \left(2 \times 1\frac{1}{2}\right) = 6 \text{ m}^2$$

$$\therefore \text{Area to be covered} = 84 - 6 = 78 \text{ m}^2$$

$$\therefore \text{Area of paper} = \text{Area to be covered} = 78 \text{ m}^2$$

$$\Rightarrow (\text{length} \times \text{breadth}) \text{ of paper} = 78$$

$$\Rightarrow \text{length of paper} =$$

$$\frac{78}{50} \times 100 \text{ m} = 156 \text{ cm}$$

$$\therefore \text{cost} = \frac{156 \times 25}{100} = ₹ 39$$

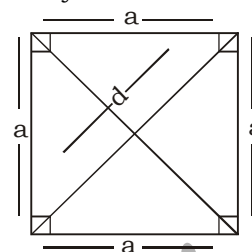
12. The dimensions of a room are 12.5 metres by 9 metres by 7 metres. There are 2 doors and 4 windows in the room, each door measures (2.5×1.2) metres and each window (1.5×1) metres. Find the cost of painting the whole room at ₹ 3.50 per square metre.

Sol. Area of 2 doors & 4 windows
= $2(2.5 \times 1.2) + 4(1.5 \times 1)$
 $\Rightarrow 6 + 6 = 12 \text{ m}^2$
Area of roof and 4 walls
= $2H(L + B) + LB$
= $2 \times 7(12.5 + 9) + 12.5 \times 9$
= $14 \times 21.5 + 112.5$
= 413.5 m^2
Area of painting = $413.5 - 12 = 401.5 \text{ m}^2$
Total Expense of painting
= 401.5×3.5
= ₹ 1405.25

Square

A square is a four-Sided flat shape where every angle is 90° and all the four sides are equal also the diagonals are equal and bisect each other at 90° .

* Every Square is a Rhombus but every Rhombus is not a square.



1. **Area** = $a^2 = (\text{side})^2$

$$= \frac{1}{2} \times (\text{diagonal})^2$$

2. **Perimeter** = $4a$

3. **Diagonal** (d) = $\sqrt{2} a$

4. **Area of Path Inside Square**

$$= 4d(x - d)$$

$$\therefore d = \text{length of Path}$$

$$x = \text{length of Square}$$

5. **Area of Path outside Square**

$$= 4d(x + d)$$

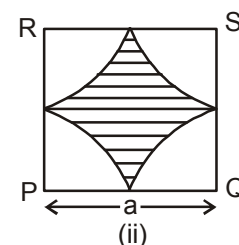
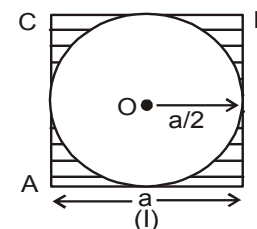
6. **Area of Path midway Square**

$$= d(2x - d)$$

7. **In circle Radius** = $\frac{\text{Side}}{2}$

8. **Circumcircle Radius** = $\frac{\text{Side}}{\sqrt{2}}$

Some-useful Results :



9. In figure (i) ABCD is a square of side 'a'

(a) O is the centre of the incircle.

In figure (ii) PQRS is a square of side 'a'

(b) P, Q, R and S are the centres of four quadrant of radius $a/2$ each.

In both case- Area of shaded

$$\text{region} = \frac{3}{14} a^2$$

10. If the additional of square increases by x times, then the area of the square becomes x^2 times.

11. If the area of the square is a cm^2 , then the area of the circle formed by the same perimeter

$$\text{is} = \frac{4a}{\pi} \text{ cm}^2.$$

EXAMPLES

1. A square field has an arm of length 125 cm. Find the area and Perimeter of a square?

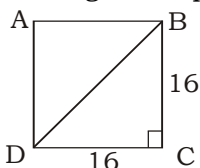
Sol. Side (a) = 125

$$\text{Area} = (\text{side})^2 = (125)^2 = 15625 \text{ cm}^2$$

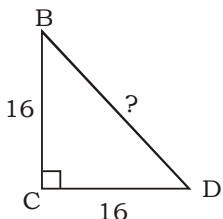
$$\text{Perimeter} = 4 \times \text{side} = 4 \times 125 = 500 \text{ cm}.$$

2. A square park has a side of 16 cm. A person cross it across diagonally. Find the distance he covered?

Sol. According to the question,



Here $\triangle BCD$ is an right angle triangle.



In Square diagonal is Angle Bisector.

$$\angle D = \angle B = 45^\circ$$

$$\sin D = \frac{P}{H} = \sin 45^\circ$$

$$= \frac{16}{x} = \frac{1}{\sqrt{2}} = \frac{16}{x}$$

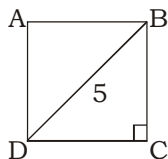
$$\Rightarrow \sqrt{2} \times 16 = 16\sqrt{2} \text{ cm}$$

Alterate:-

$$d = \sqrt{2} \times a = 16\sqrt{2}$$

3. Find the area of a Square whose diagonal is 5 cm?

Sol.



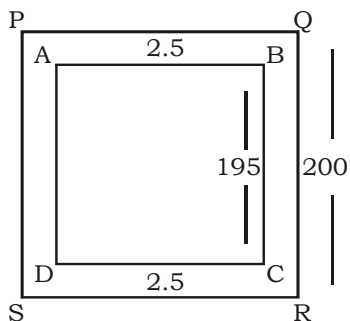
$$BD = 5 \text{ cm}$$

$$\text{Area of Square} = \frac{1}{2} \times (\text{diagonal})^2$$

$$= \frac{1}{2} \times (5)^2 = 12.5 \text{ cm}^2$$

4. A square plot is 200 m long. It has a path 2.5 m wide all round it inside. Find the area of path?

Sol.



Area of Path Inside = Area of Square PQRS - Area of Square ABCD

$$\Rightarrow (200)^2 - (195)^2$$

$$\Rightarrow 40,000 - 38025 \Rightarrow 1975 \text{ m}^2$$

Alternate:-

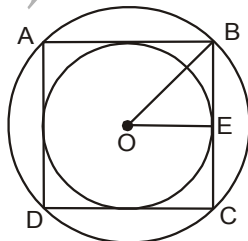
$$\text{Area of path inside} = 4d(x-d)$$

$$\text{Square} = 4 \times 2.5(200 - 2.5)$$

$$= 1975 \text{ m}^2$$

5. The length of the side of a square is 14 cm. Find out the ratio of the radii of the inscribed and circumscribed circle of the square.

Sol.



$$\text{Radius of incircle} = OE = \frac{AB}{2}$$

$$= 7 \text{ cm}$$

$$\text{Radius of circum-circle} = OB$$

$$= \frac{\text{Diagonal BD}}{2}$$

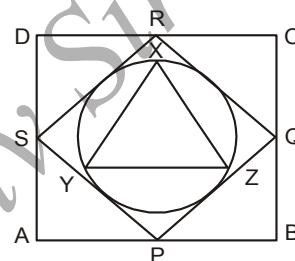
$$= \frac{\sqrt{2} \times 14}{2} = 7\sqrt{2} \text{ cm}$$

$$\therefore \text{Required ratio} = 7 : 7\sqrt{2} = 1 : \sqrt{2}$$

6. In the given figure ABCD is a square and PQRS is also a

square made by joining the mid-points of the sides of the larger square ABCD. There is inscribed a circle, In $\square PQRS$ and an equilateral $\triangle XYZ$ inscribed in the circle.

Find the ratio of the side of the square ABCD to the side of the equilateral triangle XYZ.



Sol. Let side of $\square ABCD = 2a$

$$\therefore \text{side of } \square PQRS =$$

$$= \sqrt{AP^2 + AS^2} = \sqrt{a^2 + a^2} = \sqrt{2}a$$

$$\therefore \text{radius of circle} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$$

Let side of $\triangle XYZ = b$

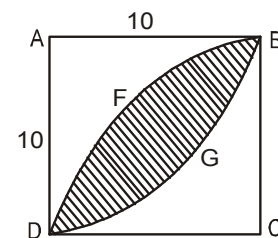
$\therefore$ radius of circumcircle of

$$\triangle XYZ = \frac{b}{\sqrt{3}}$$

$$\therefore \frac{b}{\sqrt{3}} = \frac{a}{\sqrt{2}} \Rightarrow \frac{a}{b} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\therefore \frac{2a}{b} = \frac{2\sqrt{2}}{\sqrt{3}}$$

7. In the figure, ABCD is a square with side 10 cm. BFD is an arc of a circle with centre C. BGD is an arc of a circle with centre A. What is the area of the shaded region :



Sol. Area of portion DFBGD = Area of portion DFBC - area of $\triangle BCD$

$$\frac{1}{4}\pi r^2 - \frac{1}{2}ab \sin \theta$$

$$= \frac{1}{4}\pi(10)^2 - \frac{1}{2} \times 10 \times 10$$

$$= 25\pi - 50$$

$$= \text{Area of portion DFBGD}$$

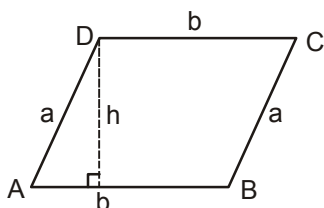
$$\therefore \text{Area of portion DFBGD}$$

$$= (25\pi - 50) + (25\pi - 50)$$

$$= 50\pi - 100$$

Parallelogram

If opposite sides of a quadrilateral are parallel, it is called parallelogram. Its opposite sides are also equal in length.



(i) **Area** = base $\times$ height = bh

(ii) **Perimeter** = $2(a + b)$

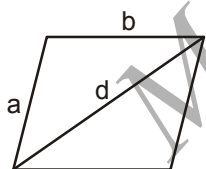
(iii) $d_1^2 + d_2^2 = 2(a^2 + b^2)$ (d_1, d_2 = length of diagonals)

(iv) **Area** = A

$$= 2\sqrt{s(s-a)(s-b)(s-d)}$$

where a & b are adjacent sides, d is the length of diagonal connecting the ends of the two sides and,

$$s = \frac{a+b+d}{2}$$



EXAMPLES

1. The area of a Parallelogram 486 cm^2 . If its altitude is 66.66% of its base then find the base and altitude?

$$\text{Sol. } 66.66\% = 66\frac{2}{3}\% = \frac{2}{3},$$

$$\therefore \text{Let Base} = x; \text{Altitude} = \frac{2}{3}x$$

Area of Parallelogram = Base $\times$ Height

$$486 = x \times \frac{2}{3}x$$

$$x^2 = \frac{486 \times 3}{2} = 729$$

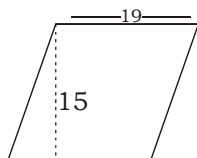
$$x = 27$$

$$\text{then Base} = 27 \text{ cm}$$

$$\text{Altitude} = 27 \times \frac{2}{3} = 18 \text{ cm}$$

2. One side of a Parallelogram is 19 cm. Its distance from the opposite side is 15 cm. Then area of the parallelogram will be:

Sol.



Area of Parallelogram = Base $\times$ Height

$$\Rightarrow 19 \times 15 \Rightarrow 285 \text{ cm}^2$$

3. In a parallelogram, the lengths of adjacent sides are 12 cm and 14 cm respectively. If the length one diagonal is 16 cm, find the length of other diagonal?

Sol. $d_1^2 + d_2^2 = 2(a^2 + b^2)$

$$d_1^2 + (16)^2 = 2(12^2 + 14^2)$$

$$d_1^2 + 256 = 2(144 + 196)$$

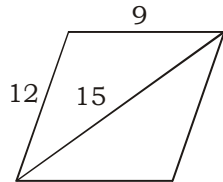
$$d_1^2 + 256 = 2 \times 340 = 680$$

$$d_1^2 = 680 - 256 = 424$$

$$d_1 = 20.6 \text{ cm}$$

4. The two adjacent sides of a parallelogram are 12 and 9 cm and the length of diagonal is 15 cm. Find the area of Parallelogram?

Sol.

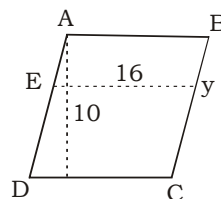


$$S = \frac{12+9+15}{2} = 18$$

$$\text{Area} = 2\sqrt{18 \times 6 \times 9 \times 3} \Rightarrow 2 \times 54 = 108 \text{ cm}^2$$

5. A parallelogram has an area of 160 cm^2 . If the distance between its opposite sides are 10 cm and 16 cm. Find the sides of the Parallelogram.

Sol.



Area of Parallelogram

$ABCD = B \times H$

$$160 = x \times 10$$

$$x = 16 \text{ cm}$$

Area of Parallelogram $ABCD = B \times H$

$$160 = y \times 16$$

$$y = 10 \text{ cm}$$

Length of the parallelogram = 16 cm

Breadth of the Parallelogram = 10 cm

6. In a $\parallel$ gm, the lengths of adjacent sides are 11 and 13 cm. If the length of one diagonal is 16 cm, find the length of other diagonal.

Sol. In a $\parallel$ gm, $d_1^2 + d_2^2 = 2(a^2 + b^2)$

$$\Rightarrow (16)^2 + d_2^2 = [(11)^2 + (13)^2] \times 2$$

$$\Rightarrow d_2^2 = 2(290) - 256 \Rightarrow d^2 = 324$$

$$\Rightarrow d = 18 \text{ cm}$$

7. Sides of a parallelogram are in the ratio 5 : 4. Its area is 1000 sq. units. Altitude on the greater side is 20 units, Altitude on the smaller side is:

Sol. Let the side of parallelogram be $5x$ and $4x$

Area of parallelogram = Base $\times$ height

$$1000 = 5x \times 20$$

$$\Rightarrow x = \frac{1000}{5 \times 20} = 10$$

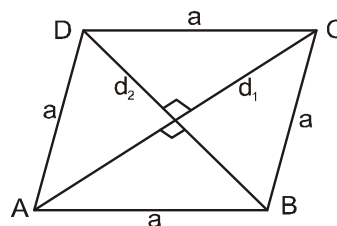
$$\therefore \text{Sides} = 50 \text{ and } 40 \text{ units}$$

$$\therefore 40 \times h = 1000$$

$$\Rightarrow h = \frac{1000}{40} = 25 \text{ units}$$

Rhombus

It is a Quadrilateral whose all four sides are equal. Diagonal bisect each other at 90° .



1. Diagonals bisect each other at 90°

2. $A = \frac{1}{2}(d_1 \times d_2)$

3. $a = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$

4. $P = 4a$

5. $d_1^2 + d_2^2 = 4a^2$

EXAMPLES

1. The area of Rhombus is 24 cm^2 has one of its diagonal is 6 cm . Find the other diagonal?

Sol. Area = $\frac{1}{2} \times d_1 \times d_2$

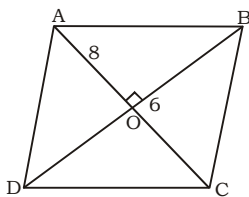
$$24 = \frac{1}{2} \times 6 \times x$$

$$\frac{24 \times 2}{6} = x$$

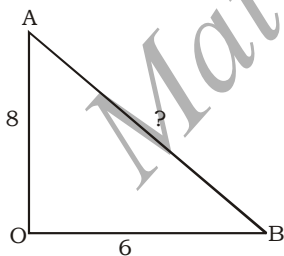
$$x = 8 \text{ cm}$$

2. Find the side of a Rhombus whose diagonals are 12 cm and 16 cm .

Sol. As diagonals of Rhombus bisect each other at 90°



In $\triangle AOB$,



$$(AB)^2 = (AO)^2 + (OB)^2$$

$$(AB)^2 = 64 + 36$$

$$AB = 10 \text{ cm}$$

Alternate:-

$$\text{Side, } a = \frac{1}{2}\sqrt{d_1^2 + d_2^2} = \frac{1}{2}\sqrt{16^2 + 12^2}$$

$$\Rightarrow \frac{1}{2} \times 20 = 10 \text{ cm}$$

3. In a Rhombus, the length of two diagonal are 120 cm & 90 cm respectively, find its perimeter.

Sol. $D_1 = 120 \text{ cm}$
 $D_2 = 90 \text{ cm}$

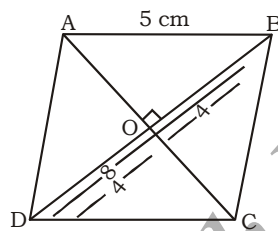
$$\text{Side (a)} = \frac{1}{2}\sqrt{D_1^2 + D_2^2} = \frac{1}{2}\sqrt{(120)^2 + (90)^2}$$

$$= \frac{1}{2} \times 150 = 75 \text{ cm}$$

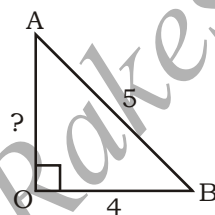
$$\text{Perimeter} = 4 \times 75 = 300 \text{ cm}$$

4. One of the diagonals of a Rhombus of side 5 cm measures 8 cm . Find the area of the Rhombus.

Sol.



In $\triangle AOB$



By phythagoras theorem,

$$(AB)^2 = (AO)^2 + (OB)^2$$

$$25 = (AO)^2 + 16$$

$$AO^2 = 9$$

$$AO = 3 \text{ cm}$$

$$\text{As } AO = 3 \text{ cm, then } AC = 2 \times AO$$

$$= 2 \times 3 = 6 \text{ cm}$$

$$D_1 = 8 \text{ cm, } D_2 = 6 \text{ cm}$$

$$\text{Area} = \frac{1}{2} \times D_1 \times D_2$$

$$= \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

Alternate : $d_1^2 + d_2^2 = 4a^2$

$$8^2 + d_2^2 = 4 \times 5^2$$

$$d_2^2 = 36$$

$$d_2^2 = 6$$

$$\text{area} = \frac{1}{2} d_1 \times d_2$$

$$= \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

5. Area of Rhombus is 256 cm^2 . One of the diagonal is half of the other. Find the perimeter and sum of the diagonals?

Sol. Let the one of the diagonals is $(d_1) = x$

other diagonal (d_2) is $= 2x$

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$256 = \frac{1}{2} \times x \times 2x$$

$$x^2 = 256$$

$$x = 16$$

$$d_1 = 16 \text{ cm, } d_2 = 32 \text{ cm}$$

$$\text{Sum of the diagonals} = (16 + 32) = 48 \text{ cm}$$

$$\text{Perimeter} = 2(\sqrt{d_1^2 + d_2^2})$$

$$= 2(\sqrt{256 + 1024})$$

$$= 2\sqrt{1280} = 71.55 \text{ cm.}$$

6. Perimeter of a Rhombus is $2p$ unit and sum of length of diagonal is m unit, then area of the Rhombus is

Sol. Side of Rhombus = $\frac{2P}{4} = \frac{P}{2}$

$$d_1 + d_2 = m \quad (\text{Given})$$

$$\left(\frac{P}{2}\right)^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2$$

$$\frac{P^2}{4} = \frac{d_1^2}{4} + \frac{d_2^2}{4}$$

$$P^2 = d_1^2 + d_2^2$$

$$\therefore (d_1 + d_2)^2 = d_1^2 + d_2^2 + 2d_1 d_2$$

$$\Rightarrow m^2 = p^2 + 2 d_1 d_2$$

$$\Rightarrow d_1 d_2 = \frac{m^2 - p^2}{2}$$

$$\text{Area of Rhombus} = \frac{1}{2} d_1 d_2$$

$$= \frac{m^2 - p^2}{4}$$

7. The perimeter of a rhombus is 146 cm and one of its diagonals is 55 cm . The other diagonal is :

Sol. Perimeter = $4 \times \text{Side}$

$$= 4 \times \frac{1}{2} \sqrt{d_1^2 + d_2^2} = 2\sqrt{d_1^2 + d_2^2}$$

$$\Rightarrow 146 = 2\sqrt{55^2 + d_2^2}$$

$$\Rightarrow 73 = \sqrt{55^2 + d_2^2}$$

$$\Rightarrow 73^2 = 55^2 + d_2^2$$

$$\Rightarrow 73^2 - 55^2 = d_2^2$$

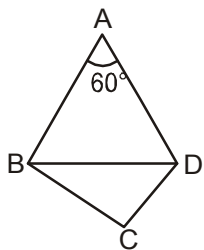
$$\Rightarrow d_2^2 = (73+55)(73-55)$$

$$= 128 \times 18$$

$$\Rightarrow d_2 = 48 \text{ cm}$$

8. The perimeter of a rhombus is 40cm and the measure of an angle is 60° , then the area of it is:

Sol.



$$\text{Side} = \frac{40}{4} = 10 \text{ cm}$$

$$AB = AD = 10 \text{ cm}$$

$$\angle ABD = \angle ADB = 60^\circ$$

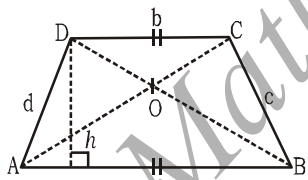
$\therefore$ Area of the rhombus

$$= 2 \times \frac{\sqrt{3}}{4} \times (AB)^2$$

$$= 2 \times \frac{\sqrt{3}}{4} \times 10 \times 10 = 50\sqrt{3} \text{ cm}^2$$

Trapezium

It is a quadrilateral, whose any two opposite sides are parallel.



- (i) **Perimeter** = $a + b + c + d$

- (ii) **Area** = $\frac{1}{2} (\text{Sum of Parallel sides}) \times \text{Distance b/w them}$

$$= \frac{1}{2} (a + b) \times h$$

- (iii) $d_1^2 + d_2^2 = c^2 + d^2 + 2(ab)$

(sum of squares of non-parallel sides) + 2 (product of parallel sides)

- (iv) If diagonals intersect at O, then

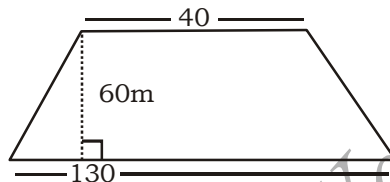
$$\frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$$

$$[\because \triangle AOB \sim \triangle COD]$$

EXAMPLES

1. A trapezium has the perpendicular distance between the two parallel sides 60m. If the lengths of the parallel sides be 40m and 130 m, then find the area of the trapezium.

Sol.



$$\text{Area} = \frac{1}{2} (a + b) \times h$$

$$= \frac{1}{2} (130 + 40) 60 = 85 \times 60$$

$$= 5100 \text{ m}^2$$

2. The cross section of a canal is a trapezium in shape. If the canal is 10 m wide at the top and 6 m wide at the bottom and the area of cross section is 640 m^2 . Find the length of canal?

Sol. Let the length of a canal = h m
Area of trapezium canal

$$= \frac{1}{2} (a + b) \times h$$

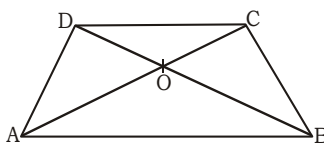
$$640 = \frac{1}{2} (10 + 6) h$$

$$1280 = 16 h$$

$$h = 80 \text{ m.}$$

3. ABCD is a trapezium with $AB \parallel CD$ whose diagonals meet at O. If $AB = 2 CD$ and length of $CD = 2.5$ cm. Find the area of $\triangle AOB$ and $\triangle COD$?

Sol.



$$AB = 5 \text{ cm}$$

$$CD = 2.5 \text{ cm}$$

$$\frac{\text{Area of } \triangle AOB}{\text{Area of } \triangle COD} = \frac{(AB)^2}{(CD)^2} = \frac{(2CD)^2}{(CD)^2} = \frac{4CD^2}{CD^2}$$

$$\Rightarrow \frac{4 \times 6.25}{6.25} = \frac{25}{6.25}$$

$$\text{Area of } \triangle AOB = 25 \text{ cm}^2$$

$$\text{Area of } \triangle COD = 6.25 \text{ cm}^2$$

4. A wall is the form of a trapezium with height 8 m and parallel sides being 6 m and 10 m. What is the cost of painting the wall, if the rate of painting is Rs. 25 per sq. m?

Sol. Area of trapezium = $\frac{1}{2} (a + b) \times h$

$$= \frac{1}{2} (6 + 10) \times 8 = 64 \text{ m}^2$$

$$\text{Total cost of painting} = \text{Rs } 25 \times 64 = \text{Rs. } 1600$$

5. Area of the trapezium formed by x -axis; y -axis and the lines $3x + 4y = 12$ and $6x + 8y = 60$ is:

Sol. For $3x + 4y = 12$

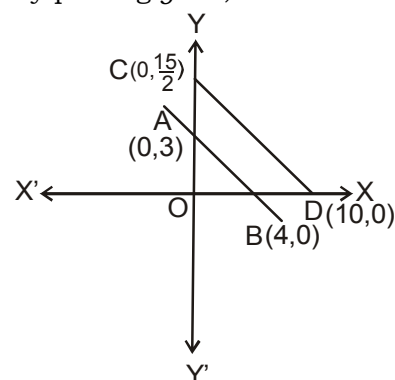
$$\text{By putting } x = 0, y = 3$$

$$\text{By putting } y = 0, x = 4$$

$$\text{For } 6x + 8y = 60,$$

$$\text{By putting } x = 0, y = \frac{15}{2}$$

$$\text{By putting } y = 0, x = 10$$



$$\therefore \text{Area of } \triangle OCD = \frac{1}{2} \times OD \times OC$$

$$= \frac{1}{2} \times 10 \times \frac{15}{2} = \frac{75}{2}$$

$$\therefore \text{Area of } \triangle OAB$$

$$= \frac{1}{2} \times OB \times OA = \frac{1}{2} \times 4 \times 3 = 6$$

$$\therefore \text{Area of trapezium} = \frac{75}{2} - 6$$

$$= \frac{75 - 12}{2} = \frac{63}{2}$$

$$= 31.5 \text{ sq. units}$$

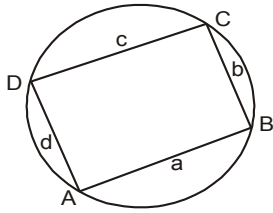
6. Find the distance between the two parallel sides of a trapezium if the area of the trapezium is 250sq.m and the two parallel sides are equal to 15m and 10m respectively.

Sol. Area = $\frac{1}{2}$ (sum of parallel sides) $\times$ (distance between them)

$$\Rightarrow 250 = \frac{1}{2}(15 + 10) \times h$$

$$\Rightarrow h = 20\text{m}$$

□ **Cyclic Quadrilateral:** A quadrilateral whose vertices lie on the circumference of the circle.



(i) **Area (A)**

$$= \sqrt{s(s-a)(s-b)(s-c)(s-d)}$$

$$\text{where, } s = \frac{a+b+c+d}{2}$$

(ii) $\angle A + \angle B + \angle C + \angle D = 2\pi$

(iii) $\angle A + \angle C = \angle B + \angle D = 180^\circ = \pi$

Triangle

A triangle is a polygon with three edges and three vertices. It is one of the basic shapes in geometry. It denoted by $\triangle ABC$.

Type of triangles:-

(1) **Scalene Triangle:-**

A scalene triangle has all its sides of different lengths. Equivalently, it has all angles of different

(i) **Area** = $\frac{1}{2} \times \text{Base} \times \text{height}$

(ii) **Area** = $\sqrt{s(s-a)(s-b)(s-c)}$

(Hero's Formula)

(iii) **Semi-perimeter (S)**

$$= \frac{(a+b+c)}{2}$$

(iv) If lengths of three medians of $\triangle ABC$ are x, y and z units, then :

$$A = \frac{4}{3} \sqrt{Sm(Sm-x)(Sm-y)(Sm-z)}$$

$$\text{where, } Sm = \frac{x+y+z}{2}$$

(v) **In circle Radius**

$$= \frac{\text{Area of } \triangle}{\text{Semi-Perimeter}}$$

$$r = \frac{\Delta}{s}$$

(vi) **Circum-circle Radius (R)**

$$= \frac{abc}{4 \times \text{area of } \triangle}$$

(a, b, c = length of sides of Triangle)

EXAMPLES

1. The area of a triangle whose sides are 15m, 16m and 17m is :

Sol. $\frac{a+b+c}{2} = \frac{15+16+17}{2} = 24\text{m}$

$$\text{Area (A)} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{24(24-15)(24-16)(24-17)}$$

$$= \sqrt{24 \times 9 \times 8 \times 7}$$

$$9 = 24 \sqrt{21} \text{m}^2$$

2. If the area of a triangle is 250m² and base : height is 4 : 5, find its height :

Sol. Let base = 4x, then height = 5x

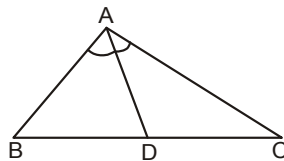
$$\therefore \frac{1}{2} \times 4x \times 5x = 250$$

$$\Rightarrow x^2 = 25$$

$$\Rightarrow x = 5$$

$$\therefore \text{height} = 5x = 5 \times 5 = 25\text{m}$$

3. In the figure given below, AD bisects $\angle BAC$. If the area of $\triangle ABD = 40\text{cm}^2$ and $AC = 3AB$, then the area of $\triangle ABC$:



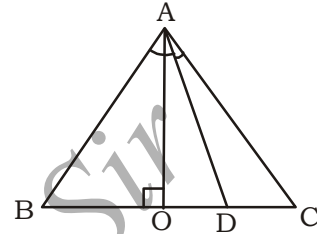
$$\frac{\text{Area of } \triangle ABD}{\text{Area of } \triangle ADC} = \frac{\frac{1}{2} \times (BD) \times (OA)}{\frac{1}{2} \times (DC) \times (OA)}$$

Sol. $\frac{\text{Area of } \triangle ABD}{\text{Area of } \triangle ADC} = \frac{BD}{DC}$

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{1}{3}$$

($\therefore$ AD is the angle bisector of $\angle BAC$)

$$\Rightarrow \text{Area of } \triangle ADC = 3 \times 40 = 120\text{cm}^2$$



$$\therefore \text{Area of } \triangle ABC = 120 + 40 = 160\text{cm}^2$$

4. What is the area of a triangle having perimeter 32cm, one side 11cm and difference of other two sides 5cm?

Sol. Let the sides of triangle be a, b and c respectively,

$$\therefore 2s = a + b + c = 32$$

$$\Rightarrow 11 + b + c = 32$$

$$\Rightarrow b + c = 32 - 11 = 21 \quad \dots(i)$$

$$\text{and } b - c = 5 \quad \dots(ii)$$

By equations (i) and (ii)

$$2b = 26 \Rightarrow b = 13$$

$$\therefore c = 13 - 5 = 8$$

$$\therefore 2s = 32 \Rightarrow s = 16$$

$$a = 11, b = 13, c = 8$$

$\therefore$ Area of triangle

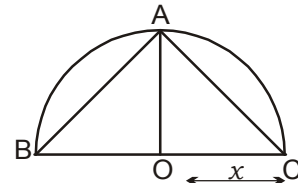
$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{16(16-11)(16-13)(16-8)}$$

$$= \sqrt{16 \times 5 \times 3 \times 8} = 8\sqrt{30} \text{ sq cm}$$

5. The area of the largest triangle that can be inscribed in a semi circle of radius x in square unit is:

Sol.



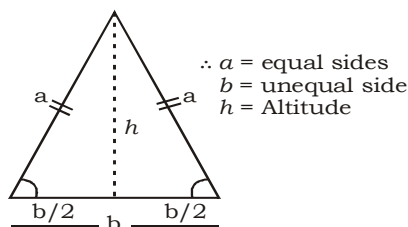
$$OA = \frac{1}{2} BC = \text{radius}$$

Area of the largest triangle

$$= \frac{1}{2} \times BC \times OA = \frac{1}{2} \times 2x \times x = x^2$$

2. Isosceles Triangle

An Isosceles triangle is a triangle with two equal sides also their opposite angles are equal.

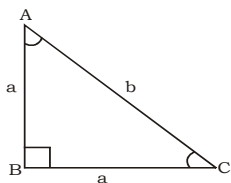


(i) **Area** = $\frac{b}{4} \sqrt{4a^2 - b^2}$

(ii) **Perimeter** = $2a + b$

(iii) **h (Altitude)** = $\frac{\sqrt{4a^2 - b^2}}{2}$

If an isosceles triangle is right angle triangle then



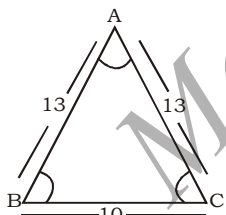
(iv) **Area** = $\frac{1}{2} a^2$

(v) **h (hypotenuse)** = $a\sqrt{2}$

EXAMPLES

1. The base and the same sides of an isosceles triangle is 10 cm and 13 cm respectively. Find its area and Perimeters ?

Sol.



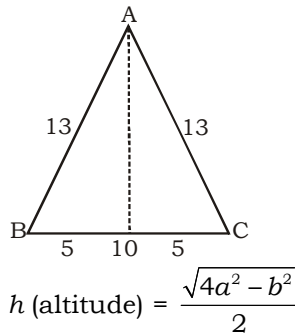
$a = 13 \text{ cm}$
 $b = 10$

$$\begin{aligned} \text{Area} &= \frac{b}{4} \sqrt{4a^2 - b^2} \\ &= \frac{10}{4} \sqrt{4 \times (13)^2 - (10)^2} \\ &= \frac{10}{4} \times 24 = 60 \text{ cm}^2 \end{aligned}$$

Perimeter = $2a + b$
 $= 2 \times 13 + 10 = 36 \text{ cm.}$

2. Find the altitude in the last question?

Sol.



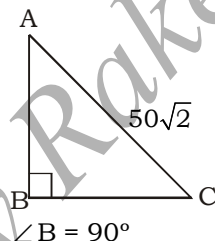
$$\begin{aligned} h \text{ (altitude)} &= \frac{\sqrt{4a^2 - b^2}}{2} \\ &= \frac{\sqrt{4 \times 169 - 100}}{2} \\ &= \frac{24}{2} = 12 \text{ cm.} \end{aligned}$$

Alternate:-

$$h = \sqrt{(13)^2 - (5)^2} = 12$$

3. A plot of land is in the shape of a right angled isosceles triangle. The length of hypotenuse is $50\sqrt{2}$ m. The cost of fencing is Rs. 3 per square meter. Find the total cost of fencing the plot ?

Sol.



$\angle B = 90^\circ$
 $\angle A = \angle C = 45^\circ$
In 45° triangle,

$$\sqrt{2} a = 50\sqrt{2}$$

$$a = 50$$

$$AB = 50 \text{ cm}$$

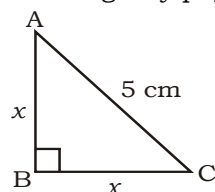
$$BC = 50 \text{ cm}$$

$$\text{Area} = \frac{1}{2} a^2 = \frac{1}{2} \times (50)^2 = 1250 \text{ cm}^2$$

Total cost of fencing the land
 $= 1250 \times 3 = \text{Rs } 3750$

4. The hypotenuse of a right angle isosceles triangle is 5 cm. Its area will be?

Sol. In triangle by pythagoras



$$\begin{aligned} \therefore x^2 + x^2 &= 5^2 \\ 2x^2 &= 25 \end{aligned}$$

$$x^2 = \frac{25}{2}$$

Area of triangle

$$= \frac{1}{2} \times x^2 = \frac{1}{2} \times \frac{25}{2} = 6.25 \text{ cm}^2$$

5. Two sides of a triangular field are 85 metres and 154 metres respectively and its perimeter is 324 metres. Find the cost of leveling the field at the rate of Rs 5 per sq. m.

Sol. P = Sum of all three sides

Third side of triangle

$$= 324 - (154 + 85)$$

$$= 85 \text{ metres}$$

Area of the field

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$S = \frac{a+b+c}{2} = \frac{p}{2} = \frac{324}{2} = 162$$

$$\begin{aligned} \text{Area} &= \sqrt{162 \times 8 \times 77 \times 77} \\ &= 2772 \text{ m}^2 \end{aligned}$$

$$\text{Cost of leveling} = 5 \times 2772$$

$$= \text{Rs. } 13,860$$

6. The perimeter of an isosceles, right-angled triangle is $2p$ unit. The area of the same triangle is:

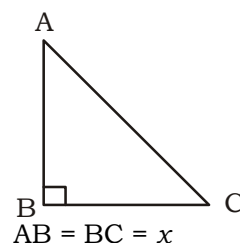
(a) $(3 - 2\sqrt{2}) p^2 \text{ sq. unit}$

(b) $(2 + \sqrt{2}) p^2 \text{ sq. unit}$

(c) $(2 - \sqrt{2}) p^2 \text{ sq. unit}$

(d) $(3 - \sqrt{2}) p^2 \text{ sq. unit}$

Sol.



$$AB = BC = x$$

$$\therefore AC = \sqrt{AB^2 + BC^2} = \sqrt{x^2 + x^2}$$

$$= \sqrt{2} x \text{ units}$$

$$\therefore 2x + \sqrt{2} x = 2p$$

$$\Rightarrow x(2 + \sqrt{2}) = 2p$$

$$\Rightarrow x = \frac{2p}{2 + \sqrt{2}} = \frac{2p(2 - \sqrt{2})}{(2 + \sqrt{2})(2 - \sqrt{2})}$$

$$= \frac{2(2-\sqrt{2})p}{4-2} = (2-\sqrt{2})p$$

$$\therefore \text{Area of triangle} = \frac{1}{2}x^2$$

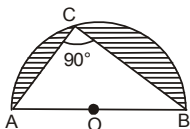
$$= \frac{1}{2} \times (2-\sqrt{2})^2 p^2$$

$$= \frac{4+2-4\sqrt{2}}{2} p^2$$

$$= (3-2\sqrt{2})p^2 \text{ sq units}$$

7. A right angled isosceles triangle is inscribed in a semi-circle of radius 7cm. The area enclosed by the semi-circle but exterior to the triangle is :

Sol.



$$\angle ACB = 90^\circ$$

$$AC = CB = X \text{ cm}$$

$$AB = 14 \text{ cm}$$

$$\text{From } \triangle ABC$$

$$AC^2 + BC^2 = AB^2$$

$$\Rightarrow x^2 + x^2 = 14^2$$

$$\Rightarrow 2x^2 = 14 \times 14$$

$$\Rightarrow x^2 = 14 \times 7$$

$$\Rightarrow x = \sqrt{14 \times 7} = 7\sqrt{2} \text{ cm}$$

$$\therefore \text{Area of } \triangle ABC = \frac{1}{2} \times AC \times BC$$

$$= \frac{1}{2} \times 7\sqrt{2} \times 7\sqrt{2} = 49 \text{ sq. cm}$$

$$\text{area of semicircle} = \frac{\pi r^2}{2}$$

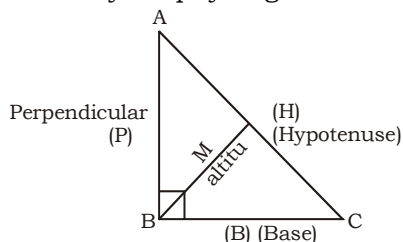
$$= \frac{22}{7} \times \frac{7 \times 7}{2} = 77 \text{ cm}^2$$

$$\text{Required area}$$

$$= 77 - 49 = 28 \text{ cm}^2$$

3. **Right angle triangle:-**

It is a triangle with an angle of 90° ($\pi/2$) radians. The sides a, b and c of such a triangle satisfy the pythagoras theorem.



$$(i) \text{ Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$(ii) \text{ Perimeter} = P + B + H$$

$$(iii) \text{ Altitude (M)} = \frac{P \times B}{H}$$

$$(iv) H^2 = P^2 + B^2$$

$$(v) \text{ In radius (r)} = \frac{P + B - H}{2} \text{ or}$$

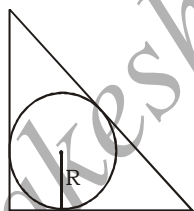
$$\frac{PB}{P+B+H}$$

$$(vi) \text{ Circum radius (R)} = \frac{H}{2}$$

EXAMPLES

1. What is the radius of the incircle of a triangle with sides 18, 24 and 30 cm?

Sol. As we know 18, 24 and 30 are triplets, then the triangle will be right angle triangle.



Incicle Radius

$$= \frac{\text{Area of triangle}}{\text{Semi-perimeter}}$$

$$S = \frac{a+b+c}{2} = \frac{18+24+30}{2} = 36 \text{ cm}$$

$$\text{Area of } \triangle = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{36 \times 18 \times 12 \times 6} = 216$$

$$\text{Incicle Radius} = \frac{\text{Area of } \triangle}{S}$$

$$= \frac{216}{36} = 6 \text{ cm.}$$

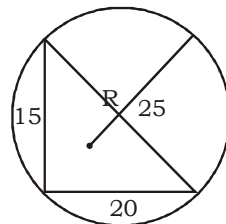
Alternate:-

$$r = \frac{P + B - H}{2}$$

$$= \frac{18 + 24 - 30}{2} = 6 \text{ cm}$$

2. What is the radius of circle drawn outside the triangle whose length of sides are 15, 20 and 25 cm?

Sol.



Circumcircle Radius

$$= \frac{abc}{4 \times \text{area of triangle}}$$

Area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$S = \frac{15+20+25}{2} = 30$$

Area of triangle

$$= \sqrt{30 \times 15 \times 10 \times 5} = 150 \text{ cm}^2$$

Circumcircle Radius

$$= \frac{abc}{4 \times \text{area of triangle}}$$

$$= \frac{15 \times 20 \times 25}{4 \times 150} = 12.5 \text{ cm}$$

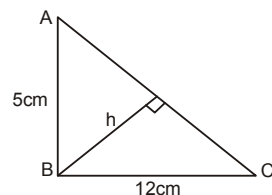
Alternate:-

$$R = \frac{\text{Hypotenuse}}{2} = \frac{H}{2}$$

$$= \frac{25}{2} = 12.5 \text{ cm}$$

3. The base and altitude of a right angled triangle are 12cm and 5cm respectively. the perpendicular distance of its hypotenuse from the opposite vertex is:

Sol.



$$AC = \sqrt{(12)^2 + (5)^2} = 13 \text{ cm}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \times 12 \times 5$$

$$= 30 \text{ cm}^2$$

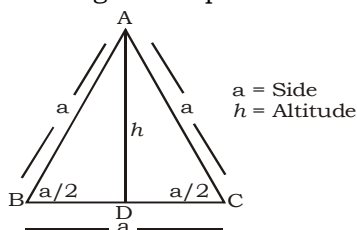
also area of $\triangle ABC$

$$= \frac{1}{2} \times (AC) \times h = \frac{13}{2} h \text{ cm}^2$$

$$\therefore \frac{13}{2} h = 30 \Rightarrow h = \frac{60}{13} = 4 \frac{8}{13} \text{ cm}$$

4. Equilateral triangle:-

It is a triangle whose all sides and angle are equal.



(i) $\text{Area} = \frac{\sqrt{3}}{4} a^2$

(ii) $\text{Perimeter (2s)} = 3a$

(iii) $\text{Altitude (h)} = \frac{\sqrt{3}}{2} a$

(iv) $\text{Incircle Radius (r)} = \frac{a}{2\sqrt{3}}$

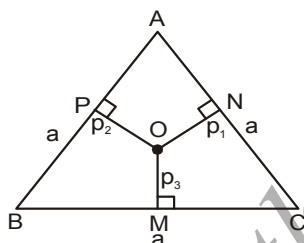
(v) $\text{Circumcircle Radius (R)} = \frac{a}{\sqrt{3}}$

We can say "r" = $\frac{R}{2}$

(vi) $\angle A = \angle B = \angle C = 60^\circ$

$$A = \frac{\sqrt{3}}{4} a^2 = \frac{h^2}{\sqrt{3}}$$

(vii) If P_1, P_2 and P_3 are perpendicular lengths from any interior point (O) of an equilateral $\triangle ABC$ to all its three sides respectively, then:-



$$P_1 + P_2 + P_3 = \frac{\sqrt{3}}{2} a = h$$

$$\Rightarrow a = \frac{2}{\sqrt{3}} (P_1 + P_2 + P_3)$$

EXAMPLES

1. The area of an equilateral triangle is $400\sqrt{3}$ sq. m. Its perimeter will be:-

Sol. $\text{Area} = \frac{\sqrt{3}}{4} a^2 = 400\sqrt{3}$

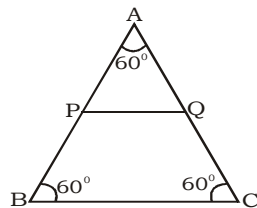
$$a^2 = \frac{400\sqrt{3} \times 4}{\sqrt{3}} = 1600$$

$$a = 40$$

$$\text{Perimeter} = 3a = 3 \times 40 = 120 \text{ m.}$$

2. ABC is an equilateral triangle. P and Q are two point on $\overline{AB}$ and $\overline{AC}$ respectively such that $\overline{PQ} \parallel \overline{BC}$. If $\overline{PQ} = 5\text{cm}$ then the area of $\triangle APQ$?

Sol.



$$\therefore \overline{PQ} \parallel \overline{BC}$$

$$\angle APQ = \angle ABC = 60^\circ$$

(corresponding anlg)

$$\angle AQP = \angle ACB = 60^\circ$$

(corresponding anlg)

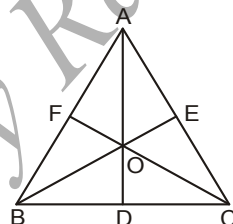
$\triangle APQ$ is an equilateral triangle

$$\text{Area of } \triangle APQ = \frac{\sqrt{3}}{4} a^2$$

$$= \frac{\sqrt{3}}{4} \times 25 = \frac{25\sqrt{3}}{4} \text{ cm}$$

3. If the circumradius of an equilateral triangle be 10cm, then the measure of its in-radius is:

Sol.



$$\text{Let } AB = x \text{ cm}$$

$$\therefore BD = \frac{x}{2}$$

$$AD = \sqrt{x^2 - \frac{x^2}{4}} = \frac{\sqrt{3}}{2} x \text{ cm.}$$

$$\therefore OD = \frac{1}{3} \times \frac{\sqrt{3}}{2} x = \frac{x}{2\sqrt{3}} \text{ cm.}$$

$$OB = \sqrt{BD^2 + OD^2}$$

$$= \sqrt{\frac{x^2}{4} + \frac{x^2}{12}} = \sqrt{\frac{4x^2}{12}} = \frac{x}{\sqrt{3}} \text{ cm.}$$

$$\therefore \frac{x}{\sqrt{3}} = 10 \Rightarrow x = 10\sqrt{3} \text{ cm.}$$

$$\therefore OD = \frac{x}{2\sqrt{3}} = \frac{10\sqrt{3}}{2\sqrt{3}} = 5 \text{ cm.}$$

Alternatively:-

$$\text{Circumradius (R)} = \frac{a}{\sqrt{3}}$$

$$\Rightarrow 10 = \frac{a}{\sqrt{3}}$$

$$\Rightarrow a = 10\sqrt{3}$$

$$\therefore \text{Inradius} = \frac{a}{2\sqrt{3}}$$

$$= \frac{10\sqrt{3}}{2\sqrt{3}} = 5 \text{ cm}$$

Alternate:-

$$r = \frac{R}{2} = \frac{10}{2} = 5$$

4. If the area of square is $3\sqrt{3}$ times the area of an equilateral triangle, then the ratio of the sides of the square to the side of the equilateral triangle is equal to :

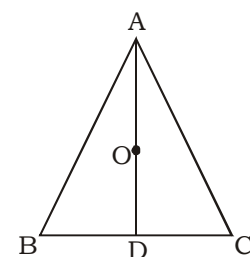
Sol. Let the side of square = x and the side of equilateral triangle = y

$$\therefore x^2 = 3\sqrt{3} \times \left(\frac{\sqrt{3}}{4} y^2 \right) \Rightarrow x^2 = \frac{9}{4} y^2$$

$$\Rightarrow \frac{x}{y} = \frac{3}{2} \Rightarrow x : y = 3 : 2$$

5. The in-radius of an equilateral triangle is of length 3 cm. Then the length of each medians is.

Sol.



$$AO : OD = 2 : 1$$

$$AO = \text{Circum radius}$$

$$OD = \text{Inradius} = 3 \text{ cm}$$

$$AD = 2 + 1 = 3$$

$$AD = 3 \times 3 = 9 \text{ cm}$$

6. The circum-circle radius of an equilateral triangle is 8 cm. The inradius of the triangle is-

Sol. Circum circle radius = $\frac{a}{\sqrt{3}}$

$$8 = \frac{a}{\sqrt{3}} \Rightarrow a = 8\sqrt{3}$$

$$\text{In radius} = \frac{a}{2\sqrt{3}} = \frac{8\sqrt{3}}{2\sqrt{3}} = 4 \text{ cm.}$$

Alternate:-

$$r = \frac{R}{2} = \frac{8}{2} = 4 \text{ cm}$$

7. The area of an equilateral triangle inscribed in a circle is $9\sqrt{3} \text{ cm}^2$. The area of the circle is :

Sol. Area of equilateral Δ
 $= \frac{\sqrt{3}}{4} (\text{side})^2 = 9\sqrt{3}$
 $\Rightarrow \text{side} = 6 \text{ cm}$

$\therefore$ circum-radius of equilateral Δ

$$= \frac{\text{side}}{\sqrt{3}} = \frac{6}{\sqrt{3}} = 2\sqrt{3} \quad \text{So, Area}$$

$$\text{of circle} = \pi \times (2\sqrt{3})^2 = 12\pi \text{ cm}^2$$

8. The area of an equilateral triangle is $4\sqrt{3} \text{ cm}^2$. the length of each side of triangle is

Sol. Area $= \frac{\sqrt{3}}{4} a^2 \Rightarrow 4\sqrt{3} = \frac{\sqrt{3}}{4} a^2$

$$a^2 = \frac{16\sqrt{3}}{\sqrt{3}} = 16 \Rightarrow a = 4 \text{ cm}$$

Each side of triangle is 4 cm.

9. From a point in the interior of an equilateral triangle, the perpendicular distance of the sides are $\sqrt{3} \text{ m}$, $2\sqrt{3} \text{ m}$ and $5\sqrt{3} \text{ m}$ respectively. The perimeter of the triangle is :

Sol. $P_1 = \sqrt{3}$,

$$P_2 = 2\sqrt{3}, P_3 = 5\sqrt{3}$$

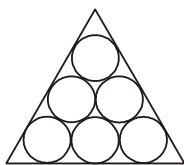
$$\therefore P = \frac{\sqrt{3}}{2} a$$

$$= P_1 + P_2 + P_3 = 8\sqrt{3}$$

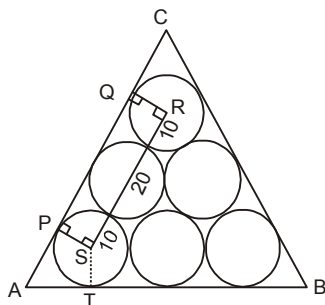
$$\Rightarrow a = 16$$

$$\therefore \text{Perimeter} = 3a = 48 \text{ m}$$

10. An equilateral triangle circumscribes all the circles, each with radius 10 cm. What is the perimeter of the equilateral triangle?



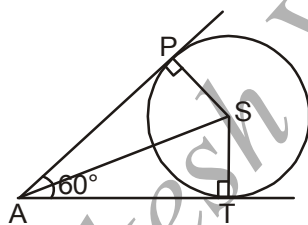
Sol.



PQRS is a rectangle.

$$\therefore PQ = 10 + 20 + 10 = 40 \text{ cm}$$

for AP,



$$\angle A = 60^\circ, \angle PST = 120^\circ$$

$$\therefore \angle PSA = \angle AST = \frac{120^\circ}{2} = 60^\circ$$

$$\text{and } \angle PAS = \angle SAT = 30^\circ$$

$$\therefore \text{In } \Delta PSA, \tan 30^\circ$$

$$= \frac{PS}{AP} = \frac{10}{AP}$$

$$\Rightarrow AP = \frac{10}{\tan 30^\circ}$$

$$\Rightarrow AP = 10\sqrt{3}$$

Similarly;

$$QC = 10\sqrt{3}$$

$$\therefore AC = PQ + AP + QC = 40 + 10\sqrt{3} + 10\sqrt{3}$$

$$= 20(2 + \sqrt{3}) \text{ cm}$$

$$\therefore AB = BC = AC$$

$$= 20(2 + \sqrt{3}) \text{ cm}$$

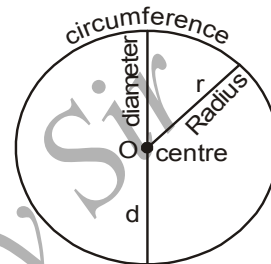
$$\therefore \text{Perimeter of } \Delta ABC$$

$$= 60(2 + \sqrt{3}) \text{ cm}$$

Circle

A circle is a set of points on a plane which lie at a fixed distance from a fixed-point.

The fixed point is known as 'centre' and the fixed distance is called the 'radius'.



- (i) Circumference or Perimeter of circle

$$(P) = 2\pi r = \pi d \quad (d = \text{diameter})$$

- (ii) Area $= A = \pi r^2 = \frac{\pi d^2}{4}$

$$\therefore \text{Diameter of the circle} = d$$

$$= \sqrt{\frac{4A}{\pi}}$$

EXAMPLES

1. The radii of two circles 7 cm and 24 cm. the area of third circle is equal to the sum of the area of the two circles. The radius of the third circle is.

Sol. Area of the $C_1 = \pi r_1^2 = 49\pi \text{ cm}^2$

$$\text{Area of the } C_2 = \pi r_2^2 = 576\pi \text{ cm}^2$$

$$\text{Area of } C_3 = \text{Area of } C_1 + \text{Area of } C_2$$

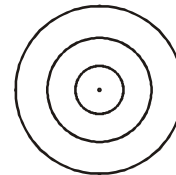
$$\pi r_3^2 = 49\pi + 576\pi = 625\pi$$

$$r_3^2 = 625$$

$$r_3 = 25 \text{ cm.}$$

2. The area of circle whose radius is 6 cm is trisected by two concentric circles. The radius of the smallest circle is

Sol.



$$\begin{aligned} \text{Area of } C_1 &= \pi r^2 \\ &= \pi (6)^2 = 36\pi \end{aligned}$$

After Trisected, the area of

$$\text{Smallest circle} = \frac{1}{3} \times 36\pi = 12\pi$$

Area of smallest circle,

$$12\pi = \pi r^2$$

$$r^2 = 12$$

$$r = 2\sqrt{3} \text{ cm}$$

3. The area of circle is increased by 22 cm^2 when its radius is increased by 1 cm. The original radius of the circle is.

Sol. $\pi(r+1)^2 - \pi r^2 = 22$
 $\pi(r^2 + 1 + 2r - r^2) = 22$
 $\pi(2r + 1) = 22$

$$\frac{22}{7}(2r + 1) = 22$$

$$2r = 7 - 1$$

$$r = 3 \text{ cm}$$

4. The area of a circle is halved when its radius is decreased by n . Find its radius :

Sol. By the question, we have

$$\pi r^2 - \pi(r-n)^2 = \frac{\pi r^2}{2}$$

$$\Rightarrow -(r-n)^2 = \frac{-r^2}{2}$$

$$\Rightarrow r^2 - (\sqrt{2}(r-n))^2 = 0$$

$$\Rightarrow [r - \sqrt{2}(r-n)][r + \sqrt{2}(r-n)] = 0$$

$$\Rightarrow r = \sqrt{2}(r-n) \text{ or } r = -\sqrt{2}(r-n)$$

$$\Rightarrow r = \sqrt{2}r - \sqrt{2}n$$

$$\Rightarrow r = \frac{\sqrt{2}n}{\sqrt{2}-1}$$

5. The area of a circular field is equal to the area of a rectangular field. The ratio of the length and the breadth of the rectangular field is 14 : 11 respectively and perimeter is 100 meters. What is the diameter of circular field ?

Sol. Let length = $14x$, then breadth = $11x$

$$\therefore 2(14x + 11x) = 100$$

$$\Rightarrow 50x = 100 \Rightarrow x = 2$$

$$\therefore \text{Area of rectangular field} = 28 \times 22 \text{ m}^2$$

$$\therefore \text{Area of circular field} = 28 \times 22 \text{ m}^2$$

$$\Rightarrow \pi r^2 = 28 \times 22 \text{ m}^2$$

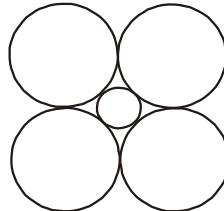
$$r^2 = \frac{28 \times 22}{22} \times 7$$

$$= 28 \times 7 = 7 \times 7 \times 4$$

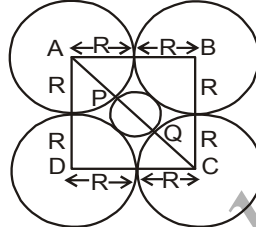
$$\Rightarrow r = 7 \times 2 = 14 \text{ m}$$

$$\therefore d = 2r = 28 \text{ m}$$

6. In the given figure, when the outer circles all have radii 'R' then the radius of the inner circle will be :



Sol.



let radius of inner circle = r

A, B, C, D are the centres of the four outer circles

$\therefore$ ABCD is a square of side $2R$

$$\therefore AC = \sqrt{2}(\text{side}) = \sqrt{2}(2R) = 2\sqrt{2}R$$

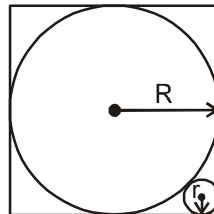
$$\therefore PQ = AC - AP - QC$$

$$= 2\sqrt{2}R - R - R = 2R(\sqrt{2} - 1)$$

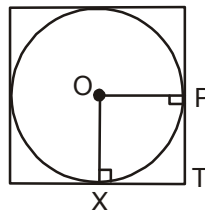
$$\Rightarrow 2r = 2R(\sqrt{2} - 1)$$

$$\Rightarrow r = R(\sqrt{2} - 1)$$

7. In the given figure, find the radius of smaller circle (r) :



Sol.

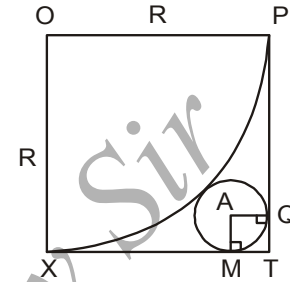


$$\text{in } \square OPTX, \angle P = \angle X = \angle T = 90^\circ$$

$\therefore \angle O = 90^\circ$ and $OP = OX = R$
 $\Rightarrow \square OPTX$, is a square of side R

$$\therefore OT = \sqrt{2}R$$

Similarly, $AQTM$ is a square of side r .



$$\therefore AT = \sqrt{2}r$$

$$\therefore OT = OA + AT$$

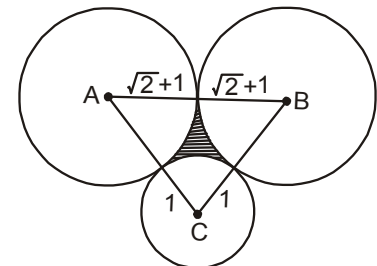
$$\Rightarrow \sqrt{2}R = (R + r) + \sqrt{2}r$$

$$\Rightarrow (\sqrt{2} - 1)R = (1 + \sqrt{2})r$$

$$\Rightarrow r = \left(\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right) R = (3 - 2\sqrt{2})R$$

8. Three circles of radius $\sqrt{2}+1$, $\sqrt{2}+1$ and 1 unit, touch each other externally, then find the perimeter of the surrounded part by three circles.

Sol.



$$AB = 2(\sqrt{2} + 1) \text{ and } AC = BC = 2 + \sqrt{2}$$

$$\therefore AB^2 = 4(\sqrt{2} + 1)^2$$

$$\text{and } AC^2 = BC^2 = (2 + \sqrt{2})^2$$

So, it is clear

$$AB^2 = AC^2 + BC^2$$

i.e. ABC is an isosceles right angled triangle.

$$\therefore \angle ACB = 90^\circ \text{ and } \angle CAB = \angle ABC = 45^\circ$$

∴ required perimeter

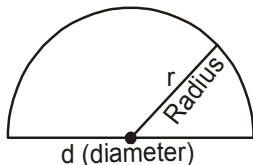
$$\begin{aligned}
 &= 2\pi(\sqrt{2}+1) \frac{45^\circ}{360^\circ} \\
 &+ 2\pi(\sqrt{2}+1) \frac{45^\circ}{360^\circ} + \\
 &2\pi \times 1 \times \frac{90^\circ}{360^\circ} \\
 &= 2 \times 2\pi(\sqrt{2}+1) \frac{1}{8} + 2\pi \frac{1}{4} \\
 &= \frac{2\pi}{4}(\sqrt{2}+1+1) \\
 &= \frac{\pi}{2}(\sqrt{2}+2)
 \end{aligned}$$

Semi Circle

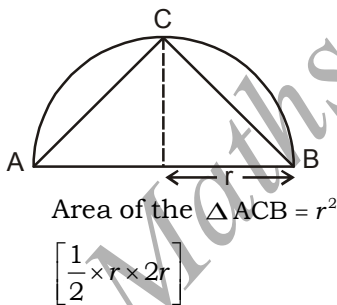
It is a figure enclosed by a diameter and the part of the circumference cut off by it.

- (i) Circumference (Perimeter) = $\pi r + 2r = \pi r + d$

(ii) Area(A) = $\frac{\pi r^2}{2}$.



- (iii) The area of largest triangle inscribed in a semi-circle of radius r is r^2 .



EXAMPLES

1. If the perimeter of a Semi-circular field 36m. Find its radius?

Sol. Circumference = $\pi r + 2r$
 $\pi r + 2r = 36$
 $r(\pi + 2) = 36$
 $r\left(\frac{22}{7} + 2\right) = 36$

$$r = \frac{36}{\frac{22}{7} + 2} = 7 \text{ cm.}$$

2. The perimeter of a Semi-circle is numerically equal to its area. The length of the diameter is:-

Sol. $\pi r + 2r = \frac{1}{2}\pi r^2$
 $r(\pi + 2) = \frac{1}{2}\pi r^2$
 $2\pi + 4 = \pi r$
 $r = \frac{4}{\pi} + 2$
 $\Rightarrow r \frac{4 \times 7}{22} + 2 = \frac{72}{22}$
 Diameter = $2 \times \frac{72}{22}$

$$= 6 \frac{6}{11} \text{ meters.}$$

3. A semi-circular shaped window has diameter 98 cm, its perimeter equals:-

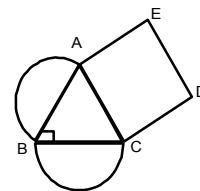
Sol. Radius = $\frac{\text{Dia}}{2} = \frac{98}{2} = 49 \text{ cm}$
 Perimeter = $\pi r + 2r$
 $= \frac{22}{7} \times 49 + 2 \times 49 = 252 \text{ cm}$

4. The perimeter of semi-circular area is 18cm, then the radius is : (using $\pi = \frac{22}{7}$)

Sol. Perimeter of semi-circular region = 18cm
 $\Rightarrow \pi r + 2r = 18$
 $\Rightarrow r(\pi + 2) = 18$
 $\Rightarrow r\left(\frac{22}{7} + 2\right) = 18$
 $\Rightarrow r\left(\frac{36}{7}\right) = 18$

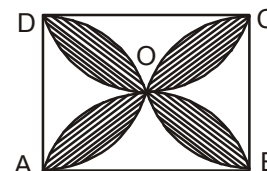
$$\Rightarrow r = \frac{18 \times 7}{36} = \frac{7}{2} = 3 \frac{1}{2} \text{ cm}$$

5. The area of the square on AC as a side is 128cm. What is the sum of the areas of semicircles drawn on AB and AC as diameters, given ABC is an isosceles right angled triangle and AC is its hypotenuse.

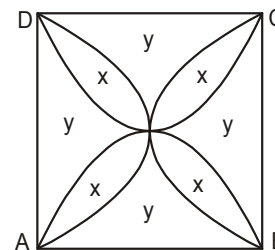


Sol. Let $AB = BC = x$,
 then $AC = \sqrt{2}x$
 But $AC = \sqrt{128} = 8\sqrt{2} \text{ cm}$
 $\sqrt{2}x = 8\sqrt{2} \Rightarrow x = 8 \text{ cm}$
 Area of semicircles
 $= \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$
 $= \frac{1}{2}\pi[2 \times 16] = 16\pi \text{ cm}^2$

6. In the given figure ABCD is a square. Four equal semicircles are drawn in such a way that they meet each other at 'O'. Sides AB, BC, CD and DA are the respective diameters of the four semicircles. Each of the side of the square is 8cm. Find the area of the shaded region.



Sol.



Let area of each shaded portion = x
 and area of each unshaded portion = y
 total area of square = $(8)^2 = 64 \text{ cm}^2$

$$\therefore 4(x + y) = 64$$

$$\Rightarrow x + y = 16 \quad \dots(i)$$

Again in a semicircle,

$$AOB = x + y + x = \frac{1}{2}\pi \times (4)^2$$

$$\Rightarrow 2x + y = 8\pi \quad \dots(ii)$$

From (i) & (ii) we get.

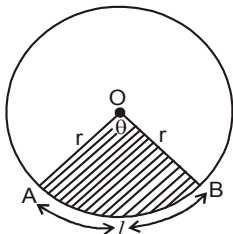
$$x = 8\pi - 16 = 8(\pi - 2)$$

$$\therefore \text{Total area of shaded region} = 32(\pi - 2)\text{cm}^2$$

Sector

A sector is a figure enclosed by two radii and an arc lying between them.

For sector AOB,



(i) $l = \text{Arc AB} = (2\pi r) \frac{\theta}{360^\circ}$

(ii) Area of sector ACBO

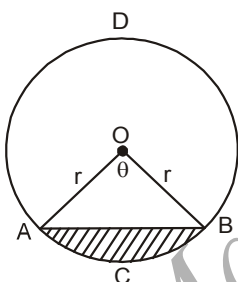
$$= \frac{1}{2} (\text{arc AB}) \times \text{radius}$$

$$= (\pi r^2) \frac{\theta}{360^\circ}$$

(iii) Perimeter (P) = Arc AB + 2r
 $= l + 2r$

Segment of a circle

A figure enclosed by a chord and an arc which it cuts off.



(i) Area of segment ACB. (minor segment) = area of sector ACBO - area of $\triangle OAB$.

(ii) Area of segment ADB (major segment) = area of circle - area of segment ACB.

(iii) Perimeter (P) = arc AB + $\theta \cdot r$

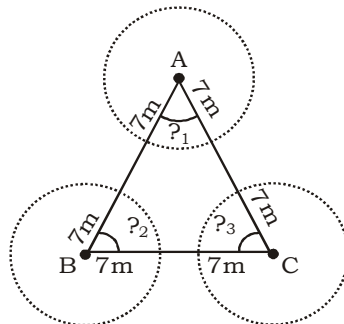
$$= 2r \left[\frac{\pi\theta}{360^\circ} + \sin\left(\frac{\theta}{2}\right) \right]$$

(iv) Arc = Angle $\times$ Radius

EXAMPLES

1. At each corner of a triangles field of sides 26m, 28m, and 30m, a cow is tethered by a rope of length 7m. The area ungrazed by the cows is.

Sol.



Area grazed by all cows

$$\text{Area of sector} = \frac{\pi r^2 \theta}{360^\circ}$$

$$\text{Now, } = \frac{\pi r^2 \theta_1}{360^\circ} + \frac{\pi r^2 \theta_2}{360^\circ} + \frac{\pi r^2 \theta_3}{360^\circ}$$

$$= \left(\frac{\pi r^2}{360^\circ} \right) (\theta_1 + \theta_2 + \theta_3)$$

$$\text{Here, } \theta_1 + \theta_2 + \theta_3 = 180^\circ$$

$$r = 7$$

$$= \frac{\pi r^2}{360^\circ} \times 180^\circ = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 7^2 = 77 \text{ cm}$$

Now, Area of $\triangle ABC$,

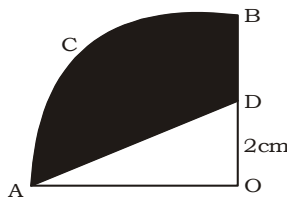
$$\text{Semi-perimeter} = \frac{26 + 28 + 30}{2} = 42$$

$$\text{Area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{42 \times 16 \times 14 \times 12} = 336$$

$$\text{Ungrazed area} = 336 - 77 = 259 \text{ m}^2$$

2. In the adjoining figure, AOB represents a quadrant of a circle of radius 4cm with centre O. Calculate the area of the shaded portion.



Sol. Area of shaded region = Area of quadrant - Area of $\triangle AOB$

$$= \frac{\pi r^2}{4} - \frac{1}{2} \times 4 \times 4$$

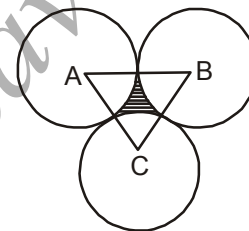
$$= \frac{\pi \times 4 \times 4}{4} - 4 = 4\pi - 4$$

$$= 4(\pi - 1)\text{cm}^2$$

$$= 4(3.14 - 1) = 4 \times 2.14$$

$$= 8.56 \text{ cm}^2$$

3. Find the area of the shaded region if the radius of each of the circle is 1cm.



Sol. ABC is an equilateral triangle with sides = 2cm

$\therefore$ Area of shaded region

$$= \text{Area of } \triangle ABC - \text{Area of 3 quadrants.}$$

$$\frac{\sqrt{3}}{4} (2)^2 - 3 \left(\pi r^2 \frac{\theta}{360^\circ} \right)$$

$$[\theta = 60^\circ \because \triangle ABC \text{ is an equilateral triangle}]$$

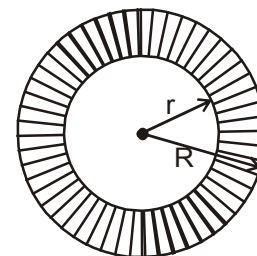
$$= \frac{\sqrt{3}}{4} \times 4 - 3 \left(\pi \times 1 \times \frac{1}{6} \right)$$

$$= \sqrt{3} - \frac{\pi}{2}$$

Ring or Circular Path :

R $\rightarrow$ outer radius

r $\rightarrow$ inner radius



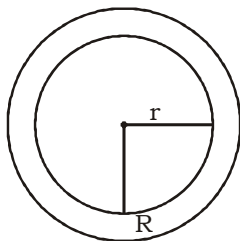
(i) Area (A) = $\pi (R^2 - r^2)$

(ii) Perimeter = $2\pi (R + r)$

EXAMPLES

1. The area of ring between two concentric circles, whose circumferences are 44 cm and 66 cm.

Sol.



C_1 (Smaller Circle);
 C_2 (Bigger Circle)

Circumference of $C_1 = 2\pi r_1$

$$2\pi r_1 = 44$$

Circumference of $C_2 = 2\pi r_2$

$$2\pi r_2 = 66$$

$$r_1 = \frac{44}{2\pi} \times 7 = 7 \text{ cm}$$

$$r_2 = \frac{66}{2\pi} \times 7 = 10.5 \text{ cm}$$

Area of ring between circles

$$\Rightarrow \pi(R_2^2 - R_1^2)$$

$$\Rightarrow \pi((10.5)^2 - (7)^2)$$

$$\Rightarrow \pi((10.5 + 7)(10.5 - 7))$$

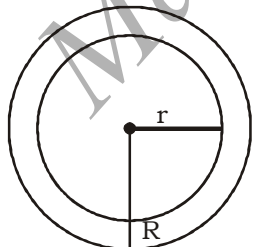
$$\Rightarrow \pi(17.5 \times 3.5)$$

$$\Rightarrow \frac{22}{7} \times 17.5 \times 3.5$$

$$\Rightarrow 192.5 \text{ cm}^2$$

2. A circular road runs round a circular ground. If the difference between the circumference of the outer and inner circle is 99m. the width of the road is:-

Sol.



Width of the Road = $2\pi R - 2\pi r$

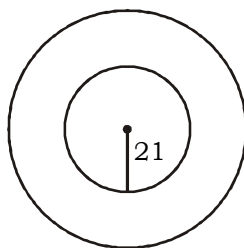
$$\Rightarrow 2\pi(R - r) = 66$$

$$\Rightarrow R - r = \frac{66}{2\pi} \times \frac{7}{2} = \frac{21}{2} = 10.5 \text{ cm}$$

Width of the road = 10.5 cm.

3. A circular Swimming pool with a diameter of 42 ft has a deck of uniform width built around it. If the area of deck is 43π sq. ft. Find its width.

Sol. Area of swimming pool
 $= \pi r^2 = \pi(21)^2$
 $= 441\pi$ sq. ft.



Area of deck = 43π Sq.ft

$$\Rightarrow \pi(R^2 - (21)^2) = 43\pi$$

$$\Rightarrow R^2 - 441 = \frac{43\pi}{\pi}$$

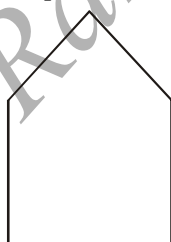
$$R^2 = 441 + 43$$

$$R^2 = 484 \Rightarrow R = 22$$

Width of the deck = $22 - 21$
 $= 1$ feet.

Polygon

It is a 2-Dimensional shapes. They are made of three or more than three straight lines, and the plane is closed.



Polygon
 $n = \text{no. of sides}$

- (i) **Sum of Exterior angles**

$$= 2\pi = 360^\circ$$

- (ii) **Sum of Interior angles**

$$= (n - 2) \times 180^\circ$$

- (iii) **Interior angle + exterior angle**
= 180°

- (iv) **Each Interior angle**

$$= \left(\frac{n-2}{n}\right) \times 180^\circ$$

- (v) **Each Exterior angle** = $\frac{360}{n}$

- (vi) **No of diagonals in polygon**

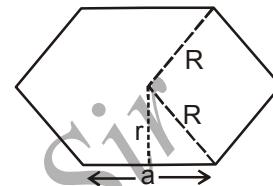
$$= \frac{n(n-3)}{2}$$

- (vii) **Perimeter (P) = $n \times a$**

(where n = no. of sides, and, a = length of each side)

- (viii) **Area (A)**

$$= \frac{1}{2} \times p \times r = \frac{1}{2} \times n \times a \times r$$



where, r = radius of inscribed circle

$$\Rightarrow A = \frac{1}{2} \times n \times a \times \sqrt{R^2 - \left(\frac{a}{2}\right)^2}$$

R = radius of circumscribed circle.

$$\text{or } A = \frac{na^2}{4} \cot \frac{\pi}{n}$$

EXAMPLES

1. An exterior angle of a regular polygon measures 36° . How many sides does the polygon have?

Sol. No of sides of polygon = n

$$\text{Each exterior angle} = \frac{360^\circ}{n}$$

$$36^\circ = \frac{360^\circ}{n}$$

$$n = \frac{360^\circ}{36^\circ} = 10$$

Sides of a polygon is 10.

2. How many sides does a polygon have if the sum of its interior angles is 1260° ?

Sol. Sum of interior angles

$$= (n - 2) \times 180$$

$$1260 = 180n - 360$$

$$1260 + 360 = 180n$$

$$n = \frac{1620}{180} = 9$$

Sides of a polygon is 9.

3. The ratio of internal and external angle is 7:1. Find the number of sides?

Sol. Ratio of I : E = 7:1

$$\begin{aligned}
 7x + x &= 180^\circ \\
 8x &= 180^\circ \\
 x &= 22.5^\circ \\
 \text{So, exterior angle is} &= 22.2^\circ \\
 \text{Each exterior angle} &= \frac{360^\circ}{n}
 \end{aligned}$$

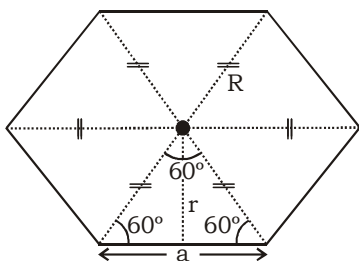
$$\begin{aligned}
 22.5 &= \frac{360^\circ}{n} \\
 n &= \frac{360}{22.5} = 16 \\
 \text{No. of sides is} &= 16.
 \end{aligned}$$

There are several other parts of polygon such as

No. of Sides	Name	Internal angle	External angle
3	Triangle	60°	120°
4	Quadrilateral	90°	90°
5	Pentagon	108°	72°
6	Hexagon	120°	60°
7	Heptagon	128.57°	51.43
8	Octagon	135°	45°
9	Nonagon	140°	40°
10	Decagon	144°	36°

Hexagon

A polygon with 6 sides is known as Hexagon.

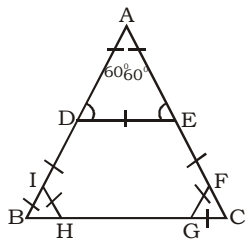


- (i) **Area** = 6 × (Area of Equilateral triangle of side a)
 $= 6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} a^2$ (a = side)
- (ii) **Perimeter** = 6a
- (iii) **Incircle radius (r)** = $\frac{\sqrt{3}}{2} a$
- (iv) **Circum Radius** = a

EXAMPLES

1. An equilateral triangle of side 6 cm has its corner cut off form a regular hexagon. Area of this regular hexagon.

Sol.



DE || BC

$$\begin{aligned}
 \angle ADE &= \angle AED = 60^\circ \\
 \therefore AD &= DE = AE \\
 \Rightarrow \text{Side of regular hexagon}
 \end{aligned}$$

$$\Rightarrow \frac{1}{3} \times 6 = 2 \text{ cm}$$

$$\text{Area} = \frac{3\sqrt{3}}{2} \times 4 = 6\sqrt{3} \text{ cm}^2$$

2. If area of a regular hexagon is $216\sqrt{3}$ sq. cm, then its perimeter.

Sol. $\text{Area} = \frac{3\sqrt{3}}{2} a^2$

$$216\sqrt{3} = \frac{3\sqrt{3}}{2} a^2$$

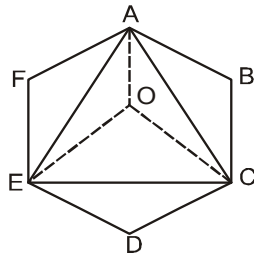
$$\frac{432\sqrt{3}}{3\sqrt{3}} = a^2$$

$$a^2 = 144 \Rightarrow a = 12$$

$$\text{Perimeter} = 6 \times 12 = 72 \text{ cm.}$$

3. Let ABCDEF be a regular hexagon. What is the ratio of the area of the triangle ACE to that of the hexagon ABCDEF?

Sol.



ABCDEF is a regular hexagon. Joining the centre O with vertices A, C and E, we get,

$$\triangle AFE = \triangle AOE$$

$$\text{similarly, } \triangle OAC = \triangle BAC$$

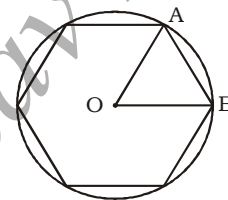
$$\text{also, } \triangle OEC = \triangle DEC$$

$$\therefore \triangle ACE = \frac{1}{2} \text{ the area of regu-}$$

lar hexagon.

4. A 6 sided regular polygon is inscribed in a circle of radius 10 cm, find the length of one side of the hexagon?

Sol.



$$\angle AOB = \frac{360^\circ}{6} = 60^\circ$$

Since OA = OB = 10 cm, triangle OAB is isosceles which gives

$$\angle OAB = \angle OBA$$

As all angle of the triangle are equal, therefore it is equilateral triangle

Hence AB = OA = OB = 10 cm

5. The area of a square is 2304 cm². Calculate the area of a regular hexagon that has the same perimeter as this square.

Sol. Perimeter of square = 4a
 Perimeter of hexagon = 6a
 Area of square = a² = 2304

$$a = 48$$

$$4a = 192 \text{ cm}$$

$$= \text{Perimeter of square}$$

$$6a = 192$$

$$a = 32$$

$$\text{Area of hexagon} = \frac{3\sqrt{3}a^2}{2}$$

$$= \frac{3\sqrt{3}}{2} \times 1024 = 1536\sqrt{3} \text{ cm}^2$$

6. A regular hexagon with an area of $150\sqrt{3}$ cm² is inscribed in a circle. Find the area not covered by hexagon?

Sol. $\text{Area of hexagon} = \frac{3\sqrt{3}}{2} a^2$

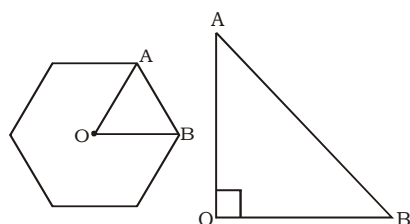
$$\frac{3\sqrt{3}}{2} a^2 = 150\sqrt{3}$$

$$a^2 = \frac{300\sqrt{3}}{3\sqrt{3}} = 100$$

$$a = 10$$

$$\text{Covered} = \pi r^2$$

As hexagon consist of 6 equilateral triangles



Side of hexagon = Radius of circle = 10 cm.

$$\begin{aligned} \text{Area not covered} &= \pi (10)^2 - 150\sqrt{3} \\ &\Rightarrow 100\pi - 150\sqrt{3} \Rightarrow 54.35 \text{ cm}^2 \end{aligned}$$

Octagon

A polygon with 8 sides is known as Octagon.

(i) **Perimeter** = $8 \times \text{side}$

(ii) **Area of regular octagon**

$$= 2(\sqrt{2} + 1)(\text{side})^2$$

EXAMPLES

1. The side of a regular octagon is 5 cm. Its area is ?

Sol. Area of regular octagon

$$= 2(\sqrt{2} + 1)(\text{side})^2$$

$$= 2(\sqrt{2} + 1)(5)^2 = 50(\sqrt{2} + 1)$$

2. If the perimeter of a regular octagon is 80 cm. Its area is ?

Sol. Perimeter of regular octagon is = 80

$$8 \times \text{side} = 80$$

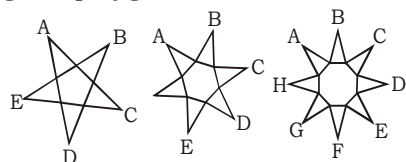
$$\text{side} = 10 \text{ cm}$$

Area of regular octagon

$$= 2(\sqrt{2} + 1)(\text{side})^2$$

$$= 2(\sqrt{2} + 1)(10)^2 = 200(\sqrt{2} + 1)$$

Star:- Sum of outer angles of a star forms by extending sides of a regular polygon.



Let outer triangles are 'n' then
sum of outer angles = $n \times 180$ -
two times sum of exterior angles
= $(n \times 180 - 2 \times 360)$
= $180(n - 4)$

If $n = 5$ then $\angle A + \angle B + \dots + \angle E$
= $180(5 - 4) = 180^\circ$

If $n = 6$ then $\angle A + \angle B + \dots + \angle F$
= $180(6 - 4) = 360^\circ$

If $n = 8$ then $\angle A + \angle B + \dots + \angle H$
= $180(8 - 4) = 720^\circ$

Miscellaneous Problem

Some Useful Results:

1. If each of the defining dimensions or sides of any 2-D figures are increased (or decreased) by $x\%$, its Perimeter also increases (or decreases) by $x\%$.

2. If all the sides of a quadrilateral are increased (or decreased) by $x\%$, its diagonals also increases (or decreases) by $x\%$.

3. The number of revolutions made by a circular wheel of radius r in travelling distance 'd' is given by:-

$$(\text{no. of revolutions}) n = \frac{d}{2\pi r}$$

4. If the length and breadth of a rectangle are increased by $x\%$ and $y\%$ respectively, then the area of rectangle will increase by:

$$\left(x + y + \frac{xy}{100}\right)\%$$

5. If the length and breadth of a rectangle are decreased by $x\%$ and $y\%$ respectively, then the area of rectangle will decrease by:

$$\left(x + y - \frac{xy}{100}\right)\%$$

6. If the length of a rectangle is increased by $x\%$, then its breadth will have to be decreased by

$$\left(\frac{100x}{100 + x}\right)\%$$

in order to maintain the same area of the rectangle.

7. If each of the defining dimensions or sides of any 2D figure (triangle, rectangle, square, circle, quadrilateral, pentagon,

hexagon etc.) is changed by $x\%$, its area changes by

$$x\left(2 + \frac{x}{100}\right)\%$$

$x \rightarrow +ve$ if increases

$x \rightarrow -ve$ if decreases

EXAMPLES

1. The length of a rectangle is increased by 50%. By what % should the width be decreased to maintain the same area ?

Sol. % decrease in breadth

$$= \left(\frac{100x}{100 + x}\right)\%$$

$$= \frac{100 \times 50}{150} = \frac{100}{3} = 33\frac{1}{3}\%$$

Alternatively :-

Let length = x and breadth = y

$$\text{New length} = x\left(\frac{150}{100}\right) = \frac{3x}{2}$$

As the area remains the same, the new breadth of the rectangle - so,

$$\frac{3x}{2} \times \text{New breadth} = xy$$

$$\Rightarrow \text{New breadth} = \frac{2y}{3}$$

$\therefore$ decrease in breadth

$$= y - \frac{2y}{3} = \frac{y}{3}$$

$\therefore$ % decrease in breadth

$$= \frac{y/3}{y} \times 100 = \frac{100}{3} = 33\frac{1}{3}\%$$

2. The length of a rectangle is increased by 60%. By what per-cent would the width be decreased so as to maintain the same area ?

Sol. Let length = width = 100m

If length = 160m, then let width = x m

$$\text{s.t. } 160x = 10000$$

$$\Rightarrow x = \frac{10000}{160} = \frac{1000}{16} = 62\frac{1}{2}$$

$\therefore$ width is reduced to $37\frac{1}{2}\%$

3. A wire is in the form of an equilateral triangle with area $4\sqrt{3}$ m². If it is change into a square, the side of a square will be.

Sol. Area of an equilateral triangle

$$= \frac{\sqrt{3}}{4} a^2$$

$$4\sqrt{3} = \frac{\sqrt{3}}{4} a^2$$

$$a^2 = \frac{16\sqrt{3}}{\sqrt{3}} \Rightarrow a = 4 \text{ m}$$

Perimeter of equilateral triangle
 $= 3a \Rightarrow 3 \times 4 = 12 \text{ cm}$

Perimeter of square $= 4a$

$$4a = 12, a = 3 \text{ m}$$

Side of a square $= 3 \text{ m}$.

4. If the ratio of areas of two squares is 225:256 then the ratio of their perimeter is:

Sol. Area of first square, $S_1 = a^2$
 $= 225$

$$a = 15$$

Area of second square,

$$S_2 = a^2 = 256$$

$$a = 16$$

Perimeter of square, $S_1 = 4a$
 $= 4 \times 15 = 60, S_2 = 4a = 4 \times 16 = 64$

Ratio of their perimeter
 $= 15 : 16$

Alternate:-

$$\frac{A_1}{A_2} = \left(\frac{P_1}{P_2} \right)^2$$

$$\frac{S_1}{S_2} \times \left(\frac{P_1}{P_2} \right)^2 \Rightarrow \frac{P_1}{P_2}$$

$$= \frac{\sqrt{225}}{\sqrt{256}} = \frac{15}{16}$$

5. A circular wire of diameter 210 cm is folded in the shape of a rectangle whose sides are in the ratio of 6:5. Find the area enclosed by the rectangle.

Sol. Perimeter of Circular wire
 $= \text{Perimeter of Rectangle}$

$$2\pi r = 2(l + b)$$

$$\Rightarrow 2\pi r = 2 \times \frac{22}{7} \times 105$$

$$= 660 \text{ cm}$$

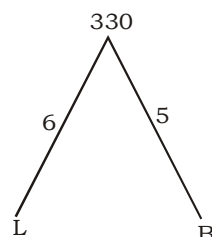
In Rectangle

$$2(l + b) = 660$$

$$l + b = 330$$

Length and breadth are in the ratio of 6:5

$\Rightarrow$



Area of rectangle $= l \times b$

$$= 180 \times 150$$

$$= 27,000 \text{ cm}^2$$

6. The diameter of a wheel is 0.14 m. How many revolutions did it makes in moving 440 m.

Sol. Circumference of a wheel
 $= \text{distance in 1 revolution}$

$$2\pi r = 2 \times \frac{22}{7} \times 0.07 = \frac{44}{100} \text{ m}$$

In 1 revolution wheel covers

$$\frac{44}{100} \text{ m}$$

In covering 440 m distance, wheel makes

$$\Rightarrow \frac{440}{\frac{44}{100}} \times 100$$

$$= 1000 \text{ revolution.}$$

Alternate:-

$$n = \frac{d}{2\pi r}$$

($\therefore n = n$: of revolution d

$$= \text{distance}) = \frac{440}{2 \times 22 \times 0.07}$$

$$= 1000 \text{ revolution.}$$

7. A metal wire when bent in the form of a square encloses an area 484 cm². If the same wire is bent in the form of a

circle, then (taking $\pi = \frac{22}{7}$)

its area is:

Sol. Side of square

$$= \sqrt{484} = 22 \text{ cm}$$

$$\therefore \text{length of wire} = 22 \times 4$$

$$= 88 \text{ cm}$$

$$\therefore 2\pi r = 88 \Rightarrow 2 \times \frac{22}{7} \times r = 88$$

$$\Rightarrow r = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

$$\therefore \text{Area} = \pi r^2$$

$$= \frac{22}{7} \times 14 \times 14 = 616 \text{ sq. cm.}$$

8. The radius of a circular wheel is 1.75 m. The number of revolutions that it will make in covering 11 km is:

Sol. Total no. of revolution

$$= \frac{d}{\text{Perimeter}} = \frac{d}{2\pi r}$$

$$= \frac{11 \times 1000 \times 7}{2 \times 22 \times 1.75} = 1000$$

9. The wheel of a motor car makes 1000 revolutions in moving 110 m. The diameter of the wheel is:-

Sol. As wheel makes 1000 revolutions in moving 110 m
 $1000 \rightarrow 110 \text{ m}$

$$1 \rightarrow \frac{110}{1000} = \text{Circumference}$$

$$2\pi r = \frac{110}{1000}$$

$$r = \frac{110 \times 7}{44 \times 1000}$$

$$r = \frac{7}{400}$$

$$\text{Diameter} = 2r = 2 \times \frac{7}{400}$$

$$= \frac{7}{200} = 0.035 \text{ m}$$

10. The cost of cultivating a square field at the rate of Rs. 190 per hectare is Rs. 1710. The cost of putting a fence around it at the Rate of 50 paise per metre.

Sol. Area of square field $= \frac{1710}{190}$

$$= 9 \text{ hectare}$$

$$1 \text{ hectare} = 10,000 \text{ m}^2$$

$$9 \text{ hectare} = 90,000 \text{ m}^2$$

Side of square

$$= \sqrt{a} = \sqrt{90,000} = 300$$

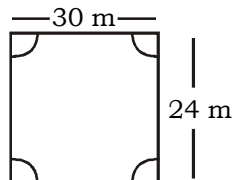
$$\text{Cost of fencing} = \frac{50}{100} \times 4a$$

$$= \frac{1}{2} \times 300 \times 4 = \text{Rs. } 600$$

11. A rectangular piece is 30 m long and 24 m wide. From its

four corners, quadrants of radii 2.1 metres have been cut. The area of the remaining part is.

Sol.



Area of Remaining part is
= Area of Rectangle - 4 × area of quarter circle

$$= L \times B - 4 \times \frac{1}{4} \times \pi r^2$$

$$= 720 - \frac{22}{7} \times 2.1 \times 2.1$$

$$= 720 - 13.86 = 706.14$$

- 12.** Area of circle is equal to the area of a rectangle having perimeter of 100cms and length is more than the breadth by 6cms. What is the diameter of the circle?

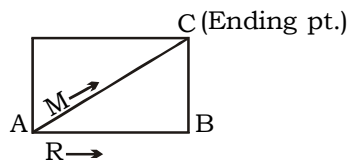
Sol. Let breadth = x ,
then length = $(x + 6)$
 $\therefore 2(x + x + 6) = 100$
 $\Rightarrow 2x + 6 = 50$
 $\Rightarrow x = 22\text{cm}$
 $\therefore$ breadth = x
= 22cm & length
= 22 + 6 = 28cm
 $\therefore$ Area of circle
= Area of rectangle
 $\Rightarrow \pi r^2 = 22 \times 28$
 $\Rightarrow r^2 = \frac{22 \times 28}{22} \times 7 = 7 \times 4 \times 7$
 $\Rightarrow r = 7 \times 2 = 14\text{cm}$
 $\therefore$ Diameter = $2r = 28\text{cm}$

- 13.** In between the race of two friends Mohit and Rakesh in a rectangular field Mohit cheated Rakesh by taking diagonal path rather than a side path. Mohit took 15 secs to reach ending point at the rate of 100 m/min whereas Rakesh took same time to. Reach the ending point at the rate of 124 m/min. Find the area of field.

Sol. Distance Covered by Mohit in 15 sec = $100 \times \frac{15}{60} = 25\text{ m}$

Distance covered by Rakesh in 15 Sec.

$$= 124 \times \frac{15}{60} = 31\text{m}$$



(Starting pt.)

Sum of length and Breadth = 31 metre.

We know that hypotenuse in a right angle triangle 25 m then length and breadth may be 24 and 7.

$$(l + b) = 31 \text{ area} = 24 \times 7 = 168 \text{ m}^2$$

- 14.** A person observed that he required 40 seconds less time to cross a circular ground along its diameter than to cover it once along a boundary. If his speed 40 m/min, then the radius of the circular ground is:

Sol. Along boundary he covers perimeter = $2\pi r$
Along diameter = $2r$
Time distance

$$= \frac{2\pi r}{\text{Speed}} - \frac{2r}{\text{Speed}}$$

$$\frac{40}{60} \text{ min} = \frac{2r}{40} (\pi - 1)$$

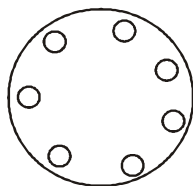
$$\frac{2}{3} = \frac{2r}{40} \left(\frac{22}{7} - 1 \right)$$

$$\frac{2 \times 40 \times 7}{3 \times 15} = 2r$$

$$r = 6.2 \text{ m (approx.)}$$

- 15.** In a circular park with a radius of 25 m there are 7 lamps whose base are circles with a radius of 1 m. The entire area of the park has grass with the exception of the base for the lamps. Calculate the total lawn area?

Sol.



$$\text{Area of the circular park} = \pi r^2$$

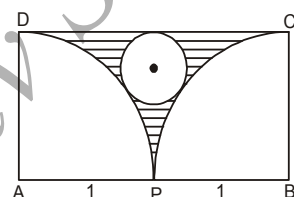
$$\Rightarrow \pi (25)^2 = 625 \pi \text{ m}^2$$

$$\text{Area of 7 lamps} = 7 \times \pi (1)^2 = 7 \pi \text{ m}^2$$

$$\text{Total lawn area} = 625 \pi - 7 \pi$$

$$= 618 \pi \text{ m}^2$$

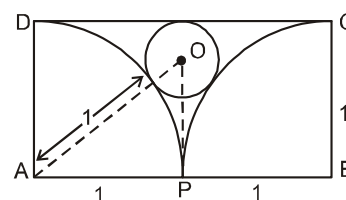
- 16.** In the following figure ABCD is a rectangle with AD and DC equal to 1 and 2 units respectively. Two quarter circles are drawn with centres at B and A respectively. Now a circle is drawn touching both the quarter circles and done of the sides of the rectangle. Find the area of the shaded region :



Sol. Let radius for the circle is ' r ' units

$$OP = (1 - r), OA = (1 + r) \text{ and } AP = 1$$

$$\text{In } \triangle AOP; OA^2 = AP^2 + OP^2$$



$$\Rightarrow (1 + r)^2 = 1^2 + (1 - r)^2$$

$$\Rightarrow r = \frac{1}{4} \text{ units}$$

$$\therefore \text{Area of smaller circle} = \pi \left(\frac{1}{4} \right)^2$$

$$= \frac{\pi}{16} \text{ square units}$$

Sum of the area of the quarter circles

$$= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2} \text{ square units}$$

Area of shaded region

$$= 2 - \left(\frac{\pi}{16} + \frac{\pi}{2} \right)$$

$$= 2 - \frac{9}{16} \pi \cong 0.23 \text{ square units}$$

Among the option choices, option (b) is closest.

EXERCISE

1. If the length of the diagonal AC of a square ABCD is 5.2 cm, then the area of the square is :
 (a) 15.12 sq. cm
 (b) 13.52 sq. cm
 (c) 12.62 sq. cm
 (d) 10.00 sq. cm
2. The length of the diagonal of a square is 'a' cm. Which of the following represents the area of the square (in sq. cm) ?
 (a) $2a$
 (b) $\frac{a}{\sqrt{2}}$
 (c) $a^2 / 2$
 (d) $a^2 / 4$
3. The breadth of a rectangular hall is three-fourth of its length. If the area of the floor is 768 sq. m., then the difference between the length and breadth of the hall is:
 (a) 8 metres
 (b) 12 metres
 (c) 24 metres
 (d) 32 metres
4. Find the length of the largest rod that can be placed in a room 16m long, 12m broad and $10\frac{2}{3}$ m high,
 (a) 123 m
 (b) 68 m
 (c) $22\frac{2}{3}$ m
 (d) $22\frac{1}{3}$ m
5. Between a square of perimeter 44 cm and a circle of circumference 44 cm, which figure has larger area and by how much ?
 (a) Square, 33cm^2
 (b) Circle, 33cm^2
 (c) Both have equal area.
 (d) square, 495cm^2
6. The perimeter of a square and a circular field are the same. If the area of the circular field is 3850 sq meter. What is the area (in m^2) of the square?
 (a) 4225
 (b) 3025
 (c) 2500
 (d) 2025
7. The perimeter of the top of a rectangular table is 28m., whereas its area is 48m^2 . What is the length of its diagonal?
 (a) 5 m
 (b) 10 m
 (c) 12 m
 (d) 12.5 m
8. The breadth of a rectangular hall is three-fourth of its length. If the area of the floor is 192 sq. m., then the difference between the length and breadth of the hall is:
 (a) 8 meters
 (b) 12 meters
 (c) 4 meters
 (d) 32 meters
9. The diagonal of a square is $4\sqrt{2}$ cm. The diagonal of another square whose area is double that of the first square is:
 (a) $8\sqrt{2}$ cm
 (b) 16 cm
 (c) $\sqrt{32}$ cm
 (d) 8 cm
10. The diagonal of a square A is (a + b). The diagonal of a square whose area is twice the area of square A is
 (a) $2(a + b)$
 (b) $2(a + b)^2$
 (c) $\sqrt{2}(a - b)$
 (d) $\sqrt{2}(a + b)$
11. The length of a rectangular garden is 12 metres and its breadth is 5 metres. Find the length of the diagonal of a square garden having the same area as that of the rectangular garden:
 (a) $2\sqrt{30}$ m
 (b) $\sqrt{13}$ m
 (c) 13 m
 (d) $8\sqrt{15}$ m
12. The areas of a square and a rectangle are equal. The length of the rectangle is greater than the length of any side of the square by 5 cm and the breadth is less by 3 cm. Find the perimeter of the rectangle.
 (a) 17 cm
 (b) 26 cm
 (c) 30 cm
 (d) 34 cm
13. The perimeter of a rectangle is 160 meter and the difference of two sides is 48 meter. Find the side of a square whose area is equal to the area of this rectangle.
 (a) 32m
 (b) 8m
 (c) 4m
 (d) 16m
14. The perimeter of two squares are 24 cm and 32 cm. The perimeter (in cm) of a third square equal in area to the sum of the areas of these squares is :
 (a) 45
 (b) 40
 (c) 32
 (d) 48
15. A wire when bent in the form of a square encloses an area of 484 sq. cm. What will be the enclosed area when the same wire is bent into the form of a circle?
 (Take $\pi = \frac{22}{7}$)
 (a) 125cm^2
 (b) 230cm^2
 (c) 550cm^2
 (d) 616cm^2
16. Find the length of the longest rod that can be placed in a hall of 10 m length, 6 m breadth and 4 m height,
 (a) $2\sqrt{38}$ m
 (b) $4\sqrt{38}$ m
 (c) $2\sqrt{19}$ m
 (d) $\sqrt{154}$ m
17. The difference of the areas of two squares drawn on two line segments of different lengths is 32sq. cm, Find the length of the greater line segment if one is longer than the other by 2 cm.
 (a) 7 cm
 (b) 9 cm
 (c) 11 cm
 (d) 16 cm
18. A took 15 sec. to cross a rectangular field diagonally walking at the rate of 52m/min and B took the same time to cross the same field along its sides walking at the rate of 68 m/ min. The area of the field is:
 (a) 30m^2
 (b) 40m^2
 (c) 50m^2
 (d) 60m^2
19. The difference between the length and breadth of a rectangle is 23 m. If its perimeter is 206 m, then its area is
 (a) 1520m^2
 (b) 2420m^2
 (c) 2480m^2
 (d) 2520m^2
20. The area (in m^2) of the square which has the same perimeter as a rectangle whose length is 48 m and is 3 times its breadth is:
 (a) 1000
 (b) 1024
 (c) 1600
 (d) 1042
21. The perimeter of two squares are 40 cm and 32 cm. The perimeter of a third square whose area is the difference of the area of the two squares is
 (a) 24 cm
 (b) 42 cm
 (c) 40 cm
 (d) 20 cm
22. The perimeter of five squares are 24 cm, 32 cm, 40 cm, 76cm and 80 cm respectively. The perimeter of another square equal in area to sum of the areas of these squares is:
 (a) 31 cm
 (b) 62 cm
 (c) 124 cm
 (d) 961 cm
23. There is a rectangular tank of length 180 m and breadth 120 m in a circular field, If the area of the land portion of the field is 40000m^2 , what is the radius of the field ? (Take $\pi = \frac{22}{7}$)
 (a) 130 m
 (b) 135 m
 (c) 140 m
 (d) 145 m

24. The length of a rectangular hall is 5m more than its breadth. The area of the hall is 750m^2 . The length of the hall is :
 (a) 15 m (b) 22.5 m
 (c) 25 m (d) 30 m
25. A cistern 6 m long and 4 m wide contains water up to a depth of 1 m 25 cm. The total area of the wet surface is
 (a) 55 m^2 (b) 53.5 m^2
 (c) 50 m^2 (d) 49 m^2
26. If the length and breadth of a rectangle are in the ratio 3 : 2 and its perimeter is 20 cm, then the area of the rectangle (in cm^2) is
 (a) 24 cm^2 (b) 36 cm^2
 (c) 48 cm^2 (d) 12 cm^2
27. The perimeter of a rectangle and a square are 160 m each. The area of the rectangle is less than that of the square by 100 sq m . The length of the rectangle is
 (a) 30m (b) 60m (c) 40m (d) 50m
28. A path of uniform width runs round the inside of a rectangular field 38 m long and 32 m wide, If the path occupies 600m^2 , then the width of the path is
 (a) 30 m (b) 5 m
 (c) 18.75 m (d) 10 m
29. The perimeter of the floor of a room is 18 m. What is the area of the walls of the room, If the height of the room is 3 m ?
 (a) 21 m^2 (b) 42 m^2
 (c) 54 m^2 (d) 108 m^2
30. A copper wire is bent in the shape of a square of area 81 cm^2 . If the same wire is bent in the form of a semicircle, the radius (in cm) of the semicircle is (take $\pi = \frac{22}{7}$)
 (a) 126 (b) 14 (c) 10 (d) 7
31. A copper wire is bent in the form of square with an area of 121 cm^2 . If the same wire is bent in the form of a circle, the radius (in cm) of the circle is
 (Take $\pi = \frac{22}{7}$)
 (a) 7 (b) 14 (c) 8 (d) 12
32. Water flows into a tank which is 200 m long and 150 m wide through a pipe of cross- section $0.3\text{m} \times 0.2\text{m}$ at 20 km/hour. Then the time (in hours) for the water level in the tank to reach 8 m is
 (a) 50 (b) 120 (c) 150 (d) 200
33. A street of width 10 metres surrounds from outside a rectangular garden whose measurement is $200\text{ m} \times 180\text{ m}$. The area of the path (in square metres) is
 (a) 8000 (b) 7000
 (c) 7500 (d) 8200
34. The area of the square inscribed in a circle of radius 8 cm is
 (a) 256 sq. cm (b) 250 sq. cm
 (c) 128 sq. cm (d) 125 sq. cm
35. Area of square with diagonal $8\sqrt{2}\text{ cm}$ is
 (a) 64 cm^2 (b) 29 cm^2
 (c) 56 cm^2 (d) 128 cm^2
36. If the area of a rectangle be $(x^2+7x+10)\text{ sq. cm}$, then one of the possible perimeter of it is
 (a) $(4x+14)\text{ cm}$ (b) $(2x+14)\text{ cm}$
 (c) $(x+14)\text{ cm}$ (d) $(2x+7)\text{ cm}$
37. If the perimeter of a square and a rectangle are the same, then the area P and Q enclosed by them would satisfy the condition
 (a) $P < Q$ (b) $P \leq Q$
 (c) $P > Q$ (d) $P = Q$
38. A cube of edge 6 cm is painted on all sides and then cut into unit cubes. The number of unit cubes with no sides painted is
 (a) 0 (b) 64 (c) 186 (d) 108
39. The length of diagonal of a square is $15\sqrt{2}\text{ cm}$. Its area is
 (a) 112.5 cm^2 (b) 450 cm^2
 (c) $\frac{255\sqrt{2}}{2}\text{ cm}^2$ (d) 225 cm^2
40. A kite in the shape of a square with a diagonal 32 cm attached to an equilateral triangle of the base 8 cm. Approximately how much paper has been used to make it? (Use $\sqrt{3} = 1.732$)
 (a) 539.712 cm^2
 (b) 538.721 cm^2
 (c) 540.712 cm^2
 (d) 539.217 cm^2
41. A lawn is in the form of a rectangle having its breadth and length in the ratio 3 : 4. The area of the lawn is $\frac{1}{12}$ hectare. The breadth of the lawn is
 (a) 25 metres (b) 50 metres
 (c) 75 metres (d) 100 metres
42. The area of a rectangle is thrice that of a square. The length of the rectangle is 20 cm and the breadth of the rectangle is $\frac{3}{2}$ times that of the side of the square. The side of the square, (in cm) is
 (a) 10 (b) 20 (c) 30 (d) 60
43. The length and breadth of a rectangular field are in the ratio 7 : 4. A path 4 m wide running all around outside has an area of 416 m^2 . The breadth (in m) of the field is
 (a) 28 (b) 14 (c) 15 (d) 16
44. How many tiles, each 4 decimeter square, will be required to cover the floor of a room 8 m long and 6 m broad?
 (a) 200 (b) 260 (c) 280 (d) 300
45. A godown is 15 m long and 12 m broad. The sum of the area of the floor and the ceiling is equal to the sum of areas of the four walls. The volume (in m^3) of the godown is:
 (a) 900 (b) 1200
 (c) 1800 (d) 720
46. Length of a side of a square inscribed in a circle is $a\sqrt{2}$ units. The circumference of the circle is
 (a) $2\pi a$ units (b) πa units
 (c) $4\pi a$ units (d) $\frac{2a}{\pi}$ units
47. The perimeter and length of a rectangle are 40 m and 12 m respectively. Its breadth will be
 (a) 10m (b) 8m (c) 6m (d) 3m
48. If each edge of a square be doubled, then the increase percentage in its area is
 (a) 200% (b) 250%
 (c) 280% (d) 300%
49. An elephant of length 4 m is at one corner of a rectangular cage of size $(16\text{ m} \times 30\text{ m})$ and faces towards the diagonally opposite corner. If the elephant starts moving towards the diagonally opposite corner it takes 15 seconds to reach this corner. Find the speed of the elephant
 (a) 1 m/sec (b) 2 m/sec
 (c) 1.87 m/sec (d) 1.5 m/sec

50. A circle is inscribed in a square of side 35 cm. The area of the remaining portion of the square which is not enclosed by the circle is
(a) 962.5 cm² (b) 262.5 cm²
(c) 762.5 cm² (d) 562.4 cm²
51. If the side of a square is $\frac{1}{2}(x+1)$ units and its diagonal is $\frac{3-x}{\sqrt{2}}$ units, then the length of the side of the square would be
(a) $\frac{4}{3}$ units (b) 1 unit
(c) $\frac{1}{2}$ units (d) 2 units
52. A rectangular carpet has an area of 120 m² and a perimeter of 46 metre. The length of its diagonal is:
(a) 17 metres (b) 21 metres
(c) 13 metres (d) 23 metres
53. If the length of a diagonal of a square is $6\sqrt{2}$ cm, then its area will be
(a) $24\sqrt{2}$ cm² (b) 24 cm²
(c) 36 cm² (d) 72 cm²
54. The length of a room is 3m more than its breadth. If the area of a floor of the room is 70 m², then the perimeter of the floor will be-
(a) 14 metres (b) 28 metres
(c) 34 metres (d) 17 metres
55. The length of a rectangle is twice the breadth. If area of the rectangle be 417.605 sq. m., then length is-
(a) 29.08 metres
(b) 29.80 metres
(c) 29.09 metres
(d) 28.90 metres
56. The area of a sector of a circle of radius 5 cm, formed by an arc of length 3.5 cm is :
(a) 8.5 cm² (b) 8.75 cm²
(c) 7.75 cm² (d) 7.50 cm²
57. The radius of a circular wheel is 1.75 m. The number of revolutions it will make in travelling 11 km is $\left(\text{use } \pi = \frac{22}{7}\right)$:
(a) 800 (b) 900
(c) 1000 (d) 1200
58. The radius of a wheel is 21 cm, How many revolutions will it make in travelling 924 metres?
 $\left(\text{use } \pi = \frac{22}{7}\right)$
(a) 7 (b) 11 (c) 200 (d) 700
59. The area (in sq. cm) of the largest circle that can be drawn inside a square of side 28 cm is :
(a) 1724 (b) 784
(c) 8624 (d) 616
60. The area of the ring between two concentric circles, whose circumference are 88 cm and 132 cm, is
(a) 78 cm² (b) 770 cm²
(c) 715 cm² (d) 660 cm²
61. The diameter of a toy wheel is 14 cm, What is the distance travelled by it in 15 revolutions?
(a) 880 cm (b) 660 cm
(c) 600 cm (d) 560 cm
62. A car can go round a circular path 8 times in 40 minutes. If the diameter of the circle is increased to 10 times the original diameter, the time required by A to go round the new path once travelling at the same speed as before is :
(a) 25 min (b) 20 min
(c) 50 min (d) 100 min
63. The base of a triangle is 15 cm and height is 12 cm. The height of another triangle of double the area having the base 20 cm is
(a) 9cm (b) 18 cm
(c) 8 cm (d) 12.5 cm
64. If a wire is bent into the shape of a square, the area of the square is 81 sq. cm. When the wire is bent into a semicircular shape, the area of the semicircle (taking $\pi = \frac{22}{7}$) is :
(a) 154 cm² (b) 77 cm²
(c) 44 cm² (d) 22 cm²
65. If the area of a triangle with base 12 cm is equal to the area of square with side 12 cm. the altitude of the triangle will be
(a) 12 cm (b) 24 cm
(c) 18 cm (d) 36 cm
66. The sides of a triangle are 3cm, 4 cm and 5 cm. The area (in cm²) of the triangle formed by joining the mid points of this triangle is:
(a) 6 (b) 3 (c) $\frac{3}{2}$ (d) $\frac{3}{4}$
67. Three circles of radius 3.5 cm each are placed in such a way that each touches the other two. The area of the portion enclosed by the circles is
(a) 1.975 cm² (b) 1.967 cm²
(c) 19.68 cm² (d) 21.22 cm²
68. The area of a circular garden is 2464 sq. m. how much distance will have to be covered if you like to cross the garden along its diameter? (use $\pi = \frac{22}{7}$)
(a) 56m (b) 48m (c) 28m (d) 24m
69. Four equal circles each of radius 'a' units touch one another. The area enclosed between them ($\pi = \frac{22}{7}$). In square units, is
(a) $3a^2$ (b) $\frac{6a^2}{7}$
(c) $\frac{41a^2}{7}$ (d) $\frac{a^2}{7}$
70. The area of the greatest circle inscribed inside a square of side 21 cm is (Take $\pi = \frac{22}{7}$)
(a) $351\frac{1}{2}$ cm² (b) $350\frac{1}{2}$ cm²
(c) $346\frac{1}{2}$ cm² (d) $347\frac{1}{2}$ cm²
71. The area of an equilateral triangle is $400\sqrt{3}$ sq. m. Its perimeter is :
(a) 120 m (b) 150 m
(c) 90 m (d) 135 m
72. From a point in the interior of an equilateral triangle, the perpendicular distance of the sides are $\sqrt{3}$ cm $2\sqrt{3}$ cm and $5\sqrt{3}$ cm. The perimeter (in cm) of the triangle is
(a) 64 (b) 32 (c) 48 (d) 24
73. The perimeter of a triangle is 30 cm and its area is 30 cm². If the largest side measures 13 cm, What is the length of the smallest side of the triangle?
(a) 3cm (b) 4cm
(c) 5cm (d) 6cm
74. Diameter of a wheel is 3 m. The wheel revolves 28 times in a minute. To cover 5.280 km distance, the wheel will take

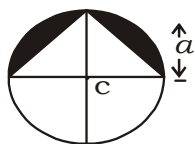
- (Take $\pi = \frac{22}{7}$):
- (a) 10 minutes (b) 20 minutes
(c) 30 minutes (d) 40 minutes
75. Find the diameter of a wheel that makes 113 revolutions to go 2 km 26 decameters.
- (Take $\pi = \frac{22}{7}$)
- (a) $4\frac{4}{13}$ m (b) $6\frac{4}{11}$ m
(c) $12\frac{4}{11}$ m (d) $12\frac{8}{11}$ m
76. The radius of a circular wheel is 1.75m. The number of revolutions that it will make in travelling 11 km, is
- (a) 1000 (b) 10,000
(c) 100 (d) 10
77. The circumference of a circle is 100 cm. The side of a square inscribed in the circle is
- (a) $\frac{100\sqrt{2}}{\pi}$ cm (b) $\frac{50\sqrt{2}}{\pi}$ cm
(c) $\frac{100}{\pi}$ cm (d) $50\sqrt{2}$ cm
78. A path of uniform width surrounds a circular park, The difference of internal and external circumference of this circular path is 132 metres. Its width is:
- (Take $\pi = \frac{22}{7}$)
- (a) 22m (b) 20m (c) 21m (d) 4m
79. Four equal sized maximum circular plates are cut off from a square paper sheet of area 784 sq. cm. The circumference of each plate is
- (Take $\pi = \frac{22}{7}$)
- (a) 22 cm (b) 44 cm
(c) 66 cm (d) 88 cm
80. The circum-radius of an equilateral triangle is 8 cm. The in-radius of the triangle is
- (a) 3.25 cm (b) 3.50 cm
(c) 4 cm (d) 4.25 cm
81. Three coins of the same size (radius 1 cm) are placed on a table such that each of them touches the other two. The area enclosed by the coins is
- (a) $\left(\frac{\pi}{2} - \sqrt{3}\right)$ cm²
(b) $\left(\sqrt{3} - \frac{\pi}{2}\right)$ cm²
(c) $\left(2\sqrt{3} - \frac{\pi}{2}\right)$ cm²
(d) $\left(3\sqrt{3} - \frac{\pi}{2}\right)$ cm²
82. The area of the largest triangle that can be inscribed in a semicircle of radius r cm, is
- (a) $2r$ cm² (b) r^2 cm²
(c) 2 cm² (d) $\frac{1}{2}r^2$ cm²
83. The area of the greatest circle, which can be inscribed in a square whose perimeter is 120 cm, is :
- (a) $\frac{22}{7} \times 15^2$ cm²
(b) $\frac{22}{7} \times \left(\frac{7}{2}\right)^2$ cm²
(c) $\frac{22}{7} \times \left(\frac{15}{2}\right)^2$ cm²
(d) $\frac{22}{7} \times \left(\frac{9}{2}\right)^2$ cm²
84. The area of the incircle of an equilateral triangle of side 42 cm is (Take $\pi = \frac{22}{7}$):
- (a) 231 cm² (b) 462 cm²
(c) $22\sqrt{3}$ cm² (d) 924 cm²
85. The number of revolutions a wheel of diameter 40 cm makes in travelling a distance of 176 m, is (Take $\pi = \frac{22}{7}$):
- (a) 140 (b) 150
(c) 160 (d) 166
86. The length of the perpendiculars drawn from any point in the interior of an equilateral triangle to the respective sides are p_1, p_2 and p_3 . The length of each side of the triangle is
- (a) $\frac{2}{\sqrt{3}}(p_1 + p_2 + p_3)$
(b) $\frac{1}{3}(p_1 + p_2 + p_3)$
(c) $\frac{1}{\sqrt{3}}(p_1 + p_2 + p_3)$
(d) $\frac{4}{\sqrt{3}}(p_1 + p_2 + p_3)$
87. A circle is inscribed in a square, An equilateral triangle of side $4\sqrt{3}$ cm is inscribed in that circle. The length of the diagonal of the square (in cm) is
- (a) $4\sqrt{2}$ (b) 8 (c) $8\sqrt{2}$ (d) 16
88. The hypotenuse of a right angle isosceles triangle is 5 cm. Its area will be
- (a) 5 sq. cm (b) 6.25 sq. cm
(c) 6.50 sq. cm (d) 12.5 sq. cm
89. From a point within an equilateral triangle, perpendiculars drawn to the three sides are 6 cm, 7 cm and 8 cm respectively, the length of the side of the triangle is :
- (a) 7 cm (b) 10.5 cm
(c) $14\sqrt{3}$ cm (d) $\frac{14\sqrt{3}}{3}$ cm
90. In an isosceles triangle, the measure of each of equal sides is 10 cm and the angle between them is 45° , then area of the triangle is
- (a) 25 cm² (b) $\frac{25}{2}\sqrt{2}$ cm²
(c) $25\sqrt{2}$ cm² (d) $2\sqrt{3}$ cm²
91. The area of circle whose radius is 6 cm is trisected by two concentric circles. The radius of the smallest circle is
- (a) $2\sqrt{3}$ cm (b) $2\sqrt{6}$ cm
(c) 2 cm (d) 3 cm
92. The area of an equilateral triangle inscribed in a circle is $4\sqrt{3}$ cm². The area of the circle is
- (a) $\frac{16}{3}\pi$ cm² (b) $\frac{22}{3}\pi$ cm²
(c) $\frac{28}{3}\pi$ cm² (d) $\frac{32}{3}\pi$ cm²
93. If the difference between the circumference and diameter of a circle is 30 cm, then the radius of the circle must be
- (a) 6cm (b) 7cm
(c) 5cm (d) 8cm
94. The base and altitude of a right angled triangle are 12 cm and 5 cm respectively. The perpendicular distance of its hypotenuse from the opposite vertex is
- (a) $4\frac{4}{13}$ cm (b) $4\frac{8}{13}$ cm
(c) 5 (d) 7 cm

95. From a point in the interior of an equilateral triangle, the length of the perpendiculars to the three sides are 6 cm, 8 cm and 10 cm respectively. The area of the triangle is

(a) 48 cm^2 (c) $16\sqrt{3} \text{ cm}^2$

(c) $192\sqrt{3} \text{ cm}^2$ (d) 192 cm^2

96. The area of the shaded region in the figure given below is



(a) $\frac{a^2}{2} \left(\frac{\pi}{2} - 1 \right)$ sq. units

(b) $a^2 (\pi - 1)$ sq. units

(c) $a^2 \left(\frac{\pi}{2} - 1 \right)$ sq. units

(d) $\frac{a^2}{b^2} (\pi - 1)$ sq. units

97. The area of a circle is increased by 22 cm, if its radius is increased by 1 cm. The original radius of the circle is

(a) 6 cm (b) 3.2 cm

(c) 3 cm (d) 3.5 cm

98. The area of the largest circle, that can be drawn inside a rectangle with sides 148 cm. by 14 cm is

(a) 49 cm^2 (b) 154 cm^2

(c) 378 cm^2 (d) 1078 cm^2

99. A circle is inscribed in an equilateral triangle of side 8 cm. The area of the portion between the triangle and the circle is

(a) 11 cm^2 (b) 10.95 cm^2

(c) 10 cm^2 (d) 10.50 cm^2

100. In a triangular field having sides 30m, 72m and 78m, the length of the altitude to the side measuring 72m is :

(a) 25 m (b) 28 m

(c) 30 m (d) 35 m

101. If the perimeter of a right-angled isosceles triangle is $4\sqrt{2}+4$ cm, the length of the hypotenuse is;

(a) 4 cm (b) 6 cm

(c) 8 cm (d) 10 cm

102. A right triangle with sides 3 cm, 4 cm and 5 cm is rotated about the side 3 cm to form a cone. The volume of the cone so formed is

(a) $16\pi \text{ cm}^3$ (b) $12\pi \text{ cm}^3$

(c) $15\pi \text{ cm}^3$ (d) $20\pi \text{ cm}^3$

103. ABC is an equilateral triangle of side 2 cm. With A, B, C as centre and radius 1 cm three arcs are drawn. The area of the region within the triangle bounded by the three arcs is

(a) $\left(3\sqrt{3} - \frac{\pi}{2} \right) \text{ cm}^2$

(b) $\left(\sqrt{3} - \frac{3\pi}{2} \right) \text{ cm}^2$

(c) $\left(\sqrt{3} - \frac{\pi}{2} \right) \text{ cm}^2$

(d) $\left(\frac{\pi}{2} - \sqrt{3} \right) \text{ cm}^2$

104. The circumference of a circle is 11 cm and the angle of a sector of the circle is 60° . The area of

the sector is (use $\pi = \frac{22}{7}$)

(a) $1\frac{29}{48} \text{ cm}^2$ (b) $2\frac{29}{48} \text{ cm}^2$

(c) $1\frac{27}{48} \text{ cm}^2$ (d) $2\frac{27}{48} \text{ cm}^2$

105. If the difference between areas of the circumcircle and the incircle of an equilateral triangle is 44 cm^2 , then the area of the triangle

is (Take $\pi = \frac{22}{7}$)

(a) 28 cm^2 (b) $7\sqrt{3} \text{ cm}^2$

(c) $14\sqrt{3} \text{ cm}^2$ (d) 21 cm^2

106. If the area of a circle inscribed in a square is $9\pi \text{ cm}^2$, then the area of the square is

(a) 24 cm^2 (b) 30 cm^2

(c) 36 cm^2 (d) 81 cm^2

107. The sides of a triangle are 6 cm, 8 cm and 10 cm. The area of the greatest square that can be inscribed in it, is

(a) 18 cm^2 (b) 15 cm^2

(c) $\frac{2304}{49} \text{ cm}^2$ (d) $\frac{576}{49} \text{ cm}^2$

108. The length of a side of an equilateral triangle is 8 cm. the area of the region lying between the circumcircle and the incircle

of the triangle is (use $\pi = \frac{22}{7}$)

(a) $50\frac{1}{7} \text{ cm}^2$ (b) $50\frac{2}{7} \text{ cm}^2$

(c) $75\frac{1}{7} \text{ cm}^2$ (d) $75\frac{2}{7} \text{ cm}^2$

109. A wire, when bent in the form of a square, encloses a region having area 121 cm^2 . If the same wire is bent into the form of a circle, then the area of the circle

is (Take $\pi = \frac{22}{7}$)

(a) 144 cm^2 (b) 180 cm^2

(c) 154 cm^2 (d) 176 cm^2

110. If the perimeter of a semicircular field is 36 m. Find its radius

(use $\pi = \frac{22}{7}$)

(a) 7 m (b) 8 m

(c) 14 m (d) 16 m

111. The perimeter (in metres) of a semicircle is numerically equal to its area (in m^2). The length of its

diameter is (Take $\pi = \frac{22}{7}$)

(a) $3\frac{3}{11}$ metres (b) $5\frac{6}{11}$ metres

(c) $6\frac{6}{11}$ metres (d) $6\frac{2}{11}$ metres

112. One acute angle of a right angled triangle is double the other. If the length of its hypotenuse is 10 cm, then its area is

(a) $\frac{25}{2}\sqrt{3} \text{ cm}^2$ (b) 25 cm^2

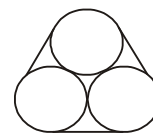
(c) $25\sqrt{3} \text{ cm}^2$ (d) $\frac{75}{2} \text{ cm}^2$

113. If a triangle with base 8 cm has the same area as a circle with radius 8 cm, then the corresponding altitude (in cm) of the triangle is

(a) 12π (b) 20π

(c) 16π (d) 32π

114. The measures (in cm) of sides of a right angled triangle are given by consecutive integers. its area (in cm^2) is
(a) 9 (b) 8 (c) 5 (d) 6
115. The area of a right-angled isosceles triangle having hypotenuse $16\sqrt{2}$ cm is
(a) 144 cm^2 (b) 128 cm^2
(c) 112 cm^2 (d) 110 cm^2
116. The area of an equilateral triangle is $4\sqrt{3} \text{ cm}^2$. The length of each side of the triangle is:
(a) 3 cm (b) $2\sqrt{2}$ cm
(c) $2\sqrt{3}$ cm (d) 4 cm
117. An equilateral triangle of side 6 cm has its corners cut off to form a regular hexagon. Area (in cm^2) of this regular hexagon will be
(a) $3\sqrt{3}$ (b) $3\sqrt{6}$
(c) $6\sqrt{3}$ (d) $\frac{5\sqrt{3}}{2}$
118. A 7 m wide road runs outside around a circular park, whose circumference is 176 m. the area of the road is : (use $\pi = \frac{22}{7}$)
(a) 1386 m^2 (b) 1472 m^2
(c) 1512 m^2 (d) 1760 m^2
119. The length (in cm) of a chord of a circle of radius 13 cm at a distance of 12 cm from its centre is
(a) 5 (b) 8 (c) 10 (d) 12
120. The four equal circles of radius 4 cm drawn on the four corners of a square touch each other externally. Then the area of the portion between the square and the four sectors is
(a) $9(\pi - 4) \text{ sq. cm}$
(b) $16(4 - \pi) \text{ sq. cm}$
(c) $99(\pi - 4) \text{ sq. cm}$
(d) $169(\pi - 4) \text{ sq. cm}$
121. If the four equal circles of radius 3 cm touch each other externally, then the area of the region bounded by the four circles is
(a) $4(9 - \pi) \text{ sq. cm}$
(b) $9(4 - \pi) \text{ sq. cm}$
(c) $5(6 - \pi) \text{ sq. cm}$
(d) $6(5 - \pi) \text{ sq. cm}$
122. The length of each side of an equilateral triangle is $14\sqrt{3}$ cm. The area of the incircle (in cm^2) is
(a) 450 (b) 308
(c) 154 (d) 77
123. Area of the incircle of an equilateral triangle with side 6 cm is
(a) $\frac{\pi}{2} \text{ sq. cm}$ (b) $\sqrt{3}\pi \text{ sq. cm}$
(c) $6\pi \text{ sq. cm}$ (d) $3\pi \text{ sq. cm}$
124. A copper wire is bent in the form of an equilateral triangle and has area $121\sqrt{3} \text{ cm}^2$. If the same wire is bent into the form of a circle. the area (in cm^2) enclosed by the wire is (take $\pi = \frac{22}{7}$)
(a) 364.5 (b) 693.5
(c) 346.5 (d) 639.5
125. At each corner of a triangular field of sides 26 m, 28 m and 30 m, a cow is tethered by a rope of length 7m, the area (in m) ungrazed by the cows is
(a) 336 (b) 259
(c) 154 (d) 77
126. In an equilateral triangle ABC, P & Q are mid point of sides AB & AC respectively such that $PQ \parallel BC$. If $PQ = 5$ cm then find the length of BC.
(a) 5 cm (b) 10 cm
(c) 15 cm (d) 12 cm
127. ABC is an equilateral triangle, P and Q are two points on $\overline{AB}$ and $\overline{AC}$ respectively such that $\overline{PQ} \parallel \overline{BC}$. If $\overline{PQ} = 5$ cm, then area of $\triangle APQ$ is :
(a) $\frac{25}{4} \text{ sq. cm}$ (b) $\frac{25}{\sqrt{3}} \text{ sq. cm}$
(c) $\frac{25\sqrt{3}}{4} \text{ sq. cm}$ (d) $25\sqrt{3} \text{ sq. cm}$
128. The area of a circle with circumference 22cm is
(a) 38.5 cm^2 (b) 39 cm^2
(c) 36.5 cm^2 (d) 40 cm^2
129. In $\triangle ABC$, O is the centroid and AD, BE, CF are three medians and the area of $\triangle AOE = 15 \text{ cm}^2$ then area of quadrilateral BDOF is
(a) 20 cm^2 (b) 30 cm^2
(c) 40 cm^2 (d) 25 cm^2
130. A straight line parallel to the base BC of the triangle ABC intersects AB and AC at the points D and E respectively. If the area of the $\triangle ABE$ be 36 sq. cm . then the area of the $\triangle ACD$ is
(a) 18 sq. cm (b) 36 sq. cm
(c) 120 sq. cm (d) 54 sq. cm
131. The length of two sides of an isosceles triangle are 15 and 22 respectively. What are the possible values of perimeter ?
(a) 52 or 59 (b) 52 or 60
(c) 15 or 37 (d) 37 or 29
132. The diameter of a wheel is 98 cm. The number of revolutions in which it will have to cover a distance of 1540 m is
(a) 500 (b) 600
(c) 700 (d) 800
133. The wheel of a motor car makes 1000 revolutions in moving 440 m. The diameter (in metre) of the wheel is
(a) 0.44 (b) 0.14
(c) 0.24 (d) 0.34
134. A bicycle wheel makes 5000 revolutions in moving 11 km . Then the radius of the wheel (in cm) is (take $\pi = \frac{22}{7}$)
(a) 70 (b) 35 (c) 17.5 (d) 140
135. Three circles of diameter 10 cm each are bound together by a rubber band as shown in the figure.



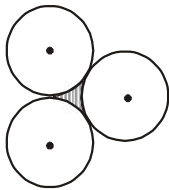
the length of the rubber band (in cm) if it is stretched is

- (a) 30 (b) $30 + 10\pi$
(c) 10π (d) $60 + 20\pi$
136. If chord of length 16 cm is at a distance of 15 cm from the centre of the circle, then the length of the chord of the same circle which is at a distance of 8 cm from the centre is equal to
(a) 10 cm (b) 20 cm
(c) 30 cm (d) 40 cm
137. A semicircular shaped window has diameter of 63 cm, its perimeter equals ($\pi = \frac{22}{7}$)
(a) 126 cm (b) 162 cm
(c) 198 cm (d) 251 cm

138. In an equilateral triangle ABC of side 10 cm, the side BC is trisected at D & E. Then the length (in cm) of AD is
 (a) $3\sqrt{7}$ (b) $7\sqrt{3}$
 (c) $\frac{10\sqrt{7}}{3}$ (d) $\frac{7\sqrt{10}}{3}$
139. The perimeter of a triangle is 40 cm and its area is 60 cm^2 . If the largest side measures 17 cm, then the length (in cm) of the smallest side of the triangle is
 (a) 4 (b) 6 (c) 8 (d) 15
140. From four corners of a square sheet of side 4 cm four pieces each in the shape of arc of a circle with radius 2 cm are cut out. The area of the remaining portion is :
 (a) $(8 - \pi) \text{ sq. cm}$
 (b) $(16 - 4\pi) \text{ sq. cm}$
 (c) $(16 - 8\pi) \text{ sq. cm}$
 (d) $(4 - 2\pi) \text{ sq. cm}$
141. If the numerical value of the perimeter of an equilateral triangle is $\sqrt{3}$ times the area of it, then the length of each side of the triangle is
 (a) 2 units (b) 3 units
 (c) 4 units (d) 6 units
142. Each side of an equilateral triangle is 6 cm. Find its area
 (a) $9\sqrt{3} \text{ sq. cm}$ (b) $6\sqrt{3} \text{ sq. cm}$
 (c) $4\sqrt{3} \text{ sq. cm}$ (d) $8\sqrt{3} \text{ sq. cm}$
143. The length of three medians of a triangle are 9 cm, 12 cm and 15 cm. The area (in sq. cm) of the triangle is
 (a) 24 (b) 72
 (c) 48 (d) 144
144. The area of the triangle formed by the straight line $3x + 2y = 6$ and the co-ordinate axes is
 (a) 3 square units
 (b) 6 square units
 (c) 4 square units
 (d) 8 square units
145. If the length of each side of an equilateral triangle is increased by 2 unit, the area is found to be increased by $3 + \sqrt{3}$ square unit. The length of each side of the triangle is
 (a) $\sqrt{3}$ units (b) 3 units
 (c) $3\sqrt{3}$ units (d) $3\sqrt{2}$ units
146. What is the area of the triangle whose sides are 9 cm, 10 cm and 11 cm?
 (a) 30 cm^2 (b) 60 cm^2
 (c) $30\sqrt{2} \text{ cm}^2$ (d) $60\sqrt{2} \text{ cm}^2$
147. The area of an isosceles triangle is 4 square units, If the length of the unequal side is 2 unit, the length of each equal side is
 (a) 4 units (b) $2\sqrt{3}$ units
 (c) $\sqrt{17}$ units (d) $3\sqrt{2}$ units
148. What is the area of a triangle having perimeter 32 cm, one side 11 cm and difference of other two sides 5 cm?
 (a) $8\sqrt{30} \text{ cm}^2$ (b) $5\sqrt{35} \text{ cm}^2$
 (c) $6\sqrt{30} \text{ cm}^2$ (d) $8\sqrt{2} \text{ cm}^2$
149. Area of equilateral triangle having side 2 cm is
 (a) 4 cm^2 (b) $\sqrt{3} \text{ cm}^2$
 (c) 3 cm^2 (d) $\sqrt{6} \text{ cm}^2$
150. The area of a circle is increased by 22 cm^2 when its radius is increased by 1 cm. The original radius of the circle is
 (a) 3 cm (b) 5 cm
 (c) 7 cm (d) 9 cm
151. The radii of two circles are 5 cm and 12 cm. The area of a third circle is equal to the sum of the area of the two circles. The radius of the third circle is :
 (a) 13 cm (b) 21 cm
 (c) 30 cm (d) 17 cm
152. The perimeter of a semicircular path is 36 m. Find the area of this semicircular path.
 (a) 42 sq. m (b) 54 sq. m
 (a) 63 sq. m (d) 77 sq. m
153. The area of a circle inscribed in a square of area 2m^2 is
 (a) $\frac{\pi}{4} \text{ m}^2$ (b) $\frac{\pi}{2} \text{ m}^2$
 (c) $\pi \text{ m}^2$ (d) $2\pi \text{ m}^2$
154. Three circles of radii 4 cm, 6 cm and 8 cm touch each other pair wise externally. The area of the triangle formed by the line-segments joining the centres of the three circles is
 (a) $144\sqrt{13} \text{ sq. cm}$
 (b) $12\sqrt{105} \text{ sq. cm}$
 (c) $4\sqrt{3} \text{ sq. cm}$
 (d) $24\sqrt{6} \text{ sq. cm}$
155. Two circles with centre A and B and radius 2 units touch each other externally at 'C'. A third circle with centre 'C' and radius '2' units meets other two at D and E. Then the area of the quadrilateral ABDE is
 (a) $2\sqrt{2} \text{ sq. units}$
 (b) $3\sqrt{3} \text{ sq. units}$
 (c) $3\sqrt{2} \text{ sq. units}$
 (d) $2\sqrt{3} \text{ sq. units}$
156. If the perimeter of a right angled triangle is 56 cm and area of the triangle is 84 sq. cm , then the length of the hypotenuse is (in cm)
 (a) 25 (b) 50 (c) 7 (d) 24
157. If the length of each median of an equilateral triangle is $6\sqrt{3} \text{ cm}$, the perimeter of the triangle is
 (a) 24 cm (b) 32 cm
 (c) 36 cm (d) 42 cm
158. The area of an equilateral triangle is $4\sqrt{3} \text{ sq. cm}$. Its perimeter is
 (a) 12 cm (b) 6 cm
 (c) 8 cm (d) $3\sqrt{3} \text{ cm}$
159. A gear 12 cm in diameter is turning a gear 18 cm in diameter. When the smaller gear has 42 revolutions. how many has the larger one made?
 (a) 28 (b) 20 (c) 15 (d) 24
160. The perimeter of a semicircle is 18 cm, then the radius is:
 (using $\pi = \frac{22}{7}$)
 (a) $5\frac{1}{3} \text{ cm}$ (b) $3\frac{1}{2} \text{ cm}$
 (c) 6 cm (d) 4 cm
161. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. The area of the circle is
 (Take $\pi = \frac{22}{7}$)
 (a) 125 cm^2 (b) 230 cm^2
 (c) 550 cm^2 (d) 616 cm^2
162. The area of a circle is 38.5 sq. cm . Its circumference (in cm) is
 (use $\pi = \frac{22}{7}$)
 (a) 22 (b) 24 (c) 26 (d) 32

163. A circle is inscribed in a square whose length of the diagonal is $12\sqrt{2}$ cm. An equilateral triangle is inscribed in that circle. The length of the side of the triangle is
 (a) $4\sqrt{3}$ cm (b) $8\sqrt{3}$ cm
 (c) $6\sqrt{3}$ cm (d) $11\sqrt{3}$ cm
164. The area (in sq. unit) of the triangle formed in the first quadrant by the line $3x+4y=12$ is
 (a) 8 (b) 12 (c) 6 (d) 4
165. The height of an equilateral triangle is 15 cm. the area of the triangle is
 (a) $50\sqrt{3}$ sq. cm
 (b) $70\sqrt{3}$ sq. cm
 (c) $75\sqrt{3}$ sq. cm
 (d) $150\sqrt{3}$ sq. cm
166. The area of an equilateral triangle is $9\sqrt{3}$ m². The length (in m) of the median is
 (a) $2\sqrt{3}$ (b) $3\sqrt{3}$
 (c) $3\sqrt{2}$ (d) $2\sqrt{2}$
167. The sides of a triangle are 16 cm, 12 cm and 20 cm. Find the area,
 (a) 64 cm² (b) 112 cm²
 (c) 96 cm² (d) 81 cm²
168. 360 sq. cm and 250 sq. cm are the area of two similar triangles. If the length of one of the sides of the first triangle be 8 cm, then the length of the corresponding side of the second triangle is
 (a) $6\frac{1}{5}$ cm (b) $6\frac{1}{3}$ cm
 (c) $6\frac{2}{3}$ cm (d) 6 cm
169. The perimeter of an isosceles triangle is 544 cm and each of the equal sides is $\frac{5}{6}$ times the base. What is the area (in cm²) of the triangle?
 (a) 38172 (b) 18372
 (c) 31872 (d) 13872
170. The altitude drawn to the base of an isosceles triangle is 8 cm and its perimeter is 64 cm. The area (in cm²) of the triangle is
 (a) 240 (b) 180
 (c) 360 (d) 120
171. Three circles of radius a, b, c touch each other externally. The area of the triangle formed by joining their centre is
 (a) $\sqrt{(a+b+c)abc}$
 (b) $(a+b+c)\sqrt{ab+bc+ca}$
 (c) $ab+bc+ca$
 (d) None of the above
172. The radii of two circles are 10 cm and 24 cm. The radius of a circle whose area is the sum of the area of these two circles is
 (a) 36 cm (b) 17 cm
 (c) 34 cm (d) 26 cm
173. A circle is inscribed in an equilateral triangle and a square is inscribed in that circle. The ratio of the areas of the triangle and the square is
 (a) $\sqrt{3}:4$ (b) $\sqrt{3}:8$
 (c) $3\sqrt{3}:2$ (d) $3\sqrt{3}:1$
174. If area of an equilateral triangle is a and height b , then value of $\frac{b^2}{a}$ is:
 (a) 3 (b) $\frac{1}{3}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$
175. If $\triangle ABC$ is similar to $\triangle DEF$ such that $BC = 3$ cm, $EF = 4$ cm and area of $\triangle ABC = 54$ cm², then the area of $\triangle DEF$ is :
 (a) 66 cm² (b) 78 cm²
 (c) 96 cm² (d) 54 cm²
176. The area of two similar triangles ABC and DEF are 20cm² and 45 cm² respectively. If $AB = 5$ cm, then DE is equal to
 (a) 6.5 cm (b) 7.5 cm
 (c) 8.5 cm (d) 5.5 cm
177. C_1 and C_2 are two concentric circles with centre at O, Their radii are 12 cm and 3 cm, respectively. B and C are the point of contact of two tangents drawn to C_2 from a point A lying on the circle C_1 . Then, the area of the quadrilateral ABOC is
 (a) $\frac{9\sqrt{15}}{2}$ sq. cm
 (b) $12\sqrt{15}$ sq. cm
 (c) $9\sqrt{15}$ sq. cm
 (d) $6\sqrt{15}$ sq. cm
178. From a point P which is at a distance of 13 cm from centre O of a circle of radius 5 cm in the same plane, a pair of tangents PQ and PR are drawn to the circle. Area of quadrilateral PQOR is
 (a) 65 cm² (b) 60 cm²
 (c) 30 cm² (d) 90 cm²
179. A circular road runs around a circular ground. If the difference between the circumference of the outer circle and the inner circle is 66 meters, the width of the road is:
 (Take $\pi = \frac{22}{7}$)
 (a) 10.5 metres (b) 7 metres
 (c) 5.25 metres (d) 21 metres
180. A person observed that he required 30 seconds less time to cross a circular ground along its diameter than to cover it once along the boundary. If his speed was 30 m/ minutes. then the radius of the circular ground is
 (Take $\pi = \frac{22}{7}$):
 (a) 5.5 m (b) 7.5 m
 (c) 10.5 m (d) 3.5 m
181. The difference of perimeter and diameter of a circle is X unit. The diameter of the circle is
 (a) $\frac{X}{\pi-1}$ unit (b) $\frac{X}{\pi+1}$ unit
 (c) $\frac{X}{\pi}$ unit (d) $\left(\frac{X}{\pi}-1\right)$ unit
182. The area of the circumcircle of an equilateral triangle is 3π sq. cm. The perimeter of the triangle is
 (a) $3\sqrt{3}$ cm (b) 9 cm
 (c) 18 cm (d) 3 cm
183. A horse is tied to a post by a rope. If the horse moves along a circular path always keeping the rope stretched and describes 88 metres when it has traced out 72° at the centre, the length of the rope is (Take $\pi = \frac{22}{7}$)
 (a) 70 m (b) 75 m
 (c) 80 m (d) 65 m
184. Three circles of radii 3.5 cm, 4.5 cm and 5.5 cm touch each other externally. Then the perimeter of the triangle formed by joining the centres of the circles, in cm is

- (a) 27
(b) $\pi [(3.5)^2 + (4.5)^2 + (5.5)^2]$
(c) 27π
(d) 13.5
185. Three sides of a triangular field are of length 15 m, 20m and 25 m long respectively. Find the cost of sowing seeds in the field at the rate of 5 rupees per sq. m
(a) Rs.300 (b) Rs.600
(c) Rs.750 (d) Rs.150
186. A chord of length 30 cm is at a distance of 8 cm from the centre of a circle. The radius of the circle is:
(a) 17 cm (b) 23 cm
(c) 21 cm (d) 19 cm
187. The radius of the incircle of a triangle whose sides are 9 cm, 12 cm and 15 cm is
(a) 9 cm (b) 13 cm
(c) 3 cm (d) 6 cm
188. The ratio of inradius and circumradius of a square is :
(a) $1 : \sqrt{2}$ (b) $\sqrt{2} : \sqrt{3}$
(c) $1 : 3$ (d) $1 : 2$
189. Three circles of equal radius 'a' cm touch each other. The area of the shaded region is :



- (a) $\left(\frac{\sqrt{3} + \pi}{2}\right) a^2 \text{sq. cm}$
(b) $\left(\frac{6\sqrt{3} - \pi}{2}\right) a^2 \text{sq. cm}$
(c) $(\sqrt{3} - \pi) a^2 \text{sq. cm}$
(d) $\left(\frac{2\sqrt{3} - \pi}{2}\right) a^2 \text{sq. cm}$
190. ABC is a right angled triangle. B being the right angle Mid- points of BC and AC are respectively B' and A'. Area of $\triangle A'B'C$ is
(a) $\frac{1}{2} \times \text{area of } \triangle ABC$
(b) $\frac{2}{3} \times \text{area of } \triangle ABC$
(c) $\frac{1}{4} \times \text{area of } \triangle ABC$
(d) $\frac{1}{8} \times \text{area of } \triangle ABC$
191. A wire of length 44 cm is first bent to form a circle and then rebent to form a square. The difference of the two enclosed areas is
(a) 44 cm^2 (b) 33 cm^2
(c) 55 cm^2 (d) 66 cm^2
192. $\angle ACB$ is an angle in the semicircle of diameter AB = 5 cm and AC : BC = 3 : 4. The area of the triangle ABC is
(a) $6\sqrt{2}$ sq. cm (b) 4 sq. cm
(c) 12 sq. cm (d) 6 sq. cm
193. If the lengths of the sides AB, BC and CA of a triangle ABC are 10 cm, 8 cm and 6 cm respectively and If M is the mid-point of BC and $MN \parallel AB$ to cut AC at N, then area of the trapezium ABMN is equal to
(a) 18 sq. cm (b) 20 sq. cm
(c) 12 sq. cm (d) 16 sq. cm
194. In an equilateral triangle of side 24 cm, a circle is inscribed touching its sides. The area of the remaining portion of the triangle is
($\sqrt{3} = 1.732$)
(a) 98.55 sq. cm (b) 100 sq. cm
(c) 101 sq. cm (d) 95 sq. cm
195. Two sides of a plot measuring 32 m and 24 m and the angle between them is a perfect right angle. The other two sides measure 25 m each and the other three angles are not right angles. The area of the plot in m^2 is
(a) 768 (b) 534
(c) 696.5 (d) 684
196. A and b are two sides adjacent to the right angle of a right angled triangle and p is the perpendicular drawn to the hypotenuse from the opposite vertex. Then p^2 is equal to
(a) $a^2 + b^2$ (b) $\frac{1}{a^2} + \frac{1}{b^2}$
(c) $\frac{a^2 b^2}{a^2 + b^2}$ (d) $a^2 - b^2$
197. A is the centre of circle whose radius is 8 and B is the centre of a circle whose diameter is 8. If these two circles touch

externally, then the area of the circle with diameter AB is

- (a) 36π (b) 64π
(c) 144π (d) 256π
198. If the numerical value of the height and the area of an equilateral triangle be same. then the length of each side of the triangle is
(a) 2 units (b) 4 units
(c) 5 units (d) 8 units
199. If the length of a side of the square is equal to that of the diameter of a circle, then the ratio of the area of the square and that of the circle ($\pi = \frac{22}{7}$)
(a) 14 : 11 (b) 7 : 11
(c) 11 : 14 (d) 11 : 7
200. The median of an equilateral triangle is $6\sqrt{3}$ cm. The area (in cm^2) of the triangle is
(a) 72 (b) 108
(c) $72\sqrt{3}$ (d) $36\sqrt{3}$
201. If the numerical value of the circumference and area of a circle is same, then the area is
(a) 6π sq. units
(b) 4π sq. units
(c) 8π sq. units
(d) 12π sq. units
202. The area of an equilateral triangle is 48 sq. cm. The length of the side is
(a) $4\sqrt{8}$ cm (b) $4\sqrt{3}$ cm
(c) 8 cm (d) $8\sqrt{3}$ cm
203. The external fencing of a circular path around a circular plot of land is 33m more than its interior fencing. The width of the path around the plot is
(a) 5.52 m (b) 5.25 m
(c) 2.55 m (d) 2.25 m
204. The perimeter of a triangle is 54 m and its sides are in the ratio 5 : 6 : 7. The area of the triangle is
(a) 18 m^2 (b) $54\sqrt{6} \text{ m}^2$
(c) $27\sqrt{2} \text{ m}^2$ (d) 25 m^2
205. A circular wire of diameter 112 cm is cut and bent in the form of a rectangle whose sides are in the ratio of 9 : 7. The smaller side of the rectangle is
(a) 77 cm (b) 97 cm
(c) 67 cm (d) 84 cm

206. If the perimeter of an equilateral triangle be 18 cm, then the length of each median is

- (a) $3\sqrt{2}$ cm (b) $2\sqrt{3}$ cm
(c) $3\sqrt{3}$ cm (d) $2\sqrt{2}$ cm

207. Two equal maximum sized circular plates are cut off from a circular paper sheet of circumference 352 cm. Then the circumference of each circular plate is

- (a) 176 cm (b) 150 cm
(c) 165 cm (d) 180 cm

208. The inradius of an equilateral triangle is $\sqrt{3}$ cm, then the perimeter of that triangle is

- (a) 18 cm (b) 15 cm
(c) 12 cm (d) 6 cm

209. The difference between the circumference and diameter of a circle is 150 m. The radius of that

circle is (Take $\pi = \frac{22}{7}$)

- (a) 25 metre (b) 35 metre
(c) 30 metre (d) 40 metre

210. The perimeters of a circle, a square and an equilateral triangle are same and their areas are C, S and T respectively. Which of the following statement is true?

- (a) $C = S = T$ (b) $C > S > T$
(c) $C < S < T$ (d) $S < C < T$

211. A horse takes $2\frac{1}{2}$ seconds to

complete a round around a circular field. If the speed of the horse was 66 m/sec, then the radius of the field is, [Given

$$\pi = \frac{22}{7}]$$

- (a) 25.62 m (b) 26.52 m
(c) 25.26 m (d) 26.25 m

212. The diameter of the front wheel of an engine is $2x$ cm and that of rear wheel is $2y$ cm to cover the same distance, find the number of times the rear wheel will revolve when the front wheel revolves 'n' times,

- (a) $\frac{n}{xy}$ times (b) $\frac{yn}{x}$ times
(c) $\frac{nx}{y}$ times (d) $\frac{xy}{n}$ times

213. A bicycle wheel has a diameter (including the tyre) of 56 cm. The number of times the wheel will rotate to cover a distance of 2.2

km is (Assume $\pi = \frac{22}{7}$)

- (a) 625 (b) 1250
(c) 1875 (d) 2500

214. If the altitude of an equilateral triangle is $12\sqrt{3}$ cm, then its area would be;

- (a) $36\sqrt{3}$ cm² (b) $144\sqrt{3}$ cm²
(c) 72 cm² (d) 12 cm²

215. Let C_1 and C_2 be the inscribed and circumscribed circles of a triangle with sides 3 cm, 4 cm and 5 cm,

then $\frac{\text{area of } C_1}{\text{area of } C_2}$ is

- (a) $\frac{9}{25}$ (b) $\frac{16}{25}$
(c) $\frac{9}{16}$ (d) $\frac{4}{25}$

216. A circular swimming pool is surrounded by a concrete wall 4m wide. If the area of the concrete wall surrounding the

pool is $\frac{11}{25}$ that of the pool, then

the radius (in m) of the pool :

- (a) 8 (b) 16 (c) 30 (d) 20

217. If the area of a circle is A, radius of the circle is r and circumference of it is c, then

- (a) $rC = 2A$ (b) $\frac{C}{A} = \frac{r}{2}$
(c) $AC = \frac{r^2}{4}$ (d) $\frac{A}{r} = C$

218. The sides of a triangle having area 7776 sq. cm are in the ratio 3 : 4 : 5. The perimeter of the triangle is:

- (a) 400 cm (b) 412 cm
(c) 424 cm (d) 432 cm

219. The perimeter of a sheet of paper in the shape of a quadrant of a circle is 75 cm. Its area would

be ($\pi = \frac{22}{7}$)

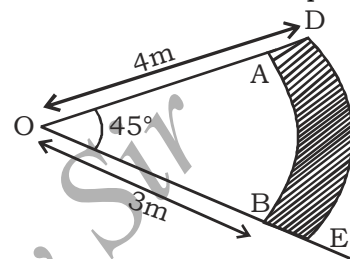
- (a) 512.25 cm² (b) 346.5 cm²
(c) 100 cm² (d) 693 cm²

220. A circle is inscribed in an equilateral triangle of side 8m. The approximate area of the

unoccupied space inside the triangle is

- (a) 21 m² (b) 11 m²
(c) 20 m² (d) 22 m²

221. In the figure, OED and OBA are sectors of a circle with centre O. The area of the shaded portion.



- (a) $\frac{11}{16}$ m² (b) $\frac{11}{8}$ m²
(c) $\frac{11}{2}$ m² (d) $\frac{11}{4}$ m²

222. If the circumference of a circle is $\frac{30}{\pi}$, then the diameter of the circle is

- (a) 30 (b) $\frac{15}{\pi}$ (c) 60π (d) $\frac{30}{\pi^2}$

223. The outer and inner diameter of a circular path be 728 cm and 700 cm respectively. The breadth of the path is

- (a) 7 cm (b) 14 cm
(c) 28 cm (d) 20 cm

224. A piece of wire when bent to form a circle will have a radius of 84 cm. If the wire is bent to form a square, the length of a side of the square is

- (a) 152 cm (b) 168 cm
(c) 132 cm (d) 225 cm

225. The area of a circle is 324 π sq. cm. The length of its longest chord (in cm.) is

- (a) 36 (b) 38 (c) 28 (d) 32

226. The circumference of a triangle is 24 cm and the circumference of its in-circle is 44 cm. Then the area of the triangle is

(taking $\pi = \frac{22}{7}$)

- (a) 56 square cm
(b) 48 square cm
(c) 84 square cm
(d) 68 square cm

227. If the length of each of two equal sides of an isosceles triangle is 10 cm. and the adjacent angle is 45° , then the area of the triangle is
 (a) $20\sqrt{2}$ square cm
 (b) $25\sqrt{2}$ square cm
 (c) $12\sqrt{2}$ square cm
 (d) $15\sqrt{2}$ square cm
228. The inner-radius of a triangle is 6 cm, and the sum of the lengths of its sides is 50 cm. The area of the triangle (in sq. cm.) is
 (a) 150 (b) 300 (c) 50 (d) 56
229. One of the angles of a right-angled triangle is 15° , and the hypotenuse is 1 m. The area of the triangle (in sq. cm.) is
 (a) 1220 (b) 1250
 (c) 1200 (d) 1215
230. If an isosceles triangle the length of each equal side is 'a' units and that of the third side is 'b' units, then its area will be
 (a) $\frac{a}{4}\sqrt{4a^2-a^2}$ sq. units
 (b) $\frac{b}{4}\sqrt{4a^2-b^2}$ sq. units
 (c) $\frac{a}{2}\sqrt{2a^2-b^2}$ sq. units
 (d) $\frac{b}{2}\sqrt{a^2-2b^2}$ sq. units
231. What is the position of the circumcentre of an obtuse-angles triangle?
 (a) It is the vertex opposite to the largest side.
 (b) It is the mid point of the largest side.
 (c) It lies outside the triangles.
 (d) It lies inside the triangles.
232. The ratio of circumference and diameter of a circle is 22 : 7. If the circumference be $1\frac{4}{7}$ m, then the radius of the circle is:
 (a) $\frac{1}{4}$ m (b) $\frac{1}{3}$ m
 (c) $\frac{1}{2}$ m (d) 1 m
233. The area of a circle whose radius is the diagonal of a square whose area is 4 is:
 (a) 4π (b) 8π
 (c) 6π (d) 16π
234. The diagonals of a rhombus are 32 cm and 24 cm respectively. The perimeter of the rhombus is:
 (a) 80 cm (b) 72 cm
 (c) 68 cm (d) 64 cm
235. The diagonals of a rhombus are 24 cm and 10 cm. The perimeter of the rhombus (in cm) is :
 (a) 68 (b) 65 (c) 54 (d) 52
236. The perimeter of a rhombus is 40 cm, If one of the diagonals be 12 cm long, what is the length of the other diagonal ?
 (a) 12 cm (b) $\sqrt{136}$ cm,
 (c) 16 cm (d) $\sqrt{44}$ cm
237. The perimeter of a rhombus is 40 m and its height is 5m its area is:
 (a) 60 m^2 (b) 50 m^2
 (c) 45 m^2 (d) 55 m^2
238. The perimeter of a rhombus is 40 cm. If the length of one of its diagonals be 16 cm, the length of the other diagonal is
 (a) 14 cm (b) 15 cm
 (c) 16 cm (d) 12 cm
239. The area of a rhombus is 150 cm^2 . The length of one of its diagonals is 10 cm. The length of the other diagonal is :
 (a) 25 cm (b) 30 cm
 (c) 35 cm (d) 40 cm
240. The area of a regular hexagon of side $2\sqrt{3}$ cm is :
 (a) $18\sqrt{3}\text{ cm}^2$ (b) $12\sqrt{3}\text{ cm}^2$
 (c) $36\sqrt{3}\text{ cm}^2$ (d) $27\sqrt{3}\text{ cm}^2$
241. Each side of a regular hexagon is 1 cm. The area of the hexagon is
 (a) $\frac{3\sqrt{3}}{2}\text{ cm}^2$ (b) $\frac{3\sqrt{3}}{4}\text{ cm}^2$
 (c) $4\sqrt{3}\text{ cm}^2$ (d) $3\sqrt{2}\text{ cm}^2$
242. The length of one side of a rhombus is 6.5 cm and its altitude is 10 cm. If the length of its diagonal be 26 cm, the length of the other diagonal will be:
 (a) 5 cm (b) 10 cm
 (c) 6.5 cm (d) 26 cm
243. The measure of each of two opposite angles of a rhombus is 60° and the measure of one of its sides is 10 cm. The length of its smaller diagonal is :
 (a) 10cm (b) $10\sqrt{3}\text{ cm}$
 (c) $10\sqrt{2}\text{ cm}$ (d) $\frac{5}{2}\sqrt{2}\text{ cm}$
244. The perimeter of a rhombus is 100 cm, If one of its diagonals is 14 cm. Then the area of the rhombus is
 (a) 144 cm^2 (b) 225 cm^2
 (c) 336 cm^2 (d) 400 cm^2
245. The ratio of the length of the parallel sides of a trapezium is 3 : 2. The shortest distance between them is 15 cm. If the area of the trapezium is 450 cm^2 the sum of the length of the parallel sides is
 (a) 15 cm (b) 36 cm
 (c) 42 cm (d) 60 cm
246. A parallelogram has sides 15 cm and 7 cm long. The length of one of the diagonals is 20 cm. The area of the parallelogram is
 (a) 42 cm^2 (b) 60 cm^2
 (c) 84 cm^2 (d) 96 cm^2
247. Sides of a parallelogram are in the ratio 5 : 4. Its area is 1000 sq. units. Altitude on the greater side is 20 units. Altitude on the smaller side is
 (a) 20 units (b) 25 units
 (c) 10 units (d) 15 units
248. The perimeter of a rhombus is 40 cm and the measure of an angle is 60° , then the area of it is:
 (a) $100\sqrt{3}\text{ cm}^2$ (b) $50\sqrt{3}\text{ cm}^2$
 (c) $160\sqrt{3}\text{ cm}^2$ (d) 100 cm^2
249. Two adjacent sides of a parallelogram are of length 15 cm and 18 cm, If the distance between two smaller sides is 12 cm, then the distance between two bigger sides is
 (a) 8 cm (b) 10 cm
 (c) 12 cm (d) 15 cm
250. A parallelogram ABCD has sides $AB = 24\text{ cm}$ and $AD = 16\text{ cm}$. The distance between the sides AB and DC is 10 cm. Find the distance between the sides AD and BC.
 (a) 15 cm (b) 18 cm
 (c) 16 cm (d) 9 cm

251. The adjacent sides of a parallelogram are 36 cm and 27 cm in length. If the distance between the shorter sides is 12 cm, then the distance between the longer sides is

- (a) 10 cm (b) 12 cm
(c) 16 cm (d) 9 cm

252. If the diagonals of a rhombus are 8 cm and 6 cm, then the area of square having same side as that of rhombus is

- (a) 25 (b) 55 (c) 64 (d) 36

253. Two circles with centres A and B and radius 2 units touch each other externally at 'C'. A third circle with centre 'C' and radius '2' units meets other two at D and E. Then the area of the quadrilateral ABDE is

- (a) $2\sqrt{2}$ sq. units
(b) $3\sqrt{3}$ sq. units
(c) $3\sqrt{2}$ sq. units
(d) $2\sqrt{3}$ sq. units

254. The perimeter of a non-square rhombus is 20 cm. One of its diagonal is 8 cm. The area of the rhombus is

- (a) 28 sq. cm (b) 20 sq. cm
(c) 22 sq. cm (d) 24 sq. cm

255. The perimeter of a rhombus is 100 cm and one of its diagonals is 40 cm. Its area (in cm^2) is

- (a) 1200 (b) 1000
(c) 600 (d) 500

256. In $\triangle ABC$, D and E are the points of sides AB and BC respectively such that $DE \parallel AC$ and $AD : BD = 3 : 2$. The ratio of area of trapezium ACED to that of $\triangle BED$ is

- (a) 4 : 15 (b) 15 : 4
(c) 4 : 21 (d) 21 : 4

257. ABCD is a trapezium in which $AB \parallel DC$ and $AB = 2 CD$. The diagonals AC and BD meet at O. The ratio of area of triangles AOB and COD is

- (a) 1 : 1 (b) 1 : $\sqrt{2}$
(c) 4 : 1 (d) 1 : 4

258. The length of each side of a rhombus is equal to the length of the side of a square whose

diagonal is $40\sqrt{2}$ cm. If the length of the diagonals of the rhombus are in the ratio 3 : 4, then its area (in cm^2) is

- (a) 1550 (b) 1600
(c) 1535 (d) 1536

259. ABCD is a parallelogram BC is produced to Q such that $BC = CQ$. Then

- (a) $\text{area}(\triangle ABC) = \text{area}(\triangle DCQ)$
(b) $\text{area}(\triangle ABC) > \text{area}(\triangle DCQ)$
(c) $\text{area}(\triangle ABC) < \text{area}(\triangle DCQ)$
(d) $\text{area}(\triangle ABC) \neq \text{area}(\triangle DCQ)$

260. ABCD is a parallelogram. P and Q are the mid-points of sides BC and CD respectively. If the area of $\triangle ABC$ is 12 cm^2 , then the area of $\triangle APQ$ is

- (a) 12 cm^2 (b) 8 cm^2
(c) 9 cm^2 (d) 10 cm^2

261. The area of a rhombus is 216 cm^2 and the length of its one diagonal is 24 cm. The perimeter (in cm) of the rhombus is

- (a) 52 (b) 60
(c) 120 (d) 100

262. One of the four angles of a rhombus is 60° . If the length of each side of the rhombus is 8 cm, then the length of the longer diagonal is

- (a) $8\sqrt{3}$ cm (b) 8 cm
(c) $4\sqrt{3}$ cm (d) $\frac{8}{\sqrt{3}}$ cm

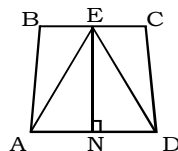
263. The diagonals of a rhombus are 12 cm and 16 cm respectively. The length of one side is

- (a) 8 cm (b) 6 cm
(c) 10 cm (d) 12 cm

264. A parallelogram has sides 60 m and 40 m and one of its diagonals is 80 m long. Its area is

- (a) $500\sqrt{15} \text{ m}^2$ (b) $600\sqrt{15} \text{ m}^2$
(c) $400\sqrt{15} \text{ m}^2$ (d) $450\sqrt{15} \text{ m}^2$

265. ABCD is a trapezium with AD and BC parallel sides. The ratio of the area of ABCD to that of $\triangle AED$ is



- (a) $\frac{AD}{BC}$ (b) $\frac{BE}{EC}$

- (c) $\frac{AD + BE}{AD + CE}$ (d) $\frac{AD + BC}{AD}$

266. perimeter of a rhombus is $2p$ unit and sum of length of diagonals is m unit, then area of the rhombus is

- (a) $\frac{1}{4} m^2 p$ sq unit
(b) $\frac{1}{4} mp^2$ sq unit
(c) $\frac{1}{4} (m^2 - p^2)$ sq unit
(d) $\frac{1}{4} (m^2 + p^2)$ sq unit

267. Area of regular hexagon with side 'a' is

- (a) $\frac{3\sqrt{3}}{4} a^2$ sq. unit
(b) $\frac{12}{2\sqrt{3}} a^2$ sq. unit
(c) $\frac{9}{2\sqrt{3}} a^2$ sq. unit
(d) $\frac{6}{\sqrt{2}} a^2$ sq. unit

268. In $\triangle ABC$, D and E are two points on the sides AB and AC respectively so that $DE \parallel BC$ and

$$\frac{AD}{BD} = \frac{2}{3}. \quad \text{Then}$$

$\frac{\text{the area of trapezium DECB}}{\text{the area of } \triangle ABC}$ is equal to

- (a) $\frac{5}{9}$ (b) $\frac{21}{25}$
(c) $1\frac{4}{5}$ (d) $5\frac{1}{4}$

269. The sides of a rhombus are 10 cm each and a diagonal measures 16 cm. Area of the rhombus is

- (a) 96 sq. cm (b) 160 sq. cm
(c) 100 sq. cm (d) 40 sq. cm

270. The lengths of two parallel sides of a trapezium are 6 cm and 8 cm. If the height of the trapezium be 4 cm, then its area is

- (a) 28 cm^2 (b) 56 cm^2
(c) 30 cm^2 (d) 36 cm^2

271. If diagonals of a rhombus are 24 cm and 32 cm, then perimeter of that rhombus is
 (a) 80 cm (b) 84 cm
 (c) 76 cm (d) 72 cm
272. The area of an isosceles trapezium is 176 cm^2 and the height is $\frac{2}{11}$ th of the sum of its parallel sides. If the ratio of the length of the parallel sides is 4 : 7, then the length of a diagonal (in cm) is
 (a) $2\sqrt{137}$ (b) 24
 (c) $\sqrt{137}$ (d) 28
273. The perimeter of a rhombus is 60 cm and one of its diagonal is 24 cm. The area of the rhombus is
 (a) 432 sq.cm (b) 216 sq.cm
 (c) 108 sq.cm (d) 206 sq.cm
274. The area of the parallelogram whose length is 30 cm, width is 20 cm and one diagonal is 40 cm is
 (a) $200\sqrt{15} \text{ cm}^2$
 (b) $300\sqrt{15} \text{ cm}^2$
 (c) $100\sqrt{15} \text{ cm}^2$
 (d) $150\sqrt{15} \text{ cm}^2$
275. The area of a rhombus is 256 sq.cm. and one of its diagonal is twice the other in length. Then length of its larger diagonal is
 (a) 32 cm (b) 48 cm
 (c) 36 cm (d) 24 cm
276. The length of two parallel sides of a trapezium are 15 cm and 20 cm. If its area is 175 sq.cm, then its height is:
 (a) 25 cm (b) 10 cm
 (c) 20 cm (d) 15 cm
277. The cost of carpentering a room is Rs. 120. If the width had been 4 metres less, the cost of the Carpet would have been Rs. 20 less. The width of the room is :
 (a) 24 m (b) 20 m
 (c) 25 m (d) 18.4 m
278. The floor of a corridor is 100 m long and 3 m wide. Cost of covering the floor with carpet 50 cm wide at the ratio of Rs. 15 per m is
 (a) Rs. 4500 (b) Rs. 9000
 (c) Rs. 7500 (d) Rs. 1900
279. A playground is in the shape of a rectangle. A sum of Rs. 1,000 was spent to make the ground usable at the rate of 25 paise per sq. m. The breadth of the ground is 50 m. If the length of the ground is increased by 20 m. what will be the expenditure (in rupees) at the same rate per sq. m.?
 (a) 1,250 (b) 1,000
 (c) 1,500 (d) 2,250
280. A hall 25 metres long and 15 metres broad is surrounded by a verandah of uniform width of 3.5 metres. The cost of flooring the verandah, at Rs. 27.50 per square metre is
 (a) Rs. 9149.50 (b) Rs. 8146.50
 (c) Rs. 9047.50 (d) Rs. 4186.50
281. The outer circumference of a circular race-track is 528 metre. The track is everywhere 14 metre wide. Cost of levelling the track at the rate of Rs.10 per sq. metre is :
 (a) Rs. 77660 (b) Rs. 67760
 (c) Rs. 66760 (d) Rs. 76760
282. The length and breadth of a rectangular field are in the ratio of 3 : 2. If the perimeter of the field is 80m, its breadth (in metres) is :
 (a) 18 (b) 16 (c) 10 (d) 24
283. The sides of a rectangular plot are in the ratio 5 : 4 and its area is equal to 500 sq.m The perimeter of the plot is :
 (a) 80 m (b) 100 m
 (c) 90 m (d) 95 m
284. ABC is a triangle with base AB, D is a point on AB such that AB = 5 and DB = 3. What is the ratio of the area of $\triangle ADC$ to the area of $\triangle ABC$?
 (a) $\frac{2}{5}$ (b) $\frac{2}{3}$
 (c) $\frac{9}{25}$ (d) $\frac{4}{25}$
285. If the area of a triangle is 1176 cm^2 and the ratio of base and corresponding altitude is 3 : 4, then the altitude of the triangle is:
 (a) 42 cm (b) 52 cm
 (c) 54 cm (d) 56 cm
286. The sides of a triangle are in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. If the perimeter of the triangle is 52 cm, the length of the smallest side is :
 (a) 24 cm (b) 10 cm
 (c) 12 cm (d) 9 cm
287. If the diagonals of two squares are in the ratio of 2 : 5. Their area will be in the ratio of
 (a) $\sqrt{2} : \sqrt{5}$ (b) 2 : 5
 (c) 4 : 25 (d) 4 : 5
288. The ratio of base of two triangles is $x : y$ and that of their areas is $a : b$. Then the ratio of their corresponding altitudes will be:
 (a) $\frac{a}{y} : \frac{b}{x}$ (b) 1 : 1
 (c) $ay : bx$ (d) $\frac{x}{a} : \frac{b}{y}$
289. The area of a field in the shape of a trapezium measures 1440 m^2 . The perpendicular distance between its parallel sides is 24m. If the ratio of the parallel sides is 5 : 3, the length of the longer parallel side is :
 (a) 75 m (b) 45 m
 (c) 120 m (d) 60 m
290. If the ratio of areas of two squares is 225 : 256, then the ratio of their perimeter is :
 (a) 225 : 256 (b) 256 : 225
 (c) 15 : 16 (d) 16 : 15
291. The area of a triangle is 216 cm^2 and its sides are in the ratio 3 : 4 : 5. The perimeter of the triangle is:
 (a) 6 cm (b) 12 cm
 (c) 36 cm (d) 72 cm
292. A circular wire of radius 42 cm is bent in the form of a rectangle whose sides are in the ratio of 6 : 5. The smaller side of the rectangle is (Take $\pi = \frac{22}{7}$):
 (a) 60 cm (b) 30 cm
 (c) 25 cm (d) 36 cm
293. The ratio of the outer and the inner perimeter of a circular path is 23 : 22, If the path is 5 meters wide the diameter of the inner circle is:
 (a) 110 m (b) 55 m
 (c) 220 m (d) 230 m
294. The angles of a triangle are in the ratio 3 : 4 : 5. The measure of the largest angle of the triangle is
 (a) 60° (b) 75°
 (c) 120° (d) 150°
295. The ratio of the area of a square to that of the square drawn on its diagonal is:
 (a) 1 : 1 (b) 1 : 2
 (c) 1 : 3 (d) 1 : 4
296. A square and an equilateral triangle are drawn on the same base. The ratio of their area is
 (a) 2 : 1 (b) 1 : 1
 (c) $\sqrt{30} : \sqrt{4}$ (d) $4 : \sqrt{3}$

297. If the area of a circle and a square are equal, then the ratio of their perimeter is
 (a) $1 : 1$ (b) $2 : \pi$
 (c) $\pi : 2$ (d) $\sqrt{\pi} : 2$
298. The area of two equilateral triangles are in the ratio $25 : 36$. Their altitudes will be in the ratio:
 (a) $36 : 25$ (b) $25 : 36$
 (c) $5 : 6$ (d) $\sqrt{5} : \sqrt{6}$
299. If the length and the perimeter of a rectangle are in the ratio $5 : 16$, then its length and breadth will be in the ratio
 (a) $5 : 11$ (b) $5 : 8$
 (c) $5 : 4$ (d) $5 : 3$
300. Through each vertex of a triangle, a line parallel to the opposite side is drawn. the ratio of the perimeter of the new triangle. thus formed, with that of the original triangle is
 (a) $3 : 2$ (b) $1 : 2$
 (c) $2 : 1$ (d) $2 : 3$
301. The ratio of the number giving the measure of the circumference and the area of a circle of radius 3 cm is
 (a) $1 : 3$ (b) $2 : 3$
 (c) $2 : 9$ (d) $3 : 2$
302. The height of an equilateral triangle is $4\sqrt{3}$ cm. The ratio of the area of its circumcircle to that of its in-circle is
 (a) $2 : 1$ (b) $4 : 1$
 (c) $4 : 3$ (d) $3 : 2$
303. The radius of circle A is twice that of circle B and the radius of circle B is twice that of circle C. Their area will be in the ratio
 (a) $16 : 4 : 1$ (b) $4 : 2 : 1$
 (c) $1 : 2 : 4$ (d) $1 : 4 : 16$
304. A circle and a square have equal areas. the ratio of a side of the square and the radius of the circle is
 (a) $1 : \sqrt{\pi}$ (b) $\sqrt{\pi} : 1$
 (c) $1 : \pi$ (d) $\pi : 1$
305. The sides of a triangle are in the ratio $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$ and its perimeter is 94 cm. The length of the smallest side of the triangle is:
 (a) 18 cm (b) 22.5 cm
 (c) 24 cm (d) 27 m
306. The sides of a quadrilateral are in the ratio $3 : 4 : 5 : 6$ and its perimeter is 72 cm. The length of its greatest side (in cm) is
 (a) 24 (b) 27 (c) 30 (d) 36
307. The ratio of the radii of two wheels is $3 : 4$. The ratio of their circumference is
 (a) $4 : 3$ (b) $3 : 4$
 (c) $2 : 2$ (d) $3 : 2$
308. The sides of a triangle are in the ratio $2 : 3 : 4$. the perimeter of the triangle is 18 cm. The area (in cm^2) of the triangle is
 (a) 9 (b) 36
 (c) $\sqrt{42}$ (d) $3\sqrt{15}$
309. The ratio of the areas of the circumcircle and the incircle of an equilateral triangle is
 (a) $2 : 1$ (b) $4 : 1$
 (c) $8 : 1$ (d) $3 : 2$
310. In $\triangle ABC$, the medians CD and BE intersect each other at O, then the ratio of the areas of $\triangle ODE$ and $\triangle OBC$ is
 (a) $1 : 4$ (b) $6 : 1$
 (c) $1 : 12$ (d) $12 : 1$
311. The ratio of the area of two isosceles triangles having the same vertical angle (i.e. angle between equal sides) is $1 : 4$. The ratio of their heights is
 (a) $1 : 4$ (b) $2 : 5$
 (c) $1 : 2$ (d) $3 : 4$
312. The ratio of length of each equal side and the third side of an isosceles triangle is $3 : 4$. If the area is $8\sqrt{5}$ units². the third side is
 (a) 3 units
 (b) $2\sqrt{5}$ square units units
 (c) $8\sqrt{2}$ units
 (d) 12 units
313. The ratio of sides of a triangle is $3 : 4 : 5$. If area of the triangle is 72 square unit then the length of the smallest side is :
 (a) $4\sqrt{3}$ unit (b) $5\sqrt{3}$ unit
 (c) $6\sqrt{3}$ unit (d) $3\sqrt{3}$ unit
314. The ratio of sides of a triangle is $3 : 4 : 5$ and area of the triangle is 72 square units. Then the area of an equilateral triangle whose perimeter is same as that of the previous triangle is
 (a) $32\sqrt{3}$ square units
 (b) $48\sqrt{3}$ square units
 (c) 96 square units
 (d) $60\sqrt{3}$ square units
315. The parallel sides of a trapezium are in a ratio $2 : 3$ and their shortest distance is 12 cm. If the area of the trapezium is 480 sq. cm., the longer of the parallel sides is of length :
 (a) 56 cm (b) 36 cm
 (c) 42 cm (d) 48 cm
316. An equilateral triangle is drawn on the diagonal of a square. The ratio of the area of the triangle to that of the square is
 (a) $\sqrt{3} : 2$ (b) $1 : \sqrt{3}$
 (c) $2 : \sqrt{3}$ (d) $4 : \sqrt{3}$
317. Two triangles ABC and DEF are similar to each other in which $AB = 10$ cm, $DE = 8$ cm. Then the ratio of the area of triangles ABC and DEF is
 (a) $4 : 5$ (b) $25 : 16$
 (c) $64 : 125$ (d) $4 : 7$
318. The ratio between the area of two circles is $4 : 7$. What will be the ratio of their radii ?
 (a) $2 : \sqrt{7}$ (b) $4 : 7$
 (c) $16 : 49$ (d) $4 : \sqrt{7}$
319. The area of a circle is proportional to the square of its radius. A small circle of radius 3 cm is drawn within a larger circle of radius 5 cm. Find the ratio of the area of the annular zone to the area of the larger circle (Area of the annular zone is the difference between the area of the larger circle and that of the smaller circle)
 (a) $9 : 16$ (b) $9 : 25$
 (c) $16 : 25$ (d) $16 : 27$
320. The diameter of two circles are the side of a square and the diagonal of the square. The ratio of the area of the smaller circle and the larger circle is
 (a) $1 : 2$ (b) $1 : 4$
 (c) $\sqrt{2} : \sqrt{3}$ (d) $1 : \sqrt{2}$
321. The ratio of the area of an equilateral triangle and that of its circumcircle is
 (a) $2\sqrt{3} : 2\pi$ (b) $4 : \pi$
 (c) $3\sqrt{3} : 4\pi$ (d) $7\sqrt{2} : 2\pi$

322. If the perimeters of a rectangle and a square are equal and the ratio of two adjacent sides of the rectangle is 1 : 2 then the ratio of area of the rectangle and that of the square is

- (a) 1 : 1 (b) 1 : 2
(c) 2 : 3 (d) 8 : 9

323. The perimeter of a rectangle and an equilateral triangle are same. Also, one of the sides of the rectangle is equal to the side of the triangle. The ratio of the area of the rectangle and the triangle is

- (a) $\sqrt{3} : 1$ (b) $1 : \sqrt{3}$
(c) $2 : \sqrt{3}$ (d) $4 : \sqrt{3}$

324. The radius of a circle is a side of a square. The ratio of the area of the circle and the square is

- (a) $1 : \pi$ (b) $\pi : 1$
(c) $\pi : 2$ (d) $2 : \pi$

325. ABC is an isosceles right angled triangle with $\angle B = 90^\circ$. On the sides AC and AB, two equilateral triangles ACD and ABE have been constructed. The ratio of area of $\triangle ABE$ and $\triangle ACD$ is

- (a) 1 : 3 (b) 2 : 3
(c) 1 : 2 (d) $1 : \sqrt{2}$

326. Two triangles ABC and DEF are similar to each other in which $AB = 10$ cm, $DE = 8$ cm. Then the ratio of the area of triangles ABC and DEF is

- (a) 4 : 5 (b) 25 : 16
(c) 64 : 125 (d) 4 : 7

327. ABC is a right angled triangle, B being the right angle. Mid-points of BC and AC are respectively B' and A'. The ratio of the area of the quadrilateral AA'B'B to the area of the triangle ABC is

- (a) 1 : 2 (b) 2 : 3
(c) 3 : 4
(d) None of the above

328. The sides of a triangle are in the

ratio $\frac{1}{4} : \frac{1}{6} : \frac{1}{8}$ and its perimeter

is 91 cm. The difference of the length of longest side and that of shortest side is

- (a) 19 cm (b) 20 cm
(c) 28 cm (d) 21 cm

329. If the arcs of unit length in two circles subtend angles of 60° and 75° at their centres, the ratio of their radii is

- (a) 3 : 4 (b) 4 : 5
(c) 5 : 4 (d) 3 : 5

330. ABCD is a parallelogram in which diagonals AC and BD intersect at O. If E, F, G and H are the mid-points of AO, DO, CO and BO respectively, then the ratio of the perimeter of the quadrilateral EFGH to the perimeter of parallelogram ABCD is

- (a) 1 : 4 (b) 2 : 3
(c) 1 : 2 (d) 1 : 3

331. If the circumference of a circle increases from 4π to 8π , what change occurs in its area ?

- (a) It doubles (b) It triples
(c) It quadruples (d) It is halved

332. If the length of a rectangle is increased by 25% and the width is decreased by 20%, then the area of the rectangle :

- (a) Increases by 5%
(b) decreases by 5%
(c) remains unchanged
(d) increases by 10%

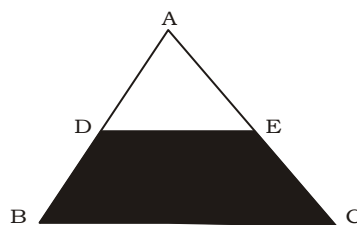
333. The area of a circle of radius 5 is numerically what percent of its circumference ?

- (a) 200% (b) 255%
(c) 240% (d) 250%

334. If the circumference and area of a circle are numerically equal, then the diameter is equal to :

- (a) area of the circle
(b) $\frac{\pi}{2}$ (c) 2π (d) 4

335. If D and E are the mid-points of the side AB and AC respectively of the $\triangle ABC$ in the given figure here, the shaded region of the triangle is what per cent of the whole triangular region ?



- (a) 50% (b) 25%
(c) 75% (d) 60%

336. The length of a rectangle is decreased by 10% and its

breadth is increased by 10%, By what percent is its area changed?

- (a) 0% (b) 1%
(c) 5% (d) 100%

337. The percentage increase in the area of a rectangle. If each of its sides is increased by 20%, is:

- (a) 40% (b) 42%
(c) 44% (d) 46%

338. If the circumference of a circle is reduced by 50%, its area will be reduced by

- (a) 12.5% (b) 25%
(c) 50% (d) 75%

339. If the side of a square is increased by 25%, then its area is increased by:

- (a) 25% (b) 55%
(c) 40.5% (d) 56.25%

340. If the radius of a circle is increased by 50% . its area is increased by :

- (a) 125% (b) 100%
(c) 75% (d) 50%

341. If the length of a rectangle is increased by 20% and its breadth is decreased by 20%, then its area

- (a) increases by 4%
(b) decreases by 4%
(c) decreases by 1%
(d) None of these

342. If each side of a rectangle is increased by 50%, its area will be increased by

- (a) 50% (b) 125%
(c) 100% (d) 250%

343. If the altitude of a triangle is increased by 10% while its area remains same, its corresponding base will have to be decreased by

- (a) 10% (b) 9%
(c) $9\frac{1}{11}\%$ (d) $11\frac{1}{9}\%$

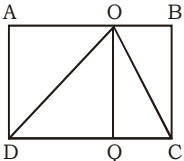
344. If the circumference of a circle is increased by 50% then the area will be increased by

- (a) 50% (b) 75%
(c) 100% (d) 125%

345. The length and breadth of a rectangle are increased by 12% and 15% respectively. Its area will be increased by :

- (a) $27\frac{1}{5}\%$ (b) $28\frac{4}{5}\%$
(c) 27% (d) 28%

346. If the sides of an equilateral triangle are increased by 20%, 30% and 50% respectively to form a new triangle the increase in the perimeter of the equilateral triangle is
(a) 25% (b) $33\frac{1}{3}\%$
(c) 75% (d) 100%
347. Each side of a rectangular field is diminished by 40%. By how much percent is the area of the field diminished?
(a) 32% (b) 64%
(c) 25% (d) 16%
348. The length of rectangle is increased by 60%. By what percent would the breadth to be decreased to maintain the same area?
(a) $37\frac{1}{2}\%$ (b) 60%
(c) 75% (d) 120%
349. The length and breadth of rectangle are increased by 20% and 25% respectively. The increase in the area of the resulting rectangle will be :
(a) 60% (b) 50%
(c) 40% (d) 30%
350. If each side of a square is increased by 10%, its area will be increased by
(a) 10% (b) 21%
(c) 44% (d) 100%
351. If the length of a rectangular plot of land is increased by 5% and the breadth is decreased by 10%. How much will its area increase or decrease?
(a) 6.5% increase
(b) 5.5% decrease
(c) 5.5% increase
(d) 6.5% decrease
352. The radius of circle is increased by 1%. How much does the area of the circle increase?
(a) 1% (b) 1.1%
(c) 2% (d) 2.01%
353. The length of a room floor exceeds its breadth by 20m. The area of the floor remains unaltered when the length is decreased by 10 m but the breadth is increased by 5 m. The area of the floor (in square meters) is:
(a) 280 (b) 325
(c) 300 (d) 420
354. In measuring the sides of a rectangle, there is an excess of 5% on one side and 2% deficit on the other. Then the error percent in the area is
(a) 3.3% (b) 3.0 %
(c) 2.9% (d) 2.7%
355. The length and breadth of a rectangle are increased by 30% and 20% respectively. The area of the rectangle so formed exceeds the area of the square by
(a) 46% (b) 66%
(c) 42% (d) 56%
356. If side of a square is increased by 40%, the percentage increase in its surface area is
(a) 40% (b) 60%
(c) 80% (d) 96%
357. If the diameter of a circle is increased by 8%, then its area is increased by :
(a) 16.64% (b) 6.64%
(c) 165 (d) 16.46%
358. One side of a rectangle is increased by 30%. To maintain the same area, the other side will have to be decreased by
(a) $23\frac{1}{13}\%$ (b) $76\frac{12}{13}\%$
(c) 30% (d) 15%
359. The length and breadth of a rectangle are doubled. Percentage increase in area is
(a) 150% (b) 200%
(c) 300% (d) 400%
360. The length of a rectangle is increased by 10% and breadth decreased by 10%. The area of the new rectangle is
(a) neither increased nor decreased
(b) increased by 1%
(c) decreased by 2%
(d) decreased by 1%
361. How many circular plates of diameter d be taken out of a square plate of side 2d with minimum loss of material?
(a) 8 (b) 6 (c) 4 (d) 2
362. What is the total area of three equilateral triangles inscribed in a semi-circle of radius 2 cm?
(a) 12 cm^2 (b) $\frac{3\sqrt{3}}{4}\text{ cm}^2$
(c) $\frac{9\sqrt{3}}{4}\text{ cm}^2$ (d) $3\sqrt{3}\text{ cm}^2$
363. The area of a sector of a circle of radius 36 cm is $72\pi\text{ cm}^2$. The length of the corresponding arc of the sector is
(a) $\pi\text{ cm}$ (b) $2\pi\text{ cm}$
(c) $3\pi\text{ cm}$ (d) $4\pi\text{ cm}$
364. A square is inscribed in a circle of diameter $2a$ and another square is circumscribing circle. The difference between the areas of outer and inner squares is
(a) a^2 (b) $2a^2$ (c) $3a^2$ (d) $4a^2$
365. ABC is a triangle right angled at A. AB = 6 cm and AC = 8 cm. Semi-circles drawn (outside the triangle) on AB, AC and BC as diameters which enclose areas x, y and z square units, respectively. What is $x+y-z$ equal to?
(a) 48 cm^2 (b) 32 cm^2
(c) 0 (d) None of these
366. Consider an equilateral triangle of a side of one unit length. A new equilateral triangle is formed by joining the mid-points of one, then a third equilateral triangle is formed by joining the mid-points of second. The process is continued. The perimeter of all triangles, thus formed is
(a) 4 (b) 5
(c) 6 (d) 7
367. What is the area of the larger segment of a circle formed by a chord of length 5 cm subtending an angle of 90° at the centre?
(a) $\frac{25}{4}\left(\frac{\pi}{2}+1\right)\text{ cm}^2$
(b) $\frac{25}{4}\left(\frac{\pi}{2}-1\right)\text{ cm}^2$
(c) $\frac{25}{4}\left(\frac{3\pi}{2}+1\right)\text{ cm}^2$
(d) None of these
368. A rectangle of maximum area is drawn inside a circle of diameter 5 cm. What is the maximum area of such a rectangle?
(a) 25 cm^2 (b) 12.5 cm^2
(c) 12 cm^2 (d) None of these
369. If AB and CD are two diameters of a circle of radius r and they are mutually perpendicular, then

- what is the ratio of the area of the circle to the area of the $\triangle ACD$?
- (a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 2π
370. What is the area of a circle whose area is equal to that of a triangle with sides 7 cm, 24 cm and 25 cm?
- (a) 80 cm^2 (b) 84 cm^2
(c) 88 cm^2 (d) 90 cm^2
371. If the area of an equilateral triangle is x and its perimeter is y , then which one of the following is correct?
- (a) $y^4 = 432x^2$ (b) $y^4 = 216x^2$
(c) $y^2 = 432x^2$ (d) None of these
372. A rectangular field is 22 m long and 10 m wide. Two hemispherical pitholes of radius 2 m are dug from two places and the mud is spread over the remaining part of the field. The rise in the level of the field is
- (a) $\frac{8}{93} \text{ m}$ (b) $\frac{13}{93} \text{ m}$
(c) $\frac{16}{93} \text{ m}$ (d) $\frac{23}{93} \text{ m}$
373. The area of an isosceles $\triangle ABC$ with $AB = AC$ is 12 sq cm and altitude $AD = 3 \text{ cm}$. What is its perimeter?
- (a) 18 cm (b) 16 cm
(c) 14 cm (d) 12 cm
374. A hospital room is to accommodate 56 patients. It should be done in such a way that every patient gets 2.2 m^2 of floor and 8.8 m^3 of space. If the length of the room is 14m, then breadth and the height of the room are respectively
- (a) 8.8 m, 4m (b) 8.4 m, 4.2m
(c) 8 m, 4 m (d) 7.8 m, 4.2m
375. How many 200 mm lengths can be cut from 10 m of ribbon?
- (a) 50 (b) 40 (c) 30 (d) 20
376. What is the area between a square of side 10 cm and two inverted semi-circular, cross-sections each of radius 5 cm inscribed in the square?
- (a) 17.5 cm^2 (b) 18.5 cm^2
(c) 20.5 cm^2 (d) 21.5 cm^2
377. The perimeter of a rectangle having area equal to 144 cm^2 and sides in the ratio 4:9 is
- (a) 52 cm (b) 56 cm
(c) 60 cm (d) 64 cm
378. One side of a parallelogram is 8.06 cm and its perpendicular distance from opposite side is 2.08 cm. What is the approximate area of the parallelogram?
- (a) 12.56 cm^2 (b) 14.56 cm^2
(c) 16.76 cm^2 (d) 22.56 cm^2
379. In the figure given below, the area of rectangle ABCD is 100 sq cm, O is any point on AB and $CD = 20 \text{ cm}$. Then, the area of $\triangle COD$ is
- 
- (a) 40 sq cm (b) 45 sq cm
(c) 50 sq cm (d) 80 sq cm
380. If an isosceles right angled triangle has area 1sq unit, then what is its perimeter?
- (a) 3 units
(b) $2\sqrt{2} + 1$ units
(c) $(\sqrt{2} + 1)$ units
(d) $2(\sqrt{2} + 1)$ units
381. A circular water fountain 6.6 m in diameter is surrounded by a path of width 1.5 m. The area of this path (in sq m) is
- (a) 13.62π (b) 13.15π
(c) 12.15π (d) None of these
382. The area of a rectangular field is 4500 sq m. If its length and breadth are in the ratio 9:5, then its perimeter is
- (a) 90 m (b) 150 m
(c) 280 m (d) 360 m
383. The area of a square inscribed in a circle of radius 8 cm is
- (a) 32 sq cm (b) 64 sq cm
(c) 128 sq cm (d) 256 sq cm
384. The short and long hands of a clock are 4 cm and 6 cm long, respectively. Then, the ratio of distances travelled by tips of short hand in 2 days and long hand in 3 days is
- (a) 4 : 9 (b) 2 : 9
(c) 2 : 3 (d) 1 : 27
385. The arc AB of the circle with centre at O and radius 10 cm has length 16 cm. What is the area of the sector bounded by the radii OA, OB and the arc AB?
- (a) 40π sq cm (b) 40 sq cm
(c) 80 sq cm (d) 20π sq cm
386. The length of a room floor exceeds its breadth by 20m. The area of the floor remains unaltered when the length is decreased by 10m but the breadth is increased by 5m. The area of the floor (in square metres) is:
- (a) 280 (b) 325
(c) 300 (d) 420
387. Find the perimeter of a square which is symmetrically inscribed in semicircle of radius 10 cm.
- (a) $\sqrt{80} \text{ cm}$ (b) 80 cm
(c) $8\sqrt{24} \text{ cm}$ (d) $16\sqrt{5} \text{ cm}$
388. Consider the following statement
- I. Area of a segment of a circle is less than area of its corresponding sector.
II. Distance travelled by a circular wheel of diameter 2d cm in one revolution is greater than 6d cm.
- Which of the above statements is/are correct?
- (a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II
389. The Perimeter of a rectangle is 82 m and its area is 400 sq m. What is the breadth of the rectangle?
- (a) 18 m (b) 16 m
(c) 14 m (d) 12 m
390. The area enclosed between the circumference of two concentric circles is 16π sq cm and their radii are in the ratio 5:3. What is the area of the outer circle?
- (a) 9π sq cm (b) 16π sq cm
(c) 25π sq cm (d) 36π sq cm
391. If the circumference of a circle is equal to the perimeter of square, then which one of the following is correct?
- (a) Area of circle = Area of square
(b) Area of circle $\geq$ Area of square
(c) Area of circle $>$ Area of square
(d) Area of circle $<$ Area of square

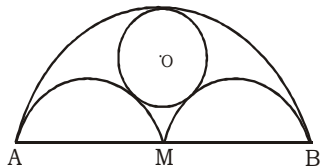
392. If the circumference of two circle are in the ratio 2:3, then what is the ratio of their areas?
 (a) 2:3 (b) 4:9
 (c) 1:3 (d) 8:27
393. If the area of a circle inscribed in an equilateral triangle is 154 sq cm, then what is the perimeter of the triangle?
 (a) 21 cm (b) $42\sqrt{3}$ cm
 (c) $21\sqrt{3}$ cm (d) 42 cm
394. In the $\triangle ABC$, the base BC is trisected at D and E. The line through D, Parallel to AB, meets AC at F and the line through E parallel to AC meets AB at G. If EG and DF intersect at H, then what is the ratio of the sum of the area of parallelogram AGHF and the area of the $\triangle DHE$ to the area of the $\triangle ABC$?
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$
395. If the area of a circle is equal to the area of a square with side $2\sqrt{\pi}$ units, then what is the diameter of the circle?
 (a) 1 unit (b) 2 units
 (c) 4 units (d) 8 units
396. A square, a circle and an equilateral triangle have same perimeter. Consider the following statements
 I. The area of square is greater than the area of the triangle.
 II. The area of circle is less than the area of triangle.
 Which of the above statement is/are correct?
 (a) Only I (b) Only II
 (c) Both I and II (d) Neither I nor II
397. If the area of a rectangle whose length is 5 more than twice its width is 75 sq units. What is the perimeter of the rectangle?
 (a) 40 units (b) 30 units
 (c) 24 units (d) 20 units
398. If the altitude of an equilateral triangle is $\sqrt{3}$ cm, then what is its perimeter?
 (a) 3 cm (b) $3\sqrt{3}$ cm
 (c) 6 cm (d) $6\sqrt{3}$ cm
399. The area of a rectangle, whose one side is a is $2a^2$. What is the area of a square having one of the diagonal of the rectangle as side?
 (a) $2a^2$ (b) $3a^2$ (c) $4a^2$ (d) $5a^2$
400. If the outer and inner diameters of a stone parapet around a well are 112 cm and 70 cm respectively. Then, what is the area of the parapet?
 (a) 264 sq cm (b) 3003 sq cm
 (c) 6006 sq cm (d) 24024 sq cm
401. If the area of a $\triangle ABC$ is equal to area of square of side length 6 cm, then what is the length of the altitude to AB, where $AB = 9$ cm?
 (a) 18 cm (b) 14 cm
 (c) 12 cm (d) 8 cm
402. What is the area of an equilateral triangle having altitude equal to $2\sqrt{3}$ cm?
 (a) $\sqrt{3}$ sq cm (b) $2\sqrt{3}$ sq cm
 (c) $3\sqrt{3}$ sq cm (d) $4\sqrt{3}$ sq cm
403. If a lawn 30 m long and 16 m wide is surrounded by a path 2 m wide, then what is the area of the path?
 (a) 200 m^2 (b) 280 m^2
 (c) 300 m^2 (d) 320 m^2
404. If a circle circumscribes a rectangle with side 16 cm and 12 cm, then what is the area of the circle?
 (a) 48π sq cm (b) 50π sq cm
 (c) 100π sq cm (d) 200π sq cm
405. The lengths of two sides of a right angled triangle which contain the right angle are a and b, respectively. Three squares are drawn on the three sides of the triangle on the outer side. What is the total area of the triangle and the three squares?
 (a) $2(a^2+b^2)+ab$
 (b) $2(a^2+b^2)+2.5ab$
 (c) $2(a^2+b^2)+0.5ab$
 (d) $2.5(a^2+b^2)$
406. A wall is of the form of a trapezium with height 4 m and parallel sides being 3 m and 5m. What is the cost of painting the wall, if the rate of painting is Rs.25 per sq m?
 (a) Rs. 240 (b) Rs. 400
 (c) Rs. 480 (d) Rs. 800
407. A grassy field has the shape of an equilateral triangle of side 6 m. If a horse with 4.2 m long rope tied at a vertex. The percentage of the total area of the field which is available for grazing is best approximated by
 (a) 50% (b) 55%
 (c) 59% (d) 62%
408. The areas of two circles are in the ratio 1:2. If the two circles are bent in the form of squares, then what is the ratio of their areas?
 (a) 1:2 (b) 1:3
 (c) $1:\sqrt{4}$ (d) 1:4
409. If the four equal circles of radius 3cm touch each other externally, then the area of the region bounded by the four circles is:
 (a) $4(9-\pi)$ sq.cm
 (b) $9(4-\pi)$ sq.cm
 (c) $5(6-\pi)$ sq.cm
 (d) $6(5-\pi)$ sq.cm
410. If the diameter of a circle circumscribing a square is $15\sqrt{2}$ cm, then what is the length of the side of the square?
 (a) 15 cm (b) 12 cm
 (c) 10 cm (d) 7.5 cm
411. Three congruent circles each of radius 4 cm touch one another. What is the area (in cm^2) of the portion included between them?
 (a) 8π (b) $16\sqrt{3}-8\pi$
 (c) $16\sqrt{3}-4\pi$ (d) $16\sqrt{3}-2\pi$
412. The two diagonals of a rhombus are of lengths 55 cm and 48 cm. If P is the perpendicular height of the rhombus, then which one of the following is correct?
 (a) $36\text{ cm} < p < 37\text{ cm}$
 (b) $35\text{ cm} < p < 36\text{ cm}$
 (c) $34\text{ cm} < p < 35\text{ cm}$
 (d) $33\text{ cm} < p < 34\text{ cm}$
413. The Perimeter of a triangular field is 240 m. If two of its sides are 78 m and 50 m, then what is the length of the perpendicular on the side of length 50 m from the opposite vertex?
 (a) 43 m (b) 52.2 m
 (c) 67.2 m (d) 70 m

414. A piece of wire 78 cm long is bent in the form of an isosceles triangle. If the ratio of one of the equal sides to the base is 5:3, then what is the length of the base?

(a) 16 cm (b) 18 cm
(c) 20 cm (d) 30 cm

415. The length of a minute hand of a wall clock is 9 cm. What is the area swept (in cm^2) by the minute hand in 20 min? (take $\pi = 3.14$)
- (a) 88.78 (b) 84.78
(c) 67.74 (d) 57.78

416. In the figure given below, AB is a line of length $2a$, with M as midpoint. Semi-circles are drawn on one side with AM, and AB as diameters. A circle with centre O and radius r is drawn such that this circle touches all the three semi-circles. What is the value of r ?



(a) $\frac{2a}{3}$ (b) $\frac{a}{2}$ (c) $\frac{a}{3}$ (d) $\frac{a}{4}$

417. A circle and a square have the same perimeter. Which one of the following is correct?

(a) The area of the circle is equal to that of square
(b) The area of the circle is larger than that of square
(c) The area of the circle is less than that of square
(d) No conclusion can be drawn

418. What is the radius of the circle inscribed in a triangle having side lengths 35 cm, 44 cm and 75 cm?

(a) 3 cm (b) 4 cm
(c) 5 cm (d) 6 cm

419. A rectangle area of 6 sq m is to be painted on a $3\text{m} \times 4\text{m}$ board leaving a border of uniform width on all sides. What should be the width of the border?

(a) 0.25 m (b) 0.5 m
(c) 1 m (d) 3 m

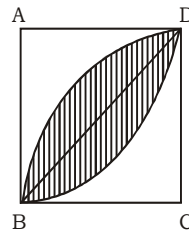
420. A wheel of a bicycle has inner diameter 50 cm and thickness 10 cm. What is the speed of the bicycle, If it makes 10 revolutions in 5 s?

(a) 5.5 m/s (b) 4.4 m/s
(c) 3.3 m/s (d) 2.2 m/s

421. If a wire of length 36 cm is bent in the form of a semi-circle, then what is the radius of the semi-circle?

(a) 9 cm (b) 8 cm
(c) 7 cm (d) 6 cm

422. In the given figure, the side of square ABCD is 7 cm. What is the area of the shaded portion, formed by the arcs BD or the circles with centre at C and A?



(a) 7cm^2 (b) 28cm^2
(c) 14cm^2 (d) 21cm^2

423. What is the maximum area of a rectangle, the perimeter of which is 18 cm?

(a) 20.25cm^2 (b) 20.00cm^2
(c) 19.75cm^2 (d) 19.60cm^2

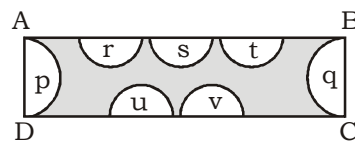
424. Three circular laminas of the same radius are cut out from a larger circular lamina. When the radius of each lamina cut out is the largest possible, then what is the ratio (approximate) of the area of the residual piece of the original lamina to its original total area?

(a) 0.30 (b) 0.35
(c) 0.40 (d) 0.45

425. A wire is in the form of a radius 42 cm. If it is bent into a square, then what is the side of the square?

(a) 66 cm (b) 42 cm
(c) 36 cm (d) 33 cm

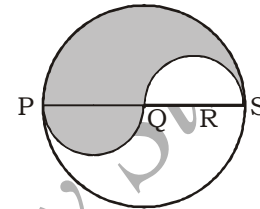
426. Seven semi-circular areas are removed from the rectangle ABCD as shown in the figure below, in which $AB = 2\text{ cm}$ and $AD = 0.5\text{ cm}$. The radius of each semi-circle, r, s, t, u and v is half of that of semi-circle p or q . What is the area of the remaining portion?



(a) $(128 - 13\pi)/128\text{cm}^2$
(b) $(125 - 13\pi)/125\text{cm}^2$

(c) $(128 - 15\pi)/128\text{cm}^2$
(d) None of these

427. PQRS is a diameter of a circle of radius 6 cm as shown in the figure above. The lengths PQ, QR and RS are equal. Semi-circles are drawn on PQ and QS as diameters. What is the perimeters of the shaded region?



(a) $12\pi\text{ cm}$ (b) $14\pi\text{ cm}$
(c) $16\pi\text{ cm}$ (d) $18\pi\text{ cm}$

428. A person rides a bicycle round a circular path of radius 50 m. The radius of the wheel of the bicycle is 50 cm. The cycle comes to the starting point for the first time in 1 h. What is the number of revolutions of the wheel in 15 min?

(a) 20 (b) 25 (c) 30 (d) 35

429. If a man walking at the rate 3 km/h crosses a square field diagonally in 1 min, then what is the area of the field?

(a) 1000 m^2 (b) 1250 m^2
(c) 2500 m^2 (d) 5000 m^2

430. The difference between the area of a square and that of an equilateral triangle on the same base is $1/4\text{ cm}^2$. What is the length of side of triangle?

(a) $(4 - \sqrt{3})^{1/2}\text{ cm}$
(b) $(4 + \sqrt{3})^{1/2}\text{ cm}$
(c) $(4 - \sqrt{3})^{-1/2}\text{ cm}$
(d) $(4 + \sqrt{3})^{-1/2}\text{ cm}$

431. A horse is tied to a pole fixed at one corner of a $50\text{ m} \times 50\text{ m}$ square field of grass by means of a 20 m long rope. What is the area of that part of the field which the horse can graze?

(a) 1256 m^2 (b) 942 m^2
(c) 628 m^2 (d) 314 m^2

432. From a rectangular metal sheet of sides 25 cm and 20 cm; a circular sheet as large as possible is cut-off. What is the area of the remaining sheet?

(a) 186 cm^2 (b) 144 cm^2
(c) 93 cm^2 (d) 72 cm^2

433. What is the area of a right angled isosceles triangle whose hypotenuse is $6\sqrt{2}$ cm?

- (a) 12 cm^2 (b) 18 cm^2
(c) 24 cm^2 (d) 36 cm^2

434. If A is the area of a triangle in cm^2 , whose sides are 9 cm, 10 cm and 11 cm, then which one of the following is correct?

- (a) $A < 40 \text{ cm}^2$
(b) $40 \text{ cm}^2 < A < 45 \text{ cm}^2$
(c) $45 \text{ cm}^2 < A < 50 \text{ cm}^2$
(d) $A > 50 \text{ cm}^2$

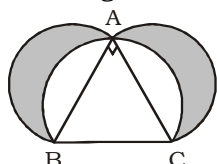
435. If x and y are respectively the areas of a square and a rhombus of sides of same length, then what is x : y?

- (a) 1 : 1 (b) $2 : \sqrt{3}$
(c) $4 : \sqrt{3}$ (d) 3 : 2

436. If the area of a circle, inscribed in an equilateral triangle is $4\pi \text{ cm}^2$, then what is the area of the triangle?

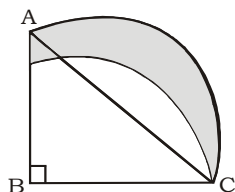
- (a) $12\sqrt{3} \text{ cm}^2$ (b) $9\sqrt{3} \text{ cm}^2$
(c) $8\sqrt{3} \text{ cm}^2$ (d) 18 cm^2

437. In the given figure, $\triangle ABC$ is a right angled triangle, right angled at A. Semi-circles are drawn on the sides AB, BC and AC. Then the area of shaded portion is equal to which one of the following?



- (a) Area of $\triangle ABC$
(b) 2 times the area of $\triangle ABC$
(c) Area of semi-circle ABC
(d) None of the above

438. In the given figure, ABC is a right angled triangle, right angled at B. BC = 21 cm and AB = 28 cm. Width AC as diameter of a semi-circle and width BC as radius a quarter circle are drawn. What is the area of the shaded portion?



- (a) 425 cm^2 (b) 425.75 cm^2
(c) 428 cm^2 (d) 428.75 cm^2

439. The Perimeter of a square S_1 is 12 m more than perimeter of the square S_2 . If the area of S_1 equals three times, the area of S_2 minus 11, then what is the perimeter of S_1 ?

- (a) 24 m (b) 32 m
(c) 36 m (d) 40 m

440. From a rectangular sheet of cardboard of size $5 \text{ cm} \times 2 \text{ cm}$, the greatest possible circle is cut-off. What is the area of the remainting part?

- (a) $(25 - \pi) \text{ cm}^2$
(b) $(10 - \pi) \text{ cm}^2$
(c) $(4 - \pi) \text{ cm}^2$
(d) $(10 - 2\pi) \text{ cm}^2$

441. A chord AB of a cricle of radius 20 cm makes a right angle at the centre of the circle. What is the area of the minor segment in cm^2 ? (take $\pi = 3.14$)

- (a) 31.4 cm^2 (b) 57 cm^2
(c) 62.8 cm^2 (d) 114 cm^2

442. The minute hand of a clock is 14 cm long. How much distance does the end of the minute hand travel in 15 min? (take $\pi = \frac{27}{7}$)

- (a) 11 cm (b) 22 cm
(c) 33 cm (d) 44 cm

443. A square of side x is taken. A rectangle is cut out from this square such that on side of the rectangle is half that of the square and the other is $\frac{1}{3}$ rd of the first side of the rectangle. What is the area of the remaining portion?

- (a) $\left(\frac{3}{4}\right)x^2$ (b) $\left(\frac{7}{8}\right)x^2$
(c) $\left(\frac{11}{12}\right)x^2$ (d) $\left(\frac{15}{16}\right)x^2$

444. A rectangle cardboard is $18 \text{ cm} \times 10 \text{ cm}$. From the four corners of the rectangle, quarter circles of radius 4 cm are cut. What is the perimeter (approximate) of the remaining portion?

- (a) 47.1 cm (b) 49.1 cm
(c) 51.0 cm (d) 53.0 cm

445. A cycle wheel makes 1000 revolution is moving 440 m. What is the diameter of the wheel?

- (a) 7 cm (b) 14 cm
(c) 28 cm (d) 21 cm

446. A circle is inscribed in a equilateral triangle of side a. What is the area of any square inscribed in this circle?

- (a) $\frac{a^2}{3}$ (b) $\frac{a^2}{4}$ (c) $\frac{a^2}{6}$ (d) $\frac{a^2}{8}$

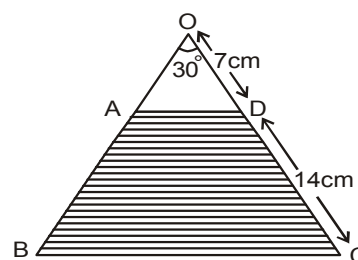
447. Consider a circle C of radius 6 cm with centre at O. What is the difference in the area of the circle C and the area of the sector of C subtending an angle of 80° at O?

- (a) $26\pi \text{ cm}^2$ (b) $16\pi \text{ cm}^2$
(c) $28\pi \text{ cm}^2$ (d) $30\pi \text{ cm}^2$

448. The ratio of the areas of the in-circle and the circum-circle of a square is:

- (a) 1 : 2 (b) $\sqrt{2} : 1$
(c) $1 : \sqrt{2}$ (d) 2 : 1

449. The diagram represents the area swept by the wiper of a car. With the dimensions given in the figure, calculate the shaded area swept by the wiper.



- (a) 102.67 cm^2 (b) 205.34 cm^2
(c) 51.33 cm^2 (d) 208.16 cm^2

450. If the length of a chord of a circle at a distance of 12 cm from the centre is 10 cm, then the diameter of the circle is :

- (a) 13 cm (b) 15 cm
(c) 26 cm (d) 30 cm

451. Area of the incircle of an equilateral triangle with side 6 cm is :

- (a) $\frac{\pi}{2} \text{ sq.cm}$ (b) $\sqrt{3}\pi \text{ sq.cm}$
(c) $6\pi \text{ sq.cm}$ (d) $3\pi \text{ sq.cm}$

452. The adjacent sides of a parallelogram are 36 cm and 27 cm in length. If the distance between the shorter sides is 12 cm, then the distance between the longer sides is :
 (a) 10 cm (b) 12 cm
 (c) 16 cm (d) 9 cm
453. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. The area of the circle is :
 (a) 125 cm^2 (b) 230 cm^2
 (c) 550 cm^2 (d) 616 cm^2
454. The perimeter of a semicircular path is 36 m. Find the area of this semicircular path.
 (a) 42 sq.m (b) 54 sq.m
 (c) 63 sq.m (d) 77 sq.m
455. Find the area of a rectangle whose area is equal to the area of a circle with perimeter equal to 24π :
 (a) 144 (b) 144π
 (c) 154π (d) none of these
456. A circle is inscribed in an equilateral triangle of side 8 cm. The area of the portion between the triangle and the circle is :
 (a) 11 cm^2 (b) 10.95 cm^2
 (c) 10 cm^2 (d) 10.50 cm^2
457. Find the ratio of the diameter of the circles inscribed in and circumscribed an equilateral triangle to its height.
 (a) 1 : 2 : 3 (b) 2 : 4 : 3
 (c) 1 : 3 : 4 (d) 3 : 2 : 1
458. Find the area of the largest (or maximum sized) square that can be made inside a right angle triangle having sides 6 cm, 8 cm & 10 cm when one of vertices of the square coincide with the vertex of right angle of the triangle?
 (a) $\frac{576}{49} \text{ cm}^2$ (b) 24 cm^2
 (c) $\frac{24}{7} \text{ cm}^2$ (d) None of these
459. Area of the trapezium formed by x -axis; y -axis and the lines $3x + 4y = 12$ and $6x + 8y = 60$ is:
 (a) 37.5 sq.unit (b) 31.5 sq.unit
 (c) 48 sq.unit (d) 36.5 sq.unit
460. A square having area 200 sq.m , is formed in such a way that the length of its diagonal is $\sqrt{2}$ times of the diagonal of the given square. Then the area of the new square formed is:
 (a) $200\sqrt{2} \text{ sq.m}$ (b) $400\sqrt{2} \text{ sq.m}$
 (c) 400 sq.m (d) 800 sq.m

ANSWER KEY

1. (b)	47. (b)	93. (b)	139. (c)	185. (c)	231. (c)	277. (a)	323. (c)	369. (b)	415. (b)
2. (c)	48. (d)	94. (b)	140. (b)	186. (a)	232. (a)	278. (b)	324. (b)	370. (b)	416. (c)
3. (a)	49. (b)	95. (c)	141. (c)	187. (c)	233. (b)	279. (a)	325. (c)	371. (a)	417. (b)
4. (c)	50. (b)	96. (c)	142. (a)	188. (a)	234. (a)	280. (d)	326. (b)	372. (c)	418. (d)
5. (b)	51. (b)	97. (c)	143. (b)	189. (d)	235. (d)	281. (b)	327. (c)	373. (a)	419. (b)
6. (b)	52. (a)	98. (b)	144. (a)	190. (c)	236. (c)	282. (b)	328. (d)	374. (a)	420. (b)
7. (b)	53. (c)	99. (b)	145. (a)	191. (b)	237. (b)	283. (c)	329. (c)	375. (a)	421. (c)
8. (c)	54. (c)	100. (c)	146. (c)	192. (d)	238. (d)	284. (d)	330. (c)	376. (d)	422. (c)
9. (d)	55. (d)	101. (a)	147. (c)	193. (a)	239. (b)	285. (d)	331. (c)	377. (a)	423. (a)
10. (d)	56. (b)	102. (a)	148. (a)	194. (a)	240. (a)	286. (c)	332. (c)	378. (c)	424. (b)
11. (a)	57. (c)	103. (c)	149. (b)	195. (d)	241. (a)	287. (c)	333. (d)	379. (c)	425. (a)
12. (d)	58. (d)	104. (a)	150. (a)	196. (c)	242. (a)	288. (c)	334. (d)	380. (d)	426. (a)
13. (a)	59. (d)	105. (c)	151. (a)	197. (a)	243. (a)	289. (a)	335. (c)	381. (c)	427. (a)
14. (b)	60. (b)	106. (c)	152. (d)	198. (a)	244. (c)	290. (c)	336. (b)	382. (c)	428. (b)
15. (d)	61. (b)	107. (d)	153. (b)	199. (a)	245. (d)	291. (d)	337. (c)	383. (c)	429. (b)
16. (a)	62. (c)	108. (b)	154. (d)	200. (d)	246. (c)	292. (a)	338. (d)	384. (d)	430. (c)
17. (b)	63. (b)	109. (c)	155. (b)	201. (b)	247. (b)	293. (c)	339. (d)	385. (c)	431. (d)
18. (d)	64. (b)	110. (a)	156. (a)	202. (d)	248. (b)	294. (b)	340. (a)	386. (c)	432. (a)
19. (d)	65. (b)	111. (c)	157. (c)	203. (b)	249. (b)	295. (b)	341. (b)	387. (d)	433. (b)
20. (b)	66. (c)	112. (a)	158. (a)	204. (b)	250. (a)	296. (d)	342. (b)	388. (c)	434. (b)
21. (a)	67. (b)	113. (c)	159. (a)	205. (a)	251. (d)	297. (d)	343. (c)	389. (b)	435. (a)
22. (c)	68. (a)	114. (d)	160. (b)	206. (c)	252. (a)	298. (c)	344. (d)	390. (c)	436. (a)
23. (c)	69. (b)	115. (b)	161. (d)	207. (a)	253. (b)	299. (d)	345. (b)	391. (c)	437. (a)
24. (d)	70. (c)	116. (d)	162. (a)	208. (a)	254. (d)	300. (c)	346. (b)	392. (b)	438. (d)
25. (d)	71. (a)	117. (c)	163. (c)	209. (b)	255. (c)	301. (b)	347. (b)	393. (b)	439. (b)
26. (a)	72. (c)	118. (a)	164. (c)	210. (b)	256. (d)	302. (b)	348. (a)	394. (b)	440. (b)
27. (d)	73. (c)	119. (c)	165. (c)	211. (d)	257. (c)	303. (a)	349. (b)	395. (c)	441. (d)
28. (b)	74. (b)	120. (b)	166. (b)	212. (c)	258. (d)	304. (b)	350. (b)	396. (a)	442. (b)
29. (c)	75. (b)	121. (b)	167. (c)	213. (b)	259. (a)	305. (c)	351. (b)	397. (a)	443. (c)
30. (d)	76. (a)	122. (c)	168. (c)	214. (b)	260. (c)	306. (a)	352. (d)	398. (c)	444. (b)
31. (a)	77. (b)	123. (d)	169. (d)	215. (d)	261. (b)	307. (b)	353. (c)	399. (d)	445. (b)
32. (d)	78. (c)	124. (c)	170. (d)	216. (d)	262. (a)	308. (d)	354. (c)	400. (c)	446. (c)
33. (a)	79. (b)	125. (b)	171. (a)	217. (a)	263. (c)	309. (b)	355. (d)	401. (d)	447. (c)
34. (c)	80. (c)	126. (b)	172. (d)	218. (d)	264. (b)	310. (a)	356. (d)	402. (d)	448. (a)
35. (a)	81. (b)	127. (c)	173. (c)	219. (b)	265. (d)	311. (c)	357. (a)	403. (a)	449. (a)
36. (a)	82. (b)	128. (a)	174. (c)	220. (b)	266. (c)	312. (a)	358. (a)	404. (c)	450. (c)
37. (c)	83. (a)	129. (b)	175. (c)	221. (d)	267. (c)	313. (c)	359. (c)	405. (c)	451. (d)
38. (b)	84. (b)	130. (b)	176. (b)	222. (d)	268. (b)	314. (b)	360. (d)	406. (b)	452. (d)
39. (d)	85. (a)	131. (a)	177. (c)	223. (b)	269. (a)	315. (d)	361. (c)	407. (c)	453. (d)
40. (a)	86. (a)	132. (a)	178. (b)	224. (c)	270. (a)	316. (a)	362. (d)	408. (a)	454. (d)
41. (a)	87. (c)	133. (b)	179. (a)	225. (a)	271. (a)	317. (b)	363. (d)	409. (b)	455. (b)
42. (a)	88. (b)	134. (b)	180. (d)	226. (c)	272. (a)	318. (a)	364. (b)	410. (a)	456. (b)
43. (d)	89. (c)	135. (b)	181. (a)	227. (b)	273. (b)	319. (c)	365. (c)	411. (b)	457. (b)
44. (d)	90. (c)	136. (c)	182. (b)	228. (a)	274. (d)	320. (a)	366. (c)	412. (a)	458. (a)
45. (b)	91. (a)	137. (b)	183. (a)	229. (b)	275. (a)	321. (c)	367. (c)	413. (c)	459. (b)
46. (a)	92. (a)	138. (c)	184. (a)	230. (b)	276. (b)	322. (d)	368. (c)	414. (b)	460. (c)

SOLUTION

1. (b) Side of a square = $\frac{\text{Diagonal}}{\sqrt{2}}$

Area of square = $\left(\frac{\text{Diagonal}}{\sqrt{2}}\right)^2$

$$= \frac{(5.2)^2}{2} = \frac{5.2 \times 5.2}{2}$$

$$= 2.6 \times 5.2 = 13.52 \text{ cm}^2$$

2. (c) Area of square

$$= \frac{\text{Diagonal}^2}{2} = \frac{a^2}{2}$$

3. (a) Let the length of rectangular hall = x

∴ Breadth of rectangular hall

$$= \frac{3}{4}x$$

According to question,

$$\text{Area} = 768 \text{ m}^2$$

$$x \times \frac{3}{4}x = 768$$

$$\frac{3}{4}x^2 = 768$$

$$x^2 = \frac{768 \times 4}{3} = 256 \times 4$$

$$x^2 = \sqrt{256 \times 4} = 32 \text{ m.}$$

Difference of length and

$$\text{breadth} = x - \frac{3}{4}x = \frac{x}{4} = \frac{32}{4} = 8 \text{ m}$$

4. (c) Since the room is in cuboid shape

Length of largest rod = Diagonal of cuboid

$$= \sqrt{l^2 + b^2 + h^2} = \sqrt{16^2 + 12^2 + \frac{32^2}{3^2}}$$

$$= \sqrt{256 + 144 + \frac{1024}{9}}$$

$$= \sqrt{\frac{2304 + 1296 + 1024}{9}}$$

$$= \sqrt{\frac{4624}{9}} = \frac{68}{3} = 22\frac{2}{3} \text{ m}$$

5. (b) Perimeter of square = 44 cm

$$4 \times \text{side} = 44$$

$$\text{side} = 11 \text{ cm}$$

$$\text{area of square} = (\text{side})^2 = (11)^2 = 121 \text{ cm}^2$$

$$\text{Circumference of circle} = 44 \text{ cm}$$

$$2\pi(\text{radius}) = 44$$

$$\text{radius} = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm}$$

$$\begin{aligned} \text{area of circle} &= \frac{22}{7} \times 7 \times 7 \\ &= 154 \text{ cm}^2 \end{aligned}$$

Option (b) is the answer. (circle, 33 cm²)

6. (b) Let the side of square = a and the radius of circle = r

$$\text{perimeter of square}$$

$$= \text{circumference of circle}$$

$$4a = 2\pi r$$

$$r = \frac{4a}{2\pi}$$

$$\text{area of circle} = 3850 \text{ m}^2$$

$$\pi \times \frac{4a}{2\pi} \times \frac{4a}{2\pi} = 3850$$

$$16a^2 = \frac{3850 \times 2 \times 2 \times 22}{7}$$

$$a^2 = 3025 \text{ m}^2$$

7. (b) $2(l + b) = 28$

$$l + b = 14$$

$$\text{and } l \times b = 48$$

$$(l + b)^2 = l^2 + b^2 + 2lb$$

$$(14)^2 = l^2 + b^2 + 48 \times 2$$

$$196 - 96 = l^2 + b^2$$

$$l^2 + b^2 = 100$$

$$\sqrt{l^2 + b^2} = 10$$

$$\text{Diagonal} = 10 \text{ m}$$

8. (c) Let the length of rectangular hall = x

$$\therefore \text{Breadth of rectangular hall}$$

$$= \frac{3}{4}x$$

According to question,

$$\text{Area} = 192 \text{ m}^2$$

$$x \times \frac{3}{4}x = 192$$

$$\frac{3}{4}x^2 = 192$$

$$x^2 = \frac{192 \times 4}{3} = 64 \times 4$$

$$x = \sqrt{64 \times 4} = 16 \text{ cm}$$

$$\text{difference of length and}$$

$$\text{breadth} = x - \frac{3}{4}x = \frac{x}{4}$$

$$= \frac{16}{4} = 4 \text{ cm}$$

9. (d) Side of the square

$$= \frac{\text{Diagonal}}{\sqrt{2}} = \frac{4\sqrt{2}}{\sqrt{2}} = 4$$

$$\text{area of the square} = 16$$

$$\text{area of new square} = 32$$

$$\text{side of new square} = \sqrt{32}$$

$$= 4\sqrt{2}$$

$$\text{Diagonal of new square}$$

$$= 4\sqrt{2} \times \sqrt{2} = 8 \text{ cm}$$

10. (d) Diagonal of square A = (a + b)
side of square

$$= \frac{\text{Diagonal}}{\sqrt{2}} = \frac{a + b}{\sqrt{2}}$$

$$\begin{aligned} \text{area of square A} &= \left(\frac{a + b}{\sqrt{2}}\right)^2 \\ &= \frac{(a + b)^2}{2} \end{aligned}$$

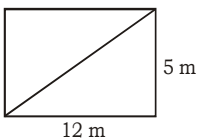
$$\text{area of square}$$

$$B = 2 \times \text{area of square A}$$

$$= 2 \times \frac{(a + b)^2}{2} = (a + b)^2$$

$$\begin{aligned} \text{side of square B} &= \sqrt{(a + b)^2} \\ &= (a + b) \end{aligned}$$

$$\text{diagonal of square B} = \sqrt{2}(a + b)$$

11. (a) 

$$\text{area of the rectangular garden}$$

$$= 12 \times 5 = 60 \text{ m}^2$$

$$\therefore \text{area of square} = 60$$

$$(\text{side})^2 = 60$$

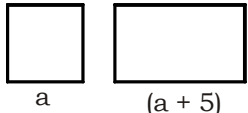
$$\text{side} = \sqrt{60}$$

$$\text{diagonal of the square}$$

$$= \sqrt{2} \text{ side}$$

$$= \sqrt{2} \times \sqrt{60} = \sqrt{120}$$

$$= 2\sqrt{30} \text{ m}$$

12. (d)  (a - 3)

According to question,

$$\begin{aligned}a^2 &= (a-3)(a+5) \\a^2 &= a^2 + 5a - 3a - 15 \\2a &= 15 \\a &= \frac{15}{2}\end{aligned}$$

$$\text{Length} = a + 5 = \frac{15}{2} + 5 = \frac{25}{2}$$

$$\begin{aligned}\text{breadth} &= a - 3 = \frac{15}{2} - 3 \\&= \frac{15-6}{2} = \frac{9}{2}\end{aligned}$$

$$\begin{aligned}\text{perimeter of the rectangle} &= 2(l+b) \\&= 2\left(\frac{25}{2} + \frac{9}{2}\right) = 34 \text{ cm}\end{aligned}$$

13. (a) According to question,
 $2(l+b) = 160$
 $l+b = 80$ (i)
 $l-b = 48$ (ii)
 on solving (i) and (ii)
 $l = 64, \quad b = 16$
 area of square = area of rectangle
 $(\text{side})^2 = 64 \times 16$

$$\text{side} = \sqrt{64 \times 16} = 32 \text{ m}$$

14. (b) Side of square, whose perimeter is 24 cm
 $= \frac{24}{4} = 6 \text{ cm}$

$$\begin{aligned}\text{So, area of square} &= 6^2 \\&= 36 \text{ cm}^2 \\ \text{Again, side of square, whose} & \\ \text{perimeter is 32 cm} &= \frac{32}{4} = 8 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{So, area of this square} &= 8^2 = 64 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{According to question,} & \\ \text{Area of new square} &= 64 + 36 = 100 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\therefore \text{side of the new square} &= \sqrt{100} = 10 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Hence perimeter of new square} &= 10 \times 4 = 40 \text{ cm}\end{aligned}$$

15. (d) $(\text{side})^2 = 484 \text{ cm}^2$
 $\text{side} = 22 \text{ cm}$
 $\text{perimeter of square} = 4 \times 22 = 88 \text{ cm}$

$$\begin{aligned}\text{According to question,} & \\ 2\pi r &= 88 \text{ cm}\end{aligned}$$

$$r = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

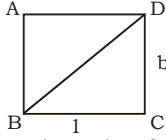
$$\begin{aligned}\text{area of circle} &= \pi r^2 = \frac{22}{7} \times 14 \times 14 \\&= 616 \text{ cm}^2\end{aligned}$$

16. (a) $l = 10 \text{ m}, b = 6 \text{ m}, h = 4 \text{ m}$
 length of diagonal (longest rod)
 $= \sqrt{100+36+16} = \sqrt{152} \text{ m}$
 $= 2\sqrt{38}$

17. (b) Let the length of smaller line segment = $x \text{ cm}$
 The length of larger line segment = $(x+2) \text{ cm}$
 According to question,
 $(x+2)^2 - x^2 = 32$
 $x^2 + 4x + 4 - x^2 = 32$

$$x = \frac{28}{4} = 7$$

$$\begin{aligned}\text{The required length} &= x + 2 \\&= 7 + 2 = 9 \text{ cm}\end{aligned}$$

18. (d) 
 $BD = \text{length of diagonal}$
 $= \text{speed} \times \text{time}$
 $= \frac{52}{60} \times 15 = 13 \text{ m}$

$$\begin{aligned}BD &= \sqrt{l^2 + b^2} \\ \Rightarrow l^2 + b^2 &= 13^2 = 169\end{aligned}$$

$$\begin{aligned}\text{Again, } l+b &= \frac{68}{60} \times 15 = 17 \\ (l+b)^2 &= l^2 + b^2 + 2lb \\ 17^2 &= 169 + 2lb\end{aligned}$$

$$lb = \frac{120}{2} = 60 \text{ m}^2$$

19. (d) Let the breadth be = $x \text{ m}$
 $\therefore \text{length} = (23+x) \text{ m}$
 $\Rightarrow 2(x+23+x) = 206$
 $4x = 206 - 46$

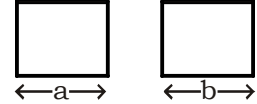
$$x = \frac{160}{4} = 40 \text{ m}$$

$$\begin{aligned}\therefore \text{length} &= 40 + 23 = 63 \text{ m} \\ \therefore \text{Required area} &= 63 \times 40 = 2520 \text{ m}^2\end{aligned}$$

20. (b) Length of rectangle = 48 m
 Breadth of rectangle = 16 m
 According to question,
 Perimeter of square = Perimeter of rectangle
 $= 2(48+16)$
 $4 \times \text{side} = 2 \times 64$

$$\text{side} = \frac{2 \times 64}{4} = 32 \text{ m}$$

$$\begin{aligned}\therefore \text{Area of the square} &= (\text{side})^2 = (32)^2 = 1024\end{aligned}$$

21. (a) 

$$\begin{aligned}4a &= 40 \\ a &= 10 \text{ cm} \quad \left| \quad \begin{aligned}4b &= 32 \\ b &= 8 \text{ cm}\end{aligned}\right.\end{aligned}$$

$$\begin{aligned}\text{area of third square} &= a^2 - b^2 = 10^2 - 8^2 \\&= 100 - 64 = 36 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{side of third square} &= \sqrt{36} = 6 \text{ cm} \\ \text{perimeter of third square} &= 4 \times 6 = 24 \text{ cm}\end{aligned}$$

22. (c) side of the square
 $= \frac{\text{perimeter}}{4}$

$\therefore$ Sides of all five squares are

$$\begin{aligned}&= \frac{24}{4}, \frac{32}{4}, \frac{40}{4}, \frac{76}{4}, \frac{80}{4} \\&= 6, 8, 10, 19, 20\end{aligned}$$

ATQ

$$\begin{aligned}\text{area of another square} &= 6^2 + 8^2 + 10^2 + 19^2 + 20^2\end{aligned}$$

$$\begin{aligned}(\text{side})^2 &= 36 + 64 + 100 + 361 + 400 \\ \text{side} &= \sqrt{961} = 31\end{aligned}$$

$$\begin{aligned}\text{perimeter of square} &= 31 \times 4 = 124\end{aligned}$$

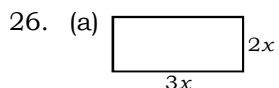
23. (c) Area of the tank
 $= \text{length} \times \text{breadth}$
 $= 180 \times 120 = 21600 \text{ m}^2$
 Total area of the circular plot
 $= 40000 + 21600 = 61600 \text{ m}^2$
 $\therefore \text{area of circle} = 61600$
 $\pi(\text{radius})^2 = 61600$

$$(\text{radius})^2 = \frac{61600 \times 7}{22}$$

$$\begin{aligned}\text{radius} &= \sqrt{2800 \times 7} \\&= \sqrt{7 \times 7 \times 400} = 7 \times 20 \\&= 140 \text{ m}\end{aligned}$$

24. (d) Let the breadth of rectangle = $x \text{ m}$
 $\therefore \text{length} = (x+5) \text{ m}$
 $\therefore \text{Area of hall} = \text{length} \times \text{breadth}$
 $750 = (x+5)x$
 $750 = 30 \times 25$
 (clearly $750 = 30 \times 25$)
 $\therefore x = 25, \text{ breadth} = 25 \text{ m}$
 $\text{length} = 25 + 5 = 30 \text{ m}$

25. (d) Required total area
= Area of four walls + area of base
= $2 \times 1.25(6 + 4) + 6 \times 4$
= 49 m^2



Ratio of length and breadth
= $3 : 2$

$$2(l + b) = 20 \text{ cm}$$

$$2(3x + 2x) = 20 \text{ cm}$$

$$2 \times 5x = 20 \text{ cm}$$

$$10x = 20$$

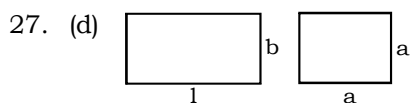
$$x = 2$$

$$\therefore \text{length} = 3 \times 2 = 6 \text{ cm,}$$

$$\text{breadth} = 2 \times 2 = 4 \text{ cm}$$

$$\text{area} = \text{length} \times \text{breadth}$$

$$= 6 \times 4 = 24 \text{ cm}^2$$



$$2(l + b) = 160 \text{ m}$$

$$l + b = 80 \text{ m}$$

$$a = 40 \text{ m}$$

ATQ

$$a^2 - lb = 100$$

$$(40)^2 - lb = 100$$

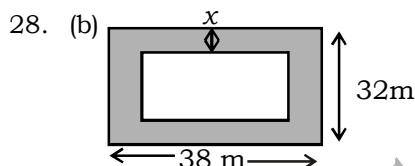
$$1600 - lb = 100$$

$$lb = 1500$$

$$\text{Clearly, } 50 + 30 = 80$$

$$\text{and } 50 \times 30 = 1500$$

$$\text{length} = 50 \text{ m}$$



$$\text{area of path} = 600 \text{ m}^2$$

$$(l + b - 2x) 2x = 600$$

$$(38 + 32 - 2x) 2x = 600$$

$$(70 - 2x) 2x = 600$$

$$(70 - 2x)x = \frac{600}{2} = 300$$

$$70x - 2x^2 = 300$$

$$2x^2 - 70x + 300 = 0$$

$$x^2 - 35x + 150 = 0$$

$$x^2 - 30x - 5x + 150 = 0$$

$$x(x - 30) - 5(x - 30) = 0$$

$$(x - 30)(x - 5) = 0$$

$$x = 30 \text{ not possible}$$

$$x = 5 \text{ (right)}$$

Alternate

$$(l + b - 2x) 2x = \text{area of path} = 600$$

take help from options to save your valuable time take option(b) $x = 5 \text{ m}$
 $(38 + 32 - 2 \times 5) 2 \times 5 = (70 - 10) \times 10 = 60 \times 10 = 600$

29. (c) Area of walls = Perimeter of base $\times$ height
 $= 18 \times 3 = 54 \text{ m}^2$

30. (d) $a^2 = 81$, $a = 9$

$$\Rightarrow \text{Perimeter of square}$$

$$= 9 \times 4 = 36 \text{ cm}$$

$$\Rightarrow 2r + \pi r = 36$$

$$r(2 + \pi) = 36$$

$$r = \frac{36}{2 + \frac{22}{7}} = 7 \text{ cm}$$

31. (a) $a^2 = 121$, $a = 11$

$$\Rightarrow \text{Perimeter of square}$$

$$= 11 \times 4 = 44 \text{ cm}$$

$$\Rightarrow \text{Circumference of circle} = 44$$

$$2\pi r = 44$$

$$2 \times \frac{22}{7} \times r = 44$$

$$\Rightarrow r = 7 \text{ cm}$$

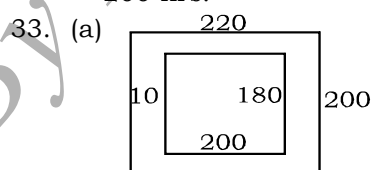
32. (d) Let the no. of hours be x

$$\Rightarrow (0.3 \times 0.2 \times 20000) \times x = 200$$

$$\times 150 \times 8$$

$$\Rightarrow x = \frac{200 \times 150 \times 8}{3 \times 2 \times 200}$$

$$= 200 \text{ hrs.}$$



Area of path

$$= 200 \times 220 - 200 \times 180$$

$$= 44000 - 36000 = 8000 \text{ m}^2$$

34. (c) Diagonal of square = diameter of circle = $8 \times 2 = 16 \text{ cm}$

$$\therefore \text{side of square} = \frac{16}{\sqrt{2}} = 8\sqrt{2} \text{ cm}$$

$$\Rightarrow \text{area of square} = (8\sqrt{2})^2 = 128 \text{ cm}^2$$

35. (a) Side of square = $\frac{8\sqrt{2}}{\sqrt{2}} = 8 \text{ cm}$

$$\therefore \text{Area of square} = 8 \times 8 = 64 \text{ cm}^2$$

36. (a) $x^2 + 7x + 10 = x^2 + 5x + 2x + 10$

$$= x(x + 5) + 2(x + 5)$$

$$= (x + 2)(x + 5)$$

$\therefore$ Two sides of rectangle

$$= (x + 2)(x + 5)$$

$$\therefore \text{Perimeter} = 2(x + 2 + x + 5)$$

$$= 2(2x + 7) = 4x + 14$$

37. (c) Let the sides of rectangle be 6 cm and 2 cm (or any other number)

$$\Rightarrow \text{Area of rectangle (Q)} = 6 \times 2$$

$$= 12 \text{ cm}^2$$

$$\therefore \text{Side of square} = 4 \text{ cm}$$

$$\Rightarrow \text{Area of square (P)}$$

$$= 4 \times 4 = 16 \text{ cm}^2$$

$$\Rightarrow P > Q$$

38. (b) No. of cubes with no side is painted = $(n - 2)^3$

Where n is the side of the bigger cube

Required number

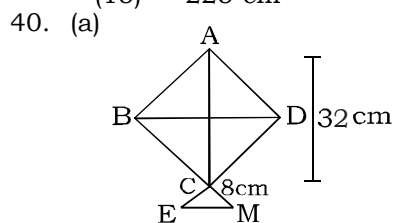
$$= (6 - 2)^3 = 64$$

39. (d) Side of square = $\frac{\text{Diagonal}}{\sqrt{2}}$

$$= \frac{15\sqrt{2}}{\sqrt{2}} = 15 \text{ cm}$$

$$\text{area of square} = (\text{side})^2$$

$$= (15)^2 = 225 \text{ cm}^2$$



$$\text{area of square} = \frac{1}{2} (\text{Diagonal})^2$$

$$= \frac{1}{2} (32)^2 = \frac{1}{2} \times 32 \times 32 = 16 \times 32$$

$$= 512 \text{ cm}^2$$

area of triangle

$$= \frac{\sqrt{3}}{4} 8^2 = \frac{1.732 \times 8 \times 8}{4}$$

$$= 1.732 \times 2 \times 8 = 27.712 \text{ cm}^2$$

$$\text{Required area} = (512 + 27.712) \text{ cm}^2 = 539.712 \text{ cm}^2$$

41. (a) Area of the lawn

$$= \frac{1}{12} \text{ hectare}$$

$$\text{length} \times \text{breadth} = \frac{1}{12} \times 10000 \text{ m}^2$$

$$4x \times 3x = \frac{10000}{12} \text{ m}^2$$

$$12x^2 = \frac{10000}{12}$$

$$x^2 = \frac{10000}{12 \times 12}$$

$$x = \frac{100}{12}$$

$$\text{Breadth} = 3x = 3 \times \frac{100}{12}$$

$$= \frac{100}{4} = 25 \text{ m}$$

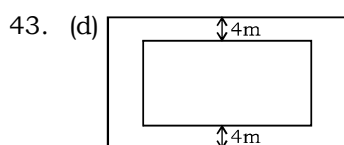
42. (a) Let the side of square = a cm

ATQ

$$1 \times b = 3a^2$$

$$20 \times \frac{3}{2}a = 3a^2$$

$$a = 10 \text{ cm}$$



$$\text{Area of path} = (l + b + 2x)2x$$

∴ where x = thickness of path

$$\text{Let } l = 7p, b = 4p$$

$$\{7p + 4p + 2(4)\}2(4) = 416$$

$$(11p + 8)8 = 416$$

$$11p + 8 = 52$$

$$11p = 44$$

$$p = \frac{44}{11} = 4, \quad p = 4$$

$$\therefore \text{breadth} = 4 \times 4 = 16 \text{ m}$$

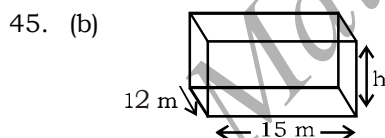
44. (d) Area of the floor = $8 \times 6 = 48 \text{ m}^2$

$$= 4800 \text{ dm}^2 \text{ (1m = 10 dm)}$$

Area of square tile

$$= 4 \times 4 = 16 \text{ dm}^2$$

$$\text{No. of tiles} = \frac{4800}{16} = 300$$



Shape of godown is cuboidal
length = 15 m, breadth = 12 m,
height = h m

$$\text{Area of four walls} = 2(l + b) \times h$$

$$\text{area of floor} = 1 \times b$$

$$\text{area of ceiling} = 1 \times b$$

ATQ

$$1 \times b + 1 \times b = 2(l + b) \times h$$

$$2(l \times b) = 2(l + b) \times h$$

$$2(15 \times 12) = 2(15 + 12) \times h$$

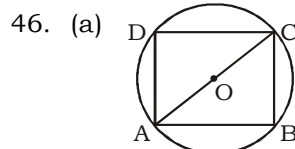
$$2 \times 180 = 2 \times 27 \times h$$

$$h = \frac{180}{27} = \frac{20}{3} \text{ m}$$

$$\text{Volume of the cuboid} = l \times b \times h$$

$$= 15 \times 12 \times \frac{20}{3}$$

$$= 60 \times 20 = 1200 \text{ m}^3$$



side of a square = AB

$$= \sqrt{2} a \text{ units}$$

$$\therefore AC = \text{Diagonal} = \sqrt{2} \times \sqrt{2} a$$

$$\therefore \text{Diameter} = 2 a \text{ units}$$

$$\text{Circumference} = \pi \times \text{diameter}$$

$$= \pi \times 2a = 2\pi a \text{ units.}$$

47. (b) Perimeter of rectangle = 40 m

$$\text{Length} = 12 \text{ metre}$$

$$\therefore 2(l + b) = 40$$

$$2(12 + b) = 40$$

$$12 + b = \frac{40}{2} = 20$$

$$b = 20 - 12 = 8 \text{ m}$$

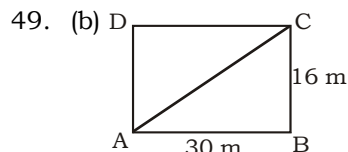
48. (d) Percentage increase in

$$\text{area} = \left(x + y + \frac{xy}{100} \right) \%$$

$$\text{Here, } x = 100\%, y = 100\%$$

$$= \left(100 + 100 + \frac{100 \times 100}{100} \right) \%$$

$$= 300\%$$



$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{30^2 + 16^2}$$

$$= \sqrt{900 + 256}$$

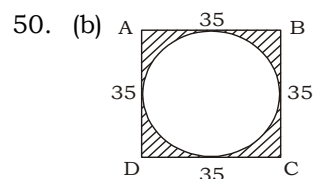
$$= \sqrt{1156} = 34 \text{ metre.}$$

$$\text{Distance travelled by elephant}$$

$$= 34 - 4 = 30 \text{ metre}$$

$$\text{speed of elephant} = \frac{30}{15}$$

$$= 2 \text{ m/s}$$



According to the question,

$$\text{Radius of circle} = \frac{35}{2}$$

Required area of shaded portion

$$= (35)^2 - \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$$

$$= 1225 - 962.5 = 262.5 \text{ m}^2$$

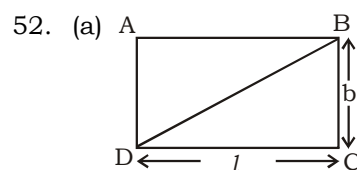
51. (b) Diagonal of square = $\sqrt{2}$ side of square

$$\text{Here } a = \frac{1}{2}(x + 1) \text{ and } d = \frac{3 - x}{\sqrt{2}}$$

$$\therefore d = \sqrt{2}a$$

$$\Rightarrow \frac{3 - x}{\sqrt{2}} = \sqrt{2} \left[\frac{1}{2}(x + 1) \right]$$

$$\therefore x = 1 \text{ unit}$$



Let ABCD is a rectangular carpet having length l metre and breadth b metre and BD is a diagonal

⇒ As we know

$$\Rightarrow \text{Area} = l \times b = 120 \quad \dots(i)$$

⇒ Perimeter

$$= 2(l + b) = 46$$

Using formula

$$\Rightarrow (l + b)^2 = l^2 + b^2 + 2lb$$

$$\Rightarrow (23)^2 = l^2 + b^2 + 2 \times 120$$

$$\Rightarrow 529 = l^2 + b^2 + 240$$

$$\Rightarrow l^2 + b^2 = 529 - 240$$

$$\Rightarrow l^2 + b^2 = 289$$

$$\Rightarrow \sqrt{l^2 + b^2} = \sqrt{289}$$

$$\text{diagonal} = 17$$

diagonal of carpet is 17 metres

53. (c) Diagonal of a square

$$= 6\sqrt{2} \text{ cm}$$

$$\text{Side of a square} = \frac{6\sqrt{2}}{\sqrt{2}} = 6 \text{ cm}$$

$$\text{Area of a square} = 6 \times 6 = 36 \text{ cm}^2$$

54. (c) Let the breadth of floor = x m
Then the length of floor = $(x+3)$ m

A.T.Q,

$$x \times (x + 3) = 70$$

$$x^2 + 3x - 70 = 0$$

$$x^2 + 10x - 7x - 70 = 0$$

$$(x + 10)(x - 7) = 0$$

$$x = 7, x = -10$$

$$\text{Breadth} = 7\text{ m}$$

$$\text{Length} = 10\text{ m}$$

$$\text{Perimeter of floor} = 2(L + B)$$

$$= 2(10 + 7) = 34\text{ m}$$

55. (d) Let the breadth of rectangle = x m

then the length of rectangle

$$= 2x\text{ m}$$

A.T.Q,

$$x \times 2x = 417.605$$

$$2x^2 = 417.605$$

$$x^2 = \frac{417.605}{2}$$

$$x = \sqrt{\frac{835.21}{400}}$$

$$x = \frac{289}{20}\text{ cm}$$

$$\text{Breadth} = \frac{289}{20}\text{ m}$$

$$\text{Length} = \frac{289}{20} \times 2 = 28.90\text{ m}$$

56. (b) Radius of circle = 5 cm
Length of arc = 3.5 cm

$$\therefore \text{Area of sector} = \frac{1}{2}lr$$

$$\frac{1}{2} \times 3.5 \times 5 = 8.75\text{ cm}^2$$

57. (c) Radius of circular wheel = 1.75 m

Circumference of circular

$$\text{wheel} = 2\pi r = 2 \times \frac{22}{7} \times 1.75\text{ m}$$

No. of revolutions

$$= \frac{\text{Distance to be covered}}{\text{Circumference of circle}}$$

$$= \frac{11000\text{ m}}{2 \times \frac{22}{7} \times 1.75\text{ m}}$$

$$= \frac{11000}{11} = 1000$$

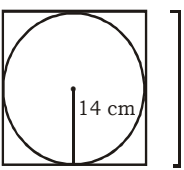
58. (d) Circumference of wheel = $2\pi r$

$$= 2 \times \frac{22}{7} \times 21\text{ cm} = 132\text{ cm}$$

No. of revolutions

$$= \frac{\text{Distance to be covered}}{\text{Circumference of circle}}$$

$$= \frac{924 \times 100}{132} = 700$$

59. (d) 

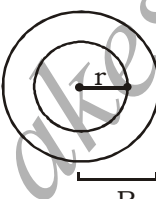
Radius of the largest circle

$$= \frac{1}{2} \times (\text{side of square})$$

$$= \frac{1}{2} \times 28 = 14\text{ cm}$$

$$\text{area of the circle} = \pi(\text{radius})^2$$

$$= \frac{22}{7} \times 14 \times 14 = 616\text{ cm}^2$$

60. (b) 

$$\therefore 2\pi r = 88$$

$$r = \frac{88 \times 7}{2 \times 22} = 14\text{ cm}$$

$$2\pi R = 132\text{ cm}$$

$$R = \frac{132 \times 7}{2 \times 22} = 21\text{ cm}$$

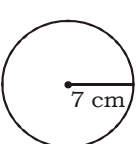
The area between two circles

$$= \pi(21)^2 - \pi(14)^2$$

$$= \pi\{21^2 - 14^2\}$$

$$= \pi(21+14)(21-14)$$

$$= \frac{22}{7} \times 35 \times 7 = 770\text{ cm}^2$$

61. (b) 

circumference of wheel

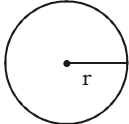
$$= 2\pi r$$

$$= 2 \times \frac{22}{7} \times 7$$

$$= 44\text{ cm}$$

$\therefore$ Total distance travelled by wheel in 15 revolutions

$$= 15 \times 44\text{ cm} = 660\text{ cm}$$

62. (c) 

$$\text{Circumference} = 2\pi r$$

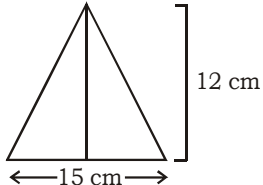
Distance covered in 1 min

$$= 2 \times \frac{8}{40} \times \pi r$$

New circumference

$$= 2 \times \pi \times r \times 10$$

$$\text{Time taken} = \frac{2\pi r \times 10 \times 40}{2\pi r \times 8} = 50\text{ min}$$

63. (b) 

area of the triangle

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 15 \times 12$$

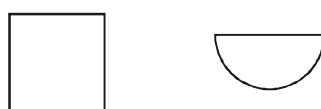
$$= 90\text{ cm}^2$$

$$\text{area of another triangle} = 2 \times 90 = 180\text{ cm}^2$$

$$= \frac{1}{2} \times \text{base} \times \text{height} = 180$$

$$= \frac{1}{2} \times 20 \times \text{height} = 180$$

$$\text{height} = \frac{180 \times 2}{20} = 18\text{ cm}$$

64. (b) 

$$\text{area of the square} = 81\text{ cm}^2$$

$$\text{side of the square} = \sqrt{81} = 9\text{ cm}$$

$$\text{perimeter of the square} = 4 \times 9 = 36\text{ cm}$$

Now, According to question,

$$\pi r + 2r = 36$$

$$r(\pi + 2) = 36$$

$$r = \frac{36}{\frac{22}{7} + 2} = \frac{36 \times 7}{22 + 14}$$

$$= \frac{36 \times 7}{36} = 7$$

area of the semi circle

$$= \frac{\pi r^2}{2} = \frac{22}{7} \times \frac{7^2}{2} = 77 \text{ cm}^2$$

65. (b) Area of square = $(12)^2$
= 144 cm²

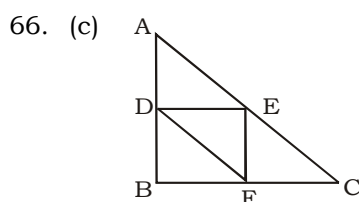
Area of triangle

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 12 \times \text{height}$$

$$= \frac{1}{2} \times 12 \times \text{height} = 144$$

$$\text{height} = \frac{144 \times 2}{12} = 24 \text{ cm}$$



$$\therefore 3^2 + 4^2 = 5^2$$

ΔABC is a right angled triangle

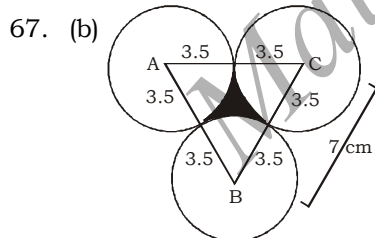
$$\text{ar (ABC)} = \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$$

$\therefore$ Required Area of ΔDEF

$$= \frac{1}{4} \times 6$$

$$= \frac{3}{2} \text{ cm}^2$$



$$AB = BC = AC = 7 \text{ cm}$$

Area enclosed
= Area of equilateral

$$\Delta ABC - \frac{1}{2} (\text{area of 1 circle})$$

$$= \frac{\sqrt{3}}{4} \times (7)^2 - \frac{1}{2} \left[\frac{22}{7} \times (3.5)^2 \right]$$

$$= 1.967 \text{ cm}^2$$

68. (a) $\pi r^2 = 2464 \text{ cm}^2$

$$\Rightarrow r = \sqrt{\frac{2464 \times 7}{22}}$$

$$= \sqrt{784} = 28 \text{ m}$$

$$\therefore \text{diameter} = 2r = 2 \times 28 = 56 \text{ cm}$$

69. (b) Required area = Area of square - Area of circle

$$= (2a)^2 - \pi (a)^2 = 4a^2 - \frac{22}{7} a^2$$

$$= \frac{28a^2 - 22a^2}{7} = \frac{6a^2}{7}$$

70. (c) Diameter of the circle

$$= \text{Side of square}$$

$$2r = 21$$

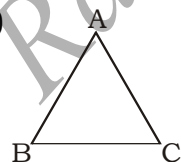
$$r = \frac{21}{2} \text{ m}$$

$$\text{Area} = \pi r^2 = \pi \left(\frac{21}{2} \right)^2$$

$$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{2} \text{ cm}^2$$

$$= 346 \frac{1}{2} \text{ cm}^2$$

71. (a)



$\therefore$ Area of an equilateral triangle = $400\sqrt{3}$

$$\frac{\sqrt{3}}{4} (\text{side})^2 = 400\sqrt{3}$$

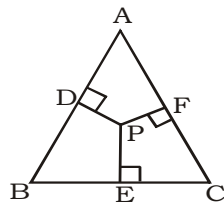
$$(\text{side})^2 = \frac{400\sqrt{3} \times 4}{\sqrt{3}} = 1600$$

$$\text{side} = 40 \text{ m}$$

$$\text{perimeter} = 3 \times \text{side}$$

$$= 3 \times 40 = 120 \text{ m}$$

72. (c)



Let P be the point inside the equilateral ΔABC

$$PD = \sqrt{3}, PE = 2\sqrt{3}, PF = 5\sqrt{3}$$

$$\text{and } AB = BC = AC = x$$

Ar (ABC)

$$= \frac{1}{2} \times x \times \sqrt{3} + \frac{1}{2} \times x \times 2\sqrt{3} + \frac{1}{2} \times x \times 5\sqrt{3}$$

$$\frac{\sqrt{3}}{4} x^2 = \frac{1}{2} \times x \times \sqrt{3} + \frac{1}{2} \times x \times 2\sqrt{3} + \frac{1}{2} \times x \times 5\sqrt{3}$$

$$\frac{\sqrt{3}}{4} x^2 = \frac{1}{2} \times x \times 8\sqrt{3}$$

$$x = 16$$

$\therefore$ perimeter of triangle

$$= 3x = 3 \times 16 = 48 \text{ cm}$$

Alternative:-

$$\text{side of equilateral } \Delta = \frac{2}{\sqrt{3}}$$

(sum of the altitudes draw from internal point)

$$\text{side} = \frac{2}{\sqrt{3}} (\sqrt{3} + 2\sqrt{3} + 5\sqrt{3})$$

$$= \frac{2}{\sqrt{3}} \times 8\sqrt{3} = 16 \text{ cm}$$

$$\text{perimeter} = 3 \times \text{sides}$$

$$= 3 \times 16 = 48 \text{ cm}$$

73. (c) Perimeter of $\Delta = 30 \text{ cm}$

$$\text{Area} = 30 \text{ cm}^2$$

Check the triplet

$$\{(5, 12, 13), (3, 4, 5)\}$$

whose largest side is 13.

$$\text{Also, } 5^2 + 12^2 = 13^2$$

$$\text{And perimeter} = 5 + 12 + 13$$

$$= 30 \text{ cm}$$

Smallest side = 5 cm

74. (b) Diameter of the wheel = 3 m

$$\text{Circumference} = \pi \times \text{diameter}$$

$$= \frac{22}{7} \times 3 = \frac{66}{7}$$

Since a wheel covers a distance equal to its circumference in one revolution therefore distance covered in 28 revolutions

$$= 28 \times \frac{66}{7} = 264 \text{ m}$$

$$264 \text{ metres covered} = 1 \text{ minutes}$$

$$1 \text{ metre covered} = \frac{1}{264} \text{ minute}$$

$$5280 \text{ metres covered} = \frac{5280}{264}$$

$$= 20 \text{ minutes}$$

75. (b) distance covered = 2 km 26 decameters

$$= (2 \times 1000 + 26 \times 10)$$

$$(1 \text{ decameter} = 10 \text{ meter})$$

$$= 2260 \text{ m}$$

Distance covered in 1 revolution

$$= \frac{\text{Total distance}}{\text{Number of revolutions}}$$

$$= \frac{2260}{113} = 20 \text{ m}$$

$$\text{Now, } \pi \times \text{diameter} = 20$$

$$\text{diameter} = \frac{20 \times 7}{22}$$

$$= \frac{70}{11} = 6\frac{4}{11} \text{ m}$$

76. (a) Distance covered in 1 revolution

= circumference of wheel

$$= 2 \times \frac{22}{7} \times 1.75 \text{ m}$$

$\therefore$ Number of revolution

$$= \frac{11 \times 1000}{2 \times \frac{22}{7} \times 1.75} = 1000$$

77. (b) Radius of circle

$$= \frac{\text{circumference}}{2\pi} = \frac{100}{2\pi}$$

When a square is inscribed in the circle, diagonal of the square is equal to diameter of the circle

$\therefore$ Diagonal of square

$$= 2 \times \frac{100}{2\pi} = \frac{100}{\pi}$$

$\therefore$ side of square

$$= \frac{\text{Diagonal}}{\sqrt{2}} = \frac{100}{\sqrt{2}\pi} = \frac{50\sqrt{2}}{\pi}$$

78. (c) Let outer Radius = R

and inner Radius = r

$$2\pi R - 2\pi r = 132$$

$$2\pi(R - r) = 132$$

$$R - r = \frac{132 \times 7}{2 \times 22} = 21$$

Hence, width of path = 21 metres.

79. (b)



side of square papersheet

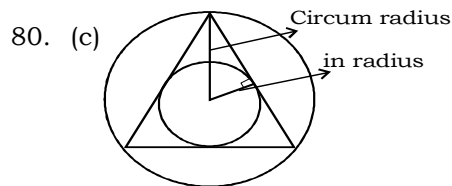
$$= \sqrt{784} = 28 \text{ cm}$$

radius of each circle

$$= \frac{28}{4} = 7 \text{ cm}$$

$\therefore$ circumference of each circular plate = $2\pi r$

$$= 2 \times \frac{22}{7} \times 7 = 44 \text{ cm}$$



80. (c)

Circum radius of equilateral tri-

$$\text{angle} = \frac{(\text{side})}{\sqrt{3}} = 8$$

$$\text{side} = 8\sqrt{3}$$

In radius of equilateral triangle

$$= \frac{\text{side}}{2\sqrt{3}} = \frac{8\sqrt{3}}{2\sqrt{3}} = 4 \text{ cm}$$

81. (b) radius of each circle = 1 cm with all the three centres an equilateral triangle of side 1 cm is formed.

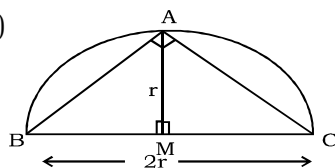
area enclosed by coins = (area of equilateral triangle) - $3 \times$ (area of sector of angle 60°)

$$= \frac{\sqrt{3}}{4} (2)^2 - 3 \times \frac{60}{360} \times \pi (1)^2$$

$$= \frac{\sqrt{3}}{4} \times 4 - 3 \times \frac{1}{6} \times \pi$$

$$= \left(\sqrt{3} - \frac{\pi}{2} \right) \text{ cm}^2$$

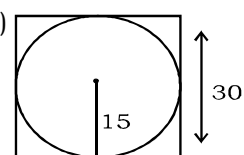
82. (b)



M is the centre, BM = CM = r
AM $\perp$ BC, (AM = r)

$$\text{area of } \triangle ABC = \frac{1}{2} r \times 2r = r^2$$

83. (a)



side of the square

$$= \frac{\text{Perimeter}}{4} = \frac{120}{4} = 30 \text{ cm}$$

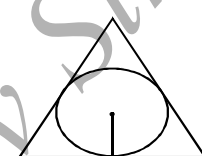
radius of the circle

$$= \frac{\text{side}}{2} = \frac{30}{2} = 15 \text{ cm}$$

area of the circle

$$= \frac{22}{7} \times (\text{radius})^2 = \frac{22}{7} \times (15)^2$$

84. (b)



radius of in circle

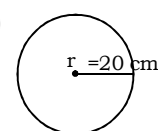
$$= \frac{\text{side}}{2\sqrt{3}} = \frac{42}{2\sqrt{3}} = \frac{21}{\sqrt{3}} \text{ cm}$$

area of incircle

$$= \frac{22}{7} \times \left(\frac{21}{\sqrt{3}} \right)^2 = \frac{22}{7} \times \frac{21 \times 21}{3}$$

$$= 22 \times 21 = 462 \text{ cm}^2$$

85. (a)



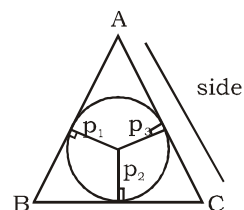
wheel of radius 20 cm

no. of revolutions

$$= \frac{\text{distance to cover}}{\text{circumference of wheel}}$$

$$= \frac{17600 \times 7}{2 \times 22 \times 20} = 140$$

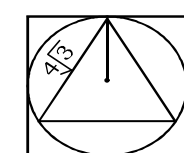
86. (a)



In an equilateral triangle

$$\text{side} = \frac{2}{\sqrt{3}} (P_1 + P_2 + P_3)$$

87. (c)



side of equilateral triangle

$$= 4\sqrt{3}$$

circumradius of triangle

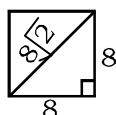
$$= \frac{\text{side}}{\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{3}} = 4$$

see the figure

side of square

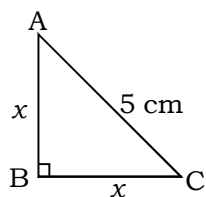
$$= 2 \times \text{circum radius}$$

$$= 2 \times 4 = 8$$



$$\text{Diagonal of square} = 8\sqrt{2} \text{ cm}$$

88. (b)



isosceles right triangle

$$\therefore x^2 + x^2 = 5^2 = 25$$

$$2x^2 = 25$$

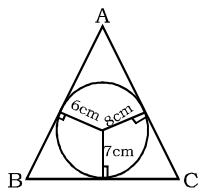
$$x^2 = \frac{25}{2}$$

Area of triangle

$$= \frac{1}{2} \times x^2 = \frac{1}{2} \times \frac{25}{2}$$

$$= 6.25 \text{ cm}^2$$

89. (c)



$$\text{length of side} = \frac{2}{\sqrt{3}} (P_1 + P_2 + P_3)$$

$$= \frac{2}{\sqrt{3}} (6 + 7 + 8) = \frac{2}{\sqrt{3}} \times 21$$

$$= \frac{42}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{42\sqrt{3}}{3} = 14\sqrt{3} \text{ cm}$$

90. (c) Area of isosceles triangle

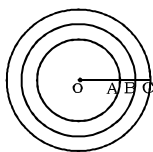
$$= \frac{1}{2} a^2 \sin \theta \quad (\theta \text{ is angle between equal sides})$$

$$= \frac{1}{2} (10)^2 \times \sin 45^\circ$$

$$= \frac{100}{2} \times \frac{1}{\sqrt{2}} = \frac{50}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= 25\sqrt{2} \text{ cm}^2$$

91. (a)



Radius of circle = 6 cm

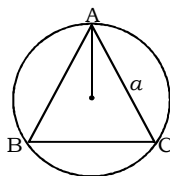
Area of smallest circle

$$= \frac{6^2 \times \pi}{3} = 12\pi$$

Radius of smallest circle

$$= \sqrt{\frac{12\pi}{\pi}} = 2\sqrt{3} \text{ cm}$$

92. (a)



$$\frac{\sqrt{3}}{4} a^2 = 4\sqrt{3}$$

$$a^2 = 4 \times 4$$

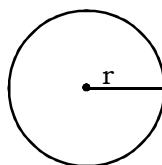
$$a = 4 \text{ cm}$$

$$\text{Circum radius} = \frac{a}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

area of circle = πr^2

$$= \pi \left(\frac{4}{\sqrt{3}} \right)^2 = \frac{16}{3} \pi \text{ cm}^2$$

93. (b)



Circumference - diameter = 30 cm

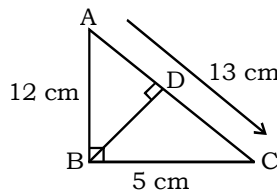
$$2\pi r - 2r = 30$$

$$2r(\pi - 1) = 30$$

$$r = \frac{30}{2\left(\frac{22}{7} - 1\right)} = \frac{30 \times 7}{2 \times 15}$$

$$= 7 \text{ cm}$$

94. (b)



$$AC = \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$$

$$= \sqrt{169} = 13 \text{ cm}$$

length of perpendicular,

$$BD = \frac{AB \times BC}{AC}$$

$\therefore$ length of perpendicular to hypotenuse to

$$= \frac{\text{perpendicular} \times \text{Base}}{\text{Hypotenuse}}$$

$$= \frac{12 \times 5}{13} = \frac{60}{13} = 4 \frac{8}{13} \text{ cm}$$

95. (c) Side of equilateral triangle

$$= \frac{2}{\sqrt{3}} (P_1 + P_2 + P_3)$$

$$= \frac{2}{\sqrt{3}} (6 + 8 + 10)$$

$$= \frac{2}{\sqrt{3}} \times 24 = \frac{48}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

Side = $16\sqrt{3}$ cm

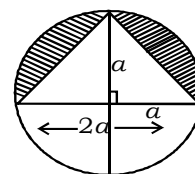
area of triangle

$$= \frac{\sqrt{3}}{4} (\text{side})^2 = \frac{\sqrt{3}}{4} \times (16\sqrt{3})^2$$

$$= \frac{\sqrt{3}}{4} \times 3 \times 16 \times 16$$

$$= 192\sqrt{3} \text{ cm}^2$$

96. (c)



area of shaded region = area of semicircle - area of triangle

$$= \frac{\pi a^2}{2} - \frac{1}{2} \times a \times 2a$$

$$= \frac{\pi a^2}{2} - a^2 = a^2 \left(\frac{\pi}{2} - 1 \right) \text{ sq units}$$

97. (c) According to question

$$\pi (R + 1)^2 - \pi R^2 = 22$$

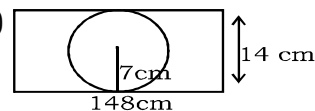
$$\pi \{(R+1)^2 - R^2\} = 22$$

$$(R + 1 + R)(R + 1 - R) = \frac{22 \times 7}{22} = 7$$

$$2R + 1 = 7$$

$$R = 3 \text{ cm}$$

98. (b)

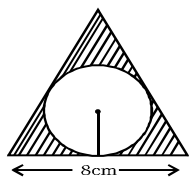


radius of largest circle

$$= \frac{\text{breadth}}{2} = \frac{14}{2} = 7 \text{ cm}$$

$$\text{area} = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$$

99. (b)



$$\text{in-radius of circle (r)} = \frac{\text{side}}{2\sqrt{3}}$$

$$= \frac{8}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$$

$$\text{area of circle} = \pi \left(\frac{4}{\sqrt{3}} \right)^2$$

$$= \frac{22}{7} \times \frac{4 \times 4}{3} = \frac{22 \times 16}{21} = 16.76$$

Required area

$$= \frac{\sqrt{3}}{4} (8)^2 - \frac{22 \times 16}{21}$$

$$= \frac{\sqrt{3}}{4} \times 64 - 16.76$$

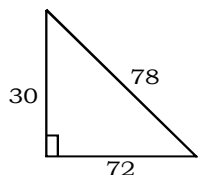
$$= 16\sqrt{3} - 16.76$$

$$= 27.71 - 16.76$$

$$= 10.95 \text{ cm}^2$$

100. (c) $\begin{matrix} 30 & : & 72 & : & 78 \\ 5 & : & 12 & : & 13 \end{matrix}$

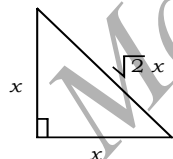
So, the triangle is right triangle



$$\frac{1}{2} \times 30 \times 72 = \frac{1}{2} \times \text{altitude} \times 72$$

$$\text{altitude} = 30 \text{ m}$$

101. (a)



$$\text{perimeter of triangle} = 4\sqrt{2} + 4$$

$$x + x + \sqrt{2}x = 4\sqrt{2} + 4$$

$$2x + \sqrt{2}x = 4\sqrt{2} + 4$$

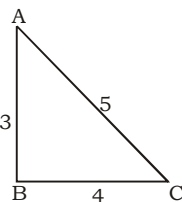
$$x(2 + \sqrt{2}) = 4(\sqrt{2} + 1)$$

$$x = \frac{4}{\sqrt{2}}$$

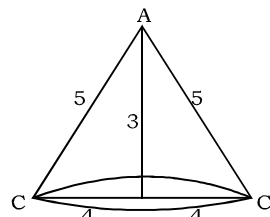
$$\text{Hypotenuse} = \sqrt{2}x$$

$$= \sqrt{2} \times \frac{4}{\sqrt{2}} = 4 \text{ cm}$$

102. (a)



cone is rotated about 3 cm



The cone so formed after rotating about Side AB.

So, slant height of cone = 5 cm

$$\therefore \text{Volume of cone} = \frac{1}{3} \times \pi r^2 h$$

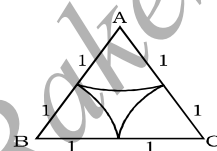
r = radius

h = height

$$\therefore \text{Volume of cone}$$

$$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3 = 16\pi \text{ cm}^3$$

103. (c)



Area of bounded region

$$= \frac{\sqrt{3}}{4} \times 2^2 - \frac{1}{2} \pi (1)^2$$

$$= \left(\sqrt{3} - \frac{\pi}{2} \right) \text{ cm}^2$$

104. (a) $2\pi r = 11$

$$\Rightarrow r = \frac{11 \times 7}{22 \times 2} = \frac{7}{4}$$

Area of sector

$$= \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}$$

$$= \frac{77}{48} = 1 \frac{29}{48} \text{ cm}^2$$

105. (c) Let the side of the triangle be 'a' cm

$$\Rightarrow \text{Circumradius} = \frac{a}{\sqrt{3}}$$

$$\text{and Inradius} = \frac{a}{2\sqrt{3}}$$

$$\Rightarrow \pi \left(\frac{a}{\sqrt{3}} \right)^2 - \pi \left(\frac{a}{2\sqrt{3}} \right)^2 = 44$$

$$\Rightarrow \pi \left(\frac{a^2}{3} - \frac{a^2}{12} \right) = 44$$

$$\Rightarrow \frac{4a^2 - a^2}{12} = \frac{44 \times 7}{22} = 14$$

$$\Rightarrow \frac{3a^2}{12} = 14$$

$$\Rightarrow a^2 = 56$$

$$\Rightarrow a = 2\sqrt{14}$$

$$\Rightarrow \text{area} = \frac{\sqrt{3}}{4} \times 2\sqrt{14} \times 2\sqrt{14}$$

$$= 14\sqrt{3} \text{ cm}^2$$

106. (c) Side of square = diameter of the circle

$$\text{area of circle} = \pi r^2 = 9\pi$$

$$\Rightarrow r = 3 \text{ cm}$$

$$\Rightarrow \text{Side of square} = 3 \times 2 = 6 \text{ cm}$$

$$\Rightarrow \text{Area} = 6 \times 6$$

$$= 36 \text{ cm}^2$$

107. (d) The given triangle is a right angled triangle

$\Rightarrow$ side of the square

$$= \frac{P \times b}{P + b} = \frac{8 \times 6}{8 + 6} = \frac{24}{7}$$

$$\Rightarrow \text{Area of square} = \left(\frac{24}{7} \right)^2$$

$$= \frac{576}{49} \text{ cm}^2$$

108. (b) Radius of circumcircle

$$= \frac{8}{\sqrt{3}} \text{ cm}$$

Radius of incircle

$$= \frac{8}{2\sqrt{3}} = \frac{4}{\sqrt{3}} \text{ cm}$$

$$\Rightarrow \text{Required area} = \pi (R^2 - r^2)$$

$$= \frac{22}{7} \left[\left(\frac{8}{\sqrt{3}} \right)^2 - \left(\frac{4}{\sqrt{3}} \right)^2 \right]$$

$$= \frac{22}{7} \left(\frac{64}{3} - \frac{16}{3} \right)$$

$$= \frac{22}{7} \times 16 = 50 \frac{2}{7} \text{ cm}^2$$

109. (c) Side of square = $\sqrt{121} = 11$ cm
Perimeter of square = Circumference of circle = 44 cm
 $\Rightarrow 2\pi r = 44$

$$\Rightarrow r = \frac{44 \times 7}{22 \times 2} = 7 \text{ cm}$$

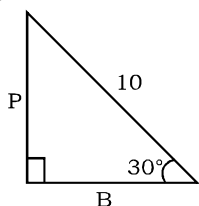
$$\Rightarrow \text{Area} = \pi r^2$$

$$= \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$$

110. (a) $2r + \pi r = 36$
 $\Rightarrow r(2 + \pi) = 36$
 $\Rightarrow r(2 + \frac{22}{7}) = 36$
 $\Rightarrow r = \frac{36 \times 7}{36} = 7 \text{ m}$

111. (c) $2r + \pi r = \frac{1}{2} \pi r^2$
 $\Rightarrow r(2 + \pi) = \frac{1}{2} \pi r^2$
 $\Rightarrow 4 + 2\pi = \pi r$
 $\Rightarrow r = \frac{4}{\pi} + 2$
 $\Rightarrow \text{Diameter} = 2\left(\frac{4}{\pi} + 2\right)$
 $= 6\frac{6}{11} \text{ m}$

112. (a) The angles of the given triangle are $90^\circ, 30^\circ$ and 60°



$$P = \frac{10}{2} = 5$$

$$B = 5\sqrt{3}$$

$$\Rightarrow \text{Area} = \frac{1}{2} \times 5\sqrt{3} \times 5$$

$$= \frac{25\sqrt{3}}{2} \text{ cm}^2$$

113. (c) Let the altitude = x cm

$$\Rightarrow \frac{1}{2} \times x \times 8 = \pi \times 8^2$$

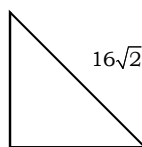
$$\Rightarrow x = \frac{\pi \times 64}{4}$$

$$\Rightarrow x = 16\pi$$

114. (d) The sides of the given triangle are 3, 4 and 5 cm

$$\text{area} = \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$$

115. (b)



$$\text{Other sides} = \frac{16\sqrt{2}}{\sqrt{2}} = 16 \text{ cm (as the isosceles } \Delta)$$

$$\Rightarrow \text{Area} = \frac{1}{2} \times 16 \times 16 = 128 \text{ cm}^2$$

116. (d) $\frac{\sqrt{3}}{4} a^2 = 4\sqrt{3}$

$$\Rightarrow a^2 = 16$$

$$\Rightarrow a = 4 \text{ cm}$$

117. (c) Side of hexagon

$$= \frac{\text{Side of equilateral triangle}}{3}$$

$$= 2 \text{ cm}$$

$$\text{Area of hexagon}$$

$$= \frac{3\sqrt{3}}{2} a^2 = \frac{3\sqrt{3}}{2} \times 4$$

$$= 6\sqrt{3} \text{ cm}^2$$

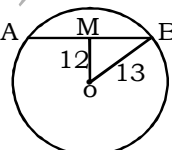
118. (a) The radius of park = $\frac{176}{2\pi}$
 $= 28 \text{ m}$

$$\Rightarrow \text{Area of road} = \pi(28 + 7)^2 - \pi(28)^2$$

$$= \pi(35 + 28)(35 - 28)$$

$$= \frac{22}{7} \times 7 \times 63 = 1386 \text{ m}^2$$

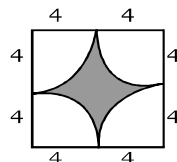
119. (c)



$$\text{In } \triangle OMB \text{ } MB = \sqrt{13^2 - 12^2} = 5$$

$$\Rightarrow AB = 5 \times 2 = 10 \text{ cm}$$

120. (b)



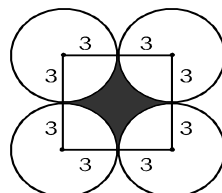
$$\text{Area of shaded portion}$$

$$= 8 \times 8 - \pi \times 4^2$$

$$= 64 - 16\pi$$

$$= 16(4 - \pi) \text{ cm}^2$$

121. (b)



$$\text{Area of the shaded portion}$$

$$= 6 \times 6 - \pi(3)^2$$

$$= 36 - 9\pi$$

$$= 9(4 - \pi) \text{ cm}^2$$

122. (c) Radius of incircle

$$= \frac{14\sqrt{3}}{2\sqrt{3}} = 7 \text{ cm}$$

$$\Rightarrow \text{Area} = \pi r^2 = \frac{22}{7} \times 7 \times 7$$

$$= 154 \text{ cm}^2$$

123. (d) Radius of incircle

$$= \frac{6}{2\sqrt{3}} = \sqrt{3} \text{ cm}$$

$$\text{Area} = \pi r^2$$

$$= 3\pi \text{ cm}^2$$

124. (c) $\frac{\sqrt{3}}{4} a^2 = 121\sqrt{3}$

$$\Rightarrow a = 22 \text{ cm}$$

$$\Rightarrow 3a = 66 \text{ cm}$$

$$\text{Circumference of circle} = 66$$

$$2\pi r = 66$$

$$r = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$$

$$\text{Area} = \pi r^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$$

$$= 346.5 \text{ cm}^2$$

125. (b) Area grazed by the cow

$$= \frac{1}{2} \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

$$= 77 \text{ m}^2$$

$$S = \frac{26 + 30 + 28}{2} = 42$$

$$\text{Area of field}$$

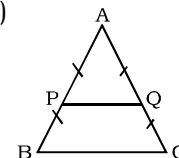
$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{42 \times 16 \times 14 \times 12} = 336 \text{ m}^2$$

$$\Rightarrow \text{Remaining area} = 336 - 77$$

$$= 259 \text{ m}^2$$

126. (b)



As P and Q are mid-point and $PQ \parallel BC$

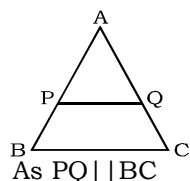
$$\Rightarrow \triangle APQ \sim \triangle ABC$$

$$\Rightarrow \frac{AP}{AB} = \frac{PQ}{BC} = \frac{1}{2}$$

$$\Rightarrow PQ = \frac{BC}{2}$$

$$\Rightarrow BC = 2PQ = 2 \times 5 = 10 \text{ cm}$$

127. (c)



As $PQ \parallel BC$
 $\Rightarrow \triangle APQ \sim \triangle ABC$
 $\Rightarrow \triangle APQ$ is also an equilateral $\triangle$

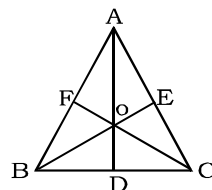
$$\Rightarrow \triangle APQ = \frac{\sqrt{3}}{4} (5)^2 = \frac{25\sqrt{3}}{4} \text{ cm}^2$$

128. (a) $2\pi r = 22$

$$\Rightarrow r = \frac{22 \times 7}{22 \times 2} = \frac{7}{2}$$

$$\Rightarrow \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} = 38.5 \text{ cm}^2$$

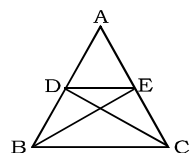
129. (b)



$$\text{ar}(\triangle AOE) = 15 \text{ cm}^2$$

$$\text{ar} \square BDOF = 2 \times \text{ar} \triangle AOE = 30 \text{ cm}^2$$

130. (b)



$$\text{ar}(\triangle ABE) = \text{ar}(\triangle ACD) = 36 \text{ cm}^2$$

131. (a) The third side will be either 15 or 22

$$\Rightarrow \text{Possible perimeter} = 15 \times 2 + 22 = 52$$

$$\text{and } 22 \times 2 + 15 = 59$$

132. (a) No. of revolutions

$$= \frac{\text{Distance}}{\text{Circumference}}$$

$$= \frac{1540 \times 100}{2 \times \frac{22}{7} \times \frac{98}{2}} = 500$$

133. (b) $2\pi r = \frac{440}{1000}$

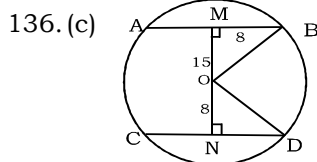
$$\Rightarrow r = \frac{22 \times 7}{50 \times 22 \times 2} = .07$$

$$\Rightarrow \text{Diameter} = .14 \text{ m}$$

134. (b) $2\pi r = \frac{11000 \times 100}{5000}$

$$\Rightarrow r = \frac{11 \times 100 \times 7}{5 \times 2 \times 22} = 35 \text{ cm}$$

135. (b) Length of rubber band
 $= 3d + 2\pi r = 30 + 10\pi$



In $\triangle OMB$

$$OB = \sqrt{15^2 + 8^2} = 17 \text{ cm}$$

$$OB = OD = \text{radius}$$

In $\triangle OND$

$$ND = \sqrt{17^2 - 8^2} = 15 \text{ cm}$$

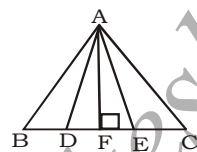
$$CD = 15 \times 2 = 30 \text{ cm}$$

137. (b) Perimeter $= 2r + \pi r$

$$= 63 + \frac{22}{7} \times \frac{63}{2}$$

$$= 63 + 99 = 162 \text{ cm}$$

138. (c)



In $\triangle AFB$

$$AF \perp BC$$

$$AF^2 = AB^2 - FB^2 = 100 - 25$$

$$AF = 5\sqrt{3}$$

In $\triangle ADF$

$$AD^2 = AF^2 + DF^2$$

$$AD^2 = 75 + \left(5 - \frac{10}{3}\right)^2$$

$$AD = \frac{10\sqrt{7}}{3}$$

139. (c) Let sides of triangle are a, b and c respectively

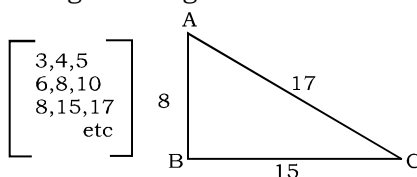
$$\therefore \text{largest side given} = 17 \text{ cm}$$

$$\text{Perimeter} = a + b + c$$

$$= 40 \text{ cm (given)}$$

$$\text{area} = 60 \text{ cm}^2 \text{ (given)}$$

In such questions take the help of triplets which form right angle triangle



So, here we have a side 17 cm

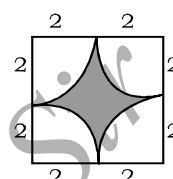
$\Rightarrow$ by triplet we get sides 8 and 15

$\Rightarrow$ check the sides perimeter
 $= 8 + 15 + 17 = 40$

$$\text{area} = \frac{1}{2} \times 8 \times 15 \Rightarrow 60$$

Hence sides are 15, 8.
 smaller side = 8 cm.

140. (b)



Area of shaded region

$$= (4)^2 - \pi (2)^2 = (16 - 4\pi) \text{ cm}^2$$

141. (c) Let the side of the triangle be a

$$\Rightarrow \text{Perimeter} = 3a$$

$$3a = \left(\frac{\sqrt{3}}{4} a^2 \right) \sqrt{3}$$

$$3 = \frac{3}{4} a$$

$$a = 4 \text{ units}$$

142. (a) Area of $\triangle = \frac{\sqrt{3}}{4} a^2$

$$= \frac{\sqrt{3}}{4} \times 36 = 9\sqrt{3} \text{ cm}^2$$

143. (b) Area of $\triangle = \frac{4}{3} (\text{Area of } \triangle \text{ formed by median as side})$

$$= \frac{4}{3} \left(\frac{1}{2} \times 9 \times 12 \right)$$

($\because 9, 12, 15$ from triplet)

$$= \frac{4}{3} \times 54 = 72 \text{ cm}^2$$

144. (a) $3x + 2y = 6$

$$\frac{x}{2} + \frac{y}{3} = 1$$

(Make R. H.S. equal to one)

$\Rightarrow$ Coordinates of $\triangle$

$$= (0, 3), (2, 0), (0, 0)$$

$$\Rightarrow \text{Area of } \triangle = \frac{1}{2} \times 3 \times 2$$

$$= 3 \text{ square units}$$

145. (a) Let each side of the triangle be a units

$$\Rightarrow \frac{\sqrt{3}}{4} ((a+2)^2 - a^2) = 3 + \sqrt{3}$$

$$\frac{1}{4} (a^2 + 4 + 4a - a^2) = 1 + \sqrt{3}$$

$$\frac{1}{4}(4 + 4a) = 1 + \sqrt{3}$$

$$1 + a = 1 + \sqrt{3}$$

$$a = \sqrt{3} \text{ units}$$

146. (c) $S = \frac{9 + 10 + 11}{2} = 15$
using heron's formula
Area = $\sqrt{s(s-a)(s-b)(s-c)}$
 $= \sqrt{15 \times 6 \times 5 \times 4}$
 $= \sqrt{1800} = 30\sqrt{2} \text{ cm}^2$

147. (c) Let the length of each equal side be a unit
 $\Rightarrow \frac{2}{4}\sqrt{4a^2 - 4} = 4$
 $\sqrt{4a^2 - 4} = 8$
 $4a^2 - 4 = 64$
 $a^2 - 1 = 16$
 $a^2 = 17$
 $a = \sqrt{17} \text{ units}$

148. (a) Sum of other two sides
 $(a + b) = 32 - 11 = 21$
and $a - b = 5$
 $\Rightarrow a = \frac{21+5}{2} = 13 \text{ cm}$
 $b = \frac{21-5}{2} = 8 \text{ cm}$
Sides of the $\Delta = 11, 8, 13 \text{ cm}$
 $S = \frac{13+8+11}{2} = 16$
 $\Rightarrow \text{area}$
 $= \sqrt{16(16-13)(16-8)(16-11)}$
 $= \sqrt{16 \times 3 \times 8 \times 5} = 8\sqrt{30} \text{ cm}^2$

149. (b) Area of $\Delta = \frac{\sqrt{3}}{4}a^2$
 $= \frac{\sqrt{3}}{4} \times (2)^2 = \sqrt{3} \text{ cm}^2$

150. (a) Let the original radius = r cm
 $\Rightarrow \pi((r+1)^2 - r^2) = 22$
 $r^2 + 1 + 2r - r^2 = \frac{22 \times 7}{22} = 7$
 $2r + 1 = 7$
 $r = 3 \text{ cm}$

151. (a) Area of two circles
 $= \pi(5^2 + 12^2) = 169\pi \text{ cm}^2$

$$\Rightarrow \pi r^2 = 169\pi$$

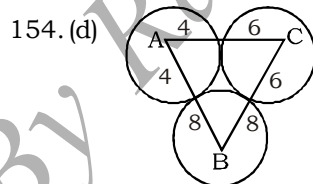
$$r^2 = 169$$

$$r = 13 \text{ cm}$$

$$\therefore \text{Radius of third circle} = 13 \text{ cm}$$

152. (d) Let the radius of the semi-circle be = r
 $\Rightarrow 2r + \pi r = 36$
 $r(2 + \pi) = 36$
 $r\left(2 + \frac{22}{7}\right) = 36$
 $r\left(\frac{36}{7}\right) = 36$
 $r = 7 \text{ m}$
 $\Rightarrow \text{Area} = \frac{\pi \times 7^2}{2}$
 $= \frac{22 \times 7 \times 7}{7 \times 2} = 77 \text{ m}^2$

153. (b) side of square = $\sqrt{\text{area}}$
 $= \sqrt{2} \text{ m} = \text{Diameter of circle}$
 $\Rightarrow \text{Radius of circle}$
 $= \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \text{ m}$
 $\therefore \text{Area} = \frac{22}{7} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{\pi}{2} \text{ m}^2$

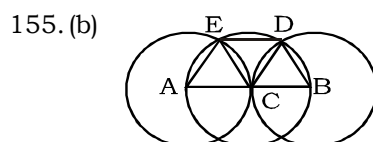


Side of $\Delta ABC = 10, 14, 12$

$$S = \frac{10+14+12}{2} = 18$$

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{18 \times 8 \times 4 \times 6} = 24\sqrt{6} \text{ cm}^2$$



Area ($\square ABDE$) = $3 \times \text{ar}(\Delta AEC)$
 $= 3 \times \frac{\sqrt{3}}{4} (2)^2$ (ADC is equilateral triangle)
 $= 3\sqrt{3} \text{ square units}$

156. (a) Check triplets
3, 4, 5
6, 8, 10
7, 24, 25
 $\Rightarrow 7, 24, 25$ fulfill the given conditions

$$\text{area} = \frac{1}{2} \times 24 \times 7 = 84 \text{ cm}^2$$

$$\text{Perimeter} = 7 + 24 + 25 = 56 \text{ cm}$$

$$\Rightarrow \text{Hypotenuse} = 25 \text{ cm}$$

157. (c) Length of median

$$= \frac{\sqrt{3}}{2} a = 6\sqrt{3}$$

$$a = 12 \text{ cm}$$

$$\therefore \text{Perimeter} = 12 \times 3 = 36 \text{ cm}$$

158. (a) Area of equilateral Δ

$$\frac{\sqrt{3}}{4} a^2 = 4\sqrt{3}$$

$$a^2 = 16$$

$$a = 4$$

$$\therefore \text{Perimeter} = 4 \times 3 = 12 \text{ cm}$$

159. (a) Distance covered by small gear
 $= 2\pi r \times 42$

$$= 84\pi \times \frac{12}{2} = 504\pi$$

$$\text{No. of revolution by big gear}$$

$$\frac{504\pi}{2\pi \times 9} = 28$$

160. (b) Perimeter of semi-circle
 $= 2r + \pi r = r(2 + \pi)$

$$\Rightarrow r(2 + \pi) = 18$$

$$r = \frac{18}{2 + \frac{22}{7}} = \frac{18 \times 7}{36}$$

$$= \frac{7}{2} \text{ cm} = 3\frac{1}{2} \text{ cm}$$

161. (d) Perimeter of circle = $2\pi r$
 $= 2(18 + 26) = 88 \text{ cm}$
 $\pi r = 44 \text{ cm}$
 $r = 14 \text{ cm}$

$$\therefore \text{Area of circle} = \frac{22}{7} \times 14 \times 14$$

$$= 616 \text{ cm}^2$$

162. (a) Area of a circle = 38.5 cm^2
 $\pi r^2 = 38.5$

$$r^2 = \frac{38.5 \times 7}{22}$$

$$r = \frac{7}{2} \text{ cm}$$

$$\text{Circumference of a circle} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} = 22 \text{ cm}$$

163.(c) Diameter of circle

$$= \frac{\text{Diagonal}}{\sqrt{2}} = \frac{12\sqrt{2}}{\sqrt{2}} = 12\text{cm}$$

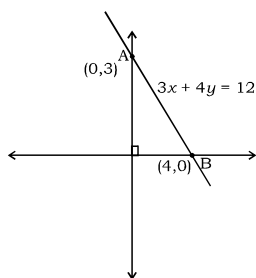
$$\text{Radius of circle} = \frac{12}{2} = 6\text{ cm}$$

Radius of circumcircle of

$$\text{equilateral } \Delta = \frac{a}{\sqrt{3}}$$

$$\Rightarrow a = \text{Radius} \times \sqrt{3} = 6\sqrt{3}\text{ cm}$$

164.(c)



$$3x + 4y = 12$$

$$\frac{3x}{12} + \frac{4y}{12} = 1$$

$\therefore$ Divide by 12 on both sides
make R.H.S = 1

$$\frac{x}{4} + \frac{y}{3} = 1$$

$\therefore$ Coordinates of point A = (0, 3)
point B = (4, 0)

$$\begin{aligned} \text{area of } \Delta OAB &= \frac{1}{2} \times 4 \times 3 \\ &= 6 \text{ sq units} \end{aligned}$$

165.(c) Height of equilateral $\Delta = 15\text{ cm}$

$$\frac{\sqrt{3}}{2}(\text{side}) = 15$$

$$\text{side} = \frac{15 \times 2}{\sqrt{3}}$$

$$\begin{aligned} \text{area} &= \frac{\sqrt{3}}{4}(\text{side})^2 \\ &= \frac{\sqrt{3}}{4} \left(\frac{15 \times 2}{\sqrt{3}} \right)^2 = \frac{\sqrt{3}}{4} \times \frac{225 \times 4}{3} \\ &= 75\sqrt{3}\text{ cm}^2 \end{aligned}$$

$$166.(b) \frac{\sqrt{3}}{4}(\text{side})^2 = 9\sqrt{3}$$

$$(\text{side})^2 = 9 \times 4 = 36$$

$$\text{side} = \sqrt{36} = 6\text{ cm}$$

$$\text{length of median} = \frac{\sqrt{3}}{2}(\text{side})$$

$$= \frac{\sqrt{3}}{2} \times 6 = 3\sqrt{3}\text{ cm}$$

Note: In an equilateral triangle,
length of median, angle bisec-

tor, altitude is equal to $\frac{\sqrt{3}}{2}$

sides

167.(c) clearly, 12 cm, 16 cm and
20 cm from a triplet

3 4 5 $\rightarrow$ triplet

$$\begin{array}{ccc} | & | & | \\ \times 4 & \times 4 & \times 4 \end{array}$$

12 16 20 $\rightarrow$ triplet

They form a right triangle,

$$\begin{aligned} \text{area of triangle} &= \frac{1}{2} \times 16 \times 12 \\ &= 96\text{ cm}^2 \end{aligned}$$

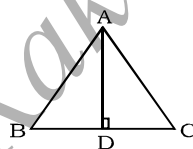
$$168.(c) \frac{(8)^2}{(x)^2} = \frac{360}{250} = \frac{36}{25}$$

$$\frac{8}{x} = \sqrt{\frac{36}{25}} = \frac{6}{5}$$

$$x = \frac{40}{6} = \frac{20}{3} = 6\frac{2}{3}\text{ cm}$$

Note: The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides

169.(d)



$$AB = AC = \frac{5}{6} BC$$

$$AB + BC + AC = 544$$

$$\frac{5}{6} BC + BC + \frac{5}{6} BC = 544$$

$$\frac{5BC + 6BC + 5BC}{6} = 544$$

$$\frac{16BC}{6} = 544$$

$$BC = \frac{544 \times 6}{16} = 204$$

$$\Rightarrow AB = AC = \frac{5}{6} \times 204 = 170\text{ cm}$$

$$\text{Area of } \Delta ABC = \frac{b}{4} \sqrt{4a^2 - b^2}$$

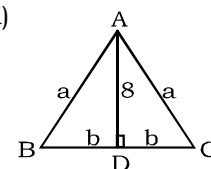
$\therefore$ where a = equal side
b = base

$$= \frac{204}{4} \sqrt{4(170)^2 - (204)^2}$$

$$= 51\sqrt{11560 - 41616}$$

$$\begin{aligned} &= 51 \times \sqrt{73984} \\ &= 51 \times 272 \\ &= 13872\text{ cm}^2 \end{aligned}$$

170.(d)



Let AB = AC = a cm

BD = DC = b cm

$\therefore$ Altitude of isosceles triangle
is also median

In right ΔADC

$$AD^2 = a^2 - b^2$$

$$64 = a^2 - b^2 \quad \dots\dots\dots(i)$$

$$\text{Perimeter} = 64$$

$$a + a + 2b = 64$$

$$2a + 2b = 64$$

$$a + b = 32 \quad \dots\dots\dots(ii)$$

$$\text{On dividing } \frac{a^2 - b^2}{a + b} = \frac{64}{32} = 2$$

$$[a^2 - b^2 = (a + b)(a - b)]$$

$$a - b = 2$$

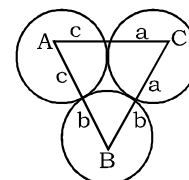
$$\therefore a + b = 32$$

$$\text{On solving } a = 17, \quad b = 15$$

$$\text{area of } \Delta ABC = \frac{1}{2} \times AD \times BC$$

$$= \frac{1}{2} \times 8 \times 30 = 120\text{ cm}^2$$

171.(a)



$$x = AB = b + c$$

$$y = BC = a + b$$

$$z = AC = a + c$$

$\therefore$ semi-perimeter(s)

$$= \frac{AB + BC + AC}{2} = \frac{2a + 2b + 2c}{2}$$

$$= a + b + c$$

area of

$$\Delta ABC = \sqrt{s(s-x)(s-y)(s-z)}$$

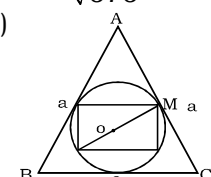
$$= \sqrt{(a+b+c)abc}$$

$$172.(d) \pi R^2 = \pi(10)^2 + \pi(24)^2$$

$$R^2 = 10^2 + 24^2 = 100 + 576$$

$$R = \sqrt{676} = 26\text{ cm}$$

173.(c)



Let the side of an equilateral triangle = 'a'
and the side of square = 'b'
In-circle radius of equilateral

$$\Delta = \frac{a}{2\sqrt{3}}$$

∴ Diagonal of square

$$= 2 \times \frac{a}{2\sqrt{3}} = \frac{a}{\sqrt{3}}$$

Now,

$$b = \frac{\text{Diagonal}}{\sqrt{2}} = \frac{a}{\sqrt{3}} = \frac{a}{\sqrt{6}}$$

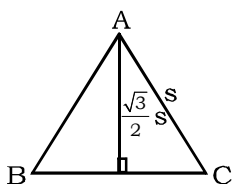
Required ratio

$$= \frac{\frac{\sqrt{3}}{4} a^2}{\left(\frac{a}{\sqrt{6}}\right)^2} = \frac{\sqrt{3}}{4} a^2 \times \frac{6}{a^2}$$

$$= \frac{3\sqrt{3}}{2} \Rightarrow 3\sqrt{3} : 2$$

174.(c) Let the side of equilateral triangle = s

$$\text{area of equilateral } \Delta = \frac{\sqrt{3}}{4} s^2$$

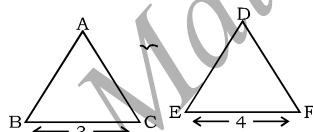


height of equilateral triangle =

$$\frac{\sqrt{3}}{2} s$$

$$\frac{b^2}{a} = \frac{\left(\frac{\sqrt{3}}{2} s\right)^2}{\frac{\sqrt{3}}{4} s^2} = \frac{\frac{3}{4} s^2}{\frac{\sqrt{3}}{4} s^2} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

175.(c)



$$\Delta ABC \sim \Delta DEF$$

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{3^2}{4^2}$$

$$\frac{54}{\text{ar}(\Delta DEF)} = \frac{9}{16}$$

$$\text{ar}(\Delta DEF) = \frac{16 \times 54}{9} = 96 \text{ cm}^2$$

$$176.(b) \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AB^2}{DE^2}$$

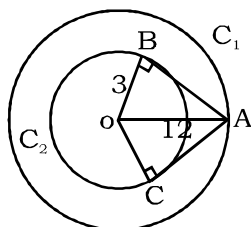
$$\frac{20}{45} = \frac{25}{DE^2}$$

$$DE^2 = \frac{45 \times 25}{20} = \frac{225}{4}$$

$$DE = \sqrt{\frac{225}{4}}$$

$$= \frac{15}{2} = 7.5 \text{ cm}$$

177.(c)



AB = AC tangents drawn from the same point equal

$$OB = OC = 3 \text{ cm}$$

$$OA = 12 \text{ cm}$$

$$\angle ABO = \angle ACO = 90^\circ$$

In Right ΔABO

$$AB = \sqrt{12^2 - 3^2} = \sqrt{135}$$

$$= \sqrt{15 \times 9} = 3\sqrt{15}$$

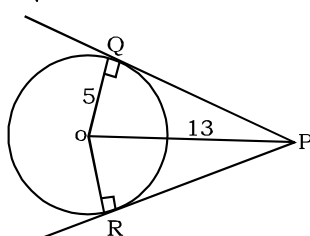
$$\text{ar}(\Delta ABOC) = 2 \times \text{ar}(\Delta ABO)$$

$$= 2 \times \frac{1}{2} \times AB \times OB$$

$$= 3\sqrt{15} \times 3$$

$$= 9\sqrt{15} \text{ cm}^2$$

178.(b)



$$\angle OQP = \angle ORP = 90^\circ$$

(radius is $\perp$ tangent)

and PQ = PR (tangent drawn from same point are equal)

$$PQ = \sqrt{OP^2 - OQ^2} = \sqrt{13^2 - 5^2}$$

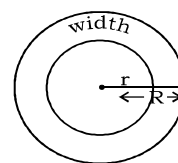
$$= 12$$

$$\text{ar}(\Delta PQOR) = 2 \times \text{ar}(\Delta PQO)$$

$$= 2 \times \frac{1}{2} \times PQ \times OQ$$

$$= 12 \times 5 = 60 \text{ cm}^2$$

179.(a)



Let radius of outer circle = R

and radius of inner circle = r

$$\text{ATQ } 2\pi R - 2\pi r = 66$$

$$2\pi(R - r) = 66$$

$$R - r = \frac{66}{2\pi} = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$$

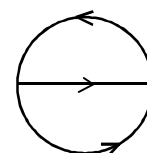
$$\text{width} = 10.5 \text{ m}$$

180.(d) Distance covered in 30 seconds

$$= 30 \text{ m/min} \times \frac{30}{60} = 15 \text{ m}$$

This is the difference of distance of the boundary and the diameter

Let 'R' be the radius



$$2\pi R - 2R = 15$$

$$2R(\pi - 1) = 15$$

$$2R = \frac{15}{\pi - 1} = \frac{15}{\frac{22}{7} - 1} = \frac{15 \times 7}{15} = 7$$

$$R = \frac{7}{2} = 3.5 \text{ m}$$

181.(a) Perimeter of the circle = circumference of circle

Let 'R' be the radius

$$\text{ATQ } 2\pi R - 2R = X$$

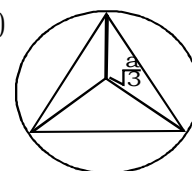
$$2R(\pi - 1) = X$$

$$2R = \frac{X}{\pi - 1}$$

$$\text{Diameter} = \frac{X}{\pi - 1}$$

∴ 2R = diameter of the circle

182.(b)



Let the side of an equilateral triangle = 'a'

$$\therefore \text{Circumcircle radius} = \frac{a}{\sqrt{3}}$$

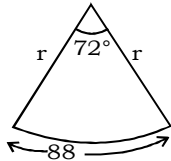
$$\text{area of circumcircle} = \pi \left(\frac{a}{\sqrt{3}} \right)^2$$

$$\frac{\pi a^2}{3} = 3\pi$$

$$a^2 = 9, \quad a = 3$$

$$\text{perimeter} = 3 \times a = 3 \times 3 = 9 \text{ cm}$$

183. (a)

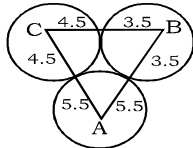


$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$

$$\frac{72}{360} \times 2 \times \frac{22}{7} \times r = 88$$

$$r = \frac{88 \times 7 \times 360}{72 \times 2 \times 22} = 70 \text{ m}$$

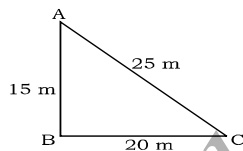
184. (a)



In $\triangle ABC$

$$\begin{aligned} \text{perimeter of } \triangle ABC &= (AB + BC + AC) \\ &= 2(3.5 + 4.5 + 5.5) \\ &= (13.5) \times 2 = 27 \end{aligned}$$

185. (c)



$\therefore 15, 20, 25$ form a triplet

Clearly, $25^2 = 15^2 + 20^2$

ABC is a right triangle

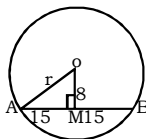
Area of Right $\triangle ABC$

$$= \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2} \times 15 \times 20 = 150$$

$$\begin{aligned} \text{Cost of sowing seeds} \\ &= 150 \times ₹ 5 = ₹ 750 \end{aligned}$$

186. (a)



$$AB = 30 \text{ cm}$$

$$OM \perp AB \text{ and } OM = 8$$

$$\therefore AM = BM = 15 \text{ cm}$$

In Right $\triangle OMA$

$$OA^2 = OM^2 + AM^2$$

$$OA^2 = 15^2 + 8^2$$

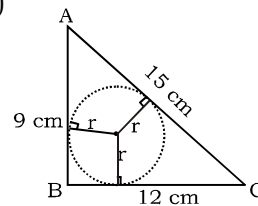
$$OA^2 = 289$$

$$OA = \sqrt{289}$$

$$OA = 17 \text{ cm}$$

$$\text{Radius of circle} = 17 \text{ cm}$$

187. (c)



Since, 9, 12, 15 forms a triplet

$$\begin{aligned} \text{area of } \triangle ABC &= \frac{1}{2} \times 9 \times 12 \\ &= 54 \text{ cm}^2 \end{aligned}$$

In-circle radius of triangle

$$= \frac{\text{area of triangle}}{\text{semiperimeter of triangle}}$$

$$= \frac{54}{\frac{9+12+15}{2}} = \frac{54 \times 2}{36} = 3 \text{ cm}$$

Alternate:

In a right triangle, with, P, B and H incircle radius

$$= \frac{P+B-H}{2}$$

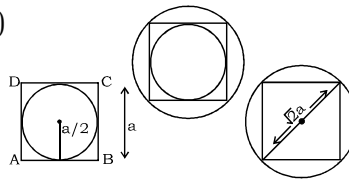
$$\text{Hence, } r = \frac{9+12-15}{2}$$

$$= \frac{6}{2} = 3 \text{ cm}$$

Also Circumcircle radius

$$= \frac{H}{2} = \frac{15}{2} = 7.5 \text{ cm}$$

188. (a)



Let the side of square = a

$$\text{In circle radius of square} = \frac{a}{2}$$

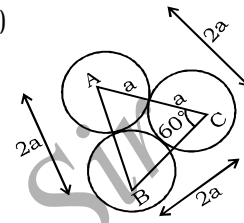
Circumcircle radius of square

$$= \frac{\text{Diagonal}}{2} = \frac{a\sqrt{2}}{2}$$

$$\therefore \frac{\text{Incircle radius}}{\text{Circumcircle radius}} = \frac{a}{\frac{a\sqrt{2}}{2}}$$

$$= \frac{1}{\sqrt{2}} = 1:\sqrt{2}$$

189. (d)



Hence

ABC is an equilateral triangle

$$AB = BC = AC = '2a' \text{ cm}$$

$$\text{area of } \triangle ABC = \frac{\sqrt{3}}{4} (2a)^2$$

$$= \frac{\sqrt{3}}{4} \times 4a^2 = \sqrt{3} a^2$$

area of 3 sectors ($\theta = 60^\circ$)

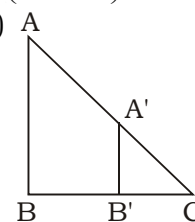
$$= 3 \times \frac{60^\circ}{360^\circ} \times \pi a^2 = \frac{\pi a^2}{2}$$

area of shaded region = area of $\triangle ABC$ - area of 3 sector

$$= \sqrt{3} a^2 - \frac{\pi a^2}{2}$$

$$= \left(\frac{2\sqrt{3} - \pi}{2} \right) a^2 \text{ cm}^2$$

190. (c)



In $\triangle ABC$ and $\triangle A'B'C$

$$\angle C = \angle C \text{ (common)}$$

$$\angle B' = \angle B (\because AB \parallel A'B')$$

$$\Rightarrow \triangle ABC \sim \triangle A'B'C$$

$$\Rightarrow \frac{\text{area of } \triangle A'B'C}{\text{area of } \triangle ABC} = \left(\frac{B'C}{BC} \right)^2$$

$$= \left(\frac{1}{2} \right)^2 = \frac{1}{4}$$

$$\Rightarrow \text{ar } \triangle A'B'C = \frac{1}{4} (\text{area } \triangle ABC)$$

191. (b) Perimeter of square = 44 cm

$$\text{Area of square} = \left(\frac{44}{4} \right)^2$$

$$= 121 \text{ cm}^2$$

Circumference of circle

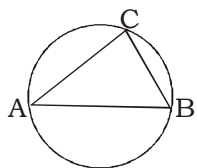
$$= 2\pi r = 44 \text{ cm}$$

$$r = \frac{22 \times 7}{22} = 7 \text{ cm}$$

$$\Rightarrow \text{area of circle} = \pi r^2 = \frac{22}{7} \times (7)^2 = 154 \text{ cm}^2$$

$$\text{Required difference} = 154 - 121 = 33 \text{ cm}^2$$

192. (d)



$\angle ACB = 90^\circ$ (angle in semi-circle)

$$AC : BC = 3 : 4$$

$$AB^2 = \sqrt{AC^2 + BC^2} = \sqrt{3^2 + 4^2}$$

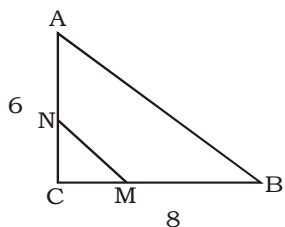
$$= 5 \text{ units}$$

$$5 \text{ units} = 5 \text{ cm}$$

$$\therefore \text{ar } \triangle ABC = \frac{1}{2} \times 3 \times 4$$

$$= 6 \text{ cm}^2$$

193. (a)



$$\text{Ar}(\triangle ABC) = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$$

$$\triangle ABC \sim \triangle MCN$$

$$\angle C = \angle C$$

$$\angle M = \angle B \quad (\because MN \parallel AB)$$

$$\therefore \frac{\text{ar}(\triangle MCN)}{\text{ar}(\triangle ABC)} = \left(\frac{CM}{BC}\right)^2$$

$$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

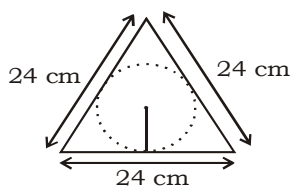
$$\text{ar}(\square MNAB)$$

$$= \text{ar}(\triangle ABC) - \text{ar}(\triangle MCN)$$

$$= 4 - 1 = 3$$

$$\therefore \text{ar}(\square MNAB) = \frac{24}{4} \times 3 = 18 \text{ cm}^2$$

194. (a)



Area of equilateral triangle

$$= \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times 24 \times 24$$

$$= \sqrt{3} \times 6 \times 24$$

$$= 144\sqrt{3} \text{ cm}^2$$

$$= 144 \times 1.732$$

$$= 249.408 \text{ cm}^2$$

Inradius of an equilateral

$$\text{triangle} = \left(\frac{\text{side}}{2\sqrt{3}}\right)$$

$$= \frac{24}{2\sqrt{3}} = 4\sqrt{3} \text{ cm}$$

Now, Area of incircle

$$= \frac{22}{7} \times (\text{Inradius})^2$$

$$= \frac{22}{7} \times 4\sqrt{3} \times 4\sqrt{3}$$

$$= \frac{22 \times 16 \times 3}{7} = \frac{1056}{7}$$

$$= 150.86 \text{ cm}^2$$

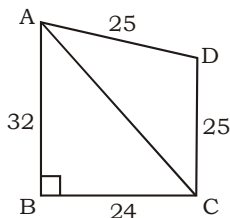
Area of remaining part = area

of $\triangle$ - area of incircle

$$= 249.408 - 150.86$$

$$= 98.548 \text{ cm}^2$$

195. (d)



$$\angle ABC = 90^\circ$$

$$AC = \sqrt{AB^2 + BC^2}$$

$$= \sqrt{32^2 + 24^2}$$

$$= \sqrt{1024 + 576}$$

$$= \sqrt{1600} = 40 \text{ m}$$

Now, area of $\triangle ABC$

$$= \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2} \times 32 \times 24 = 384 \text{ cm}^2$$

Now, In $\triangle ADC$,

$$s = \frac{25 + 25 + 40}{2} = 45 \text{ m}$$

area of $\triangle ADC$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{45(45-25)(45-25)(45-40)}$$

$$= \sqrt{45 \times 20 \times 20 \times 5}$$

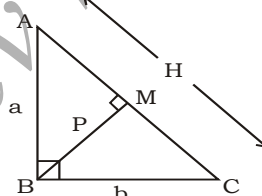
$$= 20 \times 3 \times 5$$

$$= 300 \text{ m}^2$$

Area of the plot

$$= 384 + 300 = 684 \text{ m}^2$$

196. (c)



Length of perpendicular drawn from the right angle

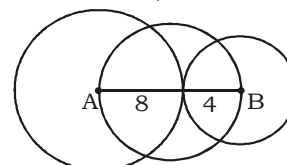
$$\text{hypotenuse, } P = \frac{a \times b}{H}$$

$$P^2 = \frac{a^2 b^2}{H^2}$$

$$P^2 = \frac{a^2 b^2}{a^2 + b^2}$$

$$(\because H^2 = a^2 + b^2)$$

197. (a)



Diameter of the circle AB

$$= 8 + 4 = 12 \text{ units}$$

$$\text{Radius} = \frac{12}{2} = 6 \text{ units}$$

$$\therefore \text{Area of circle} = \pi r^2 = \pi \times (6)^2 = 36\pi \text{ sq. units}$$

$$198. (a) \quad \frac{\sqrt{3}}{2}(\text{side}) = \frac{\sqrt{3}}{4}(\text{side})^2$$

$$\text{side} = 2 \text{ units}$$

199. (a) Let the length of side of square = a

then the diameter of circle = d

According to question,

$$a = d$$

$$\therefore \frac{\text{area of square}}{\text{area of circle}} = \frac{a^2}{\pi \left(\frac{d^2}{4}\right)}$$

$$= \frac{a^2 \times 4}{\pi d^2} = \frac{a^2 \times 4}{\pi a^2}$$

$$= \frac{4}{\pi} = \frac{4 \times 7}{22} = \frac{14}{11}$$

$$\Rightarrow 14 : 11$$

200.(d) Length of median of an

$$\text{equilateral triangle} = \frac{\sqrt{3}}{2}(\text{side})$$

Length of median, altitude, and angle bisector is

$$= \frac{\sqrt{3}}{2}(\text{side})$$

$$\therefore \frac{\sqrt{3}}{2}a = 6\sqrt{3}$$

$$a = \frac{6\sqrt{3} \times 2}{\sqrt{3}} = 12 \text{ cm}$$

$$\therefore \text{area of } \Delta ABC = \frac{\sqrt{3}}{4}a^2$$

$$= \frac{\sqrt{3}}{4} \times 12 \times 12$$

$$= \sqrt{3} \times 3 \times 12 = 36\sqrt{3} \text{ cm}^2$$

201.(b) $\pi r^2 = 2\pi r$

$$r = 2 \text{ units}$$

$$\therefore \text{Area of circle} = \pi (2)^2$$

$$= 4\pi \text{ sq. units}$$

202.(d) Area of equilateral triangle

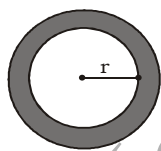
$$= \frac{\sqrt{3}}{4}(\text{side})^2$$

$$\frac{\sqrt{3}}{4}(\text{side})^2 = 48$$

$$(\text{side})^2 = \frac{48 \times 4}{\sqrt{3}} = 64\sqrt{3}$$

$$\text{side} = \left(64(3)^{\frac{1}{2}}\right)^{\frac{1}{2}} = 8(3)^{\frac{1}{4}}$$

203.(b)



$$2\pi R - 2\pi r = 33$$

$$(R - r) = \frac{33}{2\pi} = \frac{33 \times 7}{2 \times 22}$$

$$= \frac{3 \times 7}{2 \times 2} = \frac{21}{4}$$

$$\text{thickness} = 5.25 \text{ m}$$

204.(b) Ratio = 5 : 6 : 7

$$\text{sum of sides} = \text{perimeter} = 18$$

sides,

$$\frac{5}{18} \times 54 = 15 \quad \frac{6}{18} \times 54 = 18$$

$$\frac{7}{18} \times 54 = 21 \text{ metres}$$

$$S = \frac{15+18+21}{2} = 27$$

$$\therefore \text{Area of } \Delta$$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{27 \times 12 \times 9 \times 6}$$

$$= 54\sqrt{6} \text{ m}^2$$

205.(a) circumference of circle

$$= \pi \times \text{diameter}$$

$$= \frac{22}{7} \times 112 = 352 \text{ cm}$$

$$\therefore \text{perimeter of rectangle} = 352$$

$$2(l + b) = 352$$

$$l + b = \frac{352}{2} = 176$$

$$\therefore \text{smaller side} = \frac{7}{16} \times 176$$

$$= 77 \text{ cm}$$

206.(c) perimeter of equilateral

$$\text{triangle} = 18 \text{ cm}$$

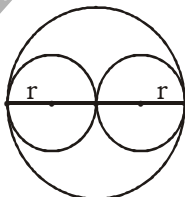
$$3 \times \text{side} = 18 \text{ cm}$$

$$\text{side} = \frac{18}{3} = 6 \text{ cm}$$

$$\text{length of median} = \frac{\sqrt{3}}{2} \text{ side}$$

$$= \frac{\sqrt{3}}{2} \times 6 = 3\sqrt{3} \text{ cm}$$

207.(a)



$$\text{Circumference of paper sheet}$$

$$= 352$$

$$2\pi R = 352$$

$$R = \frac{352}{2\pi} = \frac{352 \times 7}{2 \times 22} = 56 \text{ cm}$$

$$r = \frac{R}{2} = \frac{56}{2} = 28 \text{ cm}$$

$$\therefore \text{circumference of circular plate} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 28$$

$$= 176 \text{ cm}$$

208.(a) Inradius of equilateral

$$\text{triangle} = \frac{\text{side}}{2\sqrt{3}}$$

$$\sqrt{3} = \frac{\text{side}}{2\sqrt{3}}$$

$$\text{side} = 6 \text{ cm}$$

$$\text{perimeter of equilateral triangle}$$

$$= 3 \times 6 = 18 \text{ cm}$$

209.(b) Circumference of circle = πd

$$\therefore \pi d - d = 150$$

$$d(\pi - 1) = 150$$

$$d\left(\frac{22}{7} - 1\right) = 150$$

$$d \times \frac{15}{7} = 150$$

$$d = \frac{150 \times 7}{15} = 70$$

$$\text{Radius} = \frac{d}{2} = \frac{70}{2} = 35 \text{ m}$$

210.(b) Let radius of circle = r

$$\text{Side of square} = a$$

$$\text{Side of equilateral } \Delta = b$$

According to question,

$$2\pi R = 4a = 3b$$

$$\therefore a = \frac{\pi R}{2} \quad b = \frac{2}{3} \pi R$$

Ratio of their areas:

$$\pi R^2 : a^2 : \frac{\sqrt{3}}{4} b^2$$

$$\pi R^2 : \left(\frac{\pi R}{2}\right)^2 : \frac{\sqrt{3}}{4} \left(\frac{2}{3} \pi R\right)^2$$

$$1 : \frac{\pi}{4} : \frac{\sqrt{3}}{9} \pi$$

$$C : S : T$$

Here, we can see that $C > S > T$

Quicker Approach : When perimeter of two or more figures are same then the figure who has more vertex is greater in the area. Since, circle has infinite vertex.

Therefore, $C > S > T$

211.(d) Distance covered in 1

$$\text{revolution} = \text{Circumference of circular field} = 2\pi r$$

$$\text{Distance} = \text{speed} \times \text{time}$$

$$= 66 \text{ m/s} \times \frac{5}{2} \text{ s} = 165 \text{ m}$$

$$\therefore 2\pi r = 165$$

$$2 \times \frac{22}{7} \times r = 165$$

$$r = \frac{165 \times 7}{2 \times 22}$$

$$= 26.25 \text{ m.}$$

- 212.(c) Circumference of front wheel $\times$ no. of its revolutions = circumference of rear wheel $\times$ no. of its revolutions
 $2\pi x \times n = 2\pi y \times m$ (let 'm' is the revolution of rear wheel)

$$m = \frac{nx}{y}$$

- 213.(b) Distance to be covered in one revolution = Circumference of wheel
 $= \pi \times \text{diameter}$

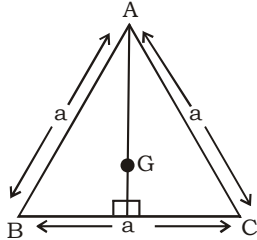
$$= \frac{22}{7} \times 56 = 176 \text{ cm}$$

$$\begin{aligned} \text{Total distance} &= 2.2 \text{ km} \\ &= (2.2 \times 1000 \times 100) \text{ cm} \\ &= 22,0000 \text{ cm} \end{aligned}$$

$\therefore$ Number of revolutions

$$= \frac{220000}{176} = 1250$$

- 214.(b)



We know that in an equilateral triangle a median also be a altitude

$\Rightarrow$ Altitude of an equilateral triangle

$$= \frac{\sqrt{3}}{2} a$$

$$\Rightarrow \frac{\sqrt{3}}{2} a = 12\sqrt{3} \text{ (given)}$$

$$\Rightarrow a = 24 \text{ cm}$$

$\Rightarrow$ Then area of an equilateral triangle

$$\begin{aligned} \text{angle} &= \frac{\sqrt{3}}{4} a^2 \\ &= \frac{\sqrt{3}}{4} \times 24 \times 24 \\ &= 144\sqrt{3} \text{ cm}^2 \end{aligned}$$

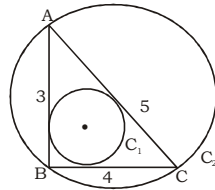
- 215.(d) Let a triangle ABC has sides of measurement 3 cm, 4cm and 5 cm using triplets (3, 4, 5)

$\Rightarrow \Delta ABC$ will be a right angled triangle

$\Rightarrow$ Inner radius of circle C_1

$$= \frac{AB + BC - CA}{2} = \frac{4+3-5}{2}$$

$$r = 1 \text{ cm}$$



$\Rightarrow$ Circum-radius of circle C_2

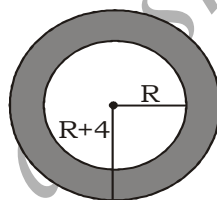
$$R = \frac{\text{Hypotenuse}}{2}$$

In a right angled triangle half of hypotenuse is circum radius

$$R = \frac{5}{2} = 2.5 \text{ cm}$$

$$\begin{aligned} \Rightarrow \frac{\text{Area of } C_1}{\text{Area of } C_2} &= \frac{\pi r^2}{\pi R^2} \\ &= \frac{1^2}{\left(\frac{5}{2}\right)^2} = \frac{4}{25} \end{aligned}$$

- 216.(d) Let the radius of Swimming Pool = R



Outer radius of Pool with concrete wall = $(R + 4)$

According to question

$$\pi R^2 \times \frac{11}{25} = \pi (R + 4)^2 - \pi R^2$$

$$R^2 \times \frac{11}{25} = R^2 + 16 + 8R - R^2$$

$$\frac{11}{25} R^2 = 16 + 8R$$

$$11R^2 - 200R - 400 = 0$$

By option (d), (In such type of equation go through the option to save your valuable time)

$$R = 20$$

$$11 \times (20)^2 - 200 \times 20 - 400 = 0$$

$$4400 - 4000 - 400 = 0$$

$$0 = 0 \text{ (satisfy)}$$

Radius of pool $R = 20 \text{ cm}$

- 217.(a) Area of circle = A

Radius of circle = r

Circumference of circle = c

$$\pi r^2 = A \quad (i)$$

$$2\pi r = c \quad (ii)$$

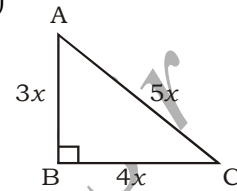
From (i) $\div$ (ii)

$$\frac{\pi r^2}{2\pi r} = \frac{A}{C}$$

$$\frac{r}{2} = \frac{A}{C}$$

$$rc = 2A$$

- 218.(d)



Area of right angled triangle = 7776

$$\Rightarrow \frac{1}{2} \times 4x \times 3x = 7776$$

$$\Rightarrow 6x^2 = 7776$$

$$\Rightarrow x^2 = 1296$$

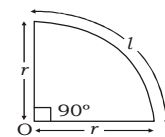
$$\Rightarrow x = 36$$

$\Rightarrow$ Perimeter of triangle

$$= 3x + 4x + 5x = 12x$$

$$= 12 \times 36 = 432 \text{ cm}$$

- 219.(b)



According to the figure,

$\Rightarrow$ Perimeter = $r + r + l$

$$\Rightarrow 75 \text{ cm} = 2r + \text{length of arc}$$

$$\Rightarrow 75 \text{ cm} = 2r + \frac{2\pi r}{4}$$

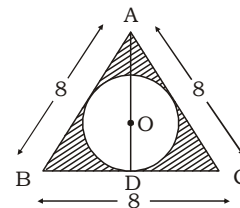
$$\Rightarrow 75 \text{ cm} = 2r + \frac{22 \times r}{7 \times 2}$$

$$\Rightarrow r = 21 \text{ cm.}$$

$\Rightarrow$ Its area

$$\begin{aligned} &= \frac{1}{4} \left[\frac{22}{7} \times 21 \times 21 \right] \\ &= 346.5 \text{ cm}^2 \end{aligned}$$

- 220.(b)



According to the question,

Here OD = radius,

$$\therefore r = \frac{a}{2\sqrt{3}} = \frac{8}{2\sqrt{3}}$$

$$r = \frac{4}{\sqrt{3}}$$

Required area of shaded portion

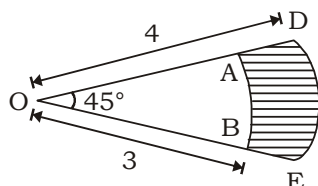
$$= \frac{\sqrt{3}}{4} \times (8)^2 - \pi \left(\frac{4}{\sqrt{3}} \right)^2$$

$$= \frac{\sqrt{3}}{4} \times 64 - \pi \times \frac{16}{3}$$

$$= \sqrt{3} \times 16 - \frac{22}{7} \times \frac{16}{3}$$

$$= 10.95 \text{ m}^2 = 11 \text{ m}^2$$

221. (d)



According to the question,

$$\text{Area of sector OED} = \pi r^2 \times \frac{\theta}{360}$$

$$= \pi \times 4 \times 4 \times \frac{45}{360} = 2\pi \text{ m}^2$$

Area of the sector OAB

$$= \pi r^2 \times \frac{\theta}{360}$$

$$= \pi \times 3 \times 3 \times \frac{45}{360}$$

$$= \frac{9}{8} \pi \text{ m}^2$$

So, Area of shaded portion = Area of OED - Area of OAB

$$= 2\pi - \frac{9}{8} \pi = \frac{16\pi - 9\pi}{8}$$

$$= \frac{7}{8} \pi = \frac{7}{8} \times \frac{22}{7} = \frac{11}{4} \text{ m}^2$$

222. (d) According to the question,

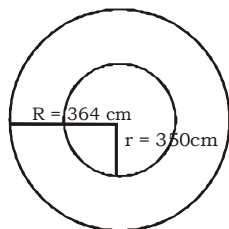
Circumference of a circle = $2\pi r$

$$2\pi r = \frac{30}{\pi}$$

$$r = \frac{15}{\pi^2}$$

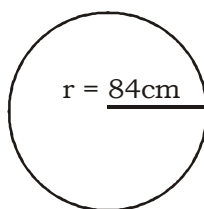
$$D = 2r = \frac{30}{\pi^2}$$

223. (b) According to the question



The breadth of the path = $(R - r)$
 $= (364 - 350) \text{ cm} = 14 \text{ cm}$

224. (c) According to the questions,



$a = ?$

Let the length of side of the square be a cm

(circumference of circle = perimeter square)

$$2\pi r = 4a$$

$$2 \times \frac{22}{7} \times 84 = 4a$$

$$132 \text{ cm} = a$$

225. (a) Area of circle = $324 \pi \text{ cm}^2$

$$\pi r^2 = 324 \pi$$

$$r = 18 \text{ cm}$$

Longest chord = diameter = $2r$

$$= 2 \times 18 = 36 \text{ cm}$$

226. (c) Circumference of a $\Delta = 24 \text{ cm}$

$$a + b + c = 24 \text{ cm}$$

$$\text{or } S = \frac{a+b+c}{2} = 12 \text{ cm}$$

Circumference of incircle

$$2\pi r \text{ (inner)} = 44 \text{ cm}$$

$$r \text{ (inner)} = 7 \text{ cm}$$

Area of $\Delta = S \times r \text{ (inner)}$

$$= 12 \times 7 = 84 \text{ cm}^2$$

227. (b) Area of $\Delta = \frac{1}{2} ab \sin \theta$

$$= \frac{1}{2} \times 10 \times 10 \times \sin 45^\circ$$

$$= 25\sqrt{2} \text{ cm}^2$$

228. (a) According to the question

$$r = \frac{\Delta}{S}$$

$$\text{semiperimeter} = \frac{50}{2} = 25$$

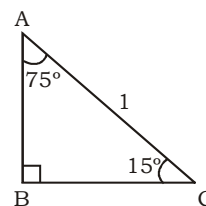
inner radius

$$= \frac{\text{Area}}{\text{Semi-perimeter}}$$

$$6 = \frac{\text{Area}}{25}$$

$$\text{Area} = 150 \text{ cm}^2$$

229. (b) According to the question,



$$\sin 15^\circ = \frac{P}{H} = \frac{AB}{1}$$

$$AB = \sin 15^\circ$$

$$\cos 15^\circ = \frac{B}{H} = \frac{BC}{1}$$

$$BC = \cos 15^\circ$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2} \times \sin 15^\circ \cos 15^\circ$$

$$= \frac{1}{4} \times \sin 2 \times 15$$

$$[\because \sin 2\theta = 2 \sin \theta \cos \theta]$$

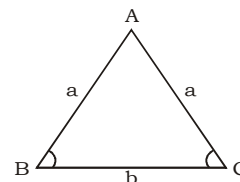
$$= \frac{1}{4} \times \sin 30^\circ$$

$$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \text{ m}^2$$

$$= \frac{1}{8} \times 100 \times 100$$

$$= 1250 \text{ cm}^2$$

230. (b) According to the question,



Let $AB = AC = a$

$BC = b$.

$$\therefore S = \frac{a+a+b}{2}$$

$$S = a + \frac{b}{2}$$

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$$

Area =

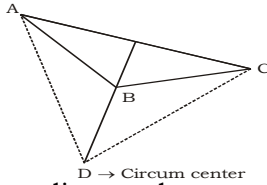
$$\sqrt{\left(a + \frac{b}{2}\right) \left(a + \frac{b}{2} - a\right) \left(a + \frac{b}{2} - a\right) \left(a + \frac{b}{2} - b\right)}$$

$$\text{Area} = \sqrt{\left(a + \frac{b}{2}\right) \left(\frac{b}{2}\right) \left(\frac{b}{2}\right) \left(a - \frac{b}{2}\right)}$$

$$\text{Area} = \frac{b}{2} \sqrt{a^2 - \frac{b^2}{4}}$$

$$\text{Area} = \frac{b}{4} \sqrt{4a^2 - b^2} \text{ sq. units.}$$

- 231.(c) As we know circumcentre always made by the intersection of half altitude
 $\Rightarrow$ In obtuse angle it will always be out.



- 232.(a) According to the question,

$$2\pi r \rightarrow \text{circumference}$$

$$2r \rightarrow \text{Diameter}$$

$$\Rightarrow \frac{2\pi r}{2r} = \frac{22}{7}$$

$$\Rightarrow \frac{1}{7} = \frac{22}{7}$$

$$\Rightarrow \frac{11}{7 \times 2r} = \frac{22}{7}$$

$$\Rightarrow \frac{1}{2r} = \frac{2}{1}$$

$$\Rightarrow R = \frac{1}{4} \text{ m}$$

- 233.(b) Given:

$$\Rightarrow \text{Area of square} = 4$$

$$\text{side}^2 = 4$$

$$\text{side} = 2$$

$$\Rightarrow \text{Diagonal of square} = \text{radius of circle}$$

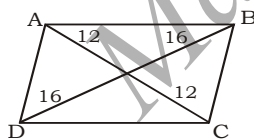
$$\sqrt{2} \text{ side} = r$$

$$\Rightarrow r = 2\sqrt{2}$$

$$\Rightarrow \text{Area of circle} = \pi r^2$$

$$\Rightarrow \pi \times (2\sqrt{2})^2 = 8\pi \text{ cm}^2$$

- 234.(a) We know that rhombus is parallelogram whose all four sides are equal and its diagonals bisect each other at 90° .



$$\therefore AB = \sqrt{16^2 + 12^2}$$

$$= \sqrt{400} = 20 \text{ cm}$$

$$= \text{side of rhombus}$$

$$\therefore \text{perimeter of the rhombus} = 20 \times 4 = 80 \text{ cm}$$

- 235.(d) If d_1 and d_2 are the lengths of diagonals of rhombus.

Then,

$$\text{Perimeter} = 2\sqrt{d_1^2 + d_2^2}$$

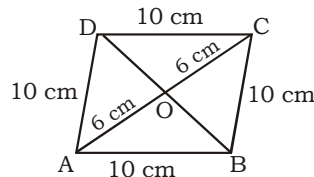
$$= 2\sqrt{24^2 + 10^2}$$

$$= 2\sqrt{676}$$

$$= 2 \times 26 = 52 \text{ cm}$$

$$236.(c) 4 \times \text{side} = 40 \text{ cm (given)}$$

$$\Rightarrow \text{side} = \frac{40}{4} = 10 \text{ cm}$$



In $\triangle AOB$,

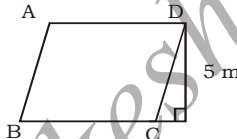
$$OB = \sqrt{10^2 - 6^2}$$

$$= \sqrt{100 - 36}$$

$$= \sqrt{64} = 8 \text{ cm}$$

$$\text{Diagonal BD} = 8 \times 2 = 16 \text{ cm.}$$

- 237.(b)



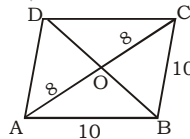
$$4 \times \text{side of rhombus} = 40 \text{ m}$$

$$\text{side of rhombus} = 10 \text{ m}$$

$$\text{Since rhombus is also a parallelogram therefore its area} = \text{base} \times \text{height}$$

$$= 10 \times 5 = 50 \text{ m}^2$$

- 238.(d)



$$\text{Perimeter of Rhombus} = 40 \text{ cm}$$

$$4 \times \text{side} = 40$$

$$\text{side} = 10 \text{ cm}$$

We know that diagonals of rhombus bisect each other at right angle,

Therefore In right $\triangle OAB$

$$OB^2 = AB^2 - OA^2$$

$$= 10^2 - 8^2 = 100 - 64 = 36$$

$$OB = \sqrt{36} = 6 \text{ cm}$$

$$\text{Diagonal BD} = 2 \times OB = 2 \times 6$$

$$= 12 \text{ cm}$$

Alternative

$$\text{Side of rhombus} = \frac{1}{2} \sqrt{d_1^2 + d_2^2}$$

$$10 = \frac{1}{2} \sqrt{16^2 + d^2}$$

$$20 = \sqrt{256 + d_2^2}$$

$$256 + d_2^2 = 400$$

$$d_2^2 = 400 - 256 = 144$$

$$d_2 = \sqrt{144} = 12 \text{ cm}$$

- 239.(b) Diagonal (d_1) = 10 cm
 area of Rhombus = 150 cm^2

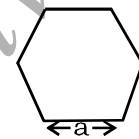
$$\frac{1}{2} \times d_1 \times d_2 = 150$$

$$\frac{1}{2} \times 10 \times d_2 = 150$$

$$d_2 = \frac{150 \times 2}{10}$$

$$= 30 \text{ cm}$$

- 240.(a)



A regular hexagon consists of 6 equilateral triangle area of regular hexagon

$$= 6 \times \frac{\sqrt{3}}{4} (\text{side})^2$$

$$= 6 \times \frac{\sqrt{3}}{4} a^2$$

$$= 6 \times \frac{\sqrt{3}}{4} \times (2\sqrt{3})^2$$

$$= 6 \times \frac{\sqrt{3}}{4} \times 12 = 18\sqrt{3} \text{ cm}^2$$

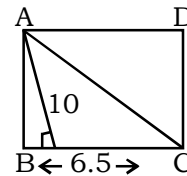
- 241.(a) area of hexagon

$$= 6 \times \frac{\sqrt{3}}{4} (\text{side})^2$$

$$= 6 \times \frac{\sqrt{3}}{4} (1)^2$$

$$= 6 \times \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{2} \text{ cm}^2$$

- 242.(a)



($\therefore$ Rhombus is a ||gm

$\therefore$ area of Rhombus = base $\times$ height)

area of Rhombus

$$= \text{base} \times \text{height}$$

$$= 6.5 \times 10 = 65 \text{ cm}^2$$

Also area of Rhombus

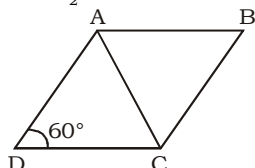
$$= \frac{1}{2} \times d_1 \times d_2$$

$$\frac{1}{2} \times 26 \times d_2 = 65$$

$$13 \times d_2 = 65$$

$$d_2 = 5 \text{ cm}$$

243. (a)



In the above figure $\triangle ADC$ is an equilateral triangle (as AC is angle bisector)

$\Rightarrow AC = 10 \text{ cm}$ (smaller diagonal)

244. (c) Side of rhombus

$$= \frac{100}{4} = 25 \text{ cm}$$

we know that in a rhombus $4a^2$

$$= d_1^2 + d_2^2$$

$$\Rightarrow d_2^2 = 4 \times (25)^2 - (14)^2 = 2500 - 196 = 2304$$

$$\Rightarrow d_2 = \sqrt{2304} = 48 \text{ cm}$$

$$\Rightarrow \text{Area} = \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times 14 \times 48 = 336 \text{ cm}^2$$

245. (d) Let the parallel sides be $3x$ and $2x$

$$\Rightarrow \frac{1}{2} (3x + 2x) \times 15 = 450$$

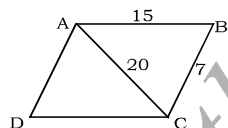
$$\Rightarrow 5x = 60$$

$$x = 12$$

$\Rightarrow$ Sum of length of parallel sides

$$= (3 + 2) \times 12 = 60 \text{ cm}$$

246. (c)



Using Hero's formula

$$S = \frac{15+7+20}{2} = 21 \text{ cm}$$

Area of $\triangle ABC$

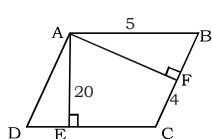
$$= \sqrt{21(21-15)(21-7)(21-20)}$$

$$= \sqrt{21 \times 6 \times 14 \times 1} = 42 \text{ cm}^2$$

$\Rightarrow$ Area of $\square ABCD$

$$= 42 \times 2 = 84 \text{ cm}^2$$

247. (b)



Area of parallelogram $= AB \times AE$

$$5x \times 20 = 1000$$

$$x = 10$$

Area of parallelogram $= AD \times AF$

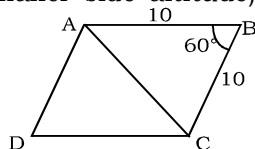
$$1000 = 4x \times AF$$

$$1000 = 4 \times 10 \times AF$$

$$AF = 25 \text{ cm}$$

(smaller side altitude)

248. (b)



as $\square ABCD$ is a rhombus

$\therefore \triangle ABC$ is an equilateral $\triangle$

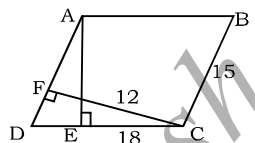
$$\Rightarrow \text{ar}(\triangle ABC) = \frac{\sqrt{3}}{4} \times 10 \times 10$$

$$= 25\sqrt{3} \text{ cm}^2$$

$$\Rightarrow \text{ar}(\square ABCD) = 25\sqrt{3} \times 2$$

$$= 50\sqrt{3} \text{ cm}^2$$

249. (b)



Area of parallelogram

$$= AD \times FC = 15 \times 12$$

$$= 180 \text{ cm}^2$$

Area of parallelogram

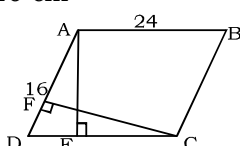
$$= DC \times AE = 180$$

$$18 \times AE = 180$$

$$AE = 10 \text{ cm}$$

$\therefore$ Distance between bigger sides $= 10 \text{ cm}$

250. (a)



$$AB = 24 \text{ cm}$$

$$AD = 16 \text{ cm}$$

$$AE = 10 \text{ cm}$$

Area of Parallelogram

$$= AE \times DC = 10 \times 24$$

$$= 240 \text{ cm}^2$$

also, area of Parallelogram

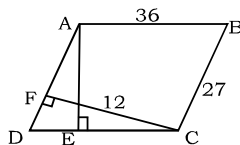
$$= FC \times AD = 240$$

$$FC \times 16 = 240$$

$$FC = 15$$

$\therefore$ Distance between AD and BC $= 15 \text{ cm}$

251. (d)



Area of parallelogram

$$= AE \times DC = CF \times AD$$

$$AE \times 36 = 12 \times 27$$

$$= AE = 9 \text{ cm}$$

$\therefore$ Distance between bigger sides $= 9 \text{ cm}$

252. (a) In a rhombus

$$4a^2 = d_1^2 + d_2^2$$

$$4a^2 = 8^2 + 6^2$$

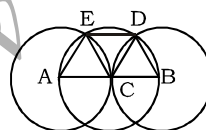
$$a^2 = \frac{100}{4} = 25$$

$$a = 5$$

$\Rightarrow$ Side of square $= 5 \text{ cm}$

$\therefore$ Area of square $= 25 \text{ cm}^2$

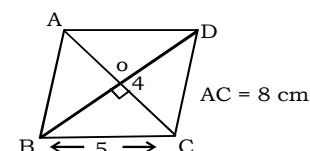
253. (b)



Area ($\square ABDE$) $= 3 \times \text{ar}(\triangle ADC)$
(ADC is an equilateral triangle)

$$= 3 \times \frac{\sqrt{3}}{4} \times 2^2 = 3\sqrt{3} \text{ unit}^2$$

254. (d) side of rhombus $= \frac{20}{4} = 5 \text{ cm}$



$$OC = 4 \text{ cm}$$

In Right $\triangle OBC$

$$OB^2 = BC^2 - OC^2$$

$$= 5^2 - 4^2 = 9$$

$$OB = \sqrt{9} = 3 \text{ cm}$$

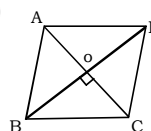
$$BD = 2 \times OB = 2 \times 3 = 6 \text{ cm}$$

area of Rhombus

$$= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

Note: In the question do not get confused with the words non-square its simply to clear that it is Rhombus

255. (c)



side of Rhombus $= \frac{100}{4} = 25 \text{ cm}$

$$BD = 40 \text{ cm}$$

$$OB = 20 \text{ cm}$$

In right $\triangle OBC$

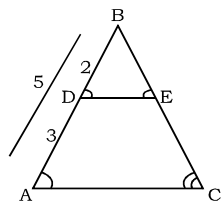
$$OC^2 = BC^2 - OB^2$$

$$OC = \sqrt{25^2 - 20^2} = 15 \text{ cm}$$

$$\therefore AC = 2 \times OC \\ = 2 \times 15 = 30 \text{ cm}$$

$$\text{Area} = \frac{1}{2} \times BD \times AC \\ = \frac{1}{2} \times 40 \times 30 \\ = 600 \text{ cm}^2$$

256. (d)



$$\therefore DE \parallel AC$$

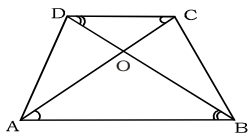
$$\therefore \triangle BDE \sim \triangle BAC$$

$$\frac{\text{ar}(\triangle BDE)}{\text{ar}(\triangle BAC)} = \frac{2^2}{5^2} = \frac{4}{25}$$

$$\text{ar(trap. ACED)} = \text{ar}(\triangle BAC) - \text{ar}(\triangle BDE) = 25 - 4 = 21$$

$$\therefore \frac{\text{ar}(\triangle ACED)}{\text{ar}(\triangle BDE)} = \frac{21}{4} = 21 : 4$$

257. (c)



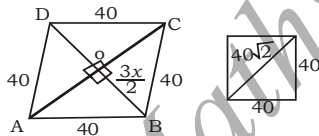
$$\therefore AB = 2 \text{ CD}$$

$$\frac{AB}{CD} = \frac{2}{1}$$

$$\frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \left(\frac{2}{1}\right)^2 = \frac{4}{1} = 4 : 1$$

$$(\triangle AOB \sim \triangle COD)$$

258. (d)



$$\text{Let } AC = 4x \text{ and } BD = 3x$$

$$\therefore OA = 2x \text{ and } OB = \frac{3x}{2}$$

$$\text{In Right } \triangle OAB$$

$$\sqrt{(2x)^2 + \left(\frac{3x}{2}\right)^2} = 40$$

$$4x^2 + \frac{9x^2}{4} = 40^2 = 1600$$

$$16x^2 + 9x^2 = 1600 \times 4$$

$$25x^2 = 6400$$

$$x^2 = \frac{6400}{25}$$

$$x = \sqrt{\frac{6400}{25}} = \frac{80}{5} = 16$$

$$\therefore AC = 4x = 4 \times 16 = 64$$

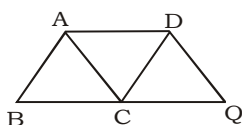
$$BD = 3x = 3 \times 16 = 48$$

$$\text{area} = \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 64 \times 48$$

$$= 1536 \text{ cm}^2$$

259. (a)



$$\text{In } \triangle ABC \text{ \& } \triangle DCQ$$

$$\angle ABC = \angle DCQ$$

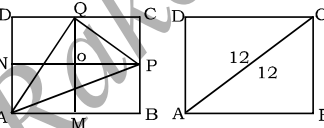
$$\angle ACB = \angle DQC$$

$$BC = CQ$$

$$\triangle ABC \cong \triangle DCQ$$

$$\text{ar}(\triangle ABC) = \text{ar}(\triangle DCQ)$$

260. (c)



$$\text{area of } ABCD = 24$$

$$\text{Draw } QM \text{ and } PN \text{ and intersect them at } O$$

$$\text{ar}(\square POQC) = \frac{1}{4} \times 24 = 6$$

$$\therefore \text{area}(\triangle PQC) = \frac{1}{2} \times 6 = 3$$

$$\text{ar}(\triangle PQC) = 3$$

$$\text{ar}(\square QMAD) = \frac{1}{2} \times 24 = 12$$

$$\text{ar}(\triangle QAD) = \frac{1}{2} \times 12 = 6$$

$$\text{ar}(\triangle ABP) = 6$$

$$\text{ar}(\triangle PQC) + \text{ar}(\triangle QAD)$$

$$+ \text{ar}(\triangle ABP) = 15$$

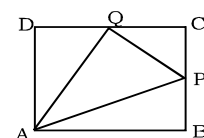
$$\text{ar}(\triangle APQ) = 24 - 15 = 9 \text{ cm}^2$$

$$\text{also}$$

$$\frac{\text{ar}(\triangle APQ)}{\text{ar}(ABCD)} = \frac{9}{24} = \frac{3}{8}$$

$$\therefore \text{always it will be } 3 : 8$$

Alternate:-



In this question

$$\text{ar}(\triangle APQ) = \frac{3}{8} (ABCD)$$

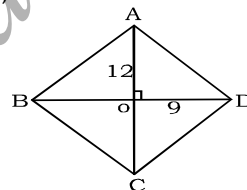
$$\frac{\text{ar}(\triangle APQ)}{\text{ar}(\triangle ABC)} = \frac{3}{8}$$

$$\therefore \text{ar}(\triangle ABC) = 12$$

$$\therefore ABCD = 2 \times 12 = 24$$

$$\text{ar}(\triangle APQ) = \frac{3}{8} \times 24 = 9 \text{ cm}^2$$

261. (b)



$$d_1 = 24 \text{ cm}$$

$$\text{area of Rhombus} = 216$$

$$\frac{1}{2} \times d_1 \times d_2 = 216$$

$$\frac{1}{2} \times 24 \times d_2 = 216$$

$$d_2 = \frac{216 \times 2}{24} = 18 \text{ cm}$$

$$OA = \frac{1}{2} \times d_1 = \frac{1}{2} \times 24 = 12 \text{ cm}$$

$$\therefore \text{Diagonals of Rhombus bisect each other at right angle}$$

$$OD = \frac{1}{2} \times d_2 = \frac{1}{2} \times 18 = 9 \text{ cm}$$

Now,

$$\text{In Right } \triangle AOD$$

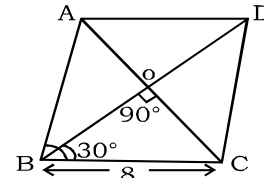
$$AD^2 = AO^2 + OD^2$$

$$= 12^2 + 9^2 = 144 + 81 = 225$$

$$AD = \sqrt{225} = 15 \text{ cm}$$

$$\therefore \text{Perimeter of Rhombus} = 4 \times AD = 4 \times 15 = 60 \text{ cm}$$

262. (a)



$$\text{Let } \angle ABC = 60^\circ$$

$$\angle OBC = 30^\circ$$

∴ Diagonals of Rhombus are the angle bisectors

In right $\triangle BOC$

$$\frac{OB}{BC} = \cos 30^\circ$$

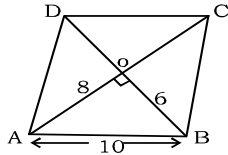
$$\frac{OB}{8} = \frac{\sqrt{3}}{2}$$

$$OB = 4\sqrt{3}$$

$$\therefore BD = 2 \times OB$$

$$= 2 \times 4\sqrt{3} = 8\sqrt{3} \text{ cm}$$

263. (c)



$$AC = 16, BD = 12 \text{ cm}$$

$$\therefore OA = 8 \text{ cm}, OB = 6 \text{ cm}$$

∴ Diagonals of rhombus bisect each other at 90°

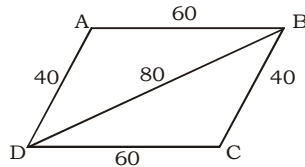
In Right $\triangle OAB$

$$AB^2 = OA^2 + OB^2$$

$$= 8^2 + 6^2 = 100$$

$$AB = \sqrt{100} = 10 \text{ cm}$$

264. (b)



$$S(\triangle ABD) = \frac{60+80+40}{2} = 90$$

$$\text{ar}(\triangle ABD)$$

$$= \frac{\sqrt{90(90-60)(90-40)(90-80)}}{4}$$

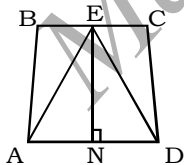
$$= \sqrt{90 \times 10 \times 30 \times 50}$$

$$= 300\sqrt{15} \text{ m}^2$$

$$\text{ar}(\square ABCD) = 2 \times \text{ar}(\triangle ABD)$$

$$= 600\sqrt{15} \text{ m}^2$$

265. (d)



Let $EN \perp AD$

$$\text{area of } \triangle AED = \frac{1}{2} \times EN \times AD$$

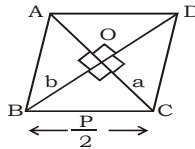
area of trapezium ABCD

$$= \frac{1}{2} (AD + BC) \times EN$$

$$\frac{\text{ar}(\square ABCD)}{\text{ar}(\triangle AED)} = \frac{\frac{1}{2} (AD + BC) \times EN}{\frac{1}{2} \times EN \times AD}$$

$$= \frac{AD + BC}{AD}$$

266. (c)



side of Rhombus

$$= \frac{\text{perimeter}}{4} = \frac{2P}{4} = \frac{P}{2}$$

Let, $AC = 2a$

$$\therefore OA = OC = a$$

$$BD = 2b$$

$$OB = OD = b$$

In Right $\triangle OBC$,

$$a^2 + b^2 = \frac{P^2}{4}$$

$$4a^2 + 4b^2 = P^2 \quad \dots\dots(i)$$

$$\text{Also, } 2a + 2b = m$$

on squaring,

$$4a^2 + 4b^2 + 8ab = m^2$$

$$4a^2 + 4b^2 = m^2 - 8ab \quad \dots\dots(ii)$$

from (i) and (ii)

$$m^2 - 8ab = P^2$$

$$8ab = m^2 - P^2$$

$$4 \times (2ab) = m^2 - P^2$$

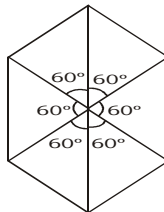
$$2ab = \frac{1}{4} (m^2 - P^2)$$

area of Rhombus

$$= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 2a \times 2b$$

$$= 2ab = \frac{1}{4} (m^2 - P^2)$$

267. (c)



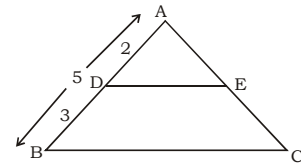
Regular hexagon has 6 equilateral triangle

$$\therefore \text{Area of Regular hexagon} = 6 \times \text{area of equilateral triangle}$$

$$= 6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} a^2$$

$$= \frac{9}{2\sqrt{3}} a^2$$

268. (b)



∴ $DE \parallel BC$

$$\therefore \angle ADE = \angle ABC \text{ and}$$

$$\angle AED = \angle ACB$$

$$\therefore \triangle ADE \sim \triangle ABC$$

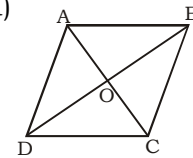
$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle ABC)} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

$$\therefore \text{area}(\square DECB) = \text{area}(\triangle ABC) - \text{area}(\triangle ADE)$$

$$= 25 - 4 = 21$$

$$\therefore \frac{\text{ar}(\square DECB)}{\text{ar}(\triangle ABC)} = \frac{21}{25}$$

269. (a)



$$AB = BC = CD = DA = 10 \text{ cm}$$

$$BD = 16 \text{ cm}$$

In $\triangle ODC$,

$$OD = 8, CD = 10, \angle DOC = 90^\circ$$

∴ OC

$$= \sqrt{CD^2 - OD^2} = \sqrt{10^2 - 8^2}$$

$$= 6 \text{ cm}$$

$$\therefore AC = 2 \times OC = 2 \times 6 = 12 \text{ cm}$$

Now, Area of Rhombus ABCD

$$= \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times 16 \times 12 = 96 \text{ cm}^2$$

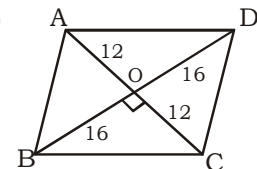
270. (a) Area of trapezium

$$= \frac{1}{2} (\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} (6 + 8) \times 4 = \frac{1}{2} \times 14 \times 4$$

$$= 28 \text{ cm}^2$$

271. (a)



$$AC = 24, BD = 32$$

$$\therefore OB = OD = 16 \text{ and}$$

$$OA = OC = 12$$

(Diagonals of Rhombus bisect each other at 90°)

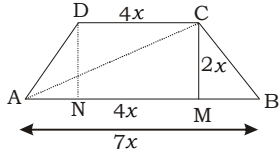
In $\triangle OBC$,

$$BC^2 = OB^2 + OC^2 = 16^2 + 12^2 = 400$$

$$BC = \sqrt{400} = 20 \text{ cm}$$

$$\text{perimeter} = 20 \times 4 = 80 \text{ cm}$$

272. (a)



$$\text{area} = \frac{1}{2} (\text{sum of parallel sides}) \times \text{distance between them}$$

$$\frac{1}{2} (7x+4x) \times 2x = 176$$

$$11x^2 = 176 \Rightarrow x^2 = 16$$

$$\Rightarrow x = 4$$

$$AB = 7 \times 4 = 28 \text{ cm}$$

$$CD = 4 \times 4 = 16 \text{ cm}$$

$$CM = 2 \times 4 = 8 \text{ cm}$$

$$AM = AN + NM$$

$$= AN + 16$$

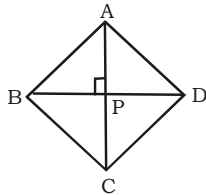
$$= 6 + 16 = 22 \quad (AN = BM = \frac{12}{2} = 6)$$

$$AC^2 = CM^2 + AM^2$$

$$AC^2 = 8^2 + 22^2$$

$$AC = \sqrt{64 + 484} \Rightarrow \sqrt{548} \Rightarrow 2\sqrt{137}$$

273. (b)



ABCD is a rhombus

$$AB = \frac{60}{4} = 15 \text{ cm}$$

$$(\text{Perimeter} = 60 \text{ cm})$$

$$AC = 24, \quad AP = 12$$

[Diagonals of rhombus bisect perpendicularly]

In $\triangle APB$

$$AB = 15, \quad AP = 12$$

$$\therefore BP = 9$$

(By pythagoras theorem)

$$BD = 9 \times 2 = 18$$

Area of rhombus

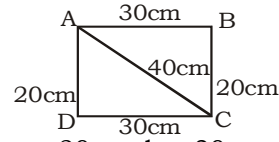
$$= \frac{1}{2} \times \text{diagonal}_1 \times \text{diagonal}_2$$

$$\Rightarrow \frac{1}{2} \times 18 \times 24 = 216 \text{ sq cm}$$

274. (d) Let ABCD is a || gm

area of $\square ABCD = 2 \times \text{area of } \triangle ADC$

For area of ($\triangle ADC$)



$$a = 20 \text{ cm}, b = 30 \text{ cm}, c = 40 \text{ cm}$$

$$S = \frac{a+b+c}{2} = \frac{20+30+40}{2}$$

$$= 45 \text{ cm}$$

area ($\triangle ADC$)

$$= \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{45(45-20)(45-30)(45-40)}$$

$$= \sqrt{45 \times 25 \times 15 \times 5}$$

$$= 75\sqrt{15} \text{ cm}^2$$

$$\text{ar } (\square ABCD) = 2 \times 75\sqrt{15}$$

$$= 150\sqrt{15} \text{ cm}^2$$

275. (a) Let the diagonal of rhombus

$$d_1 = x \text{ \& } d_2 = 2x$$

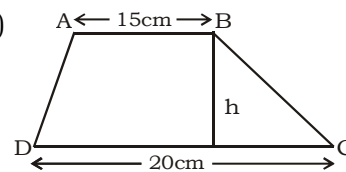
$$\text{Area of rhombus} = \frac{1}{2} d_1 d_2$$

$$256 = \frac{1}{2} (x)(2x)$$

$$16 = x$$

$$\text{Longer diagonal} = 2x = 2(16) = 32 \text{ cm}$$

276. (b)



As we know

$\Rightarrow$ Area of trapezium

$$= \frac{1}{2} (\text{sum of parallel sides}) \times \text{height}$$

$$\Rightarrow 175 = \frac{1}{2} (20+15) \times h$$

$$\Rightarrow \text{height} = 10 \text{ cm}$$

277. (a) let the rate of carpenting

$$= \text{Rs } x/\text{m}^2$$

$$\therefore \text{length} \times \text{breadth} \times x$$

$$= \text{Rs } 120 \dots (i)$$

$$\text{length} \times (\text{breadth} - 4) \times x$$

$$= \text{Rs } 100 \dots (ii)$$

$$\frac{\text{breadth}}{\text{breadth} - 4} = \frac{120}{100} = \frac{6}{5}$$

$$\text{breadth} = 24 \text{ m}$$

$$278. (b) \text{ Area of corridor} = 100 \times 3 = 300 \text{ m}^2$$

Carpet length

$$= \frac{300 \times 100}{50} = 600 \text{ cm}$$

$$\text{Cost of Carpet} = ₹ 15 \times 600 = 9000$$

279. (a) Old expenditure = ₹1000

$$\text{increase in area} = 50 \times 20 \text{ m}^2$$

$$\text{Increase in expenditure}$$

$$= 50 \times 20 \times .25 = ₹250$$

$$\Rightarrow \text{New expenditure}$$

$$= 1000 + 250 = ₹1250$$

280. (d) Area of verandah

$$= (25+3.5) \times (15+3.5) - 25 \times 15$$

$$= 527.25 - 375 = 152.25 \text{ m}^2$$

$$\text{cost of flooring} = 152.25 \times 27.5 = \text{Rs. } 4186.50 \text{ (app.)}$$

281. (b) $2\pi R_1 = 528$

$$\Rightarrow 2 \times \frac{22}{7} \times R_1 = 528$$

$$\Rightarrow R_1 = 84 \text{ cm}$$

$$\Rightarrow \text{New Radius} = R_1 - 14 = R_2$$

$$\Rightarrow R_2 = 84 - 14$$

$$\Rightarrow R_2 = 70$$

$$\text{New Radius } R_2 = 84 - 14 = 70$$

$$\text{Area of Road} = \pi (R_1^2 - R_2^2)$$

$$\Rightarrow = \pi \times 14 \times 154$$

$$\Rightarrow \text{Total expenditure}$$

$$= \frac{22}{7} \times 14 \times 154 \times 10$$

$$= \text{Rs. } 67760$$

282. (b) Since the ratio of length and breadth = 3 : 2

Let length of rectangular field = 3x

Breadth of rectangular field = 2x

Perimeter of the field = 80 m

$$2(1 + b) = 80$$

$$2(2x + 3x) = 80$$

$$2 \times 5x = 80$$

$$x = \frac{80}{10} = 8$$

then breadth = 2x

$$= 2 \times 8 = 16 \text{ cm}$$

283. (c) The sides of a rectangular plot are in the ratio = 5 : 4

Let the length of rectangular

field = $5x$
and the breadth of rectangular
field = $4x$

According to question,

$$\text{Area} = 500 \text{ m}^2$$

$$5x \times 4x = 500 \text{ m}^2$$

$$20x^2 = 500 \text{ m}^2$$

$$x^2 = \frac{500}{20} = 25$$

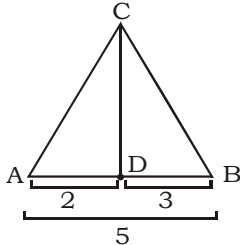
$$x = 5$$

$$\text{Length} = 5x = 5 \times 5 = 25 \text{ m}$$

$$\text{Breadth} = 4x = 4 \times 5 = 20 \text{ m}$$

$$\begin{aligned} \text{Perimeter of the rectangle} \\ &= 2(25 + 20) \\ &= 2 \times 45 = 90 \text{ m} \end{aligned}$$

284. (d)



$$AB = 5 \text{ cm}$$

$$DB = 3 \text{ cm}$$

$$\therefore AD = 2 \text{ cm}$$

$$\frac{\text{ar}(\triangle ADC)}{\text{ar}(\triangle ABC)} = \left(\frac{AD}{AB}\right)^2$$

$$= \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

285. (d) Base : Corresponding
altitude = $3 : 4$

$$\text{Let the base} = 3x$$

$$\text{altitude} = 4x$$

$$\therefore \text{area of triangle} = 1176$$

$$\frac{1}{2} \times 3x \times 4x = 1176$$

$$x^2 = \frac{1176 \times 2}{3 \times 4} = 196$$

$$x = 14$$

$$\therefore \text{altitude} = 4 \times 14 = 56 \text{ cm}$$

286. (c) According to question,

Ratio of sides of triangle are

$$= \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$$

(Take L.C.M of 2, 3, and 4
which is 12)

$$= 6 : 4 : 3$$

$$\text{Now, } 6x + 4x + 3x = 52$$

$$13x = 52$$

$$x = 4$$

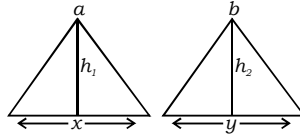
$$\therefore \text{length of smallest side} = 3x \\ = 3 \times 4 = 12 \text{ cm}$$

287. (c) Let diagonals be $2x$ and $5x$

$$\frac{A_1}{A_2} = \frac{\frac{1}{2} \times (2x)^2}{\frac{1}{2} \times (5x)^2} = \frac{4}{25}$$

$$\Rightarrow 4 : 25$$

288. (c)



$$\frac{\frac{1}{2} \times h_1 \times x}{\frac{1}{2} \times h_2 \times y} = \frac{a}{b}$$

$$\frac{h_1}{h_2} \times \frac{x}{y} = \frac{a}{b}, \quad \frac{h_1}{h_2} = \frac{ay}{bx}$$

$$ay : bx$$

289. (a) Ratio of parallel sides

$$= 5 : 3$$

Let sides are $5x$ and $3x$

$$\frac{1}{2} (\text{sum of parallel sides}) \times \text{perpendicular distance} = 1440 \text{ m}^2$$

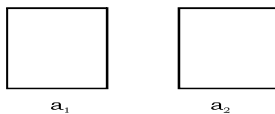
$$\frac{1}{2} (5x + 3x) \times 24 = 1440$$

$$4x \times 24 = 1440$$

$$x = \frac{1440}{4 \times 24} = 15 \text{ m}$$

$$\begin{aligned} \therefore \text{length of longer side} &= 5x \\ &= 5 \times 15 \\ &= 75 \text{ m} \end{aligned}$$

290. (c)



$$\text{ATQ, } \frac{a_1^2}{a_2^2} = \frac{225}{256}$$

$$\frac{a_1}{a_2} = \sqrt{\frac{225}{256}} = \frac{15}{16}$$

Ratio of their perimeters

$$= \frac{4a_1}{4a_2} = \frac{a_1}{a_2} = \frac{15}{16}$$

$$\Rightarrow 15 : 16$$

291. (d) Clearly, 3, 4 and 5 form a triplet therefore, consider the triangle, a right triangle

Let the sides are $3x, 4x,$ and $5x$
perimeter = $3x + 4x + 5x = 12x$

$$\text{area of triangle} = \frac{1}{2} \times 3x \times 4x$$

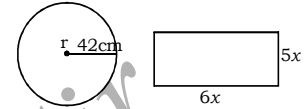
$$\frac{1}{2} \times 3x \times 4x = 216$$

$$x^2 = \frac{216 \times 2}{3 \times 4} = 36$$

$$x = \sqrt{36} = 6$$

$$\therefore \text{Perimeter} = 12 \times 6 = 72 \text{ cm}$$

292. (a)



perimeter of rectangle =
circumference of circular
wire

$$2(6x + 5x) = 2 \times \frac{22}{7} \times 42$$

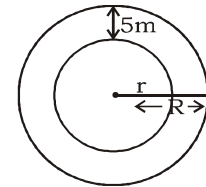
$$22x = 2 \times 22 \times 6$$

$$x = 12$$

clearly,

$$\begin{aligned} \text{smaller side of rectangle} \\ &= 5 \times 12 = 60 \text{ cm} \end{aligned}$$

293. (c)



$$\frac{2\pi R}{2\pi r} = \frac{23}{22}$$

$$\frac{R}{r} = \frac{23}{22}$$

$$\text{Let } R = 23x, r = 22x$$

$$\therefore R - r = 5$$

$$23x - 22x = 5$$

$$x = 5$$

$$\Rightarrow r = 22 \times 5 = 110$$

$$\begin{aligned} \text{diameter of inner circle} &= 2r \\ &= 2 \times 110 = 220 \text{ m} \end{aligned}$$

294. (b) Ratio of angles = $3 : 4 : 5$

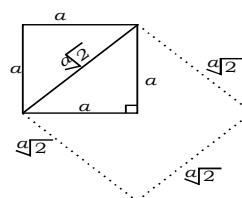
$$3 + 4 + 5 = 180^\circ$$

$$12 = 180^\circ$$

$$1 = \frac{180^\circ}{12} = 15^\circ$$

$$\begin{array}{ccc} 3 & : & 4 & : & 5 \\ \downarrow \times 15 & & \downarrow \times 15 & & \downarrow \times 15 \\ 45 & & 60 & & 75 \rightarrow \text{largest angle} \end{array}$$

295. (b)



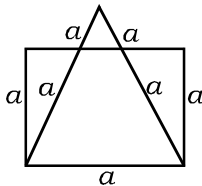
Let the side of square = a

$$\therefore \text{Diagonal} = a\sqrt{2}$$

$$\{\sqrt{a^2 + a^2} = a\sqrt{2}\}$$

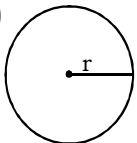
$$\begin{aligned} & \frac{\text{Area of square}}{\text{Area of square on diagonal}} \\ &= \frac{a^2}{(a\sqrt{2})^2} = \frac{a^2}{a^2 \times 2} = \frac{1}{2} = 1 : 2 \end{aligned}$$

296. (d)



$$\begin{aligned} &= \frac{\text{area of square}}{\text{area of equilateral triangle}} \\ &= \frac{a^2}{\frac{\sqrt{3}}{4} a^2} = \frac{4}{\sqrt{3}} = 4 : \sqrt{3} \end{aligned}$$

297. (d)



$$\pi r^2 = a^2$$

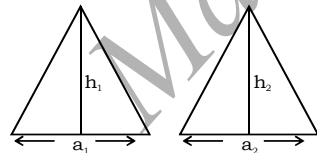
$$r^2 = \frac{a^2}{\pi}$$

$$r = \frac{a}{\sqrt{\pi}}$$

$$\begin{aligned} \text{Ratio of perimeter} &= \frac{2\pi r}{4a} \\ &= \frac{\pi r}{2a} \end{aligned}$$

$$= \frac{\pi \times \frac{a}{\sqrt{\pi}}}{2a} = \frac{\sqrt{\pi}}{2} = \sqrt{\pi} : 2$$

298. (c)



$$\begin{aligned} & \frac{\frac{\sqrt{3}}{4} (a_1)^2}{\frac{\sqrt{3}}{4} (a_2)^2} = \frac{25}{36} \end{aligned}$$

$$\frac{a_1}{a_2} = \sqrt{\frac{25}{36}} = \frac{5}{6}$$

$$\begin{aligned} \text{Ratio of altitudes} &= \frac{\frac{\sqrt{3}}{2} a_1}{\frac{\sqrt{3}}{2} a_2} \\ &= \frac{a_1}{a_2} = \frac{5}{6} = 5 : 6 \end{aligned}$$

$$299. (d) \text{ Let length} = 5x$$

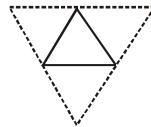
$$\frac{l}{2(l+b)} = \frac{5}{16}$$

$$8l = 5l + 5b$$

$$3l = 5b$$

$$\frac{l}{b} = \frac{5}{3} = 5 : 3$$

300. (c)



When we draw such figures as mentioned in the question the vertex of the old triangle are the mid points of the sides of new triangle and the sides of the old triangle are half of the opposite side.

$\therefore$ required ratio = 2 : 1

$$301. (b) \frac{\text{Circumference}}{\text{Area}} = \frac{2\pi r}{\pi r^2}$$

$$= \frac{2}{r} = \frac{2}{3}$$

302. (b) Ratio of area = (Ratio of

$$\text{radius})^2 = \left(\frac{\frac{a}{\sqrt{3}}}{\frac{a}{2\sqrt{3}}} \right)^2$$

$$= 4 : 1$$

303. (a) Ratio of area = (Ratio of radius)²

	A	B	C
Radius	4	2	1
Area	16	4	1

$$304. (b) \pi r^2 = a^2$$

$$\frac{a^2}{r^2} = \frac{\pi}{1}$$

$$\frac{a}{r} = \sqrt{\pi} : 1$$

$$305. (c) \text{ Ratio of sides} = \frac{1}{3} : \frac{1}{4} : \frac{1}{5}$$

$$= 20 : 15 : 12$$

$$20 + 15 + 12 = 47$$

$$\Rightarrow 47 \rightarrow 94$$

$$1 \rightarrow 2$$

$$\Rightarrow \text{Smallest side} = 12 \times 2 = 24 \text{ cm}$$

306. (a) Let the sides be 3x, 4x, 5x and 6x

$$\Rightarrow 18x \rightarrow 72, \quad x \rightarrow 4$$

$$\Rightarrow \text{Greatest side} = 6 \times 4 = 24 \text{ cm}$$

307. (b) Ratio of circumference = Ratio of radius = 3 : 4

308. (d) Let the sides be 2x, 3x and 4x

$$\Rightarrow 9x = 18 \Rightarrow x = 2$$

$\Rightarrow$ Sides are 4, 6 and 8 cm respectively

Using heron's formula

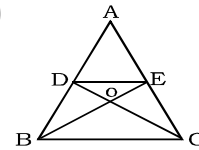
$$S = \frac{4+6+8}{2} = 9 \text{ cm}$$

$$\begin{aligned} \Rightarrow \text{area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{9 \times 5 \times 3 \times 1} \\ &= 3\sqrt{15} \text{ cm}^2 \end{aligned}$$

309. (b) Ratio of area = (Ratio of radius)²

$$= \left(\frac{\frac{a}{\sqrt{3}}}{\frac{a}{2\sqrt{3}}} \right)^2 = 4 : 1$$

310. (a)



As D and E are mid-points of AB and AC

$$\Rightarrow DE \parallel BC$$

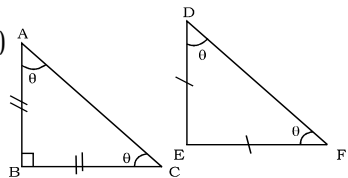
$$\Rightarrow \triangle ODE \sim \triangle OBC$$

$$\text{and also } \frac{DE}{BC} = \frac{1}{2}$$

(as D and E are mid-points)

$$\Rightarrow \frac{\text{ar}(\triangle ODE)}{\text{ar}(\triangle OBC)} = \left(\frac{1}{2} \right)^2 = \frac{1}{4}$$

311. (c)



The given angle is same

let vertical angle = θ

($\therefore \triangle ABC$ and $\triangle DEF$ are isosceles triangles)

$\Rightarrow$ when two angles are equal then third angle is also equal

$$\therefore \triangle ABC \sim \triangle DEF$$

$\triangle ABC$ is similar to $\triangle DEF$

$$\therefore \frac{\text{area of } \triangle ABC}{\text{area of } \triangle DEF}$$

$$= \left(\frac{\text{side of } \triangle ABC}{\text{side of } \triangle DEF} \right)^2$$

$$\Rightarrow \sqrt{\frac{1}{4}} = \frac{\text{side of } \triangle ABC}{\text{side of } \triangle DEF}$$

$$= \frac{\text{side of } \triangle ABC}{\text{side of } \triangle DEF} = \frac{1}{2}$$

312. (a) Let the sides be $3x$, $3x$ and $4x$

$$\Rightarrow \text{Area} = \frac{(4x)^2}{4} \sqrt{4(3x)^2 - (4x)^2}$$

$$= 4x^2 \sqrt{36x^2 - 16x^2} = 4x^2 \sqrt{20x^2}$$

$$= 8x^3 \sqrt{5} = 8\sqrt{5} = x^3 = 1$$

$$= x = 1$$

$\therefore$ 3rd side = $3 \times 1 = 3$ units

313. (c) 3, 4 and 5 from triplet
Let the sides be $3x$, $4x$ and $5x$

$$\Rightarrow \frac{1}{2} \times 3x \times 4x = 72$$

$$\Rightarrow 6x^2 = 72$$

$$\Rightarrow x^2 = 12$$

$$\Rightarrow x = 2\sqrt{3}$$

$\therefore$ Smallest side = $3 \times 2\sqrt{3}$

$$= 6\sqrt{3}$$

314. (b) Let the sides be $3x$, $4x$ and $5x$

$$\Rightarrow \text{area} = \frac{1}{2} \times 3x \times 4x = 72$$

$$\Rightarrow 6x^2 = 72$$

$$x^2 = 12$$

$$x = 2\sqrt{3}$$

$$\Rightarrow \text{Perimeter of equilateral } \triangle = 12 \times 2\sqrt{3} = 24\sqrt{3} \text{ units}$$

$$\text{Side of } \triangle = \frac{24\sqrt{3}}{3} = 8\sqrt{3} \text{ units}$$

$$\text{area of } \triangle = \frac{\sqrt{3}}{4} \times (8\sqrt{3})^2$$

$$= \frac{\sqrt{3}}{4} \times 64 \times 3 = 48\sqrt{3} \text{ unit}^2$$

315. (d) Let the parallel sides be $2x$ and $3x$

$$\Rightarrow \text{area} = \frac{1}{2} (2x + 3x) \times 12 = 480$$

$$5x = 80$$

$$x = 16$$

$\Rightarrow$ Longer parallel side

$$= 16 \times 3 = 48 \text{ cm}$$

316. (a) Let the side of square = a

$$\therefore \text{Side of equilateral } \triangle = \sqrt{2} a$$

$$\text{Required ratio} = \frac{\frac{\sqrt{3}}{4} (\sqrt{2}a)^2}{a^2}$$

$$= \frac{\sqrt{3}}{4} \times 2 = \sqrt{3} : 2$$

317. (b) Ratio of area = (Ratio of side)²

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \left(\frac{10}{8} \right)^2 = 25 : 16$$

$$318. (a) \frac{\pi r_1^2}{\pi r_2^2} = \frac{4}{7}$$

$$\frac{r_1^2}{r_2^2} = \frac{4}{7}$$

$$\frac{r_1}{r_2} = \sqrt{\frac{4}{7}} = \frac{2}{\sqrt{7}} = 2 : \sqrt{7}$$

319. (c) Required ratio =

$$\frac{\pi(5)^2 - \pi(3)^2}{\pi(5)^2} = \frac{(5)^2 - (3)^2}{(5)^2} = \frac{16}{25}$$

$$\Rightarrow 16 : 25$$

320. (a) Let side of square = a

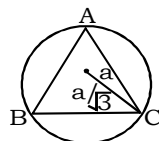
$$\text{radius of smaller circle} = \frac{a}{2}$$

$$\text{radius of larger circle} = \frac{\sqrt{2}a}{2}$$

$$\text{Required ratio} = \frac{\pi \left(\frac{a}{2} \right)^2}{\pi \left(\frac{\sqrt{2}a}{2} \right)^2}$$

$$= \frac{\frac{a^2}{4}}{\frac{2a^2}{4}} = \frac{1}{2} \Rightarrow 1 : 2$$

321. (c)



$$\text{Circumradius} = \frac{\text{side}}{\sqrt{3}} = \frac{a}{\sqrt{3}}$$

Equilateral $\triangle$

$$\text{Required ratio} = \frac{\frac{\sqrt{3}}{4} a^2}{\pi \left(\frac{a}{\sqrt{3}} \right)^2} = \frac{3\sqrt{3}}{4\pi}$$

$$= 3\sqrt{3} : 4\pi$$

322. (d) $2(l + b) = 4a$

(a = side of square)

$$2(2 + 1) = 4a$$

$$2 \times 3 = 4a$$

$$a = \frac{3}{2}$$

Required ratio

$$= \frac{l \times b}{a^2} = \frac{1 \times 2}{\left(\frac{3}{2} \right)^2} = \frac{2 \times 4}{9} = \frac{8}{9} = 8 : 9$$

323. (c) $2(l + b) = 3a$

$\therefore$ (a = side of equilateral triangle)

Let (b = a)

$$\Rightarrow 2(l + a) = 3a$$

$$2(l + a) = 3a$$

$$2l + 2a = 3a$$

$$2l = a$$

Required Ratio

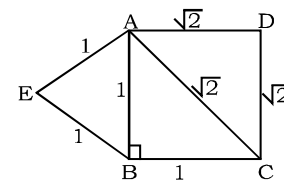
$$= \frac{l \times b}{\frac{\sqrt{3}}{4} a^2} = \frac{\frac{a}{2} \times a}{\frac{\sqrt{3}}{4} a^2} = \frac{a^2}{2} \times \frac{4}{\sqrt{3} a^2}$$

$$= \frac{2}{\sqrt{3}} = 2 : \sqrt{3}$$

$$324. (b) \text{ Required ratio} = \frac{\pi r^2}{r^2} = \frac{\pi}{1}$$

$$= \pi : 1$$

325. (c) Let $AB = 1$, $BC = 1$



$$\therefore AC = \sqrt{1^2 + 1^2} = \sqrt{2}$$

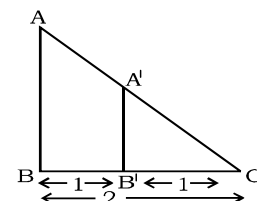
(using pythagoras)

$$= \frac{\text{ar}(\triangle ABE)}{\text{ar}(\triangle ACD)} = \frac{\frac{\sqrt{3}}{4} (1)^2}{\frac{\sqrt{3}}{4} (\sqrt{2})^2} = \frac{1}{2}$$

$$= 1 : 2$$

$$326. (b) \frac{\text{ar} \triangle ABC}{\text{ar} \triangle DEF} = \left(\frac{AB}{DE} \right)^2 = \left(\frac{10}{8} \right)^2 = \frac{25}{16}$$

327. (c)



$$A'B' \parallel AB$$

∴ A' and B' are the mid-point. By mid point theorem

$$\therefore \triangle A'B'C \sim \triangle ABC$$

$$\text{Let } BB' = B'C = 1$$

$$\therefore BC = 2$$

(B' is the mid-point of BC)

$$\frac{\text{ar}(\triangle A'B'C)}{\text{ar}(\triangle ABC)} = \left(\frac{B'C}{BC}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\text{ar}(\triangle A'B'B) = \text{ar}(\triangle ABC) - \text{ar}(\triangle A'B'C)$$

$$\frac{\text{ar}(\triangle A'B'B)}{\text{ar}(\triangle ABC)} = \frac{3}{4} = 3:4$$

$$328. (d) \text{ Ratio of sides} = \frac{1}{4} : \frac{1}{6} : \frac{1}{8}$$

$$= \frac{1}{4} \times 24 : \frac{1}{6} \times 24 : \frac{1}{8} \times 24$$

$$= 6 : 4 : 3$$

(Take L.C.M = 24)

$$\text{ATQ perimeter} = 91$$

$$6 + 4 + 3 = 91$$

$$13 \text{ units} = 91$$

$$1 \text{ unit} = \frac{91}{13} = 7$$

Diff. between longer and shorter side = 6 - 3 = 3 units

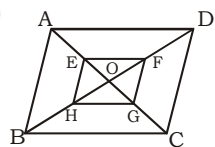
$$\Rightarrow 3 \text{ units} = 7 \times 3 = 21 \text{ cm}$$

329. (c) By using result,

$$R_1 \theta_1 = R_2 \theta_2$$

$$\frac{R_1}{R_2} = \frac{\theta_2}{\theta_1} = \frac{75^\circ}{60^\circ} = \frac{5}{4} = 5:4$$

330. (c)



In $\triangle OBC$,

H and G are the midpoints of OB and OC

$$\therefore HG = \frac{1}{2} BC$$

$$\text{similarly, } FG = \frac{1}{2} CD$$

$$\text{and } EF = \frac{1}{2} AD,$$

$$HE = \frac{1}{2} AB$$

on adding,

$$HE + HG + FG + EF$$

$$= \frac{1}{2} (AB + BC + CD + AD)$$

perimeter of EFGH.

$$= \frac{1}{2} \times \text{perimeter of } ABCD$$

$$\frac{\text{perimeter of } EFGH}{\text{perimeter of } ABCD} = \frac{1}{2}$$

$$331. (c) \text{ Old circumference} = 4\pi$$

$$2\pi r = 4\pi$$

$$r = \frac{4\pi}{2\pi} = 2\text{cm}$$

$$\text{Old area} = \pi (2)^2 = 4\pi \text{ cm}^2$$

$$\text{New circumference} = 8\pi$$

$$2\pi R = 8\pi$$

$$R = \frac{8\pi}{2\pi} = 4\text{cm}$$

$$\text{New area} = 16\pi \text{ cm}^2$$

Option (c) is the answer

(∵ area is quadruples)

$$332. (c) \text{ Length } 4 \rightarrow 5$$

$$\text{Breadth } 5 \rightarrow 4$$

$$\text{area } 20 \rightarrow 20$$

area remains unchanged

$$333. (d) \text{ Area of circle} = \pi (5)^2 = 25\pi$$

$$\text{Circumference of circle}$$

$$= 2\pi (5) = 10\pi$$

$$= \frac{25\pi}{10\pi} \times 100 = 250\%$$

$$334. (d) \text{ According to question,}$$

Circumference of a circle

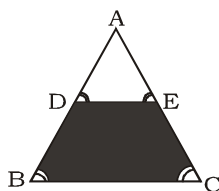
$$= \text{area of circle}$$

$$2\pi r = \pi r^2$$

$$r = 2$$

$$\therefore \text{diameter of circle} = 2r = 2 \times 2 = 4$$

335. (c)



∴ D and E are the mid points of sides AB and AC

∴ $DE \parallel BC$ (By mid point theorem)

$$\text{also } DE = \frac{1}{2} BC$$

$$\triangle ADE \sim \triangle ABC$$

$$\left\{ \begin{array}{l} \angle ADE = \angle ABC \\ \angle AED = \angle ACB \end{array} \right\}$$

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle ABC)} = \left(\frac{DE}{BC}\right)^2$$

$$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\therefore \frac{\text{ar}(\triangle DECB)}{\text{ar}(\triangle ABC)} = \frac{3}{4}$$

Percentage of ar (DECB)

$$= \frac{3}{4} \times 100 = 75\%$$

$$336. (b) \text{ Increment in breadth} = 10\%$$

$$= \frac{10}{100} = \frac{1}{10} \rightarrow \text{Increment breadth}$$

Decrement in length = 10%

$$= \frac{10}{100} = \frac{1}{10} \rightarrow \text{decrement length}$$

	length	breadth	Area
Original	10	10	100
New	9	11	99

$$\% \text{ change} = \frac{-1}{100} = 1\%$$

Alternate:-

using $x = 10\%$ (breadth),

$y = -10\%$ (length)

$$\% \text{ change} = x + y \frac{xy}{100}$$

$$= 10 - 10 + \frac{10 \times (-10)}{100} = -1\%$$

$$337. (c) \% \text{ increase} = x + y + \frac{xy}{100}$$

$$= 20 + 20 + \frac{20 \times 20}{100} = 44\%$$

338. (d) If circumference of circle is reduced by 50% then radius is reduced by 50%

$$50\% = \frac{1}{2} \rightarrow \text{decrement original}$$

	radius	Area
Original	2	4
New	1	1

(π is constant)

Reduction in area

$$= \frac{3}{4} \times 100 = 75\%$$

$$339. (d) \text{ Increase in area}$$

$$= 25 + 25 + \frac{25 \times 25}{100}$$

$$\text{use formula : } (x + y + \frac{xy}{100})$$

$$= 50 + 6.25$$

$$= 56.25\%$$

$$340. (a) \text{ Increase in area}$$

$$= 50 + 50 + \frac{50 \times 50}{100}$$

$$= 100 + 25 = 125\%$$

341. (b) using $x + y + \frac{xy}{100}$

$$= 20 - 20 + \frac{20 \times (-20)}{100}$$

$$= -4\%$$

(decrease by 4%)

342. (b) Increase in area

$$= 50 + 50 + \frac{50 \times 50}{100}$$

$$= 100 + 25 = 125\%$$

343. (c) Increase in altitude = 10%

$$= \frac{1}{10} \rightarrow \text{Increment}$$

$$= \frac{1}{10} \rightarrow \text{original}$$

altitude	base	
10	11	) -1
11	10	

Area no change

decrease in base

$$= \frac{1}{11} \times 100 = 9\frac{1}{11}\%$$

344. (d) Increase in circumference

= Increase in radius

$$= 50\% = \frac{1}{2} \rightarrow \text{Increment}$$

$$= \frac{1}{2} \rightarrow \text{Original}$$

Radius	area	
2	4	) +5
3	9	

$$\text{Increase \%} = \frac{5}{4} \times 100 = 125\%$$

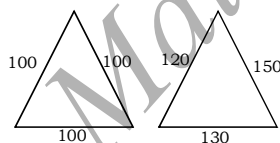
345. (b) use $x + y + \frac{xy}{100}$

percentage change

$$= 12 + 15 + \frac{12 \times 15}{100} = 27 + \frac{9}{5}$$

$$= 27 + 1 + \frac{4}{5} = 28\frac{4}{5}\%$$

346. (b)



Perimeter of equilateral triangle = $100 + 100 + 100 = 300$

Perimeter of New triangle = $120 + 150 + 130 = 400$

$$\% \text{ increase} = \frac{100}{300} \times 100 = 33\frac{1}{3}\%$$

347. (b)

Length	5	→	3
breadth	5	→	3
Area	25	→	9

$$\% \text{ decrease} = \frac{25 - 9}{25} \times 100 = 64\%$$

348. (a)

Length	5	→	8
Breadth	8	→	5
Area	40	→	40

$$\Rightarrow \% \text{ Decrease} = \frac{8 - 5}{8} \times 100$$

$$= 37\frac{1}{2}\%$$

349. (b)

Length	5	→	6
Breadth	4	→	5
Area	20	→	30

$$\% \text{ Increase} = \frac{30 - 20}{20} \times 100 = 50\%$$

350. (b)

Side	10	→	11
Area	100	→	121

$$\% \text{ Increase} = \frac{121 - 100}{100} \times 100$$

$$= 21\%$$

351. (b)

Length	20	→	21
Breadth	10	→	9
Area	200	→	189

$$\% \text{ Decrease} = \frac{200 - 189}{200} \times 100 = 5.5\%$$

352. (d)

Radius	100	→	101
Area	10000π	→	10201π

$$\% \text{ Increase} = \frac{201}{10000} \times 100 = 2.01\%$$

353. (c) Let the breadth = x cm

$$\Rightarrow \text{length} = (x + 20) \text{ cm}$$

According to the question,

$$x(x + 20) = (x + 10)(x + 5)$$

$$\Rightarrow x^2 + 20x = x^2 + 15x + 50$$

$$\Rightarrow 5x = 50$$

$$\Rightarrow x = 10$$

$$\Rightarrow \text{Area} = 10(10 + 20) = 300 \text{ m}^2$$

354. (c)

Length	20	→	21
Breadth	50	→	49
Area	1000	→	1029

$$\% \text{ error} = \frac{1029 - 1000}{1000} \times 100 = 2.9\%$$

355. (d)

Length	10	→	13
Breadth	10	→	12
Area	100	→	156

% increase in area

$$= \frac{156 - 100}{100} \times 100 = 56\%$$

356. (d) $40\% = \frac{4}{10} = \frac{2}{5}$

Side	Surface area
$40\% \left(\begin{matrix} 5 \\ 7 \end{matrix} \right)$	$\left(\begin{matrix} (5)^2 = 25 \\ (7)^2 = 49 \end{matrix} \right) 24$

$$\% \text{ increase} = \frac{24}{25} \times 100 = 96\%$$

Alternate

Percentage increase in surface area

$$= 40 + 40 + \frac{40 \times 40}{100} \%$$

$$= 80 + 16 = 96\%$$

$$[\% \text{ effect using } x + y + \frac{xy}{100}]$$

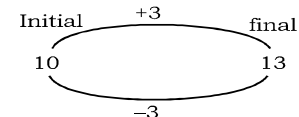
357. (a) percentage increase in area

$$= \left(8 + 8 + \frac{8 \times 8}{100} \right)$$

$$= 16 + 0.64 = 16.64\%$$

358. (a) Side of square is increased by 30%

$$= \frac{30}{100} = \frac{3}{10}$$



Other side will have to be decreased by

$$= \frac{3}{13} \times 100 = 23\frac{1}{13}\%$$

359. (c) Percentage increase in area

$$= 100 + 100 + \frac{100 \times 100}{100} = 300\%$$

Alternate

L	B	Area
1	1	1
2	2	4

Percentage increase

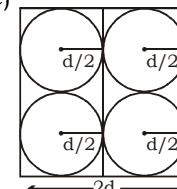
$$= \frac{3}{1} \times 100 = 300\%$$

360. (d) $x + y + \frac{xy}{100}$

$$= 10 - 10 + \frac{10 \times (-10)}{100} = -1\%$$

(Negative sign shows decrease)

361. (c)

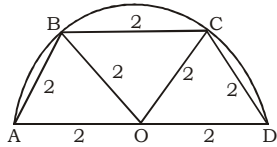


From the figure it is clear that, 4 circular plates of diameter d can be made of a. Square plate of side $2d$ with minimum loss of material.

362. (d) $\triangle AOB$, $\triangle BOC$ and $\triangle COD$ are equilateral $\triangle$.

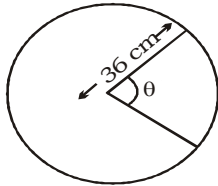
$\therefore$ Side = 2 cm

Now, total area = $3 \times \frac{\sqrt{3}}{4} (\text{Side})^2$



$$= 3 \times \frac{\sqrt{3}}{4} \times 4 = 3\sqrt{3} \text{ cm}^2$$

363. (d) Area of sector = $72\pi \text{ cm}^2$

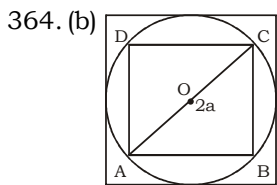


$$\Rightarrow \frac{\pi r^2 \theta}{360^\circ} = 72\pi$$

$$\therefore \theta = \frac{72 \times 360}{36 \times 36} = 20^\circ$$

Now, length of arc = $\frac{\pi r \theta}{180^\circ}$

$$= \frac{\pi \times 36 \times 20}{180} = 4\pi \text{ cm}$$



364. (b)

For inscribed circle, Diameter of circle = Diagonal of square
Since, sides of square are equal.

Now, In $\triangle ABC$ by Pythagoras theorem

$$AB^2 + BC^2 = AC^2$$

$$\Rightarrow 2AB^2 = 4a^2$$

$$AB^2 = 2a^2 \Rightarrow AB = \sqrt{2}a$$

$$\therefore \text{Area of inner square} = AB^2 =$$

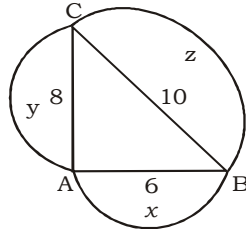
$$(\sqrt{2}a)^2 = 2a^2$$

For circumscribed square, Diameter of circle = Side of square

$$\therefore \text{Area of circumscribed square} = (2a)^2 = 4a^2$$

$$\therefore \text{Difference between areas of outer and inner squares} = 4a^2 - 2a^2 = 2a^2$$

365. (c) In $\triangle ABC$, by Pythagoras



$$BC^2 = AB^2 + AC^2 = 36 + 64 = 100 \text{ cm}$$

Now, area of semi-circle = x

$$= \frac{\pi (3)^2}{2} = \frac{9\pi}{2} \text{ cm}^2$$

Area of semi-circle = y

$$= \frac{16\pi}{2} \text{ cm}^2$$

Area of semi-circle = z

$$= \frac{25\pi}{2} \text{ cm}^2$$

Now, value of $x+y-z$

$$= \left(\frac{9\pi}{2} + \frac{16\pi}{2} \right) - \frac{25\pi}{2} = 0$$

366. (c) Perimeters of triangles,



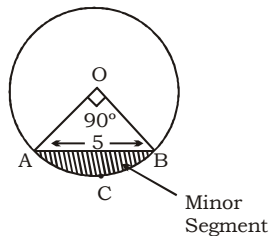
$$= (3 \times 1) + (3 \times 0.5) + (3 \times 0.25) + (3 \times 0.125) + \dots$$

$$\Rightarrow 3 + 1.5 + 0.75 + 0.375$$

$$S_n = \frac{a}{1-n} = 1 - \frac{1}{2} = \frac{3 \times 2}{2-1} = 6 \text{ units}$$

367. (c) In $\triangle AOB$,

$AO = OB = r$ (radius of circle)



Using Pythagoras theorem,

$$AB^2 = OA^2 + OB^2 \Rightarrow (5)^2 = r^2 + r^2$$

$$\therefore r^2 = \frac{25}{2} \text{ cm}$$

How, area of sector AOB

$$= \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{90^\circ}{360^\circ} \times \pi \times \frac{25}{2} = \frac{25\pi}{8} \text{ cm}^2$$

Now, area of minor segment

= Area of sector - Area of triangle

$$= \frac{25\pi}{8} - \frac{r^2}{2} = \frac{25\pi}{8} - \frac{25}{4}$$

$$= \left(\frac{25\pi - 50}{8} \right)$$

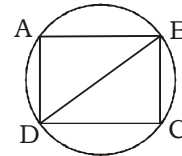
Area of major segment = Area of circle - Area of minor segment

$$= \pi r^2 - \left(\frac{25\pi - 50}{8} \right)$$

$$= \frac{100\pi - 25\pi + 50}{8} = \frac{75\pi + 50}{8}$$

$$= \frac{25}{8} (3\pi + 2) = \frac{25}{4} \left(\frac{3\pi}{2} + 1 \right) \text{ cm}^2$$

368. (c) ABCD be the rectangle inscribed in the circle of diameter 5 cm.



$\therefore$ Diameter = Diagonal of rectangle

Now, let x and y be the length and breadth of rectangle are respectively,

Now In $\triangle ABD$, $AB^2 + AD^2 = (5)^2$

$$\Rightarrow x^2 + y^2 = 25$$

Since, they form Pythagoras triplet,

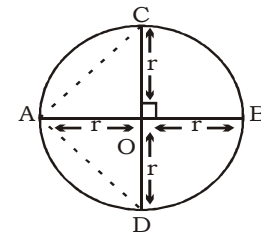
$$\therefore x = 4 \text{ and } y = 3$$

So, area of rectangle = 3×4

$$= 12 \text{ cm}^2$$

369. (b) Required ratio

$$= \frac{\text{Area of circle}}{\text{Area of } \triangle ACD}$$



$$= \frac{\pi r^2}{\frac{1}{2} \times 2r \times r} = \pi$$

370. (b) Semi-perimeter of triangle

$$= \frac{a+b+c}{2}$$

$$= \frac{7+24+25}{2} = \frac{56}{2} = 28 \text{ cm}$$

Area of circle = Area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{28(28-7)(28-24)(28-25)}$$

$$= \sqrt{28 \times 21 \times 4 \times 3}$$

$$= \sqrt{7056} = 84 \text{ cm}^2$$

371. (a) Area of equilateral triangle

$$= \frac{\sqrt{3}a^2}{4} = x \quad \dots(i)$$

$$\text{and perimeter} = 3a = y$$

$$\Rightarrow a = \frac{y}{3} \quad \dots(ii)$$

Now, putting the value of a from Eq. (ii) in Eq. (i), we get

$$\frac{\sqrt{3} \left(\frac{y}{3} \right)^2}{4} = x \Rightarrow x = \frac{\sqrt{3} \times y^2}{9 \times 4}$$

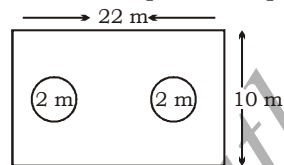
$$\Rightarrow x = \frac{y^2}{3\sqrt{3} \times 4} \Rightarrow x = \frac{y^2}{12\sqrt{3}}$$

$$\Rightarrow 12\sqrt{3}x = y^2$$

On squaring both sides, we get

$$y^4 = 432x^2$$

372. (c) Volume of mud dug out in two hemispherical pitholes



$$= 2 \times \frac{2}{3} \pi r^3 = 2 \times \frac{2}{3} \times \frac{22}{7} \times 2^3$$

$$= \frac{2 \times 2 \times 22 \times 8}{21} = \frac{704}{21} \text{ m}^3$$

Area on which the muds is spread over

$$= \text{Area of field} - \text{Area of pitholes}$$

$$= 1 \times b - 2 \times \pi r^2$$

$$= 22 \times 10 - 2 \times \frac{22}{7} \times 2^2$$

$$= 220 - \frac{176}{7} = \frac{1540-176}{7}$$

$$= \frac{1364}{7} \text{ m}^2$$

Now, let the rise in level by h m, then

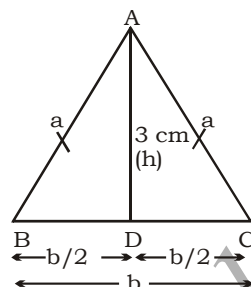
Area of remaining field $\times$ h = Volume of mud dugged out

$$\Rightarrow \frac{1364}{7} \times h = \frac{704}{21}$$

$$\therefore h = \frac{704 \times 7}{1364 \times 21} = \frac{16}{93} \text{ m}$$

373. (a) Area of the $\triangle ABC$

$$= \frac{1}{2} \times b \times h$$



$$\Rightarrow 12 = \frac{1}{2} \times b \times 3$$

$$\therefore b = \frac{12 \times 2}{3} = 8 \text{ cm}$$

$$\text{Here, } BD = CD = \frac{b}{2} = \frac{8}{2} = 4 \text{ cm}$$

In right angled $\triangle ABD$, by pythagoras theorem,

$$AB = \sqrt{BD^2 + AD^2}$$

$$\Rightarrow a = \sqrt{4^2 + 3^2} = \sqrt{16+9}$$

$$= \sqrt{25} = 5 \text{ cm}$$

Now, perimeter of an isosceles triangle

$$= 2a+b=2 \times 5+8=10+8=18 \text{ cm}$$

374. (a) Let the breadth and height of room be b and h m, respectively.

Then, according to the question,

$$\Rightarrow 1 \times b = n \text{ Area occupied by one patient}$$

$$\Rightarrow 14 \times b = 56 \times 2.2$$

$$\Rightarrow b = \frac{56 \times 2.2}{14} = 8.8 \text{ m}$$

Now, total volume of the room is equal to total patients multiplied by volume occupied by each patient.

$$\text{Then, } 14 \times 8.8 \times h = 8.8 \times 56$$

$$\therefore h = \frac{8.8 \times 56}{14 \times 8.8} = 4 \text{ m}$$

375. (a) 1 m = 1000 mm

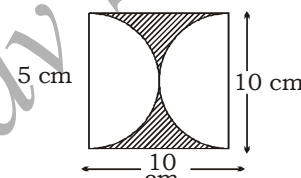
$$\therefore 10 \text{ m} = 10000 \text{ mm}$$

Number of 200 mm lengths that can be cut from 10 m of ribbon

$$= \frac{10000}{200} = 50$$

376. (d) Area between square and semi-circles

= Area of square - 2 Area of semi-circle



$$= (10)^2 - 2 \times \frac{22}{7} \times (5)^2$$

$$= 100 - 78.5 = 21.5 \text{ cm}^2$$

377. (a) Let $l = 4x$ and $b = 9x$

$$\therefore \text{Area of rectangle} = l \times b$$

$$144 = 4x \times 9x$$

$$\Rightarrow x^2 = \frac{144}{36}$$

$$\Rightarrow x^2 = 4 \Rightarrow x = 2$$

$$\text{Now, } l = 4 \times 2 = 8 \text{ cm and}$$

$$b = 9 \times 2 = 18 \text{ cm}$$

$\therefore$ Perimeter of rectangle

$$= 2(l+b) = 2(8+18)$$

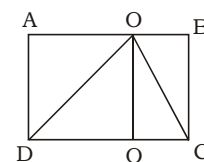
$$= 2 \times 26 = 52 \text{ cm}$$

378. (c) Area of parallelogram

$$= \text{Base} \times \text{Height}$$

$$= 8.06 \times 2.08 = 16.76 \text{ cm}^2$$

379. (c) Given that, $CD = 20 \text{ cm}$ and area of rectangle ABCD = 100 cm^2



$$\Rightarrow AD \times CD = 100 \text{ cm}^2$$

$$\Rightarrow AD \times 20 = 100$$

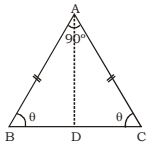
$$\therefore AD = 5 \text{ cm [in rectangle } AB = CD = 20 \text{ cm and } AD = BC = OQ = 5 \text{ cm]}$$

$$\therefore \text{Area of } \triangle ODC = \frac{1}{2} \times PQ \times CD =$$

$$\frac{1}{2} \times 5 \times 20$$

$$= 5 \times 10 = 50 \text{ cm}^2$$

380. (d) Let $AB = AC = a$
 $\therefore BC^2 = AB^2 + AC^2$
 (by Pythagoras theorem)



In $\triangle ABC$,

$$a^2 + a^2 = 2a^2 \Rightarrow BC = a\sqrt{2}$$

$90^\circ + \theta + \theta = 180^\circ$
 (since, sum of all interior angles of any triangle is 180°)

$$\Rightarrow 2\theta = 90^\circ$$

$$\therefore \theta = 45^\circ$$

Now, In $\triangle ABD$,

$$\sin 45^\circ = \frac{AD}{a} \Rightarrow AD = \frac{a}{\sqrt{2}}$$

$$\therefore \text{Area of } \triangle ABC = \frac{1}{2} \times AD \times BC$$

$$= \frac{1}{2} \times a\sqrt{2} \times \frac{a}{\sqrt{2}}$$

$$= 1 \text{ sq unit (given)}$$

$$\Rightarrow \frac{a^2}{2} = 1$$

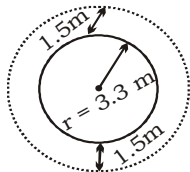
$$\therefore a = \sqrt{2}$$

$\therefore$ Perimeter of $\triangle ABC$

$$= 2a + \sqrt{2}a = 2\sqrt{2} + \sqrt{2} \cdot \sqrt{2}$$

$$= 2(1 + \sqrt{2}) \text{ units}$$

381. (c) Area of path = Area of (fountain + path) - Area of fountain

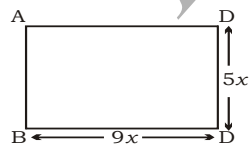


$$= \pi (3.3 + 1.5)^2 - \pi (3.3)^2$$

$$= [(4.8)^2 - (3.3)^2] \pi$$

$$= (23.04 - 10.89) \pi = 12.15 \pi \text{ m}^2$$

382. (c) Let length and breadth of a rectangular field are $9x$ and $5x$



$$\therefore \text{Area of a rectangular field} = 4500 \text{ m}^2$$

$$\Rightarrow 9x \times 5x = 4500$$

$$\Rightarrow x^2 = 100 = (10)^2$$

$$\therefore x = 10$$

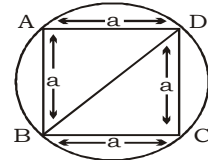
So, the length and breadth of a rectangular field are 90 m and 50 m .

$$\therefore \text{Perimeter of rectangular field} = 2 (\text{Length} + \text{Breadth})$$

$$= 2 (90 + 50) = 2 \times 140 = 280 \text{ m}$$

383. (c) Given that, radius of a circle = 8 cm

and diameter of a circle = 16 cm



$\therefore$ Length of a diameter of a square = Diameter of a circle

$$\Rightarrow a\sqrt{2} = 16$$

$$\therefore a = 8\sqrt{2} \text{ cm}$$

$$\therefore \text{Area of square ABCD} = a^2$$

$$= (8\sqrt{2})^2 = 64 \times 2$$

$$= 128 \text{ sq cm}$$

384. (d) Given that, length of hour hand = 4 cm
 and length of minute hand = 6 cm

$$\therefore \text{Hour hand rotating in 1 day} = 2 \times 360^\circ = 720^\circ$$

$$\therefore \text{Hour hand rotating in 2 days} =$$

$$2 \times 720^\circ = 1440 \times \frac{\pi}{180} \text{ radius}$$

Similarly,

$$\text{Minute hand rotating in 1 day} = 24 \times 360^\circ$$

$$\therefore \text{Minute hand rotating in 3 days}$$

$$= 72 \times 360^\circ \times \frac{\pi}{180} \text{ radius}$$

$$\therefore \text{Distance travelled by hour}$$

$$\text{hand} = 4 \times 1440^\circ \times \frac{\pi}{180^\circ} = 32\pi$$

and distance travelled by minute hand

$$= 6 \times 72^\circ \times 360^\circ \times \frac{\pi}{180}$$

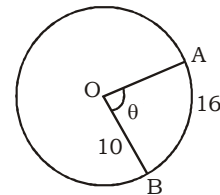
$$= 6 \times 144\pi$$

Required ratio

$$= \frac{32\pi}{6 \times 144\pi}$$

$$= \frac{1}{27}$$

385. (c) Arc of length = $2\pi r \cdot \frac{\theta}{360^\circ}$



$$\Rightarrow 16 = 2\pi r \cdot \frac{\theta}{360^\circ}$$

$$\Rightarrow \frac{\theta}{360^\circ} = \frac{16}{2\pi r}$$

Now, area of sector OAB

$$= \pi r^2 \cdot \frac{\theta}{360^\circ}$$

$$= \pi r^2 \cdot \frac{16}{2\pi r} = 8r = 8 \times 10$$

$$= 80 \text{ sq cm}$$

386. (c) Let the breadth of floor be x metre.

$$\therefore \text{Length} = (x + 20) \text{ metre}$$

$$\therefore \text{Area of the floor} = (x + 20)x \text{ sq. metre}$$

In case II,

$$(x + 10)(x + 5) = x(x + 20)$$

$$\Rightarrow x^2 + 15x + 50 = x^2 + 20x$$

$$\Rightarrow 20x = 15x + 50$$

$$\Rightarrow 5x = 50$$

$$\Rightarrow x = 10 \text{ metre}$$

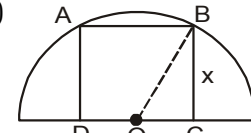
Area of the floor

$$= x(x + 20)$$

$$= 10(10 + 20)$$

$$= 300$$

387. (d)



Here ABCD is a square of side x .

$$\therefore OC = \frac{x}{2}, BC = x, \text{ and}$$

$$OB = \text{radius of circle} = 10 \text{ cm}$$

In $\triangle OCB$,

$$OB^2 = OC^2 + BC^2$$

$$\Rightarrow \left(\frac{x}{2}\right)^2 + x^2 = (10)^2$$

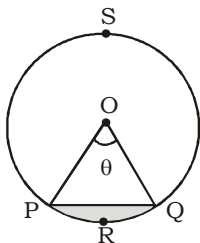
$$\Rightarrow \frac{5x^2}{4} = 100 \Rightarrow x^2 = 80$$

$$\Rightarrow x = 4\sqrt{5} \text{ cm}$$

Hence, perimeter of the square

$$ABCD = 4x = 16\sqrt{5}$$

- 388.(c) 1. We know that, Area of segment (PRQP)
= Area of sector (OPRQO) - Area of ΔOPQ
$$\text{of } \Delta OPQ = \frac{\pi r^2 \theta}{360} - \frac{1}{2} r^2 \sin \theta$$



So, the area of a segment of a circle is always less than area of its corresponding sector.

- II. Distance travelled by a circular wheel of diameter 2 d cm in one revolution

$$= 2\pi \frac{(2d)}{2} = 2 \times 3.14 \times d = 6.28d$$

which is greater than 6d cm.

- 389.(b) Given that, perimeter of a rectangle = 82 m

$$\therefore 2(\text{Length} + \text{Breadth}) = 82 \text{ m}$$

$$\Rightarrow \text{Length} + \text{Breadth} = 41 \text{ m}$$

$$\Rightarrow l + b = 41 \text{ m} \dots (i)$$

$$\text{Also, its area} = 400 \text{ m}^2$$

$$\Rightarrow l \cdot b = 400 \text{ m}^2$$

$$\text{Now, } (l - b)^2 = (l + b)^2 - 4lb$$

$$= (41)^2 - 4(400)$$

$$= 1681 - 1600 = 81$$

$$\therefore l - b = 9 \dots (iii)$$

From Eqs. (i) and (iii),

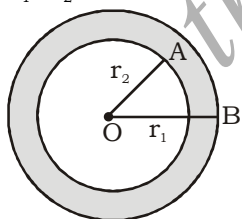
$$2l = 50 \Rightarrow l = 25 \text{ m and}$$

$$b = 16 \text{ m}$$

$$\therefore \text{Required breadth (b)} = 16 \text{ m}$$

- 390.(c) Given that, ratio of their radii = 5:3

$$\text{i.e., } r_1 : r_2 = 5 : 3$$



$$\Rightarrow \frac{r_1}{r_2} = \frac{5}{3} \dots (i)$$

$$\text{Let } r_1 = 5x \text{ and } r_2 = 3x$$

Also, given that, area enclosed between the circumferences of two concentric circles

$$= 16\pi \text{ cm}^2$$

$$\therefore \pi (r_1^2 - r_2^2) = 16\pi$$

$$\Rightarrow (5x)^2 - (3x)^2 = 16$$

$$\Rightarrow 25x^2 - 9x^2 = 16$$

$$\Rightarrow 16x^2 = 16$$

$$\Rightarrow x^2 = 1 \Rightarrow x = 1$$

$$\therefore r_1 = 5 \text{ and } r_2 = 3$$

$$\therefore \text{Area of the outer circle}$$

$$= \pi r_1^2 = \pi (5)^2 = 25\pi \text{ cm}^2$$

- 391.(c) Let the radius of a circle is r and a be the length of the side of a square.

$$\text{Given, circumference of a circle} = \text{Perimeter of a square}$$

$$\Rightarrow 2\pi r = 4a$$

$$\Rightarrow a = \frac{\pi}{2} r = 1.57r$$

$$\text{Now, area of the circle (A}_c\text{)}$$

$$= \pi r^2 = 3.14r^2$$

$$\text{and area of the square (A}_s\text{)}$$

$$= a^2 = 2.4649r^2$$

$$\therefore \text{Area of circle} > \text{Area of square}$$

- 392.(b) Let the radii of two circles are r_1 and r_2 respectively.

Given,

$$\frac{\text{Circumference of 1st circle}}{\text{Circumference of 2nd circle}} = \frac{2}{3}$$

$$\Rightarrow \frac{2\pi r_1}{2\pi r_2} = \frac{2}{3} \Rightarrow \frac{r_1}{r_2} = \frac{2}{3}$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9} \dots (i)$$

$$\therefore \frac{\text{Area of 1st circle}}{\text{Area of 2nd circle}} = \frac{\pi r_1^2}{\pi r_2^2}$$

$$= \left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9}$$

- 393.(b) We know that the radius of a circle inscribed in a

$$\text{equilateral triangle} = \frac{a}{2\sqrt{3}}$$

where, a be the length of the side of an equilateral triangle.

Given that, area of a circle inscribed in an equilateral triangle = 154 cm^2

$$\therefore \pi \left(\frac{a}{2\sqrt{3}}\right)^2 = 154$$

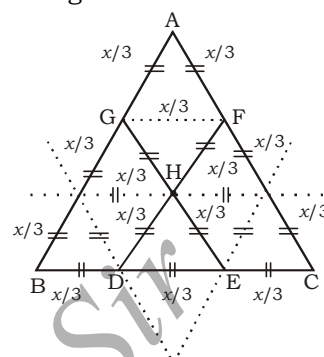
$$\Rightarrow \left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{154 \times 7}{22} = (7)^2$$

$$\Rightarrow a = 14\sqrt{3} \text{ cm}$$

$$\therefore \text{Perimeter of an equilateral triangle} = 3a = 3(14\sqrt{3})$$

$$= 42\sqrt{3} \text{ cm}$$

- 394.(b) ΔABC forms an equilateral triangle.



where, AGHF form a rhombus and ΔHDE is also an equilateral triangle.

$$\therefore \text{Area of rhombus} = (\text{Area of } \Delta AGF + \text{Area of } \Delta GFH)$$

$$= \frac{\sqrt{3}}{4} \left(\frac{x}{3}\right)^2 + \frac{\sqrt{3}}{4} \left(\frac{x}{3}\right)^2$$

$$= 2 \cdot \frac{\sqrt{3}}{4} \left(\frac{x}{3}\right)^2$$

$$\text{Now, area of } \Delta HDE = \frac{\sqrt{3}}{4} \left(\frac{x}{3}\right)^2$$

$$\text{and area of } \Delta ABC = \frac{\sqrt{3}}{4} x^2$$

By given condition,

$$\frac{\text{Area of rhombus AGHF} + \text{Area of } \Delta HDE}{\text{Area of } \Delta ABC}$$

$$= \frac{3 \times \frac{\sqrt{3}}{4} \times \left(\frac{x}{3}\right)^2}{\frac{\sqrt{3}}{4} x^2} = \frac{1}{3}$$

- 395.(c) Given that,

$$\text{Area of the circle} = \text{Area of the square} = (\text{Side})^2$$

$$\pi r^2 = (2\sqrt{\pi})^2 \Rightarrow \pi r^2 = 4\pi \Rightarrow$$

$$r^2 = \frac{4\pi}{\pi} = 4$$

$$\therefore r = \sqrt{4} = 2 \text{ units}$$

$$\therefore \text{Diameter of circle (d)} = 2 \cdot r = 2 \cdot 2 = 4 \text{ units}$$

- 396.(a) Let the radius of circle is r and the side of a square is a, then by given condition,

$$2\pi r = 4a \Rightarrow a = \frac{\pi r}{2}$$

$$\therefore \text{Area of square} = \left(\frac{\pi r}{2}\right)^2$$

$$= \frac{\pi^2 r^2}{4} = \frac{9.86r^2}{4} = 2.46r^2$$

and area of circle = $\pi r^2 = 3.14r^2$
and let the side of equilateral triangle is x.

Then, by given condition,

$$3x = 2\pi r \Rightarrow x = \frac{2\pi r}{3}$$

$$\therefore \text{Area of equilateral triangle} = \frac{\sqrt{3}}{4}$$

$$x^2 = \frac{\sqrt{3}}{4} \times \frac{4\pi^2 r^2}{9}$$

$$= \frac{\pi^2}{3\sqrt{3}} r^2 = 1.89r^2$$

Hence, Area of circle

> Area of square

> Area of equilateral triangle

397. (a) Let the width of the rectangle = x unit

$\therefore$ Length = (2x + 5) unit

According to the question,

$$\text{Area} = x(2x+5)$$

$$\Rightarrow 75 = 2x^2 + 5x$$

$$\Rightarrow 2x^2 + 5x - 75 = 0$$

$$\Rightarrow 2x^2 + 15x - 10x - 75 = 0$$

$$\Rightarrow x(2x+15) - 5(2x+15) = 0$$

$$\Rightarrow (2x+15)(x-5) = 0$$

$$\Rightarrow x = 5 \text{ and } \frac{-15}{2}$$

Since, width cannot be negative.

$\therefore$ Width = 5 units and

length = 2x + 5

$$= 2 \times 5 + 5 = 15 \text{ units}$$

$\therefore$ Perimeter of the rectangle

$$= 2(15+5) = 40 \text{ units}$$

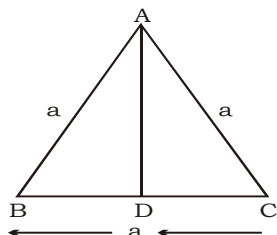
398. (c) Height of equilateral triangle,

$$(AD) = \frac{\sqrt{3}}{2} \times \text{Side}$$

$$\Rightarrow \sqrt{3} = \frac{\sqrt{3} \times \text{side}}{2}$$

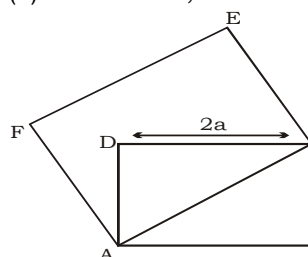
$$\Rightarrow 2\sqrt{3} = \sqrt{3} \times \text{side}$$

$$\therefore \text{Side} = \frac{2\sqrt{3}}{\sqrt{3}} = 2 \text{ cm}$$



$\therefore$ Perimeter of an equilateral triangle = $3a = 3 \times 2 = 6 \text{ cm}$

399. (d) Given that,



Area of rectangle = $2a^2 = l \times b$

$$\Rightarrow l \times b = 2a^2 = l \times a \Rightarrow l = 2a$$

Now, In $\triangle ACD$,

$$AC^2 = AD^2 + CD^2$$

$$a^2 + 4a^2 = 5a^2$$

$\therefore$ Side of square, AC

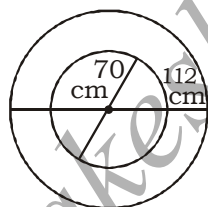
$$= a\sqrt{5} \text{ unit}$$

Hence, area of square

$$= (a\sqrt{5})^2 = 5a^2 \text{ sq units}$$

400. (c) Outer diameter = 112 cm

and inner diameter = 70 cm



$\therefore$ Required area

$$= \frac{1}{4} \pi (112^2 - 70^2)$$

$$= \frac{1}{4} (12544 - 4900) \pi$$

$$= \frac{1}{4} \times 7644 \times \frac{22}{7}$$

$$= \frac{1}{4} \times 24024 = 6006 \text{ cm}^2$$

401. (d) Let the length of altitude AB = 1

By given condition,

Area of $\triangle ABC$ = Area of square

$$\therefore \frac{1}{2} \times \text{Base} \times \text{Altitude} = (\text{Side})^2$$

$$\Rightarrow \frac{1}{2} \times 9 \times 1 = 36 \Rightarrow l = \frac{36 \times 2}{9}$$

$$\therefore l = 8 \text{ cm}$$

402. (d) Area of equilateral triangle

$$= \frac{\sqrt{3}}{4} a^2$$

$$\text{But altitude} = \frac{\sqrt{3}}{2} a$$

$$\Rightarrow 2\sqrt{3} = \frac{\sqrt{3}}{2} a$$

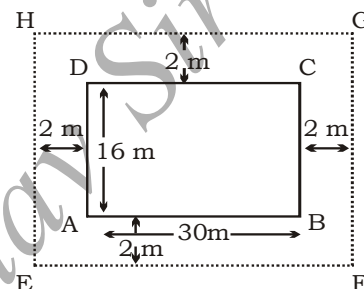
$$\Rightarrow a = 4 \text{ cm}$$

$\therefore$ Area of equilateral triangle =

$$\frac{\sqrt{3}}{4} (4)^2 = 4\sqrt{3} \text{ cm}^2$$

403. (a) Required area of the path

$$EF = 30 + 4 = 34 \text{ m}, GF = 16 + 4 = 20 \text{ m}$$

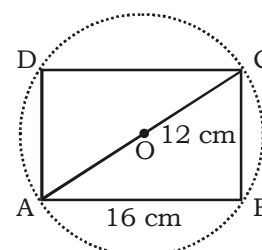


$\therefore$ Area of path = Area of EFGH - Area of ABCD

$$= 34 \times 20 - 30 \times 16$$

$$= 680 - 480 = 200 \text{ m}^2$$

404. (c) In right $\triangle ABC$,



$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (16)^2 + (12)^2$$

$$= 256 + 144 = 400$$

$$\Rightarrow AC = 20 \text{ cm}$$

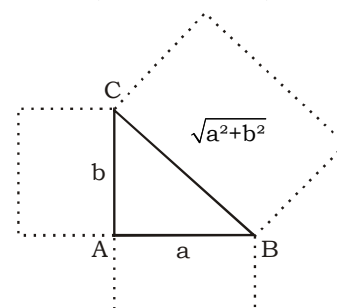
$\therefore$ AO = 10 cm (radius)

and area of circumcircle = πr^2

$$= \pi \times (10)^2 = 100\pi \text{ cm}^2$$

405. (c) In $\triangle ABC$,

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{a^2 + b^2}$$



$\therefore$ Required total area = $a^2 + b^2$

$$+ \left(\sqrt{a^2 + b^2} \right)^2 + \frac{1}{2} ab$$

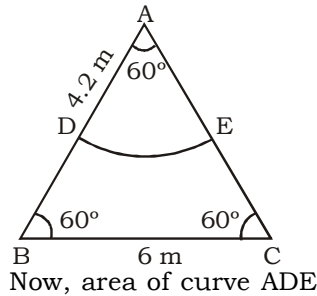
$$= 2(a^2 + b^2) + 0.5ab$$

406.(b) Area of trapezium

$$= \frac{1}{2} (3+5) \times 4 = 16 \text{ m}^2$$

$\therefore$ Total cost of painting Rs. 25 per sq m = $16 \times 25 = \text{Rs. } 400$

407.(c) Suppose a horse is tied at vertex A. Then, area available grazing field is ADE.



$$= \frac{\pi r^2 \theta}{360^\circ}$$

$$= \frac{22 \times (4.2)^2 \times 60^\circ}{7 \times 360^\circ} = 9.24 \text{ m}^2$$

and area of equilateral $\triangle ABC$

$$= \frac{\sqrt{3}}{4} (\text{side})^2 = \frac{\sqrt{3}}{4} \times (6)^2 = 15.57$$

$\therefore$ Required percentage

$$= \frac{9.24}{15.57} \times 100$$

$$= 59.34\% = 59\% \text{ (approx)}$$

408.(a) By given condition,

$$\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{1}{2}$$

$$\Rightarrow \left(\frac{r_1}{r_2} \right)^2 = \frac{1}{2} \quad \dots (i)$$

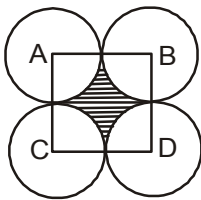
If circle or bent in the form of square, then $2\pi r_1 = 4a_1$

$$\Rightarrow a_1 = \frac{\pi r_1}{2} \text{ and } a^2 = \frac{\pi r_2}{2}$$

$$\therefore \frac{A_1}{A_2} = \frac{a_1^2}{a_2^2} = \frac{\left(\frac{\pi r_1}{2} \right)^2}{\left(\frac{\pi r_2}{2} \right)^2} = \frac{r_1^2}{r_2^2} = \frac{1}{2}$$

[from Eq. (i)]

409.(b)



Area of the shaded region

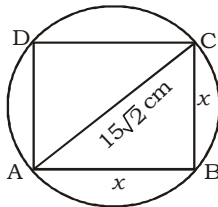
= Area of square of side 6cm - $4 \times$ a right angled sector

$$= 36 - 4 \times \frac{\pi \times 3^2}{4}$$

$$= 36 - 9\pi = 9(4 - \pi) \text{ sq. cm.}$$

410.(a) Let the sides of a square be x cm,

In $\triangle ABC$, $AC^2 = AB^2 + BC^2$



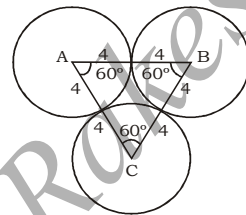
$$AC^2 = x^2 + x^2$$

$$\Rightarrow (15\sqrt{2})^2 = 2x^2 \Rightarrow 2x^2 = 225 \times 2$$

$$\Rightarrow x^2 = 225 \Rightarrow x = 15 \text{ cm}$$

Hence, length of the side of the square be 15 cm.

411.(b) Since, all sides of a $\triangle ABC$ are equal, so their all angles are equal to 60°



Area of Portion include between circles

= Area of triangle - Area of 3 sectors

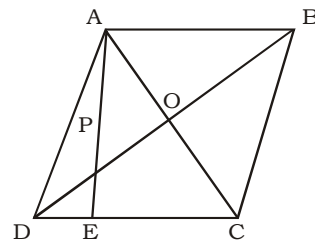
$$= \frac{\sqrt{3}}{4} (8)^2 - 3 \times \frac{60^\circ}{360^\circ} \times \pi (4)^2$$

$$= (16\sqrt{3} - 8\pi) \text{ cm}^2$$

412.(a) Area of rhombus

$$= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 55 \times 48$$

$$= 1320 \text{ cm}$$



$\therefore$ Area of rhombus

= Base $\times$ Height = DC $\times$ AE

$$\Rightarrow DC \times AE = 1320$$

$$\Rightarrow p \times \sqrt{OD^2 + OC^2} = 1320$$

$$\Rightarrow p \times \sqrt{\left(\frac{55}{2} \right)^2 + \left(\frac{48}{2} \right)^2} = 1320$$

$$\left(\because OD = \frac{1}{2} BD \text{ and } OC = \frac{1}{2} AC \right)$$

$$\Rightarrow p \times \sqrt{\frac{5329}{4}} = 1320$$

$$\Rightarrow p = \frac{1320}{36.5} = 36.16$$

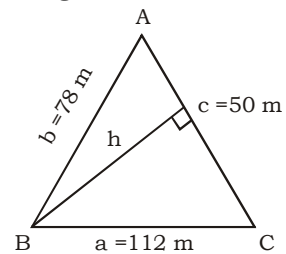
$$36 \text{ cm} < p < 37 \text{ cm}$$

413.(c) Perimeter = a + b + c

$$240 = a + 78 + 50$$

$$a = 112$$

$\therefore$ Area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$



and also, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$$\therefore \text{Area of } = \sqrt{120(120-112)(120-78)(120-50)}$$

$$= \sqrt{120 \times 8 \times 42 \times 70}$$

$$= 1680 \text{ m}^2$$

$\therefore$ Area of triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\Rightarrow 1680 = \frac{1}{2} \times 50 \times h$$

$$\therefore h = \frac{2 \times 1680}{50} = 67.2 \text{ m}$$

414.(b) Let the sides of isosceles triangle be 5x, 5x and 3x cm, respectively.

By given condition,

Perimeter of isosceles triangle = Length of wire

$$5x + 5x + 3x = 78 \Rightarrow 13x = 78$$

$$\Rightarrow x = 6 \text{ cm}$$

$$\therefore \text{Length of base} = 3 \times 6 = 18 \text{ cm}$$

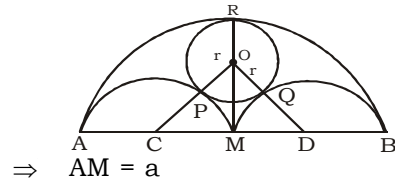
415.(b) The angle made by the minute hand in 20 min = 120° , (1 minute = 6°)

$\therefore$ The area swept by the minute hand in 20 min

$$= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{120^\circ}{360^\circ} \times 3.14 \times 9 \times 9$$

$$= 84.78 \text{ cm}^2$$

416.(c) Given, $AB = 2a$



$\Rightarrow AM = a$

and $AC = CM = MD = BD = \frac{a}{2}$

Now, $OC = OP + PC = OP + CM$

$= r + \frac{a}{2}$ and

$OD = OQ + QD = OQ + MD = r + \frac{a}{2}$

So, $\triangle OCD$ is an isosceles triangle. ($\because OC = OD$)

$\Rightarrow \angle OMC = 90^\circ$

In $\triangle OMC$,

$OC^2 = OM^2 + CM^2$

$$\Rightarrow \left(r + \frac{a}{2}\right)^2 = (a-r)^2 + \left(\frac{a}{2}\right)^2$$

$$\Rightarrow r^2 + \frac{a^2}{4} + ar$$

$$= a^2 + r^2 - 2ar + \frac{a^2}{4}$$

$$\therefore r = \frac{a}{3}$$

417.(b) Let r be the radius of circle and a be the side of square.

By given condition, $2\pi r = 4a$

$$\Rightarrow a = \frac{\pi r}{2}$$

$$\therefore \text{Area of square} = \left(\frac{\pi r}{2}\right)^2$$

$$= \frac{\pi^2 r^2}{4} = \frac{9.86 r^2}{4} = 2.46 r^2$$

$$\text{and area of circle} = \pi r^2 = 3.14 r^2$$

Hence, area of the circle is larger than that of square.

418.(d) Let $a = 35$ cm, $b = 44$ cm and $c = 75$ cm

$$\therefore s = \frac{a+b+c}{2}$$

$$= \frac{35+44+75}{2} = 77$$

Now, Area of Δ

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{77 \times 42 \times 33 \times 2}$$

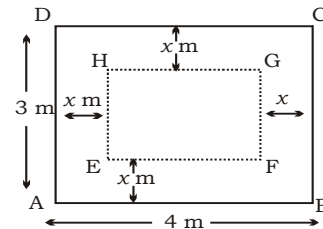
$$= \sqrt{7 \times 11 \times 2 \times 3 \times 7 \times 3 \times 11 \times 2}$$

$$= 7 \times 11 \times 2 \times 3 = 462 \text{ cm}^2$$

Hence, radius of incircle

$$= \frac{\Delta}{s} = \frac{462}{77} = 6 \text{ cm}$$

419.(b) Width of the border = x m



Given, area of EFGH = 6 m^2

$$\Rightarrow (4-2x)(3-2x) = 6$$

$$\Rightarrow 12 - 8x - 6x + 4x^2 = 6$$

$$\Rightarrow 4x^2 - 14x + 12 = 6$$

$$\Rightarrow 4x^2 - 14x + 6 = 0$$

$$\Rightarrow 2x^2 - 7x + 3 = 0$$

$$\Rightarrow 2x^2 - 6x - x + 3 = 0$$

$$\Rightarrow 2x(x-3) - 1(x-3) = 0$$

$$\Rightarrow (x-3)(2x-1) = 0$$

$$\therefore x = 3, \frac{1}{2}$$

$$\therefore x = \frac{1}{2} = 0.5 (\because x \neq 3)$$

420.(b) Inner radius, $r_1 = 25$ cm

and external radius, $r_2 = 25 + 10 = 35$ cm

Distance covered in 1 revolution

$$= 2\pi \times 35 = 70 \times \frac{22}{7}$$

$$= 220 \text{ cm.}$$

and distance covered in 10 revolutions = 2200 cm

$\therefore$ Speed of bicycle

$$= \frac{\text{Covered distance}}{\text{Time}}$$

$$= \frac{2200}{5} \text{ cm/s} = \frac{22}{5} \text{ m/s}$$

$$= 4.4 \text{ m/s}$$

421.(c) Length of wire = 36 cm

$\therefore$ Perimeter of semi-circle

$$= \pi r + 2r$$

$$\Rightarrow 36 = r \left(\frac{22}{7} + 2 \right)$$

{ $\because$ Perimeter of semi-circle = Length of wire}

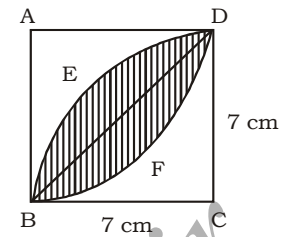
$$\Rightarrow r = \frac{36 \times 7}{36} = 7 \text{ cm}$$

Hence, radius of semi-circle = 7 cm

422.(c) Area of curve BCDE =

$$\frac{1}{4} \pi (7)^2 = \frac{22}{7 \times 4} \times 7 \times 7 = \frac{77}{2} \text{ cm}^2$$

$$\text{Area of } \triangle BCD = \frac{1}{2} \times 7 \times 7 = \frac{49}{2} \text{ cm}^2$$



$\therefore$ Required area of shaded region = 2 Area of curve BEDF

$$= 2 \left(\frac{77}{2} - \frac{49}{2} \right) = 2 \left(\frac{28}{2} \right) = 28 \text{ cm}^2$$

423.(a) Let sides of a rectangle be l and b .

Then, $2(l+b) = 18 \Rightarrow l+b = 9$

$\therefore$ Area of rectangle = $l \times b$

For maximum, area of rectangle, $l=b$

$$2l = 9 \Rightarrow l = 4.5$$

$\therefore$ Maximum area of rectangle = $l \times b = (4.5)^2 = 20.25 \text{ cm}^2$

424.(b) Let r = Radius of 3 smaller laminas

In $\triangle ADC$, $(2r)^2 = r^2 + DC^2$

$$\Rightarrow DC = \sqrt{3}r \therefore OC = \frac{2}{3} DC$$

$$= \frac{2}{3} \times \sqrt{3}r = \frac{2r}{\sqrt{3}}$$

Radius of larger circular lamina = OE

$$OC + CE = \frac{2r}{\sqrt{3}} + r = \frac{(2+\sqrt{3})r}{\sqrt{3}}$$

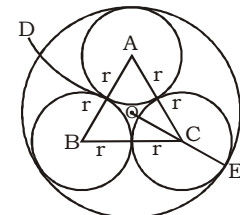
Area of 3 laminas = $3\pi r^2$

Area of larger lamina

$$= \pi \left[\frac{(2+\sqrt{3})r}{\sqrt{3}} \right]^2$$

$$= \pi \frac{(4+3+4\sqrt{3})}{3} r^2$$

$$= \frac{(7+4\sqrt{3})\pi r^2}{3}$$



Residual area

$$= \left(\frac{7+4\sqrt{3}}{3} - 3 \right) \pi r^2$$

$$= \frac{(4\sqrt{3}-2)}{3} \pi r^2$$

∴ Required ratio

$$= \frac{\left(\frac{4\sqrt{3}-2}{3}\right) \pi r^2}{\frac{7+4\sqrt{3}}{3} \pi r^2}$$

$$= \frac{4\sqrt{3}-2}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{28\sqrt{3}-48-14+8\sqrt{3}}{49-48}$$

$$= 36\sqrt{3} - 62 = 36 \times 1.732 - 62$$

$$= 62.352 - 62 = 0.35$$

425. (a) Circumference of circle
 $= 2\pi \times 42$

$$= 2 \times \frac{22}{7} \times 42 = 264 \text{ cm}$$

Perimeter of square = $4x$

$$\Rightarrow 264 = 4x$$

$$\therefore x = 66 \text{ cm}$$

426. (a) Area of 2 bigger semi-

$$\text{circles} = 2 \times \frac{\pi r^2}{2}$$

$$= 2\pi \left(\frac{0.5}{2}\right)^2 \times \frac{1}{2} = \frac{0.25\pi}{4} \text{ cm}^2$$

and area of 5 smaller semi -

$$\text{circles} = \frac{5\pi r^2}{2} = 5 \times \pi \times \frac{1}{2} \times \left(\frac{0.5}{4}\right)^2$$

$$= \frac{5\pi}{2} \times \frac{0.25}{16} = \frac{1.25}{32} \text{ cm}^2$$

Area of rectangle ABCD

$$= 2 \times 0.5 = 1 \text{ cm}^2$$

Area of remaining portion

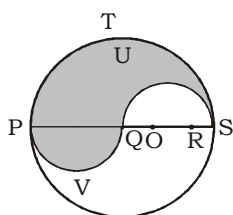
$$= 1 - \frac{0.25\pi}{4} - \frac{1.25\pi}{32}$$

$$= 1 - \frac{\pi}{16} - \frac{5\pi}{128}$$

$$= \frac{128 - 8\pi - 5\pi}{128}$$

$$= \frac{128 - 13\pi}{128} \text{ cm}^2$$

427. (a) Given, OS = 6 cm



$$\therefore PQ = QR = RS = 4 \text{ cm}$$

∴ Perimeter of shaded region

= Perimeter of semi-circle PTS

+ Perimeter of semi-circle QUS

+ Perimeter of semi-circle PVQ

$$= \pi(6) + \pi(4) + \pi(2) = 12\pi \text{ cm}$$

428. (b) Circumference of circular

$$\text{path} = 2\pi \times 50 \text{ m}$$

$$= 10000\pi \text{ cm}$$

and circumference of wheel

$$= 2\pi \times 50 = 100\pi \text{ cm}$$

∴ Distance covered in 60 min =

$$10000\pi \text{ cm}$$

Distance covered in 15 min

$$= \frac{10000}{60} \pi \times 15 = 2500\pi \text{ cm}$$

∴ Number of revolutions

$$= \frac{2500\pi}{100\pi} = 25$$

429. (b) The distance covered by a man diagonally is

$$d = \frac{3 \times 1000}{60} \times 1 = 50 \text{ m}$$

$$\therefore \text{Area of field} = \frac{1}{2} d^2$$

$$= \frac{1}{2} \times (50)^2 = 1250 \text{ m}^2$$

430. (c) Let the side of an square be a cm.

By given condition,

Area of square - Area of an

$$\text{equilateral triangle} = \frac{1}{4}$$

$$\Rightarrow a^2 - \frac{\sqrt{3}}{4} a^2 = \frac{1}{4}$$

$$\Rightarrow a^2 \left(1 - \frac{\sqrt{3}}{4}\right) = \frac{1}{4}$$

$$\Rightarrow a^2 (4 - \sqrt{3}) = 1 \Rightarrow a^2 = \frac{1}{4 - \sqrt{3}}$$

$$\therefore a = (4 - \sqrt{3})^{-1/2} \text{ cm}$$

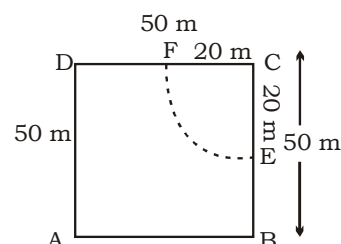
431. (d) Suppose a pole is fixed at a point C.

∴ Area of field in which the horse

can graze = Area of field in

which the horse can graze =

Area of curve CFE



$$= \frac{1}{4} (\pi r^2) = \frac{3.14 \times 20 \times 20}{4}$$

$$= 314 \text{ m}^2$$

432. (a) Here width of sheet is 20 cm, which is the maximum diameter of the circular sheet.

∴ Remaining area of sheet

= Area of rectangle sheet

- Area of circular sheet

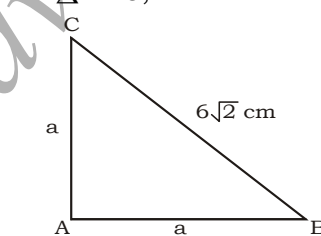
$$= 25 \times 20 - \pi(10)^2$$

$$= 500 - 314$$

$$= 186 \text{ cm}^2$$

433. (b) Let the other sides of a right isosceles triangle be a cm.

In $\triangle ABC$,



$$\sqrt{a^2 + a^2} = 6\sqrt{2}$$

$$\Rightarrow a\sqrt{2} = 6\sqrt{2}$$

$$\Rightarrow a = 6 \text{ cm}$$

$$\therefore \text{Area of } \triangle ABC = \frac{1}{2} \times a^2$$

$$= \frac{1}{2} \times 6 \times 6 = 18 \text{ cm}^2$$

434. (b) $a = 9 \text{ cm}$, $b = 10 \text{ cm}$
 and $c = 11 \text{ cm}$

$$\therefore s = \frac{9+10+11}{2} = 15 \text{ cm}$$

$$\therefore A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15(15-9)(15-10)(15-11)}$$

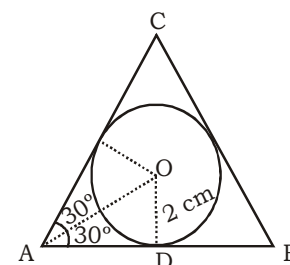
$$= \sqrt{15 \times 6 \times 5 \times 4} = 30\sqrt{2} = 42.3 \text{ cm}^2$$

435. (a) As we know that, if the length of square and rhombus are same, then the area should be same.

436. (a) Area of circle = $4\pi \text{ cm}^2$

(given)

$$\Rightarrow \pi r^2 = 4\pi \Rightarrow r = 2 \text{ cm}$$



$$\text{In } \triangle OAD, \tan 30^\circ = \frac{OD}{AD}$$

$$\Rightarrow AD = 2\sqrt{3} \text{ cm}$$

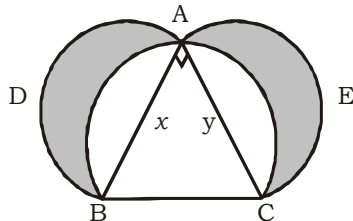
$$\text{Now, } AB = 2 AD = 4\sqrt{3} \text{ cm}$$

$$\therefore \text{Area of equilateral } \triangle ABC$$

$$= \frac{\sqrt{3}}{4} (AB)^2 = \frac{\sqrt{3}}{4} (4\sqrt{3})^2$$

$$= 12\sqrt{3} \text{ cm}^2$$

$$437. (a) \text{ In } \triangle ABC,$$



$$BC = \sqrt{x^2 + y^2}$$

$$\therefore \text{Area of } \triangle ABC$$

$$= \frac{1}{2} \times x \times y = \frac{1}{2} xy$$

$$\text{Area of semi-circle BACB}$$

$$= \frac{\pi(x^2 + y^2)}{4}$$

$$\therefore \text{Area of shaded portion}$$

$$= \text{Semi-circle ABDA} \\ + \text{Area of semi-circle AECA} - \\ (\text{Area of semi-circle BACB} - \\ \text{Area of } \triangle ABC)$$

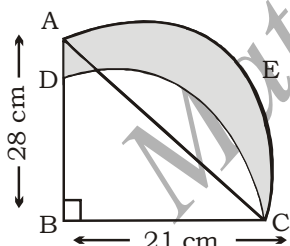
$$= \frac{\pi x^2}{4} + \frac{\pi y^2}{4} - \pi \left(\frac{x^2 + y^2}{4} \right) + \text{Area}$$

$$\text{of } \triangle ABC = \text{Area of } \triangle ABC$$

$$438. (d) \text{ In } \triangle ABC,$$

$$AC^2 = \sqrt{28^2 + 21^2} = \sqrt{784 + 441}$$

$$= \sqrt{1225} \Rightarrow AC = 35 \text{ cm}$$



$$\text{Area of shaded portion} = \text{Area of Semi-circle ACE}$$

$$+ \text{Area of } \triangle ABC - \text{Area of quadrant circle BCD}$$

$$= \frac{\pi r^2}{2} + \frac{1}{2} \times BC \times BA - \frac{\pi}{4} \times r^2$$

$$= \frac{22}{7} \times \frac{1}{2} \times \frac{35}{2} \times \frac{35}{2} + \frac{1}{2} \times 21$$

$$\times 28 - \frac{22}{7 \times 4} \times 21 \times 21$$

$$= \frac{5 \times 11 \times 35}{4} + \frac{1}{2} (21 \times 28 - 33 \times 21)$$

$$= \frac{1925}{4} + \frac{1}{2} (-105)$$

$$= 481.25 - 52.50 = 428.75 \text{ cm}^2$$

$$439. (b) \text{ Let the sides of squares } S_1 \text{ and } S_2 \text{ be } a \text{ and } b, \text{ respectively. So, perimeters of square } S_1 \text{ and } S_2 \text{ are } 4a \text{ and } 4b, \text{ respectively. By given condition,}$$

$$4a = 4b + 12$$

$$\Rightarrow a = b + 3$$

$$\dots (i)$$

$$\text{and } a^2 = 3(b)^2 - 11$$

$$\Rightarrow (b+3)^2 = 3b^2 - 11$$

$$\Rightarrow b^2 + 6b + 9 = 3b^2 - 11$$

$$\Rightarrow 2b^2 - 6b - 20 = 0$$

$$\Rightarrow 2b^2 - 10b + 4b - 20 = 0$$

$$\Rightarrow 2b(b-5) + 4(b-5) = 0$$

$$\Rightarrow (b-5)(2b+4) = 0$$

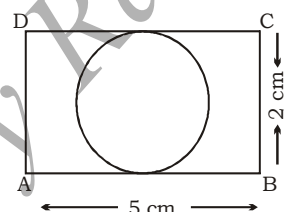
$$\Rightarrow b = 5 \text{ m } (\because b \neq -2)$$

$$\text{On putting the value of } b \text{ in Eq. (i), we get}$$

$$a = 5 + 3 = 8$$

$$\therefore \text{Perimeter of } S_1 = 4 \times 8 = 32 \text{ m}$$

$$440. (b) \text{ From a rectangular sheet of cardboard of size } 5 \times 2 \text{ cm}^2, \text{ a circle of radius } 1 \text{ cm, can be cut-off.}$$



$$\text{Area of rectangular sheet}$$

$$= 5 \times 2 = 10 \text{ cm}^2$$

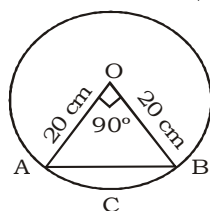
$$\text{and area of circle}$$

$$= \pi (1)^2 = \pi \text{ cm}^2$$

$$\therefore \text{Required area} = \text{Area of sheet} - \text{Area of circle}$$

$$= (10 - \pi) \text{ cm}^2$$

$$441. (d) \text{ Area of } \triangle AOB = \frac{1}{2} \times OA \times OB$$



$$= \frac{1}{2} \times 20 \times 20 = 200 \text{ cm}^2$$

$$\text{and area of sector OACBO}$$

$$= \frac{\pi r^2 \theta}{360^\circ} = \frac{3.14 \times 20 \times 20 \times 90^\circ}{360^\circ}$$

$$= \frac{3.14 \times 400}{4} = 314 \text{ cm}^2$$

$$\therefore \text{Area of minor segment}$$

$$= \text{Area of sector OACBO} - \text{Area of } \triangle AOB = 314 - 200 = 114 \text{ cm}^2$$

$$442. (b) \text{ Angle made in 60 min by minute hand of a clock} = 360^\circ$$

$$\text{and angle made in 15 min by minute hand of a clock}$$

$$= \frac{360^\circ}{60^\circ} \times 15^\circ = 90^\circ$$

$$\therefore \text{Required distance}$$

$$= \frac{2\pi(14)90^\circ}{360^\circ} = \frac{22}{7} \times \frac{14 \times 2}{4}$$

$$= 22 \text{ cm.}$$

$$443. (c) \text{ Let the length of rectangle} = \frac{x}{2}$$

$$\text{and breadth of rectangle} = \frac{x}{6}$$

$$\therefore \text{Area of rectangle}$$

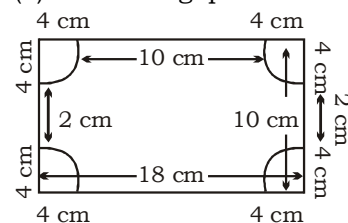
$$= \frac{x}{2} \times \frac{x}{6} = \frac{x^2}{12}$$

$$\text{and area of square} = x^2$$

$$\text{Hence, area of remaining}$$

$$\text{portion} = x^2 - \frac{x^2}{12} = \frac{11x^2}{12}$$

$$444. (b) \text{ Remaining perimeter}$$



$$= \left(\frac{2\pi r}{4} \right) 4 + 10 + 2 + 10 + 2$$

$$= 2 \times 3.14 \times 2 + 24$$

$$= 25.12 + 24 = 49.12 \text{ cm}$$

$$= 49.1 \text{ cm (approx)}$$

$$445. (b) \text{ Distance travel in 1 revolution}$$

$$= \frac{440}{1000} \text{ m}$$

$$\text{and circumference} = \pi \times d$$

$$= \frac{44000}{1000} \text{ cm}$$

$$\therefore d = \frac{44000 \times 7}{1000 \times 22} = 14 \text{ cm}$$

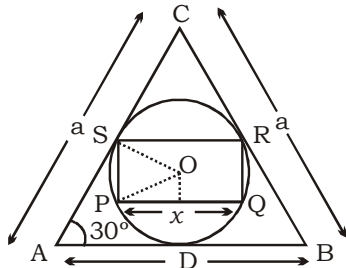
$$446. (c) \text{ Side of an equilateral triangle is } a$$

then the altitude of equilateral

triangle is $\frac{\sqrt{3}}{2}a$

$\therefore$ Radius of incircle

$$= \frac{a\sqrt{3}}{2} \times \frac{1}{3} = \frac{a}{2\sqrt{3}}$$



$\therefore$ Diameter of incircle

$$= 2 \left(\frac{a}{2\sqrt{3}} \right) = \frac{a}{\sqrt{3}}$$

Let side of a square be x .

$$\therefore \left(\frac{a}{\sqrt{3}} \right)^2 = x^2 + x^2$$

$$\Rightarrow \frac{a^2}{3} = 2x^2$$

$$\Rightarrow x^2 = \frac{a^2}{6} = \text{Area of square}$$

447. (c) Radius of circle, $r = 6$ cm

$$\therefore \text{Area of circle} = \pi r^2 = \pi \times 6^2 = 36\pi \text{ cm}^2$$

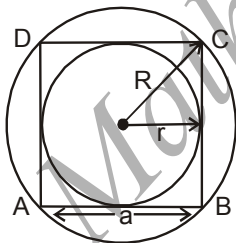
and area of sector subtending an angle of 80° at O

$$\frac{\pi r^2 \theta}{360^\circ} = \frac{\pi \times 6^2 \times 80^\circ}{360^\circ} = 8\pi \text{ cm}^2$$

$\therefore$ Required difference

$$= 36\pi - 8\pi = 28\pi \text{ cm}^2$$

448. (a)



Radius of incircle $= r = \frac{a}{2}$

and Radius of circum-circle

$$= \frac{a}{\sqrt{2}} = R$$

$$\therefore \text{Ratio of area} = \frac{\pi r^2}{\pi R^2} = \frac{r^2}{R^2}$$

$$= \frac{a^2/4}{a^2/2} = \frac{2}{4} = \frac{1}{2} = 1:2$$

449. (a) Larger Radius (R) = $14 + 7 = 21$ cm

Smaller Radius (S) = 7 cm

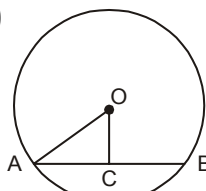
$\therefore$ Area of shaded portion

$$= \pi R^2 \frac{\theta}{360^\circ} - \pi r^2 \frac{\theta}{360^\circ}$$

$$= \pi \frac{30^\circ}{360^\circ} (21 \times 21 - 7 \times 7)$$

$$= \frac{22}{7} \times \frac{1}{12} \times 28 \times 14 = 102.67 \text{ cm}^2$$

450. (c)



OC = 12 cm AC = CB = 5 cm

$\therefore$ Radius 'OA' = $\sqrt{OC^2 + AC^2}$

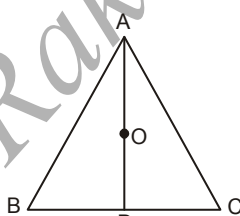
$$= \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$$

$$= \sqrt{169} = 13 \text{ cm}$$

$\therefore$ Diameter of circle

$$= 2 \times 13 = 26 \text{ cm}$$

451. (d)



DB = DC = 3 cm.

$$AD = \sqrt{AB^2 - BD^2} = \sqrt{6^2 - 3^2}$$

$$= \sqrt{36 - 9} = \sqrt{27} = 3\sqrt{3} \text{ cm.}$$

$\therefore$ OD = In-radius

$$= \frac{1}{3} \times 3\sqrt{3} = \sqrt{3} \text{ cm.}$$

$\therefore$ Area of circle = πr^2

$$= \pi \times \sqrt{3} \times \sqrt{3} = 3\pi \text{ sq. cm}$$

Alternatively :

$$\text{Inradius}(r) = \frac{a}{2\sqrt{3}} = \frac{6}{2\sqrt{3}} = \sqrt{3}$$

$$\therefore \text{Area} = \pi r^2 = \pi (\sqrt{3})^2 = 3\pi$$

452. (d) Area of parallelogram

= base $\times$ height

$$= 27 \times 12 = 324 \text{ sq. cm.}$$

Again,

$$324 = 36 \times h$$

$$\Rightarrow h = \frac{324}{36} = 9 \text{ cm}$$

453. (d) $2\pi r = 2(18 + 26)$

$$2 \times \frac{22}{7} \times r = 44 \times 2 \Rightarrow r = 14 \text{ cm}$$

$\therefore$ Area of circle = πr^2

$$= \frac{22}{7} \times 14 \times 14 = 616 \text{ sq. cm.}$$

454. (d) $\pi r + 2r = 36$

$$\Rightarrow r \left(\frac{22}{7} + 2 \right) = 36$$

$$\Rightarrow r \left(\frac{22 + 14}{7} \right) = 36$$

$$\Rightarrow r = \frac{36 \times 7}{36} = 7 \text{ metre}$$

$$\text{Area} = \frac{\pi r^2}{2} = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

$$= 77 \text{ sq. metre}$$

455. (b) Let the radius of circle be 'r'

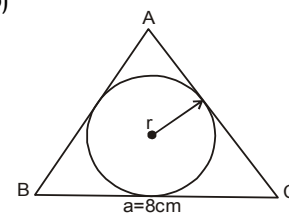
$$\Rightarrow 2\pi r = 24\pi \Rightarrow r = 12$$

$\therefore$ Area of circle

$$= \pi (12)^2 = 144\pi$$

$\therefore$ Area of the rectangle = area of circle = 144π

456. (b)



$$\text{Area of triangle} = \frac{\sqrt{3}}{4} a^2$$

$$= \frac{\sqrt{3}}{4} \times 8 \times 8 = 16\sqrt{3} \text{ cm}^2$$

radius of incircle (r)

$$= \frac{a}{2\sqrt{3}} = \frac{8}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$$

Area of inscribed circle = πr^2

$$\pi \left(\frac{4}{\sqrt{3}} \right)^2 = \frac{22}{7} \times \frac{16}{3}$$

$\therefore$ Required area

$$= \left(16\sqrt{3} - \frac{22 \times 16}{21} \right)$$

$$= \frac{16}{21}(21 \times 1.732 - 22)$$

$$= \frac{16}{21}(14.372) = 10.95 \text{ cm}^2$$

457.(b) height = h

$$= \frac{\sqrt{3}}{2}a \quad (a \text{ side of } \triangle)$$

$$\text{inradius} = \frac{a}{2\sqrt{3}}$$

incircle's diameter (d)

$$= \frac{2a}{2\sqrt{3}} = \frac{a}{\sqrt{3}}$$

$$\text{circum-radius} = \frac{a}{\sqrt{3}}$$

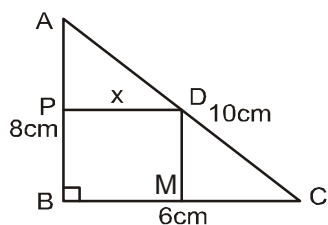
$\Rightarrow$ circumcircle's diameter

$$(D) = \frac{2a}{\sqrt{3}}$$

$\therefore d : D : H$

$$= \frac{a}{\sqrt{3}} : \frac{2a}{\sqrt{3}} : \frac{\sqrt{3}}{2}a = 2 : 4 : 3$$

458. (a)



Side of maximum sized square

$$\frac{AB \times BC}{AB + BC}$$

$$\frac{8 \times 6}{8 + 6} = \frac{24}{7}$$

$\therefore$ area of square

$$= \left(\frac{24}{7}\right)^2 = \frac{576}{49} \text{ cm}^2$$

Alternatively,

let side of largest square = x

$\therefore AP = (8 - x) \text{ cm}$ and

$MC = (6 - x) \text{ cm}$

In $\triangle ABC$ and $\triangle DMC$,

$\angle B = \angle M = 90^\circ$

$\angle C = \angle C$ (common)

$\triangle ABC \sim \triangle DMC$,

$$\Rightarrow \frac{BC}{MC} = \frac{AB}{DM} \Rightarrow \frac{6}{6-x} = \frac{8}{x}$$

$$\Rightarrow 3x = 24 - 4x \Rightarrow x = \frac{24}{7} \text{ cm}$$

$\therefore$ area of square

$$= x^2 = \frac{576}{49} \text{ cm}^2$$

459.(b) For $3x + 4y = 12$

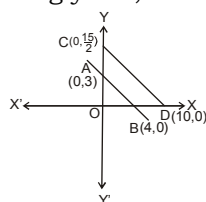
By putting $x = 0$, $y = 3$

By putting $y = 0$, $x = 4$

For $6x + 8y = 60$,

By putting $x = 0$, $y = \frac{15}{2}$

By putting $y = 0$, $x = 10$



$$\therefore \text{Area of } \triangle OCD = \frac{1}{2} \times OD \times OC$$

$$= \frac{1}{2} \times 10 \times \frac{15}{2} = \frac{75}{2}$$

$\therefore$ Area of $\triangle OAB$

$$= \frac{1}{2} \times OB \times OA = \frac{1}{2} \times 4 \times 3 = 6$$

$$\therefore \text{Area of trapezium} = \frac{75}{2} - 6$$

$$= \frac{75 - 12}{2} = \frac{63}{2} = 31.5 \text{ sq. units}$$

460. (c) Side of the first square

$$= \sqrt{\text{Area}}$$

$$= \sqrt{200} = 10\sqrt{2} \text{ metre}$$

Its diagonal = $\sqrt{2} \times \text{side}$

$$= 10\sqrt{2} \times \sqrt{2}$$

$$= 20 \text{ metre}$$

$\therefore$ Diagonal of new square

$$= \sqrt{2} \times 20 = 20\sqrt{2} \text{ metre}$$

$$\therefore \text{Its area} = \frac{1}{2} \times (\text{diagonal})^2$$

$$= \frac{1}{2} \times 20\sqrt{2} \times 20\sqrt{2}$$

$$= 400 \text{ sq. metre}$$

MENSURATION

3-D (THREE DIMENSIONAL)

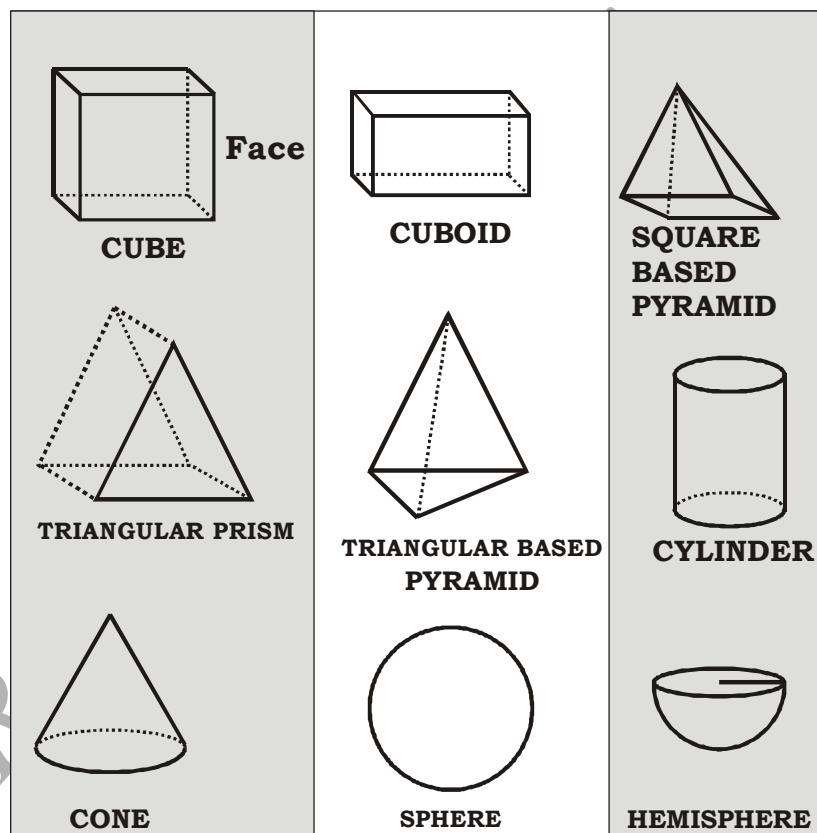
Mensuration is the branch of mathematics which deals with the study of different geometrical shapes, their areas and Volume. In the broadest sense, it is all about the process of measurement. It is based on the use of algebraic equations and geometric calculations to provide measurement data regarding the width, depth and volume of a given object or group of objects. While the measurement results obtained by the use of mensuration are estimates rather than actual physical measurements, the calculations are usually considered very accurate.

There are two types of geometric shapes:-

1. 2D
2. 3D

3D shapes: They have surface area and volume.

- (1) Cube
- (2) Rectangular Prism (Cuboid)
- (3) Cylinder
- (4) Cone
- (5) Sphere and Hemisphere
- (6) Prism
- (7) Pyramid



What is 3D....?

A **three-dimensional shape** is a solid shape that has height and depth. For example, a sphere and a cube are three-dimensional, but a circle and a square are not.

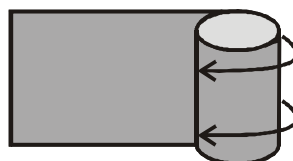
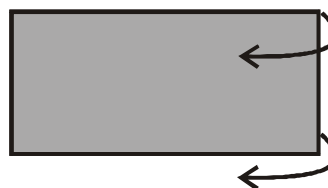
What is Volume.....?

Volume is the measure of the amount of space inside of a solid figure, like a cube, ball, cylinder or pyramid. Its units are always "cubic", that is the number of little element cubes that fit inside the figure.

Difference between Curved surface area and Total surface area

The area of all the curved surfaces of any solid. The Curved

surface area is the circumference of the base of the solid and the face parallel to it. The Total Surface area is the sum of both the curved surface area and the area of the base and top.

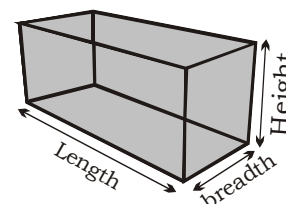


Cuboid (Parallelopiped)

A cuboid is a 3 dimensional shape.

It is a solid figure which has 6-regular faces.

12 edges, **8** vertices and **4** diagonals.



Look at this shape.

There are 3 different measurements:
Length, Breadth, Height

- Formula**
- (i) The volume is found using the formula:
Volume = Length $\times$ Breadth $\times$ Height
 Which is usually shortened to:
 $V = l \times b \times h$
 Or more simply:
 $V = lbh$
- (a) $V = l \times b \times h$ cubic units
 (b) $V = \sqrt{A_1 \times A_2 \times A_3}$ cubic units
- Where,
 A_1 = area of base or top = lb sq. units
 A_2 = area of one side face = bh sq. units
 A_3 = area of other side face = hl sq. units
- (ii) **Lateral surface Area / Curved surface area/ Area of four walls**
 = Perimeter of Base $\times$ height
 = $2(l + b) \times h$ sq. units
- (iii) **Total surface Area** = $2(lb + bh + hl)$ sq. units
- (iv) **Diagonal of cuboid**
 = $\sqrt{l^2 + b^2 + h^2}$ units
- (v) To find total surface area of a cuboid if the sum of all three sides and diagonal are given.
Total surface area = (Sum of all three sides) 2 – (Diagonal) 2
Note : For painting the surface area of a box or to know how much tin sheet is required for making a box, we use formulae (iii) i.e. Total surface Area.
 To find the length of the longest pole to be placed in a room, we use formulae (iv) i.e. Diagonal.

EXAMPLES

1. The dimensions of a cuboid are 16 cm, 18 cm and 24 cm. Find:
- (a) Volume
 (b) Surface area (c) Diagonal
- Sol.** (a) Volume = $l \times b \times h = 16 \times 18 \times 24 = 6912 \text{ cm}^3$
 (b) Surface area = $2(lb + bh + hl) = 2(16 \times 18 + 18 \times 24 + 24 \times 16) = 2208 \text{ cm}^2$

- (c) Diagonal
 = $\sqrt{l^2 + b^2 + h^2}$
 = $\sqrt{16^2 + 18^2 + 24^2}$
 = $\sqrt{1156} = 34 \text{ cm}$
2. Find the length of the longest pole that can be placed in a room 30 m long, 24 m broad and 18 m high.

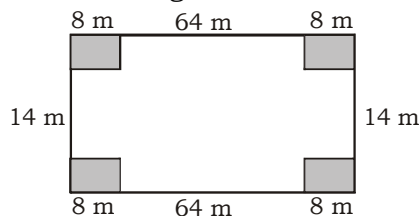
Sol. $d = \sqrt{l^2 + b^2 + h^2}$
 $d = \sqrt{900 + 576 + 324}$
 $= \sqrt{1800}$
 $d = 30\sqrt{2} \text{ m}$

3. A brick measures 20 cm $\times$ 10 cm $\times$ 7.5 cm. How many bricks will be required for a wall 20 m $\times$ 2 m $\times$ 0.75 m?

Sol. Number of bricks = $\frac{\text{Total Volume of wall}}{\text{Volume of one brick}}$
 $= \frac{20 \times 2 \times 0.75 \times 100 \times 100 \times 100}{20 \times 10 \times 7.5}$
 $= 20,000$

4. A rectangular sheet of metal is 80 m by 30 m. Equal squares of side 8 m are cut off at the corners and the remainder is folded up to form an open rectangular box. Find: (i) Volume (ii) Total surface area (iii) Surface area of box.

Sol. (i) When four square of 8 cm are removed from four corners of rectangular sheet.



Length and Breadth of remaining rectangular sheet will 64 cm and 14 cm & height of sheet will be 8 cm.

Volume of open rectangular box
 = Length $\times$ Breadth $\times$ Height
 = $64 \times 14 \times 8 = 7168 \text{ m}^3$

(ii) Surface area = $2(\text{Length} + \text{Breadth}) \times \text{Height}$
 = $2(64 + 14) \times 8 = 2 \times 78 \times 8$
 = 1248 m^2

- (iii) Total surface area = Surface area + Base area
 = $1248 + \text{Length} \times \text{Breadth}$
 = $1248 + 64 \times 14 = 1248 + 896$
 = 2144 m^2

5. In swimming pool measuring 90 m by 40 m, how much water will be displaced by 150 men, if the displacement of water by one man is 8 cm^3 , what will be the rise in water level?

Sol. Volume of water displaced by 150 men = Volume of water came out
 (Let the height raised in water = h)
 $\Rightarrow 8 \times 150 = 90 \times 40 \times h$
 $\Rightarrow h = \frac{1}{3} \text{ m} = 33.33 \text{ cm}$

6. A rectangular water reservoir is 15 m $\times$ 12 m at the base. Water flows into it through a pipe whose cross section is 5 cm by 3 cm at the rate of 16 m/s. Find the height to which water will rise in the reservoir in 25 minutes.

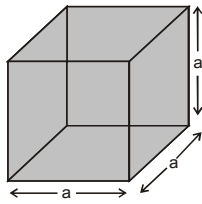
Sol. Volume of water comes out from pipe in 1 sec
 $= \frac{5}{100} \times \frac{3}{100} \times 16 \text{ m}^3 = 0.0240 \text{ m}^3$
 Volume of water comes out from pipe in 25 min = $0.0240 \times 25 \times 60 = 36 \text{ m}^3$
 $\Rightarrow$ Volume of water pour into tank = Volume of water comes out from pipe.
 $\therefore 15 \times 12 \times h = 36$
 $(\therefore h = \text{rise in level of water})$
 $\Rightarrow h = 0.2 \text{ m}$

7. The sum of length, breadth and height of a cuboid is 25 cm and its diagonal is 15 cm long. Find the total surface area of the cuboid.

Sol. We have the total surface area,
 = $(25)^2 - (15)^2 = 625 - 225$
 = 400 sq. cm.

CUBE

A cube whose length, breadth and height are all equal is called a cube. A cube has **6** equal faces, **12** equal edges, **8** vertices and **4** equal diagonals.



Consider a cube of edge a units.

It is a special type of cuboid in which $l = b = h = a$ units i.e. each face is a square.

(i) Volume = a^3 cubic units

(ii) Lateral surface Area = $4a^2$ sq. units

(iii) Total surface Area = $6a^2$ sq. units

(iv) Diagonal of cube (d) = $\sqrt{3} a$ units

(v) Face diagonal of cube = $\sqrt{2} a$ units

(vi) Volume of cube

$$= \left(\sqrt{\frac{\text{surface area}}{6}} \right)^3 \text{ cubic units}$$

EXAMPLES

1. Edge of a cube is 5 cm. Find:
(a) Volume
(b) Surface area
(c) Diagonal

Sol. Volume = $a^3 = (5)^3 = 125 \text{ cm}^3$
Surface area = $6a^2 = 6 \times (5)^2 = 150 \text{ cm}^2$

$$\text{Diagonal} = a\sqrt{3} = 5\sqrt{3} = 8.660 = \mathbf{8.66 \text{ cm}}$$

2. Three cubes of volume 1 cm^3 , 216 cm^3 and 512 cm^3 are melted to form a new cube. What is the diagonal of the new cube?

Sol. Volume of new cube = $1 + 216 + 512 = 729 \text{ cm}^3$

$$\therefore \text{Edge of new cube} = \sqrt[3]{729} = 9 \text{ cm}$$

$$\therefore \text{Surface area} = 6a^2 = 6 \times (9)^2 = 486 \text{ cm}^2$$

$$\therefore \text{Diagonal of the new cube} = a\sqrt{3} = 9\sqrt{3} = \mathbf{15.6 \text{ cm (approx)}}$$

3. The surface area of a cube is 864 cm^2 . Find the volume.

Sol. $6a^2 = 864 \Rightarrow a^2 = 144$

$$\Rightarrow a = 12 \text{ cm}$$

$$a^3 = (12)^3 = \mathbf{1728 \text{ cm}^3}$$

4. The Cost of painting the whole surface area of a cube at the rate of 13 paise per sq. cm is Rs. 343.98. Then the volume of the cube is:

Sol. Cost of painting the whole surface area = Rs. 343.98 = 34398 paise

$$\text{Total surface area} = \frac{34398}{13}$$

$$= 2646 \text{ cm}^2$$

$$6a^2 = 2646$$

$$a^2 = 441$$

$$a = 21 \text{ cm}$$

$$\therefore \text{Volume of cube} = a^3$$

$$= (21)^3 = \mathbf{9261 \text{ cm}^3}$$

5. A solid cube with an edge of 10 cm is melted to form two equal cubes. The edge of smaller cube to the bigger cube is.

Sol. Volume of larger cube = summation of volume of smaller cubes

$\therefore$ Let the edge of smaller cubes be 'a'

$$\Rightarrow (10)^3 = (a)^3 + (a)^3$$

$$\Rightarrow (10)^3 = 2(a^3)$$

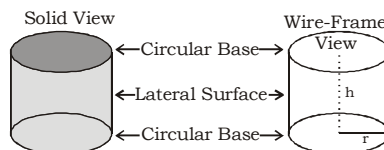
$$\Rightarrow a = \frac{10}{(2)^{1/3}}$$

$$\Rightarrow \frac{\text{edge of smaller cube}}{\text{edge of bigger cube}} = \frac{10}{\sqrt[3]{2}}$$

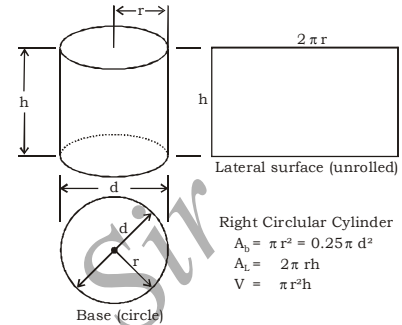
$$= \frac{1}{\sqrt[3]{2}}$$

Right Circular Cylinder

A **Right circular cylinder** is a three-dimensional object with two congruent circles as parallel bases and a lateral surface consisting of a rectangle. Volume and surface area of a **Right Circular Cylinder**, if 'r' is the radius of a circular base of the cylinder and 'h' is the height of the cylinder.



A right circular cylinder is a cylinder whose base is a circle and whose elements are perpendicular to its base.



Properties of a Right Circular Cylinder

- The axis of a right circular cylinder is the line joining the centers of the bases.
- For any oblique or non-oblique sections which do not pass any one base, the center of which is at the axis.
- A right circular cylinder can be formed by revolving a rectangle about one side as axis of revolution.
- Every section of a right circular cylinder made by a cutting plane containing two elements and parallel to the axis is a rectangle.

Formulae for Right Circular Cylinder

Area of the base, A_b

$$A_b = \pi r^2$$

$$A_b = \frac{\pi d^2}{4}$$

Lateral surface Area, A_L

$$A_L = 2\pi rh$$

$$A_L = \pi dh$$

Volume, V

$$V = A_b h$$

$$V = \pi r^2 h$$

$$V = \frac{\pi d^2 h}{4}$$

Total surface Area, A_T

Total surface area (open both ends), $A_L = A$

Total surface Area (open one end), $A = A_b + A_L$

Total surface Area (closed both ends), $A = 2A_b + A_L$

EXAMPLES

1. Find the volume of an iron rod which is 7 cm long and whose diameter is 1 cm.

Sol. Diameter = 1 cm.

$$\text{Radius} = \frac{1}{2} \text{ cm}$$

$$\text{Height} = 7 \text{ cm}$$

$$\text{Volume of Cylinder} = \pi r^2 h$$

$$= \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 7 = \frac{11}{2}$$

$$= \mathbf{5.5 \text{ cubic cm}}$$

2. Water flows at 10 km per hour through a pipe with cross section a circle of radius 35 cm, into a cistern of dimensions 25 m by 12 m by 10 m. By how much will the water level rise in the cistern in 24 minutes?

Sol. Volume flown in 24 minutes

$$= \left(\frac{22}{7} \times \frac{35}{100} \times \frac{35}{100} \times \frac{10000}{60} \times 24 \right)$$

$$= 1540 \text{ cubic m}$$

$$\text{Rise in level} = \left(\frac{1540}{25 \times 12} \right) = \mathbf{5.13 \text{ m}}$$

3. A powder tin has a square base with side 8 cm and height 13 cm. Another is cylindrical with radius of its base 7 cm and height 15 cm. Find the difference in their capacities.

Sol. Difference in capacities

$$= \frac{22}{7} \times 7 \times 7 \times 15 - 8 \times 8 \times 13$$

$$= 2310 - 832 = \mathbf{1478 \text{ cubic m}}$$

4. A metallic sphere of radius 21 cm is dropped into a cylindrical vessel, which is partially filled with water. The diameter of the vessel is 1.68 metres. If the sphere is completely submerged, find by how much the surface of water will rise.

Sol. Volume of sphere = $\frac{4}{3} \pi r^3$

$$= \left(\frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 \right)$$

$$= 38808 \text{ cubic cm.}$$

$$\therefore \frac{22}{7} \times 84 \times 84 \times h = 38808$$

$$\therefore \mathbf{h = 1.75 \text{ cm}}$$

5. The radius of a wire is decreased to one third. If volume remains same, length will increase:

Sol. Let, radius = R and length = h,
Volume = $\pi R^2 h$

$$\text{New radius} = \frac{1}{3} R$$

$$\text{Let, new length} = H$$

$$\text{Volume} = \pi \times \left(\frac{1}{3} R \right)^2 \times H$$

$$= \frac{\pi R^2 H}{9}$$

$$\therefore \pi R^2 h = \frac{\pi R^2 H}{9} \text{ or } H = 9h$$

6. A cylindrical iron rod is 70 cm long, and the diameter of its end is 2 cm. What is its weight, reckoning a cubic cm of iron to weigh 10 grams?

Sol. Volume of the iron rod = $\pi r^2 h$

$$= \frac{22}{7} \times 1 \times 1 \times 70 = 220 \text{ cm}^3$$

$$\therefore \text{weight of the cylinder}$$

$$= \frac{220 \times 10}{1000} = \mathbf{2.2 \text{ kg.}}$$

7. A cylindrical vessel, whose base is 14 dm in diameter holds 2310 litres of water. Taking a litre of water to occupy 1000 cubic cm, what is the height of the vessel in dm?

$$\text{Sol. } \frac{22}{7} \times 70 \times 70 \times h = 2310 \times 1000$$

$$[\because 1 \text{ dm} = 10 \text{ cm}]$$

$$\therefore h = \frac{2310 \times 1000 \times 7}{22 \times 70 \times 70}$$

$$= 150 \text{ cm} = \mathbf{15 \text{ dm.}}$$

8. Find how many pieces of money $\frac{3}{4}$ cm in diameter and

$\frac{1}{8}$ cm thick must be melted down to form a cube whose edge is 3 cm long?

Sol. Volume of one piece of money

$$= \pi r^2 h = \frac{22}{7} \times \frac{3}{8} \times \frac{3}{8} \times \frac{1}{8}$$

$$\therefore \frac{22}{7} \times \frac{3}{8} \times \frac{3}{8} \times \frac{1}{8} \times n = 3 \times 3 \times 3$$

$$n = \frac{3 \times 3 \times 3 \times 7 \times 8 \times 8 \times 8}{22 \times 3 \times 3 \times 1}$$

$$n = \mathbf{488.72}$$

9. The diameter of a cylindrical tank is 24.5 metres and depth 32 metres. How many metric tons of water will it hold? (One cubic metre of water weighs 1000 kg.)

Sol. Volume of the cylinder

$$= \frac{22}{7} \times \frac{24.5 \times 24.5}{2 \times 2} \times 32$$

$$= 15092 \text{ m}^3$$

$$\text{Since 1 cubic metre} = 1000 \text{ kg.}$$

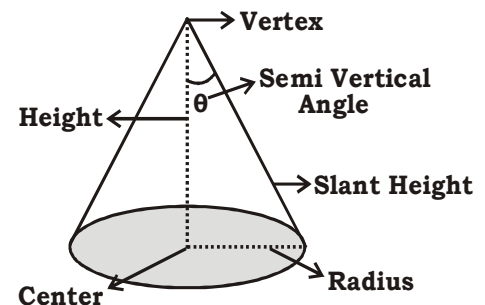
$$\therefore 1 \text{ cubic metre} = 1 \text{ metric ton}$$

$$(\because 1000 \text{ kg} = 1 \text{ metric ton})$$

$$\therefore \text{Volume of cylinder} = \mathbf{15092 \text{ metric tones}}$$

Right Circular Cone

Cone is a three dimensional geometric shape. If one end of a line is twisted about a second set line while keeping the lines other end fixed, we get a cone. The point about which the line is curved is known as the vertex and the base of the cone is a circle. The vertex is directly above the centre of the bottom.



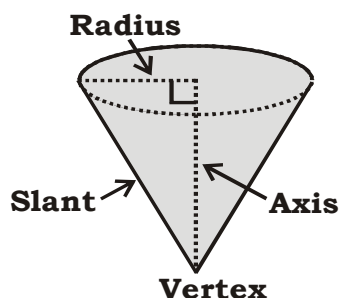
Properties of a Cone

There are number of properties of a cone. Some of them are as follows:

- Volume of a Cone
- Lateral surface Area of a Cone
- Total Surface Area of a Cone

Definition of Right Circular Cone

A right circular cone is one whose axis is perpendicular to the plane of the base. We can generate a right cone by revolving a right triangle about one of its legs.



Formulae of Right Circular Cone

For a right circular cone of radius r , height h and slant height l , we have
Lateral surface area of a right circular cone = $\pi r l$

Total surface area of a right circular cone = $\pi (r + l)r$

Volume of a right circular cone = $\frac{1}{3} \pi r^2 h$

Note:-

Area is measured in square units and volume is measured in cubic units.

Surface Area of a Right Circular Cone:-

The surface area of a right circular cone is the sum of area of base and lateral surface area of a cone. The surface area is measured in terms of square units.

Surface area of a cone = Base Area + Lateral surface Area of a cone

$$= \pi r^2 + \pi r l = \pi r(r + l)$$

Surface Area of a Right Circular Cone can be calculated by the following formula.

Area of a right circular cone
Here, $l = \sqrt{r^2 + h^2} = \pi r(r + l)$
Where, r = Radius
 h = Height and
 l = Slant height of cone

Volume of a Right Circular Cone

The volume of a cone is one third of the product of the area of base and the height of the cone. the volume of a right circular cone is measured in terms of cubic units.

Volume of a right circular cone can be calculated by the following formula.

$$\text{Volume of a right circular cone} = \frac{1}{3} \times \text{Base area} \times \text{height}$$

Where, Base area = πr^2

$$\text{Volume of a Circular cone} = \frac{1}{3} \times \pi \times \text{Radius}^2 \times \text{Height}$$

EXAMPLES

- Find the volume of a cone whose diameter of the base is 21 cm and the slant height is 37.5 cm.

$$\text{Sol. } h = \sqrt{(37.5)^2 - (10.5)^2} = 36 \text{ cm}$$

$$\therefore \text{volume} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 36 = 4158 \text{ cm}^3$$

- The radius and height of a right circular cone are in the ratio 5 : 12 and its volume is 2512 cm³. Find the slant height, radius and curved surface area of the cone. (Take $\pi = 3.14$)

$$\text{Sol. Let radius} = 5x, \text{ height} = 12x$$

$$\therefore \text{volume} = \frac{1}{3} \pi r^2 h$$

$$\frac{1}{3} \times 3.14 \times (25x^2) \times (12x) = 2512$$

$$x = 2$$

$$\therefore \text{Radius} = 5x = 10 \text{ cm \& Height} = 12x = 24 \text{ cm}$$

$$\therefore l = \sqrt{r^2 + h^2} = \sqrt{(10)^2 + (24)^2} = 26 \text{ cm}$$

$$\therefore \text{Curved surface area of cone} = \pi r l = 3.14 \times 10 \times 26 = 816.4 \text{ cm}^2$$

- If a right circular cone of vertical height 24 cm has a volume of 1232 cm³, then the area of its curved surface in cm² is:

$$\text{Sol. } \therefore \text{Volume} = \frac{1}{3} \pi r^2 h$$

$$\therefore \frac{1}{3} \pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 1232 = r^2 = 49 = r = 7$$

$$\text{Slant height} = \sqrt{(24)^2 + (7)^2} = 25 \text{ cm}$$

$\therefore$ Curved surface

$$= \left(\frac{22}{7} \times 7 \times 25 \right) = 550 \text{ cm}^2$$

- A right cylindrical vessel is full with water. How many right cones having same diameter and height as those of right cylinder will be needed to store that water?

$$\text{Sol. Volume of 1 cylinder} = \pi r^2 h$$

$$\text{Volume of 1 cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Number of cones} = \frac{\pi r^2 h}{\frac{1}{3} \pi r^2 h} = 3$$

- A cylindrical piece of metal of radius 2 cm and height 6 cm is shaped into a cone of same radius. The height of cone is:

$$\text{Sol. Volume of Cone} = \frac{1}{3} \pi r^2 h =$$

$$\frac{1}{3} \pi (2)^2 h$$

$$\text{Volume of cylinder} = \pi \times (2)^2 \times 6$$

$$\therefore \text{Volume of cone} = \text{Volume of cylinder}$$

$$\frac{1}{3} \pi \times 2^2 \times h = \pi \times 2^2 \times 6$$

$$\therefore h = 18 \text{ cm}$$

- From a solid right circular with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and base is removed. Find the volume of the remaining solid.

$$\text{Sol. Volume of remaining solid}$$

$$= \pi (r)^2 h - \frac{1}{3} \pi (r)^2 h$$

$$= \frac{22}{7} \times 6 \times 6 \times 10 - \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 10$$

$$= \frac{22}{7} \times 6 \times 6 \times 10 \times \frac{2}{3}$$

$$= \frac{5280}{7} = 754 \frac{2}{7} \text{ cm}^3$$

- The slant height of a conical tomb is $17 \frac{1}{2}$ metres. If its

diameter be 28 metres, find the cost of constructing it at Rs. 135 per cubic metre and also find the cost of white-washing its slant surface at Rs. 3.30 per square metre.

Sol. Height of the cone

$$= \sqrt{\left(\frac{35}{2}\right)^2 - 14^2} = \frac{21}{2} \text{ m}$$

Volume of the cone

$$= \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times \frac{21}{2} = 2156 \text{ m}^3$$

Cost of constructing the conical tomb = 2156×135

= Rs. 291060

Curved surface area of the conical tomb = $\pi r l$

$$= \frac{22}{7} \times 14 \times \frac{35}{2} = 770 \text{ m}^2$$

$\therefore$ Cost of white washing = 770×3.30 = Rs. 2541

8. Radius of the base of a right circular cone is 3 cm and the height of the cone is 4 cm. Find the total surface area of the cone.

Sol. Applying to the question,
Total surface area = $\pi \times r (l + r)$

$$= \frac{22}{7} \times 3 (\sqrt{4^2 + 3^2} + 3)$$

$$(\because l = \sqrt{h^2 + r^2})$$

$$= \frac{22 \times 3 \times 8}{7} = \frac{528}{7} = 75\frac{3}{7} \text{ sq. cm}$$

9. If the heights of two cones are in the ratio 1 : 4 and their diameters in the ratio 4 : 5 what is the ratio of their volumes?

Sol. We have,
Ratio of Volumes = $(4 : 5)^2 \times (1 : 4)$

$$4) = \frac{16}{25} \times \frac{1}{4}$$

$$= 4 : 25$$

[$\therefore$ ratio of diameters = ratio of radii]

10. If the volumes of the two cones are in the ratio 4 : 1 and their heights in the ratio 4 : 9, what is the ratio of their radii?

Sol. Ratio of radii = $\sqrt{(4 : 1) \times \left(\frac{1}{4} : \frac{1}{9}\right)}$

$$= \sqrt{(4 : 1) \times (9 : 4)} = 3 : 1$$

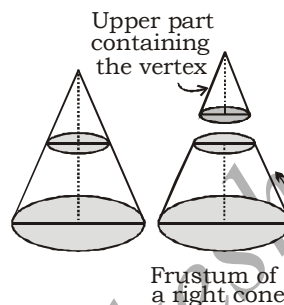
11. If the heights and the curved surface areas of two circular cylinder are in the ratio 1 : 3 and 4 : 5 respectively. Find the ratio of their radii.

Sol. Required ratio = $(4 : 5) \times \left(1 : \frac{1}{3}\right)$

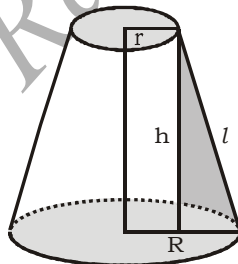
$$= (4 : 5) \times (3 : 1) = 12 : 5$$

Frustum of a right circular cone

A frustum of a cone or truncated cone is the result of cutting a cone by a plane parallel to the base and removing the part containing the apex.



The height is the line segment that joins the two bases perpendicularly.



The radii are of their bases are 'r' and 'R'.

The slant height is the shortest possible distance between the edges of the two bases.

The **slant height** of the truncated cone is obtained by applying the Pythagoras theorem for the shaded triangle:

$$l^2 = h^2 + (R - r)^2$$

$$l = \sqrt{h^2 + (R - r)^2}$$

Unfold of a Truncated Cone



Lateral Area of a Truncated Cone

$$A_L = \pi (R + r) l$$

Surface Area of a Truncated Cone

$$A_T = \pi [l(R + r) + R^2 + r^2]$$

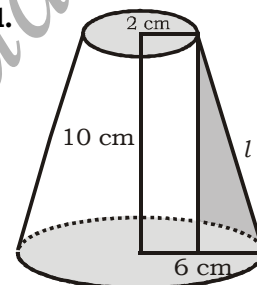
Volume of a Truncated Cone

$$V = \frac{1}{3} \cdot \pi \cdot h (R^2 + r^2 + R \cdot r)$$

EXAMPLES

1. Calculate the lateral surface area, surface area and volume of a truncated cone of radii 2 and 6 cm and height of 10 cm.

Sol.



$$l^2 = 10^2 + (6 - 2)^2$$

$$l = \sqrt{10^2 + (6 - 2)^2} = 10.77 \text{ cm}$$

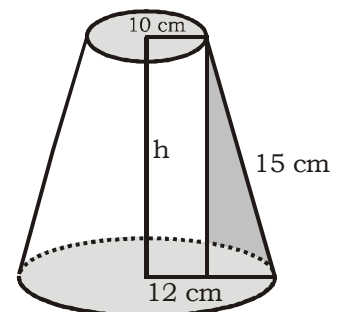
$$A_L = \pi (6 + 2) \times 10.77 = 270.78 \text{ cm}^2$$

$$A_T = 270.78 + \pi \times 6^2 + \pi \times 2^2 = 396.35 \text{ cm}^2$$

$$V = \frac{1}{3} \pi \times 10 \times (6^2 + 2^2 + 6 \times 2) = 544.54 \text{ cm}^3$$

2. Calculate the lateral surface area, surface area and volume of a truncated cone of radii 10 and 12 cm and a slant height of 15 cm.

Sol.



$$A_L = \pi (R + r) l$$

$$= \pi (12 + 10) \times 15 = 1,036.73 \text{ cm}^2$$

$$A_T = 1036.72 + \pi \times 12^2 + \pi \times 10^2$$

$$= 1803.27 \text{ cm}^2$$

$$\therefore l^2 = h^2 + (R - r)^2$$

$$15^2 = h^2 + (12 - 10)^2$$

$$h = \sqrt{15^2 - 2^2} = 14.866 \text{ cm}$$

$$V = \frac{1}{3} \pi \times 14.866 (12^2 + 10^2 + 12 \times 10)$$

$$= 5,666.65 \text{ cm}^3$$

Sphere

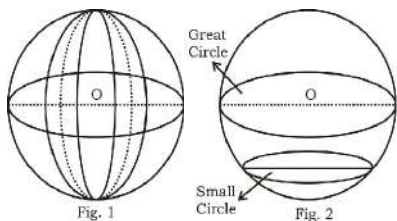
A sphere is a solid bounded by a closed surface every point of which is equidistant from a fixed point called the centre. Most familiar examples of a sphere are baseball, tennis ball, bowl, and so forth.

Terms such as radius, diameter, chord, and so forth, as applied to the sphere are defined in the same sense as for the circle.

Thus, a radius of a sphere is a straight line segment connecting its centre with any point on the sphere. Obviously, all radii of the same sphere are equal.

Diameter of the sphere is a straight line drawn from the surface and after passing through the centre ending at the surface.

The sphere may also be considered as generated by the complete rotation of a semicircle about a diameter.



Great and Small Circles:

Every section made by a plane passed through a sphere is a circle. If the plane passes through the centre of a sphere, the plane section is a great circle; otherwise, the section is a small circle (Fig. 2). Clearly any plane through the centre of the sphere contains a diameter. Hence, all great circles of a sphere are equal and have for their common centre, the centre of the sphere and have for their radius, the radius of the sphere.

Surface Area and Volume of a Sphere:

If 'r' is the radius and 'd' is the diameter of a great circle, then

(i) Surface area of a sphere = 4 times the area of its great circle = $4\pi r^2 = \pi d^2$

(ii) Volume of a sphere = $\frac{4}{3}\pi r^3$

$$= \frac{\pi}{6} d^3$$

(iii) For a spherical shell if R and r are outer and inner radii respectively, then the volume of

$$\text{a shell is} = \frac{4}{3}\pi(R^3 - r^3)$$

$$= \frac{\pi}{6}(D^3 - d^3)$$

EXAMPLES

1. The diameter of a sphere is 13.5m. Find its surface area and volume.

Sol. Here, $d = 13.5 \text{ m}$

$$\text{Surface area} = 4\pi r^2 = \pi d^2$$

$$= \pi (13.5)^2 = 572.56 \text{ sq. m.}$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{\pi}{6} d^3$$

$$= \frac{\pi}{6} (13.5)^3 = 1288.25 \text{ cu. m.}$$

2. Two spheres each of 10m diameter are melted down and recast into a cone with a height equal to the radius of its base, Find the height of the cone.

Sol. Here, $d = 10 \text{ m}$

$\therefore$ Radius of cone = height of the cone (Given)

$$\therefore r = h$$

$$\text{Volume of sphere} = \frac{\pi}{6}(d)^3$$

$$= \frac{\pi}{6} \times 10^3 = \frac{\pi}{6} \times 1000$$

$$= 523.599 \text{ cu. m.}$$

$$\text{Volume of two sphere}$$

$$= 1047.2 \text{ cu. m.}$$

$$\text{Volume of the cone}$$

$$= \text{Volume of two spheres}$$

$$\Rightarrow \frac{1}{3}\pi r^2 h = 1047.2$$

$$\Rightarrow \frac{1}{3}\pi h^3 = 1047.2$$

$$\Rightarrow h^3 = \frac{3 \times 1047.2}{\pi}$$

$$\Rightarrow h^3 = 1000$$

$$\Rightarrow h = 10 \text{ m}$$

3. How many leaden ball of a

$\frac{1}{4}$ cm. in diameter can be cast

out of metal of a ball 3 cm in diameter supposing no waste.

Sol. Here, diameter of leaden ball

$$= \frac{1}{4} \text{ cm.} = d_1$$

Diameter of metal ball

$$= 3 \text{ cm} = d_2$$

Volume of leaden ball

$$= \frac{4}{3}\pi \left(\frac{d_1}{2}\right)^3 = \frac{\pi}{6} d_1^3 = \frac{\pi}{6} \left(\frac{1}{4}\right)^3$$

$$= 0.0082 \text{ cu. m.}$$

$$\text{Volume of metal ball} = \frac{\pi}{6} d_2^3$$

$$= \frac{\pi}{6} (3)^3 = 14.137 \text{ cu. m.}$$

$\therefore$ Number of leaden ball =

$$\frac{14.137}{0.0082} = 1728.00$$

4. A metal sphere of diameter 14 cm is dropped into a cylindrical vessel, which is partly filled with water. The diameter of the vessel is 1.68 metres. If the sphere is completely submerged, find by how much the surface of water will rise.

Sol. Radius of the sphere = 7 cm

Volume of sphere

$$= \left(\frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7\right)$$

$$= 1437 \frac{1}{3} \text{ cu. m}$$

Volume of water displaced by

$$\text{sphere} = 1437 \frac{1}{3} \text{ cu cm.}$$

Let the water rise by h cm,

$$\text{Then, } \frac{22}{7} \times 28 \times 28 \times h$$

$$= 1437 \frac{1}{3}$$

$$\text{or } h = \frac{4312 \times 7}{22 \times 28 \times 28 \times 3} = \frac{7}{12}$$

$$= 0.58 \text{ cm}$$

5. Find the weight of an iron shell, the external and internal diameters of which are 13 cm and 10 cm respectively, if 1 cubic cm of iron weighs 8 gms.

Sol. Volume of iron shell

$$= \frac{4}{3}\pi \left\{ \left(\frac{13}{2} \right)^3 - (5)^3 \right\}$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{1197}{8} = 627 \text{ cu cm.}$$

$$\text{Weight of iron shell} = 627 \times 8 = 5016 \text{ gms.} = 5.016 \text{ kg.}$$

6. Find the surface area of a sphere whose volume is 310464 cu cm.

Sol. $\frac{4}{3}\pi r^3 = 310464$

$$\text{or } r^3 = \frac{310464 \times 3 \times 7}{4 \times 22} = 74088$$

$$\therefore r = 42 \text{ cm}$$

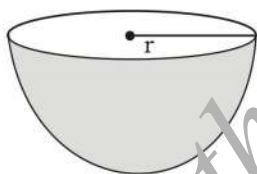
$$\therefore \text{surface area} = 4\pi r^2$$

$$= 4 \times \frac{22}{7} \times 42 \times 42$$

$$= \mathbf{22176 \text{ sq. cm.}}$$

Hemisphere

A plane through the center of the sphere cuts it into two equal parts. Each part is called hemisphere.



1. Volume of hemisphere = $\frac{2}{3}\pi r^3$
2. Curved surface area or Surface area of hemisphere = $2\pi r^2$
3. Total surface area of solid hemisphere
= $2\pi r^2 + \pi r^2 = 3\pi r^2$

EXAMPLES

1. Find the volume of a hemisphere of radius 21 cm.

Sol. According to the question, We have

$$r = 21 \text{ cm}$$

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3$$

$$\text{Volume of hemisphere}$$

$$= \left(\frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 \right)$$

$$= 19404 \text{ cm}^3$$

2. Find the curved surface area of a hemisphere of radius 21 cm.

Sol. According to the question, We have

$$\text{Curved surface area} = 2\pi r^2$$

$$= \left(2 \times \frac{22}{7} \times 21 \times 21 \right) \text{ cm}^2$$

$$= \mathbf{2772 \text{ cm}^2}$$

3. A hemisphere of lead of radius 7 cm is cast into a right circular cone of height 49 cm. Find the radius of the base.

Sol. Volume of hemisphere

$$= \frac{1}{2} \left(\frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \right)$$

$$= \frac{2156}{3} \text{ cubic cm}$$

Let, the radius of the base of cone be 'r' cm.

$$\text{Then, } \frac{1}{3} \times \frac{22}{7} \times r^2 \times 49 = \frac{2156}{3}$$

$$\text{or } r^2 = \frac{2156 \times 7 \times 3}{3 \times 22 \times 49} = 14$$

$$\therefore \text{Radius (r)} = \sqrt{14} = 3.74 \text{ cm}$$

Right Prism

A prism is a solid object with:

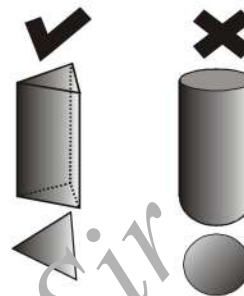
- Identical ends
- Flat faces
- Same cross section all along its length

The cross section of this object is a triangle it has the same cross section all along its length ... so it's a triangular prism.



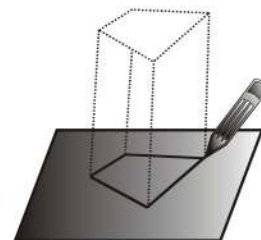
No Curves :

A prism is a polyhedron, which means all faces are flat.



For example, a **cylinder** is **not prism**, because it has curved sides.

Try drawing a shape on a piece of paper (using straight lines) Then imagine it extending up from the sheet of paper that's a prism !

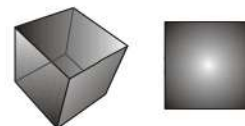


There are all Prism:

Square Cross-
Prism Section



Cube Cross-Section



(yes, a cube is a prism, because it is a square all along its length)

Triangular Cross-
Prism Section



Pentagonal Cross-
Prism Section

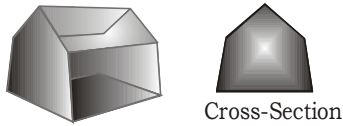


Regular and Irregular Prisms

All the previous examples are of **Regular** Prisms, because the cross section is regular (in other words it is a shape with equal edge lengths, and equal angles.)

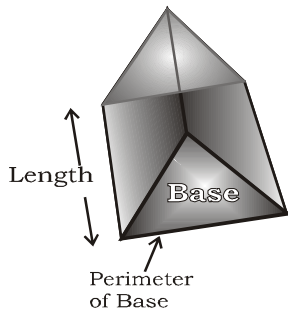
Here is an example of an **Irregular Prism**:

Irregular Pentagonal Prism:



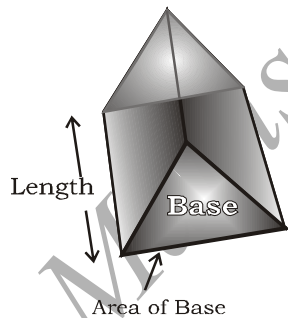
It is "irregular" because the cross-section is not "regular" in shape.

Surface Area of Prism



$$\text{Surface Area} = 2 \times \text{Base Area} + \text{Base Perimeter} \times \text{Length}$$

Volume of Prism

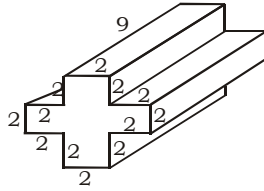


The Volume of a prism is the product of its base area and length.

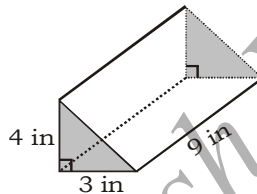
$$\begin{aligned} \text{Volume} &= \text{Base Area} \times \text{Length} \\ \text{Volume} &= \text{Base Area} \times \text{Length} \\ \text{Curved Surface Area} &= \text{Base perimeter} \times \text{Height} \\ \text{Total Surface Area} &= \text{CSA} + 2 \text{ Base Area} \end{aligned}$$

EXAMPLES

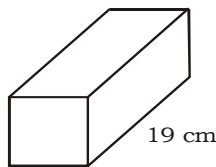
1. What is the volume of this prism ?



- Sol.** Volume = Base area $\times$ height
There are 5 squares
 $\therefore$ Base area = $5 \times 2 \times 2 = 20$
Volume = $20 \times 9 = \mathbf{180 \text{ units}}$.
2. The diagram show a prism whose cross-section is a right triangle. What is the volume of the prism ?

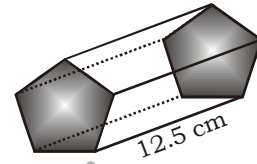


- Sol.** Volume = base area $\times$ height.
Base area = $\frac{1}{2} \times 4 \times 3 = 6 \text{ in}^2$
Volume = $6 \times 9 = \mathbf{54 \text{ in}^3}$
3. The diagram shows a prism whose cross-section is a square. The length of the base of the prism is 19 cm and its volume is $1,539 \text{ cm}^3$. What is the total surface area of the prism ?

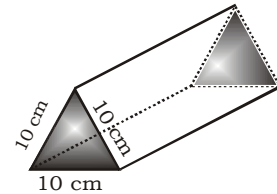


- Sol.** Volume = Base area $\times$ Height
 $1539 = \text{Base area} \times 19$
Base area = $\frac{1539}{19} = 81$
Base area = $a^2 = 81$
 $a = 9$
C.S.A = Base perimeter $\times$ Height
 $= 4 \times 9 \times 19 = 684 \text{ cm}^2$
T.S.A = C.S.A + 2 Base area
 $= 684 + 2 \times 81$
 $= 684 + 162 = \mathbf{846 \text{ cm}^2}$
4. The diagram shows a prism with a cross section that is a

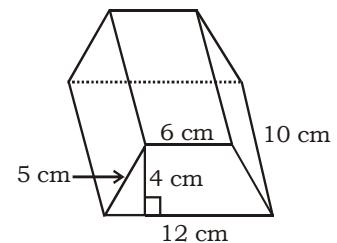
regular pentagon. If the area of the cross-section is 30 cm^2 , then what is the volume of the prism ?



- Sol.** Volume = Base area $\times$ Height
 $= 30 \times 12.5 = \mathbf{375 \text{ cm}^3}$
5. The diagram shows a prism whose cross-section is an equilateral triangle of lengths 10 cm. Given that its volume is 866 cm^3 , what is the total surface area of the prism ?



- Sol.** Volume = Base area $\times$ Height
Base area = $\frac{\sqrt{3}}{4} \times (10)^2 = 43.3 \text{ cm}^2$
Height = Volume $\div$ Base area
 $= \frac{866}{43.3} = 20$
C.S.A = Base perimeter $\times$ height
C.S.A = $3 \times 10 \times 20 = 600 \text{ cm}^2$
T.S.A = C.S.A + 2 base area
T.S.A = $600 + 2 \times 43.3$
 $= 600 + 86.6 = \mathbf{686.6 \text{ cm}^2}$
6. There is a 10 cm long prism whose cross-section is an isosceles trapezoid:



What is the total surface area of the prism ?

- Sol.** Base area = $\frac{1}{2} \times \text{Sum of } \parallel \text{ sides} \times \text{height}$

$$= \frac{1}{2} \times (6 + 12) \times 4 = 36 \text{ cm}^2$$

$$\text{Base perimeter} = 5 + 6 + 5 + 12 = 28 \text{ cm}$$

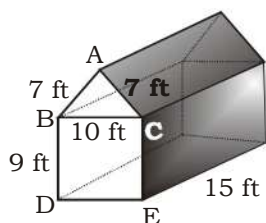
$$\text{C.S.A.} = \text{Base perimeter} \times h$$

$$\text{C.S.A.} = 28 \times 10 = 280 \text{ cm}^2$$

$$\text{T.S.A.} = \text{C.S.A.} + 2 \text{ Base area}$$

$$\text{T.S.A.} = 280 + 2 \times 36 = \mathbf{352 \text{ cm}^2}$$

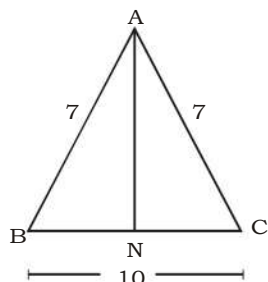
7.



The diagram shows a barn.
What is the volume of the barn?

(The length of the hypotenuse in the right triangle is rounded to the nearest foot.)

Sol. Volume = Base area $\times$ height
Total base area = Area of $\square$ BCED + Area of $\triangle$ ABC
Area of BCDE = $10 \times 9 = 90 \text{ ft}^2$



$$\begin{aligned} \text{In } \triangle ABC, \\ AB^2 &= AN^2 + BN^2 \\ AN^2 &= 49 - 25 \\ AN^2 &= 24 \end{aligned}$$

$$AN = 2\sqrt{6} \text{ ft}$$

$$\therefore \text{Area of } \triangle ABC$$

$$= \frac{1}{2} \times BC \times AN$$

$$= \frac{1}{2} \times 10 \times 2\sqrt{6}$$

$$= 10\sqrt{6} \text{ ft}^2$$

$$\therefore \text{Total base area}$$

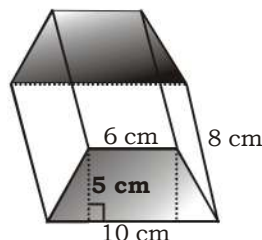
$$= [10\sqrt{6} + 90] \text{ ft}^2$$

$$\text{Volume} = (10\sqrt{6} + 90) \times 15$$

$$= 1350 + 150\sqrt{6}$$

$$= 1350 + 150 \times 2.45 = \mathbf{1717.5 \text{ ft}^3}$$

8. The diagram shows a prism whose cross-section is an isosceles trapezoid.



What is the volume of the prism?

Sol. Volume = Base area $\times$ Height

$$\text{Base area} = \frac{1}{2} (\text{Sum of || sides}) \times \text{height}$$

$$= \frac{1}{2} \times (10 + 6) \times 5$$

$$= \frac{1}{2} \times 16 \times 5 = 40 \text{ cm}^2$$

$$\text{Volume} = 40 \times 8 = 320 \text{ cm}^3$$

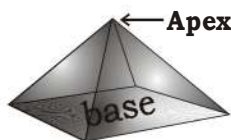
Pyramids



When we think of pyramids we think of the Great Pyramids of Egypt. They are actually Square Pyramids, because their base is a Square.



Parts of Pyramid



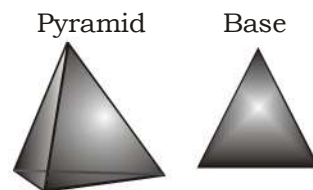
A pyramid is made by connecting a base to an apex.

Types of Pyramid

There are many types of Pyramids,

and they are named after the shape of their base.

1. Triangular Pyramid



Notice these interesting things:

- It has 4 Faces
- The 3 Side Faces are Triangles
- The Base is also a Triangle
- It has 4 Vertices (corner points)
- It has 6 Edges
- It is also a Tetrahedron (if all triangles are equilateral triangles)

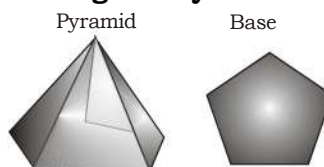
2. Square Pyramid



Notice these interesting things:

- It has 5 Faces
- The 4 Side Faces are Triangles
- The Base is a Square
- It has 5 Vertices (corner points)
- It has 8 Edges

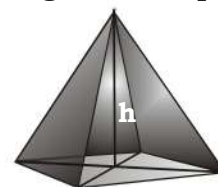
3. Pentagonal Pyramid

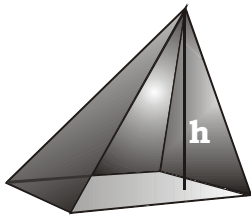


Notice these interesting things:

- It has 6 Faces
- The 5 Side Faces are Triangles
- The Base is a Pentagon
- It has 6 Vertices (corner points)
- It has 10 Edges

Right vs Oblique Pyramid:-

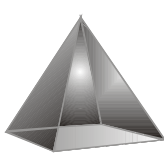




This tells us where the top (apex) of the pyramid is. When the apex is directly above the center of the base it is a **Right Pyramid**, otherwise it is an **Oblique Pyramid**.

Regular vs Irregular Pyramid:-

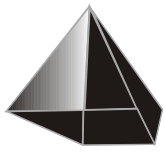
This tells us about the **shape of the base**. When the base is a regular polygon it is a **Regular Pyramid**, otherwise it is an **Irregular Pyramid**.



Regular Pyramid



Base is Regular



Irregular Pyramid



Base is Irregular

FORMULAE

(i) Volume of pyramid

$$= \frac{1}{3} \times (\text{area of base}) \times \text{height}$$

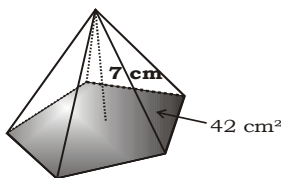
(ii) Curved surface area

$$= \frac{1}{2} \times (\text{perimeter of base}) \times \text{slant height}$$

(iii) Total surface area = curved surface area + area of the base.

EXAMPLES

1. The diagram shows a pyramid whose base is a regular pentagon of area 42 cm^2 and whose height is 7 cm . What is the volume of the pyramid?

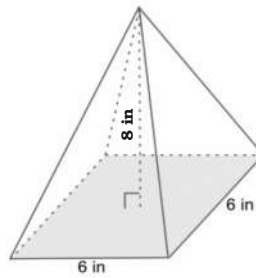


Sol. Volume = $\frac{1}{3} \times \text{Area of base} \times \text{height}$

$$\text{Volume} = \frac{1}{3} \times 42 \times 7$$

$$\text{Volume} = 98 \text{ cm}^3$$

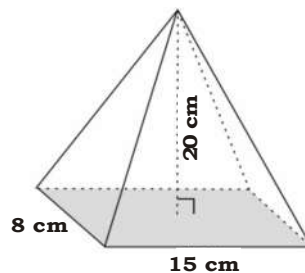
2. The diagram shows a square-based pyramid with base lengths 6 in and height 8 in . What is the volume of the pyramid?



Sol. Volume = $\frac{1}{3} \times \text{Area of base} \times \text{height}$

$$\text{Volume} = \frac{1}{3} \times 6 \times 6 \times 8 = 96 \text{ in}^3$$

3. The diagram shows a rectangular-based pyramid with base length 15 cm and width 8 cm . The height of the pyramid is 20 cm . What is the volume of pyramid?

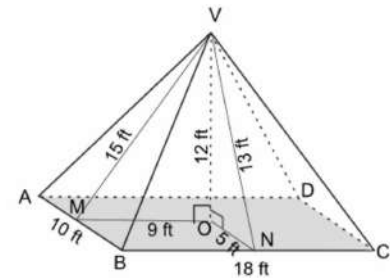


Sol. Volume = $\frac{1}{3} \times \text{Area of base} \times \text{height}$

$$\text{height} = \frac{1}{3} \times 15 \times 8 \times 20$$

$$\text{Volume} = 800 \text{ cm}^3.$$

4. The diagram shows a pyramid with vertex V and a rectangular base $ABCD$. M is the midpoint of AB , N is the midpoint of BC and O is the point at the center of the base.



$AB = 10 \text{ ft}$ $BC = 18 \text{ ft}$
 $VO = 12 \text{ ft}$ $VM = 15 \text{ ft}$
 $VN = 13 \text{ ft}$

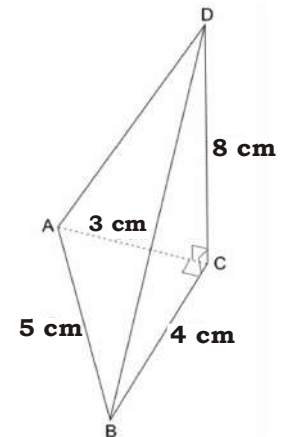
What is the total surface area of the pyramid?

Sol. Total surface area = $2 \times \text{area } \triangle ABV + 2 \times \text{area } \triangle VBC + \text{Area of } ABCD$

$$= 2 \times \frac{1}{2} \times 10 \times 15 + 2 \times \frac{1}{2} \times 18 \times 13 + 10 \times 18$$

$$= 150 + 234 + 180 = 564 \text{ ft}^2$$

5. The diagram shows a pyramid with a triangular base ABC . The point D is vertically above the point C . What is the volume of the pyramid?



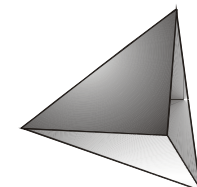
Sol. Volume = $\frac{1}{3} \times \text{Area of base} \times \text{height}$

$$= \frac{1}{3} \times \frac{1}{2} \times 3 \times 4 \times 8$$

$$\text{Volume} = 16 \text{ cm}^3$$

Tetrahedron

Tetrahedron Facts



Notice these interesting things:

- It has 4 Faces
- Each face is an Equilateral Triangle
- It has 6 Edges
- It has 4 Vertices (corner points) and at each vertex 3 edges meet
- It is one of the Platonic Solids

The tetrahedron also has a beautiful and unique property ... all four vertices are the same distance from each other!

And it is the only Platonic Solid with no parallel faces.

When we say "tetrahedron" we often mean "regular tetrahedron" (in other words all faces are the same size and shape)

$$\text{Surface Area} = \sqrt{3} \times (\text{Edge Length})^2$$

$$\text{Volume} = \frac{\sqrt{2}}{12} \times (\text{Edge Length})^3$$

EXAMPLES

1. The length of one edge of a regular tetrahedron is 9 units. What is its surface area?

Sol. Surface area

$$= \sqrt{3} \times (\text{Edge length})^2$$

$$= \sqrt{3} \times (9)^2 = 81 \times 1.732$$

$$= \mathbf{140.30}$$

Alternate:-

Calculate the area of a side, which is an equilateral triangle.

The base is 9, the height is

$$= \frac{\sqrt{3}}{2} \times 9 = \frac{9\sqrt{3}}{2}$$

$$\text{Area of triangle} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 9 \times 9 \times \frac{\sqrt{3}}{2}$$

$$= \frac{81\sqrt{3}}{4}$$

Area of 4 triangles

$$= 4 \times \frac{81 \times \sqrt{3}}{4} = 81 \times \sqrt{3}$$

$$= \mathbf{140.30}$$

2. The length of one edge of a regular tetrahedron is 9 units. What is its volume?

$$\text{Sol. Volume} = \frac{\sqrt{2}}{12} \times (\text{Edge length})^3$$

$$= \frac{\sqrt{2}}{12} \times (9)^3 = \frac{\sqrt{2}}{12} \times 729$$

$$= \mathbf{85.91 \text{ units}^3}$$

3. The total length of the edges of a tetrahedron is 24 cm. What is its surface area?

Sol. Note:- In tetrahedron has 6 edges.

$$\text{One edge length} = 24 \div 6$$

$$= 4 \text{ cm}$$

Surface area

$$= \sqrt{3} \times (\text{Edge length})^2$$

$$= \sqrt{3} \times (4)^2 = 16 \times 1.732$$

$$= \mathbf{27.71 \text{ cm}^2}$$

4. The total length of the edges of a tetrahedron is 36cm. What is its volume?

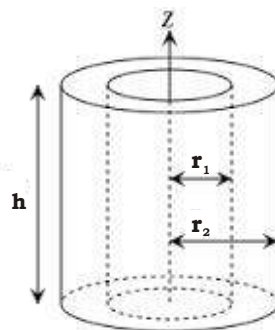
Sol. A tetrahedron has 6 edges

$$\text{So one Edge Length} = 36\text{cm} \div 6 = 6\text{cm}$$

$$\text{Volume} = \frac{\sqrt{2}}{12} \times (\text{Edge length})^3$$

$$= \frac{\sqrt{2}}{12} \times (6)^3 = \frac{\sqrt{2}}{12} \times 216 = 25.46 \text{ cm}^3$$

Hollow Cylinder



$$(i) \text{ Volume (V)} = \pi(r_2^2 - r_1^2)h$$

$$(ii) \text{ Curved surface area} = 2\pi(r_1 + r_2)h$$

$$(iii) \text{ Total surface area} = \text{inner curved surface area} + \text{Outer curved surface area} + \text{area of Base and Top} = 2\pi(r_1 + r_2)[h + (r_2 - r_1)]$$

$$(iv) \text{ Thickness (t)} = (r_2 - r_1)$$

EXAMPLES

1. A hollow garden roller 63cm wide with a girth of 440cm is made of iron 4cm thick. The volume of iron is:

Sol. Circumference = 440 cm

$$\Rightarrow 2\pi r = 440$$

$$r = \left(\frac{440}{2 \times 22} \times 7 \right) = 70 \text{ cm}$$

$$\text{Inner radius} = 70 - 4 = 66 \text{ cm}$$

Volume of Iron

$$= \pi[(70)^2 - (66)^2] \times 63$$

$$= \frac{22}{7} \times 136 \times 4 \times 63 = 58752 \text{ cm}^3$$

2. A hollow cylindrical tube open at both ends is made of iron 2 cm thick. If the external diameter be 50 cm and the length of tube be 140 cm, find the volume of iron in it.

Sol. External diameter = 50 cm

$$\therefore \text{External radius} = \frac{50}{2} = 25 \text{ cm}$$

$$\text{Volume} = \frac{22}{7} \times 140 [(25)^2 - (23)^2]$$

$$= \frac{22}{7} \times 140 \times 48 \times 2 = 42240 \text{ cm}^3$$

EXERCISE

1. If diagonal of a cube is $\sqrt{12}$ cm, then its volume in cm^3 is :
(a) 8 (b) 12 (c) 24 (d) $3\sqrt{2}$
2. How many cubes, each of edge 3 cm, can be cut from a cube of edge 15 cm ?
(a) 25 (b) 27 (c) 125 (d) 144
3. What is the volume of a cube (in cubic cm) whose diagonal measures $4\sqrt{3}$ cm?
(a) 16 (b) 27 (c) 64 (d) 8
4. A cuboidal water tank has 216 litres of water. Its depth is $\frac{1}{3}$ of its length and breadth is $\frac{1}{2}$ of $\frac{1}{3}$ of the difference of length and breadth. The length of the tank is
(a) 72 dm (b) 18 dm
(d) 6 dm (d) 2 dm
5. The volume of cuboid is twice the volume of a cube. If the dimensions of the cuboid are 9 cm, 8 cm and 6 cm, the total surface area of the cube is:
(a) 72 cm^2 (b) 216 cm^2
(c) 432 cm^2 (d) 108 cm^2
6. The length, breadth and height of a room is 5m, 4 m and 3m respectively. Find the length of the largest bamboo that can be kept inside the room.
(a) 5 m (b) 60 m
(c) 7 m (d) $5\sqrt{2}$ m
7. A wooden box measures 20 cm by 12 cm by 10 cm. Thickness of wood is 1 cm. Volume of wood to make the box (in cubic cm) is
(a) 960 (b) 519
(c) 2400 (d) 1120
8. A cuboidal block of $6 \text{ cm} \times 9 \text{ cm} \times 12 \text{ cm}$ is cut up into exact number of equal cube. The least possible number of cubes will be
(a) 6 (b) 9 (c) 24 (d) 30
9. A cistern of capacity 8000 litres measures externally 3.3 m by 2.6 m by 1.1 m and its walls are 5 cm thick. The thickness of the bottom is:
(a) 1 m (b) 13.5 m
(c) 1 dm (d) 90 cm
10. The area of three adjacent faces of a cuboid are x , y , z square units respectively. If the volume of the cuboid be v cube units. then the correct relation between v, x, y, z is
(a) $v^2 = xyz$ (b) $v^3 = xyz$
(c) $v^2 = x^3y^3z^3$ (d) $v^3 = x^2y^2z^2$
11. The largest sphere is carved out of a cube of side 7 cm. The volume of the sphere (in cm^3) will be
(a) 718.66 (b) 543.72
(c) 481.34 (d) 179.67
12. The length (in meters) of the longest rod that can be put in a room of dimensions $10 \text{ m} \times 10 \text{ m} \times 5 \text{ m}$ is
(a) $15\sqrt{3}$ (b) 15
(c) $10\sqrt{2}$ (d) $5\sqrt{3}$
13. A rectangular sheet of metal is 40 cm by 15 cm. equal squares of side 4cm are cut off at the corners and the remainder is folded up to form an open rectangular box The volume of the box is
(a) 896 cm^3 (b) 986 cm^3
(c) 600 cm^3 (d) 916 cm^3
14. The areas of three consecutive faces of a cuboid are 12 cm^2 , then the volume (in cm^3) of the cuboid is
(a) 3600 (b) 100
(c) 80 (d) $24\sqrt{3}$
15. The length of the longest rod that can be placed in a room which is 12 m long, 9 m broad and 8 m high is
(a) 27m (b) 19m
(c) 17m (d) 13m
16. The floor of a room is of size $4 \text{ m} \times 3 \text{ m}$ and its height is 3 m. The walls and ceiling of the room require painting. The area to be painted is
(a) 66 m^2 (b) 54 m^2
(c) 42 m^2 (d) 33 m^2
17. If the sum of three dimensions and the total surface area of a rectangular box are 12 cm and 94 cm^2 respectively, then the maximum length of a stick that can be placed inside the box is
(a) $5\sqrt{2}$ cm (b) 5 cm
(c) 6 cm (d) $2\sqrt{5}$ cm
18. The area of the four walls of a room is 660 m^2 and its length is twice of its breadth. If the height of the room is 11 m, then area of its floor (in m^2) is
(a) 120 (b) 150
(c) 200 (d) 330
19. If the length of the diagonal of a cube is $8\sqrt{3}$ cm, then its total surface area is
(a) 192 cm^2 (b) 512 cm^2
(c) 768 cm^2 (d) 384 cm^2
20. The maximum length of a pencil that can be kept in a rectangular box of dimensions $8 \text{ cm} \times 6 \text{ cm} \times 2 \text{ cm}$ is
(a) $2\sqrt{13}$ cm (b) $2\sqrt{14}$ cm
(c) $2\sqrt{26}$ cm (d) $10\sqrt{2}$ cm
21. The volume of a cubical box is 3.375 cubic metres. The length of edge of the box is
(a) 75 m (b) 1.5 m
(c) 1.125 m (d) 2.5 m
22. Two cubes of sides 6 cm each are kept side by side to form a rectangular parallelopiped. The area (in sq. cm) of the whole surface of the rectangular parallelopiped is
(a) 432 (b) 360
(c) 396 (d) 340
23. 2 cm of rain has fallen on a square km of land. Assuming that 50% of the raindrops could have been collected and contained in a pool having a $100 \text{ m} \times 10 \text{ m}$ base, by what level would the water level in the pool have increased ?
(a) 1 km (b) 10 m
(c) 10 cm (d) 1 m
24. A parallelopiped whose sides are in ratio 2 : 4 : 8 have the same volume as a cube. The ratio of their surface area is:
(a) 7 : 5 (b) 4 : 3
(c) 8 : 5 (d) 7 : 6
25. If two adjacent sides of a rectangular parallelopiped are 1 cm and 2 cm and the total surface area of the parallelopiped is 22 square cm,

- then the diagonal of the parallelopiped is
 (a) $\sqrt{10}$ cm (b) $2\sqrt{3}$ cm
 (c) $\sqrt{14}$ cm (d) 4 cm
26. If the sum of the length, Breadth and height of a rectangular parallelopiped is 24 cm and the length of its diagonal is 15 cm, then its total surface area is
 (a) 256 cm^2 (b) 265 cm^2
 (c) 315 cm^2 (d) 351 cm^2
27. If the total surface area of a cube is 96 cm^2 , its volume is
 (a) 56 cm^3 (b) 16 cm^3
 (c) 64 cm^3 (d) 36 cm^3
28. The length of the largest possible rod that can be placed in a cubical room is $35\sqrt{3}$ m. The surface area of the largest possible sphere that fit within the cubical room (assuming $\pi = \frac{22}{7}$) (in sq. m) is
 (a) 3,500 (b) 3,850
 (c) 2,450 (d) 4,250
29. The volume of air in a room is 204 m^3 . The height of the room is 6 m. What is the floor area of the room?
 (a) 32 m^2 (b) 46 m^2
 (c) 44 m^2 (d) 34 m^2
30. A square of side 3 cm is cut off from each corner of a rectangular sheet of length 24 cm and breadth 18 cm and the remaining sheet is folded to form an open rectangular box. The surface area of the box is
 (a) 468 cm^2 (b) 396 cm^2
 (c) 615 cm^2 (d) 423 cm^2
31. Three solid iron cubes of edges 4 cm, 5 cm and 6 cm are melted together to make a new cube. 62 cm^3 of the melted material is lost due to improper handling. The area (in cm^2) of the whole surface of the newly formed cube is
 (a) 294 (b) 343 (c) 125 (d) 216
32. Area of the floor of a cubical room is 48 sq. m . The length of the longest rod that can be kept in that room is
 (a) 9 metre (b) 12 metre
 (c) 18 metre (d) 6 metre
33. Three cubes of sides 6 cm, 8 cm and 1 cm are melted to form a new cube. The surface area of the new cube is
 (a) 486 cm^2 (b) 496 cm^2
 (c) 586 cm^2 (d) 658 cm^2
34. Some bricks are arranged in an area measuring 20 cu.m. If the length, breadth and height of each brick is 25 cm, 12.5 cm and 8 cm respectively, then the number of bricks are (suppose there is no gap in between two bricks)
 (a) 6,000 (b) 8,000
 (c) 4,000 (d) 10,000
35. The whole surface of a cube is 150 sq. cm . Then the volume of the cube is
 (a) 125 cm^3 (b) 216 cm^3
 (c) 343 cm^3 (d) 512 cm^3
36. The ratio of the length and breadth of a rectangular parallelopiped is 5 : 3 and its height is 6 cm. If the total surface area of the parallelopiped be 558 sq. cm , then its length in dm is
 (a) 9 (b) 1.5 (c) 10 (d) 15
37. If the sum of the dimensions of a rectangular parallelopiped is 24 cm and the length of the diagonal is 15 cm, then the total surface area of it is
 (a) 420 cm^2 (b) 275 cm^2
 (c) 351 cm^2 (d) 378 cm^2
38. The length, breadth and height of a cuboid are in the ratio 3 : 4 : 6 and its volume is 576 cm^3 . The whole surface area of the cuboid is
 (a) 216 cm^2 (b) 324 cm^2
 (c) 432 cm^2 (d) 460 cm^2
39. If the number of vertices, edges and faces of a rectangular parallelopiped are denoted by v, e and f respectively, the value of $(v - e + f)$ is
 (a) 4 (b) 1 (c) 0 (d) 2
40. A low land, 48 m long and 31.5m broad is raised to 6.5 dm. For this, earth is removed from a cuboidal hole, 27 m long and 18.2 m broad, dug by the side of the land. The depth of the hole will be.
 (a) 3 m (b) 2 m
 (c) 2.2 m (d) 2.5 m
41. A cuboidal shaped water tank, 2.1 m long and 1.5 m broad is half filled with water. If 630 litres more water is poured into tank, the water level will rise
 (a) 2 cm (b) 0.15 cm
 (c) 0.20 m (d) 0.18 cm
42. A solid cuboid of dimensions 8 cm $\times$ 4 cm $\times$ 2 cm is melted and cast into identical cubes of edge 2 cm. Number of such identical cubes is.
 (a) 16 (b) 4 (c) 10 (d) 8
43. A metallic hemisphere is melted and recast in the shape of cone with the same base radius (R) as that of the hemisphere. If H is the height of the cone, then:
 (a) $H = 2R$ (b) $H = \frac{2}{3}R$
 (c) $H = \sqrt{3}R$ (d) $B = 3R$
44. If the radius of a sphere is increased by 2 cm, its surface area increased by 352 cm^2 . The radius of sphere before change is :
 (a) 3 cm (b) 4 cm
 (c) 5 cm (d) 6 cm
45. The height of a conical tank is 60 cm and the diameter of its base is 64cm. The cost of painting it from outside at the rate of Rs. 35 per sq. m. is :
 (a) Rs. 52.00 approx,
 (b) Rs. 39.20 approx,
 (c) Rs. 35.20 approx,
 (d) Rs. 23.94 approx,
46. A solid metallic cone of height 10 cm, radius of base 20 cm is melted to make spherical balls each of 4 cm diameter. How many such balls can be made?
 (a) 25 (b) 75 (c) 50 (d) 125
47. A cylindrical tank of diameter 35 cm is full of water. If 11 litres of water is drawn off, the water level in the tank will drop by :
 (a) $10\frac{1}{2}$ cm (b) $12\frac{6}{7}$ cm
 (c) 14 cm (d) $11\frac{3}{7}$ cm
48. The volume of a right circular cylinder whose height is 40cm, and circumference of its base is 66 cm is:
 (a) 55440 cm^3 (b) 3465 cm^3
 (c) 7720 cm^3 (d) 13860 cm^3

49. The circumference of the base of a circular cylinder is 6π cm. The height of the cylinder is equal to the diameter of the base. How many litres of water can it hold ?
 (a) 54π cc (b) 36π cc
 (c) 0.054π cc (d) 0.54π cc
50. The volume of a right circular cylinder is equal to the volume of that right circular cone whose height is 108 cm and diameter of base is 30 cm. If the height of the cylinder is 9 cm, the diameter of its base is
 (a) 30 cm (b) 60 cm
 (c) 50 cm (d) 40 cm
51. Three solid metallic spheres of diameter 6 cm, 8 cm and 10 cm are melted and recast into a new solid sphere. The diameter of the new sphere is :
 (a) 4 cm (b) 6 cm
 (c) 8 cm (d) 12 cm
52. Find the volume of a prism which is based on a regular Hexagon & of height 10cm. If total S.A. is $156\sqrt{3}$ cm².
 (a) $60\sqrt{3}$ (b) $180\sqrt{3}$
 (c) $120\sqrt{3}$ (d) $240\sqrt{3}$
53. Three solid spheres of a metal whose radii are 1 cm, 6 cm and 8 cm are melted to form another solid sphere. The radius of this new sphere is
 (a) 10.5 cm (d) 9.5 cm
 (c) 10 cm (d) 9 cm
54. The slant height of a conical mountain is 2.5 km and the area of its base is 1.54 km². then find the height of conical mountain.
 (a) 2.2 km (b) 2.4 km
 (c) 3 km (d) 3.11 km
55. The base of a conical tent is 19.2 metres in diameter and the height is 2.8 metres. The area of the canvas required to put up such a tent (in square meters) (taking $\pi = \frac{22}{7}$) is nearly.
 (a) 3017.1 (b) 3170
 (c) 301.7 (d) 30.17
56. A hollow cylindrical tube 20 cm long, is made of iron and its external and internal diameters are 8 cm and 6 cm respectively. The volume of iron used in making the tube is ($\pi = \frac{22}{7}$)
 (a) 1760 cu.cm (b) 880 cu.cm.
 (c) 440 cu.cm (d) 220 cu.cm
57. A sphere of radius 2 cm is put into water contained in a cylinder of base- radius 4 cm. If the sphere is completely immersed in the water, the water level in the cylinder rise by
 (a) $\frac{1}{3}$ cm (b) $\frac{1}{2}$ cm
 (c) $\frac{2}{3}$ cm (d) 2 cm
58. A solid metallic spherical ball of diameter 6 cm is melted and recasted into a cone with diameter of the base as 12 cm. The height of the cone is
 (a) 6 cm (b) 2 cm
 (c) 4 cm (d) 3 cm
59. The volume of a right circular cone is 1232 cm³ and its vertical height is 24 cm. Its curved surface area is
 (a) 154 cm² (b) 550 cm²
 (c) 604 cm² (d) 704 cm²
60. The volume of a sphere is $\frac{88}{21} \times (14)^3$ cm³ The curved surface area of the sphere is
 (Take $\pi = \frac{22}{7}$)
 (a) 2424 cm² (b) 2446 cm²
 (c) 2484 cm² (d) 2464 cm²
61. The surface area of a sphere is 64π cm² Its diameter is equal to
 (a) 16 cm (b) 8 cm
 (c) 4 cm (d) 2 cm
62. The diameter of the base of a cylindrical drum is 35 dm. and the height is 24 dm. It is full of kerosene. How many tins each of size 25 cm $\times$ 22 cm $\times$ 35 cm can be filled with kerosene from the drum ?
 (use $\pi = \frac{22}{7}$)
 (a) 1200 (b) 1020
 (c) 600 (d) 120
63. A hollow iron pipe is 21 cm long and its exterior diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weights 8 g/cm³, then the weight of the pipe is (Take $\pi = \frac{22}{7}$):
 (a) 3.696 kg (b) 3.6 kg
 (c) 36 kg (d) 36.9 kg
64. The volume of a right circular cylinder, 14 cm in height, is equal to that of a cube whose edge is 11 cm Take $\pi = \frac{22}{7}$ the radius of the base of the cylinder is
 (a) 5.2 cm (b) 5.5 cm
 (c) 11.0 cm (d) 22.0 cm
65. If the volume of a right circular cylinder is $9\pi h$ m³, where h is its height (in metres) then the diameter of the base of the cylinder is equal to
 (a) 3 m (b) 6 m (c) 9 m (d) 12 m
66. Each of the measure of the radius of base of a cone and that of a sphere is 8 cm. Also, the volume of these two solids are equal. the slant height of the cone is
 (a) $8\sqrt{17}$ cm (b) $4\sqrt{17}$ cm
 (c) $34\sqrt{2}$ cm (d) 34 cm
67. A well 20 m in diameter is dug 14 m deep and the earth taken out is spread all around it to a width of 5 m to form an embankment. The height of the embankment is:
 (a) 10 m (b) 11 m
 (c) 11.2 m (d) 11.5 m
68. The diameter of the iron ball used for the shot-put game is 14 cm. It is melted and then a solid cylinder of height $2\frac{1}{3}$ cm is made. What will be the diameter of the base of the cylinder?
 (a) 14 cm (b) 28 cm
 (c) $\frac{14}{3}$ cm (d) $\frac{28}{3}$ cm
69. The sum of radii of two spheres is 10 cm and the sum of their volume is 880 cm³. What will be the product of their radii?
 (a) 21 (b) $26\frac{1}{3}$
 (c) $33\frac{1}{3}$ (d) 70

70. A rectangular paper sheet of dimensions $22 \text{ cm} \times 12 \text{ cm}$ is folded in the form of a cylinder along its length. What will be the volume of this cylinder?
(Take $\pi = \frac{22}{7}$)
(a) 460 cm^3 (b) 462 cm^3
(c) 624 cm^3 (d) 400 cm^3
71. A copper rod of 1 cm diameter and 8 cm length is drawn into a wire of uniform diameter and 18 m length. The radius (in cm) of the wire is
(a) $\frac{1}{15}$ (b) $\frac{1}{30}$ (c) $\frac{2}{15}$ (d) 15
72. 12 spheres of the same size are made by melting a solid cylinder of 16 cm diameter and 2 cm height. The diameter of each sphere is :
(a) 2 cm (b) 4 cm
(c) 3 cm (d) $\sqrt{3}$ cm
73. When the circumference of a toy ballon is increased from 20 cm to 25 cm its radius (in cm) is increased by :
(a) 5 (b) $\frac{5}{\pi}$ (c) $\frac{5}{2\pi}$ (d) $\frac{\pi}{5}$
74. If the volume and surface area of a sphere are numerically the same, then its radius is
(a) 1 unit (b) 2 units
(c) 3 units (d) 4 units
75. In a right circular cone, the radius of its base is 7 cm and its height 24 cm. A cross-section is made through the midpoint of the height parallel to the base. The volume of the upper portion is
(a) 169 cm^3 (b) 154 cm^3
(c) 1078 cm^3 (d) 800 cm^3
76. Some solid metallic right circular cones, each with radius of the base 3 cm and height 4 cm, are melted to form a solid sphere of radius 6 cm. The number of right circular cones is
(a) 12 (b) 24 (c) 48 (d) 6
77. A right circular cylinder of height 16 cm is covered by a rectangular tin foil of size $16 \text{ cm} \times 22 \text{ cm}$. The volume of the cylinder is
(a) 352 cm^3 (b) 308 cm^3
(c) 616 cm^3 (d) 176 cm^3
78. If the area of the base of a cone is 770 cm^2 and the area of its curved surface is 814 cm^2 , then find its volume.
(a) $213\sqrt{5} \text{ cm}^3$ (b) $392\sqrt{5} \text{ cm}^3$
(c) $550\sqrt{5} \text{ cm}^3$ (d) $616\sqrt{5} \text{ cm}^3$
79. The size of a rectangular piece of paper is $100 \text{ cm} \times 44 \text{ cm}$. A cylinder is formed by rolling the paper along its breadth. The volume of the cylinder is
(Use $\pi = \frac{22}{7}$)
(a) 4400 cm^3 (b) 15400 cm^3
(c) 35000 cm^3 (d) 144 cm^3
80. The radius of the base and height of a metallic solid cylinder are $r \text{ cm}$ and 6 cm respectively. It is melted and recast into a solid cone of the same radius of base. The height of the cone is:
(a) 54 cm (b) 27 cm
(c) 18 cm (d) 9 cm
81. The total surface area of a metallic hemisphere is 1848 cm^2 . The hemisphere is melted to form a solid right circular cone. If the radius of the base of the cone is the same as the radius of the hemisphere its height is
(a) 42 cm (b) 26 cm
(c) 28 cm (d) 30 cm
82. A right circular cylinder is formed by rolling a rectangular paper 12 cm long and 3 cm wide along its length. The radius of the base of the cylinder will be
(a) $\frac{3}{2\pi} \text{ cm}$ (b) $\frac{6}{\pi} \text{ cm}$
(c) $\frac{9}{2\pi} \text{ cm}$ (d) $2\pi \text{ cm}$
83. What part of a ditch, 48 metres long, 16.5 metres broad and 4 metres deep can be filled by the earth got by digging a cylindrical tunnel of diameter 4 metres and length 56 metres?
(Use $\pi = \frac{22}{7}$)
(a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{7}{9}$ (d) $\frac{8}{9}$
84. The volume of the metal of cylindrical pipe is 748 cm^3 . The length of the pipe is 14 cm and its external radius is 9 cm. its thickness is
(Take $\pi = \frac{22}{7}$)
(a) 1 cm (b) 5.2 cm
(c) 2.3 cm (d) 3.7 cm
85. Two iron sphere each of diameter 6 cm are immersed in the water contained in a cylindrical vessel of radius 6 cm. The level of the water in the vessel will be raised by
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 6 cm
86. The height of the cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume is $\frac{1}{27}$ of the volume of the cone, at what height above the base, is the section made ?
(a) 6 cm (b) 8 cm
(c) 10 cm (d) 20 cm
87. The total surface area of a solid hemisphere is $108\pi \text{ cm}^2$. The volume of the hemisphere is
(a) $72\pi \text{ cm}^3$ (b) $144\pi \text{ cm}^3$
(c) $108\sqrt{6} \text{ cm}^3$ (d) $54\sqrt{6} \text{ cm}^3$
88. A solid metallic sphere of radius 3 decimetres is melted to form a circular sheet of 1 millimetre thickness. The diameter of the sheet so formed is
(a) 26 metres (b) 24 metres
(c) 12 metres (d) 6 metres
89. Water flows through a cylindrical pipe, whose radius is 7 cm, at 5 metre per second. The time, it takes to fill an empty water tank with height 1.54 metres and area of the base (3×5) square metres, is
(take $\pi = \frac{22}{7}$)
(a) 6 minutes (b) 5 minutes
(c) 10 minutes (d) 9 minutes
90. If S denotes the area of the curved surface of a right circular cone of height h and semivertical angle α then S equals
(a) $\pi h^2 \tan^2 \alpha$
(b) $\frac{1}{3} \pi h^2 \tan^2 \alpha$
(c) $\pi h^2 \sec \alpha \tan \alpha$
(d) $\frac{1}{3} \pi h^2 \sec \alpha \tan \alpha$
91. The height and the radius of the base of a right circular cone are 12 cm and 6 cm respectively. The radius of the circular

- cross-section of the cone cut by a plane parallel to its base at a distance of 3 cm from the base is
 (a) 4 cm (b) 5.5 cm
 (c) 4.5 cm (d) 3.5 cm
92. If S_1 and S_2 be the surface areas of a sphere and the curved surface area of the circumscribed cylinder respectively, then S_1 is equal to
 (a) $\frac{3}{4} S_2$ (b) $\frac{1}{2} S_2$
 (c) $\frac{2}{3} S_2$ (d) S_2
93. The volume of a right circular cylinder and that of a sphere are equal and their radii are also equal. If the height of the cylinder be h and the diameter of the sphere d , then which of the following relation is correct?
 (a) $h = d$ (b) $2h = d$
 (c) $2h = 3d$ (d) $3h = 2d$
94. Water is being pumped out through a circular pipe whose internal diameter is 7cm. If the flow of water is 12cm per second, how many litres of water is being pumped out in one hour?
 (a) 1663.2 (b) 1500
 (c) 1747.6 (d) 2000
95. The lateral surface area of a cylinder is 1056 cm^2 and its height is 16cm. Find its volume.
 (a) 4545 cm^3 (b) 4455 cm^3
 (c) 5445 cm^3 (d) 5544 cm^3
96. The radius of the base and height of a right circular cone are in the ratio 5 : 12. If the volume of the cone is $314 \frac{2}{7} \text{ cm}^3$, the slant height (in cm) of the cone will be
 (a) 12 (b) 13 (c) 15 (d) 17
97. A solid metallic cone is melted and recast into a solid cylinder of the same base as that of the cone. If the height of the cylinder is 7 cm, the height of the cone was
 (a) 20 cm (b) 21 cm
 (c) 28 cm (d) 24 cm
98. A copper wire of length 36m and diameter 2mm is melted to form a sphere. The radius of the sphere (in cm) is
 (a) 2.5 (b) 3 (c) 3.5 (d) 4
99. The diameter of the base of a right circular cone is 4 cm and its height $2\sqrt{3}$ cm. The slant height of the cone is
 (a) 5 cm (b) 4 cm
 (c) $2\sqrt{3}$ (d) 3 cm
100. The rain water from a roof $22 \text{ m} \times 20 \text{ m}$ drains into a cylindrical vessel having a diameter of 2 m and height 3.5 m, If the vessel is just full, then the rainfall (in cm) is :
 (a) 2 (b) 2.5 (c) 3 (d) 4.5
101. From a solid cylinder of height 10 cm and radius of the base 6 cm, a cone of same height and same base is removed. The volume of the remaining solid is :
 (a) $240\pi \text{ cu. cm}$
 (b) 5280 cu. cm
 (c) $620\pi \text{ cu. cm}$
 (d) $360\pi \text{ cu. cm}$
102. Two solid right cones of equal height and of radii r_1 and r_2 are melted and made to form a solid sphere of radius R . Then the height of the cone is
 (a) $\frac{4R^2}{r_1^2 + r_2^2}$ (b) $\frac{R^3}{r_1^2 r_2^2}$
 (c) $\frac{4R^3}{r_1^2 + r_2^2}$ (d) $\frac{R^2}{r_1^2 r_2^2}$
103. The ratio of height and the diameter of a right circular cone is 3 : 2 and its volume is 1078 cc, then (taking $\pi = \frac{22}{7}$) its height is :
 (a) 7 cm (b) 14 cm
 (c) 21 cm (d) 28 cm
104. From a right circular cylinder of radius 10 cm and height 21 cm, a right circular cone of same base radius is removed. If the volume of the remaining portion is 4400 cm^3 , then the height of the removed cone (take $\pi = \frac{22}{7}$) is :
 (a) 15 cm (b) 18 cm
 (c) 21 cm (d) 24 cm
105. A child reshapes a cone made up of clay of height 24 cm and radius 6 cm into a sphere. The radius (in cm) of the sphere is
 (a) 6 (b) 12 (c) 24 (d) 48
106. A solid cylinder has total surface area of 462 sq. cm . Its curved surface area is one third of the total surface area. Then the radius of the cylinder is
 (a) 7 cm (b) 3.5 cm
 (c) 9 cm (d) 11 cm
107. The diameter of a cylinder is 7 cm and its height is 16 cm. Using the value of $\pi = \frac{22}{7}$, the lateral surface area of the cylinder is
 (a) 352 cm^2 (b) 350 cm^2
 (c) 355 cm^2 (d) 348 cm^2
108. The height of a solid right circular cylinder is 6 metres and three times the sum of the area of its two end faces is twice the area of its curved surface. The radius of its base (in meter) is
 (a) 4 (b) 2 (c) 8 (d) 10
109. A semi-circular sheet of metal of diameter 28 cm is bent into an open conical cup. The depth of the cup is approximately
 (a) 11 cm (b) 12 cm
 (c) 13 cm (d) 14 cm
110. A right angled sector of radius r cm is rolled up into a cone in such a way that the two binding radii are joined together. then the curved surface area of the cone is
 (a) $\pi r^2 \text{ cm}^2$ (b) $\frac{\pi r^2}{4} \text{ cm}^2$
 (c) $\frac{\pi r^2}{2} \text{ cm}^2$ (d) $2\pi r^2 \text{ cm}^2$
111. The radius of the base of a conical tent is 16 metre. If $427 \frac{3}{7} \text{ sq. metre}$ canvas is required to construct the tent, then the slant height of the tent is : (take $\pi = \frac{22}{7}$)
 (a) 17 metre (b) 15 metre
 (c) 19 metre (d) 8.5 metre
112. A circus tent is cylindrical up to a height of 3 m and conical above it. If its diameter is 105m and the slant height of the conical part is 63 m, then the total area of the canvas required to make the tent is (take $\pi = \frac{22}{7}$)
 (a) 11385 m^2 (b) 10395 m^2
 (c) 9900 m^2 (d) 990 m^2

113. A toy is in the form of a cone mounted on a hemisphere. The radius of the hemisphere and that of the cone is 3 cm and height of the cone is 4 cm. The total surface area of the toy (taking $\pi = \frac{22}{7}$) is
 (a) 75.43 sq. cm,
 (b) 103.71 sq. cm,
 (c) 85.35 sq. cm,
 (d) 120.71 sq. cm,
114. Marbles of diameter 1.4 cm are dropped into a cylindrical beaker containing some water and fully submerged. The diameter of the beaker is 7 cm. Find how many marbles have been dropped in it if the water rises by 5.6 cm ?
 (a) 50 (b) 150
 (c) 250 (d) 350
115. A cylindrical rod of iron whose height is eight times its radius is melted and cast into spherical balls each of half the radius of the cylinder. The number of such spherical balls is
 (a) 12 (b) 16 (c) 24 (d) 48
116. A cylinder has 'r' as the radius of the base and 'h' as the height. The radius of base of another cylinder, having double the volume but the same height as that of the first cylinder must be equal to
 (a) $\frac{r}{2}$ (b) $2r$ (c) $r\sqrt{2}$ (d) $\sqrt{2}r$
117. From a solid cylinder whose height is 12 cm and diameter 10 cm, a conical cavity of same height and same diameter of the base is hollowed out. The volume of the remaining solid is approximately ($\pi = \frac{22}{7}$)
 (a) 942.86 cm³ (b) 314.29 cm³
 (c) 628.57 cm³ (d) 450.76 cm³
118. The radius of a cylinder is 10 cm and height is 4 cm. The number of centimetres that may be added either to the radius or to the height to get the same increase in the volume of the cylinder is
 (a) 5 cm (b) 4 cm
 (c) 25 cm (d) 16 cm
119. The radius of the base of a right circular cone is doubled keeping its height fixed. The volume of the cone will be :
 (a) Three times of the previous volume
 (b) four times of the previous volume
 (c) $\sqrt{2}$ times of the previous volume
 (d) double of the previous volume
120. The base of a right circular cone has the radius 'a' which is same as that of a sphere. Both the sphere and the cone have the same volume. Height of the cone is
 (a) $3a$ (b) $4a$
 (c) $\frac{7}{4}a$ (d) $\frac{7}{3}a$
121. The circumference of the base of a 16 cm high solid cone is 33 cm. What is the volume of the cone in cm³ ?
 (a) 1028 (b) 616
 (c) 462 (d) 828
122. A solid sphere of 6 cm diameter is melted and recast into 8 solid spheres of equal volume. The radius (in cm) of each small sphere is
 (a) 1.5 (b) 3 (c) 2 (d) 2.5
123. In a cylindrical vessel of diameter 24 cm filled up with sufficient quantity of water, a solid spherical ball of radius 6 cm is completely immersed. Then the increase in height of water level is :
 (a) 1.5 cm (b) 2 cm
 (c) 3 cm (d) 4.2 cm
124. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm find the volume of wooden toy (nearly).
 (a) 104 cm³ (b) 162 cm³
 (c) 421 cm³ (d) 266 cm³
125. If a solid cone of volume 27π cm³ is kept inside a hollow cylinder whose radius and height are that of the cone, then the volume of water needed to fill the empty space is
 (a) 3π cm³ (b) 18π cm³
 (c) 54π cm³ (d) 81π cm³
126. A cylindrical can whose base is horizontal and is of internal radius 3.5 cm contains sufficient water so that when a solid sphere is placed inside, water just covers the sphere. The sphere fits in the can exactly. The depth of water in the can before the sphere was put, is
 (a) $\frac{35}{3}$ cm (b) $\frac{17}{3}$ cm
 (c) $\frac{7}{3}$ cm (d) $\frac{14}{3}$ cm
127. The radius and height of a cylinder are in the ratio 5 : 7 and its volume is 550 cm³. Calculate its curved surface area in sq. cm.
 (a) 110 (b) 444
 (c) 220 (d) 616
128. The area of the curved surface and the area of the base of a right circular cylinder are a square cm and b square cm respectively. The height of the cylinder is
 (a) $\frac{2a}{\sqrt{\pi b}}$ cm (b) $\frac{a\sqrt{b}}{2\sqrt{\pi}}$ cm
 (c) $\frac{a}{2\sqrt{\pi b}}$ cm (d) $\frac{a\sqrt{\pi}}{2\sqrt{b}}$ cm
129. The volume of a solid hemisphere is 19404 cm³. Its total surface area is
 (a) 4158 cm² (b) 2858 cm²
 (c) 1738 cm² (d) 2038 cm²
130. A solid hemisphere is of radius 11 cm. the curved surface area in sq. cm is
 (a) 1140.85 (b) 1386.00
 (c) 760.57 (d) 860.57
131. The base of a cone and a cylinder have the same radius 6 cm. They have also the same height 8 cm. The ratio of the curved surface area of the cylinder to that of the cone is
 (a) 8 : 5 (b) 8 : 3
 (c) 4 : 3 (d) 5 : 3
132. A right cylindrical vessel is full with water. How many right

- cones having the same diameter and height as that of the right cylinder will be needed to store that water ?
(take $\pi = \frac{22}{7}$)
(a) 4 (b) 2 (c) 3 (d) 5
133. A spherical lead ball of radius 10 cm is melted and small lead balls of radius 5mm are made. The total number of possible small lead balls is
(Take $\pi = \frac{22}{7}$)
(a) 8000 (b) 400
(c) 800 (d) 125
134. The number of spherical bullets that can be made out of solid cube of lead whose edge measures 44 cm each bullet being of 4 cm diameter, is (Take $\pi = \frac{22}{7}$)
(a) 2541 (b) 2451
(c) 2514 (d) 2415
135. The radius of a metallic cylinder is 3 cm and its height is 5 cm. It is melted and moulded into small cones, each of height 1 cm and base radius 1 mm. The number of such cones formed is
(a) 450 (b) 1350
(c) 8500 (d) 13500
136. A sector is formed by opening out a cone of base radius 8 cm and height 6 cm. Then the radius of the sector is (in cm)
(a) 4 (b) 8 (c) 10 (d) 6
137. A solid cone of height 9 cm with diameter of its base 18 cm is cut out from a wooden solid sphere of radius 9 cm. The percentage of wood wasted is :
(a) 25% (b) 30%
(c) 50% (d) 75%
138. The perimeter of the base of a right circular cylinder is 'a' unit. If the volume of the cylinder is V cubic unit, then the height of the cylinder is
(a) $\frac{4a^2V}{\pi}$ unit (b) $\frac{4\pi a^2}{V}$ unit
(c) $\frac{\pi a^2V}{4}$ unit (d) $\frac{4\pi V}{a^2}$ unit
139. What is the height of a cylinder that has the same volume and radius as a sphere of diameter 12 cm ?
(a) 7 cm (b) 10 cm
(c) 9 cm (d) 8 cm
140. The perimeter of the base of a right circular cone is 8 cm. If the height of the cone is 21 cm, then its volume is :
(a) $108\pi \text{ cm}^3$ (b) $\frac{112}{\pi} \text{ cm}^3$
(c) $112\pi \text{ cm}^3$ (d) $\frac{108}{\pi} \text{ cm}^3$
141. If the volume of two right circular cones are in the ratio 4 : 1 and their diameter are in the ratio 5 : 4, then the ratio of their height is :
(a) 25 : 16 (b) 25 : 64
(c) 64 : 25 (d) 16 : 25
142. The volume of a conical tent is 1232 cu. m and the area of its base is 154 sq. m. Find the length of the canvas required to build the tent, if the canvas is 2m in width.
(Take $\pi = \frac{22}{7}$)
(a) 270 m (b) 272 m
(c) 276 m (d) 275 m
143. If the ratio of the diameters of two right circular cones of equal height be 3: 4, then the ratio of their volume will be
(a) 3 : 4 (b) 9 : 16
(c) 16 : 9 (d) 27 : 64
144. The surface area of two spheres are in the ratio 4 : 9. Their volumes will be in the ratio
(a) 2 : 3 (b) 4 : 9
(c) 8 : 27 (d) 64 : 729
145. The total surface area of a sphere is 8π square unit. The volume of the sphere is
(a) $\frac{8\sqrt{2}}{3}\pi$ cubic unit
(b) $\frac{8}{3}\pi$ cubic unit
(c) $8\sqrt{3}\pi$ cubic unit
(d) $\frac{8\sqrt{3}}{5}\pi$ cubic unit
146. A semicircular sheet of metal of diameter 28 cm is bent into an open conical cup. The capacity of the cup (Take $\pi = \frac{22}{7}$) is
(a) 624.26 cm³ (b) 622.36 cm³
(c) 622.56 cm³ (d) 623.20 cm³
147. A conical flask is full of water. The flask has base radius r and height h. This water is poured into a cylindrical flask of base radius m, height of cylindrical flask is
(a) $\frac{m}{2h}$ (b) $\frac{h}{2}m^2$
(c) $\frac{2h}{m}$ (d) $\frac{r^2h}{3m^2}$
148. A solid spherical copper ball whose diameter is 14 cm is melted and converted into a wire having diameter equal to 14 cm. The length of the wire is
(a) 27 cm (b) $\frac{16}{3}$ cm
(c) 15 cm (d) $\frac{28}{3}$ cm
149. A sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is just completely submerged in water, then the rise of water level in the cylindrical vessel is
(a) 2 cm (b) 1 cm
(c) 3 cm (d) 4 cm
150. A copper sphere of diameter 18 cm is drawn into a wire of diameter 4 mm. The length of the wire in metre is :
(a) 2.43 m (b) 243 m
(c) 2430 m (d) 24.3 m
151. A rectangular block of metal has dimensions 21 cm, 77 cm and 24 cm. The block has been melted into a sphere. The radius of the sphere is (Take $\pi = \frac{22}{7}$)
(a) 21 cm (b) 7 cm
(c) 14 cm (d) 28 cm
152. The radius of cross-section of a solid cylindrical rod of iron is 50 cm. the cylinder is melted down and formed into 6 solid spherical balls of the same

- radius as that of the cylinder. The length of the rod (in metres) is
(a) 0.8 (b) 2 (c) 3 (d) 4
153. Two right circular cones of equal height and radii of their respective base 3 cm and 4 cm are melted together and made to a solid sphere of radius 5 cm. The height of a cone is
(a) 10 cm (b) 20 cm (c) 30 cm (d) 40 cm
154. The radius of the base and the height of a right circular cone are doubled. The volume of the cone will be
(a) 8 times of the previous volume
(b) three times of the previous volume
(c) $3\sqrt{2}$ times of the previous volume
(d) 6 times of the previous volume
155. If h , c , v are respectively the height, curved surface area and volume of a right circular cone then the value of $3\pi v h^3 - c^2 h^2 + 9v^2$ is
(a) 2 (b) -1 (c) 1 (d) 0
156. The total number of spherical bullets, each of diameter 5 decimeter, that can be made by utilizing the maximum of a rectangular block of lead with 11 metre length, 10 metre breadth and 5 metre width is (assume that $\pi = 3$)
(a) equal to 8800
(b) less than 8800
(c) equal to 8400
(d) greater than 9000
157. If a metallic cone of radius 30 cm and height 45 cm is melted and recast into metallic spheres of radius 5 cm, find the number of spheres,
(a) 81 (b) 41 (c) 80 (d) 40
158. A metallic sphere of radius 10.5 cm is melted and then recast into small cones each of radius 3.5 cm and height 3 cm. The number of cones thus formed is
(a) 140 (b) 132 (c) 112 (d) 126
159. A right circular cone is 3.6 cm high and radius of its base is 1.6 cm. It is melted and recast into a right circular cone with radius of its base as 1.2 cm. Then the height of the cone (in cm) is
(a) 3.6 cm (b) 4.8 cm (c) 6.4 cm (d) 7.2 cm
160. If surface area and volume of a sphere are S and V respectively, then value of $\frac{S^3}{V^2}$ is
(a) 36π units (b) 9π units (c) 18π units (d) 27π units
161. Assume that a drop of water is spherical and its diameter is one-tenth of a cm. A conical glass has a height equal to the diameter of its rim. If 32,000 drops of water fill the glass completely. Then the height of the glass (in cm) is
(a) 1 (b) 2 (c) 3 (d) 4
162. A tank 40 m long, 30 m broad and 12 m deep is dug in a field 1000 m long and 30 m wide. By how much will the level of the field rise if the earth dug out of the tank is evenly spread over the field?
(a) 2 metre (b) 1.2 metre (c) 0.5 metre (d) 5 metre
163. A sphere is cut into two hemispheres. One of them is used as bowl. It takes 8 bowlfuls of this to fill a conical vessel of height 12 cm and radius 6 cm. The radius of the sphere (in centimetre) will be
(a) 3 (b) 2 (c) 4 (d) 6
164. A ball of lead 4 cm in diameter is covered with gold. If the volume of the gold and lead are equal then the thickness of gold [given $\sqrt[3]{2} = 1.259$] is approximately
(a) 5.038 cm (b) 5.190 cm (c) 1.038 cm (d) 0.518 cm
165. A conical cup is filled with ice-cream. The ice-cream forms a hemispherical shape on its open top. The height of the hemispherical part is 7 cm. The radius of the hemispherical part equals to the height of the cone. Then the volume of the ice-cream is $\left[\pi = \frac{22}{7}\right]$
(a) 1078 cubic cm (b) 1708 cubic cm (c) 7108 cubic cm (d) 7180 cubic cm
166. A hollow sphere of internal and external diameter 6 cm and 10 cm respectively is melted into a right circular cone of diameter 8 cm. The height of the cone is
(a) 22.5 cm (b) 23.5 cm (c) 24.5 cm (d) 25.5 cm
167. A flask in the shape of a right circular cone of height 24 cm is filled with water. The water is poured in right circular cylindrical flask whose radius is $\frac{1}{3}$ rd of radius of the base of the circular cone. Then the height of the water in the cylindrical flask is
(a) 32 cm (b) 24 cm (c) 48 cm (d) 72 cm
168. A solid metallic spherical ball of diameter 6 cm is melted and recast into a cone with diameter of the base as 12 cm. The height of the cone is
(a) 2 cm (b) 3 cm (c) 4 cm (d) 6 cm
169. A hemispherical bowl of internal radius 15 cm contains a liquid. The liquid is to be filled into cylindrical shaped bottles of diameter 5 cm and height 6 cm. The number of bottles required to empty the bowl is
(a) 30 (b) 40 (c) 50 (d) 60
170. If V_1 , V_2 and V_3 be the volumes of a right circular cone, a sphere and a right circular cylinder having the same radius and same height then
(a) $V_1 = \frac{V_2}{4} = \frac{V_3}{3}$ (b) $\frac{V_1}{2} = \frac{V_2}{3} = V_3$
(c) $\frac{V_1}{3} = \frac{V_2}{2} = V_3$ (d) $\frac{V_1}{3} = V_2 = \frac{V_3}{2}$
171. If the surface area of a sphere is 346.5 cm^2 , then its radius [taking $\pi = \frac{22}{7}$]
(a) 7 cm (b) 3.25 cm (c) 5.25 cm (d) 9 cm

172. Deepali makes a model of a cylindrical kaleidoscope for her science project. She uses a chart paper to make it. If the length of the kaleidoscope is 25 cm and radius 35 cm, the area of the paper she used, in sq. cm, is $\left(\pi = \frac{22}{7}\right)$
- (a) 1100 (b) 5500
(c) 500 (d) 450
173. If the volume of a sphere is numerically equal to its surface area then its diameter is;
- (a) 4 cm (b) 6 cm
(c) 3 cm (d) 2 cm
174. 5 persons live in a tent. If each person requires 16 m^2 of floor area and 100 m^3 space for air then the height of the cone of smallest size to accommodate these persons would be?
- (a) 16 m (b) 18.75 m
(c) 10.25 m (d) 20 m
175. The numerical values of the volume and the area of the lateral surface of a right circular cone are equal. If the height of the cone be h and radius be r , the value of $\frac{1}{h^2} + \frac{1}{r^2}$ is
- (a) $\frac{9}{1}$ (b) $\frac{3}{1}$ (c) $\frac{1}{3}$ (d) $\frac{1}{9}$
176. There is wooden sphere of radius $6\sqrt{3} \text{ cm}$. The surface area of the largest possible cube cut out from the sphere will be
- (a) $464\sqrt{3} \text{ cm}^2$ (b) $646\sqrt{3} \text{ cm}^2$
(c) 864 cm^2 (d) 462 cm^2
177. If a hemisphere is melted and four spheres of equal volume are made, the radius of each sphere will be equal to
- (a) $1/4^{\text{th}}$ of the hemisphere
(b) radius of the hemisphere
(c) $1/2$ of the radius of the hemisphere
(d) $1/6^{\text{th}}$ of the radius of the hemisphere
178. The portion of a ditch 48 m long, 16.5 m wide and 4 m deep that can be filled with stones and earth available during excavation of a tunnel, cylindrical in shape, of diameter 4 m and length 56 m is $\left(\text{Take } \pi = \frac{22}{7}\right)$
- (a) $\frac{1}{9}$ Part (b) $\frac{1}{2}$ Part
(c) $\frac{1}{4}$ Part (d) $\frac{2}{9}$ Part
179. From a solid right circular cylinder of length 4 cm and diameter 6 cm, a conical cavity of the same height and base is hollowed out. The whole surface area of the remaining solid (in square cm.) is
- (a) 48π (b) 63π
(c) 15π (d) 24π
180. A spherical ball of radius 1 cm is dropped into a conical vessel of radius 3 cm and slant height 6 cm. The volume of water (in cm^3), that can just immerse the ball, is
- (a) $\frac{5\pi}{3}$ (b) 3π (c) $\frac{\pi}{3}$ (d) $\frac{4\pi}{3}$
181. If the height of a cylinder is 4 times its circumference, the volume of the cylinder in terms of its circumference, c is
- (a) $\frac{2c^3}{\pi}$ (b) $\frac{c^3}{\pi}$
(c) $4\pi c^3$ (d) $2\pi c^3$
182. The radii of a sphere and a right circular cylinder are 3 cm each. If their volumes are equal, then curved surface area of the cylinder is $\left(\text{Assume } \pi = \frac{22}{7}\right)$
- (a) $75\frac{3}{7} \text{ cm}^2$ (b) $65\frac{3}{7} \text{ cm}^2$
(c) $74\frac{3}{7} \text{ cm}^2$ (d) $72\frac{3}{7} \text{ cm}^2$
183. The radius of a hemispherical bowl is 6 cm. The capacity of the bowl is: $\left(\text{Take } \pi = \frac{22}{7}\right)$
- (a) 452.57 cm^3 (b) 452 cm^3
(c) 345.53 cm^3 (d) 495.51 cm^3
184. The total surface area of a right circular cylinder with radius of the base 7 cm and height 20 cm is:
- (a) 140 cm^2 (b) 1000 cm^2
(c) 900 cm^2 (d) 1188 cm^2
185. The radius of base and curved surface area of a right cylinder are ' r ' units and $4\pi rh$ square units respectively. The height of the cylinder is:
- (a) $4h$ units (b) $\frac{h}{2}$ units
(c) h units (d) $2h$ units
186. A hemi-spherical bowl has 3.5 cm radius. It is to be painted inside as well as outside. The cost of painting it at the rate of Rs. 5 per 10 sq. cm . will be:
- (a) Rs. 77 (b) Rs. 175
(c) Rs. 50 (d) Rs. 100
187. The volume of a right circular cone which is obtained from a wooden cube of edge 4.2 dm wasting minimum amount of wood is:
- (a) 194.04 cu. dm
(b) 19.404 cu. dm
(c) 1940.4 cu. dm
(d) 1940.4 cu. dm
188. If the radius of a sphere is increased by 2 cm, then its surface area increases by 352 cm^2 . The radius of the sphere initially was: $\left(\text{use } \pi = \frac{22}{7}\right)$
- (a) 3 cm (b) 5 cm
(c) 4 cm (d) 6 cm
189. A right triangle with sides 9 cm, 12 cm and 15 cm is rotated about the side of 9 cm to form a cone. The volume of the cone so formed is:
- (a) $432 \pi \text{ cm}^3$ (b) $327 \pi \text{ cm}^3$
(c) $334 \pi \text{ cm}^3$ (d) $324 \pi \text{ cm}^3$
190. The volume of the largest right circular cone that can be cut out of a cube of edge 7 cm ? $\left(\text{use } \pi = \frac{22}{7}\right)$
- (a) 13.6 cm^3 (b) 147.68 cm^3
(c) 89.9 cm^3 (d) 121 cm^3
191. By melting two solid metallic spheres of radii 1 cm and 6 cm,

- a hollow sphere of thickness 1 cm is made. The external radius of the hollow sphere will be
 (a) 8 cm (b) 9 cm
 (c) 6 cm (d) 7 cm
192. Water is flowing at the rate of 5 km/h through a pipe of diameter 14 cm into a rectangular tank which is 50 m long 44m wide, The time taken (in hours) for the rise in the level of water in the tank to 7 cm is
 (a) 2 (b) $1\frac{1}{2}$ (c) 3 (d) $2\frac{1}{2}$
193. The volume (in m^3) of rain water that can be collected from 1.5 hectares of ground in a rainfall of 5 cm is
 (a) 75 (b) 750
 (c) 7500 (d) 75000
194. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour, How much water (in-litres) will fall into sea in a minute ?
 (a) 4,00,000 m^3 (b) 40,00,000 m^3
 (c) 40,000 m^3 (d) 4,000 m^3
195. Water is flowing at the rate of 3 km/hr through a circular pipe of 20 cm internal diameter into a circular cistern of diameter 10 m and depth 2m. In how much time will the cistern be filled ?
 (a) 1 hour
 (b) 1 hour 40 minutes
 (c) 1 hour 20 minutes
 (d) 2 hours 40 minutes
196. Water flows at the rate of 10 metres per minute from cylindrical pipe 5 mm in diameter, How long it will take to fill up a conical vessel whose diameter at the base is 30 cm and depth 24 cm?
 (a) 28 minutes 48 seconds
 (b) 51 minutes 12 seconds
 (c) 51 minutes 24 seconds
 (d) 28 minutes 36 seconds
197. The radius of the base of conical tent is 12 m. The tent is 9 m high. Find the cost of canvas required to make the tent, if one square metre of canvas costs Rs.120 (Take $\pi = 3.14$)
 (a) Rs. 67,830 (b) Rs. 67,800
 (c) Rs. 67,820 (d) Rs. 67,824
198. A plate of square base made of brass is of length x cm and thickness 1 mm. The plate weighs 4725 gm. If 1 cubic cm of brass weighs 8.4 gram, then the value of x is:
 (a) 76 (b) 72 (c) 74 (d) 75
199. The diameter of a 120 cm long roller is 84 cm. It takes 500 complete revolutions of the roller to level a ground. The cost of levelling the ground at Rs. 1.50 sq. m. is:
 (a) Rs. 5750 (b) Rs. 6000
 (c) Rs. 3760 (d) Rs. 2376
200. Two right circular cylinders of equal volume have their heights in the ratio 1 : 2. The ratio of their radii is :
 (a) $\sqrt{2} : 1$ (b) 2 : 1
 (c) 1 : 2 (d) 1 : 4
201. If the volume of two cubes are in the ratio 27 : 1, the ratio of their edge is :
 (a) 3 : 1 (b) 27 : 1
 (c) 1 : 3 (d) 1 : 27
202. The edges of a cuboid are in the ratio 1 : 2 : 3 and its surface area is 88 cm^2 . The volume of the cuboid is :
 (a) 48 cm^3 (b) 64 cm^3
 (c) 16 cm^3 (d) 100 cm^3
203. The volume of two spheres are in the ratio 8 : 27. The ratio of their surface area is:
 (a) 4 : 9 (b) 2 : 3
 (c) 4 : 5 (d) 5 : 6
204. The base radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. The ratio of their volumes is :
 (a) 27 : 20 (b) 20 : 27
 (c) 9 : 4 (d) 4 : 9
205. The curved surface area of a cylindrical pillar is 264 m^2 and its volume is 924 m^3 (Taking $\pi = \frac{22}{7}$). find the ratio of its diameter to its height .
 (a) 7 : 6 (b) 6 : 7
 (c) 3 : 7 (d) 7 : 3
206. The ratio of the volume of two cones is 2 : 3 and the ratio of radii of their base is 1 : 2. The ratio of their height is
 (a) 3 : 8 (b) 8 : 3
 (c) 4 : 3 (d) 3 : 4
207. If the volume of two cubes are in the ratio 27 : 64, then the ratio of their total surface area is:
 (a) 27 : 64 (b) 3 : 4
 (c) 9 : 16 (d) 3 : 8
208. A hemisphere and a cone have equal base . If their heights are also equal, the ratio of their curved surface will be :
 (a) $1 : \sqrt{2}$ (b) $\sqrt{2} : 1$
 (c) 1 : 2 (d) 2 : 1
209. If the height of a given cone be doubled and radius of the base remains the same the ratio of the volume of the given cone to that of the second cone will be
 (a) 2 : 1 (b) 1 : 8
 (c) 1 : 2 (d) 8 : 1
210. Spheres A and B have their radii 40 cm and 10 cm respectively. Ratio of surface area of A to the surface area of B is :
 (a) 1 : 16 (b) 4 : 1
 (c) 1 : 4 (d) 16 : 1
211. If the radius of the base of a cone be doubled and height is left unchanged, then ratio of the volume of new cone to that of the original cone will be:
 (a) 1 : 4 (b) 2 : 1
 (c) 1 : 2 (d) 4 : 1
212. A cube of edge 5 cm is cut into cubes each of edge of 1 cm. The ratio of the total surface area of one of the small cubes to that of the large cube is equal to:
 (a) 1 : 125 (b) 1 : 5
 (c) 1 : 625 (d) 1 : 25
213. The diameter of two hollow spheres made from the same metal sheet are 21 cm and 17.5 cm respectively. The ratio of the area of metal sheets required for making the two spheres is
 (a) 6 : 5 (b) 36 : 25
 (c) 3 : 2 (d) 18 : 25
214. By melting a solid lead sphere of diameter 12 cm, three small spheres are made whose diameters are in the ratio 3 : 4 : 5. The radius (in cm) of the smallest sphere is
 (a) 3 (b) 6 (c) 1.5 (d) 4
215. A cone is cut at mid point of its height by a frustum parallel to its base. The ratio between the volumes of two parts of cone would be
 (a) 1 : 1 (b) 1 : 8
 (c) 1 : 4 (d) 1 : 7

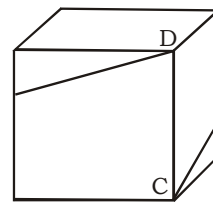
216. The ratio of the area of the incircle and the circum-circle of a square is
 (a) 1 : 2 (b) $\sqrt{2} : 1$
 (c) $1 : \sqrt{2}$ (d) 2 : 1
 (d) remains unchanged
217. The ratio of the surface area of a sphere and the curved surface area of the cylinder circumscribing the sphere is
 (a) 1 : 2 (b) 1 : 1
 (c) 2 : 1 (d) 2 : 3
218. The radii of two spheres are in the ratio 3 : 2. Their volume will be in the ratio :
 (a) 9 : 4 (b) 3 : 2
 (c) 8 : 27 (d) 27 : 8
219. The volume of a sphere and a right circular cylinder having the same radius are equal. The ratio of the diameter of the sphere to the height of the cylinder is
 (a) 3 : 2 (b) 2 : 3
 (c) 1 : 2 (d) 2 : 1
220. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their respective volume is
 (a) 1 : 2 : 3 (b) 2 : 1 : 3
 (c) 1 : 3 : 2 (d) 3 : 1 : 2
221. The radii of the base of two cylinders are in the ratio 3 : 5 and their heights in the ratio 2 : 3. The ratio of their curved surface will be :
 (a) 2 : 5 (b) 2 : 3
 (c) 3 : 5 (d) 5 : 3
222. If the radii of two spheres are in the ratio 1 : 4, then their surface area are in the ratio :
 (a) 1 : 2 (b) 1 : 4
 (c) 1 : 8 (d) 1 : 16
223. The radii of the base of two cylinders A and B are in the ratio 3 : 2 and their height in the ratio $x : 1$. If the volume of cylinder A is 3 times that of cylinder B, the value of x is
 (a) $\frac{4}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{3}{2}$
224. A solid metallic sphere of radius 8 cm is melted to form 64 equal small solid spheres. The ratio of the surface area of this sphere to that of a small sphere is
 (a) 4 : 1 (b) 1 : 16
 (c) 16 : 1 (d) 1 : 4
225. The diameter of two cylinders, whose volumes are equal, are in the ratio 3 : 2. Their heights will be in the ratio.
 (a) 4 : 9 (b) 5 : 6
 (c) 5 : 8 (d) 8 : 9
226. The radius of base and slant height of a cone are in the ratio 4 : 7. If slant height is 14 cm then the radius (in cm) of its base is (use $\pi = \frac{22}{7}$)
 (a) 8 (b) 12 (c) 14 (d) 16
227. A right circular cylinder just encloses a sphere of radius r . The ratio of the surface area of the sphere and the curved surface area of the cylinder is
 (a) 2 : 1 (b) 1 : 2
 (c) 1 : 3 (d) 1 : 1
228. The ratio of radii of two cone is 3 : 4 and the ratio of their height is 4 : 3. Then the ratio of their volume will be
 (a) 3 : 4 (b) 4 : 3
 (c) 9 : 16 (d) 16 : 9
229. If a right circular cone is separated into solids of volumes V_1, V_2, V_3 by two planes parallel to the base which also trisect the altitude, then $V_1 : V_2 : V_3$ is
 (a) 1 : 2 : 3 (b) 1 : 4 : 6
 (c) 1 : 6 : 9 (d) 1 : 7 : 19
230. The total surface area of a solid right circular cylinder is twice that of a solid sphere. If they have the same radii, the ratio of the volume of the cylinder to that of the sphere is given by
 (a) 9 : 4 (b) 2 : 1
 (c) 3 : 4 (d) 4 : 9
231. The respective height and volume of a hemisphere and a right circular cylinder are equal, then the ratio of their radii is
 (a) $\sqrt{2} : \sqrt{3}$ (b) $\sqrt{3} : 1$
 (c) $\sqrt{3} : 2$ (d) $2 : \sqrt{3}$
232. The ratio of the volume of a cube and of a solid sphere is 363 : 49. The ratio of an edge of the cube and the radius of the sphere is (take $\pi = \frac{22}{7}$)
 (a) 7 : 11 (b) 22 : 7
 (c) 11 : 7 (d) 7 : 22
233. The radius and the height of a cone are in the ratio 4 : 3. The ratio of the curved surface area and total surface area of the cone is
 (a) 5 : 9 (b) 3 : 7
 (c) 5 : 4 (d) 16 : 9
234. A sphere and a cylinder have equal volume and equal radius. The ratio of the curved surface area of the cylinder to that of the sphere is
 (a) 4 : 3 (b) 2 : 3
 (c) 3 : 2 (d) 3 : 4
235. A right circular cylinder and a cone have equal base radius and equal height. If their curved surfaces are in the ratio 8 : 5, then the radius of the base to the height are in the ratio:
 (a) 2 : 3 (b) 4 : 3
 (c) 3 : 4 (d) 3 : 2
236. A right prism with trapezium base of parallel side 8 cm & 14 cm. Height of prism is 12 cm & its volume is 1056 cm^3 then. Find the distance two parallel lines.
 (a) 8 (b) 10 (c) 16 (d) 6
237. The radii of the base of cylinder and a cone are in the ratio $\sqrt{3} : \sqrt{2}$ and their heights are in the ratio $\sqrt{2} : \sqrt{3}$. Their volumes are in the ratio of
 (a) $\sqrt{3} : \sqrt{2}$ (b) $3\sqrt{3} : \sqrt{2}$
 (c) $\sqrt{3} : 2\sqrt{2}$ (d) $\sqrt{2} : \sqrt{6}$
238. The heights of two cones are in the ratio 1 : 3 and the diameters of their base are in the ratio 3 : 5. The ratio of their volume is
 (a) 3 : 25 (b) 4 : 25
 (c) 6 : 25 (d) 7 : 25
239. A sphere and a hemisphere have the same volume. The ratio of their radii is
 (a) 1 : 2 (b) 1 : 8
 (c) $1 : \sqrt{2}$ (d) $1 : \sqrt[3]{2}$
240. The diameter of the moon is assumed to be one fourth of the diameter of the earth. Then the ratio of the volume of the earth to that of the moon is
 (a) 64 : 1 (b) 1 : 64
 (c) 60 : 7 (d) 7 : 60
241. If A denotes the volume of a right circular cylinder of same height as its diameter and B is the vol-

- ume of a sphere of same radius then $\frac{A}{B}$ is:
- (a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$
242. Diagonal of a cube is $6\sqrt{3}$ cm. Ratio of its total surface area and volume (numerically) is
(a) 2 : 1 (b) 1 : 6
(c) 1 : 1 (d) 1 : 2
243. A sphere and a hemisphere have the same volume. The ratio of their curved surface area is :
(a) $2^{\frac{3}{2}} : 1$ (b) $2^{\frac{2}{3}} : 1$
(c) $4^{\frac{2}{3}} : 1$ (d) $2^{\frac{1}{3}} : 1$
244. The volume of a cylinder and a cone are in the ratio 3 : 1. Find their diameters and then compare them when their heights are equal.
(a) Diameter of cylinder = 2 times of diameter of cone
(b) Diameter of cylinder = Diameter of cone
(c) Diameter of cylinder > Diameter of cone
(d) Diameter of cylinder < Diameter of cone
245. A solid sphere is melted and recast into a right circular cone with a base radius equal to the radius of sphere. What is the ratio of the height and radius of the cone so formed ?
(a) 4 : 3 (b) 2 : 3
(c) 3 : 4 (d) 4 : 1
246. Find the total surface area of a prism which is based on Δ of perimeter 45 cm & incircle radius 9cm, if its volume is 810 cm^3 .
(a) 405 (b) 585
(c) 616 (d) 468
247. The ratio of weights of two spheres of different materials is 8 : 17 and the ratio of weights per 1 cc of materials of each is 289 : 64. The ratio of radii of the two spheres is
(a) 8 : 17 (b) 4 : 17
(c) 17 : 4 (d) 17 : 8
248. If the ratio of volumes of two cones is 2 : 3 and the ratio of the radii of their bases is 1 : 2, then the ratio of their heights will be
(a) 8 : 3 (b) 3 : 8
(c) 4 : 3 (d) 3 : 4
249. The volumes of a right circular cylinder and a sphere are equal. The radius of the cylinder and the diameter of the sphere are equal. The ratio of height and radius of the cylinder is
(a) 3 : 1 (b) 1 : 3
(c) 6 : 1 (d) 1 : 6
250. A large solid sphere is melted and moulded to form identical right circular cones with base radius and height same as the radius of the sphere. One of these cones is melted and moulded to form a smaller solid sphere. Then the ratio of the surface area of the smaller to the surface area of the larger sphere is
(a) $1 : 3^{\frac{4}{3}}$ (b) $1 : 2^{\frac{3}{2}}$
(c) $1 : 2^{\frac{2}{3}}$ (d) $1 : 2^{\frac{4}{3}}$
251. A plane divides a right circular cone into two parts of equal volume. If the plane is parallel to the base, then the ratio, in which the height of the cone is divided, is
(a) $1 : \sqrt{2}$ (b) $1 : \sqrt[3]{2}$
(c) $1 : \sqrt[3]{2} - 1$ (d) $1 : \sqrt[3]{2} + 1$
252. A rectangle based pyramid, length and width of the base is 18cm and 10cm respectively. Find the total surface area, if its height is 12cm :
(a) 267 cm^2 (b) 564 cm^2
(c) 516 cm^2 (d) None of these
253. A cone of height 7 cm and base radius 1 cm is carved from a cuboidal block of wood $10 \text{ cm} \times 5 \text{ cm} \times 2 \text{ cm}$. [Assuming $\pi = \frac{22}{7}$] The percentage of wood wasted in the process is :
(a) $92\frac{2}{3}\%$ (b) $46\frac{1}{3}\%$
(c) $42\frac{1}{3}\%$ (d) $41\frac{1}{3}\%$
254. If the radius of a cylinder is decreased by 50% and the height is increased by 50% to form a new cylinder, the volume will be decreased by
(a) 0% (b) 25%
(c) 62.5% (d) 75%
255. Each of the height and base radius of a cone is increased by 100%. The percentage increase in the volume of the cone is
(a) 700% (b) 400%
(c) 300% (d) 100%
256. If both the radius and height of a right circular cone are increased by 20%, its volume will be increased by
(a) 20% (b) 40%
(c) 60% (d) 72.8%
257. A cone of height 15 cm and base diameter 30 cm is carved out of a wooden sphere of radius 15 cm. The percentage of used wood is :
(a) 75% (b) 50%
(c) 40% (d) 25%
258. If the height of a right circular cone is increased by 200% and the radius of the base is reduced by 50%, the volume of the cone
(a) increases by 25%
(b) increases by 50%
(c) remains unaltered
(d) decreases by 25%
259. If the height and the radius of the base of a cone are each increased by 100%, then the volume of the cone becomes
(a) double that of the original
(b) three times that of the original
(c) six times that of the original
(d) eight times that of the original
260. If the height of a cylinder is increased by 15 per cent and the radius of its base is decreased by 10 percent then by what percent will its curved surface area change ?
(a) 3.5 percent decrease
(b) 3.5 percent increase
(c) 5 percent increase
(d) 5 percent decrease

261. If the radius of a sphere is doubled, its volume becomes
(a) double (b) four times
(c) six times (d) eight times
262. If the radius of a right circular cylinder is decreased by 50% and its height is increased by 60% its volume will be decreased by
(a) 10% (b) 60%
(c) 40% (d) 20%
263. The length, breadth and height of a cuboid are in the ratio 1 : 2 : 3. If they are increased by 100%, 200% and 200% respectively. Then compared to the original volume the increase in the volume of the cuboid will be
(a) 5 times (b) 18 times
(c) 12 times (d) 17 times
264. Each of the radius of the base and the height of a right circular cylinder is increased by 10%. The volume of the cylinder is increased by
(a) 3.31% (b) 14.5%
(c) 33.1% (d) 19.5%
265. If the height of a cone is increased by 100% then its volume is increased by :
(a) 100% (d) 200%
(c) 300% (d) 400%
266. A hemispherical cup of radius 4 cm is filled to the brim with coffee. The coffee is then poured into a vertical cone of radius 8 cm and height 16 cm. The percentage of the volume of the cone that remains empty is :
(a) 87.5% (b) 80.5%
(c) 81.6% (d) 88.2%
267. The height of a circular cylinder is increased six times and the base area is decreased to one ninth of its value. The factor by which the lateral surface of the cylinder increases is
(a) 2 (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$
268. If the radius of a sphere be doubled, the area of its surface will become
(a) Double
(b) Three times
(c) Four times
(d) None of the mentioned
269. If each edge of a cube is increased by 50%, the percentage increase in its surface area is
(a) 150% (b) 75%
(c) 100% (d) 125%
270. If the radius of a sphere be doubled, then the percentage increase in volume is
(a) 500% (b) 700%
(c) 600% (d) 800%
271. Find the radius of maximum size sphere which can be inscribed or put in a cone whose base radius and height are 6cm and 8cm respectively?
(a) 4cm (b) 5cm
(c) 3cm
(d) None of these
272. If the length of each side of a regular tetrahedron is 12 cm, then the volume of the tetrahedron is
(a) $144\sqrt{2}$ cu. cm,
(b) $72\sqrt{2}$ cu. cm,
(c) $8\sqrt{2}$ cu. cm,
(d) $12\sqrt{2}$ cu. cm,
273. If the radii of the circular ends of a truncated conical bucket which is 45cm high be 28 cm and 7 cm then the capacity of the bucket in cubic centimetre is (use $\pi = \frac{22}{7}$)
(a) 48510 (b) 45810
(c) 48150 (d) 48051
274. There is a pyramid on a base which is a regular hexagon of side 2a cm. If every slant edge of this pyramid is of length $\frac{5a}{2}$ cm, then the volume of this pyramid is
(a) $3a^3$ cm³ (b) $3\sqrt{2} a^2$ cm³
(c) $3\sqrt{3} a^3$ cm³ (d) $6a^3$ cm³
275. The base of a right pyramid is a square of side 40 cm long. If the volume of the pyramid is 8000 cm³, then its height is :
(a) 5 cm (b) 10 cm
(c) 15 cm (d) 20 cm
276. The base of a right prism is a trapezium. The length of the parallel sides are 8 cm and 14 cm and the distance between the parallel sides is 8 cm, If the volume of the prism is 1056 cm³, then the height of the prism is
(a) 44 cm (b) 16.5 cm
(c) 12 cm (d) 10.56 cm
277. Each edge of a regular tetrahedron is 3 cm, then its volume is
(a) $\frac{9\sqrt{2}}{4}$ c.c. (b) $27\sqrt{3}$ c.c.
(c) $\frac{4\sqrt{2}}{9}$ c.c. (d) $9\sqrt{3}$ c.c.
278. The perimeter of the triangular base of a right prism is 15 cm and radius of the incircle of the triangular base is 3 cm. If the volume of the prism be 270 cm³ then the height of the prism is
(a) 6 cm (b) 7.5 cm
(c) 10 cm (d) 12 cm
279. The base of a solid right prism is a triangle whose sides are 9 cm, 12 cm and 15 cm, The height of the prism is 5 cm. The the total surface area of the prism is
(a) 180 cm² (b) 234 cm²
(c) 288 cm² (d) 270 cm²
280. The base of a right prism is an equilateral triangle of area 173 cm² and the volume of the prism is 10380 cm³. The area of the lateral surface of the prism is
(use $\sqrt{3} = 1.73$)
(a) 1200 cm² (b) 2400 cm²
(c) 3600 cm² (d) 4380 cm²
281. The base of a right pyramid is a square of side 16 cm long . If its height be 15 cm, then the area of the lateral surface in square cm is :
(a) 136 (b) 544
(c) 800 (d) 1280
282. Area of the base of a pyramid is 57 sq. cm. and height is 10 cm, then its volume (in cm³), is
(a) 570 (b) 390
(c) 190 (d) 590
283. The height of a right prism with a square base is 15 cm. If the area of the total surface of the prism is 608 sq. cm, its volume is
(a) 910 cm³ (b) 920 cm³
(c) 960 cm³ (d) 980 cm³

284. The base of a right prism is an equilateral triangle of side 8 cm and height of the prism is 10 cm. Then the volume of the prism is
 (a) $320\sqrt{3}$ cubic cm
 (b) $160\sqrt{3}$ cubic cm
 (c) $150\sqrt{3}$ cubic cm
 (d) $300\sqrt{3}$ cubic cm
285. A prism has as the base a right angled triangle whose sides adjacent to the right angles are 10 cm and 12 cm long. The height of the prism is 20 cm. the density of the material of the prism is 6gm/cubic cm. the weight of the prism is
 (a) 6.4 kg (b) 7.2 kg
 (c) 3.4 kg (d) 4.8 kg
286. If the slant height of a right pyramid with square base is 4 metre and the total slant surface of the pyramid is 12 square metre, then the ratio of total slant surface and area of the base is :
 (a) 16 : 3 (b) 24 : 5
 (c) 32 : 9 (d) 12 : 3
287. The length of each edge of a regular tetrahedron is 12 cm. The area (in sq. cm) of the total surface of the tetrahedron is
 (a) $288\sqrt{3}$ (b) $144\sqrt{2}$
 (c) $108\sqrt{3}$ (d) $144\sqrt{3}$
288. The base of right prism is a triangle whose perimeter is 28 cm and the inradius of the triangle is 4 cm. If the volume of the prism is 366 cc, then its height is
 (a) 6 cm (b) 8 cm
 (c) 4 cm (d) None of these
289. If the base of a right pyramid is triangle of sides 5 cm, 12 cm and 13 cm and its volume is 330 cm³, then its height (in cm) will be
 (a) 33 (b) 32 (c) 11 (d) 22
290. The base of a right pyramid is equilateral triangle of side $10\sqrt{3}$ cm. If the total surface area of the pyramid is $270\sqrt{3}$ sq. cm. its height is
 (a) $12\sqrt{3}$ cm (b) 10 cm
 (c) $10\sqrt{3}$ cm (d) 12 cm
291. A right prism stands on a base of 6 cm side equilateral triangle and its volume is $81\sqrt{3}$ cm³. the height (in cm) of the prism is
 (a) 9 (b) 10 (c) 12 (d) 15
292. A right pyramid stands on a square base of diagonal $10\sqrt{2}$ cm. If the height of the pyramid is 12 cm, the area (in cm²) of its slant surface is
 (a) 520 (b) 420 (c) 360 (d) 260
293. If the altitude of a right prism is 10 cm and its base is an equilateral triangle of side 12 cm, then its total surface area (in cm²) is
 (a) $(5 + 3\sqrt{3})$ (b) $36\sqrt{3}$
 (c) 360 (d) $72(5 + \sqrt{3})$
294. A right pyramid stands on a square base of side 16 cm and its height is 15 cm. The area (in cm²) of its slant surface is
 (a) 514 (b) 544 (c) 344 (d) 444
295. The base of a right prism is a right angled triangle whose sides are 5 cm, 12 cm and 13 cm. If the total surface area of the prism is 360 cm², then its height (in cm) is
 (a) 10 (b) 12 (c) 9 (d) 11
296. A right pyramid 6 m high has a square base of which the diagonal is $\sqrt{1152}$ m. Volume of the pyramid is
 (a) 144 m³ (b) 288 m³
 (c) 576 m³ (d) 1152 m³
297. The height of the right pyramid whose area of the base is 30 m² and volume is 500 m³ is
 (a) 50 m (b) 60 m
 (c) 40 m (d) 20 m
298. The base of a right. prism is an equilateral triangle. If the lateral surface area and volume is 120 cm², $40\sqrt{3}$ cm³ respectively then the side of base of the prism is
 (a) 4 cm (b) 5 cm
 (c) 7 cm (d) 40 cm
299. Each edge of a regular tetrahedron is 4 cm. its volume (in cubic cm) is
 (a) $\frac{16\sqrt{3}}{3}$ (b) $16\sqrt{3}$
 (c) $\frac{16\sqrt{2}}{3}$ (d) $16\sqrt{2}$
300. The base of a prism is a right angled triangle with two sides meeting at right angle are 5 cm and 12 cm. The height of the prism is 10 cm. The total surface area of the prism is
 (a) 360 sq. cm (b) 300 sq. cm
 (c) 330 sq. cm (d) 325 sq. cm
301. The radius of a cylinder is 10 cm and height is 4 cm. The number of centimetres that may be added either to the radius or to the height to get the same increase in the volume of the cylinder is :
 (a) 25 (b) 4 (c) 5 (d) 16
302. If the area of the base, height and volume of a right prism be $\left(\frac{3\sqrt{3}}{2}\right)$ p² cm², $10\sqrt{3}$ cm and 7200 cm³ respectively, then the value of P (in cm) will be?
 (a) 4 (b) $\frac{2}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{3}{2}$
303. If the base of right prism remains same and the lateral edges are halved, then its volume will be reduced by
 (a) 33.33% (b) 50%
 (c) 25% (d) 66%
304. The total surface area of a regular triangular pyramid with each edges of length 1 cm is?
 (a) $\frac{4}{2}\sqrt{2}$ cm² (b) $\sqrt{3}$ cm²
 (c) 4 cm² (d) $4\sqrt{3}$ cm²
305. Base of a right pyramid is a square of side 10 cm. If the height of the pyramid is 12 cm, then its total surface area is
 (a) 360 cm² (b) 400 cm²
 (c) 460 cm² (d) 260 cm²
306. A right prism has a triangular base whose sides are 13 cm, 20 cm and 21 cm, If the altitude of the prism is 9 cm, then its volume is
 (a) 1143 cm³ (b) 1314 cm³
 (c) 1413 cm³ (d) 1134 cm³
307. Base of a prism of height 10 cm is square. Total surface area of the prism is 192 sq. cm. The volume of the prism is
 (a) 120 cm³ (b) 640 cm³
 (c) 90 cm³ (d) 160 cm³

308. A right prism has triangular base. If v be the number of vertices, e be the number of edges and f be the number of faces of the prism. The value of $\frac{v+e-f}{2}$ is
 (a) 2 (b) 4 (c) 5 (d) 10
309. The base of a right prism is a trapezium whose lengths of two parallel sides are 10 cm and 6 cm and distance between them is 5 cm. If the height of the prism is 8 cm, its volume is:
 (a) 300 cm^3 (b) 300.5 cm^3
 (c) 320 cm^3 (d) 310 cm^3
310. Base of a right prism is a rectangle, the ratio of whose length and breadth is 3 : 2. If the height of the prism is 12 cm and total surface area is 288 sq. cm, the volume of the prism is:
 (a) 288 cm (b) 290 cm^3
 (c) 286 cm^3 (d) 291 cm^3
311. Height of a prism-shaped part of a machine is 8 cm and its base is an isosceles triangle, whose each of the equal sides is 5 cm and remaining side is 6 cm. The volume of the part is
 (a) 90 cm^3 (b) 96 cm^3
 (c) 120 cm^3 (d) 86 cm^3
312. Find the total surface area of pyramid which is based on a equal Δ of side $18\sqrt{3}$ & the height of pyramid is 12 cm.
 (a) $124\sqrt{3}$ (b) $624\sqrt{3}$
 (c) $648\sqrt{3}$ (d) $405\sqrt{3}$
313. The base of a right prism is a $\square ABCD$. If the volume of prism is 2070. Then find the lateral surface area. $AB = 9$, $BC = 14$, $CD = 13$, $DA = 12$ $\angle DAB = 90^\circ$.
 (a) 720 (b) 540
 (c) 920 (d) 960
314. Find the volue of pyramid which is based on a equilateral traingle of side 4 cm & height of pyramid is $20\sqrt{3}$ cm.
 (a) 100 (b) 160
 (c) 80 (d) 40
315. Find the total surface area of pyramid of 4 cm height which is based on a square of side 6cm.
 (a) 48 (b) 72 (c) 96 (d) 120
316. Find the value of a pyramid which is based on a square of side 10cm & lateral edge of pyramid is 12 cm.
 (a) $\frac{100}{3}$ (b) $\frac{100\sqrt{119}}{3}$
 (c) $\frac{100\sqrt{119}}{9}$ (d) $100\sqrt{119}$
317. A rectangular water tank is open at the top. Its capacity is 24 m^3 . Its length and breadth are 4 m and 3 m respectively. Ignoring the thickness of the material used for building the tank, the total cost of painting the inner and outer surfaces of the tank at the rate of Rs. 10 per m^2 is:
 (a) Rs. 400 (b) Rs. 500
 (c) Rs. 600 (d) Rs. 800
318. If V be the volume and S the surface area of a cuboid of dimensions a , b and c then $\frac{1}{V}$ is equal to:
 (a) $\frac{S}{2(a+b+c)}$
 (b) $\frac{2}{S} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$
 (c) $\frac{2S}{a+b+c}$
 (d) $2S(a+b+c)$
319. An open box is made of wood 3 cm thick. Its external length is 1.46 m, breadth 1.16 m and height 8.3 dm. The cost of painting the inner surface of the box at 50 paise per 100 cm^2 is:
 (a) Rs. 138.50 (b) Rs. 277
 (c) Rs. 415.50 (d) Rs. 554
320. The areas of three adjacent faces of a cuboid are x , y & z square units respectively. If the volume of the cuboid be v cubic units, then the correct relation between v , x , y , z is:
 (a) $v^2 = xyz$ (b) $v^3 = xyz$
 (c) $v^2 = x^3y^3z^3$ (d) $v^3 = x^2y^2z^2$
321. 1 m^3 piece of copper is melted and recast into a square cross section bar 36 m long. An exact cube is cut off from this bar. If 1 m^3 of copper cost Rs. 108, then the cost of the cube is.
 (a) 50 paise (b) 25 paise
 (c) 75 paise (d) 1 paise
322. The volume of a rectangular block of stone is 10368 dm^3 , its dimensions are in the ratio of 3:2:1, If its entire surface is polished at 2 paise per dm^2 , then what is the total cost?
 (a) Rs. 31.68 (b) Rs. 31.50
 (c) Rs. 63 (d) Rs. 63.36
323. A rectangular water tank measure $15\text{m} \times 6\text{m}$ at top and is 10m deep. It is full of water. If water is drawn out lowering the level by 1 meter how much of water has been drawn out?
 (a) 90,000 litres
 (b) 45,000 litres
 (c) 80,000 litres
 (d) 40,000 litre
324. A rectangular tank is 45 m long and 26 m broad. Water flows into it through a pipe whose cross section is 13 cm^2 , at the rate of 9 km/hour. How much will the level of the water rise in the tank in 15 min?
 (a) 0.0016m (b) 0.0020m
 (c) 0.0025m (d) 0.0018
325. The diagonals of the three faces of a cuboid are x , y and z respectively. Find the volume of cuboid?
 (a) $\frac{xyz}{2\sqrt{2}}$
 (b) $\frac{\sqrt{(y^2+z^2-x^2)(z^2+x^2-y^2)(x^2+y^2-z^2)}}{2\sqrt{2}}$
 (c) $\sqrt{\frac{(y^2+z^2-x^2)(z^2+x^2-y^2)(x^2+y^2-z^2)}{2\sqrt{2}}}$
 (d) None of these
326. The same string, when wound on the exterior four walls of a cube of side n cm, starting at point C and ending at point D, can given exactly one turn. The length of the string, (in cm) is.
 (a) $\sqrt{2}n$ (b) $\sqrt{17}n$
 (c) n (d) $\sqrt{13}n$



327. A reservoir is supplied water by a pipe 6 cm in diameter. How many pipes of 1.5 cm diameter would discharge the same quantity, supposing the velocity of water is same?
(a) 8 (b) 12 (c) 16 (d) 20
328. Given a solid cylinder of radius 10 cm and length 1000 cm, a cylindrical hole is made into it to obtain a cylindrical shell of uniform thickness and having volume equal to one-fourth of the original cylinder. The thickness of the cylindrical shell is:
(a) $5(\sqrt{5} - 2)$ cm
(b) $5(2 - \sqrt{3})$ cm
(c) 5 cm (d) $5\sqrt{2}$ cm
329. A well of radius 'r' is dug 20 m deep and the earth taken out is spread all around it to a width of 1 m to form an embankment. The height of the embankment is 5 m then find the value of 'r':
(a) $\frac{1+\sqrt{5}}{2}$ (b) $\frac{1+\sqrt{5}}{4}$
(c) $\frac{\sqrt{5}-1}{2}$ (d) $\frac{\sqrt{5}-1}{4}$
330. A cylinder is filled to $\frac{4}{5}$ th of volume. It is then tilted so that level of water coincides with one edge of its bottom and top edge of the opposite side. In the process, 30 litre of the water is spilled. What is the value of the cylinder?
(a) 75 litre (b) 96 litre
(c) Data insufficient (d) 100 litre
331. A monument has 50 cylindrical pillars each of diameter 50 cm and height 4 m. what will be the labour charges for getting these pillars cleared at the rate of 50 paise per m^2 (Use $\pi = 3.14$).
(a) Rs. 237 (b) Rs. 157
(c) Rs. 257 (d) Rs. 353
332. A right circular cylindrical tank has the storage capacity 38808 ml. If the radius of the base of the cylinder is three fourth of the height what is the radius of base?
(a) 28 cm (b) 56 cm
(c) 21 cm (d) 42 cm
333. A rectangular piece of iron sheet measuring 50 cm and 100 cm is rolled into cylinder of height 50 cm. If the cost of painting the cylinder is Rs. 50 per square meter, then what will be the cost of painting the surface of the cylinder?
(a) Rs. 25.00 (b) Rs. 37.50
(c) Rs 75.00 (d) Rs. 87.50
334. Sixteen cylindrical cans, each with a radius of 1 unit, are placed inside a cardboard box four in a row. If the cans touch the adjacent cans and or the walls of the box, then which of the following could be the interior area of the bottom of the box in square units?
(a) 16 (b) 32 (c) 64 (d) 128
335. Find the volume of a right circular cone formed by joining the edges of a sector of a circle of radius 4cm where the angle of the sector is 90° .
(a) $\frac{2\sqrt{3}}{\pi} \text{ cm}^3$ (b) $\frac{2\sqrt{2}\pi}{3} \text{ cm}^3$
(c) $\frac{\pi\sqrt{5}}{\sqrt{3}} \text{ cm}^3$ (d) $\frac{\sqrt{3}}{\pi} \text{ cm}^3$
336. A sector of circle of radius 3cm has an angle of 120° . if it is modulated into a cone, find the volume of the cone.
(a) $\frac{\pi}{\sqrt{3}} \text{ cm}^3$ (b) $\frac{2\sqrt{2}\pi}{3} \text{ cm}^3$
(c) $\frac{2\sqrt{3}}{\pi} \text{ cm}^3$ (d) $\frac{\sqrt{3}}{\pi} \text{ cm}^3$
337. If the slant height and the radius of the base of a right circular cone are H and r respectively then the ratio of the areas of the lateral surface and the base is:
(a) $2H : r$ (b) $H : r$
(c) $H : 2r$ (d) $H^2 : r^2$
338. A sector of a circle of radius 15cm has the angle 120° . It is rolled up so that two bounding radii are joined together to form a cone. the volume of the cone is.
(a) $(250\sqrt{2})\pi \text{ cm}^3$
(b) $(100\sqrt{2})\pi \text{ cm}^3$
(c) $[(250\sqrt{2})\pi / 3] \text{ cm}^3$
(d) $[(100\sqrt{2})\pi / 3] \text{ cm}^3$
339. The base radius and height of a cone is 5cm and 25cm respectively. if the cone is cut parallel to its base at a height of h from the base. If the volume of this frustrum is 110 cm^3 find the radius of smaller cone?
(a) $(104)^{1/3} \text{ cm}$ (b) $(104)^{1/2} \text{ cm}$
(c) 5 cm
(d) None of these
340. A hemispherical bowl is 176cm round the brim. supposing it to be half full, how many persons may be swerved from it in hemispherical glasses 4 cm in diameter at the top?
(a) 1372 (b) 1272
(c) 1172 (d) 1472
341. A sphere of radius 3 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vessel is 6 cm. If the sphere is submerged completely, then the surface of the water is raised by
(a) $\frac{1}{4} \text{ cm}$ (b) $\frac{1}{2} \text{ cm}$
(c) 1 cm (d) 2cm
342. Let A and B be two solid spheres area of B is 300% higher than surface area of A. The volume of A is found to be k% lower than the volume of B. The value of k must be
(a) 85.5 (b) 92.5
(c) 90.5 (d) 87.5
343. The base of a prism is a regular hexagon. If every edge of the prism measures 1 metre and height is 1 metre, then volume of the prism is
(a) $\frac{3\sqrt{2}}{2} \text{ cu m}$ (b) $\frac{3\sqrt{3}}{2} \text{ cu m}$
(c) $\frac{6\sqrt{2}}{5} \text{ cu m}$ (d) $\frac{5\sqrt{3}}{2} \text{ cu m}$
344. The base of a right prism is a pentagon whose sides are in the ratio $1:\sqrt{2}:\sqrt{2}:1:2$ and its height is 10 cm. If the longest side of the base be 6 cm, the volume of the prism is
(a) 270 cm^3 (b) 360 cm^3
(c) 540 cm^3
(d) None of these

345. There are two prism, one has equilateral triangle as a base and the other regular hexagon. If both of the prisms have equal heights and volumes, then find the ratio between the length of each side at their bases.
 (a) $1 : \sqrt{6}$ (b) $\sqrt{6} : 1$
 (c) $\sqrt{3} : 2$ (d) $2 : \sqrt{3}$
346. The base of a right prism is a trapezium. The lengths of the parallel sides are 8 cm and 14 cm and the distance between the parallel sides is 8 cm. If the volume of the prism is 1056 cm^3 , then the height of the prism is
 (a) 44 cm (b) 16.5 cm
 (c) 12 cm (d) 10.56 cm
347. If the base of a right rectangular prism is left unchanged and the measure of the lateral edges are doubled, then its volume will be
 (a) unchanged (b) tripled
 (c) doubled (d) quadrupled
348. Prism has as the base a right angled triangle whose sides adjacent to the right angles are 10 cm and 12 cm long. The height of the prism is 20 cm. The density of the material of the prism is 6 gm. cubic cm. The weight of the prism is.
 (a) 6.4 kg (b) 7.2 kg
 (c) 3.4 kg (d) 4.8 kg
349. The perimeter of the triangular base of a right prism is 15 cm and radius of the incircle of the triangular base is 3 cm. If the volume of the prism be 270 cm^3 , then the height of the prism is-
 (a) 6 cm (b) 7.5 cm
 (c) 10 cm (d) 12 cm
350. The base of a right prism is an equilateral triangle. If its height is one-fourth and each side of the base is tripled, then the ratio of the volumes of the old to the new prism is-
 (a) 4 : 3 (b) 1 : 4
 (c) 1 : 2 (d) 4 : 9
351. A right pyramid is on a regular hexagonal base. Each side of the base is 10 m and the height is 30 m. The volume of the pyramid is
 (a) 2500 m^3 (b) 2550 m^3
 (c) 2598 m^3 (d) 5196 m^3
352. There is a pyramid on a base which is a regular hexagon of side $2a$. If every slant edge of this pyramid is of length $\frac{5a}{2}$, then the volume of this pyramid is.
 (a) $3a^3$ (b) $3a^2\sqrt{2}$
 (c) $3a^3\sqrt{3}$ (d) $6a^3$
353. If the area of the base of a regular hexagonal pyramid is $96\sqrt{3} \text{ m}^2$ and the area of one of its side faces is $32\sqrt{3} \text{ m}^2$, then the volume of the pyramid is:
 (a) $380\sqrt{3} \text{ m}^3$ (b) $382\sqrt{2} \text{ m}^3$
 (c) $384\sqrt{3} \text{ m}^3$ (d) $386\sqrt{3} \text{ m}^3$
354. What part of a ditch, 48 metres long 16.5 metres broad and 4 metres deep can be filled by the sand got by digging a cylindrical tunnel of diameter 4 metres and length 56 metres?
 $\left[\text{use } \pi = \frac{22}{7} \right]$
 (a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{7}{9}$ (d) $\frac{8}{9}$
355. A cylindrical rod of iron whose height is eight times its radius is melted and cast into spherical balls each of half the radius of the cylinder. The number of such pherical balls is
 (a) 12 (b) 16 (c) 24 (d) 48
356. Water flows at the rate of 10 meters per minute from a cylindrical pipe 5 mm in diameter. How long it take to fill up a conical vessel whose diameter at the base is 30 cm and depth 24 cm?
 (a) 28 minutes 48 seconds
 (b) 51 minutes 12 seconds
 (c) 51 minutes 24 seconds
 (d) 28 minutes 36 second
357. A semi-circular sheet of metal of diameter 28 cm is bent into an open conical cup. The depth of the cup is approximately :
 (a) 11 cm (b) 12 cm
 (c) 13 cm (d) 14 cm
358. The height of a right prism with a square base 15 cm. If the total S.A. of prism of 608 cm^2 . The find its volume.
 (a) 480 (b) 460
 (c) 1500 (d) 960
359. A slab of ice 8 inches in length 11 inches in breadth, and 2 inches thick was melted and resolidified in the form of a rod of 8 inches diameter. The length of such a rod, in inches, is nearest to.
 (a) 3 (b) 3.5 (c) 4 (d) 4.5
360. A storage tank consists of a circular cylinder with a hemisphere adjoined on either side. If the external diameter of the cylinder be 14 m and its length be 50 m, then what will be the cost of painting it at the rate of Rs. 10 per sq m?
 (a) Rs. 38160 (b) Rs. 28160
 (c) Rs. 39160 (d) None of these
361. The diameter of the iron ball used for the shotput game is 14 cm. It is melted and then a solid cylinder of height $2\frac{1}{3} \text{ cm}$ is made. What will be the diameter of the base of the cylinder?
 (a) 14 cm (b) 28 cm
 (c) $\frac{14}{3} \text{ cm}$ (d) $\frac{28}{3} \text{ cm}$
362. If the area of the circular shell having inner and outer radii of 8 cm and 12 cm respectively is equal to the total surface area of cylinder of radius R_1 and height h , then h , in terms or R_1 will be.
 (a) $\frac{3R_1^2-30}{7R_1}$ (b) $\frac{R_1^2-40}{R_1^2}$
 (c) $\frac{30-R_1}{R_1^2}$ (d) $\frac{40-R_1^2}{R_1}$
363. Two solid right cones of equal heights are of radii r_1 and r_2 are melted and made to form a solid sphere of radius R . Then the height of the cone is:
 (a) $\frac{4R^2}{r_1^2+r_2^2}$ (b) $\frac{4R}{r_1+r_2}$
 (c) $\frac{4R^3}{r_1^2+r_2^2}$ (d) $\frac{R^2}{r_1^2+r_2^2}$
364. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the

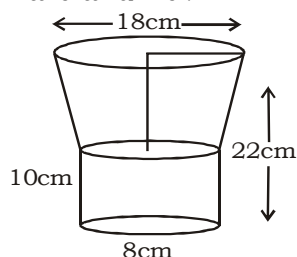
volume of the wooden by (nearly).

- (a) 104 cm^3 (b) 162 cm^3
(c) 427 cm^3 (d) 266 cm^3

365. The volume of a cylinder and a cone are in the ratio 3 : 1. Find their diameters and then compare them when their heights are equal.

- (a) Diameter of cylinder = 2 times diameter of cone
(b) Diameter of cylinder = Diameter of cone
(c) Diameter of cylinder > Diameter of cone
(d) Diameter of cylinder < Diameter of cone

366. A oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make a funnel.



- (a) 728.57 cm^3 (b) 782.57 cm^3
(c) 872.57 cm^3 (d) 827.57 cm^3

367. A conical flask has radius a cm and height h cm. It was completely filled with milk. The milk is poured into a cylindrical therefore flask whose base radius is p cm. What will be the height of the soulution level in the flask?

- (a) $\frac{a^2 h}{3p^2} \text{ cm}$ (b) $\frac{3hp^2}{a^2} \text{ cm}$
(c) $\frac{p^2}{3h^2} \text{ cm}$ (d) $\frac{3a^2}{hp^2} \text{ cm}$

368. The perimeter of an equilateral triangle is $72\sqrt{3} \text{ cm}$. Find its height.

- (a) 63 metres (b) 24 metres
(c) 18 metres (d) 36 metres

369. A pit 7.5 metre long, 6 metre wide and 1.5 metre deep is dug in a field. Find the volume of soil removed in cubic metres.

- (a) 135 m^3 (b) 101.25 m^3
(c) 50.625 m^3 (d) 67.5 m^3

370. In a shower, 10 cm of rain falls. What will be the volume of water that falls on 1 hectare area of ground?

- (a) 500 m^3 (b) 650 m^3
(c) 1000 m^3 (d) 750 m^3

371. Seven equal cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.

- (a) 750 cm^2 (b) 1500 cm^2
(c) 2250 cm^2 (d) 700 cm^2

372. In a swimming pool measuring 90 m by 40m, 150 men take a dip. If the average displacement of water by a man is 8 cubic metres, what will be rise in water level?

- (a) 30 cm (b) 50 cm
(c) 20 cm (d) 33.33 cm

373. A conical tent is to accommodate 10 persons. Each person must have 6 m^2 space to sit and 30 m^3 of air to breath. What will be the height of the cone ?

- (a) 37.5 m (b) 150 m
(c) 75 m
(d) None of these

374. A hollow spherical shell is made of a metal of density 4.9 g/cm^3 . If its internal and external radii are 10 cm and 12 cm respectively, find the weight of the shell.

(Take $\pi = 3.1416$)

- (a) 5016 gm (b) 1416.8 gm
(c) 14942.28gm (d) 5667.1 gm

375. A spherical cannon ball, 28 cm in diameter, is melted and cast into a right circular conical mould the base of which is 35 cm in diameter. Find the height of the cone correct up to two places of decimals.

- (a) 8.96 cm (b) 35.84 cm
(c) 5.97 cm (d) 17.9 cm

376. A rope is wound round the outside of a circular drum whose diameter is 70 cm and a bucket is tied to the other end of the rope. Find the number of revolutions made by the drum if the bucket is raised by 11 m.

- (a) 10 (b) 2.5 (c) 5 (d) 5.5

377. A cube whose edge is 20 cm long has circle on each of its faces painted black. What is the total area of the unpainted surface of the cube if the circles

are of the largest area possible?

- (a) 85.71 cm^2 (b) 257.14 cm^2
(c) 514.28 cm^2 (d) 331.33 cm^2

378. The areas of three adjacent faces of a cuboid are x , y , z . If the volume is V , then V^2 will be equal to

- (a) xy/z (b) yz/x^2
(c) x^2y^2/z^2 (d) xyz

379. The dimensions of a field are 20 m by 9m. A pit 10 m long, 4.5 m wide and 3m deep is dug in one corner of the field and the soil removed has been evenly spread over the remaining area of the field. What will be the rise in the height of field as a result of this operation?

- (a) 1m (b) 2m (c) 3m (d) 4m

380. A vessel is in the form of a hollow cylinder mounted on a hemispherical bowl. The diameter of the sphere is 14 cm and the total height of the vessel is 13cm. Find the capacity of the vessel.

(Take $\pi = 22/7$)

- (a) 321.33 cm^3 (b) 1642.67 cm^3
(c) 1232 cm^3 (d) 1632.33 cm^3

381. A circular tent is cylindrical to a height of 3 metres and conical above it. If its diameter is 105 m and the slant height of the conical portion is 53 m, calculate the length of the canvas 6 m wide to make the required tent.

- (a) 3894 m (b) 973.5 m
(c) 1947 m (d) 1800 m

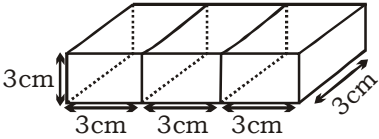
382. A steel sphere of radius 4 cm is drawn into a wire of diameter 4 mm. Find the length of wire.

- (a) 10,665 mm
(b) 42.660 mm
(c) 21,333 mm
(d) 14,220 mm

383. A cylinder and a cone having equal diameter of their bases are placed in the Qutab Minar one on the other, with the cylinder placed in the bottom. If their curved surface area are in the ratio of 8 : 5, find the ratio of their heights. Assume the height of the cylinder to be equal to the radius of Qutab Minar. (Assume Qutab Minar to

- be having same radius throughout).
- (a) 1 : 4 (b) 3 : 4
(c) 4 : 3 (d) 2 : 3
384. If the curved surface area of a cone is thrice that of another cone and slant height of the second cone is thrice that of the first, find the ratio of the area of their base.
(a) 81 : 1 (b) 9 : 1
(c) 3 : 1 (d) 27 : 1
385. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If external radius of the base of the cylinder is 5 cm and its height is 32 cm, find the uniform thickness of the cylinder.
(a) 2 cm (b) 3 cm
(c) 1 cm (d) 3.5 cm
386. A hollow sphere of external and internal radius 6 cm and 4 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone
(a) 25 cm (b) 35 cm
(c) 30 cm (d) 38 cm
387. Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.
(a) 7 : 9 (b) 49 : 81
(c) 9 : 7 (d) 27 : 23
388. If V be the volume of a cuboid of dimension x, y, z and A is its surface, then A/V will be equal to
(a) $x^2y^2z^2$
(b) $1/2(1/xy + 1/xz + 1/yz)$
(c) None of these
(d) $1/xyz$
389. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if the height of conical part is 12 cm.
(a) 1440 cm^2 (b) 385 cm^2
(c) 1580 cm^2 (d) 770 cm^2
390. A solid wooden toy is in the form of a cone mounted on a hemisphere. If the radii of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of wood used in the toy.
(a) 353.72 cm^3 (b) 266.11 cm^3
(c) 532.22 cm^3 (d) 133.55 cm^3
391. A cylindrical container whose diameter is 12 cm and height is 15 cm, is filled with ice cream. The whole ice-cream is distributed to 10 children in equal cones having hemispherical tops. If the height of the conical portion is twice the diameter of the cylindrical container its base, find the diameter of the ice-cream cone.
(a) 6 cm (b) 13 cm
(c) 3 cm (d) 18 cm
392. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the total surface area of the solid. (Use $\pi = 22/7$).
(a) 398.75 cm^2 (b) 418 cm^2
(c) 444 cm^2 (d) 412 cm^2
393. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. What is the ratio of their volumes?
(a) 2 : 1 : 3 (b) 2.5 : 1 : 3
(c) 1 : 2 : 3 (d) 1.5 : 2 : 3
394. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The cost of painting 1 cm^2 of the surface is ₹ 0.05. Find the total cost of painting the vessel all over. (Take $\pi = 22/7$)
(a) ₹ 97.65 (b) ₹ 86.4
(c) ₹ 184 (d) ₹ 96.28
395. A cylindrical cane whose base is horizontal is of internal radius 3.5 cm contain sufficient water so that when a solid sphere of max. size is placed, water just immersed it. Calculate the depth of water in the cane before the sphere was put.
(a) $\frac{5}{2}$ (b) $\frac{7}{3}$ (c) $\frac{4}{3}$ (d) $\frac{8}{3}$
396. A solid is in the form of a right circular cylinder with a hemisphere at one end and a cone at the other end. Their common diameter is 3.5 cm and the heights of conical and cylindrical portion are respectively 6 cm and 10 cm. Find the volume of the solid. (Use $\pi = 3.14$)
(a) 117 cm^3 (b) 234 cm^3
(c) 58.5 cm^3
(d) None of these
397. A right elliptical cylinder full of petrol has its widest elliptical side 2.4 m and the shortest 1.6 m. Its height is 7 m. Find the time required to empty half the tank through a hose of diameter 4 cm if the rate of flow of petrol is 120 m/min
(a) 60 min (b) 90 min
(c) 75 min (d) 70 min
398. The radius of a right circular cylinder is increased by 50%. Find the percentage increase in volume
(a) 120% (b) 75%
(c) 150% (d) 125%
399. Water flows out at the rate of 10 m/min from a cylindrical pipe of diameter 5 mm. Find the time taken to fill a conical tank whose diameter at the surface is 40 cm and depth 24 cm.
(a) 50 min (b) 102.4 min
(c) 51.2 min (d) 25.6 min
400. The section of a solid right circular cone by a plane containing vertex and perpendicular to base is an equilateral triangle of side 12 cm. Find the volume of the cone.
(a) 72 cm^3 (b) 144 cm^3
(c) 74 cm^3 (d) $72\sqrt{3}\pi \text{ cm}^3$
401. Iron weight 8 times the weight of oak. Find the diameter of an iron ball whose weight is equal to that of a ball of oak 18 cm in diameter.
(a) 4.5 cm (b) 9 cm
(c) 12 cm (d) 15 cm
402. A piece of squared timber is 7 metres long and 0.1 metre both in width and thickness. What is its weight at the rate of 950 kg per cubic metres?
(a) 66 kg (b) 67 kg
(c) 66.5 kg (d) 68.5 kg
403. How many cubic metres of masonry are there in a wall 81

- metres long, 4 metres high and 0.2 metre thick.
 (a) 64.8 cub m (b) 69 cub m
 (c) 68 cub m (d) 68.9 cub m
404. A river 10 metres deep and 200 metres wide is flowing at the rate of $4\frac{1}{2}$ km/hr. Find how many cubic m of water run into the sea per second.
 (a) 2500 cub metres
 (b) 2000 cub metres
 (c) 2200 cub metres
 (d) None of these
405. A cistern is constructed to hold 200 litres, and the base of the cistern is a square metre. what is the depth of the cistern? A cubic metre is 1000 litres.
 (a) 50 cm (b) 20 cm
 (c) 25 cm (d) 40 cm
406. A field is 500 metres long and 30 metres broad and a tank 50 metres long, 20 metres broad and 14 metres deep is dug in the field, and the earth taken out of it is spread evenly over the field, How much is the level of the field raised?
 (a) 0.5 m (b) 1.5 m
 (c) 1 m (d) 2 m
407. Find the volume and surface area of a cube, whose each edge measures 25 cm.
 (a) 15265 cu cm, 3750 sq cm
 (b) 15625 cu cm, 2500 sq cm
 (c) 15625 cu cm, 3850 sq cm
 (d) Data inadequate
408. The three co-terminus edges of a rectangular solid are 36, 75 and 80 cm respectively. Find the edge of a cube which will be of the same capacity?
 (a) 70 cm (b) 36 cm
 (c) 60 cm
 (d) Data inadequate
409. A cube of metal each edge of which measures 5 cm, weighs 0.625 kg. What is the length of each edge of a cube of the same metal which weighs 40 kg?
 (a) 20 cm (b) 25 cm
 (c) 15 cm (d) 30 cm
410. The sum of the radius of the base and the height of a solid cylinder is 37 m. If the total surface area of the cylinder be 1628 sq m, find the volume.
 (a) 4620 cu m (b) 4630 cu m
 (c) 4520 cu m (d) 4830 cu m
411. If the diameter of the base of a closed right circular cylinder is equal to its height h, then its whole surface area is:
 (a) $2\pi h^2$ (b) $\frac{4}{3}\pi h^2$
 (c) $\frac{3}{2}\pi h^2$ (d) πh^2
412. How many bullets can be made out of a cube of lead whose edge measures 22 cm, each bullet being 2 cm in diameter?
 (a) 5324 (b) 2662
 (c) 1347 (d) 2541
413. A cylindrical vessel 60 cm in diameter is partially filled with water. A sphere, 30 cm in radius is gently dropped into the vessel. To what further height will water in the cylinder rise?
 (a) 15 cm
 (b) 30 cm
 (c) 40 cm
 (d) Can't be determined
414. The difference between the outside and inside surface of a cylindrical metallic pipe, 14cm long, is 44cm^2 . If the pipe is made of 99cm^3 of metal. Find the outer radii of the pipe?
 (a) 2cm (b) 2.5cm
 (c) 4cm (d) 5cm
415. How many bullets can be made out of a cube of lead whose edge measures 22 cm, each bullet being 2 cm in diameter?
 (a) 2341 (b) 2641
 (c) 2541 (d) 2451
416. A right cylindrical vessel is full with water. How many right cones having same diameter and height as those of right cylinder will be needed to store that water?
 (a) 2 (b) 3 (c) 4 (d) 5
417. An open rectangular cistern when measured from outside is 1 m 35 cm long; 1 m 8 cm broad and 90 cm deep, and is made of iron 2.5 cm thick. Find
 (i) the capacity of the cistern,
 (ii) the volume of the iron used.
 (a) 1171625 cu cm, 140575 cu cm
 (b) 1711625 cu cm, 104575 cu cm
 (c) 1171625 cu cm, 145075 cu cm
 (d) None of these
418. Find the weight of a lead pipe 3.5 metres long, if the external diameter of the pipe is 2.4 cm and the thickness of the lead is 2 mm and 1 cc of lead weight 11.4 gm.
 (a) 5.5 kg (b) 5 kg
 (c) 8 kg (d) 10 kg
419. A closed rectangular box has inner dimensions 24 cm by 12 cm by 10 cm. Calculate its capacity and the area of tin foil needed to line its inner surface.
 (a) 2680 cu cm, 1296 sq cm
 (b) 2880 cu cm, 1396 sq cm
 (c) 2880 cu cm, 1296 sq cm
 (d) 2860 cu cm, 1296 sq cm
420. The dimension of an open box are 52 cm, 40 cm and 29 cm. Its thickness is 2 cm. If 1 cm^3 of metal used in the box weight 0.5 gm, the weight of the box is:
 (a) 8.56 kg (b) 7.76 kg
 (c) 7.756 kg (d) 6.832 kg
421. Half cubic metre of gold sheet is extended by hammering so as to cover an area of 1 hectare. Find the thickness of the gold.
 (a) 0.05 cm (b) 0.5 cm
 (c) 0.005 cm (d) 0.0005 cm
422. Two cubic metres of gold are extended by hammering so as to cover an area of twelve hectares. Find the thickness of gold.
 (a) 0.017 cm (b) 0.0017 cm
 (c) 1.7 cm (d) 0.17 cm
423. A cub of silver is drawn into a wire $\frac{1}{10}$ mm in diameter, find the length of the wire.
 ($\pi = 3.1416$)
 (a) 128 metres (b) 127.3 metres
 (c) 129.3 metres (d) 128.3 metres
424. A hollow cylindrical tube open at both ends ins made of iron 4 cm thick. If the internal diameter be 40 cm and the length of the tube be 144 cm, find the volume of iron in it.
 (a) 25344π (b) 23544π
 (c) 26344π (d) None of these
425. A hollow cylindrical tube open at both ends is made of iron 2 cm thick. If the internal diameter be 33 cm and the length of the tube be 70 cm, find the volume of iron in it.
 (a) 12400 cu cm
 (b) 15400 cu cm
 (c) 13800 cu cm
 (d) 16400 cu cm

426. One cm of rain has fallen on 2 square km of land. Assuming that 25% of the raindrops could have been collected and contained in a pool having a $50\text{ m} \times 5\text{ m}$ base, by what level would the water level in the pool have increased?
 (a) 20 m (b) 40 m
 (c) 25 m
 (d) Data inadequate
427. Two cm of rain has fallen on a square km of land. Assuming that 40% of the raindrops could have been collected and contained in a pool having a $200\text{ m} \times 20\text{ m}$ base, by what level would the water level in the pool have increased?
 (a) 2 m (b) 1 m
 (c) 4 m (d) 1.5 m
428. The length of a tank is thrice that of breadth, which is 256 cm deep and holds 3000 L water. What is the base area of the tank? ($1000\text{ L} = 1\text{ cubic metre}$)
 (a) 111775 m^2 (b) 1171.875 m^2
 (c) 1.171875 m^2
 (d) None of these
429. A lid of rectangular box of sides 39.5 cm by 9.35 cm by 9.35 cm is sealed all around with tape such that there is an overlapping of 3.75 cm of the tape. What is the length of the tape used?
 (a) 111.54 cm (b) 101.45 cm
 (c) 110.45 cm (d) None of these
430. A cistern from inside is 12.5 m long, 8.5 m broad and 4 m high and is open at top. Find the cost of cementing the inside of a cistern at Rs. 24 per sq. m:
 (a) Rs. 6582 (b) Rs. 8256
 (c) Rs. 7752 (d) Rs. 8752
431. 250 men took a dip in a water tank at a time, which is $80\text{ m} \times 50\text{ m}$. What is the rise in the water level if the average displacement of 1 man is 4 m^3 ?
 (a) 22 cm (b) 25 cm
 (c) 18 cm (d) 30 cm
432. The edge of a cube is increased by 100% the surface area of the cube is increased by:
 (a) 100% (b) 200%
 (c) 300% (d) 400%
433. The external dimensions of a wooden box closed at both ends are 24 cm, 16 cm and 10 cm respectively and thickness of the wood is 5 mm. If the empty box weighs 7.35 kg, find the weight of 1 cubic cm of wood:
 (a) 10 g (b) 12.5 g
 (c) 27 g (d) 15 g
434. The internal dimensions of a tank are 12 dm, 8 dm and 5 dm. How many cubes each of edge 7 cm can be placed in the tank with faces parallel to the sides of the tank. Find also, how much space is left unoccupied?
 (a) $35 ; 113\text{ dm}^3$
 (b) $1313 ; 31.13\text{ dm}^3$
 (c) $35 ; 31.013\text{ dm}^3$
 (d) $1309 ; 13.31\text{ dm}^3$
435. The length, breadth and height of box are 2 m, 1.5 m and 80 cm respectively. What would be the cost of canvas to cover it up fully, if one square metre of canvas costs Rs. 25.00?
 (a) Rs. 260 (b) Rs. 290
 (c) Rs. 285
 (d) None of these
436. Three cubes each of edge 3 cm long are placed together as shown in the adjoining figure. Find the surface area of the cuboid so formed:
- 
- (a) 182 sq. cm (b) 162 sq. cm
 (c) 126 sq. cm
 (d) None of these
437. A room is 36 m long, 12 m wide and 10 m high. It has 6 window, each $3\text{ m} \times 2.5\text{ m}$; one door $9.5\text{ m} \times 6\text{ m}$ and one fire chimney $4\text{ m} \times 4.5\text{ m}$. Find the expenditure of papering its walls at the rate of 70 paise per metre, if the width of the paper is 1.2 m:
 (a) Rs. 490 (b) Rs. 690
 (c) Rs. 1000
 (d) None of these
438. When each side of a cube is increased by 2 cm, the volume is increased by 1016 cm^3 . Find the side of the cube. If each side of it is decreased by 2 cm, by how much will the volume decrease?
 (a) 12 cm, 729 cm^3
 (b) 8 cm, 512 cm^3
 (c) 9 cm, 729 cm^3
 (d) 12 cm, 728 cm^3
439. Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the resulting cuboid to that of the total surface areas of the three cubes:
 (a) 5 : 7 (b) 7 : 9
 (c) 9 : 7 (d) None of these
440. A hollow square shaped tube open at both ends is made of iron. The internal square is of 5 cm side and the length of the tube is 8 cm. There are 192 cm^3 of iron in the tube. Find the thickness:
 (a) 2 cm (b) 0.5 cm
 (c) 1 cm
 (d) can't be determined
441. A cube of 11 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of base are 15 cm and 12 cm. Find the rise in water level in the vessel:
 (a) 6.85 cm (b) 7 cm
 (c) 7.31 cm (d) 7.39 cm
442. A rectangular tank 25 cm long and 20 cm wide contains water to a depth of 5 cm. A metal cube of side 10 cm is placed in the tank so that one face of the cube rests on the bottom of the tank. Find how many litres of water must be poured into the tank so as to just cover the cube?
 (a) 1 L (b) 1.5 L
 (c) 2 L (d) 2.5 L
443. A rectangular block has length 10 cm, breadth 8 cm and height 2 cm. From this block, a cubical hole of side 2 cm is drilled out. Find the volume and the surface area of the remaining solid:
 (a) 152 cm^2 , 512 cm^2
 (b) 125 cm^2 , 215 cm^2
 (c) 152 cm^2 , 240 cm^2
 (d) 125 cm^2 , 512 cm^2

444. How many bricks (number near to next hundred) will be required to build a wall 30 m long, 30 cm thick and 5 m high with a provision of 2 doors, each 4 m $\times$ 2.5 m and each brick being 20 cm $\times$ 16 cm $\times$ 8 cm when one-ninth of the wall is filled with lime?
 (a) 13500 bricks
 (b) 13600 bricks
 (c) 20050 bricks
 (d) 18500 bricks
445. A rectangular water reservoir is 15 m by 12 m at the base. Water flows into it through a pipe whose cross-section is 5 cm by 3 cm at the rate of 16 m/s second. Find the height to which the water will rise in the reservoir in 25 minutes:
 (a) 0.2 m
 (b) 2 cm
 (c) 0.5 m
 (d) None of these
446. The volume of a wall, 3 times as high as it is broad and 8 times as long as it is high, is 36.864 m³. The height of the wall is:
 (a) 1.8 m
 (b) 2.4 m
 (c) 4.2 m
 (d) None of these
447. If the areas of 3 adjacent sides of a cuboid are x , y , z respectively, then the volume of the cuboid is:
 (a) xyz
 (b) $\sqrt{xyz}$
 (c) $3xyz$
 (d) None of these
448. A cylindrical cistern whose diameter is 21 cm is partly filled with water. If a rectangular block of iron 14 cm in length, 10.5 cm in breadth and 11 cm in thickness is wholly immersed in water, by how many centimetres will the water level rise?
 (a) 14 cm
 (b) 20 cm
 (c) $\frac{14}{3}$ cm
 (d) 12 cm
449. A right circular cylindrical tunnel of diameter 4 m and length 10 m is to be constructed from a sheet of iron. The area of the iron sheet required.
 (a) $\frac{280}{\pi}$
 (b) 40π
 (c) 80π
 (d) None of these
450. A conical vessel has a capacity of 15 L of milk. Its height is 50 cm and base radius is 25 cm. How much milk can be contained in a vessel in cylindrical form having the same dimensions as that cone?
 (a) 15 L
 (b) 30 L
 (c) 45 L
 (d) None of these
451. The height of a metric cylinder is 14 cm & the different of its in curved S.A. is 44 cm². If the cylinder is made up of 99 cm³ metal the find the inner & outer radius of cylinder.
 (a) 464
 (b) 564
 (c) 660
 (d) 366
452. If the base radius and the height of a right circular cone are increased by 40% then the percentage increase in volume (approx) is:
 (a) 175%
 (b) 120%
 (c) 64%
 (d) 540%
453. From a circular sheet of paper of radius 25 cm, a sector area 4% is removed. If the remaining part is used to make a conical surface, then the ratio of the radius and height of the cone is:
 (a) 16 : 25
 (b) 9 : 25
 (c) 7 : 12
 (d) 24 : 7
454. A conical tent has 60° angle at the vertex. The ratio of its radius and slant height is:
 (a) 3 : 2
 (b) 1 : 2
 (c) 1 : 3
 (d) can't be determined
455. Water flows at the rate of 5 m per min from a cylindrical pipe 8 mm in radius. How long will it take to fill up a conical vessel whose radius is 12 cm and depth 35 cm?
 (a) 315 s
 (b) 365 s
 (c) 5 min
 (d) None of these
456. A reservoir is in the shape of a frustum of a right circular cone. It is 8 m across at the top and 4 m across at the bottom. It is 6 m deep its capacity is:
 (a) 224 m³
 (b) 176 m³
 (c) 225 m³
 (d) None of these
457. A conical vessel whose internal radius is 10 cm and height 72 cm is full of water. If this water is poured into a cylindrical vessel with internal radius 30 cm, the height of the water level rises in it is:
 (a) $2\frac{2}{3}$ cm
 (b) $3\frac{2}{3}$ cm
 (c) $5\frac{2}{3}$ cm
 (d) None of these
458. If h , c , v are respectively the height, the curved surface area and the volume of a cone then the value of $3\pi vh^3 - c^2h^2 + 9v^2$ is equal to:
 (a) 1
 (b) 2
 (c) 0
 (d) None of these
459. If P is the height of a tetrahedron & each side is of 2 cm the find the value of $3p^2$.
 (a) $6a^2$
 (b) $8a^2$
 (c) $5a^2$
 (d) $7a^2$
460. If 'h' be the height of a pyramid standing on a base which is an equilateral triangle of side 'a' units, then the slant edge is:
 (a) $\sqrt{h^2 + a^2/4}$
 (b) $\sqrt{h^2 + a^2/8}$
 (c) $\sqrt{h^2 + a^2/3}$
 (d) $\sqrt{h^2 + a^2}$
461. Find the volume of a tetrahedron whose height is $4\sqrt{3}$ cm.
 (a) 72
 (b) 108
 (c) 54
 (d) 36
462. In a shower 10 cm of rain fall the volume of water that falls on 1.5 hectares of ground is:
 (a) 1500 m³
 (b) 1400 m³
 (c) 1200 m³
 (d) 1000 m³
463. Find the total surface area of pyramid of heights 12 which is based on a rectangle of length 18 and breadth 10 cm.
 (a) 117
 (b) 564
 (c) 120
 (d) 456
464. If from a circular sheet of paper of radius 15 cm, a sector of 144° is removed and the remaining is used to make a conical surface, then the angle at the vertex will be:
 (a) $\sin^{-1}\left(\frac{3}{10}\right)$
 (b) $\sin^{-1}\left(\frac{6}{5}\right)$
 (c) $2\sin^{-1}\left(\frac{3}{5}\right)$
 (d) $2\sin^{-1}\left(\frac{4}{5}\right)$
465. Find the length of the string bound on a cylindrical tank whose base diameter and height are $5\frac{1}{11}$ cm and 48 cm

- respectively. The string makes exactly four complete turns round the cylinder, while its two ends touch the tank's top and bottom :
- (a) 75cm (b) 70cm
(c) 60cm (d) 80cm
466. A cone, a hemisphere and a cylinder stand on equal bases of radius R and have equal heights H . Their whole surfaces are in the ratio:
- (a) $(\sqrt{3} + 1) : 3 : 4$
(b) $(\sqrt{2} + 1) : 7 : 8$
(c) $(\sqrt{2} + 1) : 3 : 4$
(d) None of these
467. If l , b , p be the length, breadth and perimeter of a rectangle and b , l , p are in GP (in order), then l/b is:
- (a) $2 : 1$
(b) $(\sqrt{3} - 1) : 1$
(c) $(\sqrt{3} + 1) : 1$
(d) can't be determined
468. The height of a circular cylinder is increased by 6 times and base area is decreased by $1/9$ th times. By what factor its lateral surface area is increased ?
- (a) 2 (b) 3 (c) 6 (d) 1.5
469. A pyramid with an equal Δ based of each side 4 cm while its slant height is twice the height of pyramid. Find its volume-
- (a) $8\sqrt{3}$ (b) $\frac{8}{3\sqrt{3}}$
(c) $4\sqrt{3}$ (d) $3\sqrt{3}$
470. A cube and a sphere have equal surface areas. The ratio of their volume is :
- (a) $p : 3$ (b) $\sqrt{\pi} : \sqrt{6}$
(c) $\sqrt{6} : \sqrt{\pi}$ (d) $6 : \pi$
471. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their respective volumes is :
- (a) $1 : 2 : 3$ (b) $2 : 1 : 3$
(c) $1 : 3 : 2$ (d) $3 : 1 : 2$
472. A cube of sides 3cm is melted and smaller cubes of sides 1cm each are formed. How many such cubes are possible?
- (a) 21 (b) 23 (c) 25 (d) 27
473. A cuboidal block of $6\text{cm} \times 9\text{cm} \times 12\text{cm}$ is cut into exact number of equal cubes. The least possible number of cubes will be :
- (a) 6 (b) 9 (c) 24 (d) 30
474. If the cone is cut along its axis from the middle, the new shape we obtain after opening the paper is :
- (a) isosceles triangle
(b) right angle triangle
(c) equilateral triangle
(d) None of these
475. What is the height of the cone which is formed by joining the two ends of a sector of circle with radius r and angle 60° :
- (a) $\frac{\sqrt{35}}{6}r$ (b) $\frac{\sqrt{25}}{6}r$
(c) $\frac{r^2}{\sqrt{3}}$ (d) $\frac{35}{6}r$
476. A sphere of 20cm radius is dropped into a cylindrical vessel of 60cm diameter, which is partly filled with water, then its level rises by x cm. Find x :
- (a) $11\frac{21}{27}\text{cm}$ (b) 12cm
(c) 22.5cm (d) $11\frac{23}{27}\text{cm}$
477. A right circular cone resting on its base is cut at $\frac{4}{5}$ th its height along a parallel to the circular base. The height of original cone is 75cm and base diameter is 42cm. What is the base radius of cut out (top portion) cone ?
- (a) 4.2cm (b) 8.4cm
(c) 2.8cm (d) 3.5cm
478. A solid sphere is melted and recast into a right circular cone with a base radius equal to the radius of the sphere. What is the ratio of the height and radius of cone so formed ?
- (a) 5 : 2 (b) 4 : 3
(c) 4 : 1 (d) 3 : 2
479. 125 identical cubes are cut from a big cube and all the smaller cubes are arranged in a row to form a long cuboid. What is the percentage increase in the total surface area of the cuboid over the total surface area of the cube ?
- (a) $234\frac{1}{3}\%$ (b) $234\frac{2}{3}\%$
(c) 117% (d) None of these
480. What is the total surface area of the identical cubes of largest possible size that are cut from a cuboid of size $75\text{cm} \times 15\text{cm} \times 4.5\text{cm}$?
- (a) 20, 250cm² (b) 20, 520cm²
(c) 22, 250cm² (d) None of these
481. If the volume of a sphere, a cube, a tetrahedron and a octahedron be same then which of the following has maximum surface area ?
- (a) sphere (b) cube
(c) tetrahedron (d) octahedron
482. A spherical ball of lead 6cm in radius is melted and recast into three spherical balls. The radii of two of these balls are 3cm and 4cm. What is the radius of the third sphere ?
- (a) 6cm (b) 6.5cm
(c) 5.5cm (d) 5cm
483. The base of a right prism is a triangle whose perimeter is 45cm and the radius of incircle is 9cm. If the volume of the prism is 810cm³. Find its height :
- (a) 5cm (b) 4cm
(c) 6cm (d) 4.5cm
484. What is the semi-vertical angle of a cone whose lateral surface area is double the base area ?
- (a) 30° (b) 45°
(c) 60° (d) None of these
485. What is the number of cones of semi-vertex angle α and having r as the radius of the mid-section which can be moduled out of a cylinder of base radius r and height $2r \cot \alpha$:
- (a) 5 (b) 7 (c) 6 (d) 4
486. A water tank is 30cm long, 20cm wide and 12m deep. It is made of iron sheet which is 3m wide. The tank is open at the top. If the cost of iron sheet is ₹ 10 per meter. Find the total cost of iron required to build the tank ?
- (a) ₹ 6000 (b) ₹ 5000
(c) ₹ 5500 (d) ₹ 5800

487. A trapezium based prism with two parallel sides 8cm and 14cm respectively and distance between two parallel sides is 8cm. Find the height of the prism if the volume of the prism is 1056cm^3 ?
- (a) 11cm (b) 10cm
(c) 9cm (d) 12cm
488. From a circular sheet of paper, radius 10cm, A sector of area 40% of the sheet is removed. If the remaining part is used to make a conical surface. Then the ratio of radius and height will be :
- (a) 4 : 3 (b) 3 : 4
(c) 2 : 3 (d) 2 : 1
489. If the volume of circular cell having inner and outer radius 8cm and 12cm respectively is equal to the total surface area of cylinder of radius R_1 and height h , then h in terms of R_1 will be :
- (a) $\frac{40 - R_1}{R_1}$ (b) $\frac{40 - R_1}{R_1^2}$
(c) $\frac{40 - R_1^2}{R_1}$ (d) None of these
490. An iron pipe 20cm long has exterior diameter 25cm. If the thickness of the pipe is 1cm, then the whole surface area of the pipe ?
- (a) 3168cm^2 (b) 3186cm^2
(c) 3200cm^2 (d) 3150cm^2
491. The capacity of two hemispherical bowls are 64 litre and 216 litre respectively. Then the ratio of their internal curved surface area will be:
- (a) 2 : 3 (b) 1 : 3
(c) 16 : 81 (d) 4 : 9
492. If the length of a rectangular parallel pipe is three times of its breadth and five times of its height. If its volume is 14400cm^3 , then the total surface area will be :
- (a) 4230cm^2 (b) 4320cm^2
(c) 4203cm^2 (d) None of these
493. A right angled triangle with its sides 5cm, 12cm and 13cm is revolved about the side 12cm. Find the volume of the solid formed ?
- (a) 942cm^3 (b) 298cm^3
(c) 314cm^3 (d) 302cm^3
494. A hemisphere bowl V_1 and a hollow right circular cylinder V_2 (having length equal to its radius) have the same diameter equal to the length of a side of a hollow cubical box V_3 . Water is filled in all these vessels upto the same level and such that hemispherical bowl is full of water and the volumes of filled water are v_1 , v_2 and v_3 respectively in V_1 , V_2 and V_3 then :
- (a) $V_1 < V_2 < V_3$ (b) $V_2 < V_3 < V_1$
(c) $V_3 < V_2 < V_1$ (d) $V_3 < V_1 < V_2$
495. A vertical cone of volume V with vertex downward is filled with water up to half of its height. The volume of the water is :
- (a) $V/16$ (b) $V/8$
(c) $V/4$ (d) $V/2$
496. The heights of a cone, cylinder and hemisphere are equal. If their radii are in the ratio 2 : 3 : 1, then the ratio of their volumes is :
- (a) 2 : 9 : 2 (b) 4 : 9 : 1
(c) 4 : 27 : 2 (d) 2 : 3 : 1
497. The height of a right circular cone and the radius of its circular base are respectively 9 cm and 3 cm. The cone is cut by a plane parallel to its base so as to divide it into two parts. The volume of the frustum (i.e., the lower part) of the cone is 44 cubic cm. The radius of the upper circular surface of the frustum $\left(\text{take } \pi = \frac{22}{7}\right)$ is :
- (a) $\sqrt[3]{12}$ cm (b) $\sqrt[3]{13}$ cm
(c) $\sqrt[3]{13}$ cm (d) $\sqrt[3]{20}$ cm
498. A solid cylinder has total surface area of 462 sq. cm. Curved surface area is $\frac{1}{3}$ rd of its total surface area. The volume of the cylinder is :
- (a) 530 cm^3 (b) 536 cm^3
(c) 539 cm^3 (d) 545 cm^3
499. A solid is hemispherical at the bottom and conical above. If the surface areas of the two parts are equal, then the ratio of radius and height of its conical part is :
- (a) 1 : 3 (b) 1 : 1
(c) $\sqrt{3} : 1$ (d) $1 : \sqrt{3}$
500. The radius and the height of a cone are in the ratio 4 : 3. The ratio of the curved surface area and total surface area of the cone is :
- (a) 5 : 9 (b) 3 : 7
(c) 5 : 4 (d) 16 : 9
501. From a right circular cylinder of radius 10 cm and height 21 cm, a right circular cone of same base-radius is removed. If the volume of the remaining portion is 4400 cm^3 then the height of the removed cone is:
- $\left(\text{take } \pi = \frac{22}{7}\right)$
- (a) 15 cm (b) 18 cm
(c) 21 cm (d) 24 cm
502. A right circular cylinder and a cone have equal base radius and equal heights. If their curved surfaces are in the ratio 8 : 5, then the radius of the base to the height are in the ratio :
- (a) 2 : 3 (b) 4 : 3
(c) 3 : 4 (d) 3 : 2
503. The curved surface area of a cylindrical pillar is 264 sq.m. and its volume is 924 cu.m. The ratio of its diameter to height is :
- (a) 3 : 7 (b) 7 : 3
(c) 6 : 7 (d) 7 : 6
504. A cube of edge 6 cm is painted on all sides and then cut into unit cubes. The number of unit cubes with no sides painted is:
- (a) 0 (b) 64 (c) 186 (d) 108
505. There is a pyramid on a base which is a regular hexagon of side 2a cm. If every slant edge of this pyramid is of length $\frac{5a}{2}$ cm, then the volume of this pyramid is :
- (a) $3a^3\text{cm}^3$ (b) $3\sqrt{2}a^3\text{cm}^3$
(c) $3\sqrt{3}a^3\text{cm}^3$ (d) $6a^3\text{cm}^3$
506. The base of a right prism is an equilateral triangle of area 173 cm^2 and the volume of the prism is 10380 cm^3 . The area of the lateral surface of the prism is use $(\sqrt{3} = 1.73)$
- (a) 1200 cm^2 (b) 2400 cm^2
(c) 3600 cm^2 (d) 4380 cm^2

507. Three spherical balls of radii 1 cm, 2 cm and 3 cm are melted to form a single spherical ball. In the process, the loss of material is 25 %. The radius of the new ball is :

- (a) 6 cm (b) 5 cm
(c) 3 cm (d) 2 cm

508. The height of a cone is 40cm. The cone is cut parallel to its base such that the volume of

the small cone is $\frac{1}{64}$ of the cone. Find at which height from base the cone is cut ?

- (a) 20cm (b) 30cm
(c) 25cm (d) 22.5cm

509. A cube of side 8 metre is reduced 3 times in the ratio 2 : 1. The area of one face of the reduced cube to that of the original cube is in the ratio :

- (a) 1 : 4 (b) 1 : 8
(c) 1 : 16 (d) 1 : 64

510. The volume of the largest cylinder formed, when a rectangular sheet of paper of size 22 cm $\times$ 15cm is rolled along its

larger side, is $\left(\text{use } \pi = \frac{22}{7} \right)$:

- (a) 288.75 cm³ (b) 577.50 cm³
(c) 866.25 cm³ (d) 1155.00 cm³

511. Each of the height and radius of the base of a right circular cone is increased by 100 %.

The volume of the cone will be increased by :

- (a) 700 % (b) 500 %
(c) 300 % (d) 100 %

512. The height of a right prism with a square base is 15cm. If the area of the total surfaces of the prism is 608 sq.cm, its volume is :

- (a) 910 cm³ (b) 920 cm³
(c) 960 cm³ (d) 980 cm³

513. The internal radius and thickness of a hollow metallic pipe are 24 cm and 1 cm respectively. It is melted and recast into a solid cylinder of equal length. The diameter of the solid cylinder will be :

- (a) 7 cm (b) 14 cm
(c) 960 cm³ (d) 980 cm³

514. The radius of the base of a right circular cone is doubled. To keep the volume fixed, the height of the cone will be

- (a) One-fourth of the previous height
(b) $\frac{1}{\sqrt{2}}$ times of the previous height
(c) half of the previous height
(d) one-third of the previous height

515. If a cube maximum possible volume is cut off from a solid sphere of diameter d, then the volume of the remaining (waste) material of the sphere would be equal to :

(a) $\frac{d^3}{3} \left(\pi - \frac{d}{2} \right)$ (b) $\frac{d^3}{3} \left(\frac{\pi}{2} - \frac{1}{\sqrt{3}} \right)$

(c) $\frac{d^2}{4} (\sqrt{2} - \pi)$ (d) None of these

516. A big cube of side 9cm is formed by rearranging together 27 small but identical cubes each of side 3cm. further, if the corner cubes in the top most layer of the big cube are removed, what is the change in total surface area of the big cube?

- (a) 18cm², decreases
(b) 54cm², decreases
(c) 36cm², decreases
(d) remains the same

517. The base radius and height of a cone is 5cm and 25cm respectively. If the cone is cut parallel to its base at a height of h from the base. If the volume of this frustum is 110cm³. Find the radius of smaller cone ?®

- (a) $(104)^{1/3}$ cm (d) $(104)^{1/2}$ cm
(c) 5cm (d) None of these

518. A spherical steel ball was silver polished then it was cut into 4 similar pieces. What is the ratio of the polished area to the non-polished area :

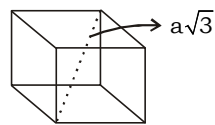
- (a) 1 : 1 (b) 1 : 2
(c) 2 : 1
(d) None of these

ANSWER KEY

1. (a)	53. (d)	105.(a)	157.(a)	209.(c)	261.(d)	313.(a)	365.(b)	417.(a)	469.(b)
2. (c)	54. (b)	106.(a)	158.(d)	210.(d)	262.(b)	314.(c)	366.(b)	418.(a)	470.(b)
3. (c)	55. (c)	107.(a)	159.(c)	211.(d)	263.(d)	315.(c)	367.(a)	419.(c)	471.(a)
4. (b)	56. (c)	108.(a)	160.(a)	212.(d)	264.(c)	316.(b)	368.(d)	420.(d)	472.(d)
5. (b)	57. (c)	109.(b)	161.(d)	213.(b)	265.(a)	317.(d)	369.(d)	421.(c)	473.(c)
6. (d)	58. (d)	110.(b)	162.(c)	214.(a)	266.(a)	318.(b)	370.(c)	422.(b)	474.(b)
7. (a)	59. (b)	111.(d)	163.(a)	215.(d)	267.(a)	319.(b)	371.(a)	423.(b)	475.(a)
8. (c)	60. (d)	112.(a)	164.(d)	216.(a)	268.(c)	320.(a)	372.(d)	424.(a)	476.(d)
9. (c)	61. (b)	113.(b)	165.(a)	217.(b)	269.(d)	321.(a)	373.(d)	425.(b)	477.(a)
10. (a)	62. (a)	114.(b)	166.(c)	218.(d)	270.(b)	322.(d)	374.(c)	426.(a)	478.(c)
11. (d)	63. (a)	115.(d)	167.(d)	219.(a)	271.(c)	323.(a)	375.(b)	427.(a)	479.(b)
12. (b)	64. (b)	116.(c)	168.(b)	220.(a)	272.(a)	324.(c)	376.(c)	428.(c)	480.(a)
13. (a)	65. (b)	117.(c)	169.(d)	221.(a)	273.(a)	325.(b)	377.(c)	429.(b)	481.(c)
14. (d)	66. (a)	118.(a)	170.(a)	222.(d)	274.(c)	326.(b)	378.(d)	430.(a)	482.(d)
15. (c)	67. (c)	119.(b)	171.(c)	223.(a)	275.(c)	327.(c)	379.(a)	431.(b)	483.(b)
16. (b)	68. (b)	120.(b)	172.(b)	224.(c)	276.(c)	328.(b)	380.(b)	432.(c)	484.(a)
17. (a)	69. (b)	121.(c)	173.(b)	225.(a)	277.(a)	329.(b)	381.(c)	433.(a)	485.(c)
18. (c)	70. (b)	122.(a)	174.(b)	226.(a)	278.(d)	330.(d)	382.(c)	434.(c)	486.(a)
19. (d)	71. (b)	123.(b)	175.(d)	227.(d)	279.(c)	331.(b)	383.(b)	435.(b)	487.(d)
20. (c)	72. (b)	124.(c)	176.(d)	228.(a)	280.(c)	332.(c)	384.(a)	436.(c)	488.(b)
21. (b)	73. (c)	125.(c)	177.(c)	229.(d)	281.(b)	333.(a)	385.(c)	437.(a)	489.(c)
22. (b)	74. (c)	126.(c)	178.(d)	230.(c)	282.(c)	334.(c)	386.(d)	438.(d)	490.(a)
23. (b)	75. (b)	127.(c)	179.(a)	231.(c)	283.(c)	335.(c)	387.(a)	439.(b)	491.(d)
24. (d)	76. (b)	128.(c)	180.(a)	232.(b)	284.(b)	336.(b)	388.(c)	440.(c)	492.(b)
25. (c)	77. (c)	129.(a)	181.(b)	233.(a)	285.(b)	337.(b)	389.(d)	441.(d)	493.(c)
26. (d)	78. (d)	130.(c)	182.(a)	234.(b)	286.(a)	338.(c)	390.(b)	442.(b)	494.(a)
27. (c)	79. (b)	131.(a)	183.(a)	235.(c)	287.(d)	339.(a)	391.(a)	443.(c)	495.(b)
28. (b)	80. (c)	132.(c)	184.(d)	236.(a)	288.(d)	340.(a)	392.(b)	444.(b)	496.(c)
29. (d)	81. (c)	133.(a)	185.(d)	237.(b)	289.(a)	341.(c)	393.(c)	445.(a)	497.(b)
30. (b)	82. (b)	134.(a)	186.(a)	238.(a)	290.(d)	342.(d)	394.(d)	446.(b)	498.(c)
31. (a)	83. (b)	135.(d)	187.(b)	239.(d)	291.(a)	343.(b)	395.(b)	447.(b)	499.(d)
32. (b)	84. (a)	136.(c)	188.(d)	240.(a)	292.(d)	344.(a)	396.(d)	448.(c)	500.(a)
33. (a)	85. (b)	137.(d)	189.(a)	241.(b)	293.(d)	345.(b)	397.(d)	449.(b)	501.(c)
34. (b)	86. (d)	138.(d)	190.(c)	242.(c)	294.(b)	346.(c)	398.(d)	450.(c)	502.(c)
35. (a)	87. (b)	139.(d)	191.(b)	243.(d)	295.(a)	347.(c)	399.(c)	451.(b)	503.(b)
36. (b)	88. (d)	140.(b)	192.(a)	244.(b)	296.(d)	348.(b)	400.(d)	452.(a)	504.(b)
37. (c)	89. (b)	141.(c)	193.(b)	245.(d)	297.(a)	349.(d)	401.(b)	453.(d)	505.(c)
38. (c)	90. (c)	142.(d)	194.(d)	246.(b)	298.(a)	350.(d)	402.(c)	454.(b)	506.(c)
39. (d)	91. (c)	143.(b)	195.(b)	247.(a)	299.(c)	351.(c)	403.(a)	455.(a)	507.(c)
40. (b)	92. (d)	144.(c)	196.(a)	248.(a)	300.(a)	352.(c)	404.(a)	456.(d)	508.(b)
41. (c)	93. (d)	145.(a)	197.(d)	249.(d)	301.(c)	353.(c)	405.(b)	457.(a)	509.(d)
42. (d)	94. (a)	146.(b)	198.(d)	250.(d)	302.(a)	354.(b)	406.(c)	458.(c)	510.(b)
43. (a)	95. (d)	147.(d)	199.(d)	251.(c)	303.(b)	355.(d)	407.(b)	459.(b)	511.(a)
44. (d)	96. (b)	148.(d)	200.(a)	252.(b)	304.(b)	356.(a)	408.(c)	460.(c)	512.(c)
45. (d)	97. (b)	149.(b)	201.(a)	253.(a)	305.(a)	357.(b)	409.(a)	461.(a)	513.(b)
46. (d)	98. (b)	150.(b)	202.(a)	254.(c)	306.(d)	358.(d)	410.(a)	462.(a)	514.(a)
47. (d)	99. (b)	151.(a)	203.(a)	255.(a)	307.(d)	359.(b)	411.(c)	463.(b)	515.(b)
48. (d)	100.(b)	152.(d)	204.(b)	256.(d)	308.(c)	360.(d)	412.(d)	464.(c)	516.(d)
49. (a)	101.(a)	153.(b)	205.(d)	257.(d)	309.(c)	361.(b)	413.(c)	465.(d)	517.(a)
50. (b)	102.(c)	154.(a)	206.(b)	258.(d)	310.(a)	362.(d)	414.(b)	466.(c)	518.(a)
51. (d)	103.(c)	155.(d)	207.(c)	259.(d)	311.(b)	363.(c)	415.(c)	467.(c)	
52. (b)	104.(c)	156.(a)	208.(b)	260.(b)	312.(c)	364.(d)	416.(b)	468.(a)	

SOLUTION

1. (a) Let the side of cube = a cm



$$\text{Diagonal of cube} = a\sqrt{3} \text{ cm}$$

$$a\sqrt{3} = \sqrt{12}$$

$$\text{on squaring, } a^2(3) = 12$$

$$a^2 = 4$$

$$a = 2 \text{ cm}$$

$$\text{volume of cube} = a^3 = 2^3 = 8 \text{ cm}^3$$

2. (c) Number of cubes = $\frac{(15)^3}{(3)^3} = 125$

3. (c) Side of the cube = $\frac{\text{Diagonal}}{\sqrt{3}}$

$$= \frac{4\sqrt{3}}{\sqrt{3}} = 4 \text{ cm}$$

$$\text{Volume of the cube} = (\text{side})^3$$

$$\Rightarrow (4)^3 = 4 \times 4 \times 4 = 64 \text{ cm}^3$$

4. (b) Let $l = 9x$, $h = 3x$, $b = x$

$$l \times b \times h = 216 \times 1000$$

$$(1 \text{ litre} = 1000 \text{ cm}^3)$$

$$9x \times x \times 3x = 216000$$

$$27x^3 = 216000$$

$$x^3 = 8000$$

$$x = 20$$

$$l = 180 \text{ cm} = 18 \text{ dm}$$

$$\text{Volume} = 2 \times \text{volume of Cuboid}$$

5. (b) $l \times b \times h = 2 \times (\text{side})^3$

$$\frac{9 \times 8 \times 6}{2} = (\text{side})^3 = 216$$

$$\text{side} = \sqrt[3]{216} = 6 \text{ cm}$$

$$\text{Total surface area of cube}$$

9

(c) Volume of the cistern

$$\Rightarrow (330 - 10) \times (260 - 10) \times (110 - x)$$

$$= 8000 \times 1000$$

$$(\text{where } x = \text{thickness of bottom})$$

$$x = 110 - 100 = 10 \text{ cm} = 1 \text{ dm}$$

10. (a) Let the length, breadth and height be l , b , h respectively

$$\Rightarrow lb = x$$

$$bh = y$$

$$lh = z$$

$$\Rightarrow l^2 b^2 h^2 = xyz$$

$$(lbh)^2 = xyz$$

$$\Rightarrow v^2 = xyz$$

11. (d) The diameter of sphere = side of cube = 7 cm

$$\text{Radius } (r) = \frac{7}{2} \text{ cm}$$

$$\text{volume of sphere} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$$

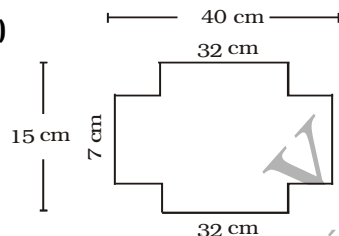
$$= 179.67 \text{ cm}^3$$

12. (b) Length of rod

$$= \sqrt{10^2 + 10^2 + 5^2} = \sqrt{225}$$

$$= 15 \text{ cm}$$

13. (a)



$$\text{Volume of the box} = l \times b \times h$$

$$= (40 - 8) \times (15 - 8) \times 4$$

$$= 32 \times 7 \times 4 = 896 \text{ cm}^3$$

14. (d) Let the three sides of the cuboid be l , b and h

$$\Rightarrow lb = bh = hl = 12$$

$$\Rightarrow l^2 b^2 h^2 = 12 \times 12 \times 12$$

$$= 1728$$

$$\Rightarrow lbh = \sqrt{1728} = 12\sqrt{12}$$

$$= 24\sqrt{3} \text{ cm}^3$$

15. (c) dimensions of room

$$\text{length}(l) = 12 \text{ cm}$$

$$\text{breadth}(b) = 9 \text{ cm}$$

$$\text{height}(h) = 8 \text{ cm}$$

$$\therefore \text{diagonal of cube} = \sqrt{l^2 + b^2 + h^2}$$

$$\therefore \text{length of longest rod}$$

$$\Rightarrow \text{length of diagonal of cuboid}$$

$$\Rightarrow \sqrt{12^2 + 9^2 + 8^2}$$

$$\Rightarrow \sqrt{144 + 81 + 64}$$

$$\Rightarrow \sqrt{289} = 17 \text{ cm}$$

16. (b) area of floor $\Rightarrow 3 \times 4 = 12 \text{ m}^2$

$$\text{height} \Rightarrow 3 \text{ m}$$

$$\therefore \text{area of walls of room}$$

$$\Rightarrow (\text{Perimeter} \times \text{height of of floor})$$

$$\Rightarrow 2(l + b) \times h$$

$$\Rightarrow l = \text{length} = 4 \text{ m}$$

$$b = \text{breadth} = 3 \text{ m}$$

$$h = \text{height} = 3 \text{ m}$$

$$\therefore \text{Area of walls} \Rightarrow 2$$

$$(4 + 3) \times 3 = 42 \text{ m}^2$$

$$\text{Area of painted part}$$

$$= 42 \text{ m}^2 + 12 \text{ m}^2 = 54 \text{ m}^2$$

17. (a) Let

$$\text{length} = l,$$

$$\text{breadth} = b$$

$$\text{height} = h$$

$$\text{given that } (l + b + h) = 12 \text{ cm}$$

$$= \text{total surface area of box}$$

$$= 2(lb + bh + hl) = 94 \text{ m}^2 \text{ (given)}$$

$$(l + b + h)^2 = l^2 + b^2 + h^2 + 2(lb + bh + hl)$$

$$(12)^2 = l^2 + b^2 + h^2 + 94$$

$$144 - 94 \Rightarrow l^2 + b^2 + h^2$$

$$50 = l^2 + b^2 + h^2$$

$$\text{diagonal of box} = \sqrt{l^2 + b^2 + h^2}$$

$$\text{length of longest rod that can be put inside the box}$$

$$= \sqrt{l^2 + b^2 + h^2} = 5\sqrt{2} \text{ cm}$$

18. (c) Let breadth = b m

$$\therefore \text{length of room} = 2b \text{ m}$$

$$(l = 2b)$$

$$\text{height} = 11 \text{ m}$$

$$\text{Area of four walls of room}$$

$$= 660 \text{ m}^2 \text{ (given)}$$

$$2(l + b) \times h = 660$$

$$2(2b + b) \times 11 = 660$$

$$3b \times 22 = 660$$

$$b = 10$$

$$\therefore \text{Breadth} = 10 \text{ m}$$

$$\text{Length} = 20 \text{ m}$$

$$\text{area of floor} = l \times b$$

$$\text{length} \times \text{breadth} =$$

$$20 \times 10 = 200 \text{ m}^2$$

19. (d) Side of cube $(a) = \frac{8\sqrt{3}}{\sqrt{3}} = 8 \text{ cm}$

$$\Rightarrow \text{Total surface area}$$

$$= 6(a)^2 = 6 \times 8^2 = 384 \text{ cm}^2$$

20. (c) Length of pencil

$$= \sqrt{a^2 + b^2 + c^2}$$

$$= \sqrt{8^2 + 6^2 + 2^2} = \sqrt{64 + 36 + 4}$$

$$= \sqrt{104} = 2\sqrt{26} \text{ cm}$$

21. (b) Edge of box

$$= \sqrt[3]{3.375} = 1.5 \text{ m}$$

22. (b) Whole surface area of cuboid

$$= 2(\text{whole surface area of cube})$$

$$- 2 \times \text{area of one face}$$

$$(\because \text{two faces of the two cubes are not visible now})$$

$$\Rightarrow \text{Required area} = 12a^2 - 2a^2$$

$$= 10a^2 = 10 \times 6^2 = 360 \text{ cm}^2$$

23. (b) Let the increase in level = x m

$$\Rightarrow \left(1000 \times 1000 \times \frac{2}{100}\right) \times \frac{1}{2} = 100 \times 10 \times x$$

$$\Rightarrow x = 10 \text{ m}$$

24. (d) Sides of parallelopiped are in ratio = 2 : 4 : 8

Let length = 2 units

breadth = 4 units

Height = 8 units

Let the side of cube = a unit

According to question,

volume of cube = volume of parallelopiped

$$a^3 = 2 \times 4 \times 8$$

$$a^3 = 64$$

$$a = \sqrt[3]{64} = 4 \text{ units}$$

Surface area of parallelopiped

Surface area of cube

$$= \frac{2(lb+bh+hl)}{6a^2}$$

$$= \frac{2(8+32+16)}{6(4)^2} = \frac{7}{6} = 7 : 6$$

25. (c) Let

length = 1 cm

breadth = 2 cm,

height = h cm

$$2(lb + bh + hl) = 22$$

$$2(2 + 2h + h) = 22$$

$$2 + 3h = 11$$

$$3h = 9$$

$$h = 3 \text{ cm}$$

$$\text{Diagonal} = \sqrt{l^2 + b^2 + h^2}$$

$$\sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$$

26. (d) $\sqrt{l^2 + b^2 + h^2} = 15$

$$l^2 + b^2 + h^2 = 225 \dots\dots\dots(i)$$

$$\therefore l + b + h = 24$$

$$(l + b + h)^2 = 576$$

$$\Rightarrow l^2 + b^2 + h^2 + 2(lb + bh + hl) = 576$$

$$= 225 + 2(lb + bh + hl) = 576$$

$$2(lb + bh + hl) = 351 \text{ cm}^2$$

27. (c) Total surface area of cube

$$= 6(\text{side})^2$$

$$6(\text{side})^2 = 96$$

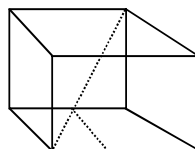
$$(\text{side})^2 = \frac{96}{6} = 16$$

$$\text{side} = \sqrt{16} = 4 \text{ cm}$$

$$\text{Volume of the cube} = (\text{side})^3$$

$$= (4)^3 = 64 \text{ cm}^3$$

28. (b)



$$\text{Diagonal} = 35\sqrt{3}$$

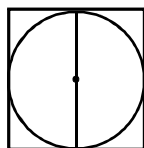
$\therefore$ The length of largest rod

$$= \text{Diagonal} = \text{side} \sqrt{3}$$

$$\text{Side} \sqrt{3} = 35\sqrt{3}$$

$$\text{side} = \frac{35\sqrt{3}}{\sqrt{3}} = 35$$

$$\text{side of cube} = 35$$



Diameter of the sphere

= side of the cube

$$= 2 \times \text{radius} = \text{side}$$

$$\text{radius} = \frac{35}{2} \text{ cm}$$

Surface area of the sphere

$$= 4\pi r^2$$

$$= 4 \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$$

$$= 3850 \text{ m}^2$$

29. (d) volume of air in room = 204 m^3

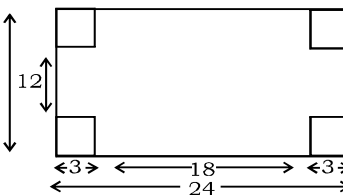
$$(\text{area of floor}) \times \text{height} = 204$$

$\therefore$ volume = area of base $\times$ height

$$(\text{area of floor}) \times 6 = 204$$

$$\text{area of floor} = \frac{204}{6} = 34 \text{ m}^2$$

30. (b)



The box will be of cuboid shape

$$\text{Length of the box, } l = 24 - (2 \times 3) = 24 - 6 = 18 \text{ cm}$$

breadth of the box, b

$$= 18 - (2 \times 3) = 18 - 6 = 12 \text{ cm}$$

Height of the box, $h = 3 \text{ cm}$

surface area of the box

$$= 2(l + b) \times h + l \times b$$

$$= 2(18 + 12) \times 3 + 18 \times 12$$

$$= 2 \times 30 \times 3 + 18 \times 12$$

$$= 180 + 216 = 396 \text{ cm}^2$$

31. (a) volume of all three cube

$$= 4^3 + 5^3 + 6^3$$

$$= 64 + 125 + 216 \text{ cm}^3$$

$$= 405 \text{ cm}^3$$

Now, 62 cm^3 is lost

$$\therefore \text{Volume of new cube} = 405 - 62 = 343$$

$$(\text{side of new cube})^3 = 343$$

$$\text{side of new cube} = \sqrt[3]{343} = 7$$

Total surface area of new cube = 6 (side)²

$$= 6 \times (7)^2 = 6 \times 49 = 294 \text{ cm}^2$$

32. (b) Area of cubical floor = 48

$$\text{Side}^2 = 48$$

$$\text{side} = \sqrt{48} = 4\sqrt{3}$$

Diagonal of cube = side $\sqrt{3}$

$$= 4\sqrt{3} \times \sqrt{3} = 12 \text{ m}$$

Length of longest rod = 12 m

33. (a) Let side of new cube = a

According to question,

$$a^3 = 6^3 + 8^3 + 1^3$$

$$= 216 + 512 + 1$$

$$= 729$$

$$a = \sqrt[3]{729} = 9$$

then surface area = 6(a)² = 6 $\times$ 9²

$$= 6 \times 81 = 486 \text{ cm}^2$$

34. (b) Volume = 20 m^3 = 20 $\times$ (100)³ cm^3

Volume of one brick = (25 $\times$ 12.5 $\times$ 8) cm^3

$\therefore$ Required number of bricks

$$= \frac{20 \times 100 \times 100 \times 100}{25 \times 12.5 \times 8} = 8000$$

35. (a) The total surface area of cube

$$6(\text{side})^2 = 150 \text{ cm}^2$$

$$6(\text{side})^2 = 150 \text{ cm}^2$$

$$(\text{side})^2 = \frac{150}{6} = 25$$

$$\text{side} = \sqrt{25} = 5 \text{ cm}$$

$\therefore$ volume of cube = (side)³

$$= (5)^3 = 125 \text{ cm}^3$$

36. (b) Let Ratio of length : breadth

$$= 5x : 3x$$

Total surface area of parallelopiped = 558 cm^2

$$2(lb + bh + hl) = 558$$

$$2(5x \times 3x + 3x \times 6 + 6 \times 5x) = 558$$

$$2(15x^2 + 18x + 30x) = 558$$

$$15x^2 + 48x = 279$$

$$15x^2 + 48x - 279 = 0$$

$$\text{On solving, } x = 3$$

$$\therefore \text{length} = 5 \times 3 = 15 \text{ cm} = \frac{15}{10}$$

$$= 1.5 \text{ dm}$$

37. (c) $l + b + h = 24 \text{ cm}$
Length of diagonal = 15 cm

$$\sqrt{l^2 + b^2 + h^2} = 15$$

$$l^2 + b^2 + h^2 = 225 \text{ cm}$$

$$(l + b + h)^2 - 2(lb + bh + hl) = 225$$

$$(24)^2 - 2(lb + bh + hl) = 225$$

$$576 - 225 = 2(lb + bh + hl)$$

$$351 = 2(lb + bh + hl)$$

$\therefore$ Total surface area = 351 cm²

38. (c) Let length = 3x,
breadth = 4x
height = 6x
 $3x \times 4x \times 6x = 576$

$$x^3 = \frac{576}{3 \times 4 \times 6} = 8$$

$$x = \sqrt[3]{8} = 2 \text{ cm}$$

$\therefore$ length = $3 \times 2 = 6 \text{ cm}$
breadth = $4 \times 2 = 8 \text{ cm}$,
height = $6 \times 2 = 12 \text{ cm}$
Total surface area = $2(lb + bh + hl)$
 $= 2(6 \times 8 + 8 \times 12 + 12 \times 6)$
 $= 2(48 + 96 + 72)$
 $= 2 \times 216 = \mathbf{432 \text{ cm}^2}$

39. (d) As we know that
A parallelopiped has vertices (v) = 8
edge (e) = 12
face (f) = 6
Put into equation (v - e + f)
 $\Rightarrow 8 - 12 + 6 \Rightarrow 2$

40. (b) According to the question.

$$1 \text{ dm} = \frac{1}{10} \text{ m}$$

Let depth of the hole = d

$$\therefore 48 \text{ m} \times 31.5 \times \frac{6.5}{10} \text{ m}$$

$$= 27 \times 18.2 \times d$$

$$d = 2 \text{ m}$$

41. (c)
 $2.1 \text{ m} \times 1.5 \text{ m} \times h = 630 \text{ lt}$
 $\frac{21}{10} \text{ m} \times \frac{15}{10} \text{ m} \times h = \frac{630}{1000} \text{ m}^3$

$$\left[\begin{array}{l} \therefore 1 \text{ m}^3 = 1000 \text{ lt} \\ 1000 \text{ cm}^3 = 1 \text{ lt} \end{array} \right]$$

$$h = \frac{1}{5} \text{ m} = 0.20 \text{ metre}$$

42. (d) Number of cubes = $\frac{8 \times 4 \times 2}{2 \times 2 \times 2}$
 $= 8$

43. (a) when we change shape of a solid figure, volume remains constant ,

$\therefore$ Volume of Hemisphere = Volume of cone

$$\frac{2}{3} \pi R^3 = \frac{1}{3} \pi R^2 h$$

$$\therefore 2R = h$$

44. (d) According to question

Let the radius of sphere = r cm

$$4\pi(r+2)^2 - 4\pi r^2 = 352$$

$$4\pi\{(r+2)^2 - r^2\} = 352$$

$$4\pi\{r^2 + 4 + 4r - r^2\} = 352$$

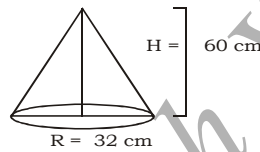
$$\pi(1+r) = \frac{352}{16} = 22$$

$$\frac{22}{7}(1+r) = 22$$

$$1+r = 7$$

$$r = 6 \text{ cm}$$

45(d)



We have to find the slant height
Take ratio of H and R

$$= \frac{60}{15} : \frac{32}{8}$$

$$L = \sqrt{15^2 + 8^2} = 17$$

$$= 17 \times 4 = 68 \text{ cm}$$

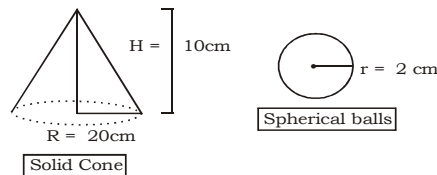
Cost of painting = Surface area of cone $\times 35$

$$= \pi R L \times 35$$

$$= \frac{22}{7} \times \frac{32 \times 68}{10000} \times 35$$

$$= \mathbf{Rs. 23.94 \text{ (approx)}}$$

46. (d)



Let the spherical balls made = 'x'
According to question,
Volume of cone = x $\times$ volume of sphere

$$\frac{1}{3} \pi R^2 H = x \times \frac{4}{3} \pi r^3$$

$$(20)^2 \times 10 = x \times 4 \times (2)^3$$

$$x = \mathbf{125}$$

47. (d) Radius of tank , r = $\frac{35}{2} \text{ cm}$

Let initial height = H

Final height = h

According to question ,

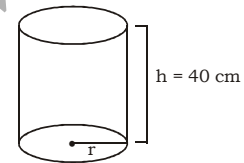
$$\pi \left(\frac{35}{2}\right)^2 \times H - \pi \left(\frac{35}{2}\right)^2 h = 11000 \text{ cm}^3$$

$$\pi \left(\frac{35}{2}\right)^2 \times (H - h) = 11000$$

$$H - h = \frac{11000 \times 2 \times 2 \times 7}{35 \times 35 \times 22}$$

$$= \frac{80}{7} = 11 \frac{3}{7} \text{ cm}$$

48. (d)



$\therefore$ circumference of its base = 66 cm

$$2\pi r = 66$$

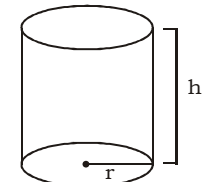
$$r = \frac{66}{2\pi} = \frac{66 \times 7}{2 \times 22} = \frac{21}{2} \text{ cm}$$

$\therefore$ volume = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 40$$

$$= \mathbf{13860 \text{ cm}^3}$$

49. (a) According to question,



$$2\pi r = 6\pi$$

$$r = 3 \text{ cm}$$

height of cylinder = diameter

$$= 2 \times r = 2 \times 3 = 6 \text{ cm}$$

volume of water

$$= \pi r^2 h = \pi (3)^2 \times 6$$

$$= \mathbf{54\pi \text{ cm}^3}$$

50. (b) Volume of the cone

$$= \frac{1}{3} \pi (15)^2 \times 108 \text{ cm}^3$$

Volume of the cylinder

$$= \pi \times r^2 \times 9 \text{ cm}^3$$

According to question,

$$\pi \times r^2 \times 9 = \frac{1}{3} \pi \times 15 \times 15 \times 108$$

$$r^2 = \frac{5 \times 15 \times 108}{9} = 900$$

$$r = \sqrt{900} = 30$$

$$\text{Diameter of base} = 2r$$

$$= 2 \times 30 = 60 \text{ cm}$$

51. (d) Volume of new solid sphere

$$= \frac{4}{3}\pi\left(\frac{6}{2}\right)^3 + \frac{4}{3}\pi\left(\frac{8}{2}\right)^3 + \frac{4}{3}\pi\left(\frac{10}{2}\right)^3$$

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left[(3)^3 + (4)^3 + (5)^3 \right]$$

$$r^3 = 216, \quad r = 6 \text{ cm}$$

$$\therefore \text{Diameter of the new sphere}$$

$$= 2 \times 6 = 12 \text{ cm}$$

52. (b) Total surface area of prism (regular hexagon)

$$= \text{Surface area} + (\text{base} + \text{top}) \text{ area}$$

$$156\sqrt{3} = \text{Perimeter of base} \times \text{height} + 2 \times \text{Base area}$$

$$156\sqrt{3} = 6 \times a \times 10 + 2 \times 6 \times \frac{\sqrt{3}}{4} \times a^2$$

$$156\sqrt{3} = 60a + 3\sqrt{3}a^2$$

$$\sqrt{3}a^2 + 20a = 52\sqrt{3}$$

$$a(\sqrt{3}a + 20) = 52\sqrt{3}$$

$$a = 2\sqrt{3}$$

$$\text{Volume of prism} = \text{Base area} \times \text{Height}$$

$$= 6 \times \frac{\sqrt{3}}{4} \times (2\sqrt{3})^2 \times 10$$

$$= 18\sqrt{3} \times 10 = 180\sqrt{3} \text{ cm}^3$$

53. (d) Volume of the new sphere

$$\Rightarrow \frac{4}{3}\pi[r_1^3 + r_2^3 + r_3^3] = \frac{4}{3}\pi R^3$$

$$R^3 = r_1^3 + r_2^3 + r_3^3$$

$$R^3 = 1^3 + 6^3 + 8^3$$

$$= 1 + 216 + 512 = 729$$

$$R = \sqrt[3]{729} = 9 \text{ cm}$$

54. (b) $l = 2.5 \text{ km}$

$$\text{area of base} = 1.54 \text{ km}^2$$

$$\pi r^2 = 1.54$$

$$r^2 = \frac{1.54 \times 7}{22}$$

$$r = \sqrt{\frac{1.54 \times 7}{22}} = 0.7 \text{ km}$$

We know that, $l^2 = r^2 + h^2$

$$h^2 = \sqrt{l^2 - r^2} = \sqrt{2.5^2 - 0.7^2}$$

$$= \sqrt{5.76} = 2.4 \text{ km}$$

55. (c) radius = $\frac{\text{diameter}}{2}$

$$= \frac{19.2}{2} = 9.6 \text{ m}$$

$$\text{height} = 2.8$$

$$l^2 = r^2 + h^2 = 9.6^2 + 2.8^2$$

$$= 92.16 + 7.84$$

$$= 100$$

$$l = \sqrt{100} = 10 \text{ m}$$

$$\text{area of the canvas} = \pi r l$$

$$= \frac{22}{7} \times 9.6 \times 10 = \mathbf{301.7}$$

56. (c) External radius $R = 4 \text{ cm}$

Internal radius $r = 3 \text{ cm}$

volume of iron used

$$= \pi R^2 h - \pi r^2 h$$

$$= \pi h (R^2 - r^2)$$

$$= \pi h (R + r) (R - r)$$

$$= \frac{22}{7} \times 20 \times (4 + 3) \times (4 - 3)$$

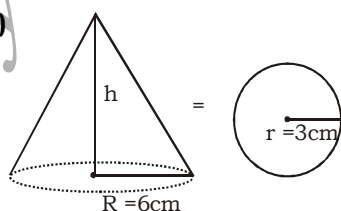
$$= \frac{22}{7} \times 20 \times 7 \times 1 = 440 \text{ cu. cm}$$

57. (c) Volume of sphere = Volume of displaced water

$$\frac{4}{3}\pi \times 2 \times 2 \times 2 = \pi \times 4 \times 4 \times h$$

$$h = \frac{2}{3} \text{ cm}$$

58. (d)



Volume of cone = volume of sphere

$$\frac{1}{3}\pi R^2 h = \frac{4}{3}\pi r^3$$

$$\frac{1}{3}\pi \times 6 \times 6 \times h = \frac{4}{3}\pi \times 3 \times 3 \times 3$$

$$h = 3 \text{ cm}$$

59. (b) Volume of a cone = $\frac{1}{3}\pi r^2 h$

$$\frac{1}{3}\pi r^2 \times 24 = 1232 \text{ cm}^2$$

$$r^2 = \frac{1232 \times 3 \times 7}{24 \times 22}$$

$$r^2 = 7 \times 7$$

$$r = \sqrt{7 \times 7} = 7 \text{ cm}$$

$$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{625}$$

$$= 25$$

$$\text{curved surface area} = \pi r l$$

$$= \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$$

60. (d) Volume of a sphere

$$= \frac{88}{21} \times (14)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (14)^3 \left\{ \frac{4}{3}\pi r^3 \right\}$$

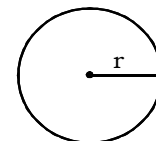
$$\text{Radius} = 14$$

$$\text{Curved surface area of sphere}$$

$$= 4\pi (\text{radius})^2$$

$$= 4 \times \frac{22}{7} \times 14 \times 14 = 2464 \text{ cm}^2$$

61. (b)



$$\text{surface area of sphere}$$

$$= 64\pi$$

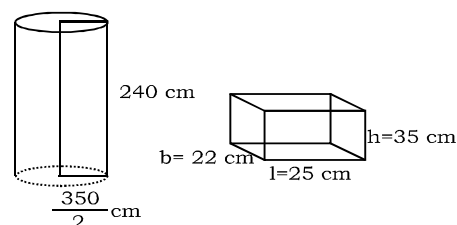
$$4\pi (\text{Radius})^2 = 64\pi$$

$$(\text{radius})^2 = \frac{64}{4} = 16$$

$$\text{radius} = \sqrt{16} = 4 \text{ cm}$$

$$\text{diameter} = 8 \text{ cm}$$

62. (a) 1 dm = 10 cm



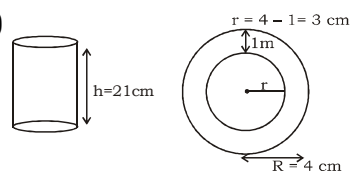
$x \times \text{volume of 1 tin} = \text{volume of cylinder}$

$$\Rightarrow x \times (25 \times 22 \times 35)$$

$$= \frac{22}{7} \times \frac{350}{2} \times \frac{350}{2} \times 240$$

$$x = 1200$$

63. (a)



Volume of hollow iron pipe

$$= \pi \{R^2 - r^2\} \times h$$

$$= \pi \{4^2 - 3^2\} \times 21$$

$$= \frac{22}{7} \times 7 \times 21 = 22 \times 21$$

$$= 462 \text{ cm}^3$$

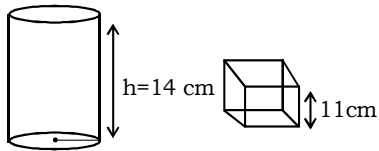
Now,

$$1 \text{ cm}^3 = 8 \text{ g}$$

$$462 \text{ cm}^3 = 8 \times 462 \text{ g}$$

$$= 3696 \text{ g} = 3.696 \text{ kg}$$

64. (b)



Volume of the cylinder = volume of cube

$$\pi r^2 h = (\text{side})^3$$

$$\frac{22}{7} \times r^2 \times 14 = 11 \times 11 \times 11$$

$$r^2 = \frac{11 \times 11 \times 11}{22 \times 2}$$

$$= \frac{121}{4}$$

$$r = \frac{11}{2} \text{ cm} = 5.5 \text{ cm}$$

65. (b) Let the radius = r

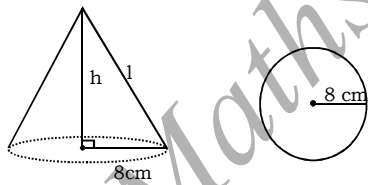
$$\pi r^2 h = 9 \pi h$$

$$r^2 = 9$$

$$r = \sqrt{9} = 3 \text{ m}$$

$$\text{diameter} = 3 \times 2 = 6 \text{ m}$$

66. (a)



volume of cone = volume of sphere

$$\frac{1}{3} \pi (8)^2 \times h = \frac{4}{3} \pi (8)^3$$

$$8 \times 8 \times h = 4 \times 8 \times 8 \times 8$$

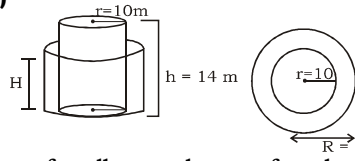
$$h = 32 \text{ cm}$$

$$\text{slant height} = (l) = \sqrt{r^2 + h^2}$$

$$= \sqrt{8^2 + 32^2}$$

$$= \sqrt{64 + 1024} = 8\sqrt{17}$$

67. (c)

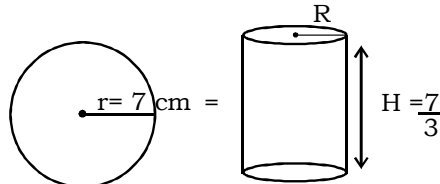


volume of well = volume of embankment

$$\pi (10)^2 \times 14 = \pi \{15^2 - 10^2\} \times H$$

$$H = \frac{100 \times 14}{125} = 11.2 \text{ m}$$

68. (b)



Volume of sphere = volume of cylinder

$$\frac{4}{3} \pi r^3 = \pi R^2 \times \frac{7}{3}$$

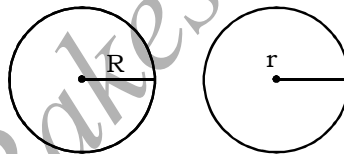
$$R^2 = 4 \times 7 \times 7 = 2 \times 2 \times 7 \times 7$$

$$R = \sqrt{2 \times 2 \times 7 \times 7} = 2 \times 7 = 14 \text{ cm}$$

diameter of base of cylinder = 2R

$$= 2 \times 14 = 28 \text{ cm}$$

69. (b)



ATQ,

$$R + r = 10$$

$$(R + r)^2 = 100$$

$$R^2 + r^2 + 2Rr = 100$$

$$R^2 + r^2 = 100 - 2Rr \quad \dots(i)$$

$$\frac{4}{3} \pi R^3 + \frac{4}{3} \pi r^3 = 880$$

$$\frac{4}{3} \pi R^3 + r^3 = 880$$

$$R^3 + r^3 = \frac{880 \times 3}{\pi \times 4} = \frac{880 \times 3 \times 7}{22 \times 4}$$

$$(R + r)(R^2 + r^2 - Rr) = 210$$

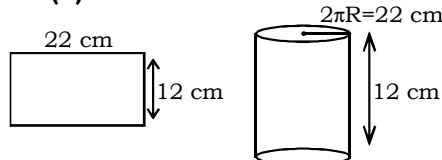
$$10 \times (100 - 2Rr - Rr) = 210$$

$$100 - 3Rr = 21$$

$$3Rr = 100 - 21 = 79$$

$$Rr = \frac{79}{3} = 26\frac{1}{3}$$

70. (b)



$\therefore$ Cylinder is folded along the length of rectangle

$$\therefore 2\pi R = 22$$

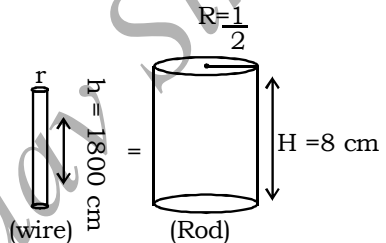
$$R = \frac{22}{2\pi} = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} \text{ cm}$$

Volume of the cylinder = $\pi R^2 H$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12$$

$$= 22 \times 7 \times 3 = 462 \text{ cm}^3$$

71. (b)



Volume of wire = volume of Rod

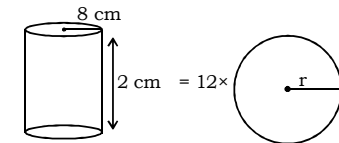
$$\pi r^2 h = \pi R^2 h$$

$$r^2 \times 1800 = \frac{1}{4} \times 8 = 2$$

$$r^2 = \frac{2}{1800} = \frac{1}{900}$$

$$r = \sqrt{\frac{1}{900}} = \frac{1}{30}$$

72. (b)



Volume of cylinder = 12 × volume of sphere

$$\pi (8)^2 \times 2 = 12 \times \frac{4}{3} \pi r^3$$

$$r^3 = \frac{8 \times 8 \times 2 \times 3}{12 \times 4}$$

$$r = \sqrt[3]{2 \times 2 \times 2} = 2 \text{ cm}$$

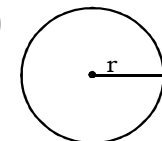
$$r = 2 \text{ cm}$$

$$d = 4 \text{ cm}$$

73. (c) $2\pi R - 2\pi r = 5$

$$(R - r) = \frac{5}{2\pi}$$

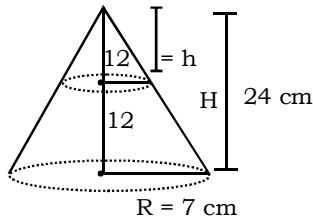
74. (c)



$$\frac{4}{3} \pi r^3 = 4 \pi r^2$$

radius (r) = 3 units

75. (b)



Volume of bigger cone

$$= \frac{1}{3} \times \pi \times (7)^2 \times 24$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$$

$$= 22 \times 7 \times 8 = 1232 \text{ cm}^3$$

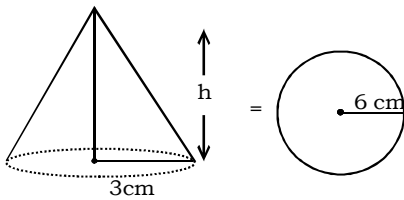
$$\frac{\text{volume of smaller cone}}{\text{volume of bigger cone}} = \frac{h^3}{H^3}$$

$$\frac{\text{Volume of smaller cone}}{1232} = \frac{12^3}{24^3}$$

volume of smaller cone = 154 cm³

∴ When the cone is cut in between then the ratio of volume of smaller cone to the bigger one is always equal to the ratio of the cubes of their heights

76. (b)



$$n = \frac{\text{Volume of sphere}}{\text{volume of cone}}$$

$$= \frac{\frac{4}{3} \pi 6^3}{\frac{1}{3} \pi 3^2 \times 4} = 24$$

77. (c) Height of cylinder = Breadth of tin foil
⇒ Circumference of the base of cylinder

$$= \text{Length of the foil} = 22 \text{ cm}$$

$$\Rightarrow 2\pi r = 22$$

$$r = \frac{22 \times 7}{22 \times 2} = \frac{7}{2} \text{ cm}$$

$$\text{Volume} = \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 16 = 616 \text{ cm}^3$$

78. (d) $\pi r^2 = 770$

$$\Rightarrow r^2 = \frac{770 \times 7}{22}$$

$$\Rightarrow r = 7\sqrt{5} \text{ cm}$$

$$\text{and } \pi r l = 814$$

$$\Rightarrow l = \frac{814 \times 7}{22 \times 7\sqrt{5}} = \frac{37}{\sqrt{5}}$$

$$l^2 = h^2 + r^2$$

$$\Rightarrow \frac{37 \times 37}{5} = h^2 + 245$$

$$\Rightarrow h^2 = \frac{1369}{5} - 245 = \frac{144}{5}$$

$$\Rightarrow h = \frac{12}{\sqrt{5}}$$

$$\text{volume} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7\sqrt{5} \times 7\sqrt{5} \times \frac{12}{\sqrt{5}}$$

$$= 616\sqrt{5} \text{ cm}^3$$

79. (b) In this case the breadth becomes the circumference of the base of the cylinder

$$\Rightarrow 2\pi r = 44$$

$$\Rightarrow r = \frac{44 \times 7}{22 \times 2} = 7 \text{ cm}$$

$$\text{New volume} = \pi r^2 h$$

$$= \frac{22}{7} \times 7 \times 7 \times 100$$

$$= 15400 \text{ cm}^3$$

$$80. (c) \pi r^2 H = \frac{1}{3} \pi r^2 h$$

$$\Rightarrow H = \frac{1}{3} h$$

$$\Rightarrow h = 3H = 3 \times 6 = 18 \text{ cm}$$

$$81. (c) 3\pi r^2 = 1848$$

$$r^2 = \frac{1848 \times 7}{3 \times 22} = 196$$

$$\Rightarrow r = 14 \text{ cm}$$

According to the question

$$\frac{2}{3} \pi r^3 = \frac{1}{3} \pi r^2 h$$

$$\Rightarrow 2r = h$$

$$\Rightarrow h = 2 \times 14 = 28 \text{ cm}$$

82. (b) The length of the paper becomes the circumference of the base of cylinder when it is rolled along its length

$$\Rightarrow 2\pi r = 12$$

$$\Rightarrow r = \frac{12}{2\pi} = \frac{6}{\pi} \text{ cm}$$

83. (b) Volume of tunnel = $\pi \times r^2 \times H$

$$= \frac{22}{7} \times \frac{4}{2} \times \frac{4}{2} \times 56 = 704 \text{ m}^3$$

$$\text{Volume of ditch} = 48 \times 16.5 \times 4 = 3168 \text{ m}^3$$

$$\text{Required part} = \frac{704}{3168} = \frac{2}{9}$$

84. (a) According to the question

$$\pi h(R^2 - r^2) = 748$$

$$\frac{R^2 - r^2}{22 \times 14} = \frac{748 \times 7}{22 \times 14}$$

$$\frac{9^2 - r^2}{2} = 17$$

$$\Rightarrow 9^2 - r^2 = 17$$

$$\Rightarrow r^2 = 81 - 17 = 64$$

$$\Rightarrow r = 8$$

$$\Rightarrow \text{Thickness} = 9 - 8 = 1 \text{ cm}$$

$$85. (b) 2 \times \left(\frac{4}{3} \times \pi \times r^3 \right) = \pi R^2 h$$

$$\Rightarrow 2 \times \frac{4}{3} \times \pi \times 27 = \pi \times 36 \times h$$

$$h = \frac{27 \times 4 \times 2}{36 \times 3}$$

$$\Rightarrow h = \frac{8 \times 27}{3 \times 36} = 2 \text{ cm}$$

86. (d)

$$\text{Ratio of height} = \sqrt[3]{\text{Ratio of volume}}$$

$$= \sqrt[3]{\frac{1}{27}}$$

$$\Rightarrow \frac{h}{H} = \frac{1}{3}$$

$$3 \text{ units} \rightarrow 30$$

$$2 \text{ units} \rightarrow 20$$

⇒ The cut is made 20 cm above the base

$$87. (b) 3\pi r^2 = 108\pi$$

$$\Rightarrow r^2 = 36$$

$$\Rightarrow r = 6 \text{ cm}$$

$$\text{Volume} = \frac{2}{3} \pi r^3 = \frac{2}{3} \times 216 \times \pi = 144\pi$$

88. (d) Radius = 3 Decimeters

$$= 30 \text{ cm}$$

$$\text{Height of circular sheet} = 1 \text{ mm} = 0.1 \text{ cm}$$

$$\Rightarrow \frac{4}{3} \pi \times (30)^3 = \pi r^2 \times \frac{1}{10}$$

$$\Rightarrow r = \sqrt{10000 \times 9 \times 4}$$

$$\Rightarrow r = 600 \text{ cm} = 6 \text{ meters}$$

89. (b) Let no. of seconds required to fill the tank = x

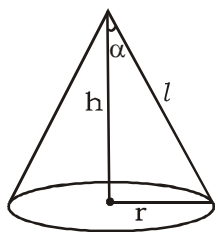
$$\Rightarrow (\pi r^2 h) \times x = 3 \times 5 \times 1.54$$

$$\Rightarrow x = \frac{3 \times 5 \times 1.54 \times 7 \times 100 \times 100}{22 \times 7 \times 7 \times 5}$$

$$= 300 \text{ seconds}$$

$$\Rightarrow \text{Time required} = 5 \text{ minutes}$$

90. (c)



$$\frac{r}{h} = \tan \alpha$$

$$\Rightarrow r = h \tan \alpha$$

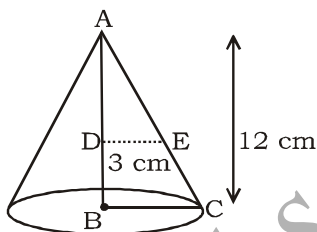
$$\text{and } \frac{l}{h} = \sec \alpha$$

$$\Rightarrow l = h \sec \alpha$$

$$\Rightarrow S = \pi \times h \tan \alpha \times h \sec \alpha$$

$$= \pi h^2 (\tan \alpha \times \sec \alpha)$$

91. (c)



As $DE \parallel BC$, $\triangle ADE \sim \triangle ABC$

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{12-3}{12} = \frac{DE}{6} \Rightarrow \frac{9 \times 6}{12} = DE$$

$$\Rightarrow DE = 4.5 \text{ cm}$$

92. (d) Height of cylinder = Diameter of sphere

$$\Rightarrow \frac{S_1}{S_2} = \frac{4\pi r^2}{2\pi r \times h} = \frac{4r^2}{4r^2} = \frac{1}{1}$$

$$\Rightarrow S_1 = S_2 \text{ (} h = 2r \text{)}$$

$$\frac{\pi r^2 h}{3}$$

$$\text{93. (d)} \quad \frac{4}{3} \pi r^3 = 1$$

$$\frac{h}{r} = \frac{4}{3}$$

$$\frac{h}{\frac{d}{2}} = \frac{4}{3}$$

$$h = \frac{4d}{6}, \quad 3h = 2d$$

94. (a) Volume of water pumped out in one hour

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 \times 3600$$

$$= 1663200 \text{ cm}^3$$

$$= 1663.2 \text{ ltr.}$$

95. (d) $2\pi rh = 1056$

$$r = \frac{1056 \times 7}{2 \times 22 \times 16} = \frac{21}{2}$$

$$\text{Volume} = \pi r^2 h = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 16 = 5544 \text{ cm}^3$$

96. (b) Let the radius and height be $5x$ and $12x$

$$\Rightarrow \frac{1}{3} \times \pi \times 25x^2 \times 12x = \frac{2200}{7}$$

$$\Rightarrow x^3 = \frac{2200 \times 7 \times 3}{7 \times 22 \times 25 \times 12} = 1$$

$$\Rightarrow x = 1$$

$$\Rightarrow \text{slant height} = \sqrt{5^2 + 12^2} = 13 \text{ cm}$$

$$\text{97. (b)} \quad \pi r^2 H = \frac{1}{3} \pi r^2 h$$

$$H = \frac{r^2 h}{r^2 \times 3} = \frac{h}{3}$$

$$\Rightarrow h = 3H = 3 \times 7 = 3 \times 7 = 21 \text{ cm}$$

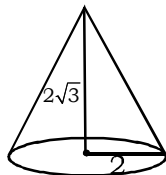
$$\text{98. (b)} \quad \pi \times r^2 \times H = \frac{4}{3} \pi r^3$$

$$\pi \times \frac{1}{10} \times \frac{1}{10} \times 36 \times 100 = \frac{4}{3} \pi r^3$$

$$r^3 = 27$$

$$r = 3 \text{ cm}$$

99. (b)



$$\text{Slant height} = \sqrt{(2\sqrt{3})^2 + 2^2}$$

$$= \sqrt{12+4} = 4 \text{ cm}$$

100. (b) Volume of vessel

= Volume of roof

$$\pi \times r^2 \times h = 22 \times 20 \times x$$

(where x is rainfall in cm)

$$\Rightarrow \frac{22}{7} \times \frac{100 \times 100 \times 350}{22 \times 20 \times 100 \times 100} = x$$

$$\Rightarrow x = 2.5 \text{ cm}$$

101. (a) Volume of remaining solid = Volume of cylinder - Volume of cone

$$\pi r^2 h - \frac{1}{3} \pi r^2 h = \frac{2}{3} \pi r^2 h$$

$$\frac{2}{3} \times \frac{22}{7} \times 36 \times 10 = 240 \pi \text{ cm}^3$$

102. (c) Let the height be H

$$\Rightarrow \frac{1}{3} \pi r_1^2 H + \frac{1}{3} \pi r_2^2 H$$

$$= \frac{4}{3} \pi R^3$$

$$\Rightarrow \frac{1}{3} \pi H (r_1^2 + r_2^2) = \frac{4}{3} \pi R^3$$

$$\Rightarrow H = \frac{4R^3}{r_1^2 + r_2^2}$$

103. (c) Let height and diameter be $3x$ and $2x$

$$\Rightarrow \frac{1}{3} \pi x^2 \times 3x = 1078$$

$$\Rightarrow x^3 = \frac{1078 \times 7}{22} = 49 \times 7$$

$$\Rightarrow x = 7$$

$$\Rightarrow \text{height} = 7 \times 3 = 21 \text{ cm}$$

104. (c) Radius of cylinder $r = 10 \text{ cm}$

height of cylinder $h = 21 \text{ cm}$

$$\text{volume of cylinder} = \pi r^2 h$$

radius of cone $\Rightarrow$ radius of cylinder = 10 cm

Let height of cone = h_1

$$\therefore \text{volume of cone} = \frac{1}{3} \pi r^2 h_1$$

$$\therefore \text{volume of remaining portion}$$

$$\Rightarrow 4400 \text{ cm}^3 \text{ (given)}$$

(after removing cone)

$$\pi r^2 h - \frac{1}{3} \pi r^2 h_1 = 4400$$

$$\pi r^2 \left(h - \frac{h_1}{3} \right) = 4400$$

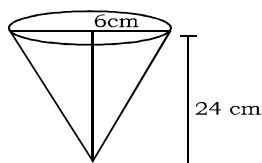
$$\frac{22}{7} \times 10 \times 10 \left(21 - \frac{h_1}{3} \right) = 4400$$

$$\Rightarrow 21 - \frac{h_1}{3} = 14$$

$$21 - 14 = \frac{h_1}{3}$$

$$h_1 = 21$$

105. (a)



radius of cone = 6 cm
height of cone = 24 cm

$$\therefore \text{volume of cone} = \frac{1}{3} \pi 6^2 \times 24 \text{ cm}^3$$

cone is converted to sphere
Let radius of sphere = r

$$\therefore \text{Volume of sphere} \Rightarrow \frac{4}{3} \pi r^3$$

volume of sphere = volume of cone

$$\therefore \frac{4}{3} \pi r^3 = \frac{1}{3} \times \pi \times 6 \times 6 \times 24$$

$$\Rightarrow r^3 \Rightarrow \frac{1}{3} \times \frac{6 \times 6 \times 24}{4} \times 3$$

$$\Rightarrow r^3 = 3 \times 3 \times 24$$

$$= 3 \times 3 \times 3 \times 8$$

$$r^3 = (3)^3 \times (2)^3$$

$$r = 3 \times 2 = 6 \text{ cm}$$

$\therefore$ radius of sphere = 6 cm

106. (a) total surface area of cylinder

$$\Rightarrow 462 \text{ (given)}$$

$$\Rightarrow (2\pi rh + 2\pi r^2) = 462 \text{ cm}^2$$

r = radius, h = height

$$2\pi r^2 = 462 - 154 = 308$$

$$\pi r^2 = 154$$

$$r^2 = \frac{154}{22} \times 7 = 49$$

$$r = 7 \text{ cm}$$

107. (a) Diameter of cylinder = 7 cm

$$\text{radius} = \frac{7}{2} \text{ cm, height} = 16 \text{ cm}$$

$\therefore$ lateral or curved surface area

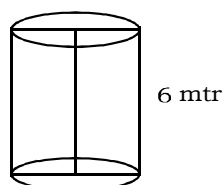
$$\Rightarrow 2\pi rh$$

$$\Rightarrow r = \text{radius}$$

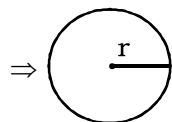
$$h = \text{height}$$

$$\therefore 2 \times \frac{22}{7} \times \frac{7}{2} \times 16 \Rightarrow 352 \text{ cm}^2$$

108. (a)



height of cylinder h = 6 mtr
Let radius of cylinder = r mtr
 $\therefore$ curved surface area = $2\pi rh$
area of end face = πr^2
 $\Rightarrow$ total area of two end faces = $2\pi r^2$



$\Rightarrow$ given that

$$3 \times 2\pi r^2 = 2 \times 2\pi rh$$

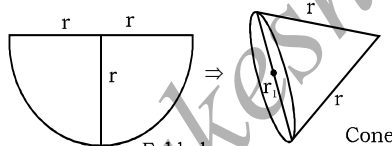
$$3r = 2h$$

$$3 \times r = 2h$$

$$r = 4 \text{ mtr}$$

$$\therefore \text{radius of base} = 4 \text{ mtr}$$

109. (b)



radius of semi-circular sheet

$$= r \Rightarrow \frac{28}{2}$$

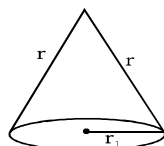
$$r = 14 \text{ cm}$$

$$\text{Circumference of sheet} = \pi r$$

$$= 14\pi \text{ cm}$$

Sheet is folded to form a cone

$$\text{Let radius of cone} = r_1$$



$\therefore$ The circumference of base of cone = circumference of sheet

$$\therefore 2\pi r_1 = 14\pi$$

$$r_1 = 7 \text{ cm}$$

$\therefore$ radius of cone = 7 cm

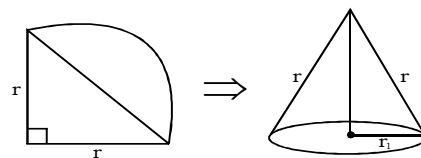
slant height = radius of semi-circular sheet

$$l = 14 \text{ cm}$$

$$\therefore \text{height} = \sqrt{14^2 - 7^2}$$

$$= \sqrt{147} = 12 \text{ cm (approx)}$$

110. (b)



$$\Rightarrow \text{Circumference of sectors} = \frac{\pi r}{2}$$

$$\Rightarrow \text{Circumference of base of cone of radius } r_1 = 2\pi r_1$$

$$\frac{\pi r}{2} = 2\pi r_1$$

$$r_1 = \frac{r}{4}$$

$$\therefore \text{radius of cone} = \frac{r}{4}$$

$$\text{curved surface area of cone} = \pi r_1 l$$

$$l = \text{slant height}$$

$$l = r$$

$$\therefore \text{surface area of cone} = \pi \times \frac{r}{4} \times r$$

$$\Rightarrow \frac{\pi r^2}{4}$$

111. (d) radius of cone = r = 16 meter (given)

Let slant height = l meter

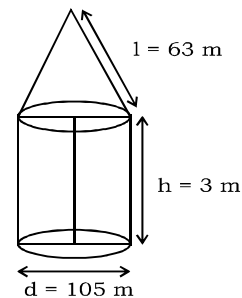
$$\text{curved surface area} = \pi r l$$

$$= 427 \frac{3}{7} \text{ m}^2 \text{ (given)}$$

$$\Rightarrow \frac{22}{7} \times 16 \times l = \frac{2992}{7}$$

$$l = \frac{2992}{22 \times 16} = 8.5 \text{ meter}$$

112. (a)



$$\therefore \text{radius of cone} = \frac{105}{2} \text{ m}$$

$$\text{slant height of cone} = 63 \text{ m}$$

$$\Rightarrow \text{curved surface area of cone} = (\pi r l)$$

$$= \frac{22}{7} \times \frac{105}{2} \times 63$$

$$= 10395 \text{ m}^2$$

$$\Rightarrow \text{radius of cylinder} = \frac{105}{2} \text{ m}$$

$$\text{height} = 3 \text{ m (given)}$$

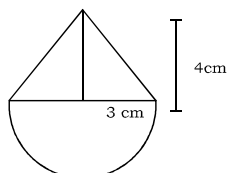
$$\therefore \text{curved surface area of cylinder} = 2\pi rh$$

$$= 2 \times \frac{22}{7} \times \frac{105}{2} \times 3 = 990 \text{ m}^2$$

$$\therefore \text{Total curved area of structure} \\ \Rightarrow \text{curved area of cone} + \text{curved area of cylinder} = 10395 + 990 = 11385 \text{ m}^2$$

$$\therefore \text{Total area of canvas} \\ = 11385 \text{ m}^2$$

113. (b)



Surface area of hemisphere

$$= 2\pi r^2 = 2 \times \frac{22}{7} \times 9 = 56.57 \text{ cm}^2$$

$$\begin{aligned} \text{height of cone} &= 4 \text{ cm} \\ \text{radius} &= 3 \text{ cm} \end{aligned}$$

$$\therefore \text{slant height} = \sqrt{16+9} = 5 \text{ cm}$$

$$\therefore \text{surface area of cone} = \pi rl$$

$$= \frac{22}{7} \times 3 \times 5$$

$$\Rightarrow 47.14 \text{ cm}^2$$

$$\therefore \text{total surface area of the toy area of cone} + \text{area of hemisphere} \\ \Rightarrow 47.14 + 56.57 \Rightarrow 103.71 \text{ cm}^2$$

114. (b) diameter of beaker = 7 cm

$$\text{radius} = \frac{7}{2} \text{ cm}$$

$$\text{level of water rises} = 5.6 \text{ cm}$$

$$\text{diameter of a marble} = 1.4 \text{ cm}$$

$$\therefore \text{radius} = \frac{1.4}{2} = 0.7 \text{ cm}$$

Let n marbles are dropped so, Volume of n marbles

$$= n \times \frac{4}{3} \pi \times (0.7)^3$$

$$\Rightarrow n \times \frac{4}{3} \pi \times (0.7)^3 = \pi \times \frac{7}{2} \times \frac{7}{2} \times 5.6$$

$$\Rightarrow n \times \frac{4}{3} \times \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$$

$$= \frac{7 \times 7 \times 56}{2 \times 2 \times 10}$$

$$\Rightarrow n = 150$$

115. (d) Let radius of iron rod = r

$$\therefore \text{height} = 8r$$

$$\therefore \text{volume of iron rod}$$

$$= \pi \times (r)^2 \times 8r \Rightarrow 8\pi r^3$$

$$\Rightarrow \text{radius of spherical ball} = \frac{r}{2}$$

volume of spherical ball

$$= \frac{4}{3} \pi \times \left(\frac{r}{2}\right)^3$$

Let n balls are cast

$$\therefore n \times \frac{4}{3} \pi \left(\frac{r^3}{8}\right) = 8\pi r^3$$

$$\Rightarrow \frac{n}{6} \Rightarrow 8 \Rightarrow n = 48$$

116. (c) Let the radius of base of second cylinder = R

$$\Rightarrow 2(\pi r^2 h) = \pi R^2 h$$

$$\Rightarrow 2r^2 = R^2$$

$$\Rightarrow R = r\sqrt{2}$$

117. (c) Volume of remaining solid = Volume of cylinder - Volume of cone

$$\pi r^2 h - \frac{1}{3} \pi r^2 h = \frac{2}{3} \pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times 5 \times 5 \times 12 \\ = 628.57 \text{ cm}^3$$

118. (a) Let the required increase = x cm

$$\Rightarrow \pi (10+x)^2 \times 4 = \pi \times 10^2 \times (4+x)$$

$$100 + x^2 + 20x = 25(4+x)$$

$$x^2 + 20x + 100 = 100 + 25x$$

$$x^2 - 5x = 0$$

$$x - 5 = 0$$

$$x = 5$$

$$\therefore \text{Required increase} = 5 \text{ cm}$$

119. (b) Let the old volume = $\frac{1}{3} \pi r^2 h$

$$\Rightarrow \text{New volume} = \frac{1}{3} \pi (2r)^2 h$$

$$= \frac{4}{3} \pi r^2 h$$

$\Rightarrow$ New volume is four times the old volume

120. (b) Let the height of cone be ' h ' cm

$$\frac{1}{3} \pi r^2 h = \frac{4}{3} \pi r^3$$

$$a^2 h = 4a^3$$

$$h = 4a$$

121. (c) Radius of base = $\frac{33}{2\pi}$

$$= \frac{33 \times 7}{2 \times 22} = \frac{21}{4} \text{ cm}$$

$$\therefore \text{Volume of cone} = \frac{1}{3} \times \pi \times r^2 \times h$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{21}{4} \times \frac{21}{4} \times 16 \\ = 462 \text{ cm}^3$$

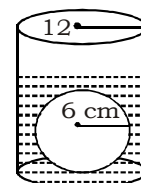
122. (a) Let the radius of small spheres be r cm

$$\Rightarrow \left(\frac{4}{3} \pi r^3\right) \times 8 = \frac{4}{3} \pi \times (3)^3$$

$$\Rightarrow 8r^3 = 3^3$$

$$\Rightarrow r = \frac{3}{2} = 1.5 \text{ cm}$$

123. (b)



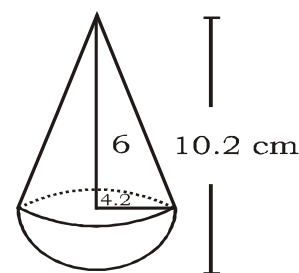
Let the increase in height = h cm

$$\Rightarrow \pi R^2 h = \frac{4}{3} \pi r^3$$

$$(12)^2 \times h = \frac{4}{3} \times 6^3$$

$$h = \frac{4}{3} \times \frac{216}{144} = 2 \text{ cm}$$

124. (c) Height of the cone = $10.2 - 4.2 = 6 \text{ cm}$



$\Rightarrow$ Volume of the toy

$$= \frac{1}{3} \pi r^2 h + \frac{4}{3} \pi r^3$$

$$= \frac{1}{3} \pi r^2 (h + 4r)$$

$$= \frac{1}{3} \times \frac{22}{7} \times (4.2)^2 (4 \times 4.2 + 6)$$

$$= \frac{1}{3} \times \frac{22}{7} \times (4.2)^2 \times 22.8$$

$$= 421 \text{ cm}^3 \text{ (approx.)}$$

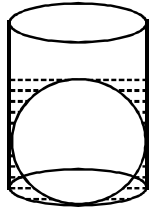
125. (c) Volume of water
= Volume of cylinder – volume of cone

$$= \pi r^2 h - \frac{1}{3} \pi r^2 h = \frac{2}{3} \pi r^2 h$$

$$= 2 \left(\frac{1}{3} \pi r^2 h \right)$$

$$= 2 \times 27 \pi = 54 \pi \text{ cm}^3$$

126. (c)



Height of water after ball is immersed = $3.5 \times 2 = 7 \text{ cm}$

$$\Rightarrow \text{Volume of water} = \pi r^2 h - \frac{4}{3} \pi r^3$$

$$\Rightarrow \pi r^2 \left(h - \frac{4}{3} r \right)$$

$$\Rightarrow \frac{22}{7} \times 3.5 \times 3.5 \left(7 - \frac{4}{3} \times 3.5 \right) =$$

Volume of water before ball was immersed

$$\frac{22}{7} \times 3.5 \times 3.5 \left(7 - \frac{4}{3} \times 3.5 \right)$$

$$= \pi (3.5)^2 \times h$$

$$= h = \frac{7}{3} \text{ cm}$$

127. (c) Let height and radius be
= $7x$ and $5x$ respectively

$$\Rightarrow \pi r^2 h = 550$$

$$\pi (5x)^2 \times 7x = 550$$

$$\frac{22}{7} \times 25x^2 \times 7x = 550$$

$$x^3 = 1$$

$$x = 1$$

$$\therefore \text{height} = 7 \text{ cm}$$

$$\text{radius} = 5 \text{ cm}$$

$$\Rightarrow \text{Curved surface area} = 2 \pi r h$$

$$= 2 \times \frac{22}{7} \times 5 \times 7 = 220 \text{ cm}^2$$

128. (c) Let the height of the cylinder be ' h ' cm and the radius be r cm

$$\Rightarrow \pi r^2 = b$$

$$\Rightarrow r = \sqrt{\frac{b}{\pi}}$$

$$\text{also, } 2 \pi r h = a$$

$$2 \pi \sqrt{\frac{b}{\pi}} \times h = a$$

$$h = \frac{a}{2 \sqrt{\pi b}} \text{ cm}$$

$$\mathbf{129. (a)} \quad \frac{2}{3} \pi r^3 = 19404$$

$$r^3 = \frac{19404 \times 7 \times 3}{22 \times 2}$$

$$\Rightarrow r = 21 \text{ cm}$$

$$\Rightarrow \text{Total surface area} = 3 \pi r^2$$

$$= 3 \times \frac{22}{7} \times 21 \times 21 = 4158 \text{ cm}^2$$

$$\mathbf{130. (c)} \quad \text{Curved surface area} = 2 \pi r^2$$

$$= 2 \times \frac{22}{7} \times 11 \times 11 = 760.57 \text{ cm}^2$$

$$\mathbf{131. (a)} \quad \text{Slant height of the cone } (l) = \sqrt{6^2 + 8^2} = 10 \text{ cm}$$

$$\Rightarrow \text{Required ratio} = \frac{2 \pi r h}{\pi r l} = \frac{2h}{l}$$

$$= \frac{2 \times 8}{10} = 8 : 5$$

132. (c) The volume of cone having same height & diameter as that of a cylinder

$$n \times \frac{1}{3} \pi r^2 h = \pi r^2 h$$

$$= \frac{1}{3} \times \text{volume of cylinder}$$

$$\text{No. of cones required} = 3$$

133. (a) Let the no. of small balls = x

$$\Rightarrow \frac{4}{3} \pi \times (10)^3 = x \times \frac{4}{3} \pi \times \left(\frac{1}{2} \right)^3$$

$$\Rightarrow 1000 = x \times \frac{1}{8}$$

$$\Rightarrow x = 8000$$

134. (a) Let the no. of balls = x

$$\Rightarrow 44 \times 44 \times 44 = x \times \frac{4}{3} \pi \times \left(\frac{4}{2} \right)^3$$

$$\Rightarrow \frac{44 \times 44 \times 44 \times 7 \times 3}{22 \times 4 \times 8} = x$$

$$\Rightarrow x = 2541$$

135. (d) Let the no. of cones = x

$$\Rightarrow \pi \times 3^2 \times 5 = x \times \frac{1}{3} \pi \times \left(\frac{1}{10} \right)^2 \times 1$$

$$\Rightarrow x = 9 \times 5 \times 3 \times 100 = 13500$$

136. (c) Slant height of cone

$$= \sqrt{8^2 + 6^2} = 10 \text{ cm}$$

Slant height of cone = radius of sector
= 10 cm

137. (d) Volume of sphere

$$= \frac{4}{3} \pi r^3 = \frac{4}{3} \times \pi \times (9)^3$$

$$= 972 \pi \text{ cm}^3$$

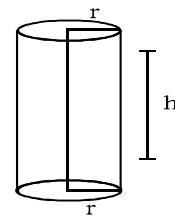
$$\text{volume of cone} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \times 9^2 \times 9$$

$$= 243 \pi \text{ cm}^3$$

$\Rightarrow$ % of wasted wood

$$= \frac{(972 - 243) \pi}{972 \pi} \times 100 = 75\%$$

138. (d)



$$2 \pi r = a, r = \frac{a}{2 \pi}$$

Volume of cylinder = V

$$\pi r^2 h = V$$

$$\pi \left(\frac{a}{2 \pi} \right)^2 \times h = V$$

$$4 \pi \frac{a^2}{4 \pi^2} = V$$

$$h = \frac{V \times 4 \pi}{a^2} = \frac{4 \pi V}{a^2}$$

139. (d) Radius of sphere = $\frac{12}{2} = 6 \text{ cm}$

Let the height of the cylinder = h

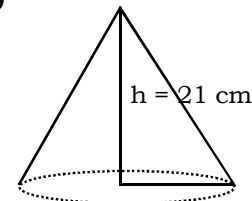
According to questions,

Volume and radius are same

$$\pi (6)^2 \times h = \frac{4}{3} \pi (6)^3$$

$$h = \frac{4 \times 6}{3} = 8 \text{ cm}$$

140. (b)



$$\text{Perimeter of base} = 8 \text{ cm}$$

$$2 \pi r = 8$$

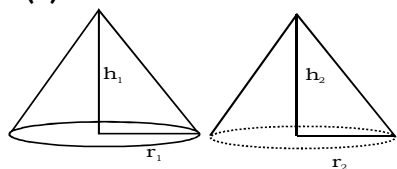
$$r = \frac{8}{2\pi}$$

$$h = 21 \text{ cm}$$

Volume of cone

$$= \frac{1}{3} \times \pi \times \frac{4}{\pi} \times \frac{4}{\pi} \times 21 = \frac{112}{\pi} \text{ cm}^3$$

141. (c)



$$\frac{V_1}{V_2} = \frac{\frac{1}{3} \pi r_1^2 h_1}{\frac{1}{3} \pi r_2^2 h_2} = \frac{r_1^2 h_1}{r_2^2 h_2} = \frac{4}{1}$$

$$\left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right) = \frac{4}{1}$$

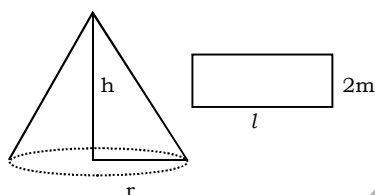
$$\therefore \frac{2r_1}{2r_2} = \frac{5}{4} \therefore \frac{r_1}{r_2} = \frac{5}{4}$$

$$\left(\frac{5}{4}\right)^2 \times \frac{h_1}{h_2} = \frac{4}{1}$$

$$\frac{25}{16} \times \frac{h_1}{h_2} = \frac{4}{1}$$

$$\frac{h_1}{h_2} = \frac{64}{25}$$

142. (d)



$$\pi r^2 = 154$$

$$r^2 = \frac{154 \times 7}{22} = 49$$

$$r = \sqrt{49} = 7 \text{ m}$$

also, volume = 1232

$$\frac{1}{3} \pi r^2 \times h = 1232$$

$$h = \frac{1232 \times 3}{\pi r^2} = \frac{1232 \times 3}{154} = 8$$

$$h = 8 \text{ m}$$

Area of canvas required = $\pi r l$

$$= \pi r \sqrt{r^2 + h^2}$$

$$= \frac{22}{7} \times 7 \times \sqrt{24^2 + 7^2}$$

$$= \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$$

$$\text{length} \times 2 = 550 \text{ m}^2$$

$$\text{length}(l) = \frac{550}{2} = 275 \text{ m}$$

143. (b) Ratio of the volume of cones

$$= \frac{\frac{1}{3} \pi r_1^2 h}{\frac{1}{3} \pi r_2^2 h} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$= 9 : 16$$

144. (c) Ratio of surface area of sphere

$$\frac{4\pi r_1^2}{4\pi r_2^2} = \frac{4}{9}$$

$$\left(\frac{r_1}{r_2}\right)^2 = \frac{4}{9}$$

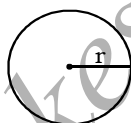
$$\frac{r_1}{r_2} = \frac{2}{3}$$

Ratio of their volume

$$= \frac{\frac{4}{3} \pi r_1^3}{\frac{4}{3} \pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

$$= 8 : 27$$

145. (a)



Total surface area of sphere

$$= 8\pi \text{ squares}$$

$$4\pi r^2 = 8\pi$$

$$r^2 = 2$$

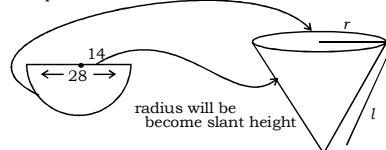
$$r = \sqrt{2} \text{ units}$$

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \pi \times (\sqrt{2})^3 = \frac{8\sqrt{2}\pi}{3} \text{ units}$$

146. (b)

This part becomes the circumference of cone



In this question just cut the semicircular paper and told it to form cone

Circumference of base of cone = $2\pi r$

$$\therefore \frac{2\pi r}{2} = \text{circumference of semi circular Sheet}$$

$$2\pi r = \pi \times 14$$

$$r = 7 \text{ cm}$$

Slant = radius of height, (l) of semicircular plate

$$l = 14 \text{ cm}$$

$$h^2 = l^2 - r^2$$

$$= 14^2 - 7^2$$

$$= 196 - 49$$

$$= 147$$

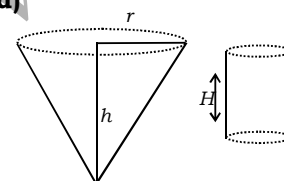
$$h = \sqrt{147} = 7\sqrt{3}$$

$$\text{Volume of cone} = \frac{1}{3} \times \frac{22}{7} \times 7$$

$$\times 7 \times 7\sqrt{3}$$

$$= 622.36 \text{ cm}^3$$

147. (d)



Volume of water in conical flask

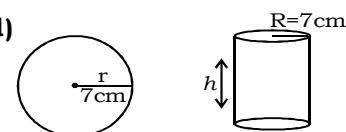
$$= \frac{1}{3} \pi r^2 h$$

If the height of water level in cylindrical flask = H units

$$\therefore \pi m^2 H = \frac{1}{3} \pi r^2 h$$

$$H = \frac{1}{3} \times \frac{\pi r^2 h}{\pi m^2} = \frac{hr^2}{3m^2}$$

148. (d)



$$\text{volume of the solid sphere} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \pi \times 7 \times 7 \times 7 \text{ cm}^3$$

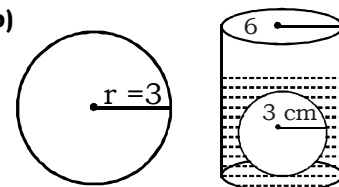
Let the length of wire = h cm

$$\pi R^2 h = \frac{4}{3} \pi \times 7 \times 7 \times 7$$

$$7 \times 7 \times h = \frac{4}{3} \times 7 \times 7 \times 7$$

$$h = \frac{28}{3} \text{ cm}$$

149. (b)

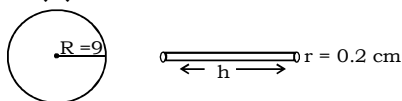


$$\begin{aligned}\text{Volume of sphere} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \pi \times 3 \times 3 \times 3 = 36 \pi \text{ cm}^3\end{aligned}$$

If the water level rises by H cm

$$\begin{aligned}\pi R^2 H &= 36 \pi \\ 6 \times 6 \times h &= 36 \\ h &= 1 \text{ cm}\end{aligned}$$

150.(b)



$$\begin{aligned}\text{Volume of sphere} &= \frac{4}{3} \pi R^3 \\ &= 972 \pi \text{ cm}^3 \\ \text{Let the length of wire} &= h \text{ cm} \\ \pi (0.2)^2 \times h &= 972 \pi \\ h &= \frac{972}{0.2 \times 0.2} = 24300 \text{ cm} \\ &= \mathbf{243 \text{ metres}}\end{aligned}$$

151. (a) Volume of sphere = volume of rectangular block

$$\begin{aligned}\frac{4}{3} \pi (\text{radius})^3 \\ &= \text{length} \times \text{breadth} \times \text{height}\end{aligned}$$

$$\frac{4}{3} \pi (\text{radius})^3 = 21 \times 77 \times 24$$

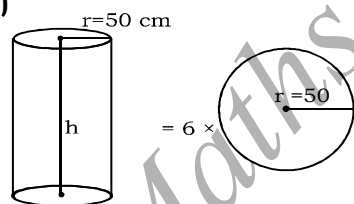
$$(\text{radius})^3 = \frac{21 \times 77 \times 24 \times 3 \times 7}{4 \times 22}$$

$$(\text{radius}) =$$

$$\sqrt[3]{7 \times 7 \times 7 \times 3 \times 3 \times 3}$$

$$\text{radius} = 7 \times 3 = 21 \text{ cm}$$

152. (d)



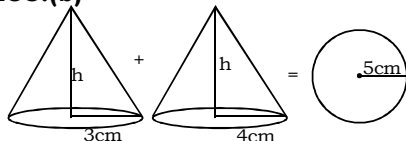
$$\begin{aligned}\text{Volume of cylinder} \\ &= 6 \times \text{volume of a sphere}\end{aligned}$$

$$\pi 50^2 h = 6 \times \frac{4}{3} \pi (50)^3$$

$$h = 6 \times \frac{4}{3} \times 50$$

$$= 400 \text{ cm} = 4 \text{ m}$$

153.(b)



volume of both the cones will be equal to the volume of sphere

$$\frac{1}{3} \pi (3)^2 h + \frac{1}{3} \pi (4)^2 h = \frac{4}{3} \pi (5)^3$$

$$\frac{1}{3} h (3^2 + 4^2) = \frac{4}{3} \times 5 \times 5 \times 5$$

$$\frac{1}{3} \times h \times 25 = \frac{4}{3} \times 5 \times 5 \times 5$$

$$h = \frac{20}{3} \times 3 = 20 \text{ cm}$$

154. (a) Volume of cone = $\frac{1}{3} \pi r^2 h$

$$\text{Now, } r_1 = 2r, h_1 = 2h$$

$\therefore$ Volume of second cone

$$= \frac{1}{3} \pi r_1^2 h_1$$

$$= \frac{1}{3} \pi (2r)^2 2h = \frac{1}{3} \pi r^2 h \times 8$$

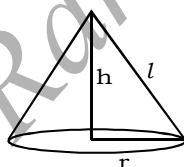
= 8 times of the previous volume

Alternate:

In the formula of volume of cone, there is power 2 on radius and power 1 on height

$$\therefore (2)^2 \times 2 = 8 \text{ times}$$

155. (d)



$$C = \pi r l$$

$$C^2 = \pi^2 r^2 l^2$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V^2 = \frac{1}{9} \pi^2 r^4 h^2$$

$$3 \pi v h^3 - c^2 h^2 + 9 v^2$$

$$\begin{aligned}3 \pi \times \frac{1}{3} \pi r^2 h \times h^3 - \pi^2 r^2 l^2 h^2 + 9 \times \frac{1}{9} \pi^2 r^4 h^2 \\ = \pi^2 r^2 h^4 - \pi^2 r^2 h^2 (r^2 + h^2) + \pi^2 r^4 h^2 \\ = \pi^2 r^2 h^4 - \pi^2 r^4 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2 \\ = 0\end{aligned}$$

156. (a) volume of rectangular block = $11 \times 10 \times 5 = 550 \text{ m}^3$

$$= 550000 \text{ dm}^3 \text{ (1 m = 10 dm)}$$

Volume of a sphere

$$= \frac{4}{3} \pi \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \text{ dm}^3$$

$$= \frac{500}{8} \text{ dm}^3$$

According to question,

$$n \times \frac{500}{8} = 550000$$

$$n = \frac{550000 \times 8}{500} = \mathbf{8800}$$

157. (a) Required number of spheres

$$= \frac{\text{volume of metallic cone}}{\text{volume of a sphere}}$$

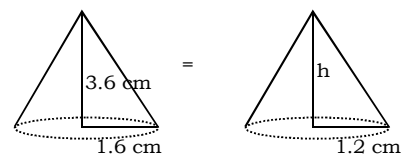
$$= \frac{\frac{1}{3} \pi \times 30 \times 30 \times 45}{\frac{4}{3} \pi \times 5 \times 5 \times 5} = 81$$

158. (d) Number of cones

$$= \frac{\text{Volume of sphere}}{\text{Volume of cone}}$$

$$\begin{aligned}&= \frac{\frac{4}{3} \pi (10.5)^3}{\frac{1}{3} \pi (3.5)^2 \times 3} \\ &= \frac{4 \times 10.5 \times 10.5 \times 10.5}{3.5 \times 3.5 \times 3} = \mathbf{126}\end{aligned}$$

159.(c)



According to question

$$\frac{1}{3} \times \pi \times 1.6 \times 1.6 \times 3.6$$

$$= \frac{1}{3} \times \pi \times 1.2 \times 1.2 \times h$$

$$h = \frac{1.6 \times 1.6 \times 3.6}{1.2 \times 1.2} = \frac{16 \times 16 \times 36}{12 \times 12 \times 10}$$

$$= \frac{64}{10} = \mathbf{6.4 \text{ cm}}$$

$$160.(a) \frac{S^3}{V^2} = \frac{(4 \pi r^2)^3}{\left(\frac{4}{3} \pi r^3\right)^2} = \frac{4^3 \times \pi^3 \times r^6}{4^2 \times \pi^2 \times r^6} \times 3^2$$

$$= 4 \times \pi \times 9 = \frac{36 \pi}{1} = 36 \pi \text{ units}$$

161. (d) Radius of sphere = $\frac{1}{20} \text{ cm}$

volume of a sphere

$$= \frac{4}{3} \pi \times \frac{1}{20} \times \frac{1}{20} \times \frac{1}{20}$$

Let the radius of cone = R
height = 2R

According to question,

$$\begin{aligned}\frac{1}{3}\pi \times R \times R \times 2R \\&= \frac{4}{3}\pi \times \frac{1}{20} \times \frac{1}{20} \times \frac{1}{20} \times 32000 \\R^3 &= \frac{2 \times 32000}{20 \times 20 \times 20} = \frac{64000}{20 \times 20 \times 20} \\R^3 &= \frac{40 \times 40 \times 40}{20 \times 20 \times 20} \\R &= \frac{40}{20} = 2\end{aligned}$$

height of glass = $2R = 2 \times 2 = 4$ cm

162. (c) Volume of earth taken out

$$= 40 \times 30 \times 12 = 14400 \text{ m}^3$$

Area of rectangular field

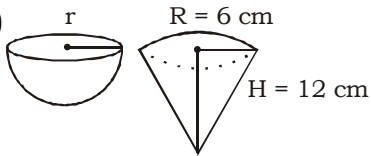
$$= 1000 \times 30 = 30000 \text{ m}^2$$

Area of region of tank = $40 \times 30 = 1200 \text{ m}^2$

$$\begin{aligned}\text{Remaining area} &= 30000 - 1200 \\&= 28800 \text{ m}^2\end{aligned}$$

$$\text{Increase in height} = \frac{14400}{28800} = \mathbf{0.5 \text{ m}}$$

163. (a)



According to question,

$$8 \times \frac{2}{3}\pi r^3 = \frac{1}{3}\pi (6)^2 \times 12$$

$$\begin{aligned}r^3 &= \frac{6 \times 6 \times 12}{8 \times 2} \\&= 3 \times 3 \times 3 \\r &= \sqrt[3]{3 \times 3 \times 3} \\&= \mathbf{3 \text{ cm}}\end{aligned}$$

164. (d)



$$\text{Volume of lead} = \frac{4}{3}\pi r^3$$

$$\text{Volume of Gold} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3$$

According to question,

$$\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^3$$

$$\frac{4}{3}\pi R^3 = \frac{8}{3}\pi r^3$$

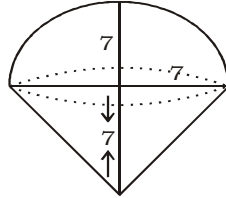
$$R^3 = 2r^3$$

$$R^3 = 2(2)^3$$

$$\begin{aligned}R &= \sqrt[3]{2} \times 2 = 1.259 \times 2 \\&= 2.518\end{aligned}$$

$$\begin{aligned}\therefore \text{Thickness} &= R - r \\&= 2.518 - 2 \\&= 0.518 \text{ cm}\end{aligned}$$

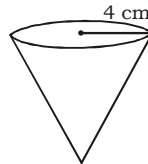
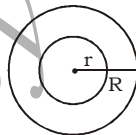
165. (a)



In the question, Radius of hemisphere = Radius of cone = height of cone = 7 cm
($\therefore$ height of hemisphere = radius of hemisphere) volume of ice cream = volume of hemisphere part + volume of conical part

$$\begin{aligned}&= \frac{2}{3} \times \frac{22}{7} \times (7)^3 + \frac{1}{3} \times \frac{22}{7} \times 7^3 \\&= \frac{22}{7} \times 7^3 = 22 \times 7^2 = 1078 \text{ cm}^3\end{aligned}$$

166. (c)



Volume of material of hollow sphere = Volume of cone

$$\frac{4}{3}\pi (5^3 - 3^3) = \frac{1}{3}\pi (4)^2 \times h$$

$$98 = 4h$$

$$h = \frac{98}{4} = \mathbf{24.5 \text{ cm}}$$

167. (d) Radius of the base of conical shape = r cm

$\therefore$ Radius of base of cylinder = $\frac{r}{3}$ cm

Volume of water = volume of cone

$$\begin{aligned}&= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 \times 24 \\&= 8\pi r^2 \text{ cm}^3\end{aligned}$$

volume of cylinder = volume of water

$$\pi \left(\frac{r}{3}\right)^2 \times H = 8\pi r^2$$

$$H = 9 \times 8 = \mathbf{72 \text{ cm}}$$

168. (b) volume of metallic sphere = volume of cone

$$\Rightarrow \frac{4}{3}\pi \times 3 \times 3 \times 3$$

$$= \frac{1}{3}\pi R^2 h$$

$$\Rightarrow \frac{4}{3}\pi \times 3 \times 3 \times 3$$

$$= \frac{1}{3} \times \pi \times 6 \times 6 \times h$$

$$h = \frac{108}{6 \times 6} = \mathbf{3 \text{ cm}}$$

169. (d) Number of bottle

$$= \frac{\text{volume of hemispherical bowl}}{\text{volume of cylindrical bottle}}$$

$$\begin{aligned}&= \frac{\frac{2}{3} \times \pi \times 15 \times 15 \times 15}{\pi \times \frac{5}{2} \times \frac{5}{2} \times 6} = \mathbf{60}\end{aligned}$$

170. (a) volume of cone V_1

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{\pi}{3} r^3 (\because h = r)$$

Volume of sphere, V_2

$$= \frac{4}{3}\pi r^3$$

Volume of cylinder V_3

$$= \pi r^2 h = \pi r^3$$

$$\therefore V_1 : V_2 : V_3 = \frac{1}{3} : \frac{4}{3} : 1$$

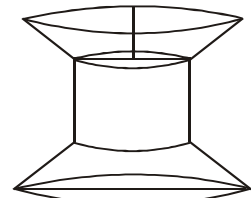
$$= 1 : 4 : 3$$

$$V_1 = \frac{V_2}{4} = \frac{V_3}{3}$$

171. (c) $4\pi \text{ radius}^2 = 346.5$

$$\text{radius}^2 = \frac{346.5 \times 7}{4 \times 22} = \mathbf{5.25 \text{ cm}}$$

172. (b)



Height of kaleidoscope = 25 cm

Radius of kaleidoscope = 35 cm
paper used = curved surface area

$$\begin{aligned}\text{of cylinder} &= 2 \times \frac{22}{7} \times 35 \times 25 \\ &= 2 \times 22 \times 5 \times 25 \\ &= 5500 \text{ cm}^2\end{aligned}$$

173. (b) According to the question,

$$\Rightarrow \frac{\text{Volume of sphere}}{\text{Surface area of sphere}}$$

$$\Rightarrow \frac{4}{3} \pi r^3 = 4 \pi r^2 \Rightarrow \frac{r}{3} = 1$$

$$\Rightarrow r = 3 \text{ cm}$$

$$\begin{aligned}\Rightarrow \text{then diameter of sphere will be} &= 2r \\ &= 2 \times 3 = 6 \text{ cm}\end{aligned}$$

174. (b) Let the height of cone h metre)

$$\begin{aligned}\Rightarrow \text{Total area of ground will be required} &= 5 \times 16 \text{ m}^2 = 80 \text{ m}^2 \\ \Rightarrow \text{Total volume of air is needed} &= 100 \times 5 \text{ m}^3 = 500 \text{ m}^3\end{aligned}$$

$$\Rightarrow \text{Total volume of air is needed} = 100 \times 5 \text{ m}^3 = 500 \text{ m}^3$$

According to the question

$$\Rightarrow \text{volume of cone} = 500 \text{ m}^3$$

$$\Rightarrow \frac{1}{3} \text{ area of ground} \times \text{height} = 500$$

$$\Rightarrow \frac{1}{3} \times \pi r^2 \times h = 500$$

$$= \frac{1}{3} \times 80 \times h = 500$$

$$\Rightarrow \text{height} = \frac{500 \times 3}{80}$$

$$\Rightarrow \text{height of cone} = 18.75 \text{ metres}$$

175. (d) Volume of cone = Lateral surface Area

$$\frac{1}{3} \pi r^2 h = \pi r l \quad [l = \sqrt{h^2 + r^2}]$$

$$\frac{rh}{3} = \sqrt{h^2 + r^2}$$

Squaring both sides

$$\frac{1}{9} = \frac{h^2 + r^2}{r^2 h^2}$$

$$\frac{1}{9} = \frac{h^2}{r^2 h^2} + \frac{r^2}{r^2 h^2}$$

$$\frac{1}{9} = \frac{1}{r^2} + \frac{1}{h^2}$$

176. (c) Diagonal of cube will be equal to diameter of sphere

$$\sqrt{3}a = 2 \times r$$

$$\sqrt{3}a = 2 \times 6\sqrt{3}$$

$$a = 12$$

$$\begin{aligned}\text{Surface area} &= 6a^2 = 6 \times 12 \times 12 \\ &\Rightarrow 864 \text{ cm}^2\end{aligned}$$

177. (c) Let hemisphere radius be = R
& Sphere radius be = r
ATQ,

$$\frac{2}{3} \pi R^3 = 4 \times \frac{4}{3} \pi r^3$$

$$2R^3 = 16r^3$$

$$\frac{R^3}{r^3} = \frac{8}{1} = \frac{R}{r} = \frac{2}{1}$$

178. (d) Let part filled be 'x'

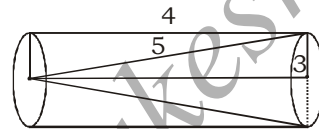
ATQ,

$$\begin{aligned}x \times (48 \text{ m} \times 16.5 \text{ m} \times 4 \text{ m}) \\ = \pi (2)^2 \times 56\end{aligned}$$

$$x = \frac{22 \times 4 \times 56}{7 \times 48 \times 16.5 \times 4}$$

$$x = \frac{2}{9}$$

179. (a) According to the question,



Whole surface of remaining Solid

$$= \pi r l + 2 \pi r h + \pi r^2$$

$$\text{Hence } l = \sqrt{h^2 + r^2}$$

$$l = \sqrt{4^2 + 3^2}$$

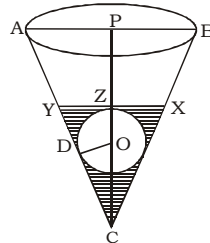
$$l = 5$$

$$\therefore = \pi r [l + 2h + r]$$

$$= \frac{22}{7} \times 3 [5 + 2 \times 4 + 3]$$

$$= \frac{22}{7} \times 3 \times 16 = 48 \pi$$

180. (a)



$\triangle ABC = \text{equilateral } \triangle$

$$\therefore \angle ACB = 60^\circ \text{ \& } \angle BCP = 30^\circ$$

$$\triangle CDO, \angle CDO = 90^\circ$$

and tangent is 90

$$OD = 1P = 1 \text{ cm}$$

$$OC = 2P = 2(1) = 2 \text{ cm}$$

$$\text{then, } CZ = OC + OZ$$

$$= 2 + 1 = 3 \text{ cm}$$

$$\triangle CZY, \angle CZY = 90^\circ$$

$$CZ = \sqrt{3}Q = 3 \text{ cm}$$

$$YZ = 1Q = \sqrt{3} \text{ cm}$$

Now, In cone XYZ

$$r = ZY = \sqrt{3} \text{ cm}$$

$$h = CZ = 3 \text{ cm}$$

Vol. of cone

$$\frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (\sqrt{3})^2 (3)$$

$$= 3 \pi \text{ cm}^2$$

$$\text{Vol. of sphere} = \frac{4}{3} \pi r_s^3$$

$$(\therefore r_s = 1 \text{ cm})$$

$$= \frac{4}{3} \pi \text{ cm}^3$$

Vol. of water that can immerse the ball

$$= \left(3\pi - \frac{4\pi}{3} \right) \text{ cm}^3 = \frac{5\pi}{3} \text{ cm}^3$$

181. (b) Here h = 4c,

$$\text{Volume of cylinder} = \pi r^2 h$$

$$= \frac{4\pi \times \pi r^2 h}{4\pi}$$

(Multiply 4π both in Numerator & denominator)

$$= \frac{(2\pi r)^2 \times (4c)}{4\pi} = \frac{c^3}{\pi}$$

182. (a) According to the question,

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\frac{4}{3} \pi r^3 = \pi r^2 h$$

$$= \frac{4}{3} r = h = \frac{4}{3} \times 3 = 4 \text{ cm}$$

$$\text{C.S.A of cylinder} = 2 \pi r h$$

$$= 2 \times \frac{22}{7} \times 3 \times 4$$

$$= \frac{44 \times 12}{7} = \frac{528}{7} = 75 \frac{3}{7} \text{ cm}^2$$

183. (a) According to question,

$$R = 6 \text{ cm.}$$

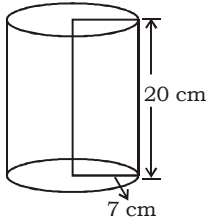
$\Rightarrow$ The capacity of the

hemispherical bowl

$$= \frac{2}{3} \pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 6^3$$

$$= 452.57 \text{ cm}^3$$

184. (d)



According to the question,
 $\Rightarrow r = 7 \text{ cm}$
 $\Rightarrow h = 20 \text{ cm}$
 $\Rightarrow$ Total surface Area of cylinder =
 curved surface Area + $2 \times$ area
 of base

$$= 2\pi rh + 2\pi r^2$$

$$= 2\pi r [r + h]$$

$$= 2 \times \frac{22}{7} \times 7 (7 + 20) = 44 \times 27$$

$$\Rightarrow \text{TSA of cylinder} = 1188 \text{ cm}^2$$

185. (d) According to question,
 Given:

$\Rightarrow$ Radius of cylinder = r
 $\Rightarrow$ CSA of cylinder = $4\pi r h$
 $\Rightarrow$ As we know
 $\Rightarrow$ Curved surface area of cylinder
 $= 2\pi R H$
 $4\pi r h = 2\pi \times r \times \text{Height}$
 $\Rightarrow \text{Height} = 2h \text{ unit}$

186. (a) According to the question,
 Radius = 3.5 cm

$\Rightarrow$ In the question it is given that A
 hemi-spherical bowl is to be
 painted inside as well as outside
 Total area that is to be painted
 $=$ Inside area of bowl + outside area
 of bowl

$$= 2\pi r^2 + 2\pi r^2$$

$$= 4\pi r^2$$

$$= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$$

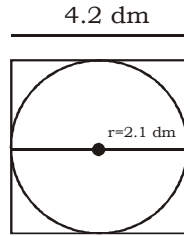
$\Rightarrow$ painting Rate = 10 cm^2 in 5 Rs.

$$1 \text{ cm}^2 \text{ will be painted} = \frac{5}{10} = \text{Rs. } \frac{1}{2}$$

so total cost will be painted in

$$= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{1}{2} = \text{Rs. } 77$$

187. (b)



$r = 2.1 \text{ dm}$
 $h = 4.2 \text{ dm}$
 (for Max.)

$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 4.2$$

$$= 19.404 \text{ dm}^3$$

188. (d) Let the initial radius = r

According to the question.

$$4\pi (r + 2)^2 - 4\pi r^2 = 352$$

$$4\pi [(r + 2)^2 - r^2] = 352$$

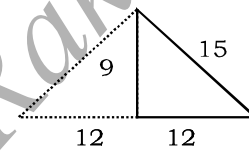
$$r^2 + 4 + 4r - r^2 = \frac{352 \times 7}{22 \times 4}$$

$$4r + 4 = 28$$

$$4r = 24$$

$$r = 6$$

189. (a)



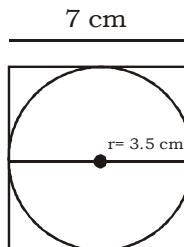
$$\text{Volume} \Rightarrow \frac{1}{3} \pi r^2 h$$

$$\Rightarrow \frac{1}{3} \pi 12 \times 12 \times 9$$

$$\Rightarrow 144 \times 3\pi$$

$$\Rightarrow 432\pi$$

190. (c)



According to the question,

$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

Height = 7 cm

$$\text{Radius} = \frac{7}{2} \text{ cm}$$

$\therefore$ Volume of cone

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 7 = 89.8 \text{ cm}^3$$

191. (b) Radius of Ist solid metallic
 spheres = $R = 6 \text{ cm}$

Radius of IInd solid metallic spheres
 $= r = 1 \text{ cm}$

Internal Radius of hollow sphere = x

External Radius of hollow sphere
 $= x + 1$

$$\text{So, } \frac{4}{3} \pi (R^3 + r^3)$$

$$= \frac{4}{3} \pi [(x + 1)^3 - x^3]$$

$$216 + 1 = x^3 + 1 + 3x(x + 1) - x^3$$

$$216 = 3x(x + 1)$$

$$72 = x^2 + x$$

$$\Rightarrow x^2 + x - 72 = 0$$

After solving,

$$x = 8 \text{ cm}$$

so, the external radius of the hollow
 sphere

$$= x + 1 = 8 + 1 = 9 \text{ cm}$$

192. (a) Let the time taken to fill
 the tank = $x \text{ hrs}$

$$\Rightarrow (\pi r^2 h) \times x = 50 \times 44 \times \frac{7}{100}$$

$$\Rightarrow x = \frac{50 \times 44 \times 7 \times 7 \times 100 \times 100}{22 \times 7 \times 7 \times 100 \times 5000}$$

$$= 2 \text{ hrs}$$

193. (b) The area of ground

$$\Rightarrow 1.5 \text{ hectares} = 1.5 \times 10000 \text{ m}^2$$

$$\Rightarrow 15000 \text{ m}^2$$

$$(\therefore 1 \text{ hectare} = 10000 \text{ m}^2)$$

$\Rightarrow$ level of rainfall

= height of water level

$$= 5 \text{ cm} = \frac{5}{100} \text{ m}$$

$\therefore$ volume of collected water

$$\Rightarrow 15000 \times \frac{5}{100} = 750 \text{ m}^3$$

194. (d) Required quantity of water

$$= \frac{3 \times 40 \times 2000}{60} = 4,000 \text{ m}^3$$

195. (b) Let r = Radius of pipe

Let R = Radius of cistern

Let H = Height of cistern

Let the no. of hours be ' x '

$$x (\pi R^2 H) = \pi r^2 h$$

$$\Rightarrow 3000 \times \pi \times \frac{10}{100} \times \frac{10}{100} \times x$$

$$= \pi \times \frac{10}{2} \times \frac{10}{2} \times 2$$

$$\Rightarrow \frac{6}{10} \times x = 1$$

$$x = \frac{10}{6} = 1 \text{ hour } 40 \text{ minutes}$$

196. (a) Diameter = 5 mm = 0.5 cm
radius = 0.25 cm

volume of water flowing from the pipe in 1 minute

$$= \pi \times 0.25 \times 0.25 \times 1000 \text{ cm}^3$$

volume of conical vessel

$$= \frac{1}{3} \pi \times 15 \times 15 \times 24 \text{ cm}^3$$

$$= \frac{1}{3} \pi \times 15 \times 15 \times 24 \text{ cm}^3$$

$$\therefore \text{Time} = \frac{\frac{1}{3} \times \pi \times 15 \times 15 \times 24}{\pi \times 0.25 \times 0.25 \times 1000}$$

$$= 28 \frac{4}{5}$$

= 28 minutes 48 second

197. (d) $r = 12 \text{ m}, h = 9 \text{ m}$

$$l = \sqrt{r^2 + h^2} = \sqrt{12^2 + 9^2} = 15 \text{ m}$$

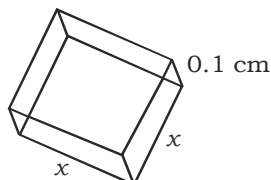
cost of canvas = curved surface area $\times$ cost of 1 m^2

$$= \pi r l \times 120$$

$$= 3.14 \times 12 \times 15 \times 120$$

$$= \text{Rs. } 67824$$

198. (d)



$$8.4 \text{ gm} = 1 \text{ cm}^3$$

$$4725 \text{ gm} = \frac{4725}{8.4} \text{ cm}^3$$

$$\text{volume} = x \times x \times 0.1 = \frac{4725}{8.4} \text{ cm}^3$$

$$= x^2 \times 0.1 = \frac{4725}{8.4} \text{ cm}^3$$

$$x = 75 \text{ cm}$$

199. (d) According to the question.
diameter = 84 cm

$$\text{radius} = 42 \text{ cm} = 0.42 \text{ m}$$

$$\text{height} = 120 \text{ cm} = 1.2 \text{ m}$$

$\therefore$ Curved surface area of cylinder

$$= 2 \pi r h = \frac{2 \times 22 \times 0.42 \times 1.2}{7}$$

Cost of levelling the ground

$$= 2 \pi r h \times 500 \times 1.5$$

$$= \frac{2 \times 22 \times 0.42 \times 1.2 \times 1.5 \times 500}{7}$$

$$= 2376$$

200. (a) since the volume of the two cylinders is same

$$\therefore \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = 1$$

$$\frac{h_1}{h_2} = \frac{1}{2}$$

$$\left(\frac{r_1}{r_2} \right)^2 = \frac{h_2}{h_1} = \frac{2}{1}$$

$$\frac{r_1}{r_2} = \sqrt{\frac{2}{1}} = \frac{\sqrt{2}}{1}$$

$\therefore$ Ratio of their radii = $\sqrt{2} : 1$

201. (a) We are given that volume of two cube are in the ratio = 27 : 1

$$\left(\frac{a_1}{a_2} \right)^3 = \frac{27}{1}$$

$$\frac{a_1}{a_2} = \sqrt[3]{\frac{27}{1}} = \frac{3}{1} = 3 : 1$$

202. (a) Ratio of edges of cuboid = 1 : 2 : 3

Let,

$$l = x, b = 2x, h = 3x$$

$$\text{Surface area} = 88 \text{ cm}^2$$

$$2(lb + bh + hl) = 88$$

$$2(2x^2 + 6x^2 + 3x^2) = 88$$

$$11x^2 = 44$$

$$\Rightarrow x^2 = 4, x = 2$$

$$\therefore l = 2 \text{ cm}, b = 4 \text{ cm}, h = 6 \text{ cm}$$

$$\therefore \text{Volume} = l b h$$

$$= 2 \times 4 \times 6$$

$$= 48 \text{ cm}^3$$

$$\text{203. (a)} \quad \frac{V_1}{V_2} = \frac{8}{27}$$

$$\frac{\frac{4}{3} \pi (r_1)^3}{\frac{4}{3} \pi (r_2)^3} = \frac{8}{27}$$

$$\left(\frac{r_1}{r_2} \right)^3 = \frac{8}{27}$$

$$\left(\frac{r_1}{r_2} \right)^2 = \left(\frac{2}{3} \right)^2$$

$$\text{Ratio of surface area} = \frac{4 \pi r_1^2}{4 \pi r_2^2}$$

$$= \left(\frac{r_1}{r_2} \right)^2 = \left(\frac{2}{3} \right)^2 = \frac{4}{9}$$

$$\text{204. (b)} \quad \frac{R_1}{R_2} = \frac{2}{3}, \quad \frac{H_1}{H_2} = \frac{5}{3}$$

$$\text{Ratio of volumes} = \frac{V_1}{V_2}$$

$$= \frac{\pi R_1^2 H_1}{\pi R_2^2 H_2} = \left(\frac{R_1}{R_2} \right)^2 \times \left(\frac{H_1}{H_2} \right)$$

$$= \left(\frac{2}{3} \right)^2 \times \left(\frac{5}{3} \right) = \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$$

$$\text{205. (d)} \quad 2 \pi r h = 264 \quad \dots (i)$$

$$\pi r^2 h = 924 \quad \dots (ii)$$

$$\text{on dividing: } \frac{2 \pi r h}{\pi r^2 h} = \frac{264}{924}$$

$$\frac{2}{r} = \frac{264}{924}$$

$$r = \frac{924 \times 2}{264} = 7 \text{ cm}$$

$$\text{diameter} = 2r = 2 \times 7 = 14 \text{ cm}$$

$$\text{putting, } r = 7 \text{ in (i)}$$

$$2 \pi r h = 264$$

$$h = \frac{264 \times 7}{2 \times 22 \times 7} = 6 \text{ cm}$$

$$\text{Required ratio} = \frac{2r}{h} = \frac{14}{6} = \frac{7}{3}$$

$$\text{206. (b)} \quad \frac{\frac{1}{3} \pi r_1^2 h_1}{\frac{1}{3} \pi r_2^2 h_2} = \frac{2}{3}$$

$$\left(\frac{1}{2^2} \right) \frac{h_1}{h_2} = \frac{2}{3}$$

$$\frac{h_1}{h_2} = \frac{8}{3}$$

$$\text{207. (c)} \quad \frac{(a_1)^3}{(a_2)^3} = \frac{27}{64}$$

$$\frac{a_1}{a_2} = \frac{3}{4}$$

Ratio of their total surface area

$$= \frac{6a_1^2}{6a_2^2} = \left(\frac{a_1}{a_2}\right)^2$$

$$= \left(\frac{3}{4}\right)^2 = \frac{9}{16} = 9 : 16$$

208. (b) Radius of both hemisphere and cone = R

Also height of hemisphere is equal to its Radius = R

∴ height of both hemisphere and cone = R

Now, In cone

$$\text{slant height, } l = \sqrt{R^2 + R^2} = \sqrt{2}R$$

$$\frac{\text{C.S.A of hemisphere}}{\text{C.S.A of cone}}$$

$$= \frac{2\pi R^2}{\pi R \times \sqrt{2}R} = \frac{\sqrt{2}}{1}$$

$$= \sqrt{2} : 1$$

209. (c) Let height of cone = h
radius of cone = r

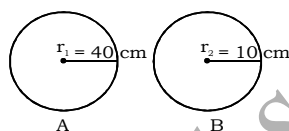
$$\text{volume of cone} = \frac{1}{3}\pi r^2 h$$

Now height is doubled,
volume of new cone

$$= \frac{1}{3}\pi r^2 (2h) = \frac{2}{3}\pi r^2 h$$

$$\text{Required ratio} = 1 : 2$$

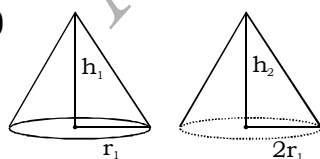
210. (d)



$$\frac{\text{surface area of A}}{\text{Surface area of B}} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{40}{10}\right)^2 = \frac{16}{1} \Rightarrow 16 : 1$$

211. (d)



$$r_2 = 2r_1$$

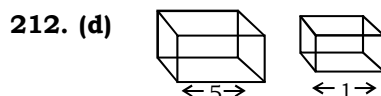
$$h_2 = h_1$$

$$\frac{\text{volume of new cone}}{\text{volume of old cone}}$$

$$= \frac{\frac{1}{3}\pi r_2^2 \times h_2}{\frac{1}{3}\pi r_1^2 \times h_1} = \frac{r_2^2 \times h_2}{r_1^2 \times h_1}$$

$$= \frac{(2r_1)^2 \times h_1}{r_1^2 \times h_1} = \frac{4}{1}$$

$$\Rightarrow 4 : 1$$



$$\text{Ratio of total surface area} = \frac{6(1)^2}{6(5)^2}$$

$$\Rightarrow = \frac{1}{25} = 1 : 25$$

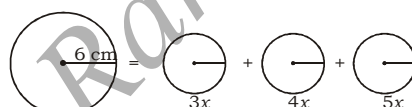
213. (b) Let $r_1 = \frac{21}{2}$ cm

$$r_2 = \frac{17.5}{2} \text{ cm}$$

$$\therefore \text{Required ratio} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2}$$

$$= \frac{21 \times 21}{17.5 \times 17.5} = \frac{36}{25} = 36 : 25$$

214. (a)



$$\frac{4}{3}\pi \{(3x)^3 + (4x)^3 + (5x)^3\}$$

$$= \frac{4}{3}\pi (6)^3$$

$$x^3 (27 + 64 + 125) = 216$$

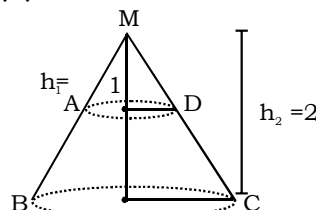
$$x^3 \times 216 = 216$$

$$x^3 = \frac{216}{216} = 1$$

$$x = \sqrt[3]{1} = 1$$

$$\text{Radius of smallest sphere} = 3x = 3 \times 1 = 3 \text{ cm}$$

215. (d)



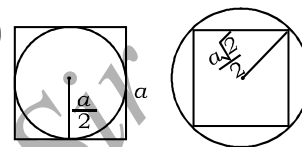
$$\frac{\text{ratio of smaller cone}}{\text{ratio of larger cone}} = \frac{h_1^3}{h_2^3}$$

$$= \frac{1^3}{2^3} = \frac{1}{8}$$

area of part (ABCD) (i.e frustum) = 8 - 1 = 7

$$\therefore \text{Required ratio} = 1 : 7$$

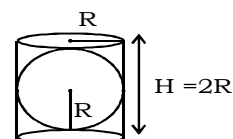
216. (a)



$$\frac{\text{area of incircle}}{\text{area of circum circle}}$$

$$= \frac{\pi \left(\frac{a}{2}\right)^2}{\pi \left(\frac{a\sqrt{2}}{2}\right)^2} = \frac{1}{2} \Rightarrow 1 : 2$$

217. (b)



(height of cylinder = 2 × R)

$$\frac{\text{Surface area of sphere}}{\text{C.S.A of cylinder}}$$

$$\frac{4\pi R^2}{2\pi R \times H} = \frac{4\pi R^2}{2\pi R(2R)}$$

$$= \frac{4\pi R^2}{4\pi R^2} = \frac{1}{1} = 1 : 1$$

218. (d) Ratio of volume = (Ratio of radius)³

$$= \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3$$

$$= \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

219. (a) $\frac{4}{3}\pi r^3 = \pi r^2 h$

$$\Rightarrow h = \frac{4}{3}r$$

$$\Rightarrow r = \frac{3}{4}h$$

$$\Rightarrow 2r(\text{diameter}) = \frac{3}{4} \times 2h = \frac{3}{2}h$$

$$\Rightarrow \frac{\text{Diameter}}{\text{Height}} = \frac{3}{2}$$

220. (a) In this case height of cylinder and cone is equal to the radius of hemisphere

$\Rightarrow h = r$ Ratio of volumes

Cone : hemisphere : cylinder

$$= \frac{1}{3}\pi r^2 \times r : \frac{2}{3}\pi r^3 : \pi r^2 \times r$$

$$= 1 : 2 : 3$$

221. (a) Ratio of curved surface area = Ratio of product of height and radius

$\Rightarrow$ Required ratio

$$\frac{C_1}{C_2} = \frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{r_1 h_1}{r_2 h_2}$$

$$= \frac{3 \times 2}{5 \times 3} = \frac{2}{5}$$

222. (d) Ratio of surface area = (Ratio of radius)²

$$= \frac{C_1}{C_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

223. (a) $\frac{\pi R^2 H}{\pi r^2 h} = \frac{3}{1}$

$$\Rightarrow \frac{3 \times 3 \times H}{2 \times 2 \times h} = \frac{3}{1}$$

$$\Rightarrow \frac{H}{h} = \frac{4}{3}$$

$$\Rightarrow \frac{x}{1} = \frac{4}{3}$$

$$\Rightarrow x = \frac{4}{3}$$

224. (c) $\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$

$$\frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = 64$$

$$\left(\frac{R}{r}\right)^3 = (4)^3$$

$$\Rightarrow R = 4r$$

$$\frac{C_1}{C_2} = \frac{4\pi R^2}{4\pi r^2} = \left(\frac{R}{r}\right)^2$$

Ratio of area = (Ratio of radius)²
 $= (4 : 1)^2$
 $= 16 : 1$

225. (a) $\frac{\pi R^2 H}{\pi R^2 h} = 1$

$$\frac{3^2 \times H}{2^2 \times h} = 1$$

$$\Rightarrow \frac{H}{h} = \frac{4}{9}$$

226. (a) Let the radius and slant height be $4x$ and $7x$

$$\Rightarrow 7x = 14 \text{ cm}$$

$$x = 2 \text{ cm}$$

$$\Rightarrow \text{Radius} = 4 \times 2 = 8 \text{ cm}$$

227. (d) Height of cylinder = Diameter of sphere

$$\Rightarrow 4\pi r^2 : 2\pi rh$$

$$\Rightarrow 4\pi r^2 : 4\pi r^2 \text{ (h=2r)}$$

$$\Rightarrow \text{Required ratio} = 1 : 1$$

228. (a) $\frac{V_1}{V_2} = \frac{r^2 h}{R^2 H} = \frac{3^2 \times 4}{4^2 \times 3} = \frac{3}{4}$

229. (d) Ratio of volume of bigger cone and smaller cones

= (Ratio of altitude)³

$$= (1 : 2 : 3)^3$$

$$= (1 : 8 : 27)$$

$$\therefore \text{Ratio of parts} = 1 : 8 - 1 : 27 - 8$$

$$= 1 : 7 : 19$$

230. (c) Let radii of cylinder and sphere be r

$\therefore$ Volume of cylinder of height (h)

$$= \pi r^2 h$$

Total surface area of cylinder

$$= 2\pi rh + 2\pi r^2$$

Total surface area of sphere

$$= 4\pi r^2$$

$\therefore$ given that

$$4\pi r^2 = 2\pi rh + 2\pi r^2$$

$$4\pi r^2 = 2\pi r(h + r)$$

$$\Rightarrow 2r = h + r$$

$$r = h$$

$\therefore$ radius of sphere or cylinder's equal to height of cylinder

$\therefore$ Ratio of volume of cylinder and sphere

$$= \pi r^2 \times r : \frac{4}{3}\pi r^3 = 3 : 4$$

231. (c) $\frac{4}{3}\pi R^3 = \pi r^2 H$

$$\frac{4}{3}R^3 = r^2 H$$

$$\frac{R^2}{r^2} = \frac{3}{4} \quad (\because H = R)$$

$$R : r = \sqrt{3} : \sqrt{4} = \sqrt{3} : 2$$

232. (b) $\frac{a^3}{\frac{4}{3}\pi r^3} = \frac{363}{49}$

$$\frac{a^3}{r^3} = \frac{363 \times 22 \times 4}{49 \times 7 \times 3}$$

$$\frac{a^3}{r^3} = \left(\frac{22}{7}\right)^3$$

$$\frac{a}{r} = \frac{22}{7}$$

233. (a) cone $\Rightarrow$ radius : height

$$4 : 3$$

$$\text{Let } 4x : 3x$$

$\therefore$ curved surface area of cone = $\pi r l$

r = radius

l = slant height

$$l = \sqrt{h^2 + r^2}$$

$$= \sqrt{(4x)^2 + (3x)^2} = 5x$$

$\therefore$ curved surface area

$$\Rightarrow \pi \times 4x \times 5x$$

$$\Rightarrow 20\pi x^2$$

$\therefore$ total surface area $\Rightarrow \pi r l + \pi r^2$

$$\Rightarrow \pi r(l + r)$$

$$\Rightarrow \pi \times 4x(5x + 4x)$$

$$\Rightarrow \pi \times 4x \times 9x$$

$$\Rightarrow 36\pi x^2$$

$\therefore$ Curved area : total area

$$20\pi x^2 : 36\pi x^2$$

$$5 : 9$$

234. (b) Let radius of sphere = r

radius of cylinder = r

$\therefore$ let height of cylinder = h

$\therefore$ given that volume of sphere = volume of cylinder

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi r^2 h$$

$$\Rightarrow \frac{4}{3}r = h$$

$\therefore$ curved surface area

cylinder : sphere

$$2 \times \pi \times r \times \frac{4r}{3} : 4\pi r^2$$

$$\Rightarrow \frac{8}{3} : 4$$

$$2 : 3$$

235. (c) radius of cone = radius of cylinder = r height of cone = height of cylinder = h

curved surface area of cylinder

curved surface area of cone

$$\Rightarrow \frac{2\pi rh}{\pi rl} = \frac{8}{5}$$

$$\Rightarrow \frac{h}{1} = \frac{4}{5}$$

$$\Rightarrow \frac{h^2}{l^2} = \frac{16}{25}$$

$$\Rightarrow l^2 = h^2 + r^2$$

$$\Rightarrow h^2 = 16$$

$$25 = 16 + r^2$$

$$r^2 = 9$$

$$r = 3$$

∴ radius : height

$$3 : 4$$

236. (a) Volume of prism = Base Area × Height

$$1056 = \frac{1}{2} (8 + 14) \times h \times 12$$

$$h = \frac{1056 \times 2}{22 \times 12} = 8 \text{ cm}$$

237. (b) Ratio of volume =

$$\frac{\pi(\sqrt{3})^2 \times \sqrt{2}}{\frac{1}{3}\pi(\sqrt{2})^2 \times \sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{2}}$$

$$= 3\sqrt{3} : \sqrt{2}$$

$$\text{238. (a)} \quad \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$$

$$\text{Ratio of volume} = \frac{(3)^2 \times 1}{(5)^2 \times 3} = 3$$

: 25

239. (d) Let the radius of sphere and hemisphere be = R and r

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{2}{3}\pi r^3$$

$$2R^3 = r^3$$

$$\frac{R^3}{r^3} = \frac{1}{2}$$

$$\Rightarrow R : r = 1 : \sqrt[3]{2}$$

240. (a) Ratio of radius of earth and moon = 4 : 1

$$\Rightarrow \text{Ratio of volume} = 4^3 : 1^3$$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = 64 : 1$$

241. (b) Let the radius of cylinder and sphere be = r cm

$$\Rightarrow \text{height of cylinder} = 2r \text{ cm}$$

$$\Rightarrow A = \pi r^2 \times 2r = 2\pi r^3$$

$$B = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{A}{B} = \frac{2\pi r^3}{\frac{4}{3}\pi r^3} = 3 : 2$$

$$\text{242. (c)} \quad \text{Side of cube} = \frac{6\sqrt{3}}{\sqrt{3}} = 6 \text{ cm}$$

$$\text{Required rate} = \frac{6 \times 6^2}{6^3} = 1 : 1$$

243. (d) Let the radius of hemisphere and sphere be 'r' and 'R'

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{2}{3}\pi r^3$$

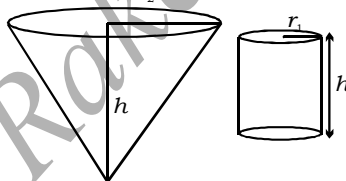
$$\frac{R^3}{r^3} = \frac{1}{2}$$

$$\frac{R}{r} = \frac{1}{\sqrt[3]{2}}$$

$$\Rightarrow \text{Ratio of curved surface area}$$

$$= \frac{4\pi R^2}{2\pi r^2} = \frac{2R^2}{r^2} = \frac{2 \times 1}{(\sqrt[3]{2})^2} = \frac{1}{2^{\frac{2}{3}}} : 1$$

$$\text{244. (b)} \quad \frac{\text{Volume of cylinder}}{\text{volume of cone}} = \frac{3}{1}$$

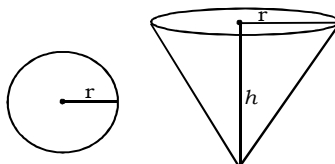


$$\frac{\pi r_1^2 h}{\frac{1}{3}\pi r_2^2 h} = \frac{3}{1}$$

$$\Rightarrow r_1 = r_2$$

Diameter of cylinder = Diameter of cone

245. (d)



Volume remains same
volume of sphere = volume of cone

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi \times r^2 \times h$$

$$4r = h$$

$$\frac{h}{r} = \frac{4}{1} = 4 : 1$$

246. (b) Area of Δ (Base Area) = Semiperimeter × in radius

$$= \left(\frac{45}{2} \times 9\right) = \text{cm}$$

Volume of prism = Base area × Height

$$810 = \frac{45}{2} \times 9 \times \text{Height}$$

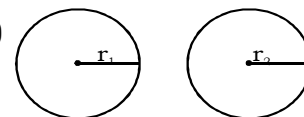
$$\text{Height} = \frac{810 \times 2}{45 \times 9} = 4 \text{ cm}$$

Total surface area = Lateral Surface Area + (Bottom + Top) Area

$$= (\text{Perimeter of Base} \times \text{Height}) + (2 \times \text{Base area})$$

$$= (45 \times 4) + 2 \times \left(9 \times \frac{45}{2}\right) = 180 + 405 = 585 \text{ cm}^2$$

247. (a)



Ratio of volume of sphere × ratio of weight per 1 cc. of material of each = Ratio of weight of two sphere

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} \times \frac{289}{64} = \frac{8}{17}$$

$$\frac{r_1^3}{r_2^3} = \frac{8 \times 64}{17 \times 289} = \frac{8 \times 8 \times 8}{17 \times 17 \times 17}$$

$$\frac{r_1}{r_2} = \frac{8}{17} \Rightarrow 8 : 17$$

$$\text{248. (a)} \quad \frac{R_1}{R_2} = \frac{1}{2}$$

$$\frac{V_1}{V_2} = \frac{2}{3}$$

$$\frac{\frac{1}{3}\pi R_1^2 H_1}{\frac{1}{3}\pi R_2^2 H_2} = \frac{2}{3}$$

$$\left(\frac{R_1}{R_2}\right)^2 \times \left(\frac{H_1}{H_2}\right) = \frac{2}{3}$$

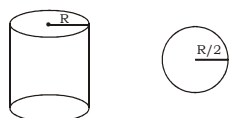
$$\left(\frac{1}{2}\right)^2 \times \frac{H_1}{H_2} = \frac{2}{3}$$

$$\frac{1}{4} \times \frac{H_1}{H_2} = \frac{2}{3}$$

$$\frac{H_1}{H_2} = \frac{2}{3} \times \frac{4}{1}$$

$$= \frac{8}{3} \Rightarrow 8 : 3$$

249. (d)



Let the Radius of cylinder = R
 $\Rightarrow$ Therefore, Radius of sphere

$$= \frac{R}{2}$$

Volume of Right circular cylinder
 $= \pi R^2 H$

$$\text{Volume of sphere} = \frac{4}{3} \pi \left(\frac{R}{2}\right)^3$$

$$= \frac{4}{3} \pi \frac{R^3}{8} = \frac{\pi R^3}{6}$$

According to question,

Volume of Cylinder = Volume of sphere

$$\pi R^2 H = \frac{\pi R^3}{6}$$

$$\frac{\pi R^2 H \times 6}{\pi R^3} = 1$$

$$\frac{H}{R} = \frac{1}{6} \Rightarrow 1 : 6$$

250. (d) Radius of larger sphere = R units

$$\text{Its volume} = \frac{4}{3} \pi R^3$$

Now cones are formed with base radius and height same as the radius of larger sphere

$$\therefore \text{Volume of smaller cone} = \frac{1}{3} \pi R^3$$

and one of the cone is converted into smaller sphere

Therefore volume of smaller sphere

$$= \frac{1}{3} \pi R^3$$

$$\therefore \frac{4}{3} \pi r^3 = \frac{1}{3} \pi R^3$$

$$\frac{r^3}{R^3} = \frac{1}{4}$$

$$\frac{r}{R} = \frac{1}{\sqrt[3]{4}}$$

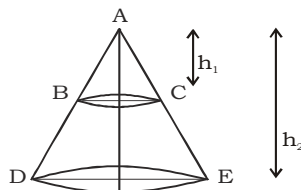
$\therefore \frac{\text{Surface area of smaller sphere}}{\text{Surface area of larger sphere}}$

$$= \frac{4 \pi r^2}{4 \pi R^2} = \frac{r^2}{R^2}$$

$$\Rightarrow \frac{(1)^2}{\left(\frac{1}{4}\right)^2} = \frac{(1)^2}{\left(\frac{2}{2}\right)^2} = \frac{1}{2^3}$$

$$\Rightarrow 1 : 2^3$$

251. (c)



$$\frac{\text{Volume of Cone ABC}}{\text{Volume of BCED}} = \frac{1}{1}$$

$$\frac{\text{Volume of Cone ABC}}{\text{Volume of Cone ADE}} = \frac{1}{1+1} = \frac{1}{2}$$

If a cone is cut in any parts parallel to its base then the ratio of volume of smaller cone to the volume of larger cone is equal to the ratio of the cubes of their corresponding heights/ radii/slant height(it is proved by similarity)

$$= \left(\frac{\text{height of Cone}(h_1)}{\text{height of Cone}(h_2)} \right)^3 = \frac{1}{2}$$

$$\frac{h_1}{h_2} = \frac{1}{\sqrt[3]{2}}$$

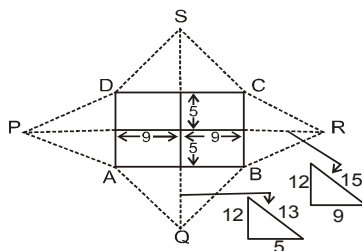
$$\Rightarrow h_1 : h_2 = h_1 : 1$$

$$= 1 : ((\sqrt[3]{2} - 1))$$

252. (b) Height (h) = 12cm (given)

$\therefore$ Slant height of

ΔSDC = Slant height of ΔQAB



$$= \sqrt{(12)^2 + (5)^2} = 13\text{cm}$$

and slant height of ΔPDA =

Slant height of ΔRCB

$$= \sqrt{(12)^2 + (9)^2} = 15\text{cm}$$

$\therefore$ Area of ΔABQ

= Area of ΔSDC

$$= \frac{1}{2} \times 18 \times 13 = 117\text{cm}^2$$

Area of ΔPDA = Area of ΔRCB

$$\frac{1}{2} \times 10 \times 15 = 75\text{cm}^2$$

& Area of $\square ABCD$ (Base area)

$$= 10 \times 18$$

$$= 180\text{cm}^2$$

Total surface area = Base area

+ area of (ΔABQ +

ΔSDC + ΔPDA + ΔRCB)

$$= 180 + 2 \times 117 + 2 \times 75 = 564\text{cm}^2$$

253. (a) volume of cone = $\frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 1^2 \times 7 = \frac{22}{3} \text{cm}^3$$

Volume of cubical block

$$= 10 \times 5 \times 2 \text{cm}^3 = 100 \text{cm}^3$$

Wastage of wood

$$= \left(100 - \frac{22}{3} \right) \text{cm}^3$$

$$= \left(\frac{300 - 22}{3} \right) = \frac{278}{3} \text{cm}^3$$

$$\% \text{ wastage} = \frac{\frac{278}{3}}{100} \times 100$$

$$= \frac{278}{3} = 92\frac{2}{3}\%$$

254. (c) Decrease in radius = 50% = $\frac{1}{2}$

increase in height = 50%

$$= \frac{1}{2} \rightarrow \text{Increment}$$

$$= \frac{1}{2} \rightarrow \text{Original}$$

(Let)

	Radius	Height	Volume
Original	2	2	$(2)^2 \times (2) = 8$
	$\downarrow 50\% \text{ decrease}$	$\downarrow 50\% \text{ increase}$	$\downarrow -5$
New	1	3	$(1)^2 \times (3) = 3$

Reduction in volume

$$= \frac{5}{8} \times 100 = 62\frac{1}{2}\%$$

255. (a) Increase in radius = 100%

$$= \frac{1}{1}$$

$$\text{Increase in height} = 100\% = \frac{1}{1}$$

	Radius	Height	Volume
Original	1	1	$(1)^2 \times (1) = 1$
New	2	2	$(2)^2 \times (2) = 8$

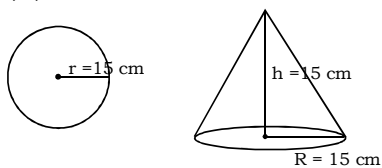
$$\% \text{ increase} = \frac{7}{1} \times 100 = 700\%$$

256. (d) 20% = $\frac{1}{5} \rightarrow$ Increment
Original

Radius	Height	Volume
5	5	$(5)^2 \times 5 = 125$
6	6	$(6)^2 \times (6) = 216$

$$= \frac{91}{125} \times 100 = 72.8\%$$

257. (d)



$$\begin{aligned} \text{volume of cone} &= \frac{1}{3} \times \pi (15)^2 \times 15 \\ &= \frac{1}{3} \pi (15)^3 \end{aligned}$$

$$\text{Volume of sphere} = \frac{4}{3} \pi (15)^3$$

Required Percentage

$$= \frac{\text{volume of cone}}{\text{volume of sphere}} \times 100$$

$$= \frac{\frac{1}{3} \times \pi \times (15)^3}{\frac{4}{3} \times \pi (15)^3} \times 100$$

$$= \frac{1}{4} \times 100 = 25\%$$

258. (d)

	Radius	Height	Volume
Original	2	1	$(2)^2 \times (1) = 4$
New	1	3	$(1)^2 \times (3) = 3$

$$\% \text{ decrease} = \frac{4-3}{4} \times 100 = 25\%$$

259. (d) height = 100% Radius = 100%

$\frac{1}{1} \rightarrow$ Increment $\frac{1}{1} \rightarrow$ Increment
 $\frac{1}{1} \rightarrow$ Original $\frac{1}{1} \rightarrow$ Original

	Height	Radius	Volume
Original	1	1	$(1)^2 \times 1 = 1$
New	2	2	$(2)^2 \times 2 = 8$

$\left[\frac{1}{3} \text{ is Constant} \right]$
= eight times that of original

260. (b) use $x + y + \frac{xy}{100}$

percentage change in area

$$\begin{aligned} &= 15 - 10 + \frac{15 \times (-10)}{100} \\ &= 5 - 1.5 = 3.5\% \\ &\quad (3.5\% \text{ increase}) \end{aligned}$$

Remember : when change in area is asked in the question, then use this formula to save your valuable time.

261. (d) Let old radius = r

$$\Rightarrow \text{volume} = \frac{4}{3} \pi r^3$$

$$\text{New radius} = 2r$$

$$\begin{aligned} \Rightarrow \text{New volume} &= \frac{4}{3} \pi (2r)^3 \\ &= \frac{4}{3} \pi \times 8r^3 \end{aligned}$$

$\Rightarrow$ Volume becomes eight times

262. (b)

	Radius	Height	Volume
Original	2	5	$(2)^2 \times (5) = 20$
New	1	8	$(1)^2 \times (8) = 8$

$\Rightarrow$ Volume decreases

$$\begin{aligned} \% \text{ decrease (\% कम)} &= \frac{20-8}{20} \times 100 \\ &= 60\% \end{aligned}$$

263. (d) Length $1 \rightarrow 2$

Breadth $2 \rightarrow 6$

Height $3 \rightarrow 9$
Volume $6 \rightarrow 108$

$\Rightarrow$ New volume = 18 times the original volume

$$\begin{aligned} \Rightarrow \text{Increase in volume} \\ &= 18 - 1 = 17 \text{ times} \end{aligned}$$

264. (c)

	Radius	Height	Volume
Original	10	10	$(10)^2 \times (10) = 1000$
New	11	11	$(11)^2 \times (11) = 1331$

$$\begin{aligned} \Rightarrow \% \text{ Increase} &= \frac{1331-1000}{1000} \times 100 \\ &= 33.1\% \end{aligned}$$

265. (a) % Change in height = % change in volume = 100%

266. (a) Volume of coffee

$$\begin{aligned} &= \frac{2}{3} \pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (4)^3 \\ &= \frac{128}{3} \pi \text{ cm}^3 \end{aligned}$$

$$\text{Volume of cone} = \frac{1}{3} \pi r^2 \times h$$

$$= \frac{1}{3} \pi (8)^2 \times 16 = \frac{1024}{3} \pi$$

$\therefore$ Required percentage

$$\begin{aligned} &= \frac{\frac{1024}{3} - \frac{128}{3}}{\frac{1024}{3}} \times 100 = 87.5\% \end{aligned}$$

267. (a) Decrease in base radius = (Decrease in base area) $^{1/2}$

$$= \left(\frac{1}{9} \right)^{\frac{1}{2}} = \frac{1}{3}$$

Let initial radius and height be $3r$ and h

$\therefore$ New radius and height are r and $6h$

old lateral surface area

$$\begin{aligned} &= 2 \times \pi \times 3r \times h \\ &= 6\pi rh \end{aligned}$$

New lateral surface area

$$\begin{aligned} &= 2 \times \pi \times r \times 6h \\ &= 12\pi rh \end{aligned}$$

$$\text{Required factor} = \frac{12\pi rh}{6\pi rh} = 2$$

268. (c) Let the original radius be 'r'

$$\Rightarrow \text{Area} = 4\pi r^2$$

$$\text{New area} = 4\pi (2r)^2 = 16\pi r^2$$

$\Rightarrow$ New area is 4 times the old area

269. (d) Edge is increased by 50% =

$$\frac{50}{100} = \frac{1}{2} \rightarrow \text{Increased}$$

$$\frac{1}{2} \rightarrow \text{Original}$$

Let original edge = 2

∴ increased edge = 3
 edge surface area

$$\begin{array}{cc} 2 & 4 \\ & \searrow \\ & 3 \end{array} \quad \begin{array}{cc} 4 & 9 \\ & \searrow \\ & 3 \end{array} \quad + [5, 6 \text{ is constant}]$$

$$\% \text{ increase} = \frac{5}{4} \times 100 = 125\%$$

270. (b) Let the initial radius = 1 unit
 New radius = 2 unit (radius is doubled)

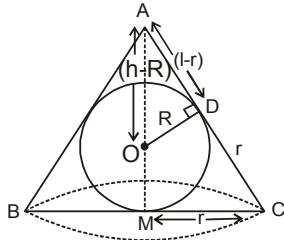
Radius : Volume

$$\begin{array}{cc} 1 & (1)^3 \\ 2 & (2)^3 \end{array} \quad \left(\frac{4}{3}\pi \text{ is constant} \right)$$

$$\begin{array}{cc} 1 & 1 \\ 2 & 8 \end{array} \quad 7$$

$$\text{Percentage increase} = \frac{7}{1} \times 100 = 700\%$$

271. (c)



Let the radius of cone = r
 and height = h, slant height = l
 and radius of sphere = R

$$\therefore R = \frac{hr}{l+r} = \frac{8 \times 6}{10+6} = 3 \text{ cm}$$

Detailed method

CD and CM are tangents at c,

∴ CD = CM

Now in $\triangle ADO$,

$$(h - R)^2 = (R)^2 + (l - r)^2$$

$$\Rightarrow h^2 + R^2 - 2Rh = R^2 + l^2 + r^2 - 2lr$$

$$\Rightarrow h^2 - 2rh = h^2 + r^2 + l^2 - 2lr$$

$$[\because l^2 = h^2 + r^2]$$

$$\Rightarrow R = \frac{r(l-r)}{h} = \frac{rh(l-r)}{h^2}$$

$$\Rightarrow R = \frac{rh(l-r)}{(l^2 - r^2)} = \frac{rh(l-r)}{(l-r)(l+r)} = \frac{rh}{l+r}$$

$$= \frac{8 \times 6}{10+6} = 3 \text{ cm}$$

272. (a) Volume of tetrahedron

$$= \frac{a^3}{6\sqrt{2}} = \frac{12^3}{6\sqrt{2}}$$

$$= \frac{1728}{6\sqrt{2}} = 144\sqrt{2} \text{ cm}^3$$

273. (a) Volume of bucket

$$= \frac{1}{3} \pi h(R^2 + r^2 + Rr)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 45 (28^2 + 7^2 + 28 \times 7)$$

$$= \frac{22}{7} \times 15 \times 1029 = 48510 \text{ cm}^3$$

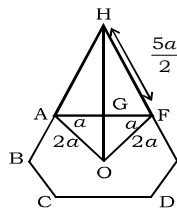
274. (c) Side of regular hexagon = 2a cm

$$\text{area of hexagon} = 6 \times \frac{\sqrt{3}}{4} \times (2a)^2$$

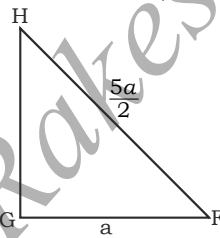
$$\Rightarrow 6\sqrt{3}a^2 \text{ cm}^2$$

$$\text{slant edge of pyramid} = \frac{5a}{2} \text{ cm}$$

∴



$$\text{slant edge} \Rightarrow \frac{5a}{2} \text{ (Given)}$$



$$\Rightarrow HF = \frac{5a}{2} \text{ (slant edge)}$$

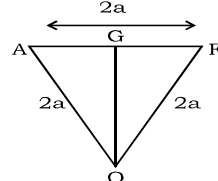
$$\Rightarrow HG = \text{slant height (l)}$$

$$\Rightarrow GF = \text{base}$$

$$\Rightarrow (a) \text{ (given)}$$

$$\text{slant height} \Rightarrow \sqrt{\left(\frac{5a^2}{2}\right) - (a)^2}$$

$$= \sqrt{\frac{25a^2}{4} - a^2} = \frac{\sqrt{21}a}{2}$$



AOF is equilateral triangle of side 2a

$$\therefore \text{Altitude GO} = \frac{\sqrt{3}}{2} \times 2a$$

$$= \sqrt{3} a$$

$$\therefore \text{Slant height} \Rightarrow \frac{\sqrt{21}}{2} a$$

$$\text{altitude} = \sqrt{3} a$$

$$\therefore \text{height of the pyramid} = h$$

$$\Rightarrow \sqrt{\left(\frac{\sqrt{21}a}{2}\right)^2 - (\sqrt{3}a)^2}$$

$$= \sqrt{\frac{21}{4}a^2 - 3a^2} = \sqrt{\frac{9a^2}{4}} = \frac{3}{2}a$$

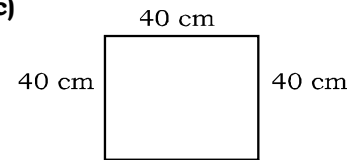
∴ Volume of pyramid

$$= \frac{1}{3} \text{ area of base} \times \text{height}$$

$$= \frac{1}{3} \times 6\sqrt{3}a^2 \times \frac{3}{2}a$$

$$= 3\sqrt{3}a^3 \text{ cm}^3$$

275. (c)



$$\Rightarrow \text{area of base} = 40 \times 40 = 1600 \text{ cm}^2$$

Let height of pyramid = h

$$\therefore \text{Volume} = \frac{1}{3} \times h \times \text{area of base}$$

$$= \frac{1}{3} \times h \times 1600$$

$$\Rightarrow 8000 \text{ (given)}$$

$$= h = 15 \text{ cm}$$

276. (c) area of trapezium

$$= \frac{1}{2} \times h (AB + CD)$$

$$= \frac{1}{2} \times 8 \times (8 + 14)$$

$$= 4 \times 22 = 88 \text{ cm}^2$$

$$= \text{volume of prism}$$

$$= \text{Height of prism} \times \text{area of base}$$

$$\Rightarrow \text{height} \times 88 = 1056 \text{ (given)}$$

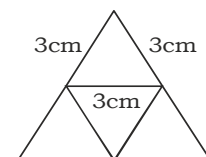
$$\Rightarrow \text{height} = \frac{1056}{88}$$

$$\Rightarrow 12 \text{ cm}$$

277. (a) Edge of regular tetrahedron

$$= 3 \text{ cm}$$

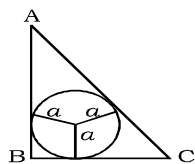
$$\therefore a = 3 \text{ cm}$$



$$\therefore \text{volume} \Rightarrow \frac{\sqrt{2}}{12} a^3 \text{ cm}^3$$

$$\Rightarrow \frac{\sqrt{2}}{12} \times (3)^3 = \frac{9}{4} \sqrt{2} \text{ cm}^3$$

278. (d)



r - inradius of incircle of triangle
Perimeter = 15 cm (given)

$$\therefore \text{Semiperimeter (S)} = \frac{15}{2} \text{ cm}$$

Inradius of any triangle

$$r \Rightarrow \frac{\Delta}{s}$$

$$r = \frac{\text{area}}{\text{semiperimeter}}$$

where Δ is the area of triangle

$\therefore r = 3$ cm given

$$3 \Rightarrow \frac{\text{area of triangle}}{\frac{15}{2}}$$

$$3 \times \frac{15}{2} = \text{area of triangle}$$

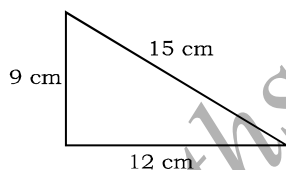
$$\Rightarrow \frac{45}{2} \text{ cm} = \text{area of triangle}$$

$$\therefore \text{volume of prism} \Rightarrow 270 \text{ cm}^3$$

$$\therefore 270 = h \times \frac{45}{2}$$

$$\Rightarrow h = 12 \text{ cm}$$

279. (c)



9, 12, 15 is a triplet which forms a right angle triangle

$\therefore$ area of base of prism

$$\Rightarrow \frac{1}{2} \times 9 \times 12 = 54 \text{ cm}^2$$

$$\text{Perimeter of triangle} = 9 + 12 + 15 = 36 \text{ cm}$$

$$\therefore \text{total surface area of prism} = \text{perimeter base} \times \text{height} + 2 \times \text{area of base}$$

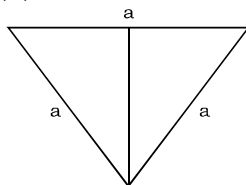
$$\Rightarrow \text{height of prism} = 5 \text{ cm given}$$

$$\therefore \text{total surface area}$$

$$= (36 \times 5) + (2 \times 54)$$

$$\Rightarrow 180 + 108 = 288 \text{ cm}^2$$

280. (c)



Let side equilateral triangle be = a

$$\therefore \text{area} = \frac{\sqrt{3}}{4} a^2$$

$$= \frac{\sqrt{3}}{4} a^2 = 173 \text{ cm}^2$$

$$\Rightarrow a^2 = \frac{173}{\sqrt{3}} \times 4$$

$$(\sqrt{3} = 1.73)$$

$$\therefore a^2 = \frac{173}{1.73} \times 4$$

$$= \frac{173}{1.73} \times 4 \times 100$$

$$a^2 = 400$$

$$a = 20 \text{ cm.}$$

$$\text{Perimeter of base} = 20 \times 3 = 60 \text{ cm}$$

$$\therefore \text{Volume of prism} = 10380$$

$$\text{cm}^3$$

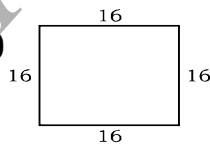
(given)

$$\text{area of base} \times \text{height} = 10380$$

$$\text{height} = \frac{10380}{173} = 60$$

$$\text{LSA} = 60 \times 60 = 3600 \text{ cm}^2$$

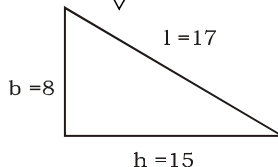
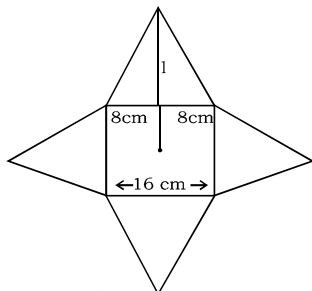
281. (b)



$$\text{Perimeter of the base} = 4 \times 16 = 64 \text{ cm}$$

Curved or lateral surface area of pyramid

$$= \frac{1}{2} \times (\text{perimeter of base}) \times \text{Slant height}$$



$$\Rightarrow \text{height of pyramid} \Rightarrow 15 \text{ cm}$$

$$\Rightarrow \text{base} = 8 \text{ cm}$$

$$\Rightarrow \text{slant height of pyramid}$$

$$l = \sqrt{(15)^2 + (8)^2} \Rightarrow 17 \text{ cm}$$

$$\Rightarrow \text{Curved surface area of pyramid}$$

$$\Rightarrow \frac{1}{2} \times 64 \times 17 \Rightarrow 544 \text{ cm}^2$$

282. (c) Volume of pyramid

$$= \frac{1}{3} \times \text{Area of base} \times \text{height}$$

$$= \frac{1}{3} \times 57 \times 10 = 190 \text{ cm}^3$$

283. (c) Let the side of square base = a cm

$$\Rightarrow 2a^2 + 4a \times h = 608$$

$$\Rightarrow 2a^2 + 4a \times 15 = 608$$

$$\Rightarrow a^2 + 30a = 304$$

$$\Rightarrow a^2 + 38a - 8a - 304 = 0$$

$$\Rightarrow a(a + 38) - 8(a + 38) = 0$$

$$\Rightarrow a = -38, 8$$

$$\Rightarrow a = 8 \text{ cm}$$

$$\therefore \text{Volume of prism} = 8 \times 8 \times 15 = 960 \text{ cm}^3$$

$$\text{284. (b) Volume of prism} = \frac{\sqrt{3}}{4} a^2 \times h$$

$$= \frac{\sqrt{3}}{4} \times (8)^2 \times 10 = 160\sqrt{3} \text{ cm}^3$$

285. (b) Volume of prism

$$= \frac{1}{2} \times 10 \times 12 \times 20 = 1200 \text{ cm}^3$$

$$\Rightarrow \text{Weight of prism} = 1200 \times 6$$

$$= 7200 \text{ gm} = 7.2 \text{ kg}$$

286. (a) Total slant surface area

$$= 4 \times \frac{1}{2} \times 4 \times a = 12$$

(where a is the side of the square base)

$$\Rightarrow a = \frac{12}{8} = \frac{3}{2} \text{ cm}$$

$$\Rightarrow \text{area of base} = \frac{9}{4} \text{ cm}^2$$

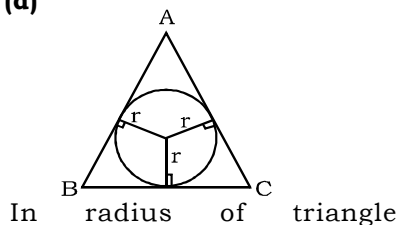
$$\therefore \text{Required ratio} = \frac{12}{9/4}$$

$$= 16 : 3$$

287 (d) Total surface area of tetrahedron

$$= \sqrt{3} a^2 = \sqrt{3} \times 12^2 = 144\sqrt{3} \text{ cm}^2$$

288. (d)



In radius of triangle

$$= \frac{\text{area of triangle}}{\text{semiperimeter}}$$

$$\therefore ar(\triangle ABC) = \text{Inradius} \times \text{semiperimeter}$$

$$= 4 \times \frac{28}{2} = 4 \times 14 = 56 \text{ cm}$$

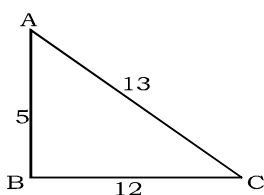
$$\text{Volume of the prism} = 366 \text{ cm}^3$$

$$(\text{area of base}) \times \text{height} = 366 \text{ cm}^3$$

$$56 \times \text{height} = 366 \text{ cm}$$

$$\text{height} = \frac{366}{56} = 6.535 \text{ cm}$$

289. (a)



Clearly the base triangle is the right triangle

$$\therefore \text{area of triangle ABC}$$

$$= \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$$

$$\text{Volume of the pyramid}$$

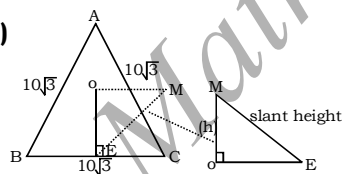
$$= \frac{1}{3} \times (\text{base area}) \times \text{height}$$

$$\frac{1}{3} \times \text{Base area} \times \text{height} = 330$$

$$\frac{1}{3} \times 30 \times \text{height} = 330$$

$$\text{height} = \frac{330 \times 3}{30} = 33 \text{ cm}$$

290. (d)



Base is equilateral triangle

In radius of equilateral triangle

$$= OE = \frac{\text{side of equilateral } \triangle}{2\sqrt{3}}$$

$$= \frac{10\sqrt{3}}{2\sqrt{3}} = 5 \text{ cm}$$

$$\text{slant length, } l = \sqrt{h^2 + OE^2}$$

$$= \sqrt{h^2 + 25}$$

$$\text{Total surface area} = 270\sqrt{3}$$

$$\frac{1}{2} (\text{perimeter of base} \times \text{slant height}) + \text{Base area} = 270\sqrt{3}$$

$$\frac{1}{2} \{30\sqrt{3} \times \sqrt{h^2 + 25}\} + \frac{\sqrt{3}}{4}$$

$$(10\sqrt{3})^2 = 270\sqrt{3}$$

$$15\sqrt{3}\sqrt{h^2 + 25} + 75\sqrt{3} = 270\sqrt{3}$$

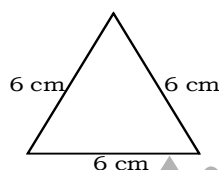
$$\sqrt{h^2 + 25} = 13$$

$$h^2 + 25 = 169$$

$$h^2 = 169 - 25 = 144$$

$$h = \sqrt{144} = 12 \text{ cm}$$

291. (a)



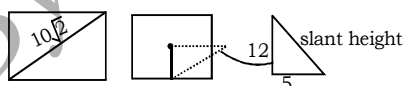
$$\text{Volume of prism} = \text{area of base} \times \text{height}$$

$$= \frac{\sqrt{3}}{4} (6)^2 \times \text{height}$$

$$\frac{\sqrt{3}}{4} \times 6 \times 6 \times \text{height} = 81\sqrt{3}$$

$$\text{height} = \frac{81\sqrt{3} \times 4}{\sqrt{3} \times 6 \times 6} = 9 \text{ cm}$$

292. (d)



$$\text{Side of square} = \frac{1}{\sqrt{2}} \times 10\sqrt{2} = 10 \text{ cm}$$

$$\text{slant height} = \sqrt{5^2 + 12^2} = 13 \text{ cm}$$

$$\text{lateral surface area} = \frac{1}{2} \times \text{perimeter of base} \times \text{slant height}$$

$$= \frac{1}{2} \times 40 \times 13 = 260 \text{ cm}^2$$

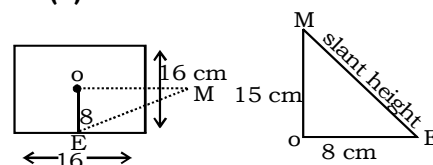
$$\text{293. (d) Total surface area of prism} = (\text{perimeter of base} \times \text{height} + 2 \times \text{base area})$$

$$= (3 \times 12 \times 10) + 2 \times \frac{\sqrt{3}}{4} \times (12)^2$$

$$= 360 + 72\sqrt{3}$$

$$= 72(5 + \sqrt{3}) \text{ cm}^2$$

294. (b)



Slant height of pyramid

$$= \sqrt{8^2 + 15^2} = 17$$

(8, 15, 17) is triplet)

lateral surface area

$$= \frac{1}{2} \times \text{perimeter of base} \times \text{slant height}$$

$$= \frac{1}{2} \times 64 \times 17$$

$$= 32 \times 17 = 544 \text{ cm}^2$$

295. (a) Perimeter of right $\triangle$

$$= (5 + 12 + 13) = 30$$

total surface area = lateral surface area + 2 $\times$ area of base

$$= (\text{perimeter of base} \times \text{height}) + 2 \times \text{area of base}$$

$$= (30 \times \text{height}) + 2 \times \frac{1}{2} \times 5 \times 12$$

$$= (30 \times \text{height}) + 60$$

ATQ,

$$30 \times \text{height} + 60 = 360$$

$$30 \times \text{height} = 360 - 60 = 300$$

$$\text{height} = 10 \text{ cm}$$

296. (d) Height of pyramid = 6 m

$$\text{Diagonal of square base} = 24\sqrt{2} \text{ m}$$

$$\text{Side of square} = 24 \text{ m}$$

$$\text{Area of square} = (24)^2 = 576 \text{ m}^2$$

Volume of the pyramid

$$= \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$= \frac{1}{3} \times 576 \times 6$$

$$= 576 \times 2 = 1152 \text{ m}^3$$

297. (a) Volume of pyramid

$$= \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$500 = \frac{1}{3} \times 30 \times \text{height}$$

$$\text{height} = \frac{500 \times 3}{30} = 50 \text{ m}$$

298. (a) Lateral surface area of prism = 120

$$\text{base perimeter} \times \text{height} = 120$$

$$\text{L.S.A of prism} = (\text{Base perimeter} \times \text{height})$$

$$3 \times (\text{side}) \times \text{height} = 120$$

$$(\text{perimeter of eq. } \triangle = 3 \times \text{side})$$

$$\text{side} \times \text{height} = \frac{120}{3} = 40 \dots (i)$$

$$\text{volume of prism} = 40\sqrt{3}$$

$$\text{area of base} \times \text{height} = 40\sqrt{3}$$

$$\frac{\sqrt{3}}{4}(\text{side})^2 \times \text{height} = 40\sqrt{3}$$

$$(\text{side})^2 \times \text{height}$$

$$= \frac{40\sqrt{3} \times 4}{\sqrt{3}} = 160 \dots (ii)$$

Dividing (ii) by (i)

$$\frac{(\text{side})^2 \times \text{height}}{\text{side} \times \text{height}} = \frac{160}{40}$$

$$\text{side} = 4 \text{ cm}$$

299.(c) Volume of tetrahedron

$$= \frac{\sqrt{2}}{12}(\text{side})^3 = \frac{\sqrt{2}}{12}(4)^3$$

$$\Rightarrow \frac{\sqrt{2} \times 4 \times 4 \times 4}{12} = \frac{16\sqrt{2}}{3} \text{ cm}^3$$

300. (a) Area of the base of prism (a right triangle)

$$= \frac{1}{2} \times 5 \times 12 = 30 \text{ cm}^2$$

Third side of the triangle

$$= \sqrt{12^2 + 5^2} = 13 \text{ cm}$$

Perimeter of the triangle

$$= 5 + 12 + 13 = 30 \text{ cm}$$

Total surface area

$$= \text{lateral surface} + 2 \times (\text{base area})$$

$$= (\text{perimeter of base} \times \text{height}) +$$

$$2 \times (\text{base area})$$

$$= (30 \times 10) + 2 \times 30$$

$$= 300 + 60 = 360 \text{ cm}^2$$

301. (c) Let radius be increased by x cm.

Volume of cylinder

$$= \pi \times 10^2 (4+x)$$

$$\therefore \pi(10+x)^2 \times 4 = \pi(10)^2 (4+x)$$

$$\Rightarrow (10+x)^2 = 25(4+x)$$

$$\Rightarrow 100 + 20x + x^2 = 100 + 25x$$

$$\Rightarrow x^2 - 5x = 0$$

$$\Rightarrow x(x-5) = 0$$

$$\Rightarrow x = 5 \text{ cm}$$

302. (a) As we know,

Volume of Right Prism = Area of the base $\times$ Height

$$\Rightarrow 7200 = \frac{3\sqrt{3}}{2} P^2 \times 100\sqrt{3}$$

$$\Rightarrow 72 \times 2 = 9P^2$$

$$\Rightarrow P^2 = 16$$

$$\Rightarrow P = 4$$

303. (b) Half of its lateral edges

$\Rightarrow$ Half of its edges

$\Rightarrow$ Half of its volume

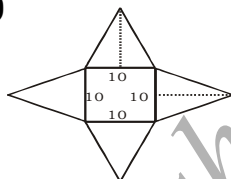
Then, volume reduced by = 50%

304. (b) Total surface area

$$= 4 \times \left[\frac{\sqrt{3}}{4} \times 1^2 \right]$$

$$= \sqrt{3} \text{ cm}^2$$

305. (a)



$$\text{Area of base} = 10 \times 10 = 100 \text{ cm}^2$$

Area of 4 Phase

$$= \left(\frac{1}{2} \times \text{Base} \times \text{slant height} \right) \times 4$$

$$\Rightarrow \left(\frac{1}{2} \times 10 \times 13 \right) \times 4$$

$$= 65 \times 4 = 260$$

$$[\text{slant height} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13]$$

Total Surface area

$$\Rightarrow 260 + 100$$

$$\Rightarrow 360 \text{ m}^2$$

306.(d) Volume of prism = (area of base $\times$ height)

Area of base (i.e area of triangle)

$\Rightarrow$ Area of base

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

= (By Hero's formula)

$$\text{So, } S = \frac{13+20+21}{2} = \frac{54}{2} = 27$$

$$\Rightarrow \sqrt{27(27-13)(27-20)(27-21)}$$

$$\Rightarrow \sqrt{27 \times 14 \times 7 \times 6}$$

$$\Rightarrow \sqrt{9 \times 3 \times 2 \times 7 \times 7 \times 2 \times 3}$$

$$\Rightarrow \sqrt{9 \times 9 \times 7 \times 7 \times 2 \times 2}$$

$$\Rightarrow 9 \times 7 \times 2$$

Volume of Prism

$$= (9 \times 7 \times 2) \times 9 = 1134 \text{ cm}^3$$

307. (d) Let the side of the square = a cm

ATQ

$$\text{T.S.A} = \text{C.S.A} + 2 \text{ base area}$$

$$\text{C.S.A} = \text{base perimeter} \times h$$

$$\text{Volume} = \text{base area} \times h$$

$$\therefore \text{T.S.A} = (\text{base perimeter} \times h) + (2 \text{ base area})$$

$$192 = 4a \times 10 + 2a^2$$

$$2a^2 + 40a - 192 = 0$$

$$a^2 + 20a - 96 = 0$$

$$a^2 + 24a - 4a - 96 = 0$$

$$a(a+24) - 4(a+24) = 0$$

$$(a+24)(a-4) = 0$$

$$\therefore a = 4, (-24)$$

$$\therefore a = 4 \text{ (Side can never be in - ve)}$$

$$\text{Volume} = \text{base area} \times h$$

$$\text{Volume} = 16 \times 10$$

$$\text{Volume} = 160 \text{ cm}^3$$

308. (c) According to the question,

V = number of vertices of prism = 6

e = edges of prism = 9

f = faces of the prism = 5

ATQ,

$$\frac{v+e-f}{2} = \frac{6+9-5}{2}$$

$$= \frac{10}{2} = 5$$

309. (c) ATQ

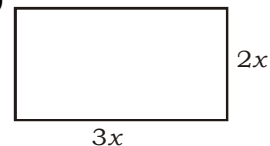
Volume of prism = Area of base $\times$ height

= trapezium area $\times$ height

$$= \frac{1}{2} (10 + 6) \times 5 \times 8$$

$$= 16 \times 5 \times 4 = 320 \text{ cm}^3$$

310. (a)



Base of prism

$\Rightarrow$ length : breadth

$$3x : 2x$$

Perimeter of base

$$= 2(3x + 2x) = 10x$$

area of base

$$\Rightarrow 2x \times 3x = 6x^2$$

height of Prism = 12 cm (given)

total surface area of prism

$$= (\text{Perimeter of base} \times \text{height}) + (2 \times \text{area of base})$$

$$288 = 10x \times 12 + 12x^2$$

$$12x^2 + 120x - 288 = 0$$

$$x^2 + 10x - 24 = 0$$

$$x = 2$$

$$\therefore \text{area of base} \Rightarrow 6 \times 4$$

$$\Rightarrow 24 \text{ cm}^2$$

$$\therefore \text{volume of prism} \Rightarrow 24 \times 12$$

$$\Rightarrow 288 \text{ cm}^3$$

311. (b) Volume of the part (prism) = Area of base $\times$ height

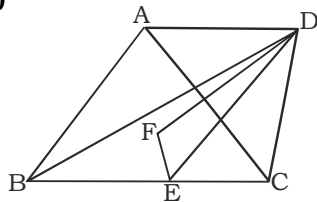
Area of base (Isoscales Δ)

$$= \frac{b}{4} \sqrt{4a^2 - b^2}$$

$$= \frac{6}{4} \sqrt{4(5)^2 - (6)^2} = 12 \text{ cm}^2$$

$$\text{Volume of prism} = 12 \times 8 = 96 \text{ cm}^3$$

312.c)



$$\text{In circle radius (EF)} = \frac{a}{2\sqrt{3}}$$

$$= \frac{18\sqrt{3}}{2\sqrt{3}} = 9 \text{ cm}$$

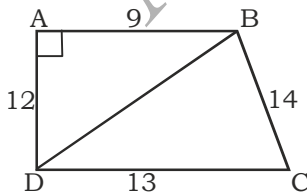
$$\text{Slant height (DE)} = \sqrt{(12)^2 + 9^2} = 15 \text{ cm}$$

Total surface area

$$= 3 \times \left(\frac{1}{2} \times 18\sqrt{3} \times 15 \right) + \frac{\sqrt{3}}{4} (18\sqrt{3})^2$$

$$= 405\sqrt{3} + 243\sqrt{3} = 648\sqrt{3} \text{ cm}^2$$

313.(a)



In ΔDAB

$$BD^2 = AD^2 + AB^2$$

$$BD^2 = (12)^2 + 9^2$$

$$BD = 15 \text{ cm}$$

$$\text{Area of } \Delta DAB = \frac{1}{2} \times AB \times AD$$

$$= \frac{1}{2} \times 9 \times 12 = 54 \text{ cm}^2$$

In ΔBDC

$$\text{Semi perimeter} = \frac{14 + 13 + 15}{2}$$

$$= 21 \text{ cm}$$

Area of Δ

$$= \sqrt{21 \times (21 - 14) \times (21 - 13) \times (21 - 15)}$$

$$= \sqrt{21 \times 7 \times 8 \times 6} = 84 \text{ cm}$$

Volume of prism = Base area $\times$ Height

$$2070 = (54 + 84) \times \text{Height}$$

$$\text{Height} = \frac{2070}{138} = 15 \text{ cm}$$

Lateral surface area = Perimeter of base $\times$ Height

$$= (9 + 14 + 13 + 12) \times 15$$

$$= 48 \times 15 = 720 \text{ cm}^2$$

314.(c) Volume of pyramid = $\frac{1}{3} \times \text{Area}$

of base $\times$ Heights

$$= \frac{1}{3} \times \frac{\sqrt{3}}{4} \times 4 \times 4 \times 20\sqrt{3}$$

$$= 80 \text{ cm}^3$$

315.(c) Slant height of pyramid

$$= \sqrt{4^2 + \left(\frac{6}{2}\right)^2} = 5 \text{ cm}$$

Total surface area = Lateral Surface area + Base area

$$= \frac{1}{2} \times \text{Perimeter of base} \times \text{Slant height} + \text{Base area}$$

$$= \left(\frac{1}{2} \times 24 \times 5 \right) + (6 \times 6)$$

$$= 60 + 36 = 96 \text{ cm}^2$$

316.(b) slant height = $\sqrt{(13)^2 - \left(\frac{10}{2}\right)^2}$

Height of pyramid

$$= \sqrt{(12)^2 - 5^2} = \sqrt{119} \text{ cm}$$

$$\text{Volume of pyramid} = \frac{1}{3} \times \text{Area}$$

of base $\times$ Height

$$= \frac{1}{3} \times 10 \times 10 \times \sqrt{119}$$

$$= \frac{100}{3} \sqrt{119} \text{ cm}^3$$

317(d) Length of tank (l) = 4m

Breadth of tank (b) = 3m

Depth of tank (d) = h

$$\therefore \text{Volume} = lbh$$

$$24 = 4 \times 3 \times h$$

$$\Rightarrow h = 2 \text{ m}$$

(As tank is open from upper sides remaining five sides of the tank will be painted)

$$\text{Area of 5 sides} = 2(hb + lh) + lb$$

$$= 2(2 \times 3 + 2 \times 4) + 4 \times 3$$

$$= 28 + 12 = 40 \text{ m}^2$$

$\therefore$ Total area to be painted inner and outer side.

$$40 \text{ m}^2 + 40 \text{ m}^2 = 80 \text{ m}^2$$

$$= 80 \text{ m}^2 \text{ cost of painting} = 80 \times 10 = 800 \text{ Rs.}$$

318. (b) $V = abc$

$$\therefore S = 2(ab + bc + ca)$$

$$\Rightarrow S = 2abc \left(\frac{1}{c} + \frac{1}{a} + \frac{1}{b} \right)$$

$$\Rightarrow S = 2V \left(\frac{1}{c} + \frac{1}{a} + \frac{1}{b} \right)$$

$$\Rightarrow \frac{1}{V} = \frac{2}{S} \left(\frac{1}{c} + \frac{1}{a} + \frac{1}{b} \right)$$

319. (b) External length (L) = 1.46 = 146 cm

External breadth (B) = 1.16 m = 116 cm

External Height (H) = 8.3 dm = 83 cm

$$\Rightarrow \text{Internal length (l)} = 146 - 6 = 140 \text{ cm}$$

$$\Rightarrow \text{Internal breadth (b)} = 116 - 6 = 110 \text{ cm}$$

$$\Rightarrow \text{Internal Height (h)} = 83 - 3 = 80 \text{ cm}$$

Total surface area of inner walls

$$= 2h(l + b) + lb$$

$$= 2 \times 80(140 + 110) + 140 \times 110$$

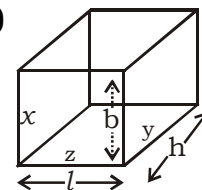
$$= 40,000 + 1540$$

$$= 55400 \text{ cm}^2$$

Cost of painting 100 cm^2 = 50 paise = 0.5 Rs.

cost of painting 55400 cm^2 = 554 $\times$ 0.5 Rs = Rs. 277 Rs.

320. (a)



$$\text{Volume} = l \times b \times h$$

$$x = hb$$

$$y = lh$$

$$z = lb$$

$$xyz = (lbh)^2$$

$$V^2 = xyz$$

- 321. (a)** Volume of bar = Volume of piece of copper
area of cross section of square $\times$ length = 1m^3
 $a^2 \times 36 = 1$

$$\Rightarrow a = \frac{1}{6}\text{m}$$

then volume of cube

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}\text{m}^3$$

Cost of cube = Volume $\times$ cost of 1m^3

$$= \frac{1}{216} \times 108 = \text{Rs. } 50 \text{ paise}$$

- 322. (d)** Let the length, breadth and height of block be $3x$, $2x$ and x

$\therefore$ Volume of block = lbh

$$3x \times 2x \times x = 10368$$

$$x = 12 \text{ dm}$$

Now, length breadth and height of block will be 36 dm, 24 dm and 12 dm respectively.

$\therefore$ Total surface area of block

$$= 2(lb + bh + hl)$$

$$= 3168 \text{ dm}^2$$

$\Rightarrow$ cost of polishing entire surface = $3168 \times 0.02 = 63.36 \text{ Rs.}$

- 323. (a)** Volume of water drawn out = (Volume of water initially in tank) - (Volume of water remained in tank)

$$= 15 \times 6 \times 10 - 15 \times 6 \times 9 \text{ m}^3$$

$$= 90,000 \text{ litre } (1\text{m}^3 = 1000 \text{ litre})$$

- 324. (c)** Volume of water will come out of pipe in 15 min

= Cross section area $\times$ Length per 15 min

$$= \frac{13}{10000} \times \frac{9000}{60} \times 15 = 2.925\text{m}^3.$$

$\therefore$ Volume of water filled in tank in 15 min

= Volume of water came out of pipe in 15 min

$$\Rightarrow 45 \times 26 \times h = 2.925$$

($\therefore$ h = level of water rise in tank)

$$\Rightarrow h = 0.0025\text{m}$$

- 325. (b)** Let the length, breadth and height of cuboid be l , b and h

$$\Rightarrow x^2 = l^2 + b^2 \dots\dots(i)$$

$$y^2 = b^2 + h^2 \dots\dots(ii)$$

$$z^2 = l^2 + h^2 \dots\dots(iii)$$

adding (i), (ii), (iii) we get,

$$x^2 + y^2 + z^2 = 2(l^2 + b^2 + h^2)$$

$$\Rightarrow l^2 + b^2 + h^2 = \frac{x^2 + y^2 + z^2}{2} \dots(iv)$$

subtracting (i) from (iv) we get,

$$h^2 = \frac{y^2 + z^2 - x^2}{2}$$

$$\Rightarrow h = \sqrt{\frac{y^2 + z^2 - x^2}{2}}$$

Subtracting (ii) from (iv) we get,

$$l^2 = \frac{x^2 + z^2 - y^2}{2} \Rightarrow l = \sqrt{\frac{x^2 + z^2 - y^2}{2}}$$

subtracting (iii) from (iv) we get

$$\therefore b^2 = \frac{x^2 + y^2 - z^2}{2}$$

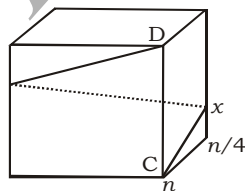
$$\Rightarrow b = \sqrt{\frac{x^2 + y^2 - z^2}{2}}$$

$$\therefore \text{Volume} = lbh$$

$$= \sqrt{\frac{(x^2 + y^2 - z^2)(y^2 + z^2 - x^2)(x^2 + z^2 - y^2)}{8}}$$

$$= \frac{\sqrt{(x^2 + y^2 - z^2)(y^2 + z^2 - x^2)(x^2 + z^2 - y^2)}}{2\sqrt{2}}$$

- 326. (b)** The string of minimum length, if starting from C, touches next corner at height $n/4$ on the completion of one turn, starting from height $n/4$ touches next corner at height $n/2$ in the second turn, and so on.



$$CX = \sqrt{n^2 + \left(\frac{n}{4}\right)^2} = \frac{\sqrt{17} \times n}{4}$$

$\Rightarrow$ Length of string

$$= 4 \times \frac{\sqrt{17} \times n}{4} = \sqrt{17}n.$$

Alternate:-

Opening up the four vertical sides of the cubes of side n ,

$$\text{Length of string} = \sqrt{(4n)^2 + (n^2)}$$

$$= \sqrt{17} \times n$$

- 327. (c)** Volume of supplied water by diameter of 6 cm = $n \times$ volume of water supplied by diameter 1.5 cm

$$\Rightarrow \pi \times \left(\frac{6}{2}\right)^2 = n \times \pi \times \left(\frac{1.5}{2}\right)^2$$

(n = no. of pipes)

$$\Rightarrow n = 16$$

- 328. (b)** Volume of cylindrical shell

$$= \frac{1}{4} \times \text{volume of solid cylinder}$$

$$\pi(R^2 - r^2) \times 1000 = \frac{1}{4} \pi R^2 \times 1000$$

$$= R^2 - r^2 = \frac{R^2}{4}$$

$$R^2 - \frac{R^2}{4} = r^2$$

$$3R^2 = 4r^2$$

$$r^2 = \frac{3 \times 10^2}{4}$$

$$r^2 = 75$$

$$r = 5\sqrt{3}$$

thickness of cylinder = radius of solid cylinder - Inner radius of cylindrical hole = $10 - 5\sqrt{3}$

$$= 5 \times (2 - \sqrt{3}) \text{ cm}$$

- 329. (b)** Let the radius of well = r
Volume of embankment (hollow cylinder)

= volume of earth taken out

$$\Rightarrow \pi((r+1)^2 - r^2) \times 5 = \pi r^2 \times 20$$

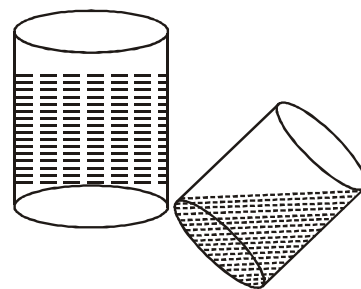
$$\Rightarrow [(r+1+r)(r+1-r)] = r^2 \times 4$$

$$\Rightarrow (2r+1) \times 1 = 4r^2$$

$$\Rightarrow 4r^2 - 2r - 1 = 0$$

$$r = \frac{2 \pm \sqrt{4+16}}{8} = \frac{\sqrt{5}+1}{4} \text{ m}$$

- 330. (d)**



Let total volume of cylinder = $5x$ litre

Volume of water = $\frac{4}{5} \times 5x = 4x$ litre

After tilting, there will be water

half of total volume $4x - \frac{5x}{2}$

$$\Rightarrow 30 = \frac{3x}{2}$$

$$\Rightarrow x = 20 \text{ litre}$$

Hence, volume of cylinder
= $5x = 100$ litre.

- 331. (b)** Curved surface area $2\pi rh$
Curved surface area of 50 pillars = $50 \times 2\pi rh$

$$= 50 \times 2 \times \frac{22}{7} \times \frac{50}{2 \times 100} \times 4 = 314 \text{ m}^2$$

Labour charge for getting these pillars cleared = 314×0.5
= Rs. 157

- 332. (c)** $r = \frac{3}{4}h$ (given)

$$\Rightarrow h = \frac{4r}{3}$$

$$\therefore \text{Volume} = \pi r^2 h = 38808 \text{ ml}$$

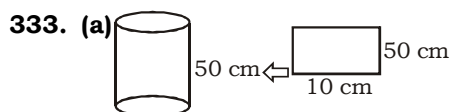
$$= 38.808 \text{ l}$$

$$= 38.808 \times 1000 \text{ cm}^3$$

$$\Rightarrow \frac{22}{7} \times r^2 \times \left(\frac{4r}{3}\right) = 38808$$

$$\Rightarrow r^3 = 9261$$

$$\Rightarrow r = 21 \text{ cm}$$



$$2\pi r = 100 \text{ cm}$$

Then, curved surface area

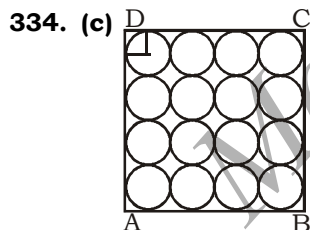
$$= 2\pi rh$$

$$100 \times 50 = 5000 \text{ cm}^2$$

Cost of painting

$$= 5000 \times \frac{50}{100 \times 100}$$

$$= \text{Rs. } 25.00$$



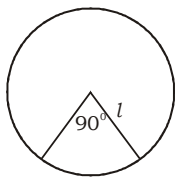
Diameter of each circle = 2 cm

$$\therefore AB = 2 \times 4 = 8$$

Similarly, CD = 8

$$\text{Area of Bottom} = AB \times CD = 64 \text{ cm}^2$$

- 335. (c)**



Slant height (l) = radius of circle
= 4 cm

r = radius of cone

$\therefore$ Perimeter of base of cone
= $2r\pi$

$$\Rightarrow 2r\pi = \frac{90^\circ}{360^\circ} \times (\text{perimeter of circle})$$

$$\Rightarrow 2r\pi = \frac{1}{4} \times 2\pi \times 4$$

$$\Rightarrow r = 1 \text{ cm}$$

$$\text{height of cone (h)} = \sqrt{l^2 - r^2}$$

$$= \sqrt{16 - 1} = \sqrt{15}$$

$\therefore$ Volume of cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (1)^2 \times \sqrt{15}$$

$$= \frac{\pi\sqrt{15}}{\sqrt{3}} \text{ cm}^3$$

- 336. (b)** Arc of Circle = $\frac{\theta}{180} \times \pi \times r$

$$= \frac{120}{180} \times \pi \times 3 = \frac{2}{3} \times \pi \times 3 = 2\pi$$

Base of cone = Arc of Circle

$$2\pi r = 2\pi$$

$$r = 1 \text{ cm}$$

$$\Rightarrow h^2 = l^2 - b^2$$

$$= (3)^2 - (1)^2 = 8$$

$$h = 2\sqrt{2} \text{ cm}$$

$$\text{Volume of Cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 1 \times h$$

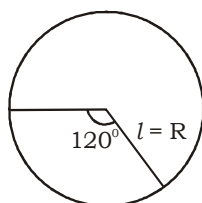
$$= \frac{1}{3} \times \pi \times 1 \times 2\sqrt{2} = \frac{2\sqrt{2}}{3} \pi$$

- 337. (b)** Slant height = H , and radius = r

$$\Rightarrow \frac{\text{curved surface area}}{\text{area of base}} = \frac{\pi r l}{\pi r^2}$$

$$= \frac{\pi r H}{\pi r^2} = \frac{H}{r} = H : r$$

- 338. (c)** Radius of cone = Slant height of Cone = 15 cm.



$\therefore$ Perimeter of base of sector = perimeter of base of cone

$$\Rightarrow \frac{120^\circ}{360^\circ} \times 2\pi (15) = 2\pi r$$

(where, r is radius. of cone)

$$\Rightarrow r = 5 \text{ cm}$$

$$h = \sqrt{l^2 - r^2} = \sqrt{(15)^2 - (5)^2}$$

$$= \sqrt{200} = 10\sqrt{2}$$

$$\therefore \text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\frac{1}{3} \times \pi \times 5 \times 5 \times 10\sqrt{2}$$

$$= [(250\sqrt{2})\pi / 3] \text{ cm}^3$$

- 339. (a)** Volume of cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 25 = 644.761$$

Volume of smaller cone

$$= 654.76 - 110 = 544.761$$

$$\therefore \frac{\text{Volume of smaller cone}}{\text{Volume of larger cone}}$$

$$= \left(\frac{\text{radius of smaller cone}}{\text{radius of large cone}} \right)$$

$$\Rightarrow \frac{544.761}{654.761} = \left(\frac{r}{5} \right)^3$$

$$\Rightarrow r = (104)^{1/3} \text{ cm}$$

- 340. (a)** Circumference of hemispherical bowl = 176

$$\Rightarrow 2\pi r = 176$$

$$\Rightarrow r = 28 \text{ cm}$$

When bowl is half full

$$= \frac{2}{3} \pi r^3 \times \frac{1}{2} = \frac{1}{3} \pi (28)^3$$

Volume of hemispherical glass

$$= \frac{2}{3} \pi (2)^3$$

No. of persons may be served =

$$\frac{\frac{1}{3} \pi (28)^3}{\frac{2}{3} \pi (2)^3} = 1372$$

- 341. (c)** Volume of cylindrical vessel = Volume of sphere

$$\pi \times (6)^2 \times h = \frac{4}{3} \times \pi \times (3)^3$$

$\therefore$ Surface of water rise (h) = 1 cm

- 428

Area of the base $\times$ Height

$$= \frac{1}{3} \times 6 \times \frac{1}{2} \times 2a \times a\sqrt{3} \times \frac{3a}{2}$$

$$= 3a^3\sqrt{3}$$

353. (c) Area of regular hexagon of

side $a = \frac{3\sqrt{3}}{2}a^2$

$$\Rightarrow \frac{3\sqrt{3}}{2}a^2 = 96\sqrt{3} \Rightarrow a = 8 \text{ m}$$

Let h be the height of the pyramid. Then area of one side face

of the pyramid $= \frac{1}{2}a \times l$, where l

is the slant height of the face.

$$\Rightarrow \frac{1}{2}a \times l = 32\sqrt{3} \Rightarrow l = 8\sqrt{3}$$

$$\Rightarrow \frac{3a^2}{4} + h^2 = l^2$$

$$\Rightarrow \frac{3 \times 64}{4} + h^2 = 64 \times 3$$

$$\Rightarrow h^2 = 64 \times 3 \left[1 - \frac{1}{4} \right] = 144$$

$$\Rightarrow h = 12 \text{ m}$$

$\therefore$ Volume of the pyramid

$$= \frac{1}{3} \times \text{Area of the base} \times h$$

$$= \frac{1}{3} \times 96\sqrt{3} \times 12 = 384\sqrt{3} \text{ m}^3$$

354. (b) Volume of tunnel $= \pi r^2 h$

$$= \frac{22}{7} \times 2 \times 2 \times 56 = 704 \text{ m}^3$$

$$\text{Volume of the ditch} = 48 \times 16.5 \times 4 = 3168 \text{ m}^3$$

$$\text{Part of the ditch filled} = \frac{704}{3168} = \frac{2}{9}$$

355. (d) Height of cylindrical rod (h) $= 8r$

$$\text{Radius of spherical ball} = \frac{r}{2}$$

Number of spherical balls

$$= \frac{\pi r^2 h}{\frac{4}{3} \pi \left(\frac{r}{2} \right)^3}$$

$$= \frac{\pi \times r^2 \times 8r}{\frac{4}{3} \times \pi \times \left(\frac{r}{2} \right)^3}$$

$$= 16 \times 3 = 48$$

356. (a) Radius of pipe (r) $= \frac{1}{4} \text{ cm}$

Radius of cone (R) $= 15 \text{ cm}$

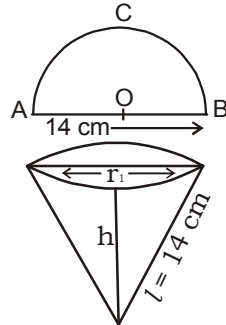
vol. passed by pipe $=$ vol of cone

$$\pi r^2 \times 1000 \times T = \frac{1}{3} \pi \times 15 \times 15 \times 24$$

[$\therefore$ speed of water $= 1000 \text{ cm/minute}$]

$$T = \frac{28800}{1000} \text{ minute} = 28 \text{ minute } 48 \text{ sec}$$

357. (b)



Length (ACB) of semi-circular sheet $= \pi r$

$$= \frac{22}{7} \times 14 = 44 \text{ cm.}$$

Slant height of the cone $= 14 \text{ cm.}$

Circumference of the base of the cone

$$= 2\pi r_1 = \frac{44}{7} r_1$$

$$\Rightarrow 44 = \frac{44}{7} r_1 \Rightarrow r_1 = 7 \text{ cm.}$$

$$\therefore h = \sqrt{14^2 - 7^2} = 7\sqrt{3} \text{ cm}$$

$$= 7 \times 1.732 = 12 \text{ cm.}$$

358. (d) Total surface area of prism

$=$ Surface area $+$ (base $+$ top) area

$608 =$ Perimeter of base $\times$ height

$+ 2 \times$ base area

$$608 = 4 \times a \times 15 + 2 \times a \times a$$

$$608 = 60a + 2a^2$$

$$a^2 + 30a = 304$$

$$a(a + 30) = 304$$

$$a = 8 \text{ cm}$$

Volume of prism $=$ Base area $\times$ Height

$$= 8 \times 8 \times 15 = 960 \text{ cm}^3$$

359. (b) A.T.Q.

$$l \times b \times h = \pi r^2 h_1$$

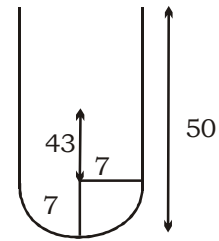
$$8 \times 11 \times 2 = \frac{22}{7} \times 4 \times 4 \times h_1$$

$$h_1 = \frac{7}{2}$$

$$h_1 = 3.5$$

360. (d) C.S.A. of storage tank $=$

C.S.A. of cylinder $+$ C.S.A. of hemisphere



$$= 2\pi rh + 2\pi r^2$$

$$= 2\pi r(h + r) = 2\pi r(43 + 7)$$

$$= 2 \times \frac{22}{7} \times 7 \times 50 = 2200 \text{ m}^2$$

361. (b) Diameter of ball $= 14 \text{ cm}$
Radius $= 7 \text{ cm}$

Height of solid cylinder $= \frac{7}{3} \text{ cm}$

A.T.Q.

Volume of ball $=$ Volume of cylinder

$$\Rightarrow \frac{4}{3} \pi r^3 = \pi r^2 h$$

$$\Rightarrow \frac{4}{3} \times \pi \times 7 \times 7 \times 7 = \pi \times r^2 \times \frac{7}{3}$$

$$\Rightarrow r^2 = 49 \times 4$$

$$\Rightarrow r = 7 \times 2 = 14 \text{ cm}$$

Diameter of the base of the cylinder

$$\Rightarrow D = 2r = 2 \times 14 = 28 \text{ cm}$$

362. (d) Inner radius $r = 8 \text{ cm}$

Outer Radius $R = 8 \text{ cm}$

Area of Circular shell

$$= \pi(R^2 - r^2) = \pi(144 - 64) = 80\pi$$

A.T.Q.

Total surface area of cylinder

$=$ area of shell

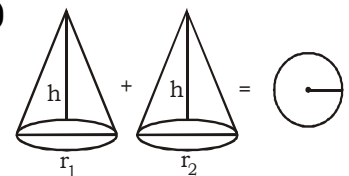
$$\Rightarrow 2\pi R_1(h + R_1) = 80\pi$$

$$\Rightarrow h + R_1 = \frac{40}{R_1}$$

$$\Rightarrow h = \frac{40}{R_1} - R_1$$

$$\Rightarrow h = \frac{40 - R_1^2}{R_1}$$

363. (c)



$$\frac{1}{3} \pi r_1^2 h + \frac{1}{3} \pi r_2^2 h = \frac{4}{3} \pi R^3$$

$$\frac{1}{3} \pi h(r_1^2 + r_2^2) = \frac{4}{3} \pi R^3$$

$$h = \frac{4R^3}{r_1^2 + r_2^2}$$

364. (d) height of cone

$$10.2 - 4.2 = 6$$

Volume of woods

$$= \frac{1}{3} \pi r^2 h + \frac{2}{3} \pi r^3 = \frac{\pi r^2}{3} (h+2r)$$

$$= \frac{\pi r^2}{3} (6 + 8.4)$$

$$\Rightarrow \frac{22}{7} \times \frac{4.2 \times 4.2}{3} \times 14.4$$

$$\Rightarrow 22 \times 0.6 \times 1.4 \times 14.4$$

$$\Rightarrow 266.112 \Rightarrow 266 \text{ (nearly)}$$

365. (b) $h_1 = h_2$

$$\frac{\pi r_1^2 h_1}{\frac{1}{3} \pi r_2^2 h_2} = \frac{3}{1}$$

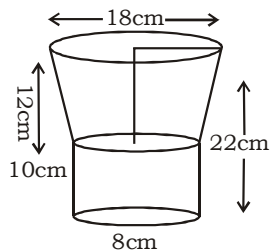
$$r_1^2 = r_2^2$$

$$r_1 = r_2$$

$$D_1 = D_2$$

(b) Diameter of cylinder = diameter of cone

366. (b)



$$R = 9$$

$$r = 4$$

$$l = \sqrt{(12)^2 + (5)^2}$$

$$l = 13$$

Area of funnel

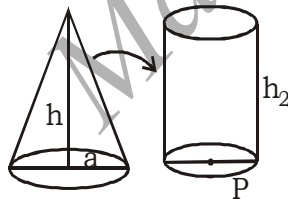
$$= 2 \pi r h \times \pi (R + r) l$$

$$= \pi [2 \times 4 \times 10 + (9 + 4) \times 13]$$

$$= 249 \pi$$

$$= 249 \times \frac{22}{7} = 782.57 \text{ cm}^3$$

367. (a)

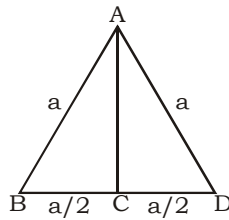


$$\pi r^2 h_2 = \frac{1}{3} \pi r^2 h$$

$$\pi p^2 h_1 = \frac{1}{3} \pi a^2 h$$

$$h_1 = \frac{a^2 h}{3 p^2}$$

368. (d)



Let one side of Δ be = a

Perimeter of equilateral triangle = $3a$

$$\therefore 3a = 72\sqrt{3} = a = 24\sqrt{3} \text{ cm}$$

Height = AC; by Pythagoras theorem

$$AC^2 = a^2 - \left(\frac{a}{2}\right)^2$$

$$AC = 36 \text{ cm}$$

369. (d) Volume of soil removed

$$= l \times b \times h$$

$$= 7.5 \times 6 \times 1.5 = 67.5 \text{ m}^3$$

370. (c) 1 hectare = 10000 m^2

$$\text{Height} = 10 \text{ cm} = \frac{1}{10} \text{ m}$$

$$\text{Volume} = 10000 \times \frac{1}{10} = 1000 \text{ m}^3$$

371. (a) Total surface area of 7 cubes

$$\Rightarrow 7 \times 6a^2 = 1050$$

But on joining end to end, 12 sides will be covered.

So there area = $12 \times a^2$

$$\Rightarrow 12 \times 25 = 300$$

So the surface area of the resulting figure = $1050 - 300 = 750$

372. (d) Let the rise in height be = h

Then, as per the question, the volume of water should be equal in both the cases.

$$\text{Now, } 90 \times 40 \times h = 150 \times 8$$

$$h = \frac{150 \times 8}{90 \times 40} = \frac{1}{3} \text{ m} = \frac{100}{3} \text{ cm}$$

$$= 33.33 \text{ cm}$$

373. (d) Area of base = $6 \times 10 = 60 \text{ m}^2$

$$\text{Volume of tent} = 30 \times 10 = 300 \text{ m}^3$$

Let the radius be = r , height = h , slant height = l

$$\pi r^2 = 60 \Rightarrow r = \frac{60}{\pi}$$

$$300 = \frac{\pi r^2 h}{3} \Rightarrow 900 = \pi \cdot \frac{60}{\pi}$$

$$h \Rightarrow h = 15 \text{ m}$$

374. (c) Volume of metal used

$$= \frac{4\pi R^3}{3} - \frac{4\pi r^3}{3}$$

$$= \frac{4\pi}{3} (12^3 - 10^3) = 3047.89 \text{ cm}^3$$

Weight = volume $\times$ density

$$\Rightarrow 4.9 \times 3047.89$$

$$\Rightarrow 14942.28 \text{ gm}$$

375. (b) The volume in both the cases will be equal. Let the height of cone be = h

$$\frac{4}{3} \times \frac{22}{7} \times (14)^3 = \frac{22}{7} \times \left(\frac{35}{2}\right)^2 \times \frac{h}{3}$$

$$\Rightarrow 4(14)^3 = h \left(\frac{35}{2}\right)^2 = h$$

$$= \frac{4 \times 14 \times 14 \times 14 \times 2 \times 2}{35 \times 35}$$

$$= h = 35.84 \text{ cm}$$

376. (c) Circumference of the circular face of the cylinder = $2\pi r$

$$\Rightarrow 2 \times \frac{22}{7} \times \frac{35}{100} = 2.2 \text{ m}$$

Number of revolutions required

$$\text{to lift the bucket by} = \frac{11}{2.2} = 5$$

377. (c) Surface area of the cube $6a^2$

$$= 6 \times (20)^2 = 2400$$

Area of 6 circles of radius 10 cm

$$= 6\pi r^2$$

$$= 6 \times \pi \times 100 = 1885.71$$

$$\text{Remaining area} = 2400 - 1884 = 514.28$$

378. (d) $x \cdot y \cdot z = l \times b \times h \times l$

$$= (lbh)^2$$

(V) Volume of a cuboid = lbh

$$\text{So } V^2 = (lbh)^2 = xyz$$

379. (a) Volume of mud dug out

$$= 10 \times 4.5 \times 3 = 135 \text{ m}^3$$

Let the remaining ground rise by = h m

$$\text{Then } \{(20 \times 9) - (10 \times 4.5)\} h = 135$$

$$135h = 135 \Rightarrow h = 1 \text{ m}$$

380. (b) Height of the cylinder

$$= 13 - 7 = 6 \text{ cm}$$

Radius of the cylinder and the hemisphere = 7 cm

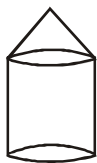
Volume of the vessel = volume of cylinder + volume of hemisphere

$$\Rightarrow \pi r^2 h + \frac{4\pi r^3}{3 \times 2}$$

$$\Rightarrow 3.14 \times (7)^2 \times 6 + \frac{4 \times 3.14 \times (7)^3}{3 \times 2}$$

$$\Rightarrow 1642.6 \text{ cm}^3$$

381. (c)



$$\text{Radius} = \frac{105}{2} = 52.5 \text{ cm}$$

Area of the entire canvas, used for the tent

$$\begin{aligned} &= \text{Area of cylinder} + \text{area of cone} \\ &= 2\pi rh + \pi r l \\ &= \pi r (2h + l) = 3.14 \times 52.5 \\ &\quad (2\sqrt{53^2 - 52.5^2} + 53) \\ &= 5 \times l \text{ (because area of canvas} = l \times b \text{ also)} \\ &= l \Rightarrow 1947 \text{ m} \end{aligned}$$

382. (c) The volume in both the cases would be the same.

$$\text{Therefore} = \frac{4\pi r^3}{3} = \pi r^2 h$$

$$\frac{4 \times 3.14 \times (4 \times 10)^3}{3} = 3.14 \times 2^2 \times h$$

$$\Rightarrow h = \frac{64000}{3} = 21333.33 \text{ mm}$$

383. (b) As the cylinder and cone have equal diameters. So they have equal area. Let cone's height be h_2 and as per question, cylinder's height be h_1 .

$$\frac{2\pi r h_1}{\pi r \sqrt{h_2^2 + r^2}} = \frac{8}{5}$$

On solving we get the desired ratio as 3 : 4

384. (a) Let the slant height of 1st cone = L

Then the slant height of 2nd cone = 3L

Let the radius of 1st cone = r_1

And let the radius of 2nd cone = r_2

$$\text{Then, } \pi r_1 L = 3 \times \pi r_2 \times 3L$$

$$\Rightarrow \pi r_1 L = 9\pi r_2 L \Rightarrow r_1 = 9r_2$$

Ratio of area of the base 81 : 1

385. (c) Let the internal radius of the cylinder = r

Then, the volume of sphere = Volume of sphere cylinder

$$\Rightarrow \frac{4\pi r^3}{3} = \pi h (5^2 - r^2)$$

$$\Rightarrow \frac{864\pi}{3} = 32\pi (25 - r^2)$$

$$\Rightarrow r^2 = 16 = r = 4 \text{ cm}$$

So thickness of the cylinder = 5 - 4 = 1 cm

386. (d) The volume in both the cases would be the same.

Let the height of the cone = h

Then, external radius = 6cm

Internal radius = 4cm

$$\Rightarrow \frac{4\pi(6^3 - 4^3)}{3} = \frac{\pi \cdot 4^2 \cdot h}{3}$$

$$\Rightarrow h = \frac{6^3 - 4^3}{4}$$

$$\Rightarrow h = \frac{216 - 64}{4} = 38 \text{ cm}$$

387. (a) Let arc side of the cube be = a units

Total surface area of 3 cubes

$$= 3 \times 6a^2 = 18a^2$$

Total surface area of cuboid

$$= 18a^2 - 4a^2 = 14a^2$$

$$\text{Ratio} = \frac{14a^2}{18a^2} = 7 : 9$$

388. (c) A = 2(xy + yz + zx)

$$V = xyz$$

$$A/V = \frac{2(xy + yz + zx)}{xyz}$$

$$= \frac{2}{z} + \frac{2}{x} + \frac{2}{y} \Rightarrow 2 \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)$$

389. (d) Radius of cylinder, hemisphere and cone = 5cm

Height of cone = 12 cm

Surface area of toy = $2\pi rh +$

$$\frac{4\pi r^2}{2} + \pi rL$$

$$L = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = 13$$

$$\text{Then } \Rightarrow (2 \times 3.14 \times 5 \times 13) + (2 \times 3.14 \times 25) + (3.14 \times 5 \times 13)$$

$$\Rightarrow 770 \text{ cm}^2$$

390. (b) Height of cone = 10.2 - 4.2 = 6cm

$$\text{Volume of wood} = \frac{\pi r^2 h}{3} + \frac{4\pi r^3}{3 \times 2}$$

$$\Rightarrow \frac{3.14 \times (4.2)^2 \times 6}{3} + \frac{4 \times 3.14 \times (4.2)^3}{3 \times 2}$$

$$\Rightarrow 266 \text{ cm}^2$$

391. (a) Volume of ice cream = $\pi r^2 h$
 $= 3.14 \times (6)^2 \times 15$
 $= 1695.6 \text{ cm}^2$

$$\text{Volume of 1 cone} = \frac{\pi r^2 \times 24}{3}$$

$$\text{Then } \Rightarrow 10 \times 3.14 \times r^2 \times 8 = 1695.6$$

$$\Rightarrow r = 3 \text{ cm (approx.)}$$

So diameter = $2 \times 3 = 6 \text{ cm}$

392. (b) Radius of cylinder and

$$\text{hemispheres} = \frac{7}{2} = 3.5 \text{ cm}$$

$$\text{Height of cylinder} = 19 - (3.5 \times 2) = 12 \text{ cm}$$

$$\text{Total surface area of solid} = 2\pi rh + 4\pi r^2$$

$$\Rightarrow 2 \times 3.14 \times 3.5 \times 12 + 4 \times 3.14 \times (3.5)^2$$

$$\Rightarrow 418 \text{ cm}^2$$

393 (c) As they stand on the same base so their radius is also same.

$$\text{Then; volume of cone} = \frac{\pi r^2 h}{3}$$

$$\text{Volume of hemisphere} = \frac{2\pi r^2 h}{3}$$

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\text{Ratio} = \frac{\pi r^2 h}{3} : \frac{2\pi r^2 h}{3} : \pi r^2 h$$

$$\Rightarrow \frac{h}{3} : \frac{2r}{3} : h \Rightarrow h : 2r : 3h$$

Radius of a hemisphere = Its height

$$\text{So } h : 2h : 3h \Rightarrow 1 : 2 : 3$$

394. (d) Total surface to be painted = external surface area + internal surface + surface area of right area

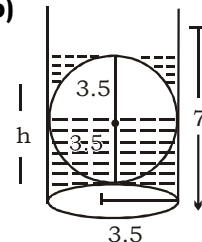
$$= 2\pi (R^2 + r^2) + 2\pi (R^2 - r^2)$$

Cost of painting

$$\Rightarrow 2 \times 3.14 \times \left\{ \left(\frac{25}{2} \right)^2 + \left(\frac{24}{2} \right)^2 + \left(\frac{25}{2} \right)^2 - \left(\frac{24}{2} \right)^2 \right\} \times 0.05$$

$$\Rightarrow 2 \times 6.28 \times (12.5)^2 \times 0.05 \times ₹ 96.28$$

395. (b)



$$\pi (3.5)^2 \times 7 - \frac{4}{3} \pi (3.5)^3$$

$$= \pi (3.5)^2 \times h$$

$$7 - \frac{4}{3} \times \frac{35}{10} = h$$

$$7 - \frac{14}{3} = h = \frac{7}{3}$$

396. (d) Radius = $\frac{3.5}{2} = 1.75$ cm

$$\text{Volume of solid} = \pi r^2 h + \frac{\pi r^2 h}{3} + \frac{2\pi r^3}{3}$$

$$\Rightarrow 3.14 \times (1.75)^2 \times (10 + \frac{6}{3} + \frac{2(1.75)}{3}) \Rightarrow 121 \text{ cm}^3$$

397. (d) Volume of the elliptical cylinder

$$= \pi \times \frac{2.4}{2} \times \frac{1.6}{2} \times 7$$

$$= 3.14 \times 1.2 \times 0.8 \times 7 \Rightarrow 9 \text{ m}^3$$

Amount of water emptied per minute = $120 \times 3.14 \times \left(\frac{2}{100}\right)^2$

Time required to empty half the tank = $\frac{4.5}{120 \times 3.14 \times (0.02)^2}$

= 70 min

398. (d) Let initial radius = r
Then volume = $\pi r^2 h$

$$\text{New radius} = r + \frac{r}{2} \Rightarrow \frac{3r}{2}$$

$$\text{New volume} = \pi \frac{9}{4} r^2 h$$

$$\text{Increased volume} = \frac{5}{4} \pi r^2 h$$

$$\text{Percent increase} = \frac{5\pi r^2 h}{4\pi r^2 h} \times 100 = 125\%$$

399. (c) Volume of the cone

$$= \frac{\pi r^2 h}{3} \Rightarrow \frac{3.14 \times 20 \times 20 \times 24}{3}$$

$$\Rightarrow 10048 \text{ cm}^3$$

Diameter of pipe = 5m

Volume of water flowing out of the pipe per minute

$$\Rightarrow 10 \times (2.5)^2 \times 3.14 \Rightarrow 196.25 \text{ cm}^3$$

Time taken to fill the tank

$$= \frac{10048}{196.25} = 51.2 \text{ mins}$$

400. (d) One side of the equilateral triangle = diameter of cone.

$$\text{Therefore radius of cone} = \frac{12}{2} = 6$$

Height of cone = Height of equilateral triangle be

$$\therefore \text{Height of cone} = \frac{\sqrt{3}a}{2} = 6\sqrt{3}$$

$$\text{Volume of cone} = \frac{\pi r^2 h}{3}$$

$$\Rightarrow \frac{\pi \times 6^2 \times 6\sqrt{3}}{3} = 72\sqrt{3}\pi \text{ cm}^3$$

401. (b) Let the radius of iron ball = r_1
Let the radius of ball = r_0
Then, as iron weights 8 times oak

$$\therefore \frac{4\pi r_0^3}{3} = \frac{8 \times 4\pi r_1^3}{3} = \frac{r_0}{r_1} = 2 \Rightarrow r_0 = 2r_1$$

So diameter of iron = $\frac{1}{2}$

diameter of oak

$$\Rightarrow \frac{1}{2} \times 18 = 9 \text{ cm}$$

402. (c) Volume of the timber = $7 \times 0.1 \times 0.1 = 0.07 \text{ cu m}$

$\therefore$ Weight of the timber = $0.07 \times 950 = 66.5 \text{ kg}$

403. (a) Volume of masonry = Length $\times$ Breadth $\times$ Height
= $81 \times 0.2 \times 4 = 64.8 \text{ m}^3$

404. (a) Speed of the river = $\frac{9}{2} \text{ km/hr}$

$$= \frac{9}{2} \times \frac{5}{18} = \frac{5}{4} \text{ m/sec.}$$

$\therefore$ required answer

$$= 10 \times 200 \times \frac{5}{4} = 2500 \text{ cub m.}$$

405. (b) 1 sq m $\times$ depth = $\frac{200}{1000}$

$$\therefore \text{depth} = \frac{1}{5} \text{ m} = \frac{1}{5} \times 100 = 20 \text{ cm.}$$

406. (c) Area of field = $500 \times 30 = 15000 \text{ m}^2$

Area of field after construction of tank = $15000 - (50 \times 20) = 1400 \text{ m}^2$

Volume of tank = Area of field after construction of tank $\times$ h
 $50 \times 20 \times 14 = 14000 \times h$
 $\Rightarrow h = 1 \text{ m}$

407. (b) Volume of cube = $a^3 = 25^3$
= 15625 cm^3
Surface = $4a^2$
= $4 \times 25 \times 25$
= 2500 cm^2

408. (c) Volume of the rectangular solid

$$= 36 \times 75 \times 80 = 21600 \text{ cu cm}$$

$$\therefore \text{Edge of the cube} = \sqrt[3]{216000} = 60 \text{ cm}$$

409. (a) Volume of the cube = $5 \times 5 \times 5 = 125 \text{ cu cm}$

$$0.625 \text{ kg} \equiv 125 \text{ cu cm}$$

$$\therefore 40 \text{ kg} \equiv \frac{125}{0.625} \times 40 = 8000 \text{ cu cm}$$

$$\therefore \text{edge} = \sqrt[3]{8000} = 20 \text{ cm.}$$

410. (a) $r + h = 37$ and $2\pi r(r + h) = 1628$

$$\text{or, } \pi r = \frac{1628}{74} = 22$$

$$\therefore r = 7 \text{ cm and } h = 37 - 7 = 30 \text{ cm}$$

$$\therefore \text{Volume} = \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 30 = 4620 \text{ cu cm.}$$

411. (c) Radius = $\frac{1}{2} h$ and height = h

$\therefore$ Whole surface

$$= 2\pi \times \left(\frac{1}{2}h\right) \times h + 2\pi \times \left(\frac{1}{2}h\right)^2$$

$$= \frac{3}{2} \pi h^2$$

412. (d) Volume of cube

$$= (22 \times 22 \times 22) \text{ cm}^3$$

Volume of 1 bullet

$$= \left(\frac{4}{3} \times \frac{22}{7} \times 1 \times 1 \times 1\right) \text{ cm}^3$$

$\therefore$ Number of bullets

$$= \left(\frac{22 \times 22 \times 22 \times 3 \times 7}{4 \times 22}\right) = 2541$$

413. (c) Let h and H be the heights of water level before and after dropping the sphere into it.

$$\text{Then, } [\pi \times (30)^2 \times H] - [\pi \times (30)^2 \times h] = \frac{4}{3} \pi \times (30)^3$$

$$\text{or } \pi \times 900 \times (H - h) = \frac{4}{3} \pi \times 27000$$

$$\text{or, } (H - h) = 40 \text{ cm.}$$

414. (b) Let outer radii = R_1 and inner radii = R_2

$$\therefore 2\pi R_1 h - 2\pi R_2 h = 44$$

[Where, h = height of pipe]

$$\Rightarrow 2 \times \frac{22}{7} \times 14 [R_1 - R_2] = 44$$

$$\Rightarrow R_1 - R_2 = \frac{1}{2} = 0.5 \text{ --- (i)}$$

and $\pi(R_1^2 - R_2^2) \times h = 99$ (given)

$$\Rightarrow \frac{22}{7} (R_1 + R_2)(R_1 - R_2) \times 14 = 99$$

$$\Rightarrow 4 \times 0.5 (R_1 + R_2) = 9$$

$$R_1 + R_2 = 4.5 \text{ --- (ii)}$$

On adding (i) and (ii) :-

$$2R_1 = 5 \Rightarrow R_1 = 2.5 \text{ cm}$$

415. (c) Required answer

$$= \frac{22 \times 22 \times 22 \times 3 \times 7}{4 \times 22 \times 1 \times 1 \times 1} = 2541$$

416. (b) Volume of 1 cylinder = $\pi r^2 h$

$$\text{Volume of 1 cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Number of cones} = \frac{\pi r^2 h}{\frac{1}{3} \pi r^2 h} = 3$$

417. (a) (i) Capacity = $(135 - 5)(108 - 5)(90 - 2.5)$ [Since cistern is open] = 1171625

$$\text{(ii) Volume of iron} = [(135 \times 108 \times 90) - (1171625)]$$

$$= 140575 \text{ cu cm}$$

418. (a) External radius of the pipe = 1.2 cm

Internal radius of the pipe

$$= (1.2 - 0.2) = 1 \text{ cm}$$

External volume

$$= \left(\frac{22}{7} \times 1.2 \times 1.2 \times 3.5 \times 100 \right)$$

$$= 1584 \text{ cu cm}$$

Internal volume

$$= \left(\frac{22}{7} \times 1 \times 1 \times 3.5 \times 100 \right)$$

$$= 1100 \text{ cu cm}$$

Volume of lead = (External Volume) - (Internal Volume)

$$= (1584 - 1100) = 484 \text{ cu cm}$$

Weight of the pipe

$$= \left(\frac{484 \times 11.4}{1000} \right) = 5.5176 \text{ kg.}$$

419. (c) Capacity = Volume = $(24 \times 12 \times 10) = 2880 \text{ cu cm}$

$$\text{Area of tin foil needed} = \text{Total surface area} = 2(lb + bh + hl)$$

$$= 2(288 + 120 + 240)$$

$$= 1296 \text{ cm}^2$$

420. (d) Volume of metal

$$= (52 \times 40 \times 29 - 48 \times 36 \times 27) = 13664 \text{ cm}^3$$

$\therefore$ Weight of metal

$$= \left(13664 \times 0.5 \times \frac{1}{1000} \right)$$

$$= 6.832 \text{ kg.}$$

421. (c) Volume of Sheet

$$= \frac{1}{2} \text{ cu m}$$

$$= \left(\frac{1}{2} \times 100 \times 100 \times 100 \right) \text{ cu cm}$$

Area of sheet = 1 hectare

= 10000 sq. metres

$$= (10000 \times 100 \times 100) \text{ sq cm.}$$

$$\text{Thickness} = \frac{\text{Volume}}{\text{Area}}$$

$$= \frac{1 \times 100 \times 100 \times 100}{2 \times 10000 \times 100 \times 100} = \frac{1}{200}$$

$$= 0.005 \text{ cm}$$

422. (b) Volume of sheet = $2 \text{ cm}^3 = 2 \times 100 \times 100 \times 100 \text{ cm}^3$

Area of sheet = 12 hectare

$$= 12 \times 10000$$

$$= 120000 \text{ m}^2 = 120000 \times 100 \times 100 \text{ cm}^2$$

$$\text{Thickness} = \frac{\text{Volume}}{\text{Area}}$$

$$= \frac{2 \times 100 \times 100 \times 100}{120000 \times 100 \times 100}$$

$$= \frac{1}{600} = 0.0017 \text{ cm}$$

$$\text{423. (b)} \quad 3.1416 \times \frac{0.01}{2} \times \frac{0.01}{2} \times h = 1$$

$$\therefore h = \frac{4}{3.1416 \times 0.01 \times 0.01}$$

$$= 12732.365 \text{ cm} = 127.3 \text{ m}$$

424. (a) Volume of hollow cylindrical tube = $\pi(R^2 - r^2)h$

$$= \frac{22}{7} ((24)^2 - (20)^2) \times 144$$

$$= \frac{22}{7} \times 44 \times 4 \times 144$$

$$= 25344\pi$$

425. (b) Volume of Hollow cylindrical Tube

$$= \pi(R^2 - r^2)h$$

$$= \frac{22}{7} \left(\left(\frac{37}{2} \right)^2 - \left(\frac{33}{2} \right)^2 \right) \times 70$$

$$= 22 \times 10 \times \left[\frac{37}{2} + \frac{33}{2} \right] \times \left[\frac{37}{2} - \frac{33}{2} \right]$$

$$= 220 \times 35 \times 2$$

$$= 15400 \text{ cm}^3$$

426. (a) Volume of collected Rain = Volume of swimming pool

$$\frac{25}{100} \left(\frac{1}{100} \times 2 \times 1000 \times 1000 \right)$$

$$= 50 \times 5 \times h$$

$$h = \frac{25 \times 2 \times 1000 \times 1000}{100 \times 100 \times 50 \times 5}$$

$$h = 20 \text{ m}$$

427. (a) Volume of collected Rain = Volume of swimming pool

$$\frac{40}{100} \left(\frac{2}{100} \times 1 \times 1000 \times 1000 \right)$$

$$= 200 \times 20 \times h$$

$$h = \frac{40 \times 2 \times 1000 \times 1000}{100 \times 100 \times 200 \times 20}$$

$$h = 2 \text{ m}$$

428. (c) Volume of the tank = 3 m^3

$$\therefore \text{Base area} \times \text{height} = 3 \text{ m}^3$$

$$\Rightarrow \text{Base area} = \frac{3}{2.56}$$

$$= 1.171875 \text{ m}^2$$

$$[\therefore \text{Volume of cuboid} = (l \times b) \times h = (\text{base area}) \times \text{height}]$$

429. (b) Total length of tape

$$= 2(l + b) + 3.75$$

$$= 2(39.5 + 9.35) + 3.75$$

$$= 101.45 \text{ cm}$$

430. (a) Area of surface to be cemented = $2 \times (l + b) \times h + (l \times b)$

i.e., area of four walls + area of floor

$$= 2 \times (21) \times 4 + (106.25)$$

$$= 274.25 \text{ m}^2$$

$$\therefore \text{Cost of cementing} = 24 \times 274.25 = \text{Rs. } 6582$$

431. (b) Total volume of water displaced by 250 men

$$= 250 \times 4 = 1000 \text{ m}^3$$

$\therefore$ Rise in water level (h)

$$= \frac{\text{Volume}}{\text{Base area}}$$

$$= \frac{1000}{80 \times 50} = 25 \text{ cm}$$

432. (c) Let each edge of smaller cube = 1 m

∴ Each edge of larger cube = 2m
and surface area of smaller cube = $6 \times (1)^2 = 6\text{m}^2$

∴ Surface area of larger cube = $6 \times (2)^2 = 24\text{m}^2$

∴ % increase in surface area =

$$\frac{24-6}{6} \times 100 = 300\%$$

Alternatively:

$$\frac{S_2}{S_1} = \left(\frac{e_2}{e_1}\right)^2 \Rightarrow \frac{S_2}{S_1} = \frac{4}{1}$$

∴ percentge increase in surface

$$\text{area} = \frac{4-1}{1} \times 100 = 300\%$$

where S = surface area, e = edge of cube.

433. (a) External volume of the box = $24 \times 16 \times 10 = 3840\text{ cm}^3$

Thickness of the wood = 5mm
= 0.5 cm

∴ Internal breadth of box

$$= 24 - 2 \times 0.5 = 23\text{ cm}$$

Internal breadth of box

$$= 16 - 2 \times 0.5 = 9\text{ cm}$$

∴ Internal volume of the box

$$= 23 \times 15 \times 9 = 3105\text{ cm}^3$$

∴ Volume of the wood = $3840 - 3105 = 735\text{ cm}^3$

Now, total weight of wood =
Volume $\times$ weight of 1 cm^3 wood
 $7350 = 735 \times \text{weight of } 1\text{ cm}^3 \text{ wood}$

∴ Weight of 1 cm^3 wood = 10 gm

434. (c) Length of tank = 120 m

But since, $\frac{120}{7} = 17\frac{1}{7}$, hence

17 cubes can be placed along
length and breadth of tank = 80
cm

But since, $\frac{80}{7} = 11\frac{3}{7}$, hence 11

cubes can be placed along
breadth and height of tank
= 50 cm.

But since, $\frac{50}{7} = 7\frac{1}{7}$, hence only

7 cubes can be placed along
height of the tank.

∴ Total volume occupied by
these cubes = $(7)^3 \times 17 \times 11 \times 7$
= 448987 cm^3

Total volume of the tank = 120
 $\times 80 \times 50 = 480000\text{ cm}^3$

∴ Area of unoccupied space =
 $480000 - 448987$

$$= 31013\text{ cm}^3 = 31.013\text{ dm}^3$$

435. (b) Surface area of the cuboid =
 $2(lb + bh + hl) = 11.6\text{ m}^2$

∴ Cost of canvas = 11.6×25
= Rs. 290

436. (c) Required surface area
= $2(9 \times 3 + 3 \times 3 + 3 \times 9) = 126\text{ cm}^2$

437. (a) Area of 4 walls
= $2(36 + 12) \times 10 = 960\text{ m}^2$

Total area of (windows + door +
chimney) = 120 m^2

∴ Net area for papering = $960 -$
 $120 = 840\text{ m}^2$

∴ Length of required paper =

$$\frac{840}{1.2} = 700\text{ m}$$

Hence, cost of papering = $700 \times$
 $0.7 = \text{Rs. } 490$

438. (d) $(x+2)^3 - x^3 = 1016$

$$\Rightarrow x = 12\text{ cm}$$

$$\text{and } x^3 - (x-2)^3 = (12)^2 - (10)^2$$

$$= 728$$

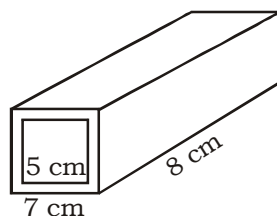
439. (b) Now let us consider that
surface area of each face of the
cube 1 cm^3 .

∴ Total surface area of the
cuboid = 14 cm^2

and Total surface area of the 3
cubes = 18 cm^2

Hence, required ratio = $14 : 18$
= $7 : 9$

440. (c) Iron used in the tube
= Difference in external and
internal volumes of the tube



$$\therefore 192 = 8x^2 - 8(5)^2$$

$$\Rightarrow x = 7\text{ cm}$$

Hence, the thickness of the tube

$$= \frac{7-5}{2} = 1\text{ cm}$$

441. (d) Base area of vessel $\times$ rise in
water level = Volume of cube

$$15 \times 12 \times h = 11 \times 11 \times 11$$

$$\Rightarrow h = 7.39\text{ cm}$$

442. (b) (Initial volume of water +
required volume of water +
volume of cube)

$$= \text{Base area of vessel} \times 10$$

$$\therefore 25 \times 20 \times 5 + \text{required volume}$$

$$\text{of water} + 1000 = 25 \times 20 \times 10$$

$$\Rightarrow \text{Required volume of water} =$$

$$1500\text{ cm}^3 = 1.5\text{ litre}$$

443. (c) Net volume = $(10 \times 8 \times 2) -$
 $(2 \times 2 \times 2) = 152\text{ cm}^3$

Net surface area = $2(10 \times 8 + 8 \times$
 $2 + 2 \times 10) + 4(2 \times 2) - 2(2 \times 2)$

$$= 240\text{ cm}^2$$

444. (b) Net volume of the wall
= Total volume - Volume taken
away due to doors

$$= (30 \times 0.3 \times 5) - 2(4 \times 2.5 \times 0.3)$$

$$= 39\text{ m}^3$$

$$\text{Volume of the bricks} = 39 \times \frac{8}{9}$$

$$\left(\text{Since } \frac{1}{9} \text{ part is lime in the wall}\right)$$

$$\therefore \text{Number of bricks}$$

$$= \frac{39 \times 8}{9 \times 0.2 \times 0.16 \times 0.08}$$

$$= 13541.66 = 13600$$

445. (a) Volume of water which flow
in 25 minutes

$$= 25 \times 60 \times 0.05 \times 0.03 \times 16$$

$$= 36\text{ m}^3$$

$$\therefore \text{Rise in water level} = \frac{36}{15 \times 12}$$

$$= \frac{1}{5}\text{ m} = 0.2\text{ m}$$

446. (b) $h : b = 3 : 1$ and $l : h = 8 : 1$
 $\Rightarrow l : h : b = 24 : 3 : 1$

$$\therefore 24x \times 3x \times x = 36.846$$

$$\Rightarrow x^3 = 0.512 \Rightarrow x = 0.8$$

$$\therefore h = 3x = 2.4\text{ m}$$

447. (b) $lb = x$, $bh = y$, $hl = z$
 $\therefore lb \times bh \times hl = xyz$

$$\Rightarrow (lbh)^2 = xyz \Rightarrow lbh = \sqrt{xyz}$$

448. (c) Volume of the block = $14 \times$
 $10.5 \times 11\text{ cm}^3$

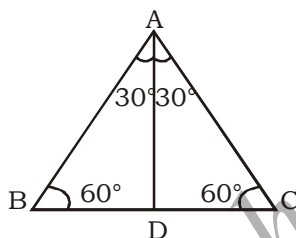
$$\begin{aligned}\text{Volume of the cylinder} &= \pi r^2 h \\ &= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times h \\ \therefore \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times h &= 14 \times 10.5 \times 11 \\ h &= \frac{14}{3} = 4\frac{2}{3} \text{ cm}\end{aligned}$$

450. (c) Since radius and height of the cylinder are same as that of cone. Therefore cylinder can contain $15 \times 3 = 45$ litre of milk.

But the remaining arc is equal

$$\therefore \frac{\text{Radius}}{\text{Height}} = \frac{24}{7}$$

$$\frac{BD}{AB} = \frac{1}{2}$$


$$= 2\frac{2}{3} \text{ cm}$$

459. (b) Side of Tetrahedron = $\frac{\sqrt{3}}{\sqrt{2}} H$

$$3P^2 = 8a^2$$

$$l = \sqrt{\frac{a^2}{3} + h^2}$$

461.(a) Side of tetrahedron = $\frac{\sqrt{3}}{\sqrt{2}} H$

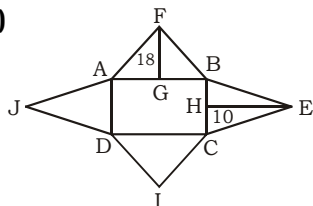
$$= \frac{\sqrt{3}}{\sqrt{2}} \times 4\sqrt{3} = 6\sqrt{2}$$

$$= 72\text{cm}^3$$

- 462. (a)** Volume of rain = 1.5 hectare $\times$ 10cm

$$= 1.5 \times 100 \times 100 \times \frac{1}{10} = 1500 \text{ cm}^3$$

- 463. (b)**



$$\text{Area of base (ABCD)} = 18 \times 10 = 180$$

In $\triangle AFB$

$$GF^2 = (12)^2 + 5^2$$

$$GF = 13 \text{ cm}$$

$$\begin{aligned} \text{Area of } \triangle AFB &= \frac{1}{2} \times 13 \times 18 \\ &= 117 \text{ cm}^2 \end{aligned}$$

In $\triangle BEC$

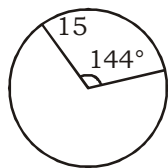
$$EH^2 = (12)^2 + 9^2$$

$$EH = 15 \text{ cm}$$

$$\begin{aligned} \text{Area of } \triangle BEC &= \frac{1}{2} \times 15 \times 10 \\ &= 75 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Total surface area of Pyramid} &= 180 + 2 \times 117 + 2 \times 75 \\ &= 180 + 234 + 150 = 564 \text{ cm}^2 \end{aligned}$$

- 464. (c)**



Remaining portion of circle of

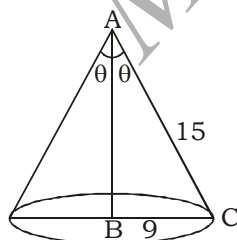
$$\text{circumference} = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{216}{360} \times 2 \times \frac{22}{7} \times 15 = 18\pi$$

$$\text{Circumference of base of cone} = 18\pi$$

$$2\pi r = 18\pi$$

$$r = 9$$



In $\triangle ABC$,

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + 9^2 = 15^2$$

$$AB^2 = 225 - 81$$

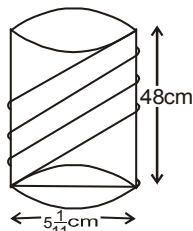
$$AB = 12 \text{ cm}$$

$$\sin \theta = \frac{BC}{AC} = \frac{9}{15}$$

$$\theta = \sin^{-1} \frac{3}{5}$$

$$2\theta = 2\sin^{-1} \frac{3}{5}$$

- 465. (d)** According to the given information, the string will be bounded on a cylindrical tank as shown in the figure,



The above figure, will look like the figure (below), when we open it.



The base circumference

$$= 2\pi r = 2 \times \frac{22}{7} \times \frac{56}{11} \times \frac{1}{2} = 16\text{cm}$$

$\therefore$ AM = length of one complete

$$\text{turn} = \sqrt{16^2 + 48^2}$$

$$= 50\text{cm}$$

$$\therefore \text{Total length} = 4 \times 20 = 80\text{cm}$$

- 466. (c)** Whole surface area of cone :

hemisphere : cylinder

$$= \pi r(l + r) : 3\pi r^2 : 2\pi r(r + h)$$

$$= \pi r(\sqrt{2}r + r) : 3\pi r^2 : 2\pi r(r + r)$$

$$= (\sqrt{2} + 1)r : 3r : 4r$$

$$= (\sqrt{2} + 1) : 3 : 4$$

- 467. (c)** Since, b, l and 2(l + b) are in GP, therefore

$$\frac{l}{b} = \frac{2(l+b)}{l}$$

$$\text{Suppose } \frac{l}{b} = x,$$

$$\text{then } x = 2\left(1 + \frac{1}{x}\right)$$

$$\Rightarrow x^2 - 2x - 2 = 0$$

$$\Rightarrow x = \sqrt{3} + 1$$

- 468. (a)** Lateral surface area of cylinder (A) = $2\pi rh$

$\therefore$ Base area is decreased by 1/9th times

$\therefore$ side (radius) will decrease by 1/3 time

$$[\because \text{area} \propto (\text{side})^2]$$

$$\therefore \text{new radius} = r' = r/3$$

and new height = $h' = 6h$

$\therefore$ new lateral surface area of cylinder

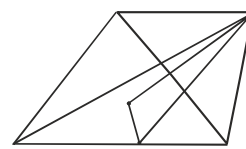
$$\Rightarrow A' = 2\pi\left(\frac{r}{3}\right) \times (6h) = 4\pi rh$$

der

$$\Rightarrow A' = 2(2\pi rh) = 2A$$

i.e. $A' = 2$ times of A.

- 469. (b)**



$$\text{Inradius} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$l = 2h$$

$$(2h)^2 = h^2 + \left(\frac{2}{\sqrt{3}}\right)^2$$

$$4h^2 = h^2 + \frac{4}{3}$$

$$3h^2 = \frac{4}{3}$$

$$h^2 = \frac{4}{9}$$

$$h = \frac{2}{3}$$

Volume of pyramid = $\frac{1}{3} \times \text{Area of base} \times \text{Height}$

$$\frac{1}{3} \times \frac{\sqrt{3}}{4} \times 4 \times 4 \times \frac{2}{3} = \frac{8}{3\sqrt{3}} \text{ cm}^3$$

- 470. (b)** Let the side of the cube and radius of the sphere be a and r respectively

$$\therefore 6a^2 = 4\pi r^2 \Rightarrow a = r\left(\frac{2}{3}\pi\right)^{\frac{1}{2}}$$

$$\frac{V_1}{V_2} = \frac{a^3}{\frac{4}{3}\pi r^3} = \frac{r^3\left(\frac{2}{3}\pi\right)^{\frac{3}{2}}}{\frac{4}{3}\pi r^3}$$

$$= \frac{2\sqrt{2}\pi^{3/2}}{3\sqrt{3}} \times \frac{3}{4\pi} = \sqrt{\pi} : \sqrt{6}$$

471. (a) $V(\text{cone}) : V(\text{hemisphere}) : V(\text{cylinder})$

$$= \frac{1}{3} \pi r^2 \cdot r : \frac{2}{3} \pi r^3 : \pi r^2 \cdot r$$

$$= 1 : 2 : 3$$

472. (d) Volume of original cube = $n \times$ volume of smaller cubes

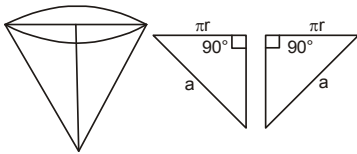
$$\Rightarrow n = \frac{3 \times 3 \times 3}{1 \times 1 \times 1} = 27$$

473. (c) $\therefore$ H.C.F of 6, 9 and 12 = 3
 $\therefore$ Least possible number of

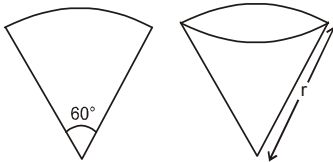
$$\text{cubes} = \frac{6 \times 9 \times 12}{3 \times 3 \times 3} = 24$$

[Note :- The cuboid is cut into smaller cubes of equal size i.e. size should be maximum]

474. (b) It will be in the form of a right angled triangle.



475. (a) Arc of sector = $2\pi r \frac{60}{360} = \frac{2\pi r}{6}$



This arc of sector will be equal to the perimeter of cone. Let the radius of cone be R, then:

$$2\pi R = \frac{2\pi r}{6} \Rightarrow R = \frac{r}{6}, \text{ and slant height of cone } (l) = \text{radius of sector} = r$$

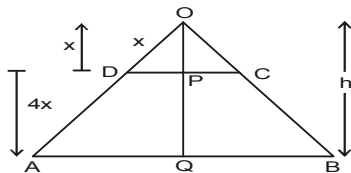
$$\therefore h = \sqrt{l^2 - R^2} = \sqrt{r^2 - \frac{r^2}{36}} = \frac{\sqrt{35}}{6} r$$

476. (d) Volume of water displaced = volume of sphere

$$\pi(30)^2 \times x = \frac{4}{3} \pi(20)^3$$

$$\Rightarrow x = \frac{320}{27} = 11 \frac{23}{27} \text{ cm}$$

477. (a) $\triangle ODC \sim \triangle OAB$



$$\therefore \frac{OP}{OQ} = \frac{1}{5} = \frac{PC}{BQ}$$

Since, the ratio in radii of the two cones is 1 : 5.

There fore the radius of smaller

$$\text{cone ODC is } \frac{21}{5} = 4.2 \text{ cm.}$$

478. (c) $\frac{4}{3} \pi r^3 = \frac{1}{3} \pi r^2 h$

(since radii of sphere and cone are same)

$$\Rightarrow 4r = h$$

$$\therefore \frac{h}{r} = \frac{4}{1} \Rightarrow h : r = 4 : 1$$

479. (b) Area of large cube = $6(5)^2 = 150 \text{ (unit)}^2$

Area of cuboid = $2(1 \times 1 + 1 \times 125 + 125 \times 1) = 502 \text{ sq. units}$

$\therefore$ % increase in surface area =

$$\frac{502 - 150}{150} \times 100 = 234 \frac{2}{3} \%$$

480. (a) H.C.F of 75, 15, 4.5 = 1.5
 $\therefore$ No. of cubes

$$= \frac{75 \times 15 \times 4.5}{1.5 \times 1.5 \times 1.5} = 1500$$

Area of each cube = $6(1.5)^2$

Area of all the 1500 cubes

$$= 1500 \times 6 \times (1.5)^2 = 20,250 \text{ cm}^2$$

481. (c) The solid with the least number of sides will have maximum surface area. So, tetrahedron will have maximum surface area. Notice that in a sphere there are infinite number of sides with least possible length. So, the surface area of the sphere will be least.

482. (d) $\frac{4}{3} \pi (r_1^3 + r_2^3 + r_3^3) = \frac{4}{3} \pi (6)^3$

$$\Rightarrow 27 + 64 + r_3^3 = 216$$

$$\Rightarrow r_3^3 = 125 \Rightarrow r = 5 \text{ cm}$$

483. (b) In radius (r) = 9cm

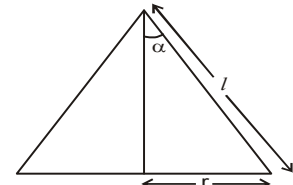
$$= \frac{\text{Area (A)}}{\text{semiperimeter (s)}}$$

$$\Rightarrow A = 9 \times \frac{45}{2} \text{ cm}^2$$

$$V = \text{Area of base (A)} \times h$$

$$\Rightarrow h = \frac{810}{9 \times 45} \times 2 = 4 \text{ cm}$$

484. (a)



Lateral surface area = $2 \times$ Base area

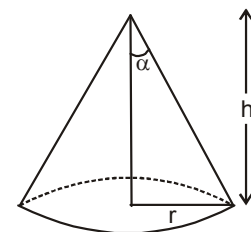
$$\Rightarrow \pi r l = 2\pi r^2 \Rightarrow l = 2r$$

$$\sin \alpha = \frac{r}{l} = \frac{r}{2r} = \frac{1}{2}$$

$$\Rightarrow \alpha = 30^\circ$$

485. (c) $\cot \alpha = \frac{h}{r} \Rightarrow h = r \cot \alpha$

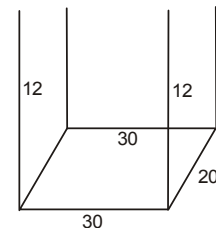
let no. of cones = n



$$\therefore n \left(\frac{1}{3} \pi r^2 \times r \cot \alpha \right) = \pi r^2 (2r \cot \alpha)$$

$$\Rightarrow n = 6$$

486. (a)



Total surface area of tank

$$TS A = 30 \times 20 + 2(12 \times 20) + 2(30 \times 12) = 1800$$

$\therefore$ area of iron sheet = T.S.A

$$\Rightarrow \text{Length} \times \text{width} = 1800$$

$$\Rightarrow \text{Length} = \frac{1800}{3} = 600 \text{ m}$$

$$\therefore \text{Cost} = 600 \times 10 = ₹ 6000$$

487. (d) Volume of prism = (Base area) $\times$ height.

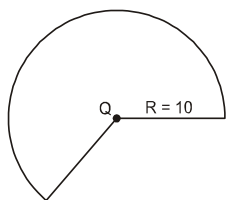
Base Area = Area of trapezium

$$= \frac{1}{2} (8 + 14) \times 8 = 88$$

$$\therefore 1056 = 88 \times \text{height}$$

$$\text{height} = \frac{1056}{88} = 12 \text{ cm}$$

488. (b)



Let radius of conical surface = r

$$\therefore 2\pi r = 60\% \text{ of } 2\pi R$$

$$\Rightarrow r = \frac{3}{5} \times 10 = 6 \text{ cm}$$

and slant height of cone = $l = R = 10 \text{ cm}$

$$\therefore \text{height}(h) = \sqrt{l^2 - r^2} = \sqrt{(10)^2 - (6)^2} = 8 \text{ cm}$$

$$\therefore r:h = 6:8 = 3:4$$

489. (c) we have,

$$2\pi R_1(R_1 + h) = \pi(12^2 - 8^2)$$

$$\Rightarrow R_1 + h = \frac{80}{2R_1} = \frac{40}{R_1}$$

$$\Rightarrow h = \frac{40}{R_1} - R_1 = \frac{40 - R_1^2}{R_1}$$

490. (a) Total surface area of pipe (hollow cylinder)

$$= 2\pi(R + r)[h + (R - r)]$$

here $R - r = \text{thickness} = 1 \text{ cm}, h = 20 \text{ cm}$

$$R = \frac{25}{2} = 12.5 \text{ cm}$$

$$\therefore r = 12.5 - 1 = 11.5$$

$$\begin{aligned} \text{Area} &= 2 \times \frac{22}{7} (12.5 + 11.5)(20 + 1) \\ &= 44 \times 72 = 3168 \text{ cm}^2 \end{aligned}$$

491. (d) $v_1 = 64 \text{ ltr}$ $v_2 = 216 \text{ ltr}$

$$\therefore \frac{r_1}{r_2} = \left(\frac{64}{216} \right)^{1/3} = \frac{4}{6} = \frac{2}{3} \quad [\therefore v \propto r^3]$$

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2} \right)^2 = \frac{4}{9}$$

$$[\therefore v \propto r^2]$$

492. (b) Let length (l) = $15R$

$\therefore$ breadth (b) = $5R$

and height (h) = $3R$

$$\therefore \text{volume (V)} = lbh = 15R \times 5R \times 3R$$

$$\Rightarrow 14400 = 225R^3$$

$$\Rightarrow R^3 = 64$$

$$\Rightarrow R = 4$$

$$\therefore l = 60 \text{ cm}, b = 20 \text{ cm}, h = 12 \text{ cm}$$

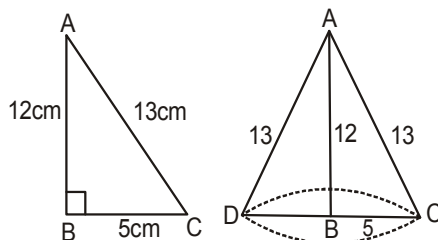
$\therefore$ Total surface area

$$= 2(lb + bh + hl)$$

$$= 2(1200 + 240 + 720)$$

$$= 4320 \text{ cm}^2$$

493. (c)



i.e. after revolution, a cone of radius 5 cm & height 12 cm is formed

$\therefore$ volume of the cone

$$= \frac{1}{3} \pi (5)^2 \times 12 = 100\pi$$

$$= 100 \times 3.14 = 314 \text{ cm}^3$$

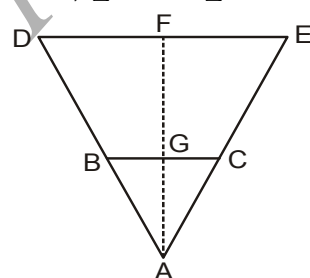
494. (a) $V_1 = \frac{2}{3} \pi r^3$, $V_2 = \pi r^2 r = \pi r^3$

$$V_3 = (2r)^3 = 8r^3$$

$$\Rightarrow V_1 < V_2 < V_3$$

495. (b) Let r be the radius and h be the height of the cone.

Since, $\triangle AGC \sim \triangle AFE$



$$\therefore \frac{AG}{AF} = \frac{GC}{FE} \Rightarrow GC = \frac{h/2}{h} \times r = \frac{r}{2}$$

$\therefore$ volume of water filled in cone

$$= \frac{1}{3} \pi \left(\frac{r}{2} \right)^2 \times \frac{h}{2} = \frac{1}{8} \left(\frac{1}{3} \pi r^2 h \right) = \frac{V}{8}$$

496. (c) Height of cone = height of cylinder = radius of hemisphere = r units

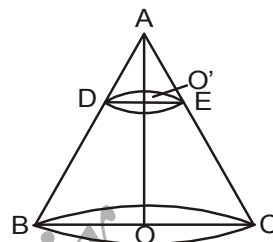
$\therefore$ Ratio of the volumes of cone, cylinder and hemisphere

$$= \frac{1}{3} \pi r_1^2 h : \pi r_2^2 h : \frac{2}{3} \pi r^3$$

$$= \frac{1}{3} \pi 2^2 r^3 : \pi 3^2 r^3 : \frac{2}{3} \pi r^3$$

$$= \frac{4}{3} : 9 : \frac{2}{3} = 4 : 27 : 2$$

497. (b)



Let $DO' = r$ cm and $OO' = h$ cm, From similar triangles ADO' and ABO ,

$$\frac{AO'}{AO} = \frac{DO'}{BO} \Rightarrow \frac{9-h}{9} = \frac{r}{3}$$

$$\Rightarrow 9-h = 3r \Rightarrow h = 9-3r$$

$$\text{volume of frustum} = \frac{1}{3} \pi h (r_1^2 + r_1 r_2 + r_2^2)$$

$$\Rightarrow 44 = \frac{1}{3} \times \frac{22}{7} (9-3r)(9+r^2+3r)$$

$$\Rightarrow 44 = \frac{22}{7} (3-r)(3^2+3r+r^2)$$

$$\Rightarrow \frac{44 \times 7}{22} = 3^3 - r^3$$

$$\Rightarrow 14 = 27 - r^3 \Rightarrow r^3 = 27 - 14 = 13$$

$$\therefore r = \sqrt[3]{13} \text{ cm}$$

498. (c) Let the height of cylinder be h cm and radius of base = r cm

$$\therefore 2\pi r^2 + 2\pi rh = 462 \text{ (i)}$$

Area of curved surface = $2\pi rh$

$$= \frac{1}{3} \times 462 = 154$$

$$\therefore 2\pi r^2 + 154 = 462$$

$$\Rightarrow 2\pi r^2 = 462 - 154 = 308$$

$$\Rightarrow 2 \times \frac{22}{7} \times r^2 = 308$$

$$\Rightarrow r^2 = \frac{308 \times 7}{2 \times 22} = 49 \Rightarrow r = 7 \text{ cm}$$

$$\therefore 2\pi rh = 154 \Rightarrow 2 \times \frac{22}{7} \times 7 \times h = 154$$

$$\Rightarrow h = \frac{154}{2 \times 22} = \frac{7}{2} \text{ cm}$$

$$\therefore \text{Volume of cylinder} = \pi r^2 h$$

$$= \frac{22}{7} \times 7 \times 7 \times \frac{7}{2} = 539 \text{ cm}^3$$

$$\begin{aligned}
 499. (d) \quad \therefore 2\pi r^2 &= \pi r \sqrt{r^2 + h^2} \\
 \Rightarrow 2r &= \sqrt{r^2 + h^2} \Rightarrow 4r^2 = r^2 + h^2 \\
 \Rightarrow 3r^2 &= h^2 \Rightarrow \sqrt{3}r = h \Rightarrow \frac{r}{h} = \frac{1}{\sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 500. (a) \quad \frac{r}{h} &= \frac{4}{3} \Rightarrow \frac{r}{4} = \frac{h}{3} = k \\
 \Rightarrow r &= 4k, h = 3k \\
 \therefore l &= \sqrt{r^2 + h^2} = \sqrt{16k^2 + 9k^2} \\
 &= \sqrt{25k^2} = 5k \\
 \therefore \frac{\text{Curved surface area}}{\text{Total surface area}} &= \frac{\pi r l}{\pi r(r + l)} = \frac{l}{r + l} = \frac{5k}{4k + 5k} = \frac{5}{9}
 \end{aligned}$$

$$\begin{aligned}
 501. (c) \quad \text{Volume of the cylinder} &= \pi r^2 h \\
 &= \frac{22}{7} \times 10 \times 10 \times 21 = 6600 \text{ cu.cm} \\
 \text{Volume of the cone} &= 6600 - 4400 = 2200 \text{ cu.cm.} \\
 \therefore 2200 &= \frac{1}{3} \pi \times 10^2 \times h \\
 \Rightarrow 2200 &= \frac{2200}{21} \times h \Rightarrow 21 \text{ cm.}
 \end{aligned}$$

$$\begin{aligned}
 502. (c) \quad \text{Radius of the base} &= r \text{ units} \\
 \text{and height} &= h \text{ units} \\
 \Rightarrow \frac{\text{Curved surface of cylinder}}{\text{Curved surface of cone}} &= \frac{2\pi r h}{\pi r l} \\
 \Rightarrow \frac{8}{5} &= \frac{2h}{l} \Rightarrow \frac{4}{5} = \frac{h}{\sqrt{h^2 + r^2}} \\
 \Rightarrow \frac{16}{25} &= \frac{h^2}{h^2 + r^2} \Rightarrow \frac{h^2 + r^2}{h^2} = \frac{25}{16} \\
 \Rightarrow 1 + \frac{r^2}{h^2} &= \frac{25}{16} \Rightarrow \frac{r^2}{h^2} = \frac{25}{16} - 1 = \frac{9}{16} \\
 \Rightarrow \frac{r}{h} &= \frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 503. (b) \quad \text{Curved surface area of cylinder} &= 2\pi r h \\
 \text{and volume} &= \pi r^2 h \\
 \therefore \frac{\pi r^2 h}{2\pi r h} &= \frac{924}{264} \Rightarrow \frac{r}{2} = \frac{924}{264} \\
 \Rightarrow r &= \frac{924 \times 2}{264} = 7 \text{ metre}
 \end{aligned}$$

$$\begin{aligned}
 \therefore 2\pi r h &= 264 \\
 \Rightarrow 2 \times \frac{22}{7} \times 7 \times h &= 264
 \end{aligned}$$

$$\begin{aligned}
 \therefore h &= \frac{264}{2 \times 22} = 6 \\
 \therefore \frac{\text{Diameter}}{\text{Height}} &= \frac{2 \times 7}{6} = \frac{7}{3}
 \end{aligned}$$

$$\begin{aligned}
 504. (b) \quad \text{Volume of bigger cube} &= 6 \times 6 \times 6 = 216 \text{ cu. cm} \\
 \text{Volume of unit cube} &= 1 \times 1 \times 1 = 1 \text{ cu.cm} \\
 \text{Number of uncoloured cubes} &= 4 \times 4 \times 4 \\
 &= 64, \text{ because edge of uncoloured cube} = 4 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 505. (c) \quad \text{Area of the base} &= 6 \times \frac{\sqrt{3}}{4} \times (2a)^2 \\
 &= 6 \times \frac{\sqrt{3}}{4} \times 4a^2 = 6\sqrt{3}a^2 \text{ sq.cm.} \\
 \text{Height} &= \sqrt{\left(\frac{5a}{2}\right)^2 - (2a)^2} \\
 &= \sqrt{\frac{25}{4}a^2 - 4a^2} = \sqrt{\frac{9a^2}{4}} = \frac{3}{2}a \text{ cm.} \\
 \therefore \text{volume of pyramid} &= \frac{1}{3} \times \text{area of base} \times \text{height} \\
 &= \frac{1}{3} \times 6\sqrt{3}a^2 \times \frac{3}{2}a = 3\sqrt{3}a^3 \text{ cm}^3
 \end{aligned}$$

$$\begin{aligned}
 506. (c) \quad \text{Volume of right prism} &= \text{Area of the base} \times \text{height} \\
 \Rightarrow 10380 &= 173 \times h \\
 \Rightarrow h &= \frac{10380}{173} = 60 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, Area of triangle} &= \frac{\sqrt{3}}{4} \times (\text{Side})^2 \\
 \Rightarrow 173 &= \frac{\sqrt{3}}{4} \times (\text{Side})^2
 \end{aligned}$$

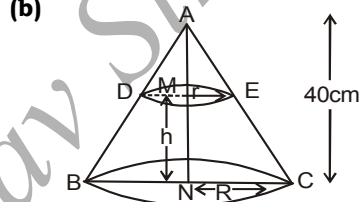
$$\begin{aligned}
 \therefore \text{Side} &= \sqrt{\frac{173 \times 4}{\sqrt{3}}} = \sqrt{\frac{173 \times 4}{1.73}} = 20 \text{ cm} \\
 \therefore \text{Perimeter} &= 3 \times 20 = 60 \text{ cm} \\
 \therefore \text{Area of the lateral surface} &= \text{Perimeter base} \times \text{height} \\
 &= 60 \times 60 = 3600 \text{ sq.cm.}
 \end{aligned}$$

$$507. (c) \quad \text{Volume of the new ball}$$

$$\begin{aligned}
 &= \frac{3}{4} \times \frac{4}{3} \pi (r_1^3 + r_2^3 + r_3^3) \\
 &= \pi (1^3 + 2^3 + 3^3) \\
 &= \pi (1 + 8 + 27) = 36\pi \text{ cubic cm} \\
 \therefore \frac{4}{3} \pi r^3 &= 36\pi \Rightarrow r^3 = \frac{36 \times 3}{4} = 27
 \end{aligned}$$

$$\therefore r = \sqrt[3]{27} = 3 \text{ cm}$$

$$508. (b)$$



Let the radius of small cone = r and height = h from the base,

$$\begin{aligned}
 \therefore \frac{1}{64} \times \frac{1}{3} \pi R^2 h &= \frac{1}{3} \pi r^2 (40 - h) \\
 \Rightarrow \frac{R^2}{r^2} &= (40 - h) \times \frac{8}{5} \text{ --- (i)}
 \end{aligned}$$

$$\begin{aligned}
 \triangle AME &\sim \triangle ANC \\
 \therefore \frac{40 - h}{40} &= \frac{r}{R} \text{ --- (ii)}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{from (i) and (ii)} \\
 \therefore \left(\frac{40}{40 - h} \right)^2 &= (40 - h) \times \frac{8}{5}
 \end{aligned}$$

$$\Rightarrow (40 - h)^3 = 25 \times 40 = 125 \times 8$$

$$\Rightarrow 40 - h = 5 \times 2 = 10$$

$$\Rightarrow h = 40 - 10 = 30 \text{ cm}$$

Short Trick:-

Bigger Smaller cone

Ratio of vol. $\frac{64}{1}$

Ratio of (height $\sqrt[3]{64} = 4$

$$\sqrt[3]{1} = 1$$

/radius / slant height)

i.e. 4 represent = 40

$$\Rightarrow 4 \cong 40 \text{ cm}$$

$$\therefore 1 \cong \frac{40}{4} = 10 \text{ cm}$$

$$\begin{aligned}
 \therefore \text{Required height} &= h \\
 &= 40 - 10 = 30 \text{ cm}
 \end{aligned}$$

$$509. (d) \quad \text{Let Edge reduced from } 2 \rightarrow 1$$

$$2 \rightarrow 1$$

$$2 \rightarrow 1$$

$$\begin{aligned} 2 &\rightarrow 1 \\ 2^3 &\rightarrow 1 \\ 8 &\rightarrow 1 \end{aligned}$$

Edge of the smaller cube = 1 metre

$\therefore$ Required ratio = 1 : 64

510. (b) $2\pi r = 22$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 22 \Rightarrow r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} \text{ cm}$$

$$h = 15 \text{ cm}$$

$\therefore$ Volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 15 = 577.5 \text{ cm}^3$$

511. (a) Volume of the original cone

$$= \frac{1}{3} \pi r^2 h$$

Volume of the new cone

$$= \frac{1}{3} \pi 4r^2 h \times 2h = 8 \left(\frac{1}{3} \pi r^2 h \right)$$

$\therefore$ Percentage increase

$$= \frac{7 \left(\frac{1}{3} \pi r^2 h \right)}{\frac{1}{3} \pi r^2 h} \times 100 = 700\%$$

512. (c) Total surface area of prism
= Curved surface area + $2 \times$ Area of base

$$\Rightarrow 608 = \text{Perimeter of base} \times \text{height} + 2 \times \text{Area of base}$$

$$\Rightarrow 608 = 4x \times 15 + 2x^2$$

(Where x = side of square)

$$\Rightarrow x^2 + 30x - 304 = 0$$

$$\Rightarrow x^2 + 38x - 8x - 304 = 0$$

$$\Rightarrow x(x + 38) - 8(x + 38) = 0$$

$$\Rightarrow (x - 8)(x + 38) = 0$$

$$\Rightarrow x = 8$$

$$\begin{aligned} \text{Volume} &= \text{Base area} \times \text{Height} \\ &= 8^2 \times 15 = 960 \text{ cm}^3 \end{aligned}$$

513. (b) If the length of the pipe be h cm, then

$$\text{Its volume} = \pi r_1^2 h - \pi r_2^2 h$$

$$= \pi h(r_1^2 - r_2^2) = \pi h(25^2 - 24^2)$$

$$= 49\pi h \text{ cu.cm.}$$

$$\Rightarrow \text{Volume of new cylinder}$$

$$\Rightarrow \pi r^2 h = 49\pi h$$

$$\Rightarrow r^2 = 49$$

$$\Rightarrow r = \sqrt{49} = 7 \text{ cm}$$

$$\Rightarrow \text{Diameter} = 14 \text{ cm}$$

514. (a) New volume = $\frac{1}{3} \pi r^2 h$

$$\therefore \frac{1}{3} \pi (2r)^2 \times h' = \frac{1}{3} \pi r^2 h$$

$$\Rightarrow 4h' = h \Rightarrow h' = \frac{h}{4}$$

515. (b) The diagonal of cube will be equal to the diameter of the sphere.

$$\therefore \text{volume of sphere} = \frac{4}{3} \pi \left(\frac{d}{2} \right)^3 = \frac{\pi d^3}{6}$$

$$\text{and each side of cube} = a = \frac{d}{\sqrt{3}}$$

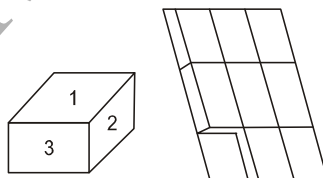
$$\therefore \text{volume of cube} = a^3 = \frac{d^3}{3\sqrt{3}}$$

$\therefore$ Remaining volume

$$= \frac{\pi d^3}{6} - \frac{d^3}{3\sqrt{3}}$$

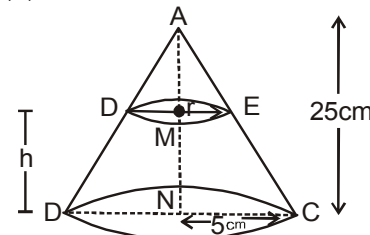
$$= \frac{d^3}{3} \left(\frac{\pi}{2} - \frac{1}{\sqrt{3}} \right)$$

516. (d)



Since, there are three faces which are visible in a corner cube. When the cube of corner is removed then the 3 faces of other cubes will be visible from outside. So, there will not be any change in the surface area of this solid figure.

517. (a)



$$\triangle AME \sim \triangle ANC$$

$$\therefore \frac{25-h}{25} = \frac{r}{5}$$

$$\Rightarrow h = 25 - 5r \text{ (i)}$$

Volume of frustum (V)

$$= \frac{1}{3} \pi [5^2 + r^2 + 5r] h$$

$$\Rightarrow 110 = \frac{1}{3} \pi [25 + r^2 + 5r] (25 - 5r)$$

$$\Rightarrow \frac{5}{3} \pi [(5-r)(5^2 + r^2 + 5r)] = 110$$

$$\Rightarrow \frac{5}{3} \pi [5^3 - r^3] = 110$$

$$\Rightarrow 5^3 - r^3 = \frac{110 \times 3}{5\pi}$$

$$\Rightarrow 125 - r^3 = \frac{110 \times 3}{5 \times \frac{22}{7}}$$

$$\Rightarrow r = (104)^{1/3} \text{ cm}$$

Short Trick :-

$$R^3 - r^3 = \frac{\text{Volume of frustum} \times 3}{R\pi}$$

518. (a)



(Front view) (Side View)

$$\text{Polished area} = 4\pi (r)^2$$

Non-polished area

$$= 4 \times \left(2 \times \frac{\pi r^2}{2} \right) = 4\pi r^2$$

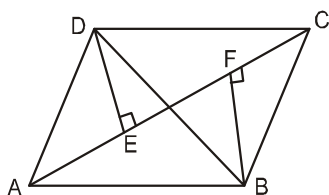
In the adjoining figure one of the four parts of the sphere is shown. (To understand it properly, take an apple and cut it in the four parts one across horizontal and another cut make vertical to it then you will notice that in a piece there are 2 semi-circles.)

Therefore required ratio = 1 : 1

QUADRILATERAL

QUADRILATERAL

A plane figure bounded by four line segments AB, BC, CD and DA is called a quadrilateral. It is denoted by symbol ' $\square$ ' i.e. $\square$ ABCD.



Pairs of consecutive (adjacent) angles:

$(\angle A, \angle B)$, $(\angle B, \angle C)$, $(\angle C, \angle D)$, $(\angle D, \angle A)$

Pairs of adjacent sides :

(AB, BC) , (BC, CD) , (CD, DA) and (DA, AB)

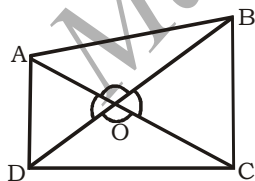
Properties

- Sum of four interior angles is 360° . i.e.

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

The figure formed by joining the mid-points of a quadrilateral is a parallelogram.

1. If the diagonals intersect at right angle then sum of square of opposite sides are equal
 $AB^2 + DC^2 = AD^2 + BC^2$



Proof:- Apply Pythagoras theorem in all right angled $\triangle$
 $OA^2 + OB^2 = AB^2$
 $OB^2 + OC^2 = BC^2$
 $OD^2 + OC^2 = DC^2$
 $OA^2 + OD^2 = AD^2$

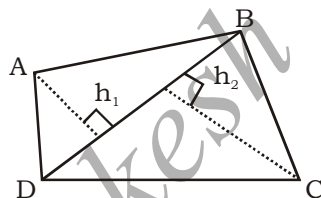
Add all

$$\begin{aligned} 2(OA^2 + OB^2 + OC^2 + OD^2) \\ = AB^2 + BC^2 + DC^2 + AD^2 \\ 2(AB^2 + DC^2) = AB^2 + BC^2 + DC^2 + AD^2 \end{aligned}$$

$$AB^2 + DC^2 = BC^2 + AD^2$$

2. Area of a quadrilateral

$= \frac{1}{2} \times \text{One diagonal} \times \text{sum of the perpendiculars drawn to the diagonals from the opposite vertices.}$



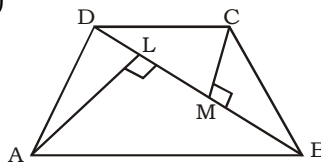
$$\begin{aligned} \text{Area } (\square ABCD) &= \text{Area } (\triangle ABD) + \text{Area } (\triangle BDC) \\ &= \frac{1}{2} \times BD \times h_1 + \frac{1}{2} \times BD \times h_2 = \end{aligned}$$

$$\frac{1}{2} \times BD \times (h_1 + h_2)$$

EXAMPLES

1. ABCD is a quadrilateral in which diagonal BD = 64 cm. $AL \perp BD$ and $CM \perp BD$, such that AL = 13.2 cm and CM = 16.8 cm. The area of the quadrilateral ABCD in square centimeters is:
 (a) 422.4 (b) 690.0 (c) 537.6 (d) 960.0

Sol. (d)



$$\text{Area of } \triangle ABD = \frac{1}{2} \times AL \times BD =$$

$$\frac{1}{2} \times 13.2 \times 64 = 422.4 \text{ cm}^2$$

$$\text{Area of } \triangle BCD = \frac{1}{2} \times CM \times BD =$$

$$\frac{1}{2} \times 16.8 \times 64 = 537.6 \text{ m}^2$$

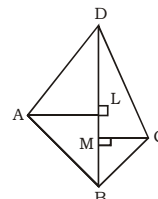
$$\begin{aligned} \Rightarrow \text{Area of quadrilateral ABCD} \\ = \text{Area of } \triangle ABD + \text{Area of } \triangle BCD = 422.4 + 537.6 = 960 \text{ cm}^2 \end{aligned}$$

Alternate

$$\text{Area of } \square ABCD = \frac{1}{2} \times BD \times (h_1 +$$

$$h_2) = \frac{1}{2} \times 64 (13.2 + 16.8) = 32 \times 30 = 960 \text{ cm}^2$$

2. In a quadrilateral ABCD, it is given that BD = 16 cm. If $AL \perp BD$ and $CM \perp BD$ such that AL = 9 cm and CM = 7 cm, then ar (quad. ABCD) = ?



- (a) 256 cm² (b) 128 cm² (c) 64 cm² (d) 96 cm²

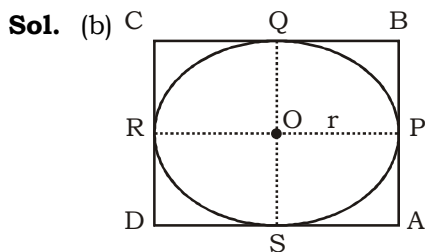
Sol. (b) Area of quadrilateral ABCD

$$= \frac{1}{2} \times BD (AL + CM)$$

$$= \frac{1}{2} \times 16 \times (9 + 7) = 128 \text{ cm}^2$$

3. ABCD is a quadrilateral such that $\angle D = 90^\circ$. A circle C (o, r) touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm. Find r.

- (a) 7 cm (b) 14 cm
 (c) 13 cm (d) 11 cm



(Two tangents drawn from an external point to a circle are equal)

$$\therefore BP = BQ = 27 \text{ cm}$$

$$\Rightarrow CQ = BC - BQ = 38 - 27 = 11 \text{ cm}$$

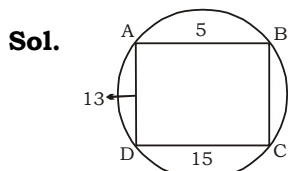
$$\Rightarrow CQ = CR = 11 \text{ cm}$$

$$\Rightarrow DR = CD - CR = 25 - 11 = 14 \text{ cm}$$

$\therefore OR \perp CD$ and $OS \perp AD$ (line drawn from centre to tangent will be perpendicular to tangent)

$$\Rightarrow \text{Radius } (r) = DR = 14 \text{ cm}$$

4. ABCD is a cyclic quadrilateral. If $AB = 5 \text{ cm}$, $CD = 15 \text{ cm}$, $DA = 13 \text{ cm}$. If diagonals intersect at right angle. Find BC ?



Using formula

$$\Rightarrow AB^2 + CD^2 = BC^2 + AD^2$$

$$5^2 + 15^2 = BC^2 + 13^2$$

$$25 + 225 = BC^2 + 169$$

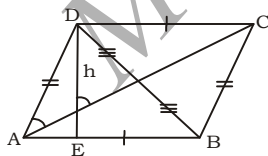
$$BC^2 = 81$$

$$BC = 9 \text{ cm}$$

TYPES OF QUADRILATERAL

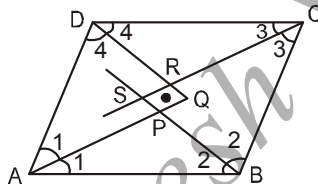
1. Parallelogram

If opposite sides of a quadrilateral are parallel, it is called a parallelogram. Its opposite sides are also equal in length and its diagonals bisect each other.

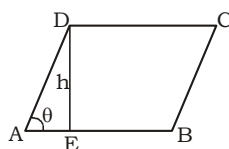


Properties

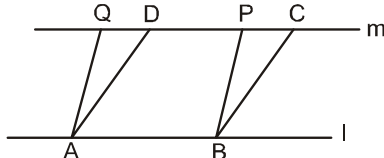
- (i) The opposite sides are equal and parallel.
- (ii) Opposite angles are equal. ($\angle A = \angle C$) and ($\angle B = \angle D$)
- (iii) Sum of any two adjacent angles are 180° .
- (iv) Diagonals bisect each other.
- (v) Diagonals need not be equal in length.
- (vi) Diagonals need not bisect at right angle.
- (vii) Each diagonal divides a ||gm into two congruent triangles. i.e. $\triangle ABC \cong \triangle ADC$ and $\triangle ABD \cong \triangle BCD$.
- (viii) Bisectors of the angles of a ||gm form a rectangle. i.e. PQRS is a rectangle.



- (ix) A ||gm inscribed in a circle is a rectangle.
- (x) A ||gm circumscribed about a circle is a rhombus.
- (xi) Area of ||gm ABCD = Base $\times$ height = $AB \times h$ = $AB \times AD \sin \theta$

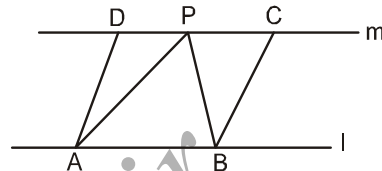


- (xii) A ||gm is a rectangle if its diagonals are equal.
- (xiii) ||gm that lie on the same base and between the same parallel lines are equal in area, i.e.



if $l \parallel m$, then $\text{ar}(\square ABCD) = \text{ar}(\square ABPQ)$

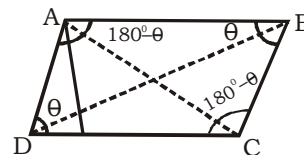
(xiv) if $l \parallel m$, and ||gm ABCD and $\triangle APB$ made on the same base AB then,



$$\text{ar}(\triangle APB) = \frac{1}{2} \text{ar}(\square ABCD)$$

(xv) Sum of square of diagonals is equal to sum of square of all sides

$$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2 = 2(AB^2 + BC^2)$$



Proof :- Apply cosine formula

$$\cos(180^\circ - \theta) = \frac{AD^2 + AB^2 - BD^2}{2AD \cdot AB} \quad \dots(i)$$

$$\cos \theta = \frac{AD^2 + DC^2 - AC^2}{2AD \cdot DC} \quad \dots(ii)$$

From (i) & (ii)

$$-\frac{AD^2 + AB^2 - BD^2}{2AD \cdot AB} = \frac{AD^2 + DC^2 - AC^2}{2AD \cdot DC}$$

$$\therefore AB = DC \text{ \& } AD = BC$$

$$BD^2 - AD^2 - AB^2 = AD^2 + DC^2 - AC^2$$

$$BD^2 - BC^2 - DC^2 = BC^2 + DC^2 - AC^2$$

$$2DC^2 + 2BC^2 = BD^2 + AC^2$$

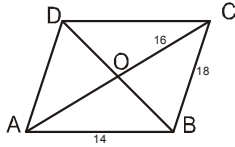
$$AC^2 + BD^2 = DC^2 + BC^2 + AB^2 + AD^2$$

EXAMPLES

1. $\square ABCD$ is a $\parallel gm$, $AB = 14\text{cm}$, $BC = 18\text{cm}$ and $AC = 16\text{cm}$. Find the length of the other diagonal?

- (a) 30cm (b) 32cm
(c) 26cm (d) 28cm

Sol. (d)



$$2(AB^2 + BC^2) = AC^2 + BD^2$$

$$\Rightarrow BD^2 = [2(196 + 324)] - 256$$

$$BD^2 = 784 \Rightarrow BD = 28\text{cm}$$

2. The perimeter of a $\parallel gm$ is 22cm. If the longer side measures 6.5cm. What is the measure of the shorter side?

- (a) 5.5cm (b) 4.5cm
(c) 6.0cm (d) 5.0cm

Sol. (b) Perimeter of $\parallel gm = 22\text{cm}$

$$\Rightarrow 2(a + b) = 22\text{cm} \Rightarrow a + b = 11$$

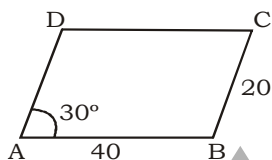
$$\Rightarrow b = 11 - a = 11 - 6.5 = 4.5\text{cm}$$

$\therefore$ shorter side, $b = 4.5\text{cm}$

3. $ABCD$ is a parallelogram, $\angle DAB = 30^\circ$, $BC = 20\text{ cm}$ and $AB = 40\text{ cm}$. Find the area of parallelogram:

- (a) 150 cm^2 (b) 200 cm^2 (c) 400 cm^2 (d) 260 cm^2

Sol. (c)



$\therefore AD \parallel BC$

$$\Rightarrow \angle ABC = 180^\circ - 30^\circ = 150^\circ$$

$\therefore$ Area of parallelogram

$$\Rightarrow AB \cdot BC \sin 150^\circ$$

$$\Rightarrow 40 \times 20 \times \sin 150^\circ$$

$$\Rightarrow 400\text{ cm}^2$$

4. If an angle of a $\parallel gm$ is two-third of its adjacent angle, then the largest angle of $\parallel gm$:

- (a) 72° (b) 60°
(c) 108° (d) 120°

Sol. (c) Since, adjacent angles of a $\parallel gm$ are supplementary.

$$\therefore x + \frac{2}{3}x = 180^\circ \Rightarrow \frac{5x}{3} = 180^\circ$$

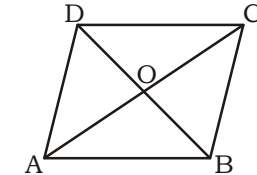
$$\Rightarrow x = 108^\circ$$

$$\therefore \frac{2}{3}x = \frac{2}{3} \times 108^\circ = 72^\circ$$

$\therefore$ Angles are $= 108^\circ, 72^\circ, 108^\circ, 72^\circ$

$\therefore$ largest angle $= 108^\circ$

5. In the given figure, $ABCD$ is a $\parallel gm$ in which diagonals AC and BD intersect at O . If $\text{ar}(\parallel gm ABCD)$ is 56cm^2 , then the $\text{ar}(\triangle OAB) = ?$



- (a) 28cm^2 (b) 22cm^2
(c) 42cm^2 (d) 14cm^2

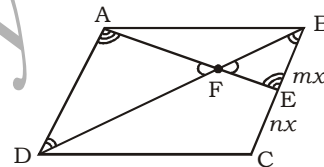
Sol. (d)

$$\text{ar}(\triangle OAB) = \frac{1}{4} \text{ar}(\parallel gm ABCD)$$

$$= \frac{1}{4} \times 56 = 14\text{cm}^2$$

6. $ABCD$ is a parallelogram. E is a point on BC such that $BE : EC = m : n$. If AE and DB intersect at F , then what is the ratio of the area of $\triangle FEB$ to the area of $\triangle AFD$?

Sol.



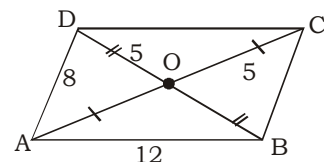
$AFD \sim EFB$

$$\frac{\text{Ar}(\triangle EFB)}{\text{Ar}(\triangle AFD)} = \frac{EB^2}{AD^2}$$

$$= \frac{mx^2}{(mx + nx)^2} = \left[\frac{m}{m + n} \right]^2$$

7. The adjacent sides of a parallelogram are 12 cm. and 8 cm. and its one diagonal is 10 cm. then other diagonal is:

Sol.



Diagonals of parallelogram bi-

sect each other

So In $\triangle ADB$ apply appollonious theorem.

$$8^2 + 12^2 = 2(5^2 + AO^2)$$

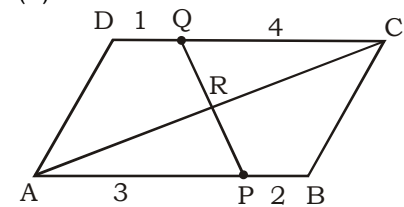
$$AO^2 = 79 \Rightarrow AO = \sqrt{79}$$

$$\therefore AC = 2\sqrt{79}$$

8. $ABCD$ is a parallelogram AB is divided at P and CD at Q so that $AP:PB = 3:2$ and $CQ:QD = 4:1$ if PQ meets AC at R then $AR = ?$

- (a) $\frac{2}{7}AC$ (b) $\frac{3}{7}AC$
(c) $\frac{4}{7}AC$ (d) $\frac{5}{7}AC$

Sol. (b)



$AB \parallel CD$

In $\triangle ARP$ and $\triangle QRC$

$\Rightarrow \angle RAP = \angle RCQ$ (alternate interior angles)

$\Rightarrow \angle RPA = \angle RQC$

$\triangle ARP \sim \triangle RCQ$ (similar triangle)

$$\Rightarrow \frac{RC}{AR} = \frac{QC}{AP} = \frac{4}{3}$$

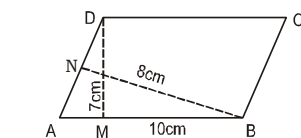
adding 1 both the sides

$$\Rightarrow \frac{RC}{AR} + 1 = \frac{4}{3} + 1$$

$$\Rightarrow \frac{RC + AR}{AR} = \frac{7}{3}$$

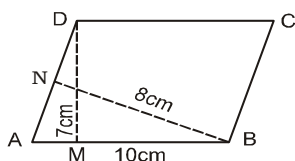
$$\Rightarrow AR = \frac{3}{7} AC$$

9. In $\parallel gm ABCD$, $AB = 10\text{cm}$. The altitude corresponding to the sides AB and AD are 7cm and 8cm respectively. Find AD :



- (a) 8.50cm (b) 8.25cm
(c) 8.75cm (d) 9.00cm

Sol. (c)



$$\begin{aligned} \therefore \text{Area of } \parallel\text{gm } ABCD &= \text{Base} \times \text{Height} \\ &= (10 \times 7) \text{cm}^2 \quad \dots(i) \\ \text{Also, ar}(\parallel\text{gm } ABCD) &= AD \times BN \\ &= (AD \times 8) \text{cm}^2 \quad \dots(ii) \end{aligned}$$

$$\Rightarrow AD = \frac{35}{4} = 8.75 \text{cm}$$

10. ABCD is a $\parallel\text{gm}$ and $\angle DAB = 60^\circ$. If the bisectors AP and BP of angles A and B respectively, meet at P on CD, then :

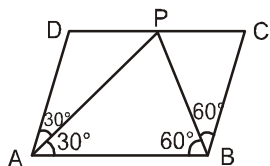
(a) $CP = 2DP$ (b) $CP = \frac{1}{2}DP$

(c) $CP = \frac{1}{3}DP$ (d) $CP = DP$

Sol. (d) $\angle DAB = 60^\circ$

$$\Rightarrow \angle B = 120^\circ$$

$$\therefore \angle ABP = \angle PBC = \frac{120^\circ}{2} = 60^\circ$$



$\angle DPA = \angle BAP = 30^\circ$ [$\because AB \parallel DC$ and AP intersects them]

Thus, In $\triangle ADP$,

$$\angle DPA = \angle DAP = 30^\circ$$

$$\Rightarrow AD = DP \quad \dots(i)$$

Similarly, $\angle BPC = \angle ABP = 60^\circ$

$$\therefore \text{In } \triangle BPC, \angle BPC = \angle PBC = 60^\circ$$

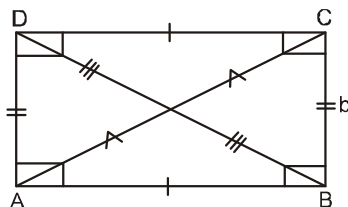
$$\Rightarrow BC = CP = AD \quad \dots(ii) \quad (\because BC = AD)$$

$$\therefore \text{from (i) and (ii) } CP = DP.$$

2. Rectangular

A parallelogram is called a rectangle if its all angles are 90° . Hence every rectangle is a parallelogram but every parallelogram is not a rectangle.

If diagonals of a parallelogram are equal (i.e. $AC = BD$) then it is a rectangle.



Properties :

(i) Diagonals are equal and bisect each other, but not necessarily bisect at right angles.

(ii) For the given perimeter of rectangles, a square has maximum area.

(iii) The figure formed by joining the mid-points of the adjacent sides of a rectangle is rhombus.

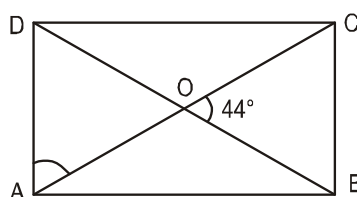
(iv) Area of rectangle ABCD = length $\times$ breadth = $l \times b$

(v) Diagonals of a rectangle = $\sqrt{l^2 + b^2}$

(vi) Bisectors of the angles of a rectangle form another rectangle.

EXAMPLES

1. The diagonals of rectangle ABCD meet at O. If $\angle BOC = 44^\circ$, then $\angle OAD$ is equal to :



(a) 90° (b) 60° (c)

100° (d) 68°

Sol. (d) The diagonals of a rectangle bisect each other.

$$\therefore OA = OD \Rightarrow \angle ODA = \angle OAD$$

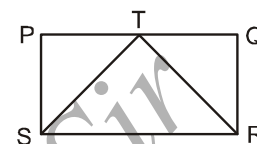
$$\text{But, } \angle AOD = 44^\circ$$

(vertically opposite angle to $\angle BOC$)

$$\therefore \angle OAD = \frac{1}{2}(180^\circ - 44^\circ)$$

$$= \frac{1}{2}(136^\circ) = 68^\circ$$

2. In the given figure, what is the ratio of the area of the $\triangle STR$ to the area of the rectangle PQRS?



(a) 1 : 4 (b) 1 : 2 (c) 1 :

(d) 2 : 1

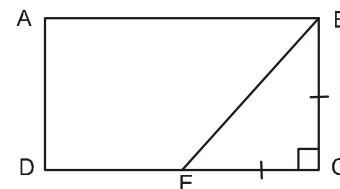
Sol. (b)

$$\frac{\text{Area of } (\triangle STR)}{\text{Area of (quadrilateral PQRS)}} =$$

$$\frac{\frac{1}{2}(SR \times PS)}{(SR \times PS)}$$

$$= \frac{1}{2} = 1 : 2$$

3. The diagram below, ABCD is a rectangle. The area of isosceles right angle $\triangle BCE$ is 14cm^2 and $DE = 3EC$. What is area of ABCD ?



(a) 56 (b) 84 (c)

112 (d) $3\sqrt{28}$

Sol. (c) Area of $(\triangle BCE) =$

$$\frac{1}{2} \times x \times x$$

$=$

$$14 \Rightarrow x^2 = 28$$

$$\therefore \text{Area of } (\square ABCD) = (DE + EC) \times x$$

$=$

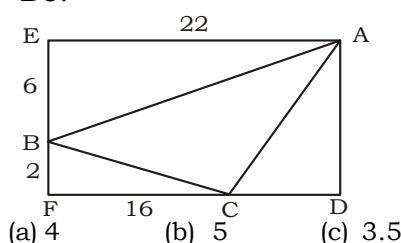
$$4EC \times x = 4x \cdot x$$

$$\Rightarrow 4x^2 = 4 \times 28 =$$

$$112$$

4. In the given figure EADF is a rectangle and ABC is a triangle whose vertices lie on the sides

of $\square EADF$. $AE = 22$, $BE = 6$, $CF = 16$ and $BF = 2$. Find the length of the line joining the mid-points of the sides AB and BC .



(d) $4\sqrt{2}$

Sol. (b) $EF = AD = 8$ ($\because EADF$ is a rectangle)
 $CD = 22 - 16 = 6$

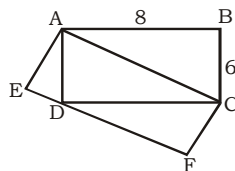
So, In right angled $\triangle ADC$,

$$AC = \sqrt{8^2 + 6^2} = 10$$

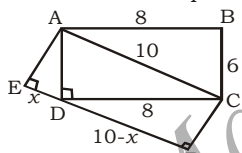
$\therefore$ length of the line joining the mid-points of AB & $BC =$

$$\frac{1}{2}(AC) = 5$$

5. $ABCD$ is a rectangle of dimensions 8 units and 6 units. $AEFC$ is a rectangle drawn in such way that diagonal AC of the first rectangle is one side and side opposite to it is touching the first rectangle at D as shown in the figure. What is the ratio of the area of rectangle $ABCD$ to that of $AEFC$?



Sol.



$$AC = \sqrt{(8)^2 + (6)^2} = 10$$

$AC = EF$ ($AEFC$ is rectangle)

Let $ED = x$

then, $DF = 10 - x$

In $\triangle AED$, $AE^2 = AD^2 - ED^2$

$$= 36 - x^2 \quad \dots(i)$$

In $\triangle CFD$, $CF^2 = CD^2 - DF^2$

$$= 64 - (10 - x)^2 \quad \dots(ii)$$

Both (i) & (ii) is equal

$$36 - x^2 = 64 - (100 + x^2 - 20x)$$

$$20x = 72 \Rightarrow x = \frac{18}{5}$$

$$AE^2 = AD^2 - ED^2 \\ = 6^2 - \left(\frac{18}{5}\right)^2 = \frac{24}{5}$$

$$\frac{\text{area of } \square ABCD}{\text{area of } \square AEFC} = \frac{8 \times 6}{10 \times \frac{24}{5}} = \frac{1}{1}$$

6. If the perimeter of a rectangle is P unit and its diagonal is d unit, then the difference between the length and width of the rectangle is-

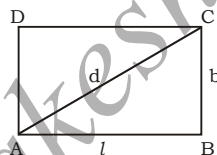
$$(a) \sqrt{\frac{8d^2 - P^2}{4}} \text{ unit}$$

$$(b) \sqrt{\frac{8d^2 - P^2}{2}} \text{ unit}$$

$$(c) \sqrt{\frac{8d^2 + P^2}{2}} \text{ unit}$$

$$(d) \sqrt{\frac{8d^2 + P^2}{4}} \text{ unit}$$

Sol. (a)



$$\text{Perimeter } (P) = 2(l+b)$$

$$\text{diagonal } (d) = \sqrt{l^2 + b^2}$$

$$\Rightarrow d^2 = l^2 + b^2$$

$$\Rightarrow \frac{P^2}{4} = l^2 + b^2 + 2lb$$

from eq. [(ii) - (i)] we get

$$\frac{P^2}{4} - d^2 = 2lb$$

Formula

$$(l - b)^2 = l^2 + b^2 - 2lb$$

Now, putting value of P and d

$$\therefore (l - b)^2 = d^2 - \frac{P^2}{4} + d^2 = \frac{8d^2 - P^2}{4}$$

$$\Rightarrow l - b = \sqrt{\frac{8d^2 - P^2}{4}} \text{ unit}$$

7. If l , b and p be the length, breadth and perimeter of a rectangle and b , l and p are in GP

(in order) then $\frac{l}{b}$ is-

$$(a) 2:1 \quad (b) (\sqrt{3} - 1):1$$

$$(c) (\sqrt{3} + 1):1 \quad (d) 2:\sqrt{3}$$

Sol. (c) $P = 2(l+b)$

$\therefore b$, l and P are in G.P.

$$\Rightarrow l^2 = bp$$

$$\Rightarrow l^2 = b \times 2(l+b)$$

$$\Rightarrow \frac{l^2}{b^2} = 2\left(\frac{l}{b} + 1\right)$$

$$\Rightarrow \left(\frac{l}{b}\right)^2 - 2\left(\frac{l}{b}\right) - 2 = 0$$

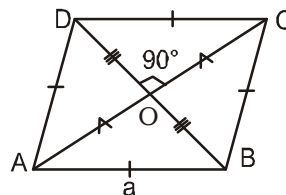
$$\frac{l}{b} = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2+2\sqrt{3}}{2}$$

$$= (\sqrt{3} + 1):1$$

(Negative factor can not consider other wise value of l/b will be negative which is not possible)

3. Rhombus

If all the sides of a parallelogram are equal it is rhombus. Diagonals of a rhombus bisect each other at right angle. i.e.



... (i) **Properties**

(i) $AB = BC = CD = DA = a$ (say)

(ii) Diagonals bisect each other at right angle, but they are not necessarily equal.

(iii) A rhombus may or may not be a square but all squares are rhombus.

(iv) The figure formed by joining the mid-points of the adjacent sides of a rhombus is a rectangle.

(v) A || gm is a rhombus if its diagonals are perpendicular to each other.

(vi) (a) Area of rhombus = $\frac{1}{2} \times$ product of diagonals
 $= \frac{1}{2} \times d_1 \times d_2$

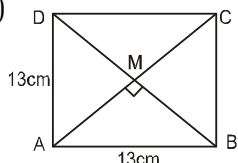
(b) Area of rhombus = Product of adjacent sides $\times$ sine of the include angle.

- (vii) $AC = d_1$ and $BD = d_2$ (say)
then, $d_1^2 + d_2^2 = AB^2 + BC^2 + CD^2 + DA^2$
 $\Rightarrow d_1^2 + d_2^2 = 4a^2$

- (viii) A rhombus is a square if its diagonals are equal.
i.e. if $d_1 = d_2 \Rightarrow ABCD$ is a square.

EXAMPLES

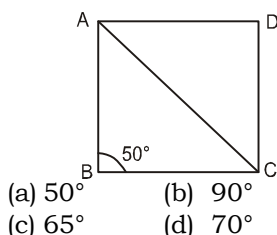
1. The length of a side of a rhombus is 13cm and one of its diagonal is 24cm. The length of the other diagonal is:
(a) 14cm (b) 12cm
(c) 20cm (d) 10cm

Sol. (d) 
Let $BD = 24\text{cm}$, $\therefore BM = 12\text{cm}$

$$\therefore AM = \sqrt{13^2 - 12^2} = 5\text{cm}$$

$$\therefore AC = 2AM = 10\text{cm}$$

2. ABCD is a rhombus with $\angle ABC = 50^\circ$, then $\angle ACD$ is :



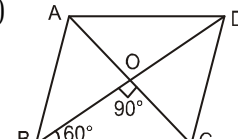
Sol. (c) $AB = BC$

$$\therefore \angle BAC = \angle BCA = \frac{1}{2}(180^\circ - 50^\circ) = 65^\circ$$

$$\angle ACD = \angle ACB = 65^\circ$$

3. ABCD is a rhombus whose side $AB = 4\text{cm}$ and $\angle ABC = 120^\circ$, then the length of diagonal BD is:

- (a) 1cm (b) 2cm
(c) 3cm (d) 4cm

Sol. (d) 

From $\triangle BOC$

$$\cos 60^\circ = \frac{BO}{4}$$

$$BO = \frac{1}{2} \times 4 = 2\text{cm}$$

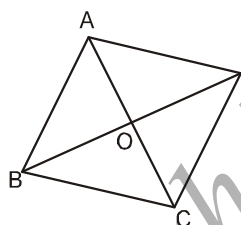
$$\therefore BD = 2 \times 2 = 4\text{cm}$$

4. If the diagonals of a rhombus are 8 and 6, then the square of its sides is:

- (a) 25 (b) 55
(c) 64 (d) 36

Sol. (a) $BO = 4$ units, $OC = 3$ units
 $\angle BOC = 90^\circ$

$$\therefore BC = \sqrt{4^2 + 3^2} = 5\text{ units}$$



$$\therefore BC^2 = 25\text{ sq. units}$$

5. If the perimeter of rhombus is 150 cm and length of one diagonal is 50 cm. Then find the length of second diagonal and area of rhombus.

Sol. Perimeter $= 4a = 150 \Rightarrow 2a = 75\text{ cm}$.

$$4a^2 = d_1^2 + d_2^2 \Rightarrow (2a)^2 = (d_1)^2 + (d_2)^2$$

$$\Rightarrow (75)^2 = (50)^2 + d_2^2$$

$$\Rightarrow (75)^2 - (50)^2 = d_2^2$$

$$\Rightarrow d_2^2 = (75 + 50)(75 - 50) = 125 \times 25 = 25 \times 25 \times 5$$

$$d_2 = 25\sqrt{5}$$

$$\Rightarrow \text{Area} = \frac{1}{2} d_1 d_2 = \frac{1}{2} \times 50 \times 25\sqrt{5} = 625\sqrt{5}\text{ cm}^2$$

6. Perimeter of a rhombus is $2p$ unit and sum of length of diagonals is m unit, then area of the rhombus is-

(a) $\frac{1}{4} m^2 p$ sq. unit

(b) $\frac{1}{4} m p^2$ sq. unit

(c) $\frac{1}{4} (m^2 - p^2)$ sq. unit

(d) $\frac{1}{4} (p^2 - m^2)$ sq. unit

Sol. (c) Sum of diagonal length = m (given)

$$\text{Perimeter} = 2p \text{ (given)}$$

$$\text{area of Rhombus} = ?$$

$$4a^2 = d_1^2 + d_2^2$$

$$\therefore 4a = 2P \text{ (perimeter)}$$

$$\Rightarrow a = \frac{P}{2}$$

$$\text{and } d_1 + d_2 = m$$

$$\text{Squaring both sides}$$

$$\Rightarrow d_1^2 + d_2^2 + 2d_1 d_2 = m^2$$

$$\Rightarrow 4a^2 + 2d_1 d_2 = m^2$$

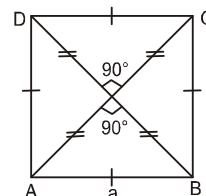
$$\Rightarrow 2d_1 d_2 = m^2 - 4a^2$$

$$\Rightarrow 2d_1 d_2 = m^2 - 4 \left(\frac{P}{2} \right)^2 \text{ (Put } a = P/2)$$

$$\Rightarrow \frac{1}{2} d_1 d_2 = \frac{m^2 - P^2}{4}$$

4. Square

A square is a rectangle with adjacent sides of equal length or a rhombus with each angle 90° .



Properties

(i) $AB = BC = CD = AD = a$ (say)
& $\angle A = \angle B = \angle C = \angle D = 90^\circ$

(ii) Diagonals are equal and bisect each other at right angle.

(iii) The figure formed by joining the mid-points of the adjacent sides of a square is a square.

(iv) Every square is a rhombus but every rhombus is not a square.

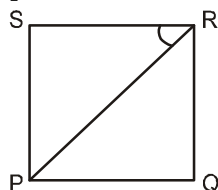
(v) Perimeter of square $= 4 \times \text{side of square} = 4 \times a$

(vi) Area $= (\text{side})^2 = a^2 = \frac{d^2}{2}$, and

$$\text{diagonal}(d) = a\sqrt{2}.$$

EXAMPLES

1. PQRS is a square. The $\angle$ SRP is equal to :



- Sol.** PQRS is a square, $SP = SR$ and $\angle S = 90^\circ$
and $\angle SRP = \angle SPR =$

$$\frac{1}{2}(90^\circ) = 45^\circ$$

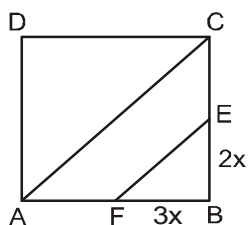
Hence, $\angle SRP = 45^\circ$

2. ABCD is a square, F is the mid-point of AB and E is a point on BC such that BE is one-third of BC. If area of $\triangle FBE = 147\text{m}^2$, then the length of AC is :

(a) $21\sqrt{2}\text{ m}$ (b) 63m (c)

$63\sqrt{2}\text{ m}$ (d) $42\sqrt{2}\text{ m}$

- Sol.** (d)



Let the side of square be $6x$ then,

Area of $\triangle FBE =$

$$\frac{1}{2} \times 3x \times 2x = 147$$

$$\Rightarrow x^2 = 49 \Rightarrow x = 7$$

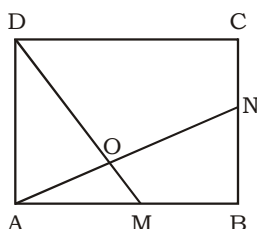
$\therefore$ side of square = 42m

$$\Rightarrow AC^2 = (42)^2 + (42)^2 = 2(42)^2$$

$$\Rightarrow AC = 42\sqrt{2}\text{ m}$$

3. ABCD is a square. M is the mid-point of AB and N is the mid-point of BC. DM and AN are joined and they meet at O. Then which of the following is correct?
(a) $OA : OM = 1:2$
(b) $AN = MD$
(c) $\angle ADM = \angle ANB$
(d) $\angle AMD = \angle BAN$

- Sol.** (b)



ABCD is a square and M, N are mid Points of AB and BC respectively

If $AB = 2x$

then, $BN = x$

$$\therefore AN = \sqrt{AB^2 + BN^2}$$

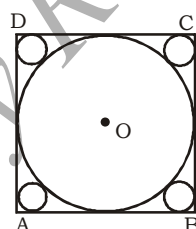
$$= \sqrt{4x^2 + x^2} = \sqrt{5x^2} = \sqrt{5} x$$

$$\text{Similarly: } MD = \sqrt{AD^2 + AM^2}$$

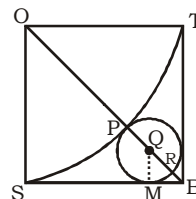
$$= \sqrt{4x^2 + x^2} = \sqrt{5x^2} = \sqrt{5} x$$

Hence, $AN = MD$

4. ABCD is square, in which a circle is inscribed touching all the sides of square. In the four corners of square 4 smaller circles of equal radii are drawn, containing maximum possible area. What is the ratio of the area of larger circle to that of sum of the areas of four smaller circles?



Sol. $AB = BC = CD = DA$



Let $OS = SB = R$ (radius of the larger circle) then $OB = R\sqrt{2}$

Similarly $PQ = MQ = QR = r$ (radius of the smaller circle)

then $BQ = r\sqrt{2}$

$$BP = r + r\sqrt{2}$$

$$\text{and } BP = OB - OP = R\sqrt{2} - R$$

$$\Rightarrow R\sqrt{2} - R = r + r\sqrt{2}$$

$$\Rightarrow R(\sqrt{2}-1) = r(\sqrt{2}+1)$$

$$\Rightarrow r = R(\sqrt{2}-1)^2$$

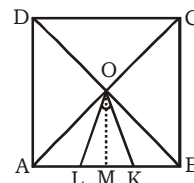
$$\Rightarrow r = R(3-2\sqrt{2})$$

$$\Rightarrow \therefore$$

$$\frac{\text{Area of larger circle}}{\text{Area of 4 smaller circles}} = \frac{\pi R^2}{4\pi r^2} = \frac{R^2}{4r^2} = \frac{1}{4(3-2\sqrt{2})^2} = \frac{1}{4(17-12\sqrt{2})}$$

5. ABCD is a square. The diagonals AC and BD meet at O. Let K, L be the points on AB such that $AO = AK$ and $BO = BL$. If $\theta = \angle LOK$, then what is the value of $\tan \theta$?

Sol.



Let sides of square be a

$$OB = BL = \frac{a}{\sqrt{2}} \text{ (given)}$$

$$LM = BL - BM = \frac{a}{\sqrt{2}} - \frac{a}{2}$$

$$OM = \frac{a}{2}$$

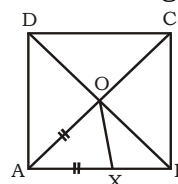
In $\triangle OML$

$$\tan \frac{\theta}{2} = \frac{ML}{OM}$$

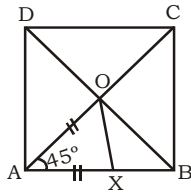
$$\tan \frac{\theta}{2} = \frac{\frac{a}{\sqrt{2}} - \frac{a}{2}}{\frac{a}{2}} = \frac{\frac{\sqrt{2}-1}{2}}{\frac{1}{2}} = \sqrt{2}-1$$

$$\tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} = \frac{2(\sqrt{2}-1)}{1 - (2+1-2\sqrt{2})} = \frac{(2\sqrt{2}-2)}{2\sqrt{2}-2} = 1$$

6. In the figure given below, ABCD is a square in which $AO = AX$. What is angle XOB?



Sol.



$\angle XAO = 45^\circ$ (AC is diagonal angle bisect of square ABCD)
 $AO = AX$ (given)

$$\angle AOX = \frac{180 - 45}{2} = \frac{135}{2}$$

$\angle AOB = 90^\circ$ (Diagonals bisect at right angle)
 $\angle XOB = \angle AOB - \angle AOX$

$$\angle XOB = 90 - \frac{135}{2}$$

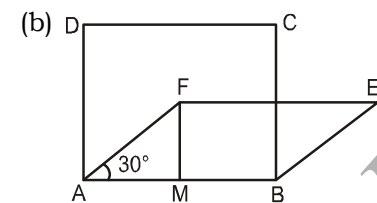
$$= \frac{180 - 135}{2} = \frac{45}{2} = 22.5^\circ$$

7. A square and a rhombus have the same base and the rhombus is inclined at 30° . What is the ratio of the area of the square to the area of the rhombus :

(a) $\sqrt{2} : 1$ (b) $2 : 1$ (c) $1 :$

1 (d) $2 : \sqrt{3}$

Sol.



ABCD is a square and ABEF is a rhombus

$$\sin 30^\circ = \frac{FM}{AF} = \frac{1}{2}$$

$$\Rightarrow FM = \frac{AF}{2}, AF = AB = a$$

Area of square = a^2 ($AB = AD = a$)

Area of rhombus = $AB \times FM$

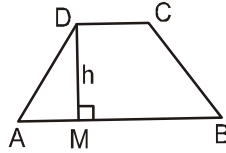
$$= a \times \left(\frac{a}{2} \right) = \frac{a^2}{2}$$

$$\therefore \frac{\text{Area of square}}{\text{Area of rhombus}} = \left(\frac{a^2}{a^2/2} \right)$$

$$= \frac{2}{1}$$

5. Trapezium

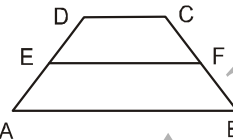
If two sides of a quadrilateral are parallel and other two sides are non parallel then it is called a trapezium.



Properties

- (i) $\angle A + \angle D = \angle B + \angle C = 180^\circ$
 (ii) If E and F are the mid-points of two non-parallel sides AD and BC respectively, then -

$$\text{Median}(EF) = \frac{1}{2}(AB + DC)$$



- (iii) **Area of trapezium**

$$= \frac{1}{2} (\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} \times (AB + CD) \times DM$$

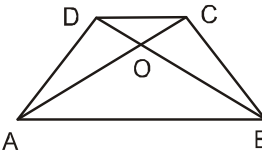
$$= \frac{1}{2} (AB + CD) \times h$$

- (iv) Sum of square of diagonals = (sum of squares of non-parallel side) + 2(product of || sides)

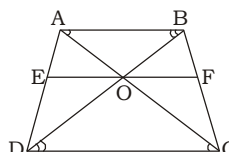
$$AC^2 + BD^2 = BC^2 + AD^2 + 2AB \cdot CD$$

- (v) By joining the mid-points of adjacent sides of a trapezium four similar triangles are obtained.

$$(vi) \frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{DC}$$



- (vii) Any line in a trapezium, parallel to parallel sides divides the non-parallel sides in equal ratio.



$$\frac{AE}{ED} = \frac{BF}{FC}$$

Proof: In $\triangle DBA$, $OE \parallel AB$ then

By Thales theorem $\frac{DE}{EA} =$

$$\frac{DO}{OB} \quad \dots (i)$$

& In $\triangle BDC$, $OF \parallel DC$ then

$$\frac{DO}{OB} = \frac{CF}{FB} \quad \dots (ii)$$

By (i) & (ii)

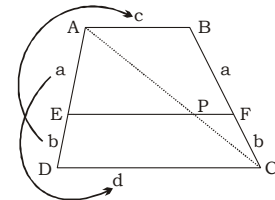
$$\frac{DE}{EA} = \frac{CF}{FB}$$

- (viii) When Only one pair of sides is parallel

Case I

The length of parallel sides $EF =$

$$\frac{ad + bc}{a + b}$$



Proof :- In $\triangle ABC$,

$$\frac{PF}{c} = \frac{b}{a+b}$$

($\triangle CFP \sim \triangle CBA$)

$$\therefore PF = \frac{bc}{a+b}$$

In $\triangle ADC$,

$$\frac{EP}{d} = \frac{a}{a+b}$$

($\triangle AEP \sim \triangle ADC$)

$$\therefore EP = \frac{ad}{a+b}$$

$$\therefore EF = EP + PF$$

$$= \frac{ad}{a+b} + \frac{bc}{a+b}$$

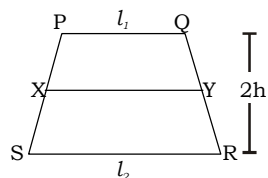
$$= \frac{ad+bc}{a+b}$$

Case II

If E and F are mid-points of AD and BC respectively then

$$a = b \Rightarrow EF = \frac{d+c}{2}$$

- (ix) The line joining the mid-point of non-parallel sides divides the trapezium into two trapezium then ratio of their area.



$$\frac{\text{Area of Trapezium PQYX}}{\text{Area of Trapezium XYRS}} =$$

$$\frac{3l_1 + l_2}{3l_2 + l_1}$$

Proof :-

$$\frac{\text{Area of PQYX}}{\text{Area of XYRS}} =$$

$$\frac{\frac{1}{2} \left(l_1 + \frac{l_1 + l_2}{2} \right) \times h}{\frac{1}{2} \left(l_2 + \frac{l_1 + l_2}{2} \right) \times h} = \frac{3l_1 + l_2}{3l_2 + l_1}$$

The figure formed by joining the mid-point

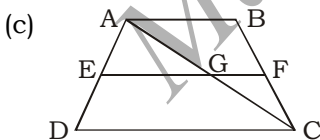
- Quadrilateral is Parallelogram
- Rectangle is Rhombus
- Rhombus is Rectangle
- Square is Square
- Area becomes half of basic figure

EXAMPLES

1. The parallel sides of a trapezium are p and q respectively. The line joining the mid points of its non-parallel sides will be

- (a) $\sqrt{pq}$ (b) $\frac{2pq}{p+q}$
 (c) $\frac{(p+q)}{2}$ (d) $\frac{1}{2}(p-q)$

Sol. (c)



In a trapezium ABCD

$$\begin{cases} DC = p \\ AB = q \end{cases}$$

$$AB \parallel CD$$

E and F are mid points of its non parallel sides a line is drawn from A to C, which intersects EF at G

In $\triangle ADC$

$$EF \parallel DC$$

$$EG = \frac{1}{2} DC$$

...(i)

Similarly, In $\triangle ABC$,

$$AB \parallel GF$$

$$GF = \frac{1}{2} AB$$

...(ii)

Adding eq. (i) and (ii) we get,

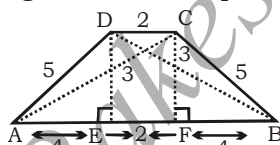
$$EG + GF = \frac{1}{2} DC + \frac{1}{2} AB$$

$$EF = \frac{1}{2} (DC + AB)$$

$$EF = \frac{1}{2} (p + q)$$

2. In a trapezium, the two non-parallel sides are equal in length, each being of 5cm. The parallel sides are at a distance of 3cm. If the smaller side of the parallel sides is of length 2cm., then the sum of the diagonal of the trapezium is:

Sol.



In $\triangle ACF$

$$AC = \sqrt{6^2 + 3^2} = \sqrt{45}$$

$$= \sqrt{9 \times 5} = 3\sqrt{5}$$

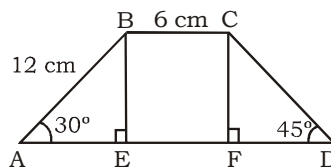
In $\triangle BDE$,

$$DB = \sqrt{6^2 + 3^2} = 3\sqrt{5}$$

sum of diagonals

$$= 3\sqrt{5} + 3\sqrt{5} = 6\sqrt{5} \text{ cm}$$

3. In a trapezium ABCD, $\angle BAE = 30^\circ$, $\angle CDF = 45^\circ$, $BC = 6$ cm and $AB = 12$ cm. Find the area of ABCD.



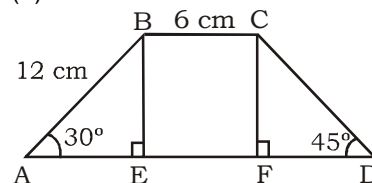
$$(a) 18 (3 + \sqrt{3}) \text{ cm}^2$$

$$(b) 36 \sqrt{3} \text{ cm}^2$$

$$(c) 12 (3 + 2\sqrt{3}) \text{ cm}^2$$

$$(d) \text{None of these}$$

Sol. (a)



Trapezium ABCD,

In $\triangle ABE$,

$$\frac{BE}{AB} = \sin 30^\circ$$

$$\therefore BE = 6 \text{ cm}$$

$$BE = CF$$

$$\cos 30^\circ = \frac{AE}{AB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AE}{12}$$

$$\Rightarrow AE = 6\sqrt{3} \text{ cm.}$$

In $\triangle CFD$,

$$\tan 45^\circ = \frac{CF}{FD}$$

$$\Rightarrow 1 = \frac{6}{FD}$$

$$\Rightarrow FD = 6 \text{ cm}$$

$$\therefore BC = EF = 6 \text{ cm.}$$

Now the area of trapezium =

$$\frac{1}{2} \times (\text{Sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} \times (BC + AD) \times BE$$

$$(\therefore AD$$

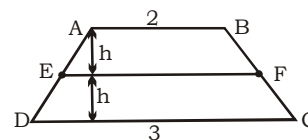
$$= AE + EF + FD = 6\sqrt{3} + 6 + 6)$$

$$= \frac{1}{2} (18 + 6\sqrt{3}) \times 6$$

$$\text{Area} = 18 (3 + \sqrt{3}) \text{ cm}^2$$

4. ABCD is a trapezium with parallel sides $AB = 2$ cm and $DC = 3$ cm E and F are the mid-points of the non-parallel sides. The ratio of area of ABFE to the area of EFCD is:

Sol.



By property of trapezium

$$EF = \frac{1}{2} (AB + DC) =$$

$$\frac{1}{2} (2 + 3) = \frac{5}{2}$$

$$\frac{\text{Area of trapezium ABFE}}{\text{Area of trapezium EFCD}} =$$

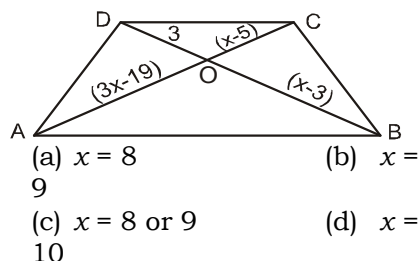
$$\frac{\frac{1}{2} \left(2 + \frac{5}{2} \right) \times h}{\frac{1}{2} \left(3 + \frac{5}{2} \right) \times h} = \frac{9}{11}$$

Alternate

$$\frac{\text{Area of trapezium ABFE}}{\text{Area of trapezium EFCD}} =$$

$$\frac{3a+b}{a+3b} = \frac{9}{11}$$

5. In the given figure, $AB \parallel CD$, find the value of x :



- Sol.** (c) The diagonals of a trapezium divide each other proportionally,

$$\begin{aligned} \therefore \frac{AO}{CO} &= \frac{BO}{OD} \Rightarrow \frac{3x-19}{x-5} = \frac{x-3}{3} \\ \Rightarrow 9x - 57 &= x^2 - 8x + 15 \\ \Rightarrow x^2 - 17x + 72 &= 0 \\ \Rightarrow x &= 8 \text{ or } x = 9 \end{aligned}$$

6. The parallel sides of a trapezium are in a ratio 2 : 3 and their shortest distance is 12cm. If the area of the trapezium is 480sq.cm., the longer of the parallel sides is of length:

- (a) 56cm (b) 36cm
(c) 42cm (d) 48cm

- Sol.** (d) Sides of the trapezium = $2x$ and $3x$ cm

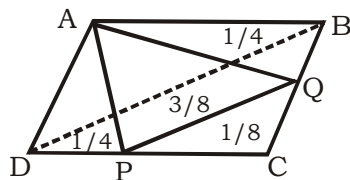
$$\therefore \frac{1}{2} (2x + 3x) \times 12 = 480$$

$$\Rightarrow 5x = \frac{480}{6} = 80$$

$$\Rightarrow x = \frac{80}{5} = 16$$

$$\text{Longer side} = 16 \times 3 = 48 \text{ cm}$$

- If P and Q are mid-point of DC & BC respectively then Area of $\triangle APQ = \frac{3}{8}$ Area of $\square ABCD$



Proof

$$\text{Area of } \triangle ABQ = \frac{1}{4} \text{ Area of } \square ABCD$$

[$\triangle ABQ$ & $\square ABCD$ lie in between same $\parallel$ lines & $QB = \frac{BC}{2}$]

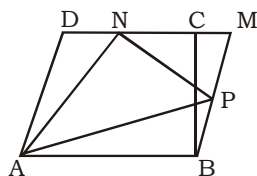
$$\text{Similarly Area of } \triangle ADP = \frac{1}{4} \text{ Area of } \square ABCD$$

$$\text{Area of } \triangle PQC = \frac{1}{4} \text{ Area of } \triangle BDC =$$

$$\frac{1}{8} \text{ Area of } \square ABCD \text{ [mid-point theorem]}$$

$$\begin{aligned} \therefore \text{Area of } \triangle APQ &= \text{ar} (\square ABCD - \triangle ADP - \triangle PQC - \triangle ABQ) = \\ &= \square ABCD - \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{8} \right) \square ABCD \\ &= \frac{3}{8} \square ABCD \end{aligned}$$

7. Parallelograms ABCD and ABMN are on the base AB, where $AB \parallel DM$. If the area of $\parallel$ gm ABMN is 80 sq. unit, what will be the area of $\triangle APN$?



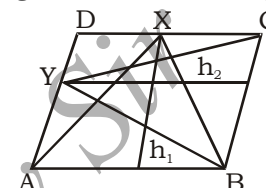
- (a) 20 sq. unit (b) 30 sq. unit
(c) 40 sq. unit (d) 160 sq. unit

- Sol.** (c) Parallelogram ABCD and ABMN are on the base AB where, So, area of ABCD = area of ABMN = 80 We know that

$$\text{Area of } \triangle APN = \frac{1}{2} \times \text{area of parallelogram ABMN}$$

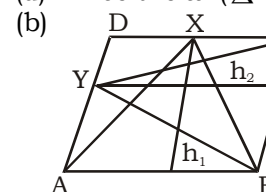
$$= \frac{1}{2} \times 80 = 40 \text{ sq. unit}$$

8. Two points X and Y are one the sides DC and AD of the parallelogram ABCD. The ar ($\triangle ABX$) is-



- (a) $\frac{1}{2}$ of ar ($\triangle BYC$)
(b) equal to ar ($\triangle BYC$)
(c) $\frac{1}{3}$ of ar ($\triangle BYC$)
(d) Twice the ar ($\triangle BYC$)

Sol.



$$\text{Area of } \triangle AXB = \frac{1}{2} \times AB \times h_1$$

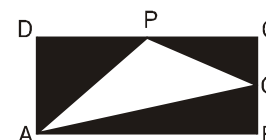
$$= \frac{1}{2} \times (\text{area of parallelogram ABCD})$$

$$\text{Area of } \triangle BYC = \frac{1}{2} \times BC \times h_2$$

$$= \frac{1}{2} \times (\text{area of parallelogram ABCD})$$

$$\Rightarrow \text{Area of } \triangle AXB = \text{Area of } \triangle BYC$$

9. In the given figure, ABCD is a rectangle. P and Q are the mid-points of sides CD and BC respectively. Then the ratio of area of shaded portion : area of unshaded portion is :

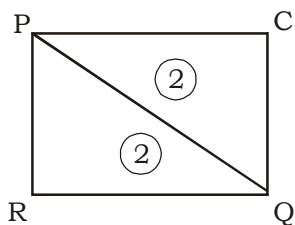


- (a) 5 : 4 (b) 3 : 5
(c) 5 : 3 (d) 5 : 8

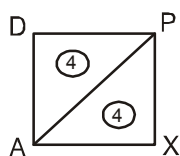
- Sol.** (c) Let total area of rectangle ABCD = 16 units

$$\therefore \text{area of } \square AXPD =$$

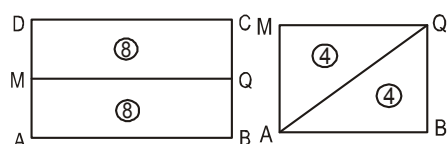
$$\frac{16}{2} = 8 \text{ units}$$



$\therefore$ area of $\triangle PCQ = 2$ units



$\therefore$ area of $\triangle ADP = 4$ units

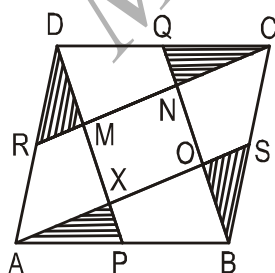


$\therefore$ area of $\triangle ABQ = 4$ units

$\therefore$ total area of shaded portion =
area of $\triangle ADP$ + area of $\triangle ABQ$
+ area of $\triangle PCQ$
= $4 + 2 + 4 = 10$ units
 $\therefore$ area of unshaded portion =
area of $\triangle APQ$
= $16 - \text{Area of shaded portion}$
= $16 - 10 = 6$ units

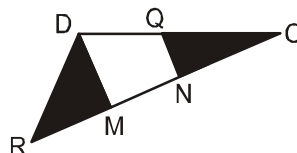
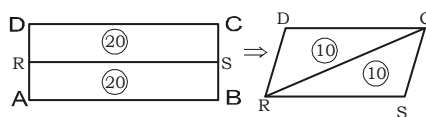
$\therefore$ Required ratio = $10 : 6 = 5 : 3$

10. In the $\parallel\text{gm}$ ABCD, P, Q, R and S are mid-points of sides AB, CD, DA and BC respectively. AS, BQ, CR and DP are joined. Find the ratio of the area of the shaded region to the area of the $\parallel\text{gm}$ ABCD.



- (a) $1/5$ (b) $1/4$
(c) $4/15$ (d) $1/6$

- Sol. (a) Let total area of $\parallel\text{gm}$ ABCD = 40 units



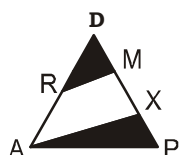
In $\triangle DMC$, Q is the mid point of DC and $QN \parallel DM$
[$\because$ DMNQ is also a $\parallel\text{gm}$]

$\Rightarrow$ N is the mid-point of MC

$\therefore$ ar ($\triangle QCN$) : ar ($\triangle DMC$) = $1 : 4$

Let ar ($\triangle QCN$) = 1 unit

$\Rightarrow$ ar (quadrilateral DMNQ) = $4 - 1 = 3$ units



Similarly, In $\triangle DAX$

ar ($\triangle DRM$) : ar ($\triangle DAX$) = $1 : 4$

ar ($\triangle DRM$) = 1 unit

$\Rightarrow$ ar ($\triangle DAX$) = 4 units

$\Rightarrow$ ar ($\square$ RMXA) = $4 - 1 = 3$ units

$\therefore$ ar ($\triangle DRM$) + ar ($\triangle QNC$) = $1 + 1 = 2$ units

and ar ($\triangle DRC$) = $4 + 1 = 5$ units

but from (i) ar ($\triangle DRC$) = 10 unit (2 times)

$\therefore$ ar ($\triangle DRM$) + ar ($\triangle QNC$) = $2 \times (2 \text{ times})$
= 4 units

Similarly, ar ($\triangle APX$) + ar ($\triangle BOS$) = 2×2
= 4 units

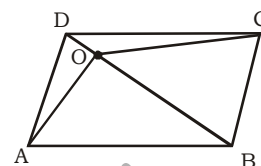
$\therefore$ total shaded area = $4 + 4$

= 8 units

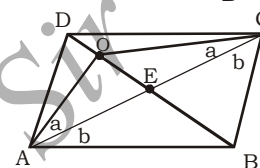
& area of $\parallel\text{gm}$ ABCD = 40 units

$\therefore$ Required ratio = $\frac{8}{40} = \frac{1}{5}$

11. In the below figure, ABCD is a parallelogram. If area of $\triangle OAB = 19 \text{ cm}^2$ then, find the area of $\triangle OBC$.



Sol.



Line joining A and C cuts BD at E. AC diagonals of parallelogram bisect each other,

OE and BE will be median of $\triangle OAC$ and $\triangle ABC$ respectively,

Let Area of $\triangle OAE$ = Area of $\triangle OEC$ = a

Then, Area of $\triangle AEB$ = Area of $\triangle EBC$ = b

Area of $\triangle OAB$ = Area of $\triangle OAE$ + Area of $\triangle AEB$ = (a + b)

Similarly,

Area of $\triangle OBC$ = Area of $\triangle OEC$ + Area of $\triangle EBC$ = (a + b)

Hence, if O is any point on diagonal BD in parallelogram ABCD then,,

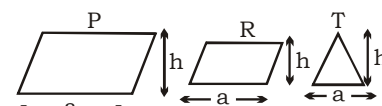
Area of $\triangle OAB$ = Area of $\triangle OBC$

Area of $\triangle OBC = 19 \text{ cm}^2$

12. If P, R, T are the area of a Parallelogram, a rhombus and a triangle standing on the same base and between the same parallels, Which of the following is true?

- (a) $R < P < T$ (b) $P > R > T$
(c) $R = P = T$ (d) $R = P = 2T$

Sol. (d)



Let base side be a and height be h

area of parallelogram

(P) = ah (i)

area of rhombus

(R) = ah (ii)

area of triangle (T) = $\frac{1}{2} ah$

$\Rightarrow R = P = 2T$

If ABCD is a square or rectangle

If 'P' is point inside a rectangle then

$$AP^2 + PC^2 = BP^2 + PD^2$$

$$(AP^2 - BQ^2 = AP^2 - BP^2 \dots\dots(i))$$

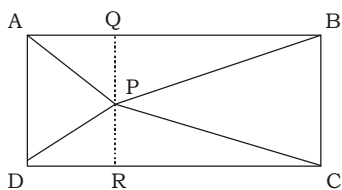
$$(Subtract DR^2 - CR^2 = PD^2 - PC^2 \dots\dots(ii))$$

$$AQ = DR \text{ \& } BQ = CR$$

$$(1) = (2)$$

$$AP^2 - BP^2 = PD^2 - PC^2$$

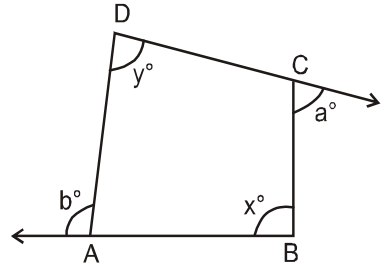
$$AP^2 + PC^2 = PD^2 + BP^2$$



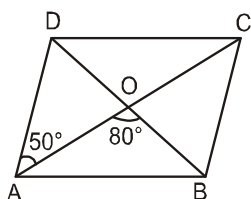
S.No.	Property	Rhombus	Square	Rectangle	Parallelogram
1.	Opposite sides are equal	✓	✓	✓	✓
2.	Opposite sides are parallel	✓	✓	✓	✓
3.	Opposite angles are equal	✓	✓	✓	✓
4.	All sides are equal	✓	✓	×	×
5.	All angles are equal and right angle	×	✓	✓	✓
6.	Diagonals bisect each other	✓	✓	✓	×
7.	Diagonals bisect each other at right angles	✓	✓	×	✓
8.	Diagonals bisect vertex angles	✓	✓	×	×
9.	Diagonals are equal	×	✓	✓	×
10.	Diagonals form four triangles of equal area	✓	✓	✓	✓
11.	Diagonals form four congruent triangles	✓	✓	×	×
12.	Area	$\frac{1}{2}d_1 \times d_2$	Side ²	$l \times b$	base $\times$ height
13.	The figure formed by joining the mid-points	Rectangle	Square	Rhombus	Parallelogram



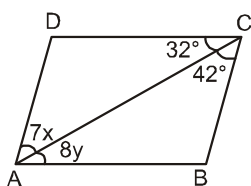
EXERCISE

1. The side AB of a parallelogram ABCD is produced to E in such way that BE = AB, DE intersects BC at Q. The point Q divides BC in the ratio
(a) 1 : 2 (b) 1 : 1
(c) 2 : 3 (d) 2 : 1
2. ABCD is a rhombus. A straight line through C cuts AD produced at P and AB produced at Q. If $DP = \frac{1}{2} AB$, then the ratio of the length of BQ and AB is
(a) 2:1 (b) 1:2
(c) 1:1 (d) 3:1
3. In a quadrilateral ABCD, with unequal sides if the diagonals AC and BD intersect at right angles then
(a) $AB^2 + BC^2 = CD^2 + DA^2$
(b) $AB^2 + CD^2 = BC^2 + DA^2$
(c) $AB^2 + AD^2 = BC^2 + CD^2$
(d) $AB^2 + BC^2 = 2(CD^2 + DA^2)$
4. The ratio of the angles $\angle A$ and $\angle B$ of a non-square rhombus ABCD is 4 : 5, then the value of $\angle C$ is :
(a) 50° (b) 45° (c) 80° (d) 95°
5. ABCD is a rhombus whose side AB = 4 cm and $\angle ABC = 120^\circ$, then the length of diagonal BD is equal to :
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 4 cm
6. The tangents at two points A and B on the circle with centre O intersect at P. If in quadrilateral PAOB, $\angle AOB : \angle APB = 5 : 1$, then measure of $\angle APB$ is :
(a) 30° (b) 60° (c) 45° (d) 15°
7. ABCD is a trapezium whose side AD is parallel to BC, Diagonals AC and BD intersect at O. If $AO = 3$, $CO = x - 3$, $BO = 3x - 19$ and $DO = x - 5$, the value(s) of x will be :
(a) 7, 6 (b) 12, 6
(c) 7, 10 (d) 8, 9
8. Q is a point in the interior of a rectangle ABCD, if QA = 3 cm, QB = 4 cm and QC = 5 cm then the length of QD (in cm) is
(a) $3\sqrt{2}$ (b) $5\sqrt{2}$
(c) $\sqrt{34}$ (d) $\sqrt{41}$
9. ABCD is a rectangle where the ratio of the length of AB and BC is 3 : 2 . If P is the mid- point of AB, then the value of $\sin \angle CPB$ is
(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{5}$
10. Inside a square ABCD , BEC is an equilateral triangle. If CE and BD intersect at O, then $\angle BOC$ is
(a) 60° (b) 75° (c) 90° (d) 120°
11. The diagonals AC and BD of a cyclic quadrilateral ABCD intersect each other at the point P. Then, it is always true that
(a) BP . AB = CD . CP
(b) AP . CP = BP . DP
(c) AP . BP = CP . DP
(c) AP . CD = AB . CP
12. If the opposite sides of a quadrilateral and also its diagonals are equal, then each of the angles of the quadrilateral is
(a) 90° (b) 120°
(c) 100° (d) 60°
13. In a rhombus ABCD, $\angle A = 60^\circ$ and AB = 12 cm. Then the diagonal BD is:
(a) $2\sqrt{3}$ cm (b) 6 cm
(c) 12 cm (d) 10 cm
14. If PQRS is a rhombus and $\angle SPQ = 50^\circ$, then $\angle RSQ$ is:
(a) 75° (b) 45°
(c) 55° (d) 65°
15. The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. The area of the field is?
(a) 252 m^2 (b) 1152 m^2
(c) 96 m^2 (d) 156 m^2
16. In trapezium ABCD, $AB \parallel CD$ and $AB = 2 CD$. Its diagonals intersect at O. If the area of $\triangle AOB = 84 \text{ cm}^2$, then the area of $\triangle COD$ is equal to
(a) 21 cm^2 (b) 72 cm^2
(c) 42 cm^2 (d) 26 cm^2
17. Quadrilateral ABCD is circumscribed about a circle. If the lengths of AB, BC, CD are 7 cm, 8.5 cm and 9.2 cm respectively, then the length (in cm) of DA is
(a) 16.2 (b) 7.7
(c) 10.2 (d) 7.2
18. If the ratio of the angles of a quadrilateral is 2 : 7 : 2 : 7, then it is a
(a) trapezium (b) square
(c) parallelogram
(d) rhombus
19. The angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4, the largest angle is :
(a) 120° (b) 134°
(c) 144° (d) 150°
20. The sides BA and DC of a quadrilateral ABCD are produced as shown in figure. Then the true statement is :

(a) $x^\circ + y^\circ = a^\circ + b^\circ$
(b) $x^\circ + a^\circ = y^\circ + b^\circ$
(c) $2x^\circ + y^\circ = a^\circ + b^\circ$
(d) $x + \frac{1}{2}y^\circ = \frac{a^\circ + b^\circ}{2}$
21. If ABCD is a rhombus, then :
(a) $AC^2 + BD^2 = 4AB^2$
(b) $AC^2 + BD^2 = AB^2$
(d) $AC^2 + BD^2 = 2AB^2$
(d) $2(AC^2 + BD^2) = 3AB^2$
22. Two parallelograms stand on equal bases and between the same parallel lines. The ratio of their areas is :
(a) 1 : 1 (b) $\sqrt{2} : 1$
(c) 1 : 3 (d) 1 : 2
23. The diagonals AC and BD of a ||gm ABCD intersect each other at the point O such that $\angle DAC = 50^\circ$ and $\angle AOB =$

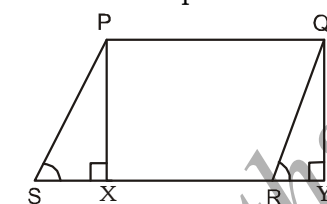
80°. Then $\angle DBC = ?$



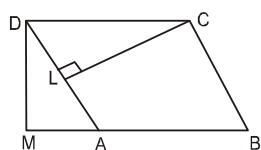
- (a) 50° (b) 40° (c) 45° (d) 30°
24. If ABCD is a $\parallel$ gm with two adjacent angles A and B are equal to each other, then the $\parallel$ gm is a :
- (a) square (b) rhombus
(c) rectangle (d) both (a) and (c)
25. In the adjoining figure, ABCD is a parallelogram, then the value of x and y are :



- (a) 6, 4 (b) 5, 4
(c) 4, 5 (d) None of these
26. PQRS is a $\parallel$ gm. PX and QY are respectively, the perpendicular from P and Q to SR and SR produced. Then PX is equal to :



- (a) QY (b) 2QY
(c) $\frac{1}{2}$ QY (d) XR
27. ABCD is a $\parallel$ gm $CL \perp AD$ and $DM \perp BA$. If $CD = 16$ units, $DM = 12$ units and $CL = 15$ units, then $AD = ?$



- (a) 12.8 units (b) 13.6 units
(c) 11.1 units (d) 12.4 units

28. If a square and a rhombus stand on the same base and between two parallel lines then the ratio of the areas of the square and the rhombus is:

- (a) 2 : 1 (b) 1 : 4
(c) 1 : 4 (d) 1 : 1

29. If area of a $\parallel$ gm with sides a and b is A and that of a rectangle with sides a and b is B, then :

- (a) $A > B$ (b) $A < B$
(c) $A = B$ (d) none of these.

30. ABCD is a quadrilateral in which diagonal $BD = 64$ cm, $AL \perp BD$, such that $AL = 13.2$ cm and $CM = 16.8$ cm. The area of the quadrilateral ABCD in square centimetres is:

- (a) 422.4 (b) 690.0
(c) 537.6 (d) 960.0

31. ABCD is cyclic trapezium whose sides AD and BC are parallel to each other. If $\angle ABC = 72^\circ$, then the measure of the $\angle BCD$ is :

- (a) 162° (b) 18°
(c) 108° (d) 72°

32. If an exterior angle of a cyclic quadrilateral be 50° , then the interior opposite angle is :

- (a) 130° (b) 40° (c) 50° (d) 90°

33. In a quadrilateral ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively, then $\angle AOB$ is equal to:

- (a) $\angle C + \angle D$
(b) $2\angle C + 2\angle D$

- (c) $\frac{1}{2}(\angle C + \angle D)$

- (d) $\frac{1}{2}(\angle C - \angle D)$

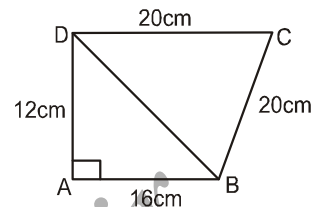
34. In a parallelogram ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively, then $\angle AOB$ is equal to:

- (a) 60° (b) 120°
(c) 100° (d) 90°

35. The measures of the angles of a quadrilateral taken in order are proportionates to :
- (a) parallelogram

- (b) trapezium
(c) rectangle
(d) rhombus

36. Find the area of $\square ABCD$:



- (a) $4(24 + 25\sqrt{3})\text{cm}^2$

- (b) $4(25 + 24\sqrt{3})\text{cm}^2$

- (c) $2(24 + 25\sqrt{3})\text{cm}^2$

- (d) None of these

37. ABCD is a trapezium in which $AB \parallel DC$ and $AB = 8$ cm, $BC = 10$ cm, $CD = 12$ cm, $AD = 16$ cm, then $AC^2 + BD^2$ is equal to :

- (a) 458cm^2 (b) 448cm^2
(b) 546cm^2 (d) 548cm^2

38. If O is a point within a rectangle ABCD then :

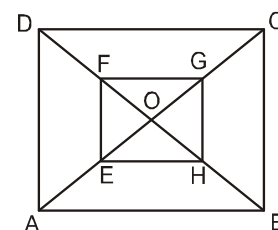
- (a) $OA^2 + OC^2 = OB^2 + OD^2$
(b) $OA^2 + OB^2 = OC^2 + OD^2$

- (c) $OA + OC = OB + OD$

- (d) $OA \times OC = OB \times OD$

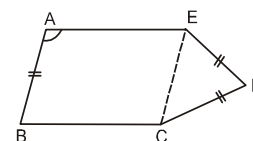
39. In the given figure, ABCD is a $\parallel$ gm and E, F, G, H are the mid-points of AO, DO, CO and BO respectively, then

$$\frac{EF + FG + GH + HE}{AD + DC + CB + BA} = ?$$



- (a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

40. In the given figure $AE = BC$ and $AE \parallel BC$ and the three sides AB, CD and ED are equal in length. If $\angle A = 102^\circ$, find measure of $\angle BCD$:

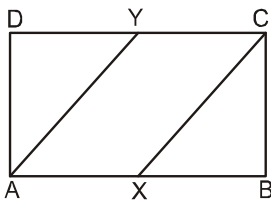


- (a) 138° (b) 162°
 (c) 88°
 (d) None of these

41. If ABCD is a rectangle. P, Q are the mid-points of BC and AD respectively and R is any point on PQ, then $\triangle ARB$ equals :

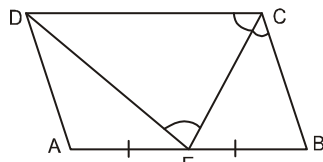
- (a) $\frac{1}{6}(\square ABCD)$ (b) $\frac{1}{3}(\square ABCD)$
 (c) $\frac{1}{4}(\square ABCD)$ (d) $\frac{1}{2}(\square ABCD)$

42. ABCD is a $\parallel$ gm and X, Y are the mid-points of sides AB and CD respectively. Then quadrilateral AXC Y is :



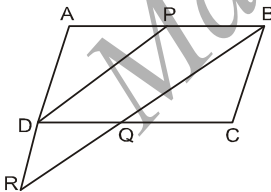
- (a) parallelogram
 (b) rhombus
 (c) square
 (d) rectangle

43. ABCD is a $\parallel$ gm, E is the mid-point of AB and CE bisects $\angle BCD$. Then $\angle DEC$ is :



- (a) 60° (b) 90°
 (c) 100° (d) 120°

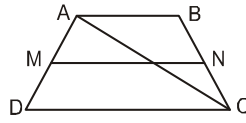
44. P is a mid-point of side AB to a $\parallel$ gm ABCD. A line through B parallel to PD meets DC at Q and AD produced at R. Then BR is equal to :



- (a) BQ (b) $\frac{1}{2}$ BQ
 (c) 2BQ
 (d) None of these

45. ABCD is a trapezium in which $AB \parallel CD$. M and N are the mid-

points of AD and BC respectively. If $AB = 14$ cm and $MN = 15$ cm, find CD.

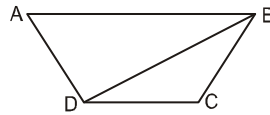


- (a) 16 cm (b) 18 cm
 (c) 8 cm (d) 10 cm

46. In a quadrilateral ABCD, $\angle B = 90^\circ$, and $AD^2 = AB^2 + BC^2 + CD^2$. Then $\angle ACD$ is equal to :

- (a) 60° (b) 90° (c) 30° (d) 45°

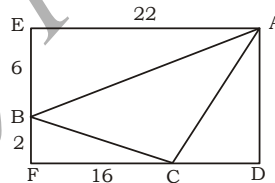
47. In the quadrilateral ABCD



$AB + BC + CD + DA$ is :

- (a) greater than $2BD$
 (b) less than $2BD$
 (c) equal to $2BD$
 (d) none of these

48. In the given figure EADF is a rectangle and ABC is a triangle whose vertices lie on the sides of $\square EADF$. $AE = 22$, $BE = 6$, $CF = 16$ and $BF = 2$. Find the length of the line joining the mid-points of the sides AB and BC.



- (a) 4 (b) 5
 (c) 3.5 (d) $4\sqrt{2}$

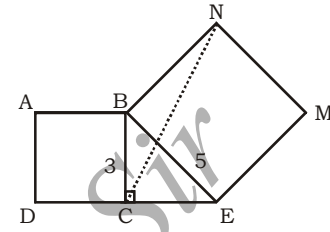
49. If the length of the side PQ of the rhombus PQRS is 6 cm and $\angle PQR = 120^\circ$, then the length of QS, in cm is

- (a) 4 (b) 6 (c) 3 (d) 5

50. ABCD is a square. M is the mid-point of AB and N is the mid-point of BC. DM and AN are joined and they meet at O. Then which of the following is correct ?

- (a) $OA : OM = 1 : 2$
 (b) $AN = MD$
 (c) $\angle ADM = \angle ANB$
 (d) $\angle AMD = \angle BAN$

51. In the given figure, there is a square of 3 cm. If an another square of 5 cm with side BE is formed. In triangle BCE, C is right angle. Find the length of CN?

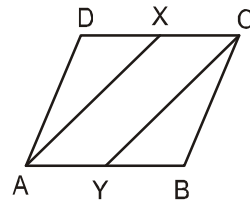


- (a) $\sqrt{56}$ cm (b) $\sqrt{57}$ cm
 (c) $\sqrt{58}$ cm (d) $\sqrt{59}$ cm

52. ABCD is a quadrilateral inscribed in a circle with centre O. If $\angle COD = 120^\circ$ and $\angle BAC = 30^\circ$, then $\angle BCD$ is:

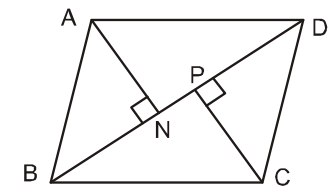
- (a) 75° (b) 90° (c) 120° (d) 60°

53. In the given figure, ABCD is a $\parallel$ gm and line segments AX, CY bisect the angles A and C respectively, then which one is true:



- (a) $AX \parallel CY$
 (b) $AX \parallel CY$ is a trapezium
 (c) AX is not parallel to CY
 (d) None of these.

54. In the given figure, $AN \perp BD$ and $CP \perp BD$ and ABCD is a parallelogram, then :



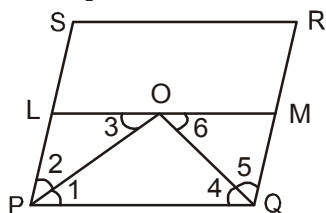
- (a) $AN \neq CP$ (b) $AN = CP$

- (c) $AN = \frac{1}{2} CP$

- (d) none of these.

55. In the given figure, PQRS is a $\parallel$ gm, PO and QO are respectively, the angle bisectors of

$\angle P$ and $\angle Q$. Line LOM is drawn parallel to PQ, then :



- (a) $LO = 2OM$ (b) $LO = \frac{1}{2} OM$

(c) $LO = OM$
(d) None of these.

56. The diagonals of a $\parallel$ gm ABCD intersect at O. A line through O intersects AB at X and DC at Y, then:

- (a) $OX = 2OY$ (b) $OX = OY$
(c) $OY = 2OX$
(d) None of these.

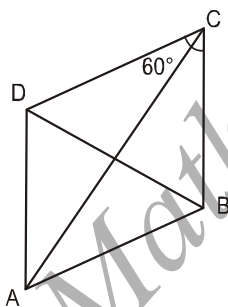
57. In a $\parallel$ gm ABCD, the bisector of $\angle A$ also bisects BC at X, then :

- (a) $AD = 2AB$
(b) $AD = AB$
(c) $AD = 3AB$
(d) None of these

58. Diagonals of a $\parallel$ gm are 8m and 6m respectively. If one of side is 5m, then the area of $\parallel$ gm is:

- (a) $18m^2$ (b) $30m^2$
(c) $24m^2$ (d) $48m^2$

59. ABCD is rhombus in which $\angle C = 60^\circ$, then $AC : BD = ?$



- (a) $\sqrt{3} : 1$ (b) $\sqrt{3} : 2$
(c) $3 : 1$ (d) $3 : 2$

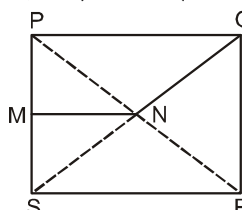
60. In a $\parallel$ gm, the adjacent side are 36cm and 27cm in length. If the distance between the longer sides is 12cm, then the distance between the smaller sides is :

- (a) 12 cm (b) 16 cm
(c) 14 cm (d) 15 cm

61. The length of the diagonal BD of the $\parallel$ gm ABCD is 18cm. If P and Q are the centroid of $\triangle ADC$ and $\triangle ABC$, then length of PQ is :

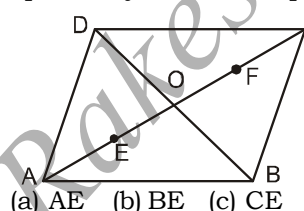
- (a) 5.5 cm (b) 7cm
(c) 5 cm (d) 6cm

62. PQRS is a square, M is the mid-point of side PS and N is the intersecting point of its diagonals. Then the ratio Area ($\square PQNM$) : Area ($\square PQRS$) is :



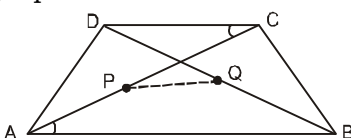
- (a) 5 : 8 (b) 3 : 8
(c) 1 : 4 (d) none of these

63. In the adjoining figure ABCD is a $\parallel$ gm and E, F are the centroids of $\triangle ABD$ and $\triangle BCD$ respectively, then EF equals :



- (a) AE (b) BE (c) CE (d) DE

64. ABCD is a trapezium and P, Q are the mid-points of the diagonals AC and BD. Then PQ is equal to :

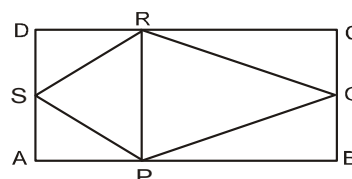


- (a) $\frac{1}{2}(AB)$ (b) $\frac{1}{2}(CD)$

- (c) $\frac{1}{2}(AB - CD)$

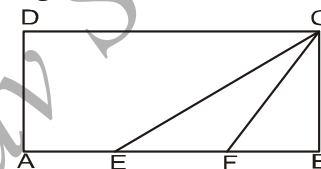
- (d) $\frac{1}{2}(AB + CD)$

65. ABCD is a $\parallel$ gm. P, Q, R and S are points on sides AB, BC, CD and DA respectively such that $AP = DR$. If the area of the $\parallel$ gm ABCD is $20cm^2$, then the area of quadrilateral PQRS is:



- (a) $10cm^2$ (b) $8cm^2$
(c) $12cm^2$ (d) $8.5cm^2$

66. In the given figure, ABCD is a rectangle with $AE = EF = FB$. What is the ratio of the area of the $\triangle CEF$ to that of the rectangle?



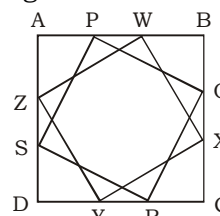
- (a) 2 : 5 (b) 2 : 3
(c) 1 : 4 (d) 1 : 6

67. Side AB of rectangle of ABCD is divided into four equal parts by points x, y, z. The ratio of

the $\frac{\text{Area}(\triangle XYZ)}{\text{Area}(\text{Rectangle ABCD})}$ is :

- (a) $1/7$ (b) $1/6$
(c) $1/9$ (d) $1/8$

68. In the adjoining figure ABCD, PQRS and WXYZ are three squares. Find number of triangles and quadrilaterals in the figure:



- (a) 24 and 16 (b) 28 and 15
(c) 27 and 16
(d) none of the above

69. Two light rods $AB = a+b$, $CD = a-b$ symmetrically lying on a horizontal AB. There are perpendicular distance between rods is a. The length of AC is given by

- (a) a (b) b
(c) $\sqrt{a^2 - b^2}$ (d) $\sqrt{a^2 + b^2}$

70. If PQRS be a rectangle such $PQ = \sqrt{3} QR$. Then, what is $\angle PRS$ equal to?

- (a) 60° (b) 45° (c) 30° (d) 15°

71. In a trapezium, the two non-parallel sides are equal in length, each being of 5 cm. The parallel sides are at a distance of 3 cm apart. If the smaller side of the parallel sides is of length 2 cm, then the sum of the diagonals of the trapezium is

(a) $10\sqrt{5}$ cm (b) $6\sqrt{5}$ cm

(c) $5\sqrt{5}$ cm (d) $3\sqrt{5}$ cm

72. The area of a rectangle lies between 40 cm^2 and 45 cm^2 . If one of the sides is 5 cm, then its diagonal lies between

(a) 8 cm and 10 cm

(b) 9 cm and 11 cm

(c) 10 cm and 12 cm

(d) 11 cm and 13 cm

73. Let ABCD be a parallelogram. Let P, Q, R and S be the mid-points of sides AB, BC, CD and DA, respectively. Consider the following statements.

I. Area of triangle APS < Area of triangle DSR, if $BD < AC$.

II. Area of triangle ABC = 4 (Area of triangle BPQ).

Select of correct answer using the codes given below.

(a) Only I (b) Only II

(c) Both I and II

(d) Neither I nor II

74. Consider the following statements

I. Let ABCD be a parallelogram which is not a rectangle.

Then, $2(AB^2 + BC^2) \neq AC^2 + BD^2$

II. If ABCD is a rhombus with $AB = 4$ cm, then $AC^2 + BD^2 = n^3$ for some positive integer n.

Which of the above statements is/are correct?

(a) Only I (b) Only II

(c) Both I and II

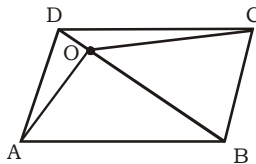
(d) Neither I nor II

75. ABCD is parallelogram. E is a point on BC such that $BE:EC = m:n$. AC and DB intersect at F, then what is the ratio of the area of $\triangle FEB$ and $\triangle AFD$?

(a) m/n (b) $(m/n)^2$

(c) $(n/m)^2$ (d) $[m/(m+n)]^2$

76. In the below figure, ABCD is a parallelogram. If area of $\triangle OAB = 19\text{ cm}^2$ then, find the area of $\triangle OBC$.



(a) 19 (b) 20 (c) 38 (d) $\frac{19}{2}$

77. If the diagonals of a rhombus are 4.8 cm and 1.4 cm, then what is the perimeter of the rhombus?

(a) 5 cm (b) 10 cm

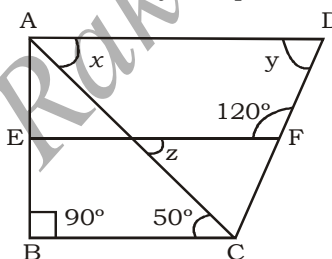
(c) 12 cm (d) 20 cm

78. ABCD is a trapezium with parallel sides $AB = 2\text{ cm}$ and $DC = 3\text{ cm}$. E and F are the mid-points of the non-parallel sides. The ratio of area of ABFE to area of EFCD is

(a) 9:10 (b) 8:9

(c) 9:11 (d) 11:9

79. In the figure given above, ABCD is a trapezium. EF is parallel to AD and BC. $\angle y$ is equal to



(a) 30° (b) 45° (c) 60° (d) 65°

80. The locus of a point in rhombus ABCD which is equidistant from A and C is

(a) a fixed point on diagonal BD

(b) diagonal BD

(c) diagonal AC

(d) None of the above

81. If two parallel lines are cut by two distinct transversals, then the quadrilateral formed by the four lines is always a

(a) square

(b) parallelogram

(c) rhombus (d) trapezium

82. ABCD is a parallelogram. If the bisectors of the $\angle A$ and $\angle C$ meet the diagonal BD at points P and Q respectively, then which one of the following is correct?

(a) PCQA is a straight line

(b) $\triangle APQ$ is similar to $\triangle PCQ$

(c) $AP = CP$ (d) $AP = AQ$

83. The sides of a parallelogram are 12 cm and 8 cm long and one of the diagonals is 10 cm long. If d is the length of other diagonal, then which one of the following is correct?

(a) $d < 8\text{ cm}$

(b) $8\text{ cm} < d < 10\text{ cm}$

(c) $10\text{ cm} < d < 12\text{ cm}$

(d) $d > 12\text{ cm}$

84. Let LMNP be a parallelogram and NR be perpendicular to LP. If the area of the parallelogram is six times the area of $\triangle RNP$ and $RP = 6\text{ cm}$ what is LR equal to?

(a) 15 cm (b) 12 cm

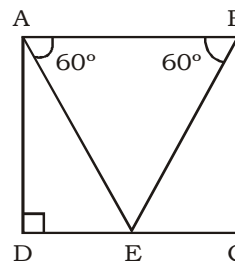
(c) 9 cm (d) 8 cm

85. Two poles of heights 6 m and 11 m stand vertically upright on a place ground. If the distance between their feet is 12 m, what is the distance between their tops?

(a) 11 m (b) 12 m

(c) 13 m (d) 14 m

86. In the given figure, ABCD is a quadrilateral with AB parallel to DC and AD parallel to BC, ADE is a right angle. If the perimeter of the $\triangle ABE$ is 6 units, what is the area of the quadrilateral?



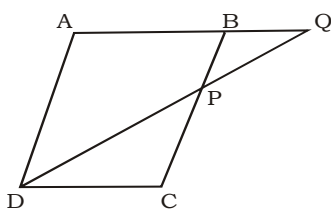
(a) $2\sqrt{3}$ sq units

(b) 4 sq units

(c) 3 sq units

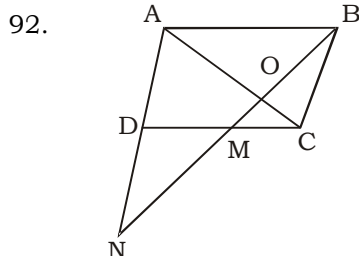
(d) $4\sqrt{3}$ sq units

87. In the figure given below, ABCD is a parallelogram. P is a point in BC such that $PB:PC = 1:2$. DP produced meets AB produced at Q. If the area of the $\triangle BPQ$ is 20 sq units, what is the area of the $\triangle DCP$?

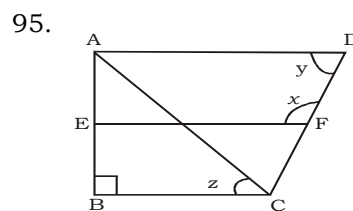


- (a) 20 sq units (b) 30 sq units
(c) 40 sq units
(d) None of the above
88. ABCD is a square, P, Q, R and S are points on the sides AB, BC, CD and DA respectively such that $AP = BQ = CR = DS$. What is $\angle SPQ$ equal to?
(a) 30° (b) 45° (c) 60° (d) 90°
89. Two similar parallelograms have corresponding sides in the ratio $1:k$. What is the ratio of their areas?
(a) $1:3k^2$ (b) $1:4k^2$
(c) $1:k^2$ (d) $1:2k^2$
90. ABC is a triangle in which $AB = AC$. Let BC be produced to D. From a point E on the line AC let EF be a straight line such that EF is parallel to AB. Consider the quadrilateral ECDF thus formed. If $\angle ABC = 65^\circ$ and $\angle EFD = 80^\circ$, then what is $\angle D$ equal to?
(a) 43° (b) 41° (c) 37° (d) 35°
91. Two sides of a parallelogram are 10 cm and 15 cm. If the altitude corresponding to the side of length 15 cm is 5 cm, then what is the altitude to the side

of length 10 cm?
(a) 5 cm (b) 7.5 cm
(c) 10 cm (d) 15 cm



92. In the figure given above, M is the mid-point of the side CD of the parallelogram ABCD. What is $ON:OB$?
(a) $3:2$ (b) $2:1$
(c) $3:1$ (d) $5:2$
93. An equilateral triangle and a regular hexagon are inscribed in a given circle. If a and b are the lengths of their sides respectively, then which one of the following is correct?
(a) $a^2 = 2b^2$ (b) $b^2 = 3a^2$
(c) $b^2 = 2a^2$ (d) $a^2 = 3b^2$
94. In a cricket match, the first 5 batsmen of a team scored runs :30, 40, 50, 30 and 40. If these data represent a four sided figure with 50 as its one of the diagonals, then what does second diagonal represent?
(a) 30 runs (b) 40 runs
(c) 50 runs (d) 70 runs

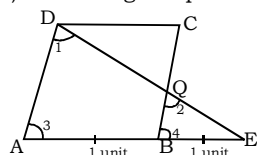


95. ABCD is a trapezium in which EF is parallel to BC. $\angle x = 120^\circ$ and $\angle z = 50^\circ$, then what is $\angle y$?
(a) 50° (b) 60° (c) 70° (d) 80°
96. An obtuse angle made by a side of a parallelogram PQRS with other pair of parallel sides is 150° . If the perpendicular distance between these parallel sides (PQ and SR) is 20 cm, what is the length of the sides RQ?
(a) 40 cm (b) 50 cm
(c) 60 cm (d) 70 cm
97. ABCD be a quadrilateral the diagonals AC and BD meet at O. the perpendicular drawn from A to CD, meet CD at E. Further, $AO:OC = BO:OD$, $AB = 30$ cm, $CD = 40$ cm, and the area of the quadrilateral ABCD is 1050 sq. cm.
(a). What is BE equal to?
(b). What is the area of the $\triangle ADC$ is equal to?
(c). What is $\angle AEB$ equal to?

ANSWER KEY

1. (b)	11. (b)	21. (a)	31. (d)	41. (c)	51. (c)	61. (d)	71. (b)	81. (d)	91. (b)
2. (a)	12. (a)	22. (a)	32. (c)	42. (a)	52. (b)	62. (b)	72. (b)	82. (b)	92. (b)
3. (b)	13. (c)	23. (d)	33. (c)	43. (b)	53. (a)	63. (a)	73. (b)	83. (d)	93. (d)
4. (c)	14. (d)	24. (d)	34. (d)	44. (c)	54. (b)	64. (c)	74. (b)	84. (b)	94. (c)
5. (d)	15. (a)	25. (a)	35. (b)	45. (a)	55. (c)	65. (a)	75. (d)	85. (c)	95. (b)
6. (a)	16. (a)	26. (a)	36. (a)	46. (b)	56. (b)	66. (d)	76. (a)	86. (a)	96. (a)
7. (d)	17. (b)	27. (a)	37. (d)	47. (a)	57. (a)	67. (d)	77. (b)	87. (d)	97. (*)
8. (a)	18. (c)	28. (d)	38. (a)	48. (b)	58. (c)	68. (c)	78. (c)	88. (d)	
9. (d)	19. (c)	29. (b)	39. (b)	49. (b)	59. (a)	69. (d)	79. (c)	89. (c)	
10. (b)	20. (a)	30. (d)	40. (b)	50. (b)	60. (b)	70. (c)	80. (a)	90. (d)	

1. (b) According to questions



$AD \parallel BC$ and $AB \parallel DC$

Point B is the midpoint of AE

$\angle 1 = \angle 2$ (Alternate Angle)

$\angle 3 = \angle 4$ (Alternate Angle)

$\therefore \triangle EQB \sim \triangle EDA$

$$\therefore \frac{EB}{EA} = \frac{EQ}{ED} = \frac{QB}{AD}$$

$$\Rightarrow \frac{1}{2} = \frac{QB}{AD}$$

$$\Rightarrow \frac{QB}{AD} = \frac{1}{2}$$

If $AD = 2$

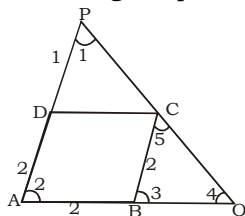
$QB = 1$

Then $QC = 1$

$\therefore Q$ divides BC in the ratio (1:1)

2. (a) According to question,

Given:



ABCD is a rhombus

$AB = BC = CD = DA$

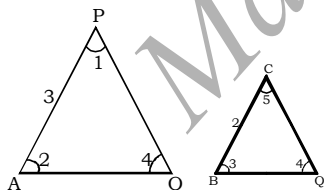
$$\Rightarrow DP = \frac{1}{2} AB$$

$$\Rightarrow \frac{DP}{AB} = \frac{1}{2}$$

In a rhombus $\angle 2 = \angle 3$

$\therefore \triangle APQ \sim \triangle BCQ$

($\because \angle Q$ is common and $\angle 2 = \angle 3$)



$$\Rightarrow \frac{AP}{BC} = \frac{AQ}{BQ} \quad \frac{AQ}{BQ} = \frac{3}{2}$$

$$\Rightarrow \frac{AB+BQ}{BQ} = \frac{3}{2} \quad (\because AQ = AB + BQ)$$

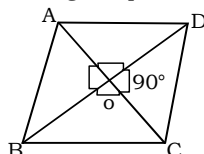
$$\Rightarrow \frac{AB}{BQ} + 1 = \frac{3}{2}$$

$$\Rightarrow \frac{AB}{BQ} = \frac{3}{2} - 1$$

$$\Rightarrow \frac{AB}{BQ} = \frac{1}{2}$$

$$\Rightarrow \therefore \frac{BQ}{AB} = \frac{2}{1}$$

3. (b) According to question



$$OB^2 + OC^2 = BC^2$$

$$OB^2 + OA^2 = AB^2$$

$$OA^2 + OD^2 = AD^2$$

[By pythagoras theorem]

$$OC^2 + OD^2 = CD^2$$

Add equation (i),(ii),(iii) and (iv)

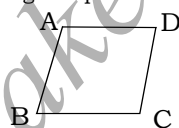
$$\Rightarrow 2(OB^2 + OC^2 + OD^2 + OA^2) = BC^2 + AB^2 + AD^2 + CD^2$$

$$\Rightarrow 2BC^2 + 2AD^2 = BC^2 + AB^2 + AD^2 + CD^2$$

$$BC^2 + AD^2 = AB^2 + CD^2$$

$$\text{or } AB^2 + CD^2 = BC^2 + DA^2$$

4. (c) According to question.



Given:

Ratio of $\angle A$ and $\angle B$ is 4 : 5

$$\Rightarrow \frac{\angle A}{\angle B} = \frac{4}{5}$$

We know that $\angle A + \angle B = 180^\circ$

$$9 \text{ units} = 180^\circ$$

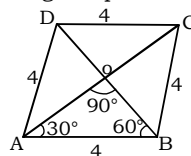
$$1 \text{ unit} = 20^\circ$$

$$\angle A = 4 \text{ units} = 4 \times 20^\circ = 80^\circ$$

$$\angle A = \angle C = 80^\circ$$

[Opposite $\angle$ of rhombus are equal]

5. (d) According to question,



Given :

$$\angle B = 120^\circ$$

In a rhombus diagonal are angle bisector and diagonal cut at right triangle.

$$\therefore \sin 30^\circ = \frac{P}{H} = \frac{BO}{AB} = \frac{1}{2} = \frac{BO}{4}$$

$$BO = 2 \text{ cm}$$

$$\therefore BD = 2 \times BO$$

$$= 2 \times 2 = 4 \text{ cm}$$

Alternate

$$\angle ABD = \frac{1}{2} \angle ABC$$

$$= \frac{1}{2} \times 120^\circ = 60^\circ$$

$$\therefore \angle A = \angle ABD = \angle ADB = 60^\circ$$

$\therefore \triangle ABD$ = equilateral triangle

So, $AB = BD = 4 \text{ cm}$

6. (a)

According to question

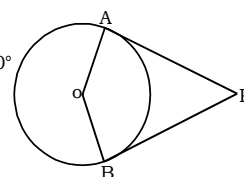
Given: PAOB is quadrilateral

$$\therefore \angle AOB : \angle APB$$

$$5x : 1x$$

Note: In Quadrilateral Sum of opposite angle is 180°

$$\therefore \angle AOB + \angle APB = 180^\circ$$



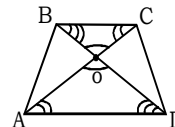
$$\text{Then } 5x + x = 180^\circ$$

$$6x = 180^\circ$$

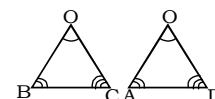
$$x = 30^\circ$$

$$\therefore \angle APB = 30^\circ$$

7. (d) According to question,



$\triangle AOD \sim \triangle COB$



$$\therefore \frac{OB}{OD} = \frac{OC}{OA}$$

$$\Rightarrow \frac{3x-19}{x-5} = \frac{x-3}{3}$$

$$\Rightarrow 9x - 57 = x^2 - 8x + 15$$

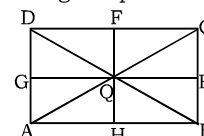
$$\Rightarrow x^2 - 17x + 72 = 0$$

$$\Rightarrow x(x-8) - 9(x-8) = 0$$

$$\Rightarrow (x-8)(x-9) = 0$$

$$\Rightarrow x = 8 \text{ or } 9$$

8. (a) According to question



Given:

$$QA = 3 \text{ cm}$$

$$QB = 4 \text{ cm}$$

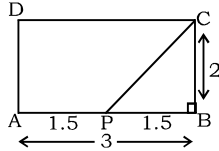
$$QC = 5 \text{ cm}$$

$$QD = ?$$

As we know that
 $\Rightarrow QD^2 + QB^2 = QA^2 + QC^2$
(By using Pythagoras theorem)
 $\Rightarrow QD^2 + (4)^2 = (3)^2 + (5)^2$
 $\Rightarrow QD^2 + 16 = 9 + 25$
 $\Rightarrow QD^2 = 34 - 16$
 $\Rightarrow QD^2 = 18$

$$QD = \sqrt{18}, \quad QD = 3\sqrt{2}$$

9. (d) According to question



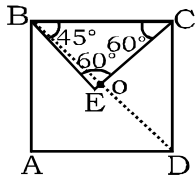
$\Rightarrow$ In $\triangle CBP$
 $\Rightarrow CP^2 = BP^2 + BC^2$
 $\Rightarrow CP^2 = (1.5)^2 + (2)^2$
 $\Rightarrow CP^2 = 2.25 + 4$
 $\Rightarrow CP^2 = 6.25$
 $\Rightarrow CP = \sqrt{6.25}$
 $\Rightarrow CP = 2.5$

$$\therefore \sin \angle CPB = \frac{BC}{CP}$$

$$\sin \angle CPB = \frac{2}{2.5}$$

$$\sin \angle CPB = \frac{4}{5}$$

10. (b) According to question



ABCD is a square and BCE is an equilateral triangle

$$\therefore \angle CEB = 60^\circ$$

If BD is a diagonal

$$\therefore \angle CBD = 45^\circ$$

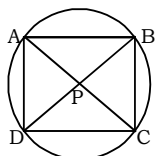
Then In $\triangle BOC$

$$\angle CBO + \angle BOC + \angle BCE = 180^\circ$$

$$\angle BOC = 180^\circ - 60^\circ - 45^\circ$$

$$\angle BOC = 75^\circ$$

11. (b) According to question



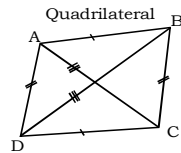
ABCD is a cyclic quadrilateral.

$$\therefore AP \times PC = DP \times BP \text{ (theorem)}$$

AC and BD are chords of circle

$$AP \cdot CP = BP \cdot DP$$

12. (a) According to question



$$AB = CD$$

$$BC = AD$$

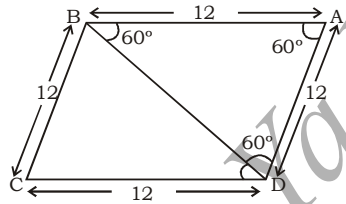
$$AC = BD$$

$$\angle DCB = 90^\circ$$

Note: Only rectangle follows these condition

$\therefore$ angles of the quadrilateral is same as each angle of rectangle = 90°

13. (c) We know that in a Rhombus diagonal bisect the angle.



$$\therefore \angle A = 60^\circ$$

$$\text{then } \angle B = 180^\circ - 60^\circ$$

$$\angle ABC = 120^\circ$$

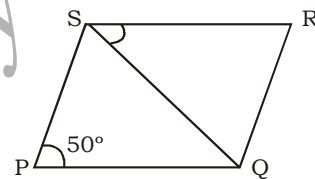
$$\text{Now } \angle ABD = \angle CBD = \frac{120}{2} = 60^\circ$$

$$\Rightarrow \text{So } \angle ABD = \angle BDA = \angle BAD = 60^\circ$$

$$\Rightarrow \text{So } \triangle ABD \text{ is an equilateral triangle then } AD = AB = BD = 12 \text{ cm}$$

$$\Rightarrow \text{So Diagonal } BD = 12 \text{ cm}$$

14. (d) According to the question,



$$\angle P = 50^\circ, \quad \angle R = 50^\circ$$

$$\text{then } \angle PSR = 180^\circ - 50^\circ = 130^\circ$$

$$\text{then } \angle RSQ = \frac{130^\circ}{2} = 65^\circ$$

15. (a) Let ABCD is a quadrilateral and its BD diagonal = 24 metres

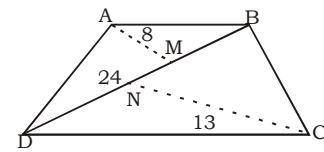
$$\text{And, } AM = 8 \text{ metres}$$

$$CN = 13 \text{ metres}$$

$$\text{Area of } \square ABCD = \text{ar}(\triangle ABD) + \text{ar}(\triangle CBD)$$

$$= \left(\frac{1}{2} BD \times AM \right) + \left(\frac{1}{2} BD \times CN \right)$$

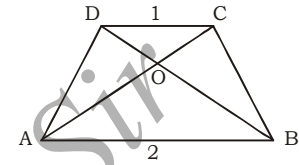
$$= \frac{1}{2} \times BD [AM + CN]$$



$$= \frac{1}{2} \times 24 [8 + 13] = 12 \times 21$$

$$\text{Area of } \square ABCD = 252 \text{ m}^2$$

16. (a)



$$\frac{\text{area of } \triangle COD}{\text{area of } \triangle AOB} = \frac{CD^2}{AB^2}$$

$$\frac{\text{area of } \triangle COD}{84} = \left(\frac{1}{2} \right)^2 \Rightarrow \frac{1}{4}$$

$$\text{area of } \triangle COD = 21 \text{ cm}^2$$

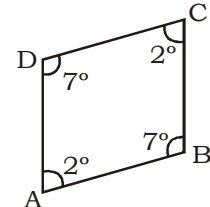
17. (b) $AB + CD = BC + DA$ (Property)

$$\Rightarrow 7 + 9.2 = x + 8.5$$

$$\Rightarrow 16.2 = x + 8.5,$$

$$x = 7.7$$

18. (c) ATQ



As we know that in a parallelogram opposite angle are same.

$$\therefore \angle A = \angle C$$

$$\angle B = \angle D$$

Note:- Parallelogram is rhombus but rhombus is not a parallelogram

19. (c) Angles be $x, 2x, 3x, 4x$

$$\therefore x + 2x + 3x + 4x = 360^\circ$$

$$\Rightarrow 10x = 360^\circ \Rightarrow x = 36^\circ$$

$$\therefore \text{largest angle} = 4x = 144^\circ$$

20. (a) $\angle DAB = 180^\circ - b$ and

$$\angle BCD = 180^\circ - a$$

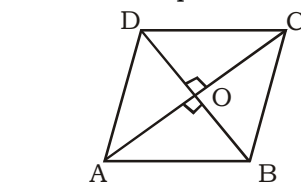
But,

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\Rightarrow (180^\circ - b^\circ) + x^\circ + (180^\circ - a^\circ) + y^\circ = 360^\circ$$

$$\Rightarrow x^\circ + y^\circ = a^\circ + b^\circ$$

21. (a) Since diagonals bisect each other at right angles and all sides are equal.



$$AO^2 + BO^2 = AB^2$$

$$\left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2 = AB^2$$

$$AC^2 + BD^2 = 4AB^2$$

22.(a) Since, their base and perpendicular heights are same.

23.(d) $\angle ACB = \angle DAC = 50^\circ$ (Alternate interior $\angle$ s)

$$\angle BOC = 180^\circ - 80^\circ = 100^\circ$$

$\therefore$ Now in $\triangle BOC$,

$$\begin{aligned}\angle DBC &= 180^\circ - (100^\circ + 50^\circ) \\ &= 30^\circ\end{aligned}$$

24.(d) $\angle A + \angle B = 180^\circ$ and $\angle A = \angle B$

$$\Rightarrow \angle A = \angle B = 90^\circ$$

So, the given $||gm$ may be a square or a rectangle as in both the cases the adjacent angles are equal.

25.(a) $7x = 42 \Rightarrow x = 6$

and $8y = 32 \Rightarrow y = 4$

26.(a) In $\triangle PSX$ and $\triangle QRY$

$$\angle X = \angle Y = 90^\circ \text{ and }$$

$$SX = RY$$

$$[\because SX = SY - XY \text{ and } RY = SY - SR = SY - PQ = SY - XY]$$

$$\text{and } PS = QR \text{ (sides of a } ||gm)$$

$$\therefore \triangle PSX \cong \triangle QRY$$

(R.H.S axiom)

$$\therefore PX = QY$$

27.(a) Area of $||gm$ ABCD

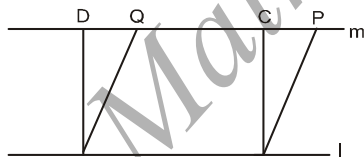
$$= \text{Base} \times \text{height}$$

$$\Rightarrow AB \times DM = AD \times CL$$

$$\Rightarrow 16 \times 12 = AD \times 15$$

$$\Rightarrow AD = 12.8 \text{ units}$$

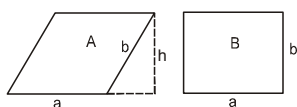
28.(d)



area of square ABCD = area of rhombus ABPQ

they lie on the same base AB and between two parallel lines ($l || m$).

29.(b)

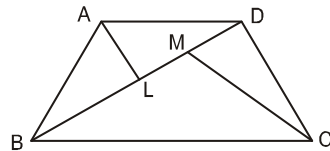


$$B = ab$$

$$A = ah \Rightarrow A < ab \text{ } [\because h < b]$$

$$\Rightarrow A < B$$

30.(d)



$$\begin{aligned}\text{Area of quadrilateral ABCD} \\ &= \text{Area of } \triangle ABD + \text{Area of } \triangle CBD\end{aligned}$$

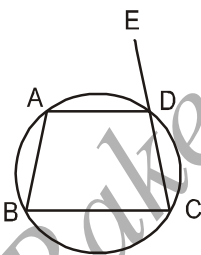
$$= \frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CM$$

$$= \frac{1}{2} \times BD(AL + CM)$$

$$= \frac{1}{2} \times 64(13.2 + 16.8)$$

$$= \frac{1}{2} \times 64 \times 30 = 960 \text{sq.cm.}$$

31.(d)



$$\angle ABC + \angle CDA = 180^\circ$$

$$\Rightarrow \angle CDA = 180^\circ - 72^\circ = 108^\circ$$

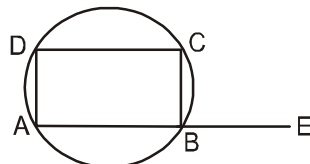
$$\therefore \angle ADE = 180^\circ - 108^\circ = 72^\circ$$

$$AD || BC$$

$$\angle BCD = \angle ADE = 72^\circ$$

(corresponding angles)

32.(c)



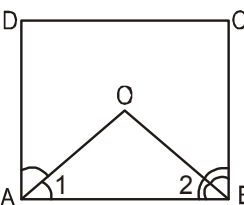
$$\angle ABC + \angle ADC = 180^\circ$$

$$\therefore \angle ABC = 180^\circ - 50^\circ = 130^\circ$$

$$\therefore \angle ADC = 180^\circ - 130^\circ = 50^\circ$$

$$\angle CBE = 50^\circ = \angle ADC$$

33.(c)



$$\angle AOB = 180^\circ - (\angle 1 + \angle 2)$$

$$= 180^\circ - \left(\frac{1}{2}\angle A + \frac{1}{2}\angle B\right)$$

$$= 180^\circ - \frac{1}{2} [360^\circ - (\angle C + \angle D)]$$

$$[\because \angle A + \angle B + \angle C + \angle D = 360^\circ]$$

$$\therefore \angle AOB = 180^\circ - 180^\circ + \frac{1}{2}(\angle C$$

$$+ \angle D) = \frac{1}{2}(\angle C + \angle D)$$

34.(d) $\angle AOB = \frac{1}{2}(\angle A + \angle B)$

$$= \frac{1}{2}(180^\circ) \text{ } [\because \angle C + \angle D = \angle A$$

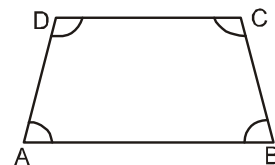
$$+ \angle B = 180^\circ] = 90^\circ$$

35.(b) $x + 2x + 3x + 4x = 360$

$$\Rightarrow x = 36$$

$\therefore$ The angles of quadrilateral (in order) are

$$36^\circ, 72^\circ, 108^\circ, 144^\circ$$



Since, opposite angles are supplementary,

Therefore, $AB || CD$. Hence, it is a trapezium.

36.(a) $BD = \sqrt{12^2 + 16^2} = 20 \text{cm}$

($\triangle ABD$ is a right angle triangle)

$\therefore \triangle BCD$ is an equilateral triangle.

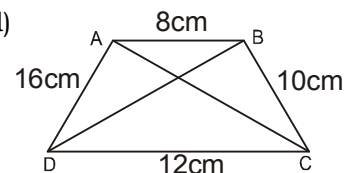
$\therefore$ Area of $\square ABCD$ = Area of $\triangle ABD$ + Area of $\triangle BCD$

$$= \frac{1}{2} \times 16 \times 12 + \frac{\sqrt{3}}{4} (20)^2$$

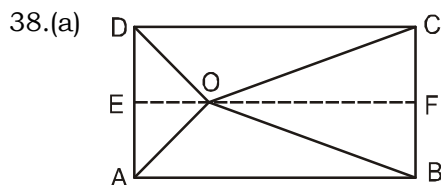
$$= 96 + 100\sqrt{3}$$

$$= 4(24 + 25\sqrt{3}) \text{ cm}^2$$

37.(d)



$$\begin{aligned}
 AC^2 + BD^2 &= (AD^2 + BC^2) + 2 \\
 &\quad (AB \times CD) \\
 &= (256 + 100) + 2(8 \times 12) \\
 &= 356 + 192 = 548 \text{ cm}^2
 \end{aligned}$$



Draw $EF \parallel AB$

In right angled $\triangle EOA$ and $\triangle OCF$.

$$OA^2 = OE^2 + AE^2 \text{ and } OC^2 = OF^2 + CF^2$$

$$\therefore OA^2 + OC^2$$

$$= OE^2 + AE^2 + OF^2 + CF^2 \quad \dots(i)$$

Similarly in the right angled $\triangle DEO$ and $\triangle OBF$,

$$OB^2 + OD^2 = OE^2 + OF^2 + DE^2 + BF^2$$

$$\Rightarrow OB^2 + OD^2 = OE^2 + OF^2 + CF^2 + AE^2 \quad \dots(ii)$$

$$(\because DE = CF \text{ and } BF = AE)$$

$$\therefore \text{from (i) and (ii)}$$

$$OA^2 + OC^2 = OB^2 + OD^2$$

39. (b) By mid-point theorem

$$\frac{EF}{AD} = \frac{FG}{DC} = \frac{GH}{CB} = \frac{HE}{BA} = \frac{1}{2}$$

$$\therefore \frac{EF + FG + GH + HE}{AD + DC + CB + BA}$$

$$\therefore \frac{\frac{1}{2}(AD + DC + CB + BA)}{(AD + DC + CB + BA)} = \frac{1}{2}$$

40.(b) $\angle BCE = 102^\circ$, $AB = CD = ED$ (given)

$$\therefore CD = ED = CE [\because AB = CE]$$

$\triangle ECD$ is an equilateral triangle.

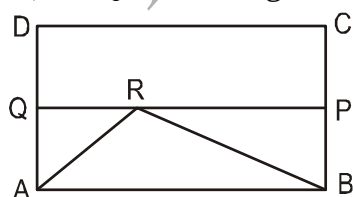
$$\therefore \angle ECD = 60^\circ$$

$$\angle BCD = 102^\circ + 60^\circ$$

$$= 162^\circ$$

41.(c) $AB \parallel PQ \parallel CD$.

So, $ABPQ$ is a rectangle



$$\therefore \triangle ARB = \frac{1}{2}(\square ABPQ)$$

$$= \frac{1}{2} \times \left(\frac{1}{2} \times \square ABCD \right)$$

$$= \frac{1}{4}(\square ABCD)$$

$$42.(a) \quad AX = \frac{1}{2}AB \text{ and } CY = \frac{1}{2}DC$$

but, $AB = DC$

$$\Rightarrow \frac{1}{2}AB = \frac{1}{2}DC$$

$$\Rightarrow AX = CY$$

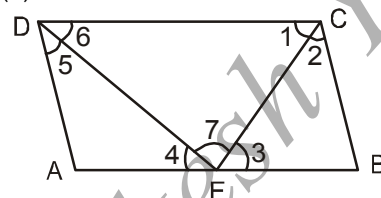
also, $AB \parallel DC$ [$\because ABCD$ is a $\parallel$ gm]

Thus in quadrilateral $AXCY$,

$$\Rightarrow AX \parallel CY \text{ and } AX = CY$$

Hence, quadrilateral $AXCY$ is a $\parallel$ gm.

43.(b)



$AB \parallel DC$ and EC cuts them

$$\Rightarrow \angle 3 = \angle 1$$

$$\Rightarrow \angle 3 = \angle 2 (\because \angle 1 = \angle 2)$$

$$\Rightarrow EB = BC$$

$$\Rightarrow AE = AD$$

Now, $AE = AD$

$$\Rightarrow \angle 4 = \angle 5 \text{ and } \angle 4 = \angle 6$$

$$\Rightarrow DE \text{ bisects } \angle ADC,$$

Again, $\angle ADC + \angle BCD = 180^\circ$ (Co.Int. Angles)

$$\Rightarrow \frac{1}{2}\angle ADC + \frac{1}{2}\angle BCD = 90^\circ$$

$$\Rightarrow \angle 6 + \angle 1 = 90^\circ$$

$$\Rightarrow \angle 1 + \angle 6 + \angle 7 = 180^\circ$$

$$\Rightarrow \angle 7 = 180^\circ - 90^\circ$$

$$\Rightarrow \angle DEC = 90^\circ$$

44.(c) In $\triangle ARB$, P is the mid-point of AB and $PD \parallel BR$

$$\Rightarrow D \text{ is the mid-point of } AR.$$

$$\therefore ABCD \text{ is a } \parallel \text{ gm}$$

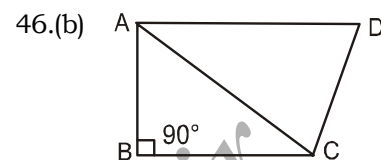
$$\Rightarrow DC \parallel AB \Rightarrow DQ \parallel AB$$

$\therefore Q$ is the mid-point of $RB \Rightarrow BR = 2BQ$.

$$45.(a) \quad MN = \frac{1}{2}(AB + CD) \Rightarrow 2 \times 15 =$$

$$14 + CD$$

$$\Rightarrow CD = 16 \text{ cm.}$$



$$AD^2 = AB^2 + BC^2 + CD^2$$

$$\therefore \text{In } \triangle ABC, \angle B = 90^\circ$$

$$\therefore AC^2 = AB^2 + BC^2$$

$$\therefore AD^2 = AC^2 + CD^2$$

$$\Rightarrow \angle ACD = 90^\circ$$

47.(a) As we know that sum of two sides of a triangle is greater than the third side.

$$\therefore \text{In } \triangle ABD$$

$$AB + DA > BD \quad (i)$$

and In $\triangle BDC$

$$BC + CD > BD \quad (ii)$$

$$\therefore \text{from (i) and (ii) } AB + BC + CD + DA > 2BD.$$

48.(b) $EF = AD = 8$ ($\because EADF$ is a rectangle)

$$CD = 22 - 16 = 6$$

So, In right angled $\triangle ADC$,

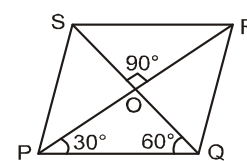
$$AC = \sqrt{8^2 + 6^2} = 10$$

$\therefore$ length of the line joining the

$$\text{mid-points of } AB \text{ \& } BC = \frac{1}{2}(AC)$$

$$= 5$$

49.(b)



$$\angle PQO = \frac{1}{2}\angle PQR = 60^\circ$$

From $\triangle POQ$,

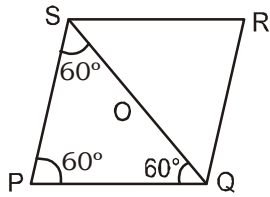
$$\angle OPQ = 180^\circ - 90^\circ - 60^\circ = 30^\circ$$

$$\sin \angle OPQ = \frac{OQ}{PQ}$$

$$\Rightarrow OQ = PQ \sin 30^\circ = 6 \times \frac{1}{2} = 3$$

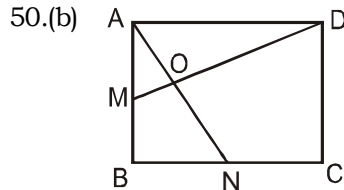
$$\therefore QS = 2 \times 3 = 6 \text{ cm}$$

Alternatively



$\triangle SPQ$ is an equilateral triangle

$\therefore QS = 6 \text{ cm}$



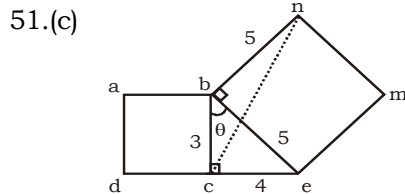
If $AB = 2x$, then $BN = x$

$$\therefore AN = \sqrt{4x^2 + x^2} = \sqrt{5}x$$

Similarly

$$MD = \sqrt{4x^2 + x^2} = \sqrt{5}x$$

$AN = MD$



In $\triangle CBE$

$$\cos(90^\circ + \theta) = \frac{bc^2 + bn^2 - cn^2}{2bc \times bn}$$

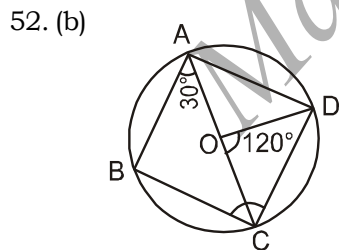
$$-\sin\theta = \frac{(3^2 + 5^2 - cn^2)}{2 \times 3 \times 5}$$

$$-\sin\theta = \frac{34 - cn^2}{30}$$

$$-\frac{4}{5} = \frac{34 - cn^2}{30}$$

$$-24 - 34 = -cn^2$$

$$cn = \sqrt{58}$$



$$\angle COD = 120^\circ$$

$$\angle BAC = 30^\circ$$

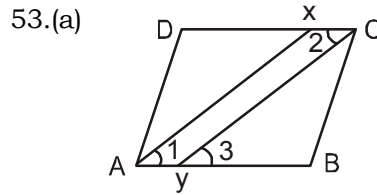
$$\angle CAD = \frac{1}{2} \times 120^\circ = 60^\circ$$

(angle made on other part of circle is half of angle made at centre by same arc)

$$\therefore \angle BAD = 90^\circ$$

$$\therefore \angle BCD = 180^\circ - 90^\circ = 90^\circ$$

(cyclic quadrilateral)



ABCD is a ||gm (given)

$$\therefore \angle A = \angle C$$

$$\Rightarrow \frac{1}{2} \angle A = \frac{1}{2} \angle C$$

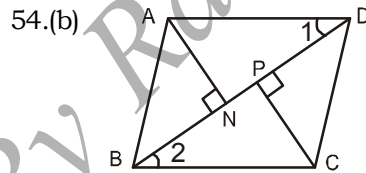
$$\Rightarrow \angle 1 = \angle 2$$

Now, $AB \parallel DC$ and the transversal CY intersects then:

$$\therefore \angle 2 = \angle 3$$

from (i) and (ii), we get $\angle 1 = \angle 3$
Thus, transversal AB intersects AX and YC at A and Y such that $\angle 1 = \angle 3$ i.e. corresponding angles are equal.

$$\therefore AX \parallel CY.$$



In $\triangle ADN$ and $\triangle CBP$,

$$\angle 1 = \angle 2 \quad [\because AD \parallel BC]$$

$$\angle AND = \angle CPB \text{ and }$$

$AD = BC$ ($\because$ opposite sides of a ||gm are equal)

So, by AAS criterion of congruence. $\therefore AN = CP$

55.(c) PQRS is a ||gm.

$$\therefore PS \parallel QR$$

$$\Rightarrow PL \parallel QM \text{ and } LM \parallel PQ$$

(given)

$$\Rightarrow PQML \text{ is a } ||gm$$

$$\Rightarrow PL = QM \text{ (Opposite sides of a } ||gm \text{ are equal)}$$

$$\angle 1 = \angle 2$$

...(i)

[OP is the bisector of $\angle P$]

$$\text{and } \angle 1 = \angle 3$$

...(ii)

$$[\because PQ \parallel LM]$$

from (i) and (ii) $\angle 2 = \angle 3$

$$\therefore \text{In } \triangle OPL, \angle 2 = \angle 3$$

$$\Rightarrow OL = PL \quad \dots(\text{iii})$$

Similarly,

$$\angle 4 = \angle 5 \text{ and } \angle 4 = \angle 6$$

$$\therefore \angle 5 = \angle 6$$

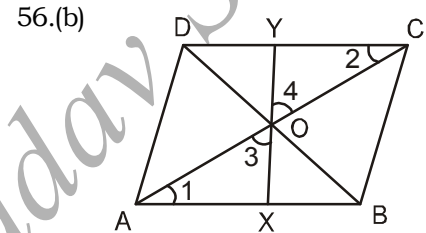
$$\therefore \text{In } \triangle QOM, \angle 5 = \angle 6$$

$$\Rightarrow OM = QM$$

$$\Rightarrow OM = PL \quad \dots(\text{iv})$$

$$[\because PL = QM]$$

$$\therefore \text{from (iii) and (iv), } OL = OM$$



In $\triangle OAX$ and $\triangle OCY$.

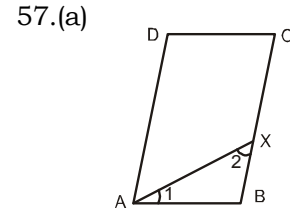
$$\angle 1 = \angle 2 \quad (\because AB \parallel DC)$$

$\angle 3 = \angle 4$ (vertically opposite angles)

and $OA = OC$ ($\because$ diagonals of a ||gm bisect each other)

$$\therefore \triangle OAX \cong \triangle OCY$$

$$\Rightarrow OX = OY$$



$$\angle B = 180^\circ - \angle A = 180^\circ - 2\angle 1$$

In $\triangle ABX$,

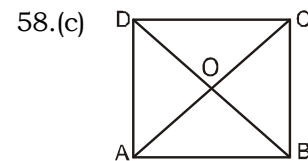
$$\angle 1 + \angle 2 + \angle B = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 + 180^\circ - 2\angle 1 = 180^\circ$$

$$\Rightarrow \angle 1 = \angle 2$$

$$\Rightarrow AB = BX \Rightarrow 2BX = 2AB$$

$$\Rightarrow BC = 2AB \Rightarrow AD = 2AB$$



Let $BD = 6 \text{ cm}$ and $AC = 8 \text{ cm}$

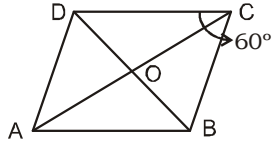
$$\therefore AO = 4 \text{ m and } BO = 3 \text{ m}$$

$$\text{let } AB = 5 \text{ m } \therefore \angle AOB = 90^\circ$$

$$\Rightarrow \angle BOC = \angle AOD = \angle DOC = 90^\circ$$

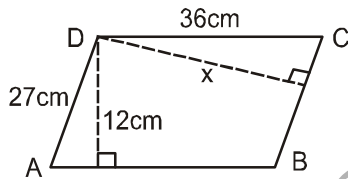
Here, ABCD is a rhombus
 $\therefore$ Area of rhombus ABCD
 $= \frac{AC \times BD}{2} = \frac{6 \times 8}{2} = 24\text{m}^2$

59.(a)



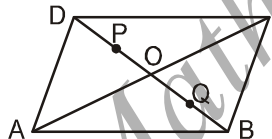
In $\triangle BDC$, $BC = CD$
 $\Rightarrow$ Let, $\angle BDC = \angle DBC = x^\circ$
 $\therefore x + x + 60^\circ = 180^\circ \Rightarrow x = 60^\circ$
 $\therefore \triangle BDC$ is an equilateral triangle
 $\therefore BD = BC = a$
 In $\triangle AOB$, $\angle AOB = 90^\circ$
 $\therefore AB^2 = OA^2 + OB^2$
 $\Rightarrow OA^2 = AB^2 - OB^2$
 $\Rightarrow OA^2 = a^2 - \left(\frac{a}{2}\right)^2$
 $\Rightarrow OA = \frac{\sqrt{3}a}{2}$
 $\Rightarrow AC = 2(OA) = \sqrt{3}a$
 $\therefore AC : BD = \sqrt{3}a : a = \sqrt{3} : 1$

60.(b)



let distance = x
 Area of $\parallel\text{gm} = \text{Base} \times \text{Height}$
 $36 \times 12 = x \times 27$
 $\Rightarrow x = 16$

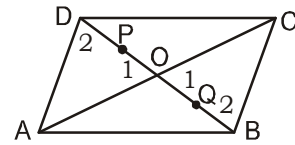
61.(d)



Since, diagonals of a $\parallel\text{gm}$ bisect each other.

$\therefore BO = OD = \frac{18}{2} = 9\text{cm}$
 $P = \text{centroid of } \triangle ADC$
 $\therefore OP = \frac{1}{3}OD = \frac{1}{3} \times 9 = 3\text{cm}$
 $Q = \text{centroid of } \triangle ABC$
 $\therefore OQ = \frac{1}{3}OB = \frac{1}{3} \times 9 = 3\text{cm}$
 $\therefore PQ = OP + OQ = 3 + 3 = 6\text{cm}$

Alternatively

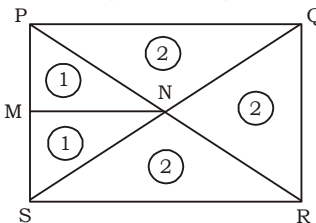


6 units = 18

1 unit = 3

$\therefore 2 \text{ unit} = 6$

62.(b) Let $\text{ar}(\square PQRS) = 8 \text{ units}$



Area of $PQNM = \text{area of } \triangle PNQ + \text{area of } \triangle PNM$
 $= 2 + 1 = 3 \text{ units}$

$\therefore \text{area}(\square PQNM) : \text{area}(\square PQRS) = 3 : 8$

63.(a) $AE : EO = 2 : 1$ and

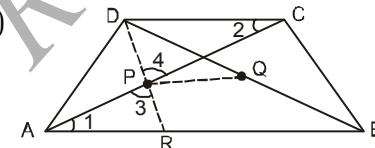
$CF : FO = 2 : 1$

$\therefore OE = \frac{1}{3}AO$ and $OF = \frac{1}{3}OC$

$\therefore EF = OE + OF$

$= \frac{1}{3}(AO + OC) = \frac{1}{3}AC = AE$

64.(c)



In $\triangle APR$ and $\triangle DPC$,
 $\angle 1 = \angle 2$ (alternate angles)
 $AP = CP$ ($\because P$ is mid-point of AC)
 and $\angle 3 = \angle 4$ (vertically opposite angles)

So, $\triangle APR \cong \triangle DPC$ (ASA)

$\Rightarrow AR = DC$ and $PR = DP$

Again, P & Q are the mid-points of sides DR and DB respectively.

In $\triangle DRB$,

$PQ = \frac{1}{2}BR$

$\therefore PQ = \frac{1}{2}(AB - AR)$

$\therefore PQ = \frac{1}{2}(AB - CD)$

($\because AR = DC$)

65.(a) Area of $(\triangle PRS + \triangle PQR) =$

$\frac{1}{2}$ (area of $\square APRD$)

$+ \frac{1}{2}$ (area of $\square BPRC$)

$= \frac{1}{2}(AP \times AD) + \frac{1}{2}(PB \times BC)$

$= \frac{1}{2}(AP \times AD) + \frac{1}{2}(PB \times AD)$ ($\because BC = AD$)

$= \frac{1}{2}AD (AP + PB)$

$= \frac{1}{2}(AD \times AB)$

$= \frac{1}{2}$ (area of $\square ABCD$)

$= \frac{1}{2} \times 20 = 10\text{cm}^2$

66.(d) Let $BC = x$ and

$FB = EF = AE = y$

$\therefore AB = CD = 3y$

Now, Area of

$\triangle CBF = \frac{1}{2}xy$

and area of $\triangle CBE =$

$\frac{1}{2}x \times 2y = xy$

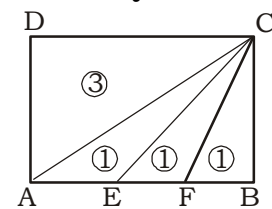
$\therefore$ area of $\triangle CEF$

$= xy - \frac{1}{2}xy = \frac{1}{2}xy$

and area of rectangle $ABCD = 3xy$

$\therefore$ Required ratio $= \frac{1}{2}xy : 3xy$
 $= 1 : 6$

Alternatively



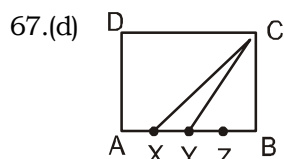
Let $\text{ar}(\square ABCD) = 6 \text{ units}$

Base and height are same

$\text{ar}(\triangle CAE) = \text{ar}(\triangle CEF) =$

$\text{ar}(\triangle CFB) = 1 \text{ unit}$

$\therefore$ required ratio $= 1 : 6$



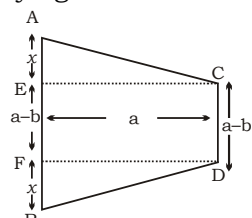
Let, $AB = 4x$ units and
 $BC = y$ units
 $\therefore \square ABCD = 4xy$ sq. units
 In $\triangle XYZ$
 $XY = x$ units
 Height = y units

$$\therefore \text{Area of } \triangle XYZ = \frac{1}{2}xy$$

$$\therefore \frac{\text{Ar}(\triangle XYZ)}{\text{Ar}(\text{Rectangle } ABCD)} = \frac{\frac{1}{2}xy}{4xy} = \frac{1}{8}$$

68.(c) Calculate them manually

69.(d) They are symmetrically lying on horizontal place.



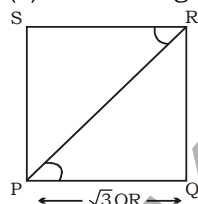
$\therefore AC = BD$
 $\therefore AE = BF = x$ (say)
 Now, $AB = (a-b) + 2x$
 i.e., $a+b = a-b+2x \Rightarrow$
 $2b = 2x$

$\therefore x = b$

Now In $\triangle ACE$,
 $x^2 + a^2 = AC^2$

$$\Rightarrow AC^2 = b^2 + a^2 \Rightarrow AC = \sqrt{b^2 + a^2}$$

70.(c) In rectangle PQRS,



$PQ \parallel RS$

$\therefore \angle RPQ = \angle PRS$ (alternate angles)(i)

Now In $\triangle PQR$,

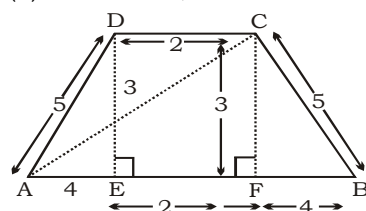
$$\tan \angle QPR = \frac{RQ}{PQ}$$

$$\Rightarrow \tan \angle QPR = \frac{QR}{\sqrt{3}QR}$$

$$\Rightarrow \angle QPR = 30^\circ$$

$$\therefore \angle PRS = 30^\circ \quad [\text{from Eq. (i)}]$$

71. (b) In $\triangle BCF$,



By Pythagoras theorem,
 $(5)^2 = (3)^2 + (BF)^2 \Rightarrow BF = 4$ cm

$$\therefore AB = 2+4+4=10 \text{ cm}$$

Now In $\triangle ACF$,

$$AC^2 = CF^2 + FA^2$$

$$\Rightarrow AC^2 = 3^2 + 6^2, AC = \sqrt{45} \text{ cm}$$

$$\text{Similarly, } BD = \sqrt{45} \text{ cm}$$

$$\therefore \text{Sum of diagonal} = 2 \times \sqrt{45} = 2$$

$$\times 3\sqrt{5} = 6\sqrt{5} \text{ cm}$$

72. (b) Area of rectangle lies between 40 cm^2 and 45 cm^2

Now, one side = 5 cm

Since, area cannot be less than 40 cm^2 .

$\therefore$ Other side cannot be less than

$$= \frac{40}{5} = 8 \text{ cm}$$

Since, area cannot be greater than 45 cm^2

$\therefore$ Other side cannot be greater

$$\text{than } = \frac{45}{5} = 9 \text{ cm}$$

$\therefore$ Minimum value of diagonal =

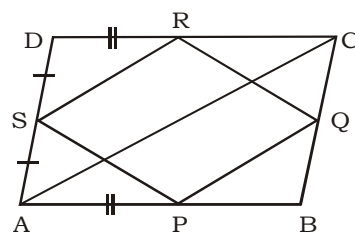
$$\sqrt{8^2 + 5^2} = \sqrt{89}$$

Maximum value of diagonal

$$= \sqrt{9^2 + 5^2} = \sqrt{106} = 10.3 \text{ cm}$$

So, diagonal lies between 9 cm and 11 cm.

73. (b) Area of $\triangle APS$ = Area of $\triangle DSR$

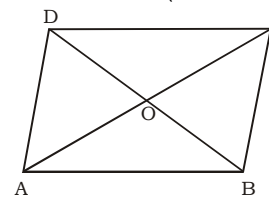


$\therefore AS = SD$ and $AP = DR$

$$\therefore \text{ar}(\triangle ABC) = 4 \text{ ar}(\triangle BPQ)$$

74. (b) I. ABCD is parallelogram, then

$$AC^2 + BD^2 = 2(AB^2 + BC^2)$$



II. ABCD is a rhombus and diagonals AC and BD bisect each other

$$\therefore AO = OC$$

$$\text{and } OB = OD$$

$$\text{In } \triangle AOB, AB^2 = AO^2 + OB^2$$

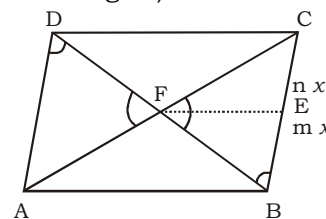
$$(4)^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$$\therefore AC^2 + BD^2 = 64$$

$$= (4)^3 \text{ i.e., } n^3$$

75. (d) In $\triangle AFD$ and $\triangle BFE$,

$\angle AFD = \angle BFE$ (vertically opposite angles)

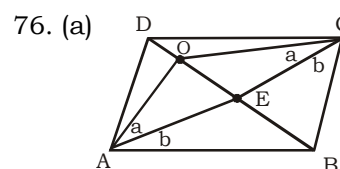


and $\angle ADF = \angle FBE$ (alternate angles)

$$\therefore \triangle AFD \sim \triangle BFE$$

$$\text{So, } \frac{\text{ar}(\triangle FEB)}{\text{ar}(\triangle AFD)} = \frac{EB^2}{AD^2}$$

$$= \frac{m^2 x^2}{(mx + nx)^2} = \frac{m^2}{(m+n)^2} = \left[\frac{m}{(m+n)} \right]^2$$



Line joining A and C cuts BD at E.

AC diagonals of parallelogram bisect each other,

OE and BE will be median of $\triangle OAC$ and $\triangle ABC$ respectively,

Let Area of $\triangle OAE$ = Area of $\triangle OEC = a$

Then,

Area of $\triangle AEB$ = Area of $\triangle EBC$ = b

Area of $\triangle OAB$ = Area of $\triangle OAE$
 + Area of $\triangle AEB$ = $(a + b)$

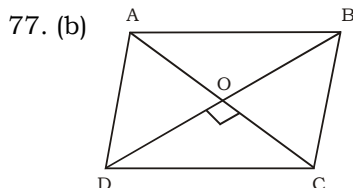
Similarly,

Area of $\triangle OBC$ = Area of $\triangle OEC$
 + Area of $\triangle EBC$ = $(a + b)$

Hence, if O is any point on diagonal BD in parallelogram ABCD then,,

Area of $\triangle OAB$ = Area of $\triangle OBC$

Area of OBC = 19 cm^2



$$\text{Here, } OD = \frac{BD}{2} = \frac{4.8}{2} = 2.4 \Rightarrow OC$$

$$= \frac{AC}{2} = \frac{1.4}{2} = 0.7$$

Since, In rhombus diagonal bisect at 90° . Then, In $\triangle ODC$.
 $OD^2 + OC^2 = CD^2$

$$\Rightarrow CD = \sqrt{OD^2 + OC^2} = \sqrt{(2.4)^2 + (0.7)^2}$$

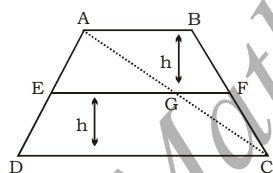
$$CD = \sqrt{6.25} \Rightarrow CD = 2.5 = \frac{5}{2}$$

$$\therefore \text{Perimeter of rhombus} = 4a$$

$$= 4 \times \frac{5}{2} = 10 \text{ cm}$$

78. (c) Join AC.

In $\triangle ACD$, $EG \parallel DC$ and E and G are mid-points of AD and AC, respectively.



$$\therefore EG = \frac{1}{2} DC = \frac{3}{2}$$

Similarly, In $\triangle ABC$

$$GF = \frac{1}{2} AB = 1$$

$$EF = EG + GF = 1 + \frac{3}{2} = \frac{5}{2}$$

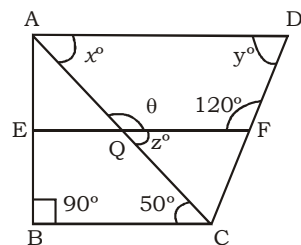
$$\therefore \text{Area of trapezium} = \frac{1}{2} (\text{Sum}$$

of parallel sides $\times$ Height)

Now, Required ratio

$$= \frac{\text{Area of ABFE}}{\text{Area of EFCD}} = \frac{\frac{1}{2} \left(2 + \frac{5}{2} \right) \times h}{\frac{1}{2} \left(3 + \frac{5}{2} \right) \times h} = \frac{9}{11}$$

79. (c) From figure,



$BC \parallel EF \parallel AD$

$\therefore x^\circ = z^\circ = 50^\circ$ (corresponding interior angle)

$\therefore \theta + z^\circ = 180^\circ$ (linear pair)

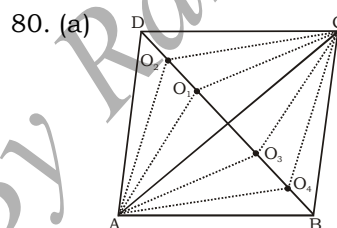
$\theta = 180^\circ - 50^\circ = 130^\circ$

In quadrilateral AQFD,

$$x^\circ + y^\circ + 120^\circ + \theta = 360^\circ$$

$$50^\circ + y^\circ + 120^\circ + 130^\circ = 360^\circ$$

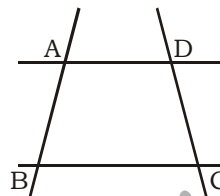
$$y = 360^\circ - 300^\circ = 60^\circ$$



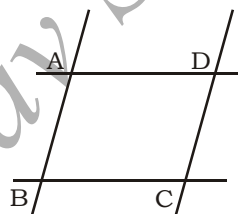
We know that, in a rhombus ABCD diagonals are bisect each other at point O. Which means that the distance of O from four vertices of rhombus i.e., A, B, C and D are equal. Even, If we take any fixed point on diagonal BD of rhombus ABCD and join with vertices A and C, we get which is equidistant from A and C. (by property of congruent triangle). Hence, the locus of a point in rhombus ABCD which is equidistant from A and C is a fixed point on diagonal BD.

81. (d) If two parallel lines are cut by two distinct transversals, then the quadrilateral formed by the four lines is always a Trapezium'.

Case I. If two distinct transversals (are not parallel), then always $\rightarrow$ (Trapezium)



Case II. If two distinct transversals are parallel, then always (Trapezium + Parallelogram)

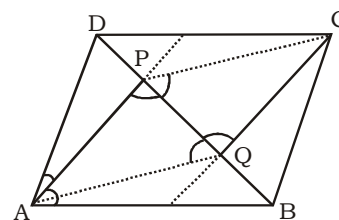


82. (b) Line segment AP and CQ bisects the $\angle A$ and $\angle C$, respectively.

Then, $AP \parallel CQ$

Now, In $\triangle APQ$ and $\triangle CQP$,

$$\therefore AP \parallel QC$$



$\therefore \angle APQ = \angle PQC$ (alternate angle)

$PQ = PQ$ (common)

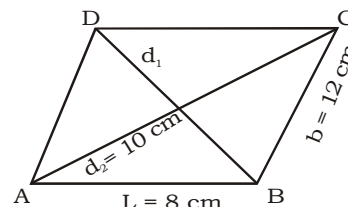
Also, $PC \parallel AQ$

$\therefore \angle CPQ = \angle PQA$ (alternate angle)

$\therefore \triangle APQ \sim \triangle CQP$ (by ASA)

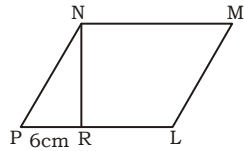
Hence, $\triangle APQ$ is similar to $\triangle PCQ$.

83. (d) In parallelogram, $d_1^2 + d_2^2 = 2(1^2 + b^2)$



$$\begin{aligned}\therefore d^2 + (10)^2 &= 2(64 + 144) \\ \Rightarrow d^2 &= 2 \times 208 - 100 \\ \Rightarrow d^2 &= 416 - 100 = 316 \Rightarrow d = \sqrt{316} \\ \Rightarrow d &= 17.76 \text{ cm} \\ \therefore d &> 12\end{aligned}$$

84. (b) By given condition,

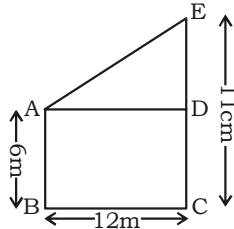


Area of parallelogram = 6 × Area of $\triangle NPR$

$$\therefore NR \times PL = 6 \times \frac{1}{2} \times NR \times PR$$

$$\begin{aligned}\Rightarrow PL &= 3PR \text{ (here, } PL = PR + RL) \\ \Rightarrow PR + RL &= 3PR \\ \Rightarrow RL &= 2PR = 2 \times 6 = 12 \text{ cm}\end{aligned}$$

85. (c) $AD = BC = 12 \text{ m}$
and $ED = 11 - 6 = 5 \text{ cm}$
Since, AE is distance between top point of AB and CE.



$$\begin{aligned}\text{In } \triangle ADE, \\ AE^2 &= AD^2 + ED^2 = 12^2 + 5^2 \\ &= 144 + 25 = 169\end{aligned}$$

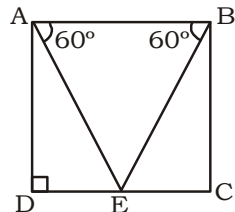
$\therefore AE = 13 \text{ m}$
Hence, the distance between their tops = 13 m

86. (a) $AB \parallel DC$ and $AD \parallel BC$
In $\triangle ABE$,

$$\begin{aligned}\angle EAB &= \angle ABE = 60^\circ \\ \Rightarrow \angle AEB &= 60^\circ \\ \Rightarrow \triangle ABE &\text{ is an equilateral triangle.}\end{aligned}$$

Now, perimeter of $\triangle ABE = 6$

$$\begin{aligned}\Rightarrow AB + BE + EA &= 6 \\ \Rightarrow AB &= 2 \text{ units} \\ AB &= BE = EA = 2\end{aligned}$$



and In $\triangle ADE$, $AE^2 = AD^2 + ED^2$

$$\Rightarrow 4 = AD^2 + 1 \text{ (since, E is mid-point of CD)}$$

$$\Rightarrow AD = \sqrt{3} \text{ units}$$

Hence, area of quadrilateral ABCD = $AB \times AD$

$$= 2 \times \sqrt{3} = 2\sqrt{3} \text{ sq units}$$

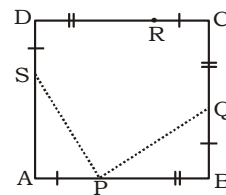
87. (d) We know that, ratio of the areas of two similar triangles is equal to the ratios of squares of their corresponding sides.

$$\therefore \frac{\text{Area}(\triangle BPQ)}{\text{Area}(\triangle DPC)} = \frac{PB^2}{PC^2}$$

$$\Rightarrow \frac{20}{\text{Area}(\triangle DPC)} = \frac{1}{4}$$

$$\Rightarrow \text{Area}(\triangle DPC) = 80 \text{ sq units}$$

88. (d) In $\triangle APS$ and $\triangle PBQ$,

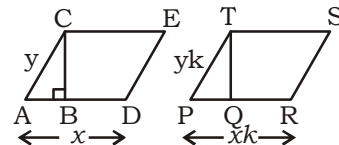


$PB = AS$
 $AP = BQ$ (given)
and $\angle A = \angle B = 90^\circ$
(since, ABCD is a square)
So, $\triangle APS$ and $\triangle PBQ$ are congruent.

$$\begin{aligned}\therefore SP &= PQ \\ \angle SPA &= \angle BQP \text{ and} \\ \angle ASP &= \angle BPQ \\ \therefore \angle SPQ &= 90^\circ \text{ (by RHS rule)}\end{aligned}$$

$$\triangle APS \cong \triangle PBQ$$

89. (c) Let the sides of a parallelogram are x, y and xk, yk .



Since, sides of two parallelogram are in $1 : k$

$$\therefore \triangle ABC \sim \triangle PQT$$

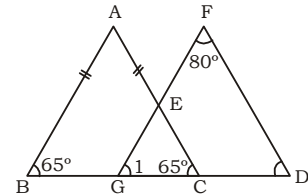
$$\Rightarrow \frac{AC}{PT} = \frac{BC}{QT} \Rightarrow \frac{BC}{QT} = \frac{y}{yk} = \frac{1}{k}$$

Let $BC = z$ and $QT = zk$
 $\therefore$ Ratio of areas of two similar parallelograms

$$= \frac{x \times z}{xk \times zk} = \frac{1}{k^2}$$

90. (d) $\angle B = \angle C = 65^\circ$

Here, $GF \parallel AB$, Which is intersects



Now $\angle B = \angle G$

$$\angle 1 = \angle B = 65^\circ \text{ (corresponding angles)}$$

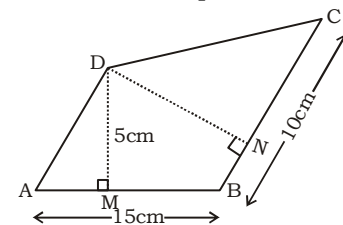
In $\triangle FGD$,

$$\angle 1 + \angle F + \angle D = 180^\circ$$

$$\Rightarrow 65^\circ + 80^\circ + \angle D = 180^\circ$$

$$\Rightarrow \angle D = 35^\circ$$

91. (b) Area of parallelogram = Base $\times$ Height
 $= 15 \times 5 = 75 \text{ sq cm}$

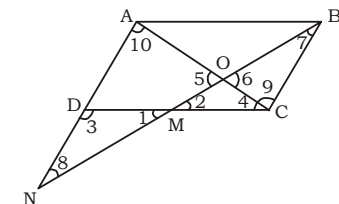


Area of parallelogram = Base $\times$ Height = $10 \times DN$

$$\Rightarrow 10 \times DN = 75$$

$$\therefore DN = \frac{75}{10} = 7.5 \text{ cm}$$

92. (b) In $\triangle DMN$ and $\triangle BMC$,
 $DM = MC$ (mid-point) (given)



$$\angle 1 = \angle 2$$

Since, $BC \parallel AD$ and intersects by CD.

$$\triangle DMN \cong \triangle BMC$$

$$DN = BC = AD$$

$$\text{So, } AN = 2BC \Rightarrow \frac{AN}{BC} = \frac{2}{1}$$

In $\triangle OAN$ and $\triangle OBC$,

$$\angle 5 = \angle 6$$

(vertically opposite angle)

$$\angle 7 = \angle 8$$

(alternate interior angle)

$$\angle 9 = \angle 10 \text{ (alternate interior angle)}$$

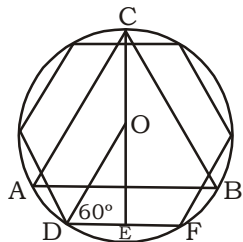
$$\therefore \triangle OAN \sim \triangle OBC$$

So, the sides will be in same ratio

$$\frac{AN}{BC} = \frac{ON}{OB}$$

$$\Rightarrow \frac{2}{1} = \frac{ON}{OB} \text{ [from Eq. (i)]}$$

93. (d) We know altitude of equilateral $\triangle ABC$ is $\frac{\sqrt{3}}{2}a$.



$$\therefore \text{Length of } OC = \frac{\sqrt{3}}{2}a \times \frac{2}{3} = \frac{a}{\sqrt{3}} = \text{radius}$$

$$\text{Also } DF = b \Rightarrow DE = \frac{b}{2}$$

$$\text{In } \triangle ODE, \cos 60^\circ = \frac{DE}{OD}$$

$$\frac{1}{2} = \frac{b/2}{a/\sqrt{3}}$$

$$\Rightarrow \frac{1}{2} = \frac{\sqrt{3}b}{2a} \Rightarrow a = \sqrt{3}b$$

$$\therefore a^2 = 3b^2$$

Alternatively

Circum radius of equilateral tri-

$$\text{angle} = \frac{a}{\sqrt{3}}$$

Circum radius of regular hexagon = side(b)

$$\frac{a}{\sqrt{3}} = b$$

$$a = \sqrt{3}b \Rightarrow a^2 = 3b^2$$

$$94. (c) (50)^2 = (30)^2 + (40)^2$$

$$\Rightarrow 2500 = 900 + 1600$$

$$\Rightarrow 2500 = 2500$$

It means given scores are the sides of a rectangle. So, other diagonal should be 50 runs.

95. (b) ABCD is a trapezium.

$\therefore AD \parallel BC$ and $EF \parallel BC$ (given)

Hence, $EF \parallel AD$

$$\therefore \angle x + \angle y = 180^\circ$$

(interior angles)

$$\therefore \angle y = 180^\circ - 120^\circ = 60^\circ$$

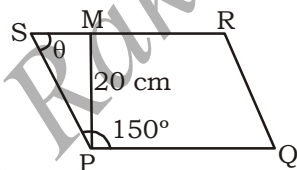
96. (a) Given that, $\angle SPQ = 150^\circ$ and $PM = 20$ cm in parallelogram PQRS,

$$\angle RSP + \angle SPQ = 180^\circ$$

(interior angles)

$$\Rightarrow \angle RSP = 180^\circ - 150^\circ = 30^\circ$$

$$\Rightarrow \theta = 30^\circ$$



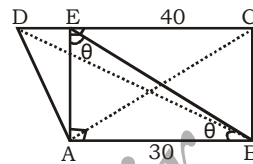
In $\triangle PSM$,

$$\sin \theta = \sin 30^\circ = \frac{PM}{SP}$$

$$\Rightarrow \frac{1}{2} = \frac{20}{SP} \Rightarrow SP = 40 \text{ cm}$$

$$\therefore RQ = SP = 40 \text{ cm}$$

97.



In trapezium diagonal intersect proportionally

$\therefore$ ABCD is a trapezium where $DC \parallel AB$

$$\text{Area of trapezium } ABCD = \frac{1}{2} (AB + CD) \times AE$$

$$1050 = \frac{1}{2} (30 + 40) \times AE$$

$$\Rightarrow AE = 30 \text{ cm.}$$

(a) $\angle EAB = 90^\circ$ ($AB \parallel DC$)

In right $\triangle EAB$,

$$BE = \sqrt{AE^2 + AB^2} = \sqrt{30^2 + 30^2} = 30\sqrt{2} \text{ cm}$$

(b). Area of $\triangle ADC = \frac{1}{2} \times CD \times AE$

$$= \frac{1}{2} \times 40 \times 30 = 600 \text{ cm}^2.$$

(c). In $\triangle AEB$, $AE = AB$

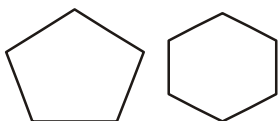
so $\angle AEB = \angle EBA$

$$2\theta = 90^\circ \Rightarrow \theta = 45^\circ$$

POLYGON

POLYGONS

A closed-figure bounded by three or more than three straight lines.

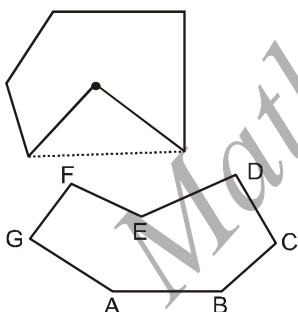


e.g.

No. of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

- **Convex Polygon:-** A polygon in which none of its interior angle is more than 180° and all line inside the figure, is known as a 'convex polygon'.

- **Concave Polygon:-** A polygon in which atleast one interior angle is more than 180° and at least one diagonal lies outside the figure, then it is said to be 'concave'.



- **Regular Polygon:-** A polygon in which all the sides are equal and also the interior angles are equal, is called a 'Regular polygon'.

If n = total no. of sides of a regular polygon, then:-

1. Sum of interior angles
= $(n-2) \times 180^\circ$

2. Each exterior angle = $\left(\frac{360^\circ}{n}\right)$

3. Sum of all exterior angle
= 360°

4. Each interior angle = $180^\circ - \text{exterior angle}$
 $\Rightarrow \text{interior angle} + \text{exterior angle} = 180^\circ$

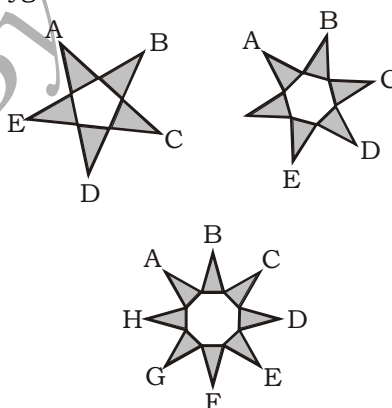
5. Number of diagonals
= $\frac{n(n-3)}{2}$

6. Perimeter = $n \times a$
where a = length of side

7. Area of regular polygon =
 $\frac{na^2}{4} \times \cot\left(\frac{180^\circ}{n}\right)$

Star

Sum of outer angles of a star Star forms by extending sides of a regular polygon.



Let outer triangles are 'n' then sum of outer angles = $n \times 180^\circ - \text{two times sum of exterior angles} = (n \times 180^\circ - 2 \times 360^\circ) = 180^\circ (n - 4)$

If $n = 5$ then $\angle A + \angle B + \dots + \angle E = 180^\circ (5 - 4) = 180^\circ$

If $n = 6$ then $\angle A + \angle B + \dots + \angle F = 180^\circ (6 - 4) = 360^\circ$

If $n = 8$ then $\angle A + \angle B + \dots + \angle H = 180^\circ (8 - 4) = 720^\circ$

Some important points about Polygon.

- A convex polygon in which there is maximum number of sides, it has the greater enclosed area when the perimeter of the polygon is constant.
- Note:** Which of the following figures will have maximum area if the perimeter of all figures is same
Circle > Octagon > Hexagon > Square > Rhombus
Circle is a polygon with infinite sides of minimum possible length.

Example

1. Each interior angle of a regular polygon is 140° . The number of sides is:

Sol. Each interior angle = 140°

$\therefore$ Exterior angle = $180^\circ - 140^\circ = 40^\circ$

$\therefore$ Number of sides =

$\frac{360^\circ}{\text{Exterior angle}} = \frac{360^\circ}{40^\circ} = 9$

2. Each interior angle of a regular hexagon is:

Sol. Exterior angle of a regular hexagon =

$\frac{360^\circ}{6} = 60^\circ$

Interior angle = $180^\circ - 60^\circ = 120^\circ$

3. If one of the interior angles of a regular polygon is equal to $\frac{5}{6}$ times of one of the interior angles of a regular pentagon, then the number of sides of the polygon is:

Sol. Exterior angle of pentagon =
 $\frac{360^\circ}{5} = 72^\circ$

$\therefore$ Interior angle of pentagon = $180^\circ - 72^\circ = 108^\circ$

$\therefore$ Interior angle of required polygon
= $\frac{5}{6} \times 108^\circ = 90^\circ$

$\therefore$ Each exterior angle of the required polygon
= $180^\circ - 90^\circ = 90^\circ$

∴ Number of sides

$$= \frac{360^\circ}{\text{Exterior angle}} = \frac{360^\circ}{90^\circ} = 4$$

4. The sum of the interior angles of a polygon is 1260° . The number of sides of the polygon is:

Sol. Sum of all the interior angles = $(n - 2) 180^\circ$

$$\Rightarrow (n - 2) 180^\circ = 1260$$

$$\Rightarrow n - 2 = 7$$

$$\Rightarrow n = 9 \quad (n \rightarrow \text{number of sides})$$

5. If each interior angle of a regular polygon is 3 times its exterior angle, the number of sides of the polygon is:

Sol. Interior angle = $3 \times$ exterior angle

$$\begin{array}{ccc} \text{Interior angle} & : & \text{exterior angle} \\ 3 & : & 1 \end{array}$$

$$\text{Interior angle} + \text{exterior angle} = 180^\circ$$

$$3x + x = 180^\circ$$

$$4x = 180^\circ$$

$$x = 45^\circ$$

$$\text{exterior angle} = 45^\circ$$

$$\text{Number of sides} = \frac{360^\circ}{\text{exterior angle}}$$

$$= \frac{360^\circ}{45}$$

$$\text{Number of sides} = 8$$

6. Difference between the interior and exterior angles of regular polygon is 60° . The number of sides in the polygon is:

$$(a) 5 \quad (b) 6$$

$$(c) 8 \quad (d) 9$$

Sol. Go through option

$$\text{Exterior angle} = \frac{360^\circ}{6} = 60^\circ$$

$$\text{Interior angle} = 180^\circ - 60^\circ = 120^\circ$$

$$\text{Difference between interior angle and exterior angle}$$

$$= 120^\circ - 60^\circ = 60^\circ$$

Alternate

$$x + y = 180^\circ \dots\dots(i)$$

$$x - y = 60^\circ \dots\dots(ii)$$

Solve equation (i) and (ii) we get

$$x = 120^\circ \text{ and } y = 60^\circ$$

$$\therefore \text{Number of sides} = \frac{360^\circ}{60^\circ} = 6$$

7. A polygon has 54 diagonals. The number of sides in the polygon is:

$$(a) 7 \quad (b) 9 \quad (c) 12$$

$$(d) \text{None of these}$$

$$\text{Sol. } \frac{n(n-3)}{2} = 54$$

$$\Rightarrow n^2 - 3n = 108$$

$$\Rightarrow n^2 - 3n - 108 = 0$$

$$\Rightarrow n^2 - 12n + 9n - 108 = 0$$

$$\Rightarrow n(n - 12) + 9(n - 12) = 0$$

$$\Rightarrow (n + 9)(n - 12) = 0$$

$$\Rightarrow n = 12, n \neq -9$$

No. of side always +ve

So, number of sides = 12

Alternate

$$\frac{n(n-3)}{2} = 54$$

$$n(n - 3) = 108$$

$$\text{We take option (a) } = 12$$

$$12 \times (12 - 3) = 12 \times 9 = 108$$

satisfy the equation

So, number of sides = 12

8. The ratio between the number of sides of two regular polygon 1 : 2 and the ratio between their interior angle is 3 : 4. The number of sides of these polygons are respectively:

$$(a) 3, 6 \quad (b) 4, 8$$

$$(c) 6, 9 \quad (d) 5, 10$$

Sol. Go through options. Let us consider the correct option (d).

$$\text{Number of sides } 5 : 10$$

$$\text{Exterior angle} = 72^\circ : 36^\circ$$

$$\text{Interior angle} = 108^\circ : 144^\circ$$

$$3 : 4$$

Hence, option (d) is correct.

Alternatively:-

Let side = n & $2n$

$$\frac{(n-2)180}{2n} = \frac{3}{4}$$

$$\frac{2n-4}{2n-2} = \frac{3}{4}$$

$$8n - 16 = 6n - 6$$

$$n = 5$$

$$2n = 10$$

9. The sum of the all interior angles of a regular polygon is four times the sum of its exterior angles. The polygon is:

$$(a) \text{hexagon} \quad (b) \text{triangle}$$

$$(c) \text{decagon} \quad (d) \text{nonagon}$$

Sol. Sum of a exterior angles = 360°

$$\therefore \text{Sum of interior angles} = 4 \times 360^\circ = 1440^\circ$$

$$\therefore 1440^\circ = (n - 2) 180^\circ$$

$$\Rightarrow n = 10 \text{ (no. of sides)}$$

$$\therefore \text{The polygon is decagon}$$

10. The ratio of the measure of an interior angle of a regular nonagon to the measure of its exterior angle is:

$$(a) 3 : 5 \quad (b) 5 : 2$$

$$(c) 7 : 2 \quad (d) 4 : 5$$

$$\text{Sol. Exterior angle} = \frac{360^\circ}{9} = 40^\circ$$

$$\text{Interior angle } 180^\circ - 40^\circ = 140^\circ$$

$$\text{Interior angle} : \text{exterior angle} = 140^\circ : 40^\circ = 7 : 2$$

EXERCISE

1. Raju has drawn an angle of measure $45^{\circ}27'$ when he was asked to draw an angle of 45° . The percentage error in his drawing is
(a) 0.5% (b) 1.0%
(c) 1.5% (d) 2.0%
2. In a regular polygon, the exterior and interior angles are in the ratio 1 : 4. The number of sides of the polygon is
(a) 5 (b) 10 (c) 3 (d) 8
3. The difference between the interior and exterior angles at a vertex of a regular polygon is 150° . The number of sides of the polygon is
(a) 10 (b) 15 (c) 24 (d) 30
4. Each interior angle of a regular polygon is 144° . The number of sides of the polygon is
(a) 8 (b) 9 (c) 10 (d) 11
5. If the sum of the interior angles of a regular polygon be 1080° , the number of sides of the polygon is
(a) 6 (b) 8 (c) 10 (d) 12
6. The number of sides in two regular polygons are in the ratio of 5 : 4. The difference between their Interior angles of the polygon is 6° . Then the number of sides are
(a) 15, 12 (b) 5, 4
(c) 10, 8 (d) 20, 16
7. Each internal angle of regular polygon is two times its external angle. Then the number of sides of the polygon is :
(a) 8 (b) 6 (c) 5 (d) 7
8. Ratio of the number of sides of two regular polygons is 5 : 6 and the ratio of their each interior angle is 24 : 25. Then the number of sides of these two polygons are
(a) 10, 12 (b) 20, 24
(c) 15, 18 (d) 35, 42
9. Measure of each interior angle of a regular polygon can never be:
(a) 150° (b) 105°
(c) 108° (d) 144°
10. Each interior angle of a regular polygon is three times its exterior angle, then the number of sides of the regular polygon is:
(a) 9 (b) 8 (c) 10 (d) 7
11. The sum of all interior angles of a regular polygon is twice the sum of all its exterior angles. The number of sides of the polygon is
(a) 10 (b) 8 (c) 12 (d) 6
12. The ratio between the number of sides of two regular polygons is 1 : 2 and the ratio between their interior angles is 2 : 3. The number of sides of these polygons is respectively
(a) 6, 12 (b) 5, 10
(c) 4, 8 (d) 7, 14
13. Each internal angle of regular polygon is two times its external angle. Then the number of sides of the polygon is:
(a) 8 (b) 6 (c) 5 (d) 7
14. The sum of interior angles of a regular polygon is 1440° . The number of sides of the polygon is
(a) 10 (b) 12 (c) 6 (d) 8
15. An interior angle of a regular polygon is 5 times its exterior angle. Then the number of sides of the polygon is
(a) 14 (b) 16 (c) 12 (d) 18
16. In a regular polygon, if one of its internal angle is greater than the external angle by 132° , then the number of sides of the polygon is
(a) 14 (b) 12 (c) 15 (d) 16
17. If the ratio of an external angle and an internal angle of a regular polygon is 1 : 17, then the number of sides of the regular polygon is
(a) 20 (b) 18 (c) 36 (d) 12
18. The ratio of each interior angle to each exterior angle of a regular polygon is 3:1. The number of sides of the polygon is:
(a) 6 (b) 7 (c) 8 (d) 9
19. The interior angle of regular polygon exceeds its exterior angle by 108° . The number of sides of the polygon is
(a) 10 (b) 14 (c) 12 (d) 16
20. If the sum of all interior angles of a regular polygon is 14 right angles, then its number of sides is
(a) 6 (b) 8 (c) 9 (d) 7
21. How many diagonals are there in a octagon ?
(a) 10 (b) 14 (c) 18 (d) 20
22. A polygons has 44 diagonals. The number of sides of the polygon is :
(a) 11 (b) 10 (c) 13 (d) 12
23. The angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4, the largest angle is :
(a) 120° (b) 134°
(c) 144° (d) 150°
24. Each interior angle of a regular polygon is 120° . The number of sides is :
(a) 7 (b) 6 (c) 5 (d) 8
25. Each interior angle of a regular octagon is :
(a) 120° (b) 90°
(c) 135° (d) None of these
26. The sum of the interior angles of polygon is 1440° . the number of sides of the polygon is :
(a) 9 (b) 10 (c) 8 (d) 12
27. One angle of a pentagon is 140° . If the remaining angles are in the ratio 1 : 2 : 3 : 4, the size of the greatest angle is :
(a) 150° (b) 180°
(c) 160° (d) 170°
28. Each interior angle of a regular polygon is 144° . The number of sides of the polygon is :
(a) 8 (b) 12 (c) 10 (d) 11
29. A regular polygon is inscribed in a circle. If a side subtends an angle of 36° at the centre, then the number of sides of the polygon is :
(a) 5 (b) 10 (c) 12 (d) 9
30. The difference between an exterior angle of (n - 1) sided regular polygon and an exterior angle of (n + 2) sided regular polygon is 6° , then the value of n is :
(a) 15 (b) 14 (c) 12 (d) 13
31. If a regular polygon has each of its angles equal to $\frac{3}{5}$ times of two right angles, then the number of sides is
(a) 3 (b) 5 (c) 6 (d) 8

ANSWER KEY

1. (b)	4. (c)	7. (b)	10. (b)	13. (b)	16. (c)	19. (a)	22. (a)	25. (c)	28. (c)
2. (b)	5. (b)	8. (a)	11. (d)	14. (a)	17. (c)	20. (c)	23. (c)	26. (b)	29. (b)
3. (c)	6. (a)	9. (b)	12. (c)	15. (c)	18. (c)	21. (d)	24. (b)	27. (c)	30. (d)
									31. (b)

SOLUTION

1. (b) According to question
Angle of measure = $45^\circ 27'$

$$= 45^\circ + \frac{27}{60}$$

Asked to draw an angle of $= 45^\circ$

$$\text{Error} = 45^\circ + \frac{27}{60} - 45^\circ = \frac{27}{60}$$

$$\text{Error \%} = \frac{\left(\frac{27}{60}\right)}{45} \times 100$$

$$= \frac{27}{60 \times 45} \times 100 = 1.0\%$$

2. (b) According to question

$$\frac{\text{Exterior angle}}{\text{Interior angle}} = \frac{1}{4} = \frac{x}{4x}$$

As we know that

Interior angle + Exterior angle = 180°

$$\therefore x + 4x = 180^\circ$$

$$5x = 180^\circ$$

$$x = 36^\circ$$

$\therefore$ No. of sides

$$= \frac{360^\circ}{\text{Exterior angle (बाह्य कोण)}}$$

$$\text{No. of sides} = \frac{360^\circ}{36^\circ} = 10$$

No. of sides = 10

3. (c) According to question

Given :

$$\text{Interior angle} - \text{Exterior angle} = 150^\circ \quad \dots\dots\dots(i)$$

We know,

$$\text{Interior angle} + \text{Exterior angle} = 180^\circ \quad \dots\dots\dots(ii)$$

Solve equation (i) and (ii)

$$\text{Interior angle} = 165^\circ$$

$$\text{Exterior angle} = 15^\circ$$

$$\therefore \text{No. of sides} = \frac{360^\circ}{\text{Exterior angle}}$$

$$= \frac{360^\circ}{15^\circ} = 24$$

4. (c) According to question

Given:

$$\text{Interior angle} = 144^\circ$$

$$\text{Exterior angle} = 180^\circ - 144^\circ = 36^\circ$$

$$\begin{aligned} \therefore \text{no. of sides} &= \frac{360^\circ}{\text{Exterior angle}} \\ &= \frac{360^\circ}{36^\circ} = 10 \end{aligned}$$

5. (b) According to question

Sum of interior angle

$$= (n - 2) \times 180^\circ$$

Given:

$$\text{Sum of interior angle} = 1080^\circ$$

$$(n - 2) \times 180^\circ = 1080^\circ$$

$$(n - 2) = \frac{1080^\circ}{180}$$

$$(n - 2) = 6$$

$$n = 6 + 2 = 8$$

$$\text{No. of sides } n = 8$$

6. (a) Let the no. of sides is $5x$ and $4x$
According to questions

$$\left(180^\circ - \frac{360^\circ}{5x}\right) - \left(180^\circ - \frac{360^\circ}{4x}\right) = 6^\circ$$

$$180^\circ - \frac{360^\circ}{5x} - 180^\circ + \frac{360^\circ}{4x} = 6^\circ$$

$$\frac{360^\circ}{4x} - \frac{360^\circ}{5x} = 6^\circ$$

$$360^\circ \left(\frac{1}{4x} - \frac{1}{5x} \right) = 6^\circ$$

$$\frac{1}{20x} = \frac{1}{60}, \quad x = 3$$

No. of sides is $5x$ and $4x = 15, 12$

7. (b) According to questions

Given:

$$\text{Internal Angle} = 2 \text{ (External Angle)}$$

As we know that

$$\text{Internal Angle} + \text{External Angle} = 180^\circ$$

$$\therefore 2 \text{ External Angle} + \text{External Angle} = 180^\circ$$

$$3 \text{ External Angle} = 180^\circ$$

$$\text{External Angle} = \frac{180^\circ}{3} = 60^\circ$$

$$\text{No. of sides} = \frac{360^\circ}{\text{External angle}}$$

$$= \frac{360^\circ}{60^\circ} = 6 \text{ (no. of sides)}$$

8. (a) Let the number of sides be $5x$ and $6x$

As we know that

Each interior angle

$$= \frac{(2n - 4) \times 90^\circ}{n}$$

Given:

$$\frac{n_1}{n_2} = \frac{5x}{6x}$$

$$\frac{n_1}{n_2} = \frac{5}{6}$$

$$\text{Interior angle}_1 = \frac{24}{25}$$

$$\text{Interior angle}_2 = \frac{24}{25}$$

$\therefore$ Using Interior angle formula

$$\frac{(n_1 - 2)180^\circ}{n_1} = \frac{24}{25}$$

$$\frac{5x - 2}{\frac{5x}{6x - 2}} = \frac{24}{25}$$

$$x = 2$$

Then, No. of sides = $5 \times 2 = 10$,
 $6 \times 2 = 12$
 $= 10, 12$

9. (b) According to question

$n \rightarrow$ No. of sides

Interior angle

$$= \frac{(2n - 4) \times 90}{n} = 180 - \frac{360}{n}$$

$$(a) \quad 150^\circ = 180^\circ - \frac{360^\circ}{n}$$

$$\frac{360^\circ}{n} = 30^\circ$$

$$n = 12$$

(b) $105^\circ = 180^\circ - \frac{360^\circ}{n}$

$$\frac{360^\circ}{n} = 75^\circ$$

$$n = \frac{24}{5}$$

(c) $108^\circ = 180^\circ - \frac{360^\circ}{n}$

$$\frac{360^\circ}{n} = 72^\circ$$

$$n = 5$$

(d) $144^\circ = 180^\circ - \frac{360^\circ}{n}$

$$\frac{360^\circ}{n} = 36^\circ$$

$$n = 10$$

$\therefore$ Only 105° angle which can never be interior angle of regular polygon

10. (b) According to question.

Given:

Interior Angle = $3 \times$ exterior angle

As we know that

Interior Angle + Exterior Angle = 180°

3 Exterior Angle + Exterior Angle = 180°

4 exterior = 180°

Exterior angle = $\frac{180^\circ}{4} = 45^\circ$

$\therefore$ No. of Sides = $\frac{360^\circ}{\text{Exterior angle}}$

No. of Sides = $\frac{360^\circ}{45^\circ} = 8$

11. (d) Sum of all interior angle of regular polygon = $(n-2) \times 180$

Sum of all exterior angle of polygon = 360

$(n-2) \times 180 = 2 \times 360$

$n = 6$

12. (c) Let the sides be x and $2x$

According to question

$$\frac{180^\circ - \frac{360^\circ}{n_1}}{\frac{360^\circ}{n_2}} = \frac{2}{3}$$

$$\frac{180^\circ - \frac{360^\circ}{x}}{\frac{360^\circ}{2x}} = \frac{2}{3}$$

$$540^\circ - \frac{1080^\circ}{x} = 360^\circ - \frac{360^\circ}{x}$$

$$180^\circ = \frac{720^\circ}{x}$$

$$x = 4$$

$\therefore$ Sides be x and $2x = 4, 8$

Alternate

In this question go through option.

Option: (c) 4, 8

Given: $\frac{n_1}{n_2} = \frac{1}{2}$

(n = no. of sides)

$$\frac{I_1}{I_2} = \frac{2}{3} \text{ (I = Interior Angle)}$$

$\therefore$ Through option $n_1 = 4$
 $n_2 = 8$

$$E_1 = \frac{360^\circ}{n_1} = \frac{360^\circ}{4} = 90^\circ$$

$$E_2 = \frac{360^\circ}{n_2} = \frac{360^\circ}{8} = 45^\circ$$

As we know that

$$I + E = 180^\circ$$

$$I_1 + E_1 = I_1 + 90^\circ = 180^\circ$$

$$I_1 = 90^\circ$$

$$I_2 + E_2 = I_2 + 45^\circ = 180^\circ$$

$$I_2 = 180^\circ - 45^\circ = 135^\circ$$

$$\frac{I_1}{I_2} = \frac{90^\circ}{135^\circ} = \frac{2}{3} \text{ (satisfied)}$$

13. (b) According to question

Given:

Internal angle = $2 \times$ External Angle

As we know that

$$\text{Internal angle} + \text{Exterior Angle} = 180^\circ$$

$$2 \times \text{External angle} + \text{Exterior Angle} = 180^\circ$$

$$3 \text{ Exterior Angle} = 180^\circ$$

$$\text{Exterior Angle} = 60^\circ$$

$$\text{No. of sides} = \frac{360^\circ}{\text{Exterior Angle}}$$

$$\text{No. of sides} = \frac{360^\circ}{60^\circ}$$

$$\text{No. of sides} = 6$$

14. (a) If the number of sides of regular Polygon be = n Sum of the interior angle = $(n-2) \times 180^\circ$

$$\therefore (n-2) \times 180^\circ = 1440^\circ$$

$$n-2 = \frac{1440^\circ}{180^\circ}$$

$$n-2 = 8 \quad n = 10$$

15. (c) According to question
sum of interior angles = $5 \times$ sum of exterior angles
As we know that
Exterior angle + Interior angle = 180°

$$\text{Exterior angle} + 5 \text{ Exterior angle} = 180^\circ$$

$$6 \text{ Exterior angle} = 180^\circ$$

$$\text{Exterior angle} = 30^\circ$$

$$\therefore \text{no. of sides} = \frac{360^\circ}{\text{External angle}} = \frac{360^\circ}{30^\circ} = 12$$

16. (c) According to question
Interior angle - Exterior angle = 132°

As we know that

$$\text{Interior angle} + \text{Exterior angle} = 180^\circ \dots (i)$$

$$\text{Interior angle} - \text{Exterior angle} = 132^\circ \dots (ii)$$

$$2 \text{ Interior angle} = 312^\circ$$

$$\text{Interior angle} = 156^\circ$$

Put this value in equation (i) and (ii)

$$\therefore \text{Exterior angle} = 180^\circ - 156^\circ = 24^\circ$$

$$\therefore \text{no. of sides}$$

$$= \frac{360^\circ}{\text{External angle}}$$

$$\text{no. of sides} = \frac{360^\circ}{24^\circ} = 15$$

17. (c) According to question

$$\frac{\text{External angle}}{\text{Internal angle}} = \frac{1}{17}$$

As we know that

$$\text{External angle} + \text{Internal angle} = 180^\circ$$

$$\therefore 18 \text{ units} = 180^\circ$$

$$1 \text{ unit} = \frac{180^\circ}{18} = 10^\circ$$

$$\therefore \text{External angle} = 10^\circ \times 1 = 10^\circ$$

$$\therefore \text{no. of sides}$$

$$= \frac{360^\circ}{\text{External angle}}$$

$$\text{no. of sides} = \frac{360^\circ}{10^\circ} = 36$$

18. (c) Interior angle + exterior angle
 $= 180^\circ$
 $3x + x = 180^\circ$
 $4x = 180^\circ$
 $x = 45^\circ$
 each exterior angle $= 45^\circ$

$$\text{No. of sides} = \frac{360^\circ}{\text{exterior angle}}$$

$$= \frac{360^\circ}{45^\circ} = 8$$

19. (a) Let internal angle $= x$
 External angle $= y$
 $x - y = 108$ (i)
 $x + y = 180$ (ii)
 from equation (i) and (ii)
 $y = 36^\circ$

$$\text{No. of sides} = \frac{360^\circ}{\text{exterior angle}}$$

$$= \frac{360^\circ}{36^\circ}$$

$$n = 10$$

Thus, side of polygon is 10

20. (c) Sum of Interior angles
 $= (n - 2) \times 180$
 Hence, $(n - 2) \times 180$
 $= 14 \times \text{Right angle triangle } (90^\circ)$

$$n - 2 = \frac{14 \times 90}{180}$$

$$n = 7 + 2 = 9$$

21. (d) no. of diagonals of a poly-
 gon of n sides

$$= \frac{n(n-3)}{2} = \frac{8(8-3)}{2} = 20$$

22. (a) $= \frac{n(n-3)}{2} = 44 \Rightarrow n(n-3)$
 $= 88 = 11 \times 8$
 $\therefore n = 11$

23. (c) angles be $x, 2x, 3x, 4x$
 $\therefore x + 2x + 3x + 4x =$
 $360^\circ \Rightarrow 10x = 360^\circ \Rightarrow x = 36^\circ$
 $\therefore \text{largest angle} = 4x = 144^\circ$

24. (b) Each interior angle $= 120^\circ$
 $\therefore \text{Exterior angle}$
 $= 180^\circ - 120^\circ = 60^\circ$
 $\therefore \text{no. of sides}$

$$= \frac{360^\circ}{\text{exterior angle}} = \frac{360}{60} = 6$$

25. (c) Exterior angle of a regular
 octagon $= \frac{360}{8} = 45^\circ$

$$\therefore \text{interior angle} = 180^\circ - 45^\circ$$

$$= 135^\circ$$

26. (b) Sum of all interior angles
 $= (n - 2) \times 180^\circ$
 $\Rightarrow 1440 = (n - 2) \times 180$
 $\Rightarrow n = 10 (n \rightarrow \text{no. of sides})$

27. (c) Sum of interior angles of
 pentagon $= (n - 2) \times 180^\circ$
 $= (5 - 2) \times 180^\circ = 540^\circ$
 $\Rightarrow 140^\circ + x + 2x + 3x + 4x$
 $= 540$

$$\Rightarrow 10x = 400 \Rightarrow x = 40$$

$$\therefore \text{largest angle} = 4x = 4 \times 40 = 160^\circ$$

28. (c) If the number of sides of
 the polygon be n , then
 Interior angle + exterior angle
 $= 180^\circ$
 Exterior angle $= 180^\circ - 144^\circ$
 $= 36^\circ$

$$\text{Number of sides} = \frac{360^\circ}{36^\circ} = 10$$

29. (b) Let no. of sides $= n$
 each equal side subtends equal
 angle at the centre.

$$\therefore n \times 36 = 360^\circ \Rightarrow n = \frac{360}{36} = 10$$

30. (d) $\frac{360^\circ}{n-1} - \frac{360^\circ}{n+2} = 6$

$$\Rightarrow 60 \left(\frac{1}{n-1} - \frac{1}{n+2} \right) = 1$$

$$\Rightarrow 60(n+2 - n+1) = (n-1)(n+2)$$

$$\Rightarrow 60 \times 3 = n^2 + n - 2$$

$$\Rightarrow n^2 + n - 182 = 0 \Rightarrow (n+14)(n-13) = 0$$

$$\Rightarrow n = -14 \text{ or } n = 13$$

$$\Rightarrow n = 13 (\because n \text{ can not be negative})$$

31. (b) Each interior angle of a
 regular polygon

$$= 180 \times \frac{3}{5} = 108^\circ$$

$$\therefore \text{Each exterior angle} = 180^\circ - 108^\circ = 72^\circ$$

$$\therefore \text{No. of sides} = \frac{360}{72} = 5$$

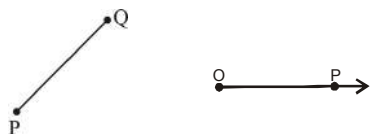


LINES & ANGLES

A line is made up of an infinite number of points and it has only length. i.e. A line has no end points on either side.

A **line segment** has two end points.

A **ray** has one end point.

**Line segment line ray**

□ A line segment PQ is generally denoted by $\overline{PQ}$

□ A line AB is denoted by $\overleftrightarrow{AB}$

□ The ray OP is denoted by $\overrightarrow{OP}$

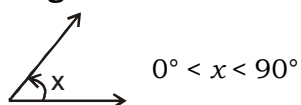
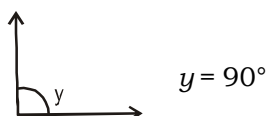
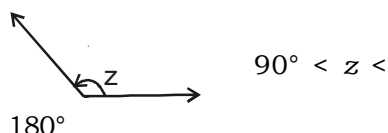
□ **Collinear points** : Three or more than three points are said to be collinear if there is a line which contains them all.



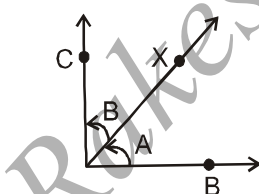
Here A, B and C are collinear points

□ **Concurrent lines**- Three or more than three lines are said to be concurrent if there is a point which lies on all of them.

AB, CD and EF are concurrent lines.

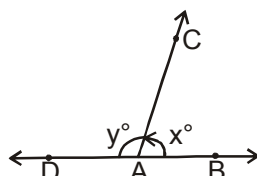
Related Angles**Acute angle -****Right Angle -****Obtuse Angle -****Straight Angle -****Reflex Angle**

Complementary Angles:- When the sum of the measures of the two angles is 90° , the angles are called "Complementary Angles".



i.e. $\angle A + \angle B = 90^\circ$ or
 $\angle A = 90^\circ - \angle B =$
 complement of $\angle A$

Supplementary Angles - Sum of two angles which are supplementary is 180°



Sum of two angles which are supplementary is

180° . i.e. $\angle x + \angle y = 180^\circ$

EXAMPLES

1. If one of angle is four times the complement of the other angle. Find the angles.

Sol. Let the one angle = θ
 Other angle = $90^\circ - \theta$
 A.T.O

$$\theta = 4(90^\circ - \theta)$$

$$\theta = 360^\circ - 4\theta$$

$$5\theta = 360^\circ$$

$$\theta = 72^\circ$$

One angle = 72°

Other angle = $90^\circ - 72^\circ = 18^\circ$

2. One angle of the complementary is 45° . Find the other angles.

Sol. Sum of the complementary angles = 90°

$$\text{Angle}_1 + \text{Angle}_2 = 90^\circ$$

$$45^\circ + \text{Angle}_2 = 90^\circ$$

$$\text{Angle}_2 = 90^\circ - 45^\circ = 45^\circ$$

3. If two supplementary angles are $(7x + 58)^\circ$ and $(16x + 53)^\circ$ then $5x$ is:

Sol. Sum of supplementary angles = 180°

$$7x + 58^\circ + 16x + 53^\circ = 180^\circ$$

$$23x + 111 = 180^\circ$$

$$23x = 69$$

$$x = 3$$

$$5x = 5 \times 3 = 15^\circ$$

4. Ratio of two complementary angles are 11: 19 then both angles are:

Sol. Sum of two complementary angle = 90°

$$11x + 19x = 90^\circ$$

$$30x = 90^\circ$$

$$x = 3$$

$$\text{Ist angle} = 11x = 11 \times 3 = 33^\circ$$

$$\text{IInd angle} = 19x = 19 \times 3 = 57^\circ$$

5. Angle Q_1 & Q_2 are complementary angles Q_1 is

$\frac{9}{11}$ times of its supplementary angle, then Q_2 is

Sol. $Q_1 = (180^\circ - Q_1) \frac{9}{11}$

$$11 \times Q_1 = 180 \times 9 - 9 Q_1$$

$$20Q_1 = 1620$$

$$Q_1 = 81^\circ$$

$$Q_2 = 90^\circ - 81^\circ = 9^\circ$$

6. What is the complementary angle of $37^\circ 49' 13''$

Sol. Complementary angle = $90^\circ - 37^\circ 49' 13''$
 $= 89^\circ 59' 60'' - 37^\circ 49' 13''$
 $= 52^\circ 10' 47''$

7. Angles a and b are linear pair. b is 24° more than one fifth of its supplementary angle. Find the complementary of half of a :

Sol. $b - \left(\frac{180^\circ - b}{5}\right) = 24^\circ$

$$5b - 180^\circ + b = 120^\circ$$

$$6b = 300^\circ$$

$$b = 50^\circ$$

$$a = 180^\circ - 50^\circ = 130^\circ$$

$$\text{Complementary of } \left(\frac{a}{2}\right) = 90^\circ$$

$$- 65^\circ = 25^\circ$$

8. Rakesh has drawn an angle of measure $45^\circ 36'$, when he was asked to draw an angle of 45° , the percentage error in his drawing:

Sol. Percentage error

$$= \frac{45^\circ 36' - 45^\circ}{45 \times 60} \times 100$$

$$= \frac{36}{45 \times 60} \times 100 = \frac{4}{3} = 1.33\%$$

9. Ratio of supplementary angles are $14 : 22$. Find the smaller angles.

Sol. Sum of supplementary angles = 180°

$$14x + 22x = 180^\circ$$

$$36x = 180^\circ$$

$$x = 5$$

$$\text{Smaller angle} = 14x = 14 \times 5 = 70^\circ$$

10. Angle Q_1 & Q_2 are supplementary angles Q_2 is $\frac{3}{7}$ times of its complementary angle, then Q_1

Sol. $Q_2 = \frac{3}{7}(90^\circ - Q_2)$

$$7Q_2 = 270^\circ - 3Q_2$$

$$10Q_2 = 270^\circ$$

$$Q_2 = 27^\circ$$

$$Q_1 + Q_2 = 180^\circ$$

$$Q_1 = 180^\circ - 27^\circ = 153^\circ$$

11. Angle Q_1 & Q_2 are supplementary angles Q_2 is $46\frac{2}{3}$ greater than the complementary angle, then Q_1 is:

Sol. $Q_2 - (90^\circ - Q_2) = 46\frac{2}{3}$

$$Q_2 - 90^\circ + Q_2 = \frac{140^\circ}{3}$$

$$= Q_2 = \frac{140^\circ + 270^\circ}{2 \times 3}$$

$$Q_2 = \frac{410}{2 \times 3}$$

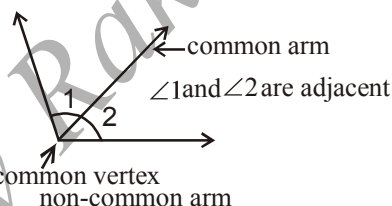
$$Q_2 = \frac{205}{3} = 68\frac{1}{3}$$

$$Q_1 = 180^\circ -$$

$$\frac{205}{3} = \frac{335}{3} = 111\frac{2}{3}$$

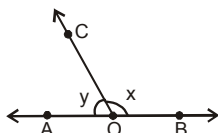
Adjacent Angles

Two angles are said to be adjacent if



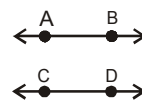
1. They have a common vertex;
2. They have a common arm; and
3. The non-common arms are on either side of the common arm.

□ **Linear pair-** A linear pair is a pair of adjacent angles whose non-common sides are opposite rays.



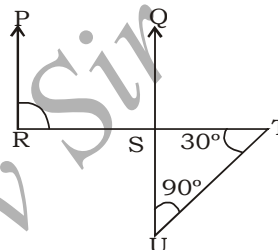
i.e. linear pair Angles are supplementary.

□ **Parallel Lines-** If two lines have no point in common they are said to be Parallel lines.



EXAMPLES

1. In this figure, $PR \parallel QU$, $\angle STU = 30^\circ$ and $\angle SUT = 90^\circ$ then the value of $\angle PRS = ?$



Sol. In $\triangle SUT$

$$\angle S + \angle U + \angle T = 180^\circ$$

[sum of angle of $\triangle$ is 180°]

$$\angle S + 90^\circ + 30^\circ = 180^\circ$$

$$\angle S = 60^\circ$$

$$\angle QST + \angle TSU = 180^\circ$$

[Linear angle]

$$\angle QST + 60 = 180^\circ$$

$$\angle QST = 120^\circ$$

$$\angle RSQ + \angle QST = 180^\circ$$

$$\angle RSQ + 120^\circ = 180^\circ$$

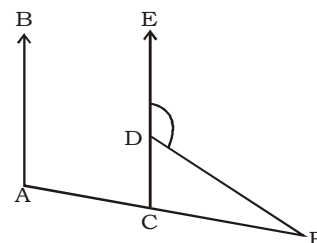
$$\angle RSQ = 60^\circ$$

$$\angle PRS + \angle RSQ = 180^\circ$$

$$\angle PRS + 60^\circ = 180^\circ$$

$$\angle PRS = 120^\circ$$

2. In the figure, $AB \parallel CE$, $\angle BAF = 115^\circ$, $\angle AFD = 30^\circ$ then the value of $\angle EDF = ?$



Sol. $\angle BAC + \angle ECA = 180^\circ$

[Linear angle]

$$115^\circ + \angle ECA = 180^\circ$$

$$\angle ECA = 65^\circ$$

$$\angle ACE + \angle ECF = 180^\circ$$

$$65^\circ + \angle ECF = 180^\circ$$

$$\angle ECF = 115^\circ$$

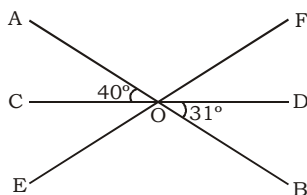
In $\triangle DCF$

$$\angle D + \angle C + \angle F = 180^\circ$$

$$\angle D + 115^\circ + 30^\circ = 180^\circ$$

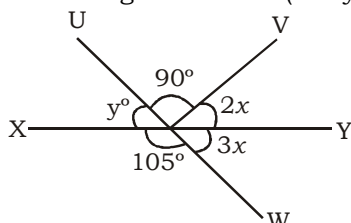
$$\begin{aligned}\angle D &= 35^\circ \\ \angle EDF + \angle CDF &= 180^\circ \\ \angle EDF + 35^\circ &= 180^\circ \\ \angle EDF &= 145^\circ\end{aligned}$$

3. In the following figure. Find the value of $\angle BOC$?



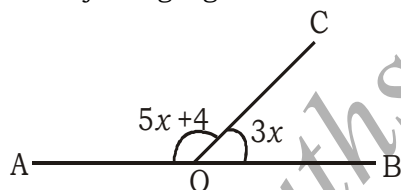
Sol. $\angle BOC + \angle BOD = 180^\circ$
[CD is a straight line]
 $\angle BOC + 31^\circ = 180^\circ$
 $\angle BOC = 149^\circ$

4. In the following figure U W is a straight line find $(x + y)$



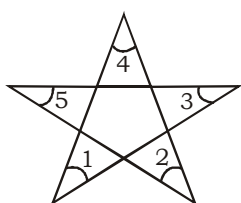
Sol. $\angle UOV + \angle VOY + \angle YOW = 180^\circ$
[UW is straight line]
 $90^\circ + 2x + 3x = 180^\circ$
 $5x = 90^\circ$
 $x = 18^\circ$
 $\angle XOY + \angle YOW = 180^\circ$
 $y^\circ + 105^\circ = 180^\circ$
 $y = 75^\circ \Rightarrow x + y = 18^\circ + 75^\circ = 93^\circ$

5. What is the value of x in the adjoining figure

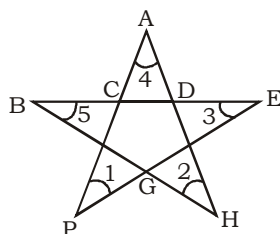


Sol. $\angle AOC + \angle BOC = 180^\circ$ [AB is a straight line]
 $5x + 4 + 3x = 180$
 $8x = 180 - 4$
 $8x = 176$
 $x = 22^\circ$

6. In the given figure. Find $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = ?$



Sol.

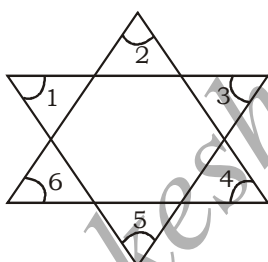


$$\begin{aligned}\angle ADC &= \angle 2 + \angle 5 \\ \angle ACD &= \angle 1 + \angle 3 \\ \text{[Sum of two Interior angle is equal to opposite of exterior angle]}\end{aligned}$$

In $\triangle ACD$

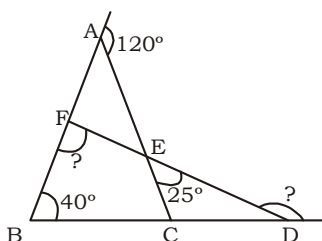
$$\begin{aligned}\angle A + \angle C + \angle D &= 180^\circ \\ \angle 4 + \angle 1 + \angle 3 + \angle 2 + \angle 5 &= 180^\circ\end{aligned}$$

7. In the given figure. Find $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = ?$



Sol. Sum of two triangle = $180 + 180 = 360^\circ$
 $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360^\circ$

8.



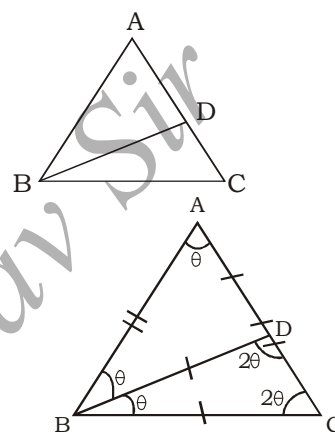
Sol. $\angle B + \angle BCA = \angle A$
[Sum of two interior angle is equal to opposite of exterior angle]
 $40^\circ + \angle BCA = 120^\circ$
 $\angle BCA = 80^\circ$
 $\angle BCE + \angle ECD = 180^\circ$
[Linear angle]
 $80^\circ + \angle ECD = 180^\circ$
 $\angle ECD = 100^\circ$
 $\angle D = \angle E + \angle C$ [use above property exterior angle]
 $\angle D = 25^\circ + 100^\circ = 125^\circ$

$$\angle EDC = 180^\circ - 125^\circ = 55^\circ$$

In $\triangle FBD$

$$\begin{aligned}\angle F + \angle B + \angle D &= 180^\circ \\ \angle F + 40^\circ + 55^\circ &= 180^\circ \\ \angle F &= 85^\circ\end{aligned}$$

9. In the given figure, $AB = AC$, $AD = BD = BC$, find $\angle C = ?$

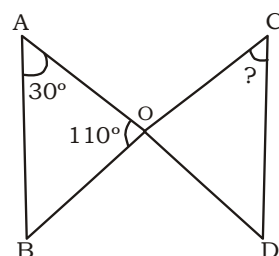


Sol.

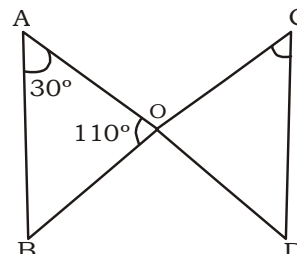
Let $\angle A = \theta$

$$\begin{aligned}\angle A &= \angle ABD = \theta \text{ [AD = BD]} \\ \angle BDC &= \angle A + \angle ABD \text{ [Sum of two interior angle is equal to opposite of exterior angle]} \\ \angle C &= \angle BDC = 2\theta \text{ [BD = BC]} \\ \therefore \angle DBC &= \theta \\ B &= \angle C = 2\theta \text{ [AB = AC]} \\ \text{In } \triangle ABC \\ \angle A + \angle B + \angle C &= 180^\circ \\ \theta + 2\theta + 2\theta &= 180^\circ \\ \theta &= 36^\circ \\ \angle C &= 2\theta = 2 \times 36^\circ = 72^\circ\end{aligned}$$

10. In the given figure $AB \parallel CD$, $\angle A = 30^\circ$, $\angle O = 110^\circ$, find $\angle C = ?$

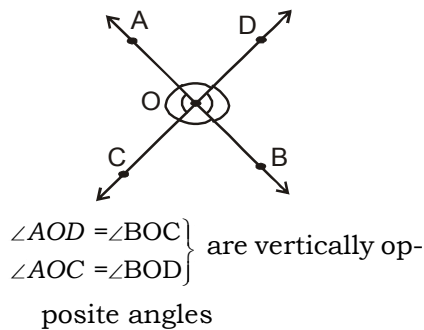


Sol.

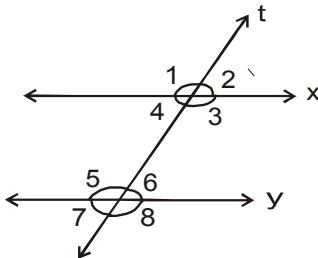


In ABO
 $\angle A + \angle B + \angle O = 180^\circ$
 $30^\circ + \angle B + 110^\circ = 180^\circ$
 $\angle B = 40^\circ$
 $\angle B = \angle C = 40^\circ$
 [Alternate Angle]

Vertically opposite Angles



Angles made by a transversal line -



Interior Angles- $\angle 3, \angle 4, \angle 5, \angle 6$

Exterior Angles- $\angle 1, \angle 2, \angle 7, \angle 8$

Pairs of corresponding angles-

$$\angle 1 = \angle 5, \angle 2 = \angle 6, \\ \angle 4 = \angle 7, \angle 3 = \angle 8$$

Pairs of alternate interior angles-

$$\angle 3 = \angle 5, \angle 4 = \angle 6$$

Pairs of alternate exterior angles-

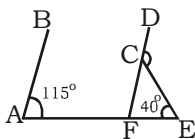
$$\angle 1 = \angle 8, \angle 2 = \angle 7$$

Pairs of interior angles on the same side of the transversal-

$$\angle 3 \text{ and } \angle 6, \angle 4 \text{ and } \angle 5$$

EXAMPLES

1. In the figure, $AB \parallel CD$ and $\angle BAE = 115^\circ$, $\angle AEC = 40^\circ$, then the value of $\angle DCE$?



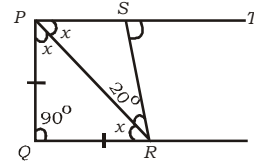
Sol. $\angle BAE = \angle CFE = 115^\circ$ [Corresponding angles]

$\angle CFE + \angle CEF = \angle DCE$ [Sum of interior angle is equal to opposite of exterior angle]

$$115^\circ + 40^\circ = \angle DCE$$

$$\angle DCE = 155^\circ$$

2. In this figure, PQRS is a quadrilateral. $QR \parallel PS$. If $PQ = QR$ then then value of $\angle TSR$?



Sol. In $\triangle PQR$

$$\angle P + \angle Q + \angle R = 180^\circ$$

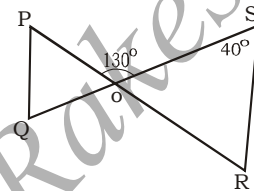
$$x + 90^\circ + x = 180^\circ$$

$$x = 45^\circ$$

$$\angle SRQ = 45^\circ + 20^\circ = 65^\circ$$

$$\angle TSR = \angle QRS = 65^\circ$$
 [Alternate angle]

3. Two line PR and QS intersect each other at O. $\angle POS = 130^\circ$ & $\angle QSR = 40^\circ$, $PQ \parallel RS$. Find $\angle QPO$ & $\angle POQ$.



Sol. $\angle ORS + \angle OSR = \angle POS$ [sum of interior angle is equal to opposite of exterior angle]

$$\angle ORS + 40^\circ = 130^\circ$$

$$\angle ORS = 90^\circ$$

$$\angle POS + \angle SOR = 180^\circ$$
 [Linear angle]

$$130^\circ + \angle SOR = 180^\circ$$

$$\angle SOR = 50^\circ$$

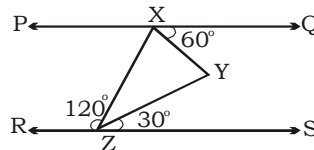
$$\angle POQ = \angle SOR$$
 [vertically opposite angle]

$$PQ \parallel RS$$

$$\angle QPO = \angle SRO$$
 [Alternative angle]

$$\angle QPO = 90^\circ$$

4. In this figure, $PQ \parallel RS$, then $\angle XYZ = ?$



Sol. $PQ \parallel RS$

$$\angle QXZ = \angle XZR = 120^\circ$$
 [Alternate angle]

$$\angle YXZ = 120^\circ - 60^\circ = 60^\circ$$

$$\angle RZX + \angle XZY + \angle YZS = 180^\circ$$
 [Linear angle]

$$120^\circ + \angle XZY + 30^\circ = 180^\circ$$

$$\angle XZY = 30^\circ$$

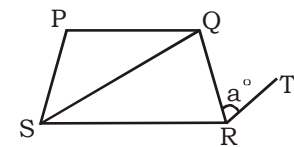
In $\triangle XYZ$

$$\angle X + \angle Y + \angle Z = 180^\circ$$
 [Sum of angle of $\triangle$ is 180°]

$$60^\circ + \angle Y + 30^\circ = 180^\circ$$

$$\angle Y = 90^\circ$$

5. In the figure, $SQ \parallel RT$ and $\angle SPQ = 130^\circ$, $\angle PQS = 40^\circ$, $\angle PSR = 40^\circ$, $\angle QRS = 90^\circ$, then the value of a.



Sol. In $\triangle PQS$

$$\angle P + \angle Q + \angle S = 180^\circ$$

$$130^\circ + 40^\circ + \angle S = 180^\circ$$

$$\angle S = 10^\circ$$

$$\angle QSR + \angle PSQ = \angle PSR$$

$$\angle QSR + 10^\circ = 40^\circ$$

$$\angle QSR = 30^\circ$$

In $\triangle QSR$

$$\angle Q + \angle S + \angle R = 180^\circ$$

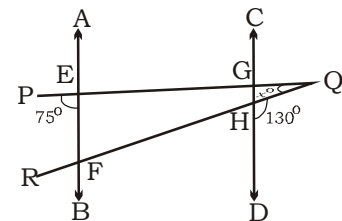
$$\angle Q + 30^\circ + 90^\circ = 180^\circ$$

$$\angle Q = 60^\circ$$

$$\angle SQR = \angle QRT = 60^\circ$$
 [Alternate angle]

$$a = 60^\circ$$

6. In the adjoining figure $AB \parallel CD$ and PQ, QR intersects AB and CD both at E, F and G, H respectively. Given that $\angle PEB = 75^\circ$, $\angle QHD = 130^\circ$ and $\angle PQR = x^\circ$. Find x:



Sol. $\angle PEF = \angle EGH = 75^\circ$ [Corresponding angle]

$\angle EGH + \angle HGQ = 180^\circ$ [Linear angle]

$$\angle HGQ = 180^\circ - 75^\circ$$

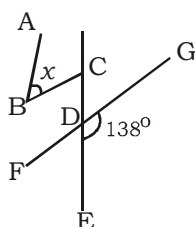
$$\angle HGQ = 105^\circ$$

$\angle HGQ + \angle GQH = \angle DHQ$
[Sum of interior angle is equal to opposite of exterior angle]

$$105^\circ + x = 130^\circ$$

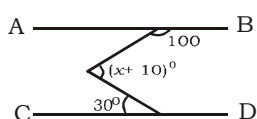
$$x = 25^\circ$$

7. In this given figure $AB \parallel CE$ and $BC \parallel FG$. Find the value of x° :

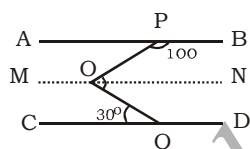


- Sol.** $\angle FDC = \angle GDE = 138^\circ$ [Vertically opposite angle]
 $\angle BCD + \angle FDC = 180^\circ$
 $\angle BCD + 138^\circ = 180^\circ$
 $\angle BCD = 42^\circ$
 $\angle ABC = \angle BCD = x = 42^\circ$ [Alternate angle]

8. $AB \parallel CD$, shown in the figure. Find the value of x .



Sol.



$\angle OPB + \angle PON = 180^\circ$ [Linear angle]

$$\angle PON = 80^\circ$$

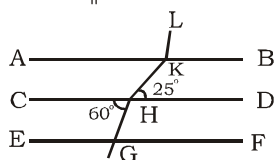
$\angle NOQ = \angle OQC = 30^\circ$ [Alternate angle]

$$\angle POQ = (x + 10)^\circ$$

$$80^\circ + 30^\circ = x + 10^\circ$$

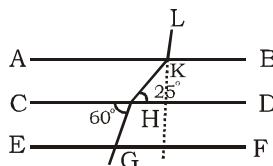
$$x = 100^\circ$$

9. In the given figure $AB \parallel CD \parallel EF$ and $GH \parallel KL$. Find $\angle HKL$



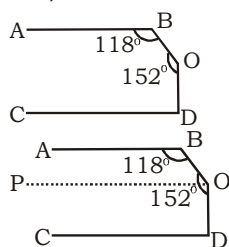
Sol. $\angle AKH = \angle KHD = 25^\circ$ [Alternate angle]

$$\angle EGH = \angle AKL = 180^\circ - 60^\circ = 120^\circ$$
 [Corresponding angle]



$$\angle HKL = 120^\circ + 25^\circ = 145^\circ$$

10. $AB \parallel CD$. $\angle ABO = 118^\circ$, $\angle BOD = 152^\circ$, find $\angle CDO$:



Sol.

$\angle ABO + \angle BOP = 180^\circ$ [Linear angle]

$$118^\circ + \angle BOP = 180^\circ$$

$$\angle BOP = 62^\circ$$

$$\angle POD = 152^\circ - \angle BOP$$

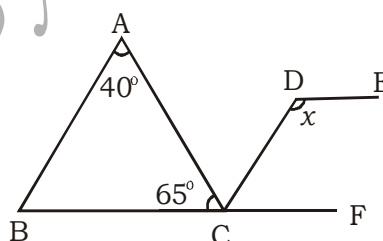
$$= 152^\circ - 62^\circ = 90^\circ$$

$$\angle CDO + \angle POD = 180^\circ$$

$$\angle CDO + 90^\circ = 180^\circ$$

$$\angle CDO = 90^\circ$$

11. In the figure $AB \parallel DC$ and $DE \parallel BF$. Find the value of x & $\angle ACD$



Sol. In $\triangle ABC$

$\angle A + \angle B + \angle C = 180^\circ$ [Sum of angle of $\triangle$ is 180°]

$$40^\circ + \angle B + 65^\circ = 180^\circ$$

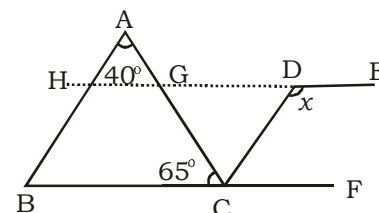
$$\angle B = 75^\circ$$

$\angle ABC = \angle DCF = 75^\circ$ [Corresponding angle]

$\angle EDC + \angle DCF = 180^\circ$ [Interior angle on same side]

$$\angle EDC + 75^\circ = 180^\circ$$

$$\angle EDC = 105^\circ$$



$\angle DGC = \angle BCG = 65^\circ$ [Alternate angle]

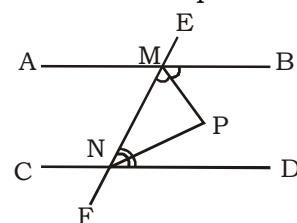
$\angle DGC + \angle GCD = \angle CDE$
[Sum of interior angle is equal to opposite of exterior angle]

$$65^\circ + \angle GCD = 105^\circ$$

$$\angle GCD = 40^\circ$$

12. Two parallel lines AB and CD are intersected by a transversal EF at M & N respectively. The lines MP and NP are the bisectors of interior angles $\angle BMN$ and $\angle DNM$ on the same side of the transversal. Then $\angle MPN$ is equal to:

Sol.



$$\angle BMN = 2a, \angle DNM = 2b$$

$$2a + 2b = 180^\circ$$

$$a + b = 90^\circ$$

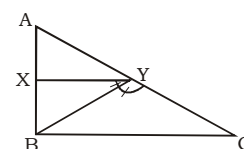
$$\angle MPN = 180^\circ - \angle NMP - \angle MNP$$

$$\angle MPN = 180^\circ - (a + b)$$

$$\angle MPN = 180^\circ - 90^\circ$$

$$\angle MPN = 90^\circ$$

13. In a $\triangle ABC$, a line XY parallel to BC intersects AB at X and AC at Y . If BY bisects $\angle XYZ$, then $\angle CBY : \angle CYB$ is



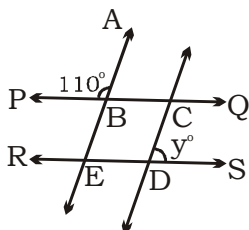
Sol.

$\angle XYB = \angle YBC$ [Alternate angle]

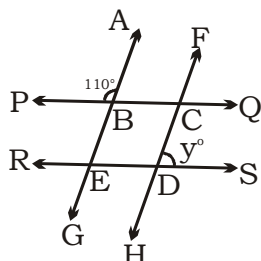
$\angle XYB = \angle BYC$ [Internal bisector]

$$\angle CBY : \angle CYB = 1 : 1$$

14. In the adjoining figure $AE \parallel CD$ and $BC \parallel ED$, then find Y .

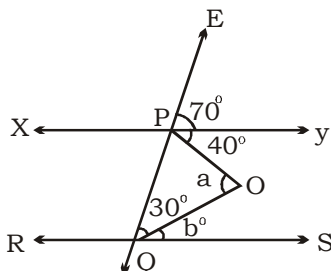


Sol.



$$\begin{aligned}\angle ABP + \angle ABC &= 180^\circ \\ 110^\circ + \angle ABC &= 180^\circ \\ \angle ABC &= 70^\circ \\ \angle ABC &= \angle FCQ = \angle CDS = 70^\circ \text{ [Corresponding angle]}\end{aligned}$$

15. In the given figure $XY \parallel RS$, Find the value of a & b .



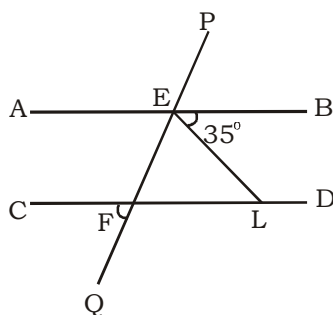
Sol. $\angle EPY = \angle PQS = 70^\circ$ [Corresponding angle]
 $\angle PQO + \angle OQS = \angle PQS$
 $30^\circ + b = 70^\circ$
 $b = 40^\circ$

$$\begin{aligned}\angle EPY + \angle YPO + \angle OPQ &= 180^\circ \\ 70^\circ + 40^\circ + \angle OPQ &= 180^\circ \\ \angle OPQ &= 70^\circ\end{aligned}$$

In $\triangle OPQ$

$$\begin{aligned}\angle P + \angle O + \angle Q &= 180^\circ \\ 70^\circ + \angle a + 30^\circ &= 180^\circ \\ \angle a &= 80^\circ \\ a &= 80^\circ, b = 40^\circ\end{aligned}$$

16. AB and CD are two parallel lines. PQ cuts AB & CD at E & F respectively. EL is the bisector of $\angle BEF$. It $\angle LEB = 35^\circ$ then $\angle CFQ$ will be:



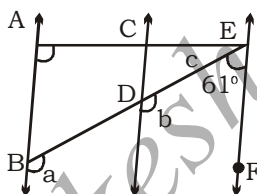
$$\angle BEL = \angle LEF = 35^\circ \text{ [Angle bisector]}$$

$$\angle BEF = \angle DFQ = 70^\circ \text{ [Corresponding angle]}$$

$$\angle DFQ + \angle CFQ = 180^\circ$$

$$\angle CFQ = 180^\circ - 70^\circ = 110^\circ$$

17. In the given figure. $AB \parallel CD \parallel EF$. Also $EA \perp AB$. If $\angle BEF = 61^\circ$. Find the value of a , b & c .



Sol. $\angle A = \angle C = \angle E = 90^\circ$

$$\angle CED + \angle DEF = 90^\circ$$

$$c + 61^\circ = 90^\circ$$

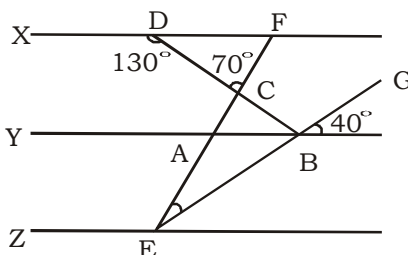
$$c = 29^\circ$$

$$61^\circ + b = 180^\circ \text{ [Linear angle]}$$

$$b = 119^\circ$$

$$a = b = 119^\circ \text{ [Corresponding angle]}$$

18. Three straight lines x , y and z are parallel and the angles are shown in the figure above $\angle AEB$ equal to?



$$\angle XDC = \angle DFC + \angle DCF$$

$$\text{[Sum of two interior angle is opposite to exterior angle]}$$

$$\angle DFC = 60^\circ$$

$$\angle DFC = \angle CAB = 60^\circ \text{ [Alternate angle]}$$

$$\angle B = \angle ABE = 40^\circ \text{ [Vertically opposite angle]}$$

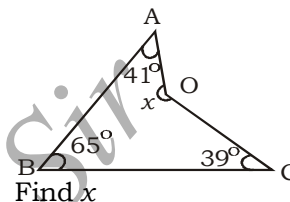
$$\angle AEB + \angle ABE = \angle FAB$$

$$\text{[Sum of two interior angle is opposite to exterior angle]}$$

$$\angle AEB + 40^\circ = 60^\circ$$

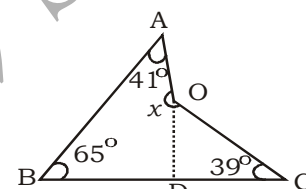
$$\angle AEB = 20^\circ$$

19.



Find x

Sol.



$$\angle ADC = \angle A + \angle B \text{ [Sum of two interior angle is equal to opposite of exterior angle]}$$

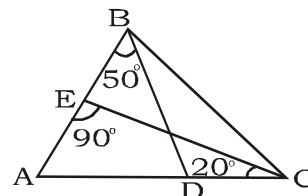
$$\angle ADC = 41^\circ + 65^\circ = 106^\circ$$

$$\angle AOC = \angle C + \angle D$$

$$= 39^\circ + 106^\circ = 145^\circ$$

$$\text{Exterior angle of } AOC = 360^\circ - 145^\circ = 215^\circ$$

20. In the given figure. $CE \perp AB$, $\angle ACE = 20^\circ$ and $\angle ABD = 50^\circ$. Find $\angle BDA$



Sol.

In $\triangle AEC$

$$\angle A + \angle E + \angle C = 180^\circ$$

$$\angle A + 90^\circ + 20^\circ = 180^\circ$$

$$\angle A = 70^\circ$$

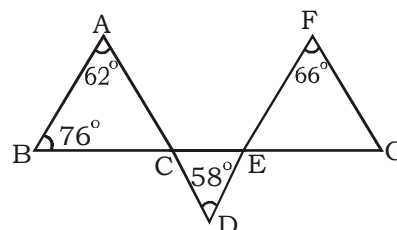
In $\triangle BAD$

$$\angle B + \angle A + \angle D = 180^\circ$$

$$50^\circ + 70^\circ + \angle D = 180^\circ$$

$$\angle D = 60^\circ$$

21. In the adjoining figure $\angle CAB = 62^\circ$, $\angle CBA = 76^\circ$, $\angle ADE = 58^\circ$ and $\angle DFG = 66^\circ$, Find $\angle FGE$.



Sol. In $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$62^\circ + 76^\circ + \angle C = 180^\circ$$

$$\angle C = 42^\circ$$

$$\angle C = \angle DCE = 42^\circ \text{ [Vertically opposite angle]}$$

In $\triangle CDE$

$$\angle C + \angle D + \angle E = 180^\circ$$

$$42^\circ + 58^\circ + \angle E = 180^\circ$$

$$\angle E = 80^\circ$$

$$\angle E = \angle FEG = 80^\circ \text{ [Vertically opposite angle]}$$

In $\triangle FGE$

$$\angle F + \angle G + \angle E = 180^\circ$$

$$66^\circ + \angle G + 80^\circ = 180^\circ$$

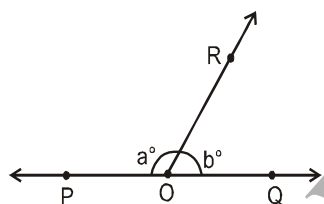
$$\angle G = 34^\circ$$

$$\angle FGE = 34^\circ$$

EXERCISE

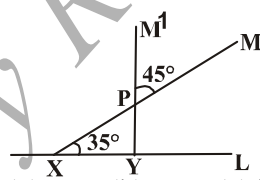
- Find the measures of an angle which is complement of itself.
(a) 40° (b) 30° (c) 45° (d) 50°
- AB is a straight line and O is a point on AB, if line OC is drawn not coinciding with OA or OB, then $\angle AOC$ and $\angle BOC$ are :
(a) equal
(b) complementary
(c) supplementary
(d) together equal to 100
- An angle is equal to one-third of its supplement. Find its measure :
(a) 45° (b) 50°
(c) 55°
(d) None of these
- The ratio of two complementary angle is 1 : 5. What is the difference between the two angles?
(a) 60° (b) 90° (c) 120°
(d) Cannot be determined with the given data
- Which of the following statements are true :
(a) Angles forming a linear pair are supplementary.
(b) If two adjacent angles are equal, then measures of each angle will only be 90° .
(c) Angles forming a linear pair can both be acute angles.
(d) If angles forming a linear pair are equal, then each of these angles is of measure 90° .
(a) Only a (b) a, b and d
(c) c and d (d) a and d
- If one angle of a linear pair is acute, then its other angle will be :
(a) acute (b) obtuse
(c) right angle
(d) None of these

- In figure $\angle POR$ and $\angle QOR$ form a linear pair. If $a - b = 80^\circ$, find the value of 'a' and 'b':



- $a = 125^\circ$, $b = 55^\circ$
- $a = 110^\circ$, $b = 70^\circ$
- $a = 130^\circ$, $b = 50^\circ$
- $a = 75^\circ$, $b = 105^\circ$

- The angle between lines L and M measures 35° . If line M is rotated 45° counter clockwise about point P to line M' what is the angle in degrees between lines L and M' :



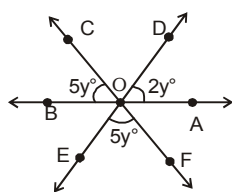
- 90° (b) 80° (c) 75° (d) 60°

- In the given figure, $\angle a$ is greater than one-sixth of right angle, then :



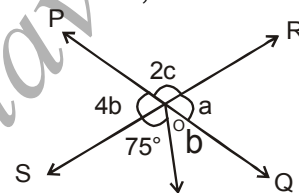
- $b > 165^\circ$ (b) $b < 165^\circ$
(c) $b \leq 165^\circ$ (d) $b \geq 165^\circ$

- Determine the value of y.



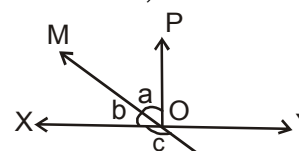
- 17° (b) 15° (c) 20° (d) 25°

- In the given two straight lines PQ and RS intersect each other at 'O'. If $\angle SOT = 75^\circ$, find the value of a, b and c:



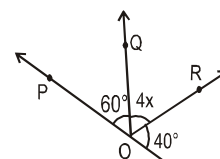
- $a = 84^\circ$, $b = 21^\circ$, $c = 48^\circ$
(b) $a = 48^\circ$, $b = 20^\circ$, $c = 50^\circ$
(c) $a = 72^\circ$, $b = 24^\circ$, $c = 54^\circ$
(d) $a = 64^\circ$, $b = 28^\circ$, $c = 45^\circ$

- In the given figure XY and MN intersect at O. If $\angle POY = 90^\circ$ and $a:b = 2:3$, then find c:



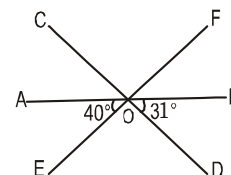
- 113° (b) 54°
(c) 126° (d) 48°

- In the given figure POS is a line, find x :



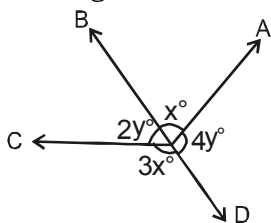
- 20° (b) 80° (c) 10° (d) 100°

- In the following figure find the value of $\angle BOC$:

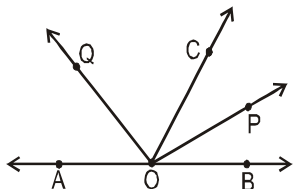


- 101° (b) 149°
(c) 71° (d) 140°

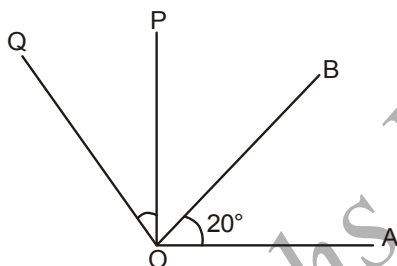
15. Find y , if $x^\circ = 36^\circ$, as per the given diagram :



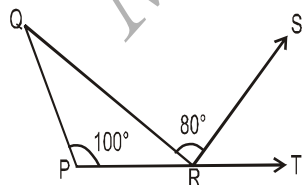
- (a) 36° (b) 16° (c) 12° (d) 42°
 16. In figure OP bisects $\angle BOC$ and OQ, $\angle AOC$. Find the value of $\angle POQ$:



- (a) 60° (b) 75° (c) 90°
 (d) None of these
 17. A, O, B, are three points on a line segment and C is a point not lying on AOB. If $\angle AOC = 40^\circ$ and OX, OY are the internal and external bisectors of $\angle AOC$ and $\angle BOC$ respectively, then $\angle XOY$ is :
 (a) 70° (b) 80° (c) 72° (d) 68°
 18. In the figure, $OP \perp OA$ and $OQ \perp OB$. Find $\angle POQ$ if $\angle AOB = 20^\circ$

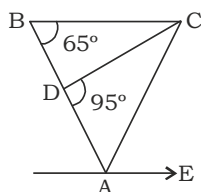


- (a) 20° (b) 30° (c) 40°
 (d) None of these
 19. In the figure $\angle PRQ = \angle SRT$. If $\angle QPR = 100^\circ$ and $\angle QRS = 80^\circ$, Find $\angle PQR$

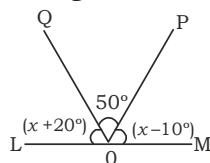


- (a) 20° (b) 30° (c) 40° (d) 60°
 20. In the figure given below, ABC is a triangle. BC is parallel to AE. If $BC = AC$, then what is

the value of $\angle CAE$?

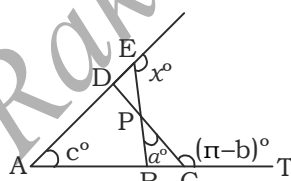


- (a) 20° (b) 30° (c) 40° (d) 50°
 21. In the figure given below LOM is a straight line.



What is the value of x° ?

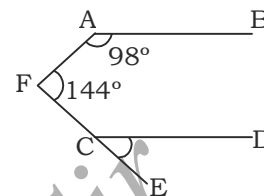
- (a) 20° (b) 60° (c) 40° (d) 50°
 22. In a $\triangle ABC$, $\frac{1}{2} \angle A + \frac{1}{3} \angle C + \frac{1}{2} \angle B = 80^\circ$, then what is the value of $\angle C$?
 (a) 35° (b) 40° (c) 60° (d) 70°
 23. The angles x° , a° , c° and $(\pi - b)^\circ$ are indicated in the figure given below.



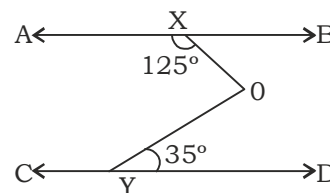
Which one of the following is correct ?

- (a) $x = a + c - b$
 (b) $x = b - a - c$
 (c) $x = a + b + c$
 (d) $x = a - b - c$
 24. Two transversals S and T cut a set of distinct parallel lines. S cuts the parallel lines in points A, B, C, D and T cuts the parallel line in points E, F, G and H, respectively. If $AB = 4$, $CD = 3$ and $EF = 12$, then what is the length of GH?
 (a) 4 (b) 6 (c) 8 (d) 9
 25. AB is a straight line. C is a point whose distance from AB is 3 cm. What is the number of points which are at a distance of 1 cm from AB and 5 cm from C ?
 (a) 1 (b) 2 (c) 3 (d) 4

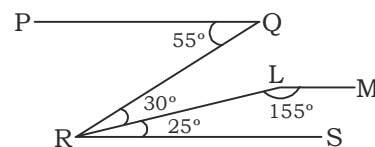
26. In the figure given below, AB is parallel to CD. If $\angle BAF = 98^\circ$ and $\angle AFC = 144^\circ$, then what is $\angle ECD$ equal to ?



- (a) 62° (b) 64° (c) 82° (d) 84°
 27. In a $\triangle ABC$, side AB is extended beyond B, side BC beyond C and side CA beyond A. What is the sum of the three exterior angles ?
 (a) 270° (b) 305°
 (c) 360° (d) 540°
 28. In the figure given below, AB is parallel to CD, What is the value $\angle XOY$?

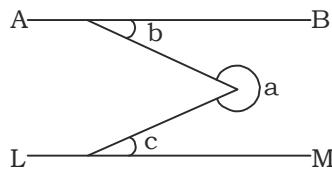


- (a) 80° (b) 90°
 (c) 95° (d) 100°
 29. In the figure given below, PQ is parallel to RS, What is the angle between the lines PQ and LM ?



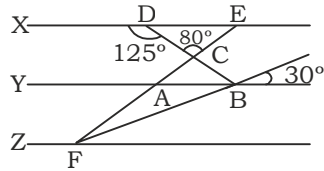
- (a) 175° (b) 177°
 (c) 179° (d) 180°
 30. The line segments AB and CD intersect at O. OF is the internal bisector of obtuse $\angle BOC$ and OE is the internal bisector of acute $\angle AOC$. If $\angle BOC = 130^\circ$, what is the measure of $\angle FOE$?
 (a) 90° (b) 110°
 (c) 115° (d) 120°
 31. In the figure given below, AB is parallel to LM. What is the

angle a equal to ?



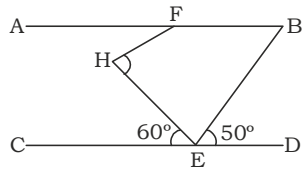
- (a) $\pi + b + c$ (b) $2\pi - b + c$
(c) $2\pi - b - c$ (d) $2\pi + b - c$

32. Three straight lines X, Y and Z are parallel and the angles are as shown in the figure above. What is $\angle AFB$ equal to?



- (a) 20° (b) 15° (c) 30° (d) 10°

33. In the figure given below, AB is parallel to CD and BE is parallel to FH. What is the $\angle FHE$ equal to ?



- (a) 110° (b) 120°
(c) 125° (d) 130°

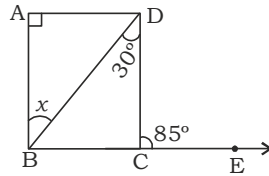
34. Let AB and AC be two rays intersecting at A. If D, E be the points lying on AB, AC respectively and P be the point such that P divides the line DE such that $PD : PE = AD : AE$. Then, what is the locus of the point P?

- (a) The angle bisector of angle A
(b) The angle trisector of angle A
(c) The perpendicular bisector of angle A
(d) None of the above

35. The length of a line segment AB is 2 unit. It is divided into two parts at the point C such that $AC^2 = AB \times CB$. What is the length of CB ?

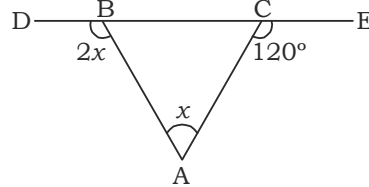
- (a) $3 + \sqrt{5}$ units
(b) $3 - \sqrt{5}$ units
(c) $2 - \sqrt{5}$ units
(d) $\sqrt{3}$ units

36. In figure $AD \parallel BE$, $\angle DCE = 85^\circ$ and $\angle BDC = 30^\circ$, then what is the value of x ?



- (a) 30° (b) 35° (c) 45° (d) 55°

37. In the figure given below, what is the value of x ?

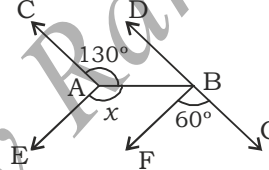


- (a) 30° (b) 40° (c) 45° (d) 60°

38. LM is a straight line and O is a point on LM. Line ON is drawn not coinciding with OL or OM. If $\angle MON$ is one-third of $\angle LON$, then what is $\angle MON$ equal to ?

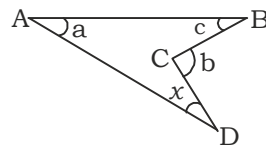
- (a) 45° (b) 30° (c) 60° (d) 75°

39. In the figure given below, $AC \parallel BD$ and $AE \parallel BF$, What is $\angle x$?



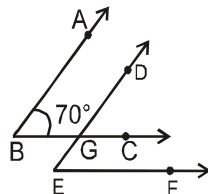
- (a) 130° (b) 110° (c) 70° (d) 50°

40. What is the value of x in the figure given above ?



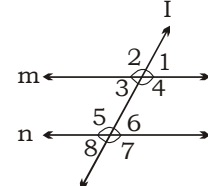
- (a) $b - a - c$ (b) $b - a + c$
(c) $b + a - c$ (d) $\pi - (a + b - c)$

41. In the given figure, the arms of two angles are parallel. If $\angle ABC = 70^\circ$, then find - $\angle DGC$ and $\angle DEF$



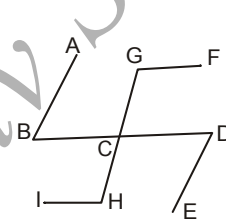
- (a) $70^\circ, 60^\circ$ (b) $60^\circ, 50^\circ$
(c) $70^\circ, 70^\circ$ (d) None of these

42. In the given figure $m \parallel n$ and $\angle 1 = 65^\circ$, find $\angle 5$ and $\angle 8$:-



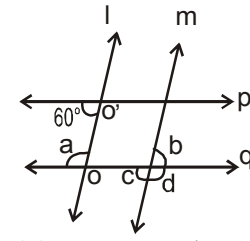
- (a) $125^\circ, 55^\circ$ (b) $115^\circ, 65^\circ$
(c) $105^\circ, 75^\circ$ (d) None of these

43. In the given diagram $AB \parallel GH$, $DE \parallel GF$, $BD \parallel HI$, $\angle FGC = 80^\circ$. Find the value of $\angle CHI$:



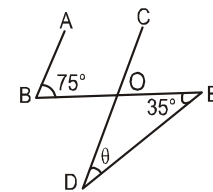
- (a) 80° (b) 120°
(c) 100° (d) 160°

44. Lines $l \parallel m, p \parallel q$. Find a, b, c, d:



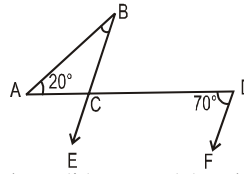
- (a) $a = 120^\circ, b = 60^\circ, c = 60^\circ, d = 120^\circ$
(b) $a = 120^\circ, b = 120^\circ, c = 60^\circ, d = 120^\circ$
(c) $a = 120^\circ, b = 60^\circ, c = 120^\circ, d = 60^\circ$
(d) None of these

45. In figure, $AB \parallel CD$. Find θ :



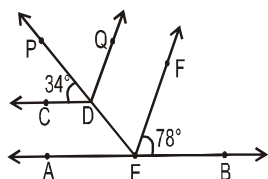
- (a) 30° (b) 35°
(c) 40° (d) 45°

46. From the given figure, find $\angle ABC$, if $BE \parallel DF$



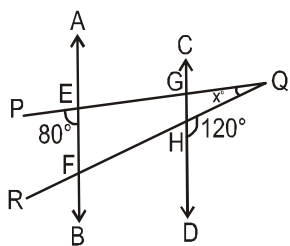
- (a) 50° (b) 40° (c) 35°
(d) None of these

47. In the figure, $AB \parallel CD$ and $EF \parallel DQ$, the value of $\angle PDQ$ is:



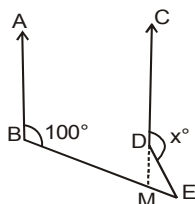
- (a) 68° (b) 78° (c) 56°
(d) None of these

48. In the given figure $AB \parallel CD$, given that $\angle PEB = 80^\circ$, $\angle QHD = 120^\circ$ and $\angle PQR = x^\circ$, find the value of x :



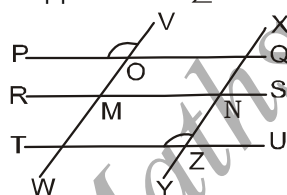
- (a) 40° (b) 20° (c) 100° (d) 30°

49. In the given figure $AB \parallel CD$, $\angle ABE = 100^\circ$, $\angle MED = 25^\circ$. Find $\angle CDE$:



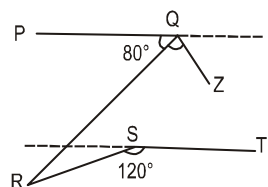
- (a) 125° (b) 55° (c) 65° (d) 75°

50. In the given figure, $\angle XZT = 130^\circ$, PQ , RS and TU are parallel. $VW \parallel XY$. Find $\angle VOP$:



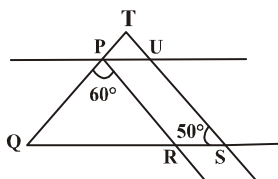
- (a) 125° (b) 130°
(c) 120° (d) 135°

51. From the following figure, find $\angle RQZ$ if $\angle RQZ = 2\angle QRS$ and $PQ \parallel ST$:



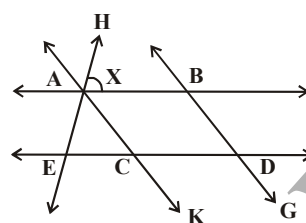
- (a) 20° (b) 30° (c) 40° (d) 60°

52. In the given figure, $PR \parallel TS$ and $PU \parallel RS$. Find $\angle TPU$:



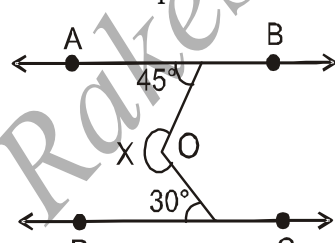
- (a) 60° (b) 70°
(c) 80° (d) 100°

53. In the given figure, $AB \parallel CD$ and $AC \parallel BD$. If $\angle EAC = 40^\circ$, $\angle FDG = 55^\circ$, $\angle HAB = x$, then the value of x is:



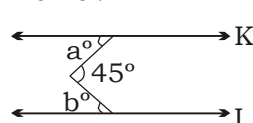
- (a) 95° (b) 70°
(c) 35° (d) 85°

54. In the given figure, $AB \parallel CD$, then X is equal to:



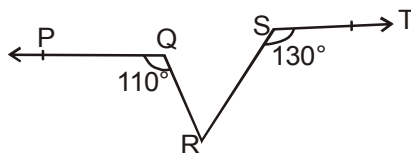
- (a) 290° (b) 300°
(c) 280° (d) 285°

55. In the figure below, lines K and L are parallel. The value of $a^\circ + b^\circ$ is:



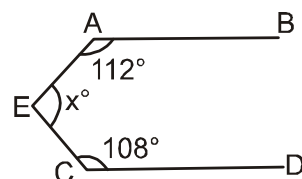
- (a) 45° (b) 180°
(c) 180° (d) 360°

56. In the figure, if $PQ \parallel ST$, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$, find $\angle QRS$.



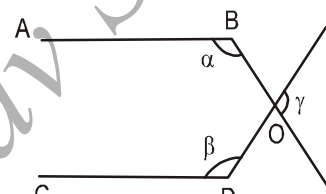
- (a) 40° (b) 50° (c) 60° (d) 70°

57. In the figure, $AB \parallel CD$, the value of x is:



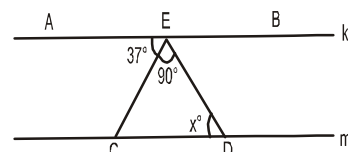
- (a) 220° (b) 140°
(c) 150° (d) none of these

58. If $AB \parallel CD$, then find the value of $\alpha + \beta + \gamma$:



- (a) 180° (b) 270°
(c) 360° (d) 90°

59. In the figure below, if $AB \parallel CD$ and $CE \perp ED$, then the value of x is:



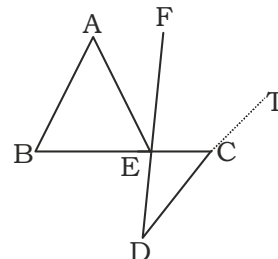
- (a) 53° (b) 63° (c) 37° (d) 45°

60. If a straight line L makes an angle θ ($\theta > 90^\circ$) with the positive direction of x -axis, then the acute angle made by a straight line L_1 , perpendicular to L , with the y -axis is:

- (a) $\frac{\pi}{2} + \theta$ (b) $\frac{\pi}{2} - \theta$

- (c) $\pi + \theta$ (d) $\pi - \theta$

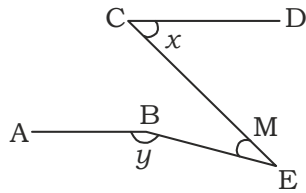
61. In the figure given below, AB is parallel to CD . $\angle ABC = 65^\circ$, $\angle CDE = 15^\circ$ and $AB = AE$.



What is the value of $\angle AEF$?

- (a) 30° (b) 35° (c) 40° (d) 45°

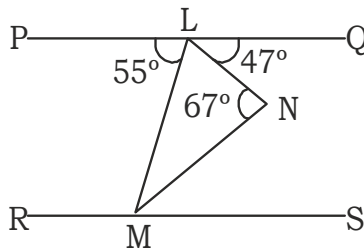
62. In the figure given below, AB is parallel to CD. If $\angle DCE = x$ and $\angle ABE = y$, then what is $\angle CEB$ equal to ?



- (a) $y - x$ (b) $(x + y)/2$
(c) $x + y - (\pi/2)$ (d) $x + y - \pi$

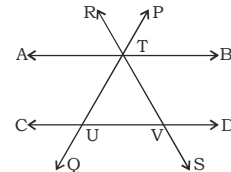
63. In the figure given below, PQ is parallel to RS. What is the

$\angle NMS$ equal to ?



- (a) 20° (b) 23° (c) 27° (d) 47°

64. In the given figure, If $AB \parallel CD$, $\angle PTB = 55^\circ$ and $\angle DVS = 45^\circ$, then what is the sum of the measures of $\angle CUQ$ and $\angle RTP$?



- (a) 180° (b) 135°
(c) 110° (d) 100°

65. In given that $AB \parallel DE$. $\angle ABC = 115^\circ$, $\angle CDE = 140^\circ$. Then, what is the value of $\angle BCD$?



- (a) 45° (b) 55° (c) 65° (d) 75°

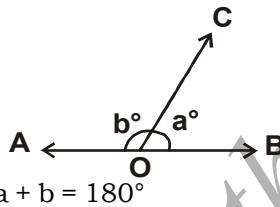
ANSWER KEY

1. (c)	8. (b)	15. (a)	22. (c)	29. (d)	36. (b)	43. (a)	50. (b)	57. (b)	64. (b)
2. (c)	9. (b)	16. (c)	23. (c)	30. (a)	37. (d)	44. (a)	51. (c)	58. (c)	65. (d)
3. (a)	10. (b)	17. (a)	24. (d)	31. (c)	38. (a)	45. (c)	52. (b)	59. (a)	
4. (a)	11. (a)	18. (a)	25. (d)	32. (b)	39. (b)	46. (a)	53. (d)	60. (d)	
5. (d)	12. (c)	19. (b)	26. (a)	33. (a)	40. (a)	47. (a)	54. (d)	61. (b)	
6. (b)	13. (a)	20. (d)	27. (c)	34. (a)	41. (c)	48. (b)	55. (a)	62. (d)	
7. (c)	14. (b)	21. (b)	28. (b)	35. (b)	42. (b)	49. (a)	56. (c)	63. (a)	

SOLUTION

1. (c) Let the measure of angle = x°
measure of its complement = x°
 $\therefore x^\circ + x^\circ = 90^\circ \Rightarrow x^\circ = 45^\circ$

2. (c)



$$\therefore a + b = 180^\circ$$

3. (a) Let the measure of angle = x°
 $\therefore$ its supplement = $(180 - x)^\circ$

$$x = \frac{1}{3} (180 - x) \Rightarrow x = 45^\circ$$

4. (a) Given that,

$$\frac{\alpha}{\beta} = \frac{1}{5} \Rightarrow \alpha = k \text{ and } \beta = 5k$$

or complementary angles,

$$\alpha = 90^\circ - \beta \Rightarrow k = 90^\circ - 5k$$

$$\Rightarrow k = 15^\circ$$

$$\therefore a = 15^\circ \text{ and } \beta = 75^\circ$$

$$\therefore \text{Difference between angles} = 75^\circ - 15^\circ = 60^\circ$$

5. (d)
6. (b) Let $X < 90^\circ$ and other angle = y
 $\therefore X + Y = 180^\circ$ (linear pair)
 $\therefore Y = 180^\circ - X, \therefore Y > 90^\circ$
($\because X < 90^\circ$)

7. (c) $\angle POR = a^\circ$ and $\angle QOR = b^\circ$
form a linear pair

$$\therefore a + b = 180^\circ \text{ (1)}$$

$$a - b = 80^\circ \text{ (2) given}$$

$$(1) + (2)$$

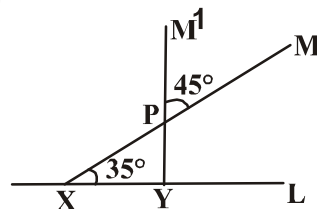
$$2a = 260^\circ \Rightarrow a = 130^\circ$$

$$\therefore \text{Put in (1)}$$

$$b = 180^\circ - a = 180^\circ - 130^\circ = 50^\circ$$

$$\therefore a = 130^\circ \text{ and } b = 50^\circ$$

8. (b)



$$\angle PYX = 180^\circ - 35^\circ - \angle XPY$$

$$= 180^\circ - 35^\circ - 45^\circ = 100^\circ$$

$$\therefore \angle PYL = 180^\circ - \angle PYX$$

$$= 80^\circ$$

9. (b) $a > \frac{90^\circ}{6} \Rightarrow a > 15^\circ$
 $a + b = 180^\circ \Rightarrow b < 165^\circ (\angle a > 15^\circ)$

10. (b) $\angle COD = \angle EOF = 5Y^\circ$ (vertically opposite angle)

$$\therefore \angle AOD + \angle DOC + \angle COB = 180^\circ$$

$$\Rightarrow 2y^\circ + 5y^\circ + 5y^\circ = 180^\circ$$

$$\Rightarrow 12y^\circ = 180^\circ$$

$$y = 15^\circ$$

11. (a) $\angle ROQ = \angle POS$ (vertically opposite angles)

$$\therefore a = 4b$$

$$75^\circ + b + a = 180^\circ$$

$$\Rightarrow b + 4b = 180^\circ - 75^\circ = 105^\circ$$

$$\Rightarrow b = 21^\circ$$

$$\therefore a = 4b = 4 \times 21 = 84^\circ$$

$$a + 2c = 180^\circ$$

$$\Rightarrow 2c = 180^\circ - 84^\circ$$

$$\Rightarrow 2c = 96^\circ$$

$$\Rightarrow c = 48^\circ$$

$$\therefore a = 84^\circ, b = 21^\circ, c = 48^\circ$$

12. (c) $\angle POX = 90^\circ$
By line property:-
 $\angle a + \angle b = 90^\circ$
 $a : b = 2 : 3$
So, $\angle a = 36^\circ$ and $\angle b = 54^\circ$
Here MN is a line so
 $\Rightarrow 54^\circ + \angle C = 180^\circ$
 $\Rightarrow \angle C = 126^\circ$

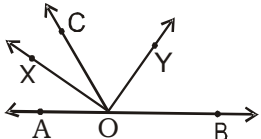
13. (a) by line property,
 $60^\circ + 4x + 40^\circ = 180^\circ$
 $\Rightarrow 4x = 80^\circ$
 $\Rightarrow x = 20^\circ$

14. (b) $\angle AOC = \angle BOD = 31^\circ$ (vertically opposite)
 $\therefore \angle BOC = 180^\circ - \angle AOC = 149^\circ$
15. (a) $2y + 3x = 180^\circ \Rightarrow y = 36^\circ$
($\because x = 36^\circ$)

$$\text{or } x + 4y = 180^\circ \Rightarrow y = 36^\circ$$

$$(\because x = 36^\circ)$$

16. (c) $\angle BOC = 2\angle POC$
 $\therefore OP$ bisects $\angle BOC$
 $\angle AOC = 2\angle QOC$
 $\therefore OQ$ bisects $\angle AOC$
Since, ray OC stands on line AB.
Therefore,
 $\angle AOC + \angle BOC = 180^\circ$
 $\Rightarrow 2\angle QOC + 2\angle POC = 180^\circ$
 $\Rightarrow \angle QOC + \angle POC = 90^\circ$
 $\Rightarrow \angle POQ = 90^\circ$

17. (a) 
 $\therefore \angle AOC = 40^\circ$
 $\therefore \angle BOC = 180^\circ - 40^\circ = 140^\circ$
 $\therefore OY$ is the bisector of $\angle BOC$

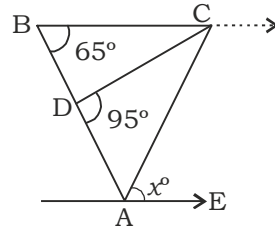
$$\therefore \angle BOY = \frac{1}{2} \angle BOC$$

$$= \frac{1}{2} \times 140 = 70^\circ$$

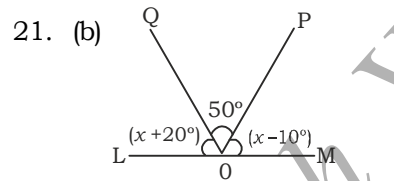
18. (a) $\angle BOP = 90^\circ - \angle AOB$
 $= 90^\circ - 20^\circ = 70^\circ$
 $\therefore \angle POQ = 90^\circ - \angle BOP$
 $= 90^\circ - 70^\circ = 20^\circ$

19. (b) $\angle PRQ + \angle QRS + \angle SRT = 180^\circ$
 $\therefore \angle PRQ + 80^\circ + \angle PRQ = 180$
($\because \angle PRQ = \angle SRT$)
 $\Rightarrow \angle PRQ = 50^\circ$
 $\Rightarrow \angle PQR = 180^\circ - \angle QPR - \angle PRQ$

- $= 180^\circ - 100^\circ - 50^\circ = 30^\circ$
20. (d) Given that, $BC \parallel AE$
 $\angle CBA + \angle EAB = 180^\circ$
 $\Rightarrow \angle EAB = 180^\circ - 65^\circ = 115^\circ$
 $\therefore BC = AC$
Hence, $\triangle ABC$ is an isosceles triangle.



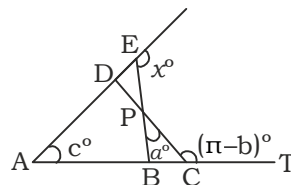
- $\Rightarrow \angle CBA = \angle CAB = 65^\circ$
Now, $\angle EAB = \angle EAC + \angle CAB$
 $\Rightarrow 115^\circ = x + 65^\circ \Rightarrow x = 50^\circ$



- $\angle LOQ + \angle QOP + \angle POM = 180^\circ$
 $\therefore (x+20)^\circ + 50^\circ + (x-10)^\circ = 180^\circ$
 $\Rightarrow 2x + 60^\circ = 180^\circ \Rightarrow 2x = 120^\circ$
 $\therefore x = 60^\circ$

22. (c) Given that,
 $\Rightarrow \frac{1}{2} \angle A + \frac{1}{3} \angle C + \frac{1}{2} \angle B = 80^\circ$
 $\Rightarrow 3\angle A + 2\angle C + 3\angle B = 480^\circ$
 $\Rightarrow 3(\angle A + \angle B) + 2\angle C = 480^\circ$
Also In $\triangle ABC$,
 $\angle A + \angle B + \angle C = 180^\circ$
 $\Rightarrow 3(\angle A + \angle B) + 3\angle C = 540^\circ \dots (ii)$
On subtracting Eq. (i) from (ii), we get
 $\angle C = 60^\circ$

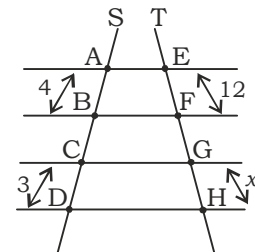
23. (c) $\angle PCT + \angle PCB = \pi$ (linear pair)
 $\angle PCB = \pi - (\pi - b^\circ) = b^\circ \dots (i)$



- In $\triangle BPC$,
 $\angle PCB + \angle BPC + \angle PBC = \pi$
 $\Rightarrow \angle PBC = \pi - \angle PCB - \angle BPC$
 $= \pi - b^\circ - a^\circ \dots (ii)$
 $\therefore \angle ABE + \angle EBC = \pi$
($\because \angle PBC = \angle EBC$) (Linear pair)
 $\Rightarrow \angle ABE = \pi - \angle PBC = \pi - (\pi - b^\circ - a^\circ)$
 $= a^\circ + b^\circ \dots (iii)$

- Now in $\triangle ABE$,
Sum of two interior angles = Exterior angle
 $\angle EAB + \angle ABE = \angle BES$
 $\Rightarrow c^\circ + b^\circ + a^\circ = x^\circ$
 $x^\circ = a^\circ + b^\circ + c^\circ$

24. (d) From figure,

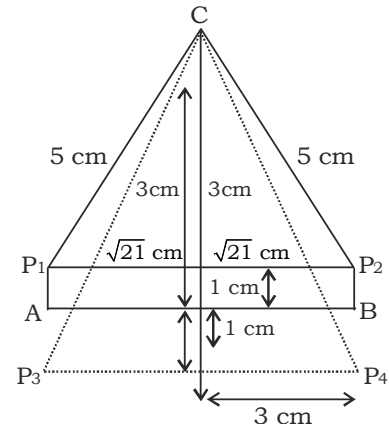


By proportionality law,

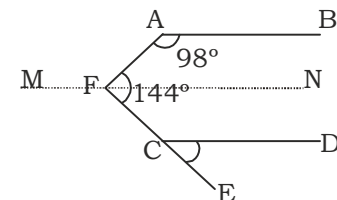
$$\frac{AB}{CD} = \frac{EF}{GH} \Rightarrow \frac{4}{3} = \frac{12}{x}$$

$$x = 3 \times 3 = 9$$

25. (d) Required number of points = 4 (P_1, P_2, P_3, P_4)

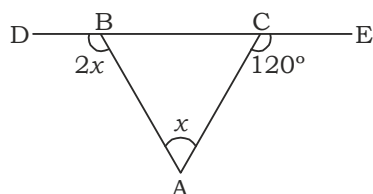


26. (a) Draw a line MN parallel to AB and CD.



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37. (d) $\angle ABC = 180^\circ - \angle DBA$



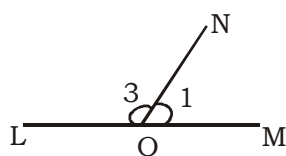
$$= 180^\circ - 2x$$

$$\text{and } \angle ACB = 180^\circ - \angle ACE \\ = 180^\circ - 120^\circ = 60^\circ$$

We know that,

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ \\ \Rightarrow 180^\circ - 2x + 60^\circ + x = 180^\circ \\ \Rightarrow 240^\circ - 180^\circ = x \\ x = 60^\circ$$

38. (a)



$$\angle MON = \frac{1}{3} \angle LON$$

$$\frac{\angle MON}{\angle LON} = \frac{1}{3}$$

$$\angle MON + \angle LON = 180^\circ$$

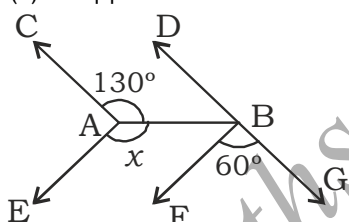
$$x + 3x = 180^\circ$$

$$4x = 180^\circ$$

$$x = 45^\circ$$

$$\angle MON = x = 45^\circ$$

39. (b) $AC \parallel BD$



$$\therefore \angle DBA = 180^\circ - 130^\circ = 50^\circ$$

Since, DBG is a straight line,

$$\therefore \angle DBA + \angle ABF + \angle FBG = 180^\circ$$

$$\Rightarrow 50^\circ + \angle ABF + 60^\circ = 180^\circ$$

$$\Rightarrow \angle ABF = 70^\circ$$

Since, $AE \parallel BF$

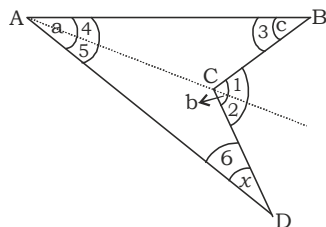
$$\therefore x = 180^\circ - \angle ABF$$

$$x = 180^\circ - 70^\circ = 110^\circ$$

40. (a) $\angle 1 = \angle 3 + \angle 4$

$$\angle 2 = \angle 5 + \angle 6$$

(exterior angle is equal to sum of two opposite interior angles)



$$\Rightarrow \angle 1 + \angle 2 = \angle 3 + \angle 4 + \angle 5 + \angle 6$$

$$= b + c + a + x$$

$$x = b + c - a$$

41. (c) $\angle DGC = \angle DEF = \angle ABC = 70^\circ$
(corresponding angles)

42. (b) $\angle 1 = \angle 3$ (vertically opposite angles)

$$\angle 3 = \angle 8 \text{ (corresponding angles)}$$

$$\therefore \angle 1 = \angle 8 \Rightarrow \angle 8 = 65^\circ$$

Now,

$$\angle 5 + \angle 8 = 180^\circ \Rightarrow \angle 5 = 115^\circ$$

$$\text{Thus, } \angle 5 = 115^\circ \text{ and } \angle 8 = 65^\circ$$

43. (a) $\because GF \parallel HI$

$$\therefore \angle CHI = \angle FGC = 80^\circ$$

44. (a) $60^\circ + a = 180^\circ \Rightarrow a = 120^\circ$

$$\therefore \angle loq = 180^\circ - a = 60^\circ$$

$$\therefore \angle c = \angle loq = 60^\circ \text{ (Alternate angles)}$$

$$\text{and } \angle b = \angle c = 60^\circ$$

$$\text{(Vertically opposite angles)}$$

$$\text{and } \angle d = 180^\circ - \angle c = 120^\circ$$

$$\therefore a = 120^\circ, b = 60^\circ, c = 60^\circ, \\ d = 120^\circ$$

45. (c) $\angle COE = \angle ABE = 75^\circ$
(corresponding angles)

$$\therefore \angle DOE = 180^\circ - \angle COE$$

$$= 180^\circ - 75^\circ = 105^\circ$$

In $\triangle DOE$

$$105^\circ + 35^\circ + \theta = 180^\circ$$

$$\Rightarrow \theta = 180^\circ - 140^\circ = 40^\circ$$

46. (a) $BE \parallel DF$

$$\therefore \angle ACE = \angle CDF = 70^\circ$$

(Corresponding angle)

$$\therefore \angle ACB = 180^\circ - \angle ACE \\ = 110^\circ$$

$$\therefore \angle ABC = 180^\circ - 20^\circ - 110^\circ \\ = 50^\circ$$

47. (a) $CD \parallel AB$

$$\therefore \angle AED = \angle PDC = 34^\circ$$

$$\therefore \angle DEF = 180^\circ - 78^\circ - 34^\circ \\ = 68^\circ$$

$$\therefore QD \parallel EF$$

$$\therefore \angle PDQ = \angle DEF = 68^\circ$$

(Corresponding angle)

48. (b) $\angle PEF = \angle PGH = 80^\circ$
(Corresponding angle)

$$\Rightarrow \angle QGH = 100^\circ$$

$$\angle QHD = 120^\circ$$

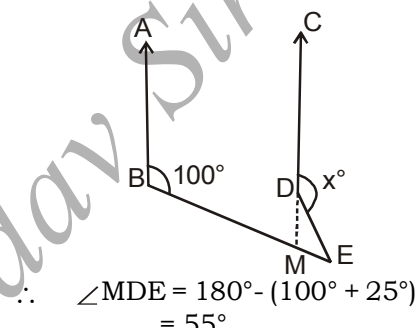
$$\Rightarrow \angle CHQ = 60^\circ$$

$$\therefore \angle x + 100^\circ + 60^\circ = 180^\circ$$

$$\Rightarrow x = 20^\circ$$

49. (a) Extend CD to M, then $\angle DME = \angle ABE = 100^\circ$

$$\angle MED = 25^\circ$$



$$\therefore \angle MDE = 180^\circ - (100^\circ + 25^\circ) \\ = 55^\circ$$

$$\therefore \angle CDE = 180^\circ - 55^\circ = 125^\circ$$

Alternate:

$$\angle CDE = \angle DME + \angle MED = 125^\circ$$

50. (b) $RS \parallel TU$

$$\therefore \angle XNR = \angle XZT = 130^\circ \text{ (corresponding angles)}$$

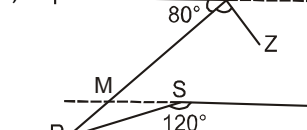
$$\therefore VW \parallel XY$$

$$\therefore \angle OMR = \angle XNR = 130^\circ \text{ (corresponding angles)}$$

$$\therefore PQ \parallel RS$$

$$\therefore \angle VOP = \angle OMR = 130^\circ \text{ (corresponding angles)}$$

51. (c)



$$\angle MSR = 180^\circ - \angle RST \\ = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore PQ \parallel ST$$

$$\therefore \angle QMS = \angle PQR = 80^\circ \text{ (alternate angles)}$$

$$\therefore \angle RMS = 180^\circ - \angle QMS = 100^\circ$$

$$\angle RMS + \angle MSR + \angle SRM = 180^\circ$$

$$\Rightarrow \angle SRM = 180^\circ - 100^\circ - 60^\circ = 20^\circ$$

$$\therefore \angle RQZ = 2 \angle QRS = 2 \angle SRM$$

$$\therefore \angle RQZ = 2 \times 20^\circ = 40^\circ$$

52. (b) $\because PR \parallel TS$

$$\therefore \angle PRQ = \angle USR = 50^\circ$$

In $\triangle PQR$:

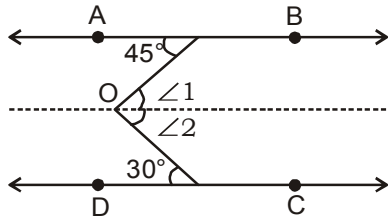
$$\angle PQR = 180^\circ - (50^\circ + 60^\circ) = 70^\circ$$

$$\therefore \angle TPU = \angle PQR = 70^\circ$$

$$[\because PU \parallel RS \parallel QS]$$

53. (d) $\angle DCK = \angle FDG = 55^\circ$
(corresponding angle)
 $\therefore \angle ACE = \angle DCK = 55^\circ$
(vertically opposite angle)
So, $\angle AEC = 180^\circ - (40^\circ + 55^\circ) = 85^\circ$
 $\therefore \angle HAB = \angle AEC = 85^\circ$
(corresponding angle)
Hence, $x = 85^\circ$

54. (d)



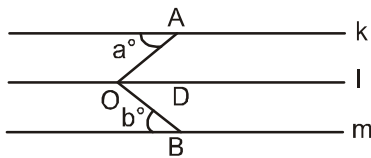
Through O, draw a line parallel to both AB & CD
Then, $\angle 1 = 45^\circ$
(alternate angle)
and $\angle 2 = 30^\circ$
(alternate angle)

$$\therefore \angle BOC = \angle 1 + \angle 2 = 45^\circ + 30^\circ = 75^\circ$$

$$\text{So, } x = 360^\circ - 75^\circ = 285^\circ$$

Hence, $x = 285^\circ$

55. (a)



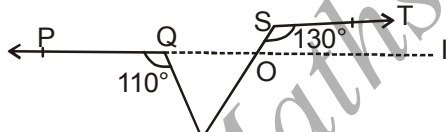
$$\angle BOA = 45^\circ$$

$$\therefore k \parallel l \parallel m$$

$$\therefore \angle DOB = b^\circ \text{ and } \angle AOD = a^\circ$$

$$\therefore a^\circ + b^\circ = \angle AOB = 45^\circ$$

56. (c)



draw a line QI, then

$$\angle RQO = 180^\circ - 110^\circ = 70^\circ$$

$$\angle ROI = 130^\circ$$

(corresponding angle)

$$\text{So, } \angle ROQ = 180^\circ - 130^\circ = 50^\circ$$

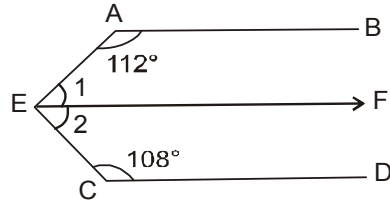
In $\triangle QRO$

$$\Rightarrow \angle RQO + \angle ROQ + \angle QRO = 180^\circ$$

$$\Rightarrow \angle QRO = 180^\circ - (70^\circ + 50^\circ)$$

$$\Rightarrow \angle QRO = 60^\circ = \angle QRS$$

57. (b)



Draw EF parallel to both AB and CD

$\therefore AB \parallel EF$ and AE transversal cuts them at A and E respectively.

$$\therefore \angle BAE + \angle FEA = 180^\circ$$

$$\Rightarrow 112^\circ + \angle 1 = 180^\circ$$

$$\angle 1 = 68^\circ$$

Again $EF \parallel CD$ and transversal cuts them at C and E.

$$\therefore \angle FEC + \angle ECD = 180^\circ$$

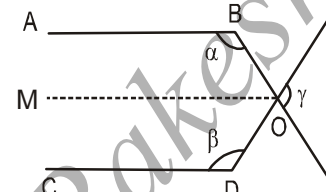
$$\Rightarrow \angle 2 + 108^\circ = 180^\circ$$

$$\Rightarrow \angle 2 = 72^\circ$$

$$\text{Now, } x = \angle 1 + \angle 2$$

$$\Rightarrow x = 72^\circ + 68^\circ = 140^\circ$$

58. (c) Draw $OM \parallel AB \parallel CD$



$$\therefore AB \parallel OM$$

$$\therefore \angle BOM = 180^\circ - \alpha$$

Also,

$$\therefore OM \parallel CD$$

$$\therefore \angle DOM = 180^\circ - \beta$$

$$\therefore \gamma = \angle BOD \text{ (vertically opposite angle)}$$

$$\Rightarrow \gamma = \angle BOM + \angle DOM$$

$$\Rightarrow \gamma = 180^\circ - \alpha + 180^\circ - \beta$$

$$\Rightarrow \alpha + \beta + \gamma = 360^\circ$$

$$59. (a) \angle AEC + \angle CED + \angle DEB = 180^\circ$$

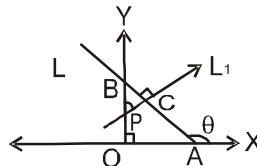
$$\Rightarrow 37^\circ + 90^\circ + \angle DEB = 180^\circ$$

$$\Rightarrow \angle DEB = 180^\circ - 127^\circ = 53^\circ$$

$$\therefore EB \parallel CD$$

$$\therefore \angle DEB = \angle EDC = 53^\circ$$

60. (d)



$$\angle BCP = 180^\circ - \angle BCL_1 = 90^\circ$$

In $\triangle BOA$,

$$\angle O = 90^\circ, \angle A = 180^\circ - \theta$$

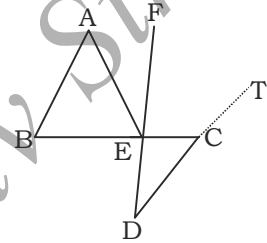
$$\therefore \angle OBA = 180^\circ - (90^\circ + 180^\circ - \theta) = \theta - 90^\circ$$

In $\triangle BPC$,

$$\angle BPC = 180^\circ - (90^\circ + \theta - 90^\circ) = 180^\circ - \theta$$

$$= \pi - \theta$$

61. (b)



Given that,

$$\angle ABC = 65^\circ \text{ and } \angle CDE = 15^\circ$$

Here,

$$\angle ABC + \angle TCB = 180^\circ$$

$$(\because AB \parallel CD)$$

$$\therefore \angle TCB = 180^\circ - 65^\circ = 115^\circ$$

$$\therefore \angle TCB + \angle DCB = 180^\circ$$

(linear pair)

$$\therefore \angle DCB = 65^\circ$$

Now, In $\triangle CDE$

$$\angle CED = 180^\circ - (\angle ECD + \angle EDC) = 180^\circ - (65^\circ + 15^\circ) = 100^\circ$$

$$\therefore \angle DEC + \angle FEC = 180^\circ$$

$$\Rightarrow \angle FEC = 180^\circ - 100^\circ = 80^\circ$$

Given that, $AB = AE$

i.e.,

$\triangle ABE$ is an isosceles triangle.

$$\therefore \angle ABE = \angle AEB = 65^\circ$$

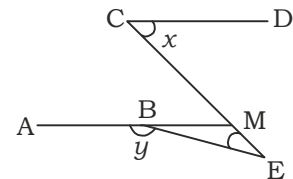
$$\therefore \angle AEB + \angle AEF + \angle FEC = 180^\circ$$

(Straight line)

$$65^\circ + x^\circ + 80^\circ = 180^\circ$$

$$\therefore x^\circ = 180^\circ - 145^\circ = 35^\circ$$

62. (d) Here, we produced line AB to AM.



Since, AM is parallel to CD.

$$\therefore \angle DCM = \angle BMC = x$$

(alternate angle)

Also, ABM is a straight line.

$$\therefore \angle EBM = \pi - y$$

$$\Rightarrow \angle MBE + \angle MEB = \angle CMB$$

$$\pi - y + \angle MEB = x$$

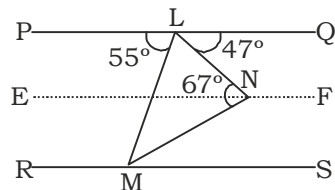
$$\angle MEB = x + y - \pi$$

63. (a) $\angle PLM = \angle LMS$
 $= 55^\circ$ (Alter. angle)

Let draw EF line which is parallel to PQ and bisects by LN.

Then, $\angle QLN = \angle LNE = 47^\circ$

$$\therefore \angle ENL + \angle MNE = 67^\circ$$



$$\Rightarrow 47^\circ + \angle MNE = 67^\circ$$

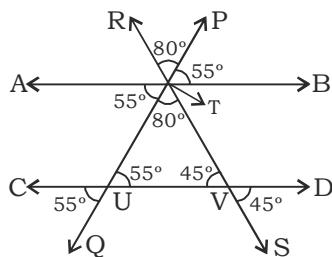
$$\angle MNE = 67^\circ - 47^\circ$$

$$\angle MNE = 20^\circ$$

Similarly, $EF \parallel RS$,

Then $\angle ENM = \angle NMS = 20^\circ$

64. (b) $\angle PTB = 55^\circ$



Then, $\angle TUV = 55^\circ$
 (Corresponding angle)

Also, $\angle PTB = \angle UTA = 55^\circ$
 (Vertically opposite angle)

$\angle CUQ = \angle UTA = 55^\circ$ (Corresponding angle)

Also, given $\angle DVS = 45^\circ$

Then, $\angle UVT = 45^\circ$
 (Vertically opposite angle)

In $\triangle UTV$,

$$\angle T = 180^\circ - (55^\circ + 45^\circ) = 80^\circ$$

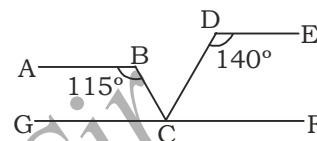
$$\Rightarrow \angle T = \angle PTR = 80^\circ$$

(Vertically Opposite angle)

$$\therefore \angle CUQ + \angle RTP = 55^\circ + 80^\circ$$

$$= 135^\circ$$

65. (d) Draw a line GF through C parallel to AB and DE.
 Now,



$$\therefore \angle BCG = 180^\circ - \angle ABC$$

$$= 180^\circ - 115^\circ = 65^\circ$$

$$\text{and } \angle DCF = 180^\circ - \angle CDE$$

$$= 180^\circ - 140^\circ = 40^\circ$$

$$\text{Now, } \angle BCG + \angle BCD + \angle DCF = 180^\circ$$

$$\Rightarrow 65^\circ + \angle BCD + 40^\circ = 180^\circ$$

$$\Rightarrow \angle BCD = 180^\circ - 105^\circ = 75^\circ$$



TRIANGLE

A **triangle** is closed figure with three sides or a closed plane figure having **three sides** and **three angles**.

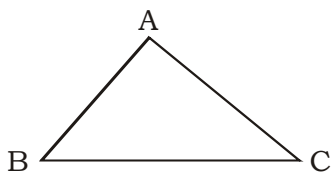
Classification of Triangles

There are many different kinds of triangles. The following table outlines some basic types of triangles.

On the basis of sides

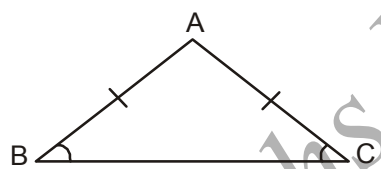
- (i) **Scalene Triangle** - All three sides are of different lengths is called a Scalene Triangle.

$$AB \neq BC \neq CA$$



- (ii) **Isosceles Triangle** :- Two sides are of equal length, is called a Isosceles Triangle.

$$AB = AC$$

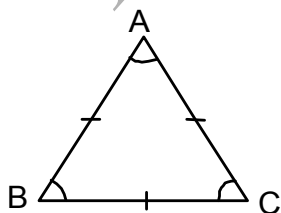


→ Angles opposite to equal sides are equal.

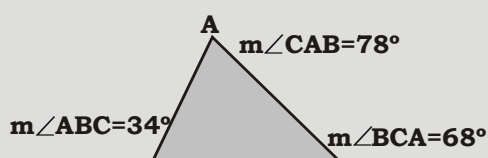
$$\text{i.e. } \angle B = \angle C$$

- (iii) **Equilateral Triangle** - A triangle having all sides equal is called Equilateral triangle.

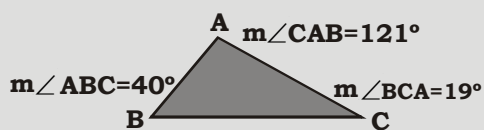
$$AB = BC = CA$$



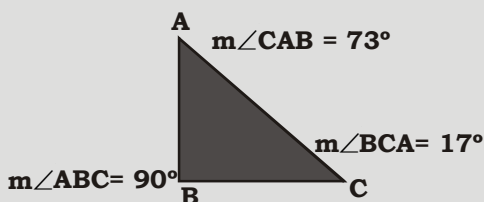
- All angles are equal and is equal to 60°
 $\angle A = \angle B = \angle C = 60^\circ$



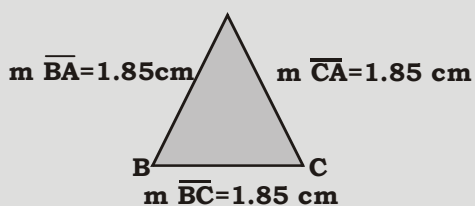
Acute triangles are triangles in which the measure of all three are less than 90° degrees.



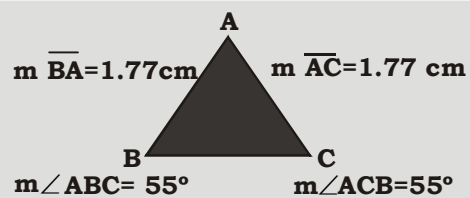
Obtuse triangles are triangles in which the measure of one angle is greater than 90° degrees.



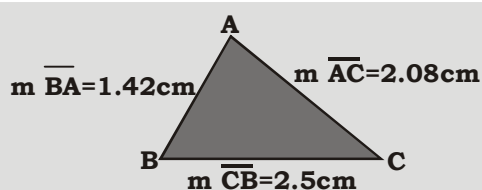
Right triangles are triangles in which the measure of one angle equals 90° degrees.



Equilateral triangles are triangles in which all three sides are the same length.



Isosceles triangles are triangles in which two of the sides are the same length.

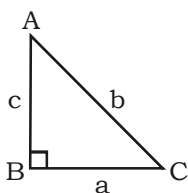


Scalene triangles are triangles in which none of the same length.

On the basis of angles

- (i) **Right-angled Triangle** - In a triangle, in which one of the

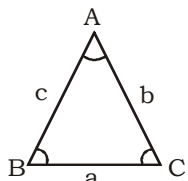
angles measures 90° . Rest two angles are complementary to each other.



Largest side, $b^2 = a^2 + c^2$

$$\therefore \angle B = \angle A + \angle C$$

- (ii) **Acute-angled Triangle** - A triangle in which every angle is more than 0° and less than 90° .



Largest side, Let $b^2 < (a^2 + c^2)$

$\therefore$ According to cosine formula

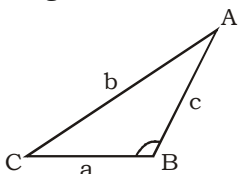
$$b^2 = a^2 + c^2 - 2ac \cos B$$

Here $\cos$ gives +ve value because $B < 90^\circ$

Each angle is less than the sum of the other two.

- (iii) **Obtuse-angled triangle** - A triangle in which one of the angles is more than 90° .

Largest side, $b^2 > a^2 + c^2$



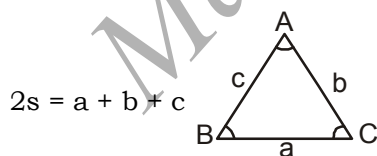
$\therefore$ According to cosine formula

$$b^2 = a^2 + c^2 + 2ac \cos B$$

Here $\cos$ gives negative value So, -ve becomes +ve because $B > 90^\circ$

$$\therefore \angle B > \angle A + \angle C$$

Perimeter of a triangle (2s) : The sum of lengths of three sides of a triangle is called its perimeter.

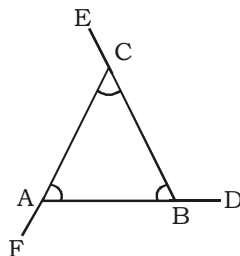


Some basic theorem on the basis of Triangle

Theorem – 1

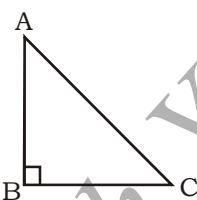
"A triangle is a closed figure that has

three sides and three angles. The sum of the three angles of every triangle is always equals to **180°** ."



$$i.e., \angle CAB + \angle ABC + \angle BCA = 180^\circ$$

"A right triangle always has one angle equal to **90°** and two angles whose sum is **90°** , making a total of **180°** ."



$$\angle B = 90^\circ$$

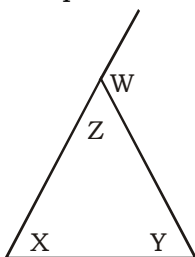
$$\therefore \angle A + \angle C = 90^\circ$$

$$\text{Then, } \angle A + \angle B + \angle C = 180^\circ$$

Theorem – 2

Exterior Angle Theorem

At each vertex of a triangle, an exterior angle of the triangle may be formed by extending ONE SIDE of the triangle. See picture below.



Calculating the Angles

We can use equations to represent the measures of the angles described above. One equation might tell us the sum of the angles of the triangle. For example,

$$x + y + z = 180^\circ$$

We know this is true, because the sum of the angles inside a triangle is always 180° . What is w?

We don't know yet. But, we may observe that the measure of angle w plus the measure of angle z = 180° , because they are a pair of supplementary angles. Notice how Z and W together make a straight line? That's 180° . So, we can make a new equation:

$$w + z = 180$$

Then, if we combine the two equations above, we can determine that the measure of angle w = x + y. Here's how to do that:

$$x + y + z = 180^\circ \quad \dots(i)$$

$$w + z = 180^\circ \quad \dots(ii)$$

Now, rewrite the second equation as $z = 180 - w$ and the substitute that for z in the first equation.

$$x + y + (180 - w) = 180$$

$$x + y - w = 0$$

$$x + y = w$$

The measure of the exterior angle equals the total of the other two interior angles. In fact, there is a theorem called the Exterior Angle Theorem.

Theorem – 3

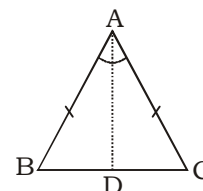
Angles Opposite to Equal Sides of a Triangles are Equal

Angles Opposite to equal sides of an isosceles triangle are equal.

Given : $\triangle ABC$ such that $AB = AC$

To Prove : $\angle ABC = \angle ACB$

Construction : Draw AD, the bisector of $\angle BAC$ to meet BC at D.



Proof

In $\triangle ABD$ and $\triangle ADC$,
 $AB = AC$ (Given)
 $AD = AD$ (Common)

$\angle BAD = \angle CAD$... (by construction)

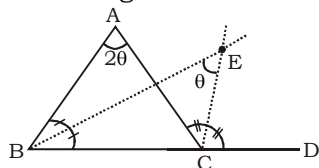
$\therefore \triangle ABD \cong \triangle ADC$... (SAS Cong. Axiom)

$\therefore \angle ABD = \angle ACD$ (CPCT)

$$\Rightarrow \angle ABC = \angle ACB$$

Theorem – 4

The angle between internal bisector of a base angle and external bisector of the other base angle is half of the remaining vertex angle.

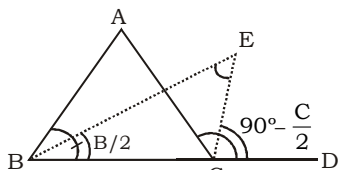


Proof

$$\angle ACD = 180 - C$$

$$\angle ECD = \frac{180 - C}{2} = 90^\circ - \frac{C}{2}$$

In $\triangle BEC$ exterior angle,



$$\angle ECD = \angle EBC + \angle BEC$$

$$90^\circ - \frac{C}{2} = \frac{B}{2} + \angle BEC$$

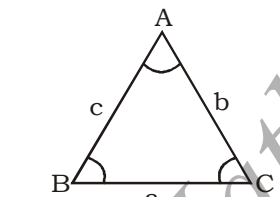
$$\angle BEC = 90 - \left(\frac{B+C}{2}\right) = 90 -$$

$$\left(\frac{180-A}{2}\right) = \frac{A}{2}$$

Theorem – 5

Cosine Formula

If two sides and angle between sides are given then we can find the opposite side by cosine formula.



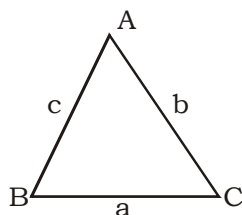
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

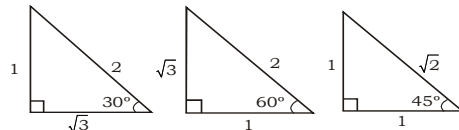
Theorem – 6

Sine Formula



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Some Important Facts About Right Angle Triangle



EXAMPLES

1. The sides of a triangle are 5cm, 6 cm and 4 cm respectively the triangle is:

Sol. $6^2 < 5^2 + 4^2$

$\therefore$ The triangle is Acute angled triangle

2. The sides of a triangle are 7cm, 2cm and 10cm respectively the triangle is:

Sol. 10^2 (largest side) $> 7^2 + 2^2$

$\therefore$ The triangle is Obtuse angled triangle

3. The length of the three sides of a right angled triangle are $(x-2)$ cm, x cm & $(x+2)$ cm respectively. Then the value of x is:

Sol. $(x+2)^2 = (x-2)^2 + x^2$

$$\Rightarrow x^2 + 4 + 4x = x^2 + 4 - 4x + x^2$$

$$\Rightarrow x^2 = 8x$$

$$\Rightarrow x = 8$$

4. If two angles of a right angled triangle other than right angle are $(2x+21)^\circ$ and $(3x+34)^\circ$ then x is:

Sol. $2x + 21^\circ + 3x + 34^\circ = 90^\circ$

$$5x = 35^\circ \text{ or } x = 7^\circ$$

5. In a $\triangle ABC$, $\angle A + \angle B = 113^\circ$, $\angle B + \angle C = 105^\circ$, then find $\angle B$.

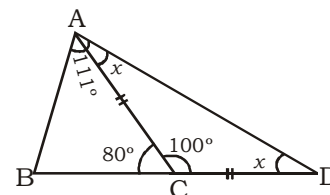
Sol. $\angle A + \angle B + \angle B + \angle C = 113^\circ + 105^\circ$

$$180^\circ + \angle B = 218^\circ$$

$$\angle B = 218^\circ - 180^\circ = 38^\circ$$

6. In a triangle ABC, BC is produced to D so that $CD = AC$. If $\angle BAD = 111^\circ$ and $\angle ACB = 80^\circ$, then the measure of $\angle ABC$ is:

Sol. In $\triangle ABC$, $2x = 180^\circ - 100^\circ = 80^\circ$
 $\Rightarrow x = 40^\circ$

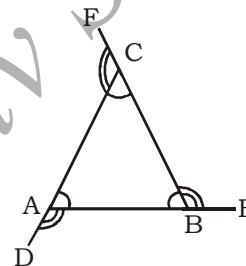


In $\triangle ABD$,

$$\angle ABC = 180^\circ - (111^\circ + 40^\circ) = 29^\circ$$

7. If the sides of a triangle are produced then the sum of the exterior angles i.e.

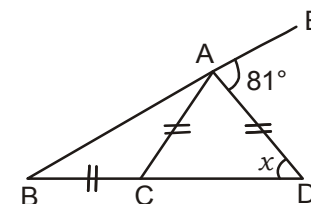
$\angle DAB + \angle EBC + \angle FCA$ is equal to



Sol. Sum of exterior angles = 360°

$$\therefore \angle DAB + \angle EBC + \angle FCA = 360^\circ$$

8. In the given figure, $BC = AC = AD$, $\angle EAD = 81^\circ$. Find the value of x :



Sol. In $\triangle ACD$,

$$\therefore \angle ACD = \angle ADC = x$$

$$\therefore \angle CAD = (180^\circ - 2x)$$

In $\triangle ABC$,

$$\angle ABC = \angle BAC = \frac{x}{2}$$

$$(\because \angle ABC + \angle BAC = \angle ACD = x)$$

$$\therefore \angle BAC + \angle CAD + 81^\circ = 180^\circ$$

$$\frac{x}{2} + (180^\circ - 2x) + 81^\circ = 180^\circ$$

$$\frac{3}{2}x = 81^\circ \Rightarrow x = 54^\circ$$

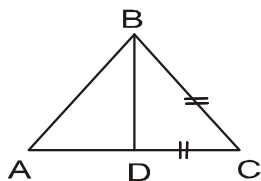
9. In the given triangle ABC, $BC = CD$ and $(\angle ABC - \angle BAC) = 30^\circ$. The measure of $\angle ABD$ is:

Sol. Given,

$$\therefore BC = CD$$

$$\therefore \angle BDC = \angle DBC$$

$$\& \angle ABC - \angle BAC = 30^\circ$$

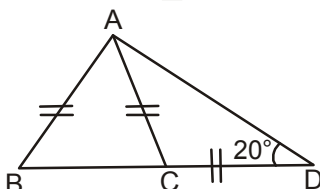


$$\begin{aligned}\angle ABD &= \angle ABC - \angle DBC \\ &= \angle ABC - \angle BDC \\ (\because \angle DBC &= \angle BDC) \\ &= \angle ABC - (\angle ABD \\ &+ \angle BAD)\end{aligned}$$

$$\begin{aligned}\Rightarrow 2(\angle ABD) &= \angle ABC - \angle BAD \\ &= \angle ABC - \angle BAC \\ (\because \angle BAD &= \angle BAC) \\ &= 30^\circ\end{aligned}$$

$$\Rightarrow \angle ABD = 15^\circ$$

10. Consider $\triangle ABD$ such that $\angle ADB = 20^\circ$ and C is a point on BD such that $AB = AC$ and $CD = CA$. Then the measure of $\angle ABC$ is :

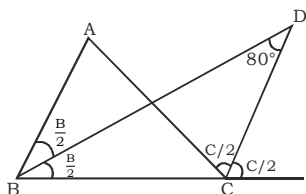


Sol.

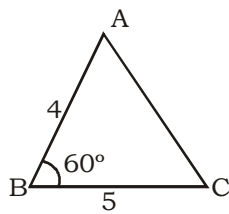
$$\begin{aligned}\angle ADB &= 20^\circ \\ \Rightarrow \angle ADB &= \angle CAD = 20^\circ \\ (\because AC &= CD) \\ \therefore \angle ACB &= 20^\circ + 20^\circ = 40^\circ \text{ (exterior} \\ &\text{angle of } \triangle ACD) \\ \therefore \angle ABC &= \angle ACB = 40^\circ (\because AB = AC)\end{aligned}$$

11. In $\triangle ABC$, the bisectors of the internal angle $\angle B$ & external angle $\angle C$ intersect at D. If $\angle BDC = 80^\circ$, then $\angle A$ is :

Sol. $\angle A = 2 \angle BDC$.
 $\angle A = 2 \times 80 = 160^\circ$.



12.



$$AC = ?$$

Sol. By Cosine formula,
 $AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cos B$
 $AC^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \cos 60^\circ$

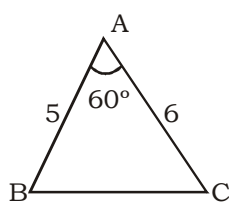
$$AC^2 = 16 + 25 - 40 \times \frac{1}{2}$$

$$AC^2 = 41 - 20$$

$$AC^2 = 21$$

$$\therefore AC = \sqrt{21}$$

13.



$$\text{Find } BC = ?$$

Sol. By using Cosine formula,
 $BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cos 60^\circ$

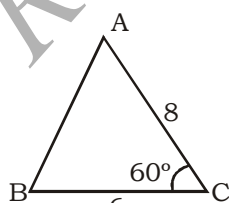
$$BC^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \frac{1}{2}$$

$$BC^2 = 25 + 36 - 30$$

$$BC^2 = 31$$

$$\therefore BC = \sqrt{31}$$

14.



$$\text{Find } AB = ?$$

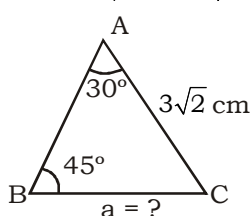
Sol. By using Cosine formula,
 $AB^2 = AC^2 + BC^2 - 2AC \cdot BC \cos 60^\circ$

$$AB^2 = 8^2 + 6^2 - 2 \times 8 \times 6 \times \frac{1}{2}$$

$$AB^2 = 64 + 36 - 48$$

$$\therefore AB = \sqrt{52} = 2\sqrt{13}$$

15.



Sol. By using Sine formula,

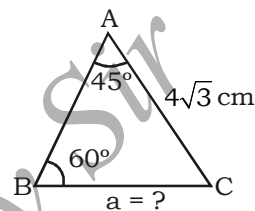
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\Rightarrow \frac{a}{\sin 30^\circ} = \frac{3\sqrt{2}}{\sin 45^\circ}$$

$$\Rightarrow a = 3\sqrt{2} \times \sqrt{2} \times \frac{1}{2}$$

$$\therefore a = 3 \text{ cm}$$

16.



Sol. By using sine formula,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

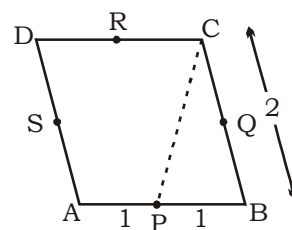
$$\Rightarrow \frac{a}{\sin 45^\circ} = \frac{4\sqrt{3}}{\sin 60^\circ}$$

$$\Rightarrow a = \frac{4\sqrt{3}}{\sqrt{3}} \times 2 \times \frac{1}{\sqrt{2}}$$

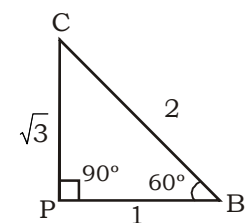
$$\therefore a = 4\sqrt{2} \text{ cm}$$

17. A, B, C, D are the vertex of a Rhombus and P, Q, R, S are the mid points of AB, BC, CD & DA. Find the largest angle of the rhombus ? & $CP \perp AB$

Sol. Let the side of rhombus = 2 units



As we know that,



$$\therefore \text{Largest angle} = \angle DAB$$

$$= 180^\circ - 60^\circ = 120^\circ$$

EXERCISE

1. In a triangle ABC, if AB, BC and AC are the three sides of the triangle, then which of the statement is necessarily true?

(a) $AB + BC < AC$
(b) $AB + BC > AC$
(c) $AB + BC = AC$
(d) $AB - BC > AC$

2. The sides of a triangle are 12 cm, 8 cm and 6 cm respectively, the triangle is :

(a) Acute Angled
(b) Obtuse Angled
(c) Right Angled
(d) Can't be determined

3. In a $\triangle ABC$, $\angle BAC > 90^\circ$, then $\angle ABC$ and $\angle ACB$ must be:

(a) Acute (b) Obtuse
(c) One Acute and One Obtuse
(d) Can't be determined

4. In a $\triangle ABC$, $\angle A = x$, $\angle B = y$ and $\angle C = (y + 20)^\circ$. If $4x - y = 10$, then the triangle is:

(a) Right-angled
(b) Obtuse-angled
(c) Equilateral
(d) None of these

5. If one angle of a triangle is equal to the sum of the other two, then the triangle is:

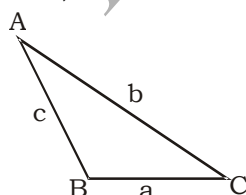
(a) Right-angled
(b) Obtuse-angled
(c) Acute-angled
(d) None of these

6. If each angle of a triangle is less than the sum of the other two, then the triangle is :

(a) Right-angled
(b) Acute-angled
(c) Obtuse-angled
(d) None of these

7. ABC is a triangle. It is given that $a + c > 90^\circ$, then b is

(a) greater than 90°
(b) less than 90°
(c) equal to 90°
(d) can't be said



8. A man goes 150 m due east and then 200 m due north, How far is he from the starting point ?

(a) 200 m (b) 350 m
(c) 250 m (d) 175 m

9. In a $\triangle ABC$, $\angle A : \angle B : \angle C = 2 : 3 : 4$. Then find the smallest angle is :

(a) 45° (b) 40° (c) 60° (d) 65°

10. The sum of two angles of a triangle is equal to its third angle. Determine the measure of third angle :

(a) 100° (b) 80°
(c) 120° (d) 90°

11. Of the three angles of a triangle, one is twice the smallest and another is three times the smallest. Then find smallest angle :

(a) 60° (b) 90° (c) 30° (d) 45°

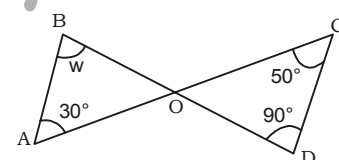
12. In a $\triangle ABC$, If $2\angle A = 3\angle B = 6\angle C$, then $\angle A$ is equal to :

(a) 60° (b) 30°
(c) 90° (d) 120°

13. The degree measures each of the three angles of a triangle is an integer. Which of the following could not be the ratio of their measures ?

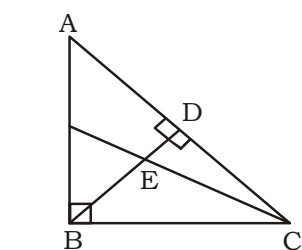
(a) 2 : 3 : 4 (b) 3 : 4 : 5
(c) 5 : 6 : 7 (d) 6 : 7 : 8

14. In the given figure below, what is the value of w ?



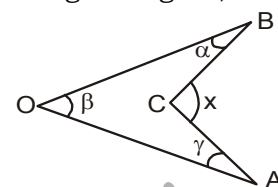
(a) 100° (b) 110°
(c) 120° (d) 130°

15. $AB \perp BC$, $BD \perp AC$ and CE bisects $\angle C$, $\angle A = 30^\circ$. then, what is $\angle CED$?



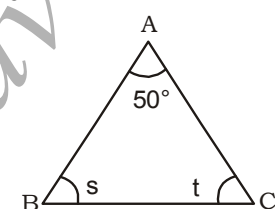
(a) 30° (b) 60° (c) 45° (d) 65°

16. In the given figure, $x = ?$



(a) $\alpha + \beta - \gamma$ (b) $\alpha - \beta + \gamma$
(c) $\alpha + \beta + \gamma$ (d) $\alpha + \gamma - \beta$

17. In the figure below, if $s < 50^\circ < t$, then

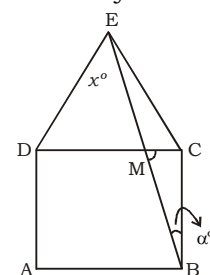


(a) $t < 80$ (b) $s + t < 130$
(c) $50 < t < 80$
(d) $t > 80$

18. The sum of two angles of a triangle is 80° and their difference is 20° , then the smallest angle:

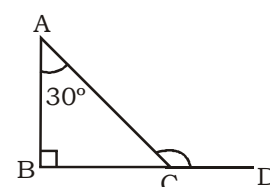
(a) 50° (b) 100° (c) 30°
(d) None of these

19. In the given diagram, equilateral triangle EDC surmounts square ABCD. Find the $\angle BED$ represented by x. Where, $\angle EBC = \alpha^\circ$:



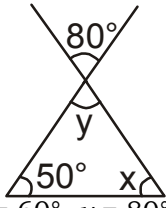
(a) 45° (b) 60° (c) 30°
(d) None of these

20. In the given figure, if $\angle ABC = 90^\circ$, and $\angle A = 30^\circ$, then $\angle ACD =$



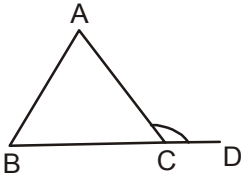
(a) 120° (b) 100° (c) 110°
(d) None of these

21. Value of x and y is :



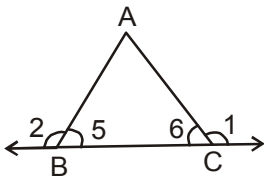
- (a) $x = 60^\circ, y = 80^\circ$
 (b) $x = 80^\circ, y = 50^\circ$
 (c) $x = 50^\circ, y = 80^\circ$
 (d) None of these

22. In the triangle ABC, side BC is produced to D, $\angle ACD = 100^\circ$ if $BC = AC$, then find $\angle ABC$ is :



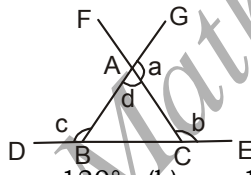
- (a) 40° (b) 50° (c) 80°
 (d) Can't be determined

23. In the given figure, the side BC of a $\triangle ABC$ is produced on both sides, then $\angle 1 + \angle 2$ is equal to:



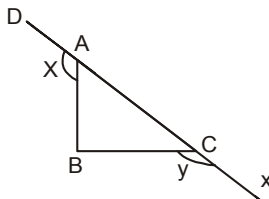
- (a) $\angle A + 180^\circ$
 (b) $180^\circ - \angle A$
 (c) $\frac{1}{2}(\angle A + 180^\circ)$
 (d) $\angle A + 90^\circ$

24. It is given that $d = 70^\circ$, $b = 120^\circ$, then :



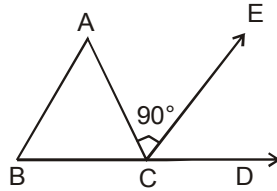
- (a) $c = 130^\circ$ (b) $a = 110^\circ$
 (c) Both (a) and (b) are correct
 (d) Both (a) and (b) are wrong

25. It is given that $AB \perp BC$. Which one of the following is true :



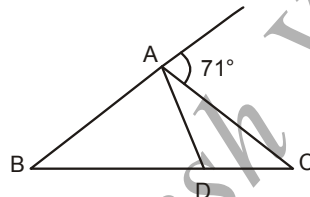
- (a) $x + y = 180^\circ$
 (b) $x + y = 270^\circ$
 (c) $x + y = 300^\circ$
 (d) can not be said

26. In the given figure, $AC \perp CE$ and $\angle A : \angle B : \angle C = 3 : 2 : 1$, find the value of $\angle ECD$:



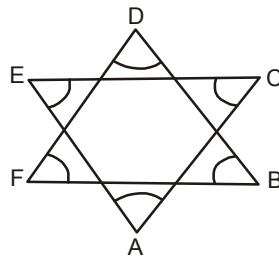
- (a) 50° (b) 45° (c) 55° (d) 60°

27. In the given figure, if $AD = BD = AC$, then the value of $\angle C$ will be :



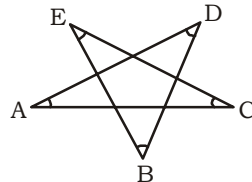
- (a) $\frac{124^\circ}{3}$ (b) $\frac{142^\circ}{3}$
 (c) 39° (d) None of these

28. In the given figure, $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F =$



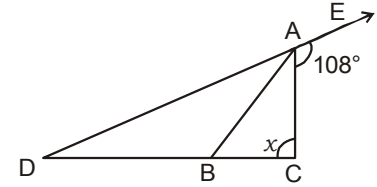
- (a) 360° (b) 720°
 (c) 180° (d) 300°

29. In the given figure, $\angle A + \angle B + \angle C + \angle D + \angle E =$



- (a) 900° (b) 720°
 (c) 180° (d) 540°

30. In the given figure, AB divides $\angle DAC$ in the ratio 1: 3 and $AB = DB$. The value of x :



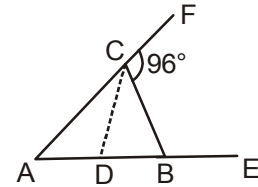
- (a) 90° (b) 80°
 (c) 100° (d) 70°

31. The side BC of $\triangle ABC$ is produced to D. If $\angle ACD = 108^\circ$ and

$$\angle B = \frac{1}{2} \angle A \text{ then } \angle A \text{ is :}$$

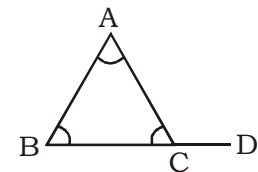
- (a) 36° (b) 108°
 (c) 59° (d) 72°

32. In the given figure below, if $AD = CD = BC$, and $\angle BCF = 96^\circ$, How much is $\angle DBC$?



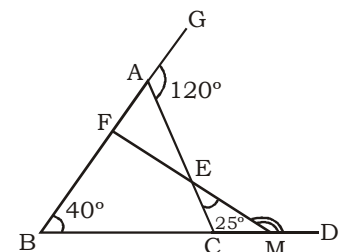
- (a) 32° (b) 84° (c) 64°
 (d) Can't be determined

33. In the given figure BC is produced to D and $\angle BAC = 40^\circ$ and $\angle ABC = 70^\circ$. Find the value of $\angle ACD$



- (a) 30° (b) 40°
 (c) 70° (d) 110°

34. In the given figure, $\angle CAG = 120^\circ$, $\angle CEM = 25^\circ$, $\angle ABC = 40^\circ$, Find $\angle EMD = ?$



- (a) 125° (b) 140°
 (c) 70° (d) 110°

35. In $\triangle ABC$, the line BC extend up to D. If $\angle ACD = 170^\circ$ and $\angle B = \frac{2}{3} \angle A$ then $\angle A$ is :

(a) 10° (b) 68°
(c) 102° (d) Not

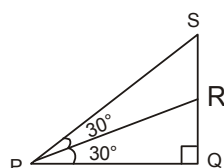
36. In $\triangle ABC$, value of exterior angle at B and C are 111° & 148° respectively then $\angle A$ is :

(a) 32° (b) 69°
(c) 79° (d) 101°

37. In a $\triangle ABC$, $\angle BAC = 75^\circ$, $\angle ABC = 45^\circ$ BC is produced to D. If $\angle ACD = x$, then $\frac{x}{3}\%$ of 60° is:

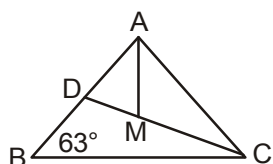
(a) 30° (b) 48° (c) 15° (d) 24°

38. In the given figure which of the following statements is true ?



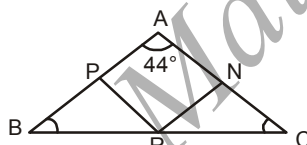
(a) $PQ = QS$ (b) $PR = RS$
(c) $QR = RS$ (d) $PS < RS$

39. In the given figure, $AM = AD$, $\angle B = 63^\circ$ and CD is an angle bisector of $\angle C$, then $\angle MAC = ?$



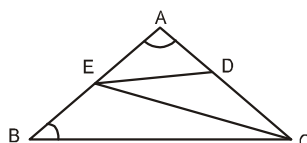
(a) 27° (b) 37° (c) 63°
(d) None of these

40. If $\angle A = 44^\circ$, $BP = BR$ and $CN = RC$ then $\angle PRN = ?$



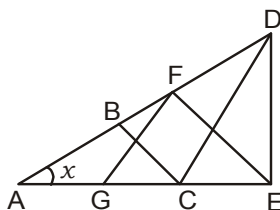
(a) 58° (b) 78° (c) 68° (d) 66°

41. In the given figure, if $\angle B = \angle C = 78^\circ$, $BC = EC$, $CD = BC$ and DE not parallel to BC, then $\angle EDB = ?$



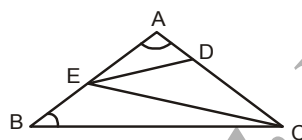
(a) 18° (b) 12° (c) 22°
(d) None of these

42. In the given figure, if $AB = BC = CD = EF = DE = GA = FG$, then $x = ?$



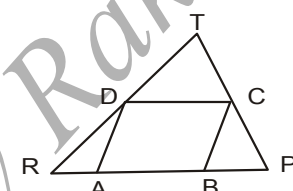
(a) $\frac{153^\circ}{7}$ (b) 28°
(c) $\frac{180^\circ}{7}$ (d) None of these

43. In the given figure, if $AD = DE = EC = BC$ then $\angle A : \angle B =$



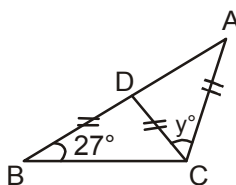
(a) $1 : 3$ (b) $2 : 5$
(c) $3 : 1$ (d) $1 : 2$

44. In the given figure, ABCD is a rhombus and $AR = AB = BP$, then the value of $\angle RTP$ is :



(a) 60° (b) 90° (c) 120° (d) 75°

45. In the following figure ADBC, $BD = CD = AC$, $\angle ABC = 27^\circ$, $\angle ACD = Y$. Find the value of y :

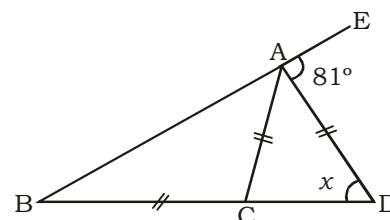


(a) 27° (b) 54° (c) 72° (d) 58°

46. ABC is an isosceles triangle with $AB = AC$. Side BA is produced to D such that $AB = AD$. Find $\angle BCD$:

(a) 60° (b) 90° (c) 120°
(d) can't be determined

47. In the given figure, $BC = AC = AD$ and $\angle EAD = 81^\circ$. Find the value of x :



(a) 45° (b) 54° (c) 63° (d) 36°

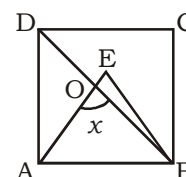
48. ABC is an equilateral triangle and CD is the internal bisector of $\angle C$. If DC is produced to E such that $AC = CE$ then $\angle CAE$ is equal to :

(a) 45° (b) 75° (c) 30° (d) 15°

49. In $\triangle ABC$, $\angle ABC = 120^\circ$ then relation between sides is :

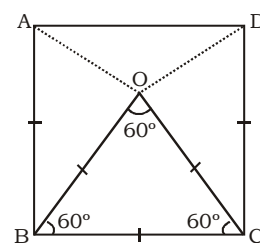
(a) $b^2 = a^2 + c^2 + ac$
(b) $b^2 = a^2 + c^2 - ac$
(c) $b^2 = a^2 + c^2 - 2ac$
(d) $b^2 = a^2 + c^2 + 2ac$

50. In the figure $\triangle ABE$ is an equilateral triangle in a square ABCD. Find the value of angle x in degrees :



(a) 60° (b) 45° (c) 75° (d) 90°

51. ABCD is a square in which $\triangle OBC$ is an equilateral triangle then find $\angle DOA$.



(a) 60° (b) 45°
(c) 150° (d) 90°

52. In $\triangle ABC$, $\angle A = 110^\circ$ and D, E are two points on BC is such that $\angle BDA = 140^\circ$, $\angle CEA = 120^\circ$ and $\angle EAC = 2 \times \angle BAD$. Find the angle $\angle ABD$

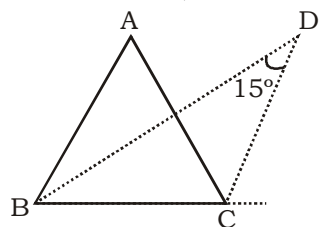
(a) 30° (b) 40° (c) 60° (d) 80°

53. In an obtuse angle $\triangle ABC$ the external angle bisector of $\angle A$ intersect the extended part of line CB at M and the external angle bisector of $\angle C$ intersect the ex-

tended part of line AB at N. $MA = AC = CN$, then find $\angle B = ?$

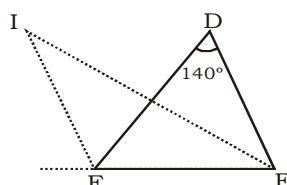
- (a) 108° (b) 110°
(c) 112° (d) 114°

54. In $\triangle ABC$, the bisectors of the internal $\angle B$ and external $\angle C$ at D. If $\angle BDC = 15^\circ$, then $\angle A$ is:



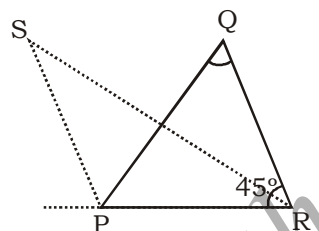
- (a) 45° (b) 75° (c) 30° (d) 15°

55. In $\triangle DEF$, the bisectors of the External $\angle E$ & internal $\angle F$ intersect at I, If $\angle EDF = 140^\circ$, then $\angle I$ is:



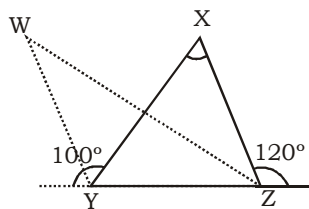
- (a) 45° (b) 70° (c) 30° (d) 15°

56. In $\triangle PQR$, the bisector of the external $\angle P$ & internal $\angle R$ intersect at S. If external angle of $\angle P = 100^\circ$ & $\angle PRQ = 45^\circ$ then $\angle PSR =$:



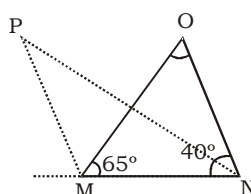
- (a) 27.5° (b) 37.5°
(c) 30° (d) 15°

57. In $\triangle xyz$, the bisector of the external $\angle y$ & internal $\angle z$ meet at w. If external angle of $\angle y$ & $\angle z$ are 100° and 120° respectively then $\angle YWZ$ is :



- (a) 45° (b) 40° (c) 30° (d) 20°

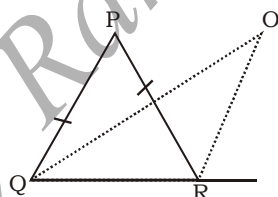
58. In $\triangle MNO$, the bisector of the external $\angle M$ & Internal $\angle N$ are 65° & 40° and meet at point P. Find the ratio of $\angle O : \angle P$ is:-



- (a) $2 : 1$ (b) $5 : 1$
(c) $3 : 1$ (d) $4 : 1$

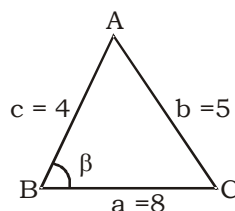
59. In $\triangle PQR$, the bisector of internal $\angle Q$ & external $\angle R$ meet at

O. If $\angle QOR = \frac{31^\circ}{2}$, $PQ = PR$, then external angle of $\angle Q$ is:



- (a) 105.5° (b) 106.5°
(c) 106° (d) 105°

60. In $\triangle ABC$, $AB = 4$ cm, $BC = 8$ cm & $AC = 5$ cm, then find $\cos \beta$?

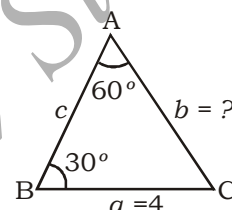


- (a) $\frac{55}{64}$ (b) $\frac{64}{55}$
(c) 120 (d) 60

61. In $\triangle ABC$, $AB = 5$ cm, $BC = 8$ cm and $\angle ABC = 60^\circ$, then find AC ?

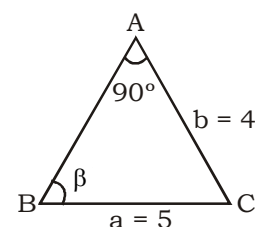
- (a) 7 (b) $\sqrt{55}$
(c) 120 (d) $\sqrt{56}$

62. In $\triangle ABC$, $BC = 4$ cm and $\angle A = 60^\circ$, $\angle B = 30^\circ$, Find AC ?



- (a) $\frac{4}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{3}{\sqrt{4}}$ (d) $\frac{\sqrt{3}}{2}$

63. In $\triangle ABC$, $AC = 4$ cm, $BC = 5$ cm, $\angle A = 90^\circ$, Find $\sin \beta$.



- (a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{\sqrt{4}}$ (d) $\frac{2}{5}$

64. If the angles of triangle are 60° , 90° and 30° , then what is the ratio of the sides opposite to these angles ?

- (a) $\sqrt{3} : \sqrt{2} : 1$ (b) $1 : \sqrt{2} : 2$
(c) $2 : \sqrt{3} : 1$ (d) $\sqrt{3} : 2 : 1$

65. Three sides of a triangle are 7 cm, $4\sqrt{3}$ cm and $\sqrt{13}$ cm then the smallest angle is :

- (a) 15° (b) 30° (c) 45° (d) 60°

ANSWER KEY

1. (b)	8. (c)	15. (b)	22. (b)	29. (c)	36. (c)	43. (a)	50. (c)	57. (d)	64. (d)
2. (b)	9. (b)	16. (c)	23. (a)	30. (a)	37. (d)	44. (b)	51. (c)	58. (a)	65. (b)
3. (a)	10. (d)	17. (d)	24. (c)	31. (d)	38. (b)	45. (c)	52. (a)	59. (a)	
4. (a)	11. (c)	18. (c)	25. (b)	32. (c)	39. (c)	46. (b)	53. (a)	60. (a)	
5. (a)	12. (c)	19. (a)	26. (d)	33. (d)	40. (c)	47. (b)	54. (c)	61. (a)	
6. (b)	13. (d)	20. (a)	27. (b)	34. (a)	41. (b)	48. (d)	55. (b)	62. (a)	
7. (b)	14. (b)	21. (c)	28. (a)	35. (c)	42. (c)	49. (b)	56. (a)	63. (a)	

SOLUTION

1. (b) The sum of the smaller sides cannot be equal to or less than the largest side.

2. (b) $12^2 > 8^2 + 6^2$

$\therefore$ the triangle is Obtuse angled triangle.

3. (a) In $\triangle ABC$,

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

$$\therefore \angle BAC > 90^\circ$$

$$\therefore \angle ABC + \angle ACB < 90^\circ$$

4. (a) $x + y + (y + 20^\circ) = 180^\circ$

$$\Rightarrow x + 2y = 160^\circ$$

$$4x - y = 10^\circ \Rightarrow y = 70^\circ, x = 20^\circ$$

$\therefore$ The angles of the triangle are $20^\circ, 70^\circ, 90^\circ$.

$\therefore$ The triangle is right angled.

5. (a) Let, ABC be a triangle.

$$\angle A = \angle B + \angle C \text{ (Given)}$$

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle A = 180^\circ$$

$$\Rightarrow \angle A = 90^\circ$$

$\therefore$ The triangle is right angled.

6. (b) Let $\angle A < \angle B + \angle C$. Then

$$\angle A < 180^\circ - \angle A \quad [\because \angle A + \angle B + \angle C = 180^\circ]$$

$$\Rightarrow 2\angle A < 180^\circ$$

$$\Rightarrow \angle A < 90^\circ$$

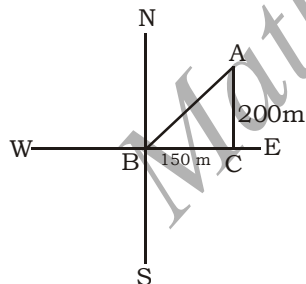
Similarly, $\angle B < 90^\circ$ and $\angle C < 90^\circ$

7. (b) If $a + b + c = 180^\circ$

$$\Rightarrow b = 180^\circ - (a + c)$$

$$\therefore b < 90^\circ (\because a + c > 90^\circ)$$

8. (c) In right angle $\triangle ABC$,



By Pythagoras theorem,

$$AB^2 = BC^2 + AC^2$$

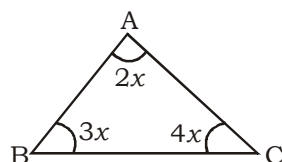
$$\Rightarrow AB^2 = (150)^2 + (200)^2$$

$$\Rightarrow AB^2 = 22500 + 40000$$

$$\Rightarrow AB^2 = 62500$$

$$\Rightarrow AB = 250 \text{ m}$$

9. (b)



As we know that,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\therefore 2x + 3x + 4x = 180^\circ$$

$$9x = 180^\circ$$

$$x = 20^\circ$$

$$\therefore \text{Smallest angle} = \angle A = 2x = 2 \times 20^\circ = 40^\circ$$

10. (d) Let ABC be a triangle,

$$\angle A + \angle B = \angle C$$

We know that,

$$(\angle A + \angle B + \angle C) = 180^\circ$$

$$\Rightarrow \angle C + \angle C = 180^\circ$$

$$\Rightarrow 2\angle C = 180^\circ$$

$$\Rightarrow \angle C = 90^\circ$$

11. (c) Let the smallest angle = x then, other two angles = $2x$ and $3x$

$$\therefore x + 2x + 3x = 180^\circ$$

$$\Rightarrow 6x = 180^\circ$$

$$\Rightarrow x = 30^\circ$$

Hence, Smallest angle = 30°

12. (c) Let, $2\angle A = 3\angle B = 6\angle C = K$

$$\therefore \angle A = \frac{K}{2}, \angle B = \frac{K}{3}, \angle C = \frac{K}{6}$$

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$\frac{K}{2} + \frac{K}{3} + \frac{K}{6} = 180^\circ \Rightarrow K = 180^\circ$$

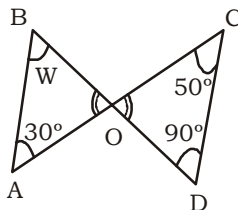
$$\therefore \angle A = \frac{180^\circ}{2} = 90^\circ$$

13. (d) By option (d)

$$\therefore (6 + 7 + 8)x = 180^\circ$$

$$\Rightarrow x = \frac{180^\circ}{21} \neq \text{integer}$$

14. (b)



In $\triangle OCD$,

$$\therefore \angle OCD + \angle CDO + \angle COD = 180^\circ$$

$$\therefore 50^\circ + 90^\circ + \angle COD = 180^\circ$$

$$\Rightarrow \angle COD = 180^\circ - 140^\circ = 40^\circ$$

$$\angle COD = \angle BOA = 40^\circ$$

(Vertically opposite angle)

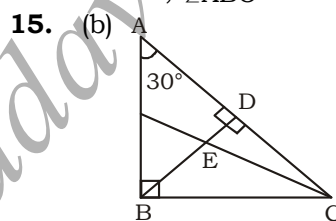
In $\triangle BOA$,

$$\therefore \angle BOA + \angle OAB + \angle ABO = 180^\circ$$

$$\therefore 40^\circ + 30^\circ + \angle ABO = 180^\circ$$

$$\Rightarrow \angle ABO = 180^\circ - 70^\circ = 110^\circ$$

Hence, $\angle ABO = w = 110^\circ$



In $\triangle ABC$,

$$\angle B = 90^\circ, \angle A = 30^\circ$$

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$\therefore 30^\circ + 90^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 120^\circ = 60^\circ$$

$\therefore$ CE Bisects $\angle BCD$ (given)

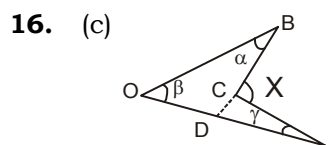
$$\therefore \angle ECD = \frac{60^\circ}{2} = 30^\circ$$

In $\triangle CDE$,

$$\therefore \angle CED + \angle EDC + \angle ECD = 180^\circ$$

$$\therefore \angle CED + 90^\circ + 30^\circ = 180^\circ$$

$$\Rightarrow \angle CED = 180^\circ - 120^\circ = 60^\circ$$



Produce BC to D,

$$\therefore \angle CDA = \alpha + \beta \text{ (exterior angle)}$$

In $\triangle ADC$, x is an exterior angle.

$$\therefore x = \angle CDA + \angle CAD$$

$$\Rightarrow x = \alpha + \beta + \gamma$$

17. (d) In $\triangle ABC$,

$$\therefore s + t + 50^\circ = 180^\circ$$

$$\Rightarrow s + t = 130^\circ$$

$$\Rightarrow t = 130^\circ - s$$

$$\therefore s < 50^\circ$$

$$\therefore t > 130^\circ - 50^\circ$$

$$\Rightarrow t > 80^\circ$$

18. (c) Let, ABC be a triangle
 $\angle A + \angle B = 80^\circ$... (i)
 $\angle A - \angle B = 20^\circ$... (ii)
 On adding equation (i) & (ii),
 $2\angle A = 100^\circ$

$$\Rightarrow \angle A = 50^\circ$$

$$\therefore \angle B = 80^\circ - \angle A$$

$$\angle B = 80^\circ - 50^\circ$$

$$\Rightarrow \angle B = 30^\circ$$

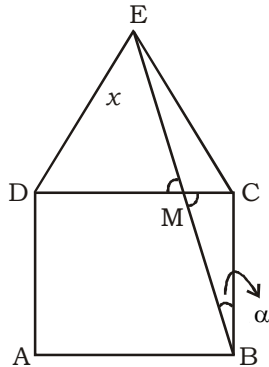
$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$\therefore \angle C = 180^\circ - (50^\circ + 30^\circ)$$

$$= 100^\circ$$

Hence, Smallest angle = $\angle B = 30^\circ$

19. (a) According to the question,
 $\angle BCD = 90^\circ$ ($\because$ each angle of square is 90°)
 $\angle ECD = 60^\circ$ ($\because$ each angle of an equilateral triangle is 60°)
 $\therefore \angle BCE = \angle BCD + \angle ECD$
 $= 90^\circ + 60^\circ = 150^\circ$



$$\therefore \angle EBC + \angle BEC + \angle BCE = 180^\circ$$

$$\therefore \angle EBC + \angle BEC = 180^\circ - 150^\circ$$

$$\Rightarrow \angle EBC + \angle BEC = 30^\circ$$

$$\Rightarrow 2\angle BEC = 30^\circ \quad (\because BC = EC)$$

$$\Rightarrow \angle BEC = 15^\circ$$

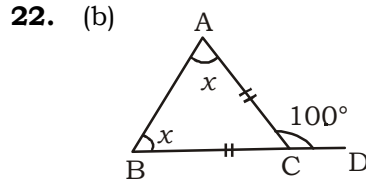
$$\therefore \angle DEC = 60^\circ$$

$$\therefore \angle DEM = 60^\circ - 15^\circ$$

$$\Rightarrow \angle DEM = x^\circ = 45^\circ$$

20. (a)
 $\therefore \angle B + \angle A = \angle ACD$
 (exterior angle)
 $\therefore 90^\circ + 30^\circ = \angle ACD$
 $\Rightarrow \angle ACD = 120^\circ$

21. (c) $y = 80^\circ$ (vertically opposite angle)
 $\therefore x = 180^\circ - 50^\circ - 80^\circ = 50^\circ$



- In $\triangle ABC$,
 $\therefore AC = BC$
 $\therefore \angle CAB = \angle ABC = x$
 $\angle ABC + \angle BAC = \angle ACD$
 (exterior angle)

$$\Rightarrow x + x = 100^\circ$$

$$\Rightarrow x = 50^\circ$$

Hence, $\angle ABC = x = 50^\circ$

23. (a) $\angle 1 = \angle A + \angle 5$ (i)
 and $\angle 2 = \angle A + \angle 6$ (ii)
 (exterior angle)
 $\Rightarrow \angle 1 + \angle 2 = \angle A + \angle 5 + \angle A + \angle 6$
 $\Rightarrow \angle 1 + \angle 2 = \angle A + (\angle A + \angle 5 + \angle 6)$
 $\Rightarrow \angle 1 + \angle 2 = \angle A + 180^\circ$
 ($\because \angle A + \angle 5 + \angle 6 = 180^\circ$)

24. (c) $a = 180^\circ - d = 180^\circ - 70^\circ$
 $= 110^\circ$
 and $c = d + \angle ACB$
 $= 70^\circ + (180^\circ - b)$
 $= 70^\circ + (180^\circ - 120^\circ)$
 $= 70^\circ + 60^\circ = 130^\circ$

25. (b) $\therefore \angle BCA = 180^\circ - Y$
 $\therefore 90^\circ + (180^\circ - y) = x$
 (exterior angle)

- $$\Rightarrow x + y = 270^\circ$$
26. (d) $\therefore \angle A : \angle B : \angle C = 3 : 2 : 1$
 $\therefore 3x + 2x + x = 180^\circ$
 $\Rightarrow 6x = 180^\circ \Rightarrow x = 30^\circ$
 $\therefore \angle C = x = 30^\circ$
 $\Rightarrow \angle ECD = 180^\circ - \angle ACB - \angle ACE$
 $\Rightarrow \angle ECD = 180^\circ - 90^\circ - 30^\circ = 60^\circ$

27. (b)
 $\therefore AD = BD$
 $\therefore \angle 1 = \angle 2 = x$ (let)
 $\therefore \angle 3 = \angle 1 + \angle 2 = 2x$ (external angle)
 $\therefore \angle 4 = \angle 3 = 2x$
 ($\because AD = AC$)

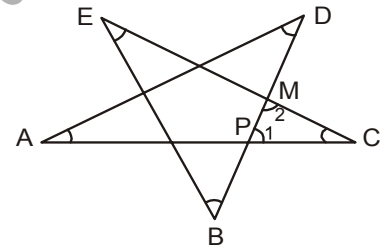
In $\triangle ABC$,
 71° is an external angle.
 $\therefore \angle 1 + \angle 4 = 71^\circ$

$$\Rightarrow x + 2x = 71^\circ \Rightarrow x = \frac{71^\circ}{3}$$

$$\therefore \angle C = \angle 4 = 2x = \frac{142^\circ}{3}$$

28. (a) In $\triangle AEC$,
 $\angle A + \angle E + \angle C = 180^\circ$... (i)
 In $\triangle BFD$,
 $\angle B + \angle F + \angle D = 180^\circ$... (ii)
 On Adding (i) and (ii), we get
 $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360^\circ$

29. (c) $\angle 2 = \angle E + \angle B$ (exterior angle of $\triangle MEB$)
 and $\angle 1 = \angle A + \angle D$ (exterior angle of $\triangle APD$)



In $\triangle PMC$,
 $\angle 1 + \angle 2 + \angle C = 180^\circ$
 $\therefore \angle A + \angle B + \angle C + \angle D + \angle E = 180^\circ$

30. (a) $\angle DAC = 180^\circ - 108^\circ = 72^\circ$
 Let, $\angle DAB = y^\circ$
 $\therefore \angle BAC = 3y^\circ$
 $\therefore y + 3y = 72^\circ \Rightarrow y = 18^\circ$
 Now, $DB = AB$
 $\Rightarrow \angle ADB = \angle DAB = y^\circ = 18^\circ$
 Now, in $\triangle ACD$, $\angle CAE$ is exterior angle
 $\therefore 108^\circ = \angle ADC + x^\circ$
 $108^\circ = 18^\circ + x$
 $x = 90^\circ$

Alternate

$$\therefore \angle DAC = 180^\circ - 108^\circ = 72^\circ$$

$$\angle DAB : \angle BAC = 1 : 3 \text{ (given)}$$

$$\therefore \angle DAB = y \text{ and } \angle BAC = 3y$$

$$\angle DAB + \angle BAC = 72^\circ$$

$$\Rightarrow y + 3y = 72^\circ$$

$$\begin{aligned}
 \Rightarrow 4y &= 72^\circ \\
 \Rightarrow y &= 18^\circ \\
 \therefore DB &= AB \text{ (given)} \\
 \Rightarrow \angle ADB &= \angle DAB = y = 18^\circ \\
 \therefore \angle BAC &= 3y = 3 \times 18^\circ = 54^\circ \\
 \text{and } \angle ABC &= 2y = 2 \times 18^\circ = 36^\circ \\
 \text{In } \triangle ABC, \\
 \angle ABC + \angle BAC + \angle ACB &= 180^\circ \\
 \Rightarrow 36^\circ + 54^\circ + \angle ACB &= 180^\circ \\
 \Rightarrow \angle ACB &= 90^\circ \\
 \text{Hence, } x &= 90^\circ
 \end{aligned}$$

31. (d)

$$\begin{aligned}
 \angle ACD &= \angle ABC + \angle BAC \\
 \Rightarrow 108^\circ &= \frac{\angle A}{2} + \angle A \\
 \Rightarrow \frac{3\angle A}{2} &= 108^\circ \\
 \Rightarrow \angle A &= \frac{108 \times 2}{3} = 72^\circ
 \end{aligned}$$

32. (c)

$$\begin{aligned}
 x + y &= 180^\circ - 96 = 84 \quad \dots(i) \\
 \text{Also for } \triangle CDB, \\
 4x + y &= 180^\circ \quad \dots(ii) \\
 \text{Subtract eqn. (i) from eqn. (ii),} \\
 3x &= 96 \text{ or } x = 32^\circ \\
 \therefore \angle DBC &= 2x = 64^\circ
 \end{aligned}$$

33. (d) As we know that,

$$\begin{aligned}
 \Rightarrow \angle A + \angle B + \angle C &= 180^\circ \\
 \Rightarrow \angle C &= 180^\circ - 70^\circ - 40^\circ \\
 \Rightarrow \angle C &= 70^\circ
 \end{aligned}$$

$$\therefore \angle ACD = 180^\circ - 70^\circ = 110^\circ$$

Alternate

As we know that

$$\begin{aligned}
 \angle ACD &= \angle A + \angle B \\
 \therefore \angle ACD &= 40^\circ + 70^\circ = 110^\circ
 \end{aligned}$$

34. (a) According to the question,

$$\begin{aligned}
 \angle CAG &= \angle ABC + \angle ACB = 120^\circ \\
 (\because \angle CAG \text{ is exterior angle}) \\
 \Rightarrow \angle ACB &= 120^\circ - 40^\circ \\
 \Rightarrow \angle ACB &= 80^\circ \\
 \therefore \angle ECM &= 180^\circ - 80^\circ = 100^\circ \\
 \angle EMD &= \angle ECM + \angle CEM \\
 \Rightarrow \angle EMD &= 100 + 25^\circ \\
 \therefore \angle EMD &= 125^\circ
 \end{aligned}$$

35. (c) According to the question,

$$\begin{aligned}
 \angle ACD &= 170^\circ \\
 \therefore \frac{\angle B}{\angle A} &= \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{Let, } \angle A &= 3x \text{ \& } \angle B = 2x \\
 \angle ACD &= \angle A + \angle B \\
 3x + 2x &= 170^\circ \\
 5x &= 170^\circ \\
 x &= 34^\circ \\
 \therefore \angle A &= 3x = 3 \times 34 = 102^\circ
 \end{aligned}$$

36. (c) According to the question,

$$\begin{aligned}
 \therefore \angle ABD &= 111^\circ \\
 \& \angle ACE &= 148^\circ \\
 \therefore \angle ACB &= 180^\circ - 148^\circ = 32^\circ \\
 \& \angle ABC &= 180^\circ - 111^\circ = 69^\circ \\
 \text{In } \triangle ABC, \\
 \therefore \angle A + \angle B + \angle C &= 180^\circ \\
 \therefore \angle A &= 180^\circ - 69^\circ - 32^\circ \\
 \text{Hence, } \angle A &= 79^\circ
 \end{aligned}$$

37. (d) According to the question,

$$\begin{aligned}
 \angle ACD &= x = \angle A + \angle B \\
 (\because \angle ACD \text{ is an exterior angle}) \\
 &= 75^\circ + 45^\circ \\
 \angle ACD &= 120^\circ \\
 \therefore \frac{x}{3} \% \text{ of } 60^\circ &= \frac{120}{3} \% \text{ of } 60^\circ \\
 &= 40\% \text{ of } 60^\circ \\
 &= \frac{40}{100} \times 60^\circ = 24^\circ
 \end{aligned}$$

38. (b) $\angle PSQ = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$

In $\triangle PSR$,
 $\therefore \angle PSR = \angle RPS$
 $\therefore PR = RS$

39. (c)

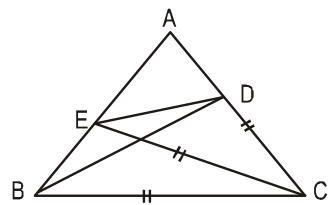
In $\triangle BDC$, $\angle 1$ is an external angle

$$\begin{aligned}
 \therefore \angle 1 &= 63^\circ + x \\
 \text{and in } \triangle AMC, \angle 2 &\text{ is an external angle} \\
 \therefore \angle 2 &= x + y \\
 \therefore AM &= AD \text{ (given)} \\
 \therefore \angle 1 &= \angle 2 \\
 \Rightarrow 63^\circ + x &= x + y \\
 \Rightarrow y &= 63^\circ
 \end{aligned}$$

$\therefore \angle MAC = 63^\circ$

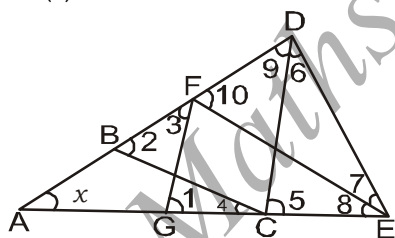
40. (c) $\therefore BP = BR$
 $\therefore \angle BPR = \angle BRP = x$ (let)
 and $CN = RC$
 $\therefore \angle CRN = \angle RNC = y$ (let)
 $\therefore \angle PBR = 180^\circ - 2x$ and
 $\angle NCR = 180^\circ - 2y$
 In $\triangle ABC$,
 $\angle A + \angle B + \angle C = 180^\circ$
 $\Rightarrow 44^\circ + 180^\circ - 2x + 180^\circ - 2y$
 $= 180^\circ$
 $\Rightarrow x + y = 112^\circ$
 Hence, $\angle PRN = 180^\circ - (x + y)$
 $= 180^\circ - 112^\circ = 68^\circ$

41. (b)



$\therefore BC = EC$
 $\therefore \angle CEB = \angle ECB = 78^\circ$
 In $\triangle BCD$,
 $\therefore CD = BC$
 $\therefore \angle DBC = \angle CDB$
 $\therefore \angle CDB + \angle DBC + \angle DCB = 180^\circ$
 $\Rightarrow 2\angle CDB + 78^\circ = 180^\circ$
 $\Rightarrow \angle CDB = 51^\circ$
 In $\triangle BEC$,
 $\therefore EC = BC$
 $\therefore \angle EBC = \angle BEC = 78^\circ$
 $\angle EBC + \angle BEC + \angle ECB = 180^\circ$
 $\Rightarrow 78^\circ + 78^\circ + \angle ECB = 180^\circ$
 $\Rightarrow \angle ECB = 24^\circ$
 $\therefore \angle ACB = 78^\circ$
 $\therefore \angle DCE + \angle ECB = 78^\circ$
 $\Rightarrow \angle DCE = 54^\circ$
 In $\triangle EDC$,
 $\therefore DC = EC$
 $\therefore \angle EDC = \angle DEC$
 $\angle DCE + \angle EDC + \angle DEC = 180^\circ$
 $2\angle EDC = 180^\circ - 54^\circ = 126^\circ$
 $\angle EDC = 63^\circ$
 $\therefore \angle EDC = \angle EDB + \angle CDB$
 $\Rightarrow 63^\circ = \angle EDB + 51^\circ$
 Hence, $\angle EDB = 12^\circ$

42. (c)



$\therefore AB = BC$
 $\therefore \angle 4 = x$
 $\therefore \angle 2 = x + \angle 4 = 2x$
 (exterior angle) [and $BC = CD$]
 $\therefore \angle 9 = \angle 2 = 2x$
 $\therefore \angle 3 = x$ ($\therefore FG = GA$)
 $\therefore \angle 1 = x + \angle 3 = 2x$
 (exterior angle)
 $\therefore EF = FG$ $\angle 8 = \angle 1 = 2x$
 $\angle 5 = \angle A + \angle 9 = x + 2x$

$= 3x$ (exterior angle)

$\therefore CD = DE$

$\therefore \angle 7 + \angle 8 = \angle 5$

$\Rightarrow \angle 7 = 3x - 2x = x$

$\angle 10 = \angle A + \angle 8 = 3x$

(exterior angle)

$\therefore DE = EF$

$\therefore \angle 9 + \angle 6 = \angle 10$

$\Rightarrow \angle 6 = 3x - 2x = x$

Now in $\triangle ADE$,

$\Rightarrow \angle A + \angle D + \angle E = 180^\circ$

$\Rightarrow \angle x + 3x + 3x = 180^\circ$

$\Rightarrow x = \frac{180^\circ}{7}$

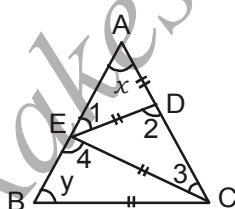
Alternate:-

If the figure is given in the form of zig-zag way then calculate how many sides are equal. In above figure seven sides are equal.

$$\therefore \angle DAE = \frac{180^\circ}{\text{No. of sides equal}}$$

$$= \frac{180^\circ}{7}$$

43. (a)



$\therefore AD = DE$

$\therefore \angle 1 = x$

$\therefore \angle 2 = \angle 1 + x = 2x$

(exterior angle)

$\therefore DE = EC$

$\therefore \angle 3 = \angle 2 = 2x$

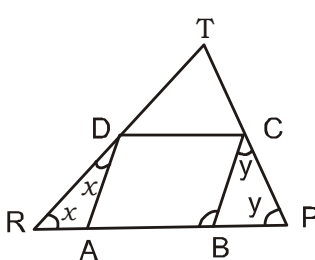
In $\triangle AEC$, $\angle 4$ is an exterior angle.

$\therefore \angle 4 = x + \angle 3 = 3x$

$\therefore y = \angle 4 = 3x$ ($\therefore EC = BC$)

$\therefore \angle A : \angle B = x : y = x : 3x$
 $= 1 : 3$

44. (b)



Let $\angle ARD = x$ and $\angle BPC = y$

$\angle ARD = \angle RDA$ ($\therefore AR = AD$)

$\therefore \angle DAB = \angle ARD + \angle RDA$

$= x + x = 2x$ (exterior angle)

Similarly,

$\angle BPC = \angle BCP = y$

$\therefore \angle ABC = y + y = 2y$

(exterior angle)

$\therefore 2x + 2y = 180^\circ$ ($\therefore ABCD$ is a rhombus)

$x + y = 90^\circ$

Now in $\triangle RTP$,

$\angle RTP = 180^\circ - (x + y)$

$= 180^\circ - 90^\circ$

$= 90^\circ$

45. (c) $\therefore BD = CD$

$\therefore \angle DBC = \angle BCD = 27^\circ$

In $\triangle BDC$,

$\therefore \angle BDC + \angle DBC + \angle BCD = 180^\circ$

$\angle BDC + 27^\circ + 27^\circ = 180^\circ$

$\angle BDC = 126^\circ$

$\angle ADC = 180^\circ - 126^\circ = 54^\circ$

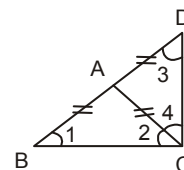
In $\triangle ADC$,

$\therefore CD = AC$

$\therefore \angle ADC = \angle CAD$

$\angle ACD = y^\circ = 180^\circ - (54^\circ + 54^\circ)$
 $= 72^\circ$

46. (b)



$\angle 1 = \angle 2$ ($\therefore AB = AC$)

and $\angle 3 = \angle 4$

($\therefore AB = AC = AD$)

In $\triangle BDC$,

$\angle B + \angle C + \angle D = 180^\circ$

$\Rightarrow \angle 1 + \angle 2 + \angle 4 + \angle 3 = 180^\circ$

$\Rightarrow \angle 2 + \angle 2 + \angle 4 + \angle 4 = 180^\circ$

$\Rightarrow 2(\angle 2 + \angle 4) = 180^\circ$

$\Rightarrow \angle 2 + \angle 4 = 90^\circ \Rightarrow \angle C = 90^\circ$

$\therefore \angle BCD = 90^\circ$

47.

(b) According to the question,

$BC = AC = AD$

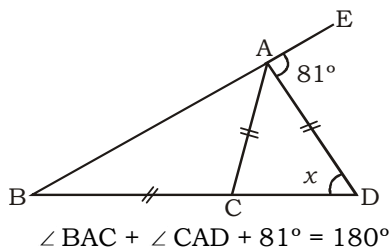
$\angle EAD = 81^\circ$

$\angle ACD = \angle ADC = x$

$\angle CAD = 180^\circ - 2x$

$\angle ABC = \angle BAC = \frac{x}{2}$

[$\therefore \angle ACD = \angle BAC + \angle ABC$]

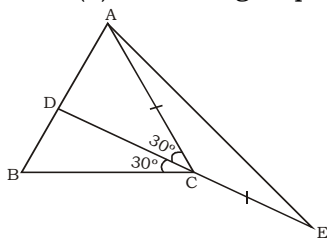


$$\frac{x}{2} + 180^\circ - 2x + 81 = 180^\circ$$

$$-\frac{3x}{2} = -81$$

$$\therefore x = 54^\circ$$

48. (d) According to question,

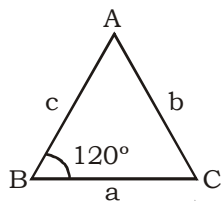


$$\begin{aligned} \therefore \angle A &= \angle B = \angle C = 60^\circ \\ \angle ACE &= 180^\circ - 30^\circ = 150^\circ \\ \angle CAE &= \angle CEA \quad (\because CE = AC) \end{aligned}$$

In $\triangle CAE$,

$$\begin{aligned} \angle CAE + \angle CEA + \angle ACE &= 180^\circ \\ \Rightarrow 2\angle CAE &= 180^\circ - 150^\circ \\ \Rightarrow \angle CAE &= 15^\circ \end{aligned}$$

49. (a) By using cosine formula,
 $b^2 = a^2 + c^2 - 2ac \cos B$

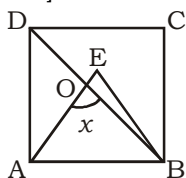


$$b^2 = a^2 + c^2 - 2ac \cos 120^\circ$$

$$b^2 = a^2 + c^2 - 2ac \times \left(-\frac{1}{2}\right)$$

$$b^2 = a^2 + c^2 + ac$$

50. (c) According to the question,
 $\triangle ABE$ is an equilateral triangle
 $\angle BAE = \angle ABE = \angle AEB = 60^\circ$
 As we know that,
 $\angle ABD = 45^\circ$ [$\because$ BD is diagonal of a square]

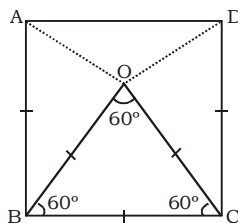


In $\triangle ABO$,

$$\begin{aligned} \angle BAO + \angle ABO + \angle AOB &= 180^\circ \\ 60 + x + 45^\circ &= 180 \\ x &= 75^\circ \end{aligned}$$

51. (c) According to the question,
 $\angle BCD = 90^\circ$
 $\angle OBA = \angle DCO = 90^\circ - 60^\circ = 30^\circ$
 In $\triangle OBA$,
 $\therefore OB = AB$

$$\therefore \angle BOA = \angle BAO = \frac{180^\circ - 30^\circ}{2} = 75^\circ$$



Same in $\triangle COD$

$$\begin{aligned} \angle COD &= \angle ODC = 75^\circ \\ \therefore \angle DOA &= 360^\circ - 75^\circ - 75^\circ - 60^\circ \\ \Rightarrow \angle DOA &= 150^\circ \end{aligned}$$

52. (a) According to the question,
 $\angle BDA = 140^\circ$
 $\angle CEA = 120^\circ$
 $\therefore \angle EAC = 2 \times \angle BAD$

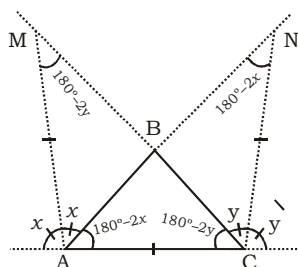
$$\Rightarrow \frac{\angle EAC}{\angle BAD} = \frac{2}{1}$$

$$\begin{aligned} \angle ADE &= 180^\circ - 140^\circ = 40^\circ \\ \angle AED &= 180^\circ - 120^\circ = 60^\circ \\ \angle DAE &= 180^\circ - 40^\circ - 60^\circ \\ \angle DAE &= 80^\circ \\ \therefore \angle EAC &= 20^\circ \\ \angle BAD &= 10^\circ \end{aligned}$$

In $\triangle BDA$,

$$\angle ABD = 180^\circ - 10^\circ - 140^\circ = 30^\circ$$

53. (a) According to the question,



$$\begin{aligned} \text{In } \triangle MAC, \\ \Rightarrow 180^\circ - 2y + 180^\circ - 2y + x + 180^\circ - 2x &= 180^\circ \\ x + 4y &= 360^\circ \quad \dots(i) \end{aligned}$$

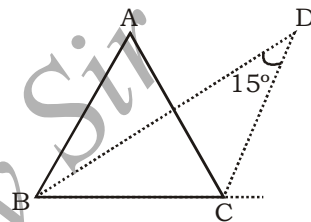
$$\begin{aligned} \text{In } \triangle NAC, \\ 180^\circ - 2x + 180^\circ - 2x + 180^\circ - 2y + y &= 180^\circ \\ 4x + y &= 360^\circ \quad \dots(ii) \end{aligned}$$

Solve Eq. (i) and (ii), We get
 $x + y = 144^\circ$

In $\triangle ABC$,

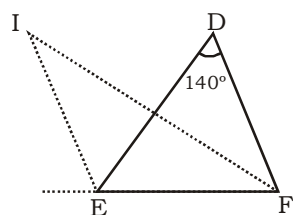
$$\begin{aligned} \therefore \angle A + \angle B + \angle C &= 180^\circ \\ \angle B + 180^\circ - 2x + 180^\circ - 2y &= 180^\circ \\ \Rightarrow \angle B &= 2(x + y) - 180^\circ \\ \Rightarrow \angle B &= 288^\circ - 180^\circ \\ \Rightarrow \angle B &= 108^\circ \end{aligned}$$

54. (c)



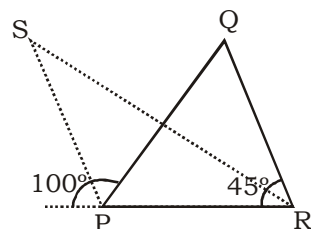
$$\begin{aligned} \therefore \angle BDC &= 15^\circ \\ \therefore \angle BAC &= 2 \times \angle BDC \\ &= 2 \times 15^\circ = 30^\circ \end{aligned}$$

55. (b)



$$\begin{aligned} \therefore \angle EDF &= 140^\circ \\ \therefore \angle EIF &= \frac{\angle EDF}{2} = \frac{140^\circ}{2} = 70^\circ \end{aligned}$$

56. (a)



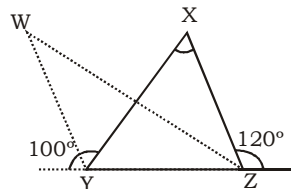
$$\angle QPR = 180^\circ - 100^\circ = 80^\circ$$

In $\triangle PQR$

$$\begin{aligned} \therefore \angle P + \angle Q + \angle R &= 180^\circ \\ \Rightarrow 80^\circ + \angle Q + 45^\circ &= 180^\circ \\ \Rightarrow \angle Q &= 55^\circ \end{aligned}$$

$$\angle PSR = \frac{\angle Q}{2} = \frac{55}{2} = 27.5^\circ$$

57. (d)



$$\angle YZX = 180^\circ - 120^\circ = 60^\circ$$

$$\angle XYZ = 180^\circ - 100^\circ = 80^\circ$$

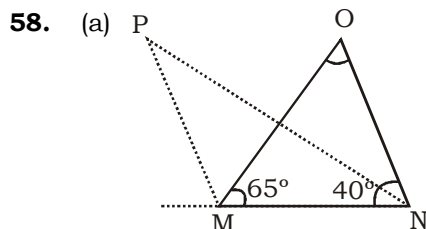
In Δxyz ,

$$\angle x + \angle y + \angle z = 180^\circ$$

$$\angle x + 80^\circ + 60^\circ = 180^\circ$$

$$\angle x = 40^\circ$$

$$\therefore \angle YWZ = \frac{40}{2} = 20^\circ$$



In ΔMNO

$$\angle M + \angle N + \angle O = 180^\circ$$

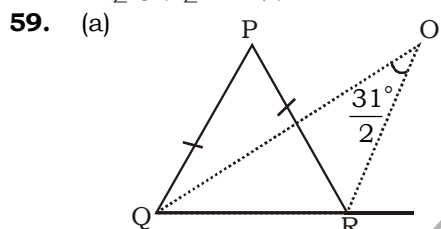
$$65^\circ + 40^\circ + \angle O = 180^\circ$$

$$\angle O = 75^\circ$$

$$\angle P = \frac{\angle O}{2} = \frac{75^\circ}{2}$$

$$\text{Required Ratio} = \frac{\angle O}{\angle P} = \frac{75}{\frac{75}{2}} \times 2 = \frac{2}{1}$$

$$\therefore \angle O : \angle P = 2 : 1$$



$$\angle P = 2 \times \angle O = 2 \times \frac{31}{2} = 31^\circ$$

In ΔPQR ,

$$\angle P + \angle Q + \angle R = 180^\circ$$

$$\left[\begin{array}{l} \because PQ = PR \\ \therefore \angle Q = \angle R \end{array} \right]$$

$$31^\circ + \angle Q + \angle Q = 180^\circ$$

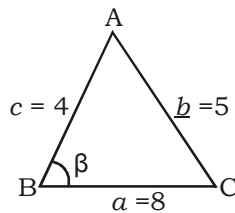
$$\Rightarrow 2\angle Q = 149^\circ$$

$$\Rightarrow \angle Q = 74.5^\circ$$

$\therefore$ External angle of $\angle Q$

$$= 180^\circ - 74.5^\circ = 105.5^\circ$$

60. (a)



By using Cosine formula,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\Rightarrow \cos \beta = \frac{64 + 16 - 25}{2 \times 8 \times 4}$$

$$\Rightarrow \cos \beta = \frac{55}{64}$$

61. (a) By using Cosine formula,

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\Rightarrow \cos 60^\circ = \frac{5^2 + 8^2 - b^2}{2 \times 5 \times 8}$$

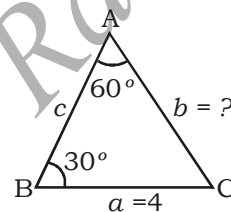
$$\Rightarrow \frac{1}{2} = \frac{25 + 64 - b^2}{80}$$

$$\Rightarrow 40 = 89 - b^2$$

$$\Rightarrow b^2 = 49$$

$$\Rightarrow b = 7$$

62. (a) Using sine formula,



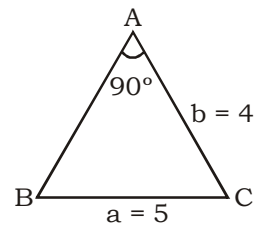
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{4}{\sin 60^\circ} = \frac{AC}{\sin 30^\circ}$$

$$\Rightarrow \frac{4 \times 2}{\sqrt{3}} = \frac{AC \times 2}{1}$$

$$\Rightarrow AC = \frac{4}{\sqrt{3}}$$

63. (a)



Using sine formula,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\Rightarrow \frac{5}{\sin 90^\circ} = \frac{4}{\sin B}$$

$$\Rightarrow \frac{5}{1} = \frac{4}{\sin \beta}$$

$$\Rightarrow \sin \beta = \frac{4}{5}$$

64. (d) According to the sine formula,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k \text{ (Let)}$$

$$\frac{a}{\sin 60^\circ} = \frac{b}{\sin 90^\circ} = \frac{c}{\sin 30^\circ} = k$$

$$a = k \sin 60^\circ = k \times \frac{\sqrt{3}}{2}$$

$$b = k \sin 90^\circ = k \times 1$$

$$c = k \sin 30^\circ = k \times \frac{1}{2}$$

$$a : b : c$$

$$\frac{\sqrt{3}}{2}k : k : \frac{k}{2}$$

$$\sqrt{3} : 2 : 1$$

65. (b) By using cosine formula,

$$\cos \theta = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \cos \theta = \frac{49 + 48 - 13}{2 \times 7 \times 4\sqrt{3}}$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = 30^\circ$$

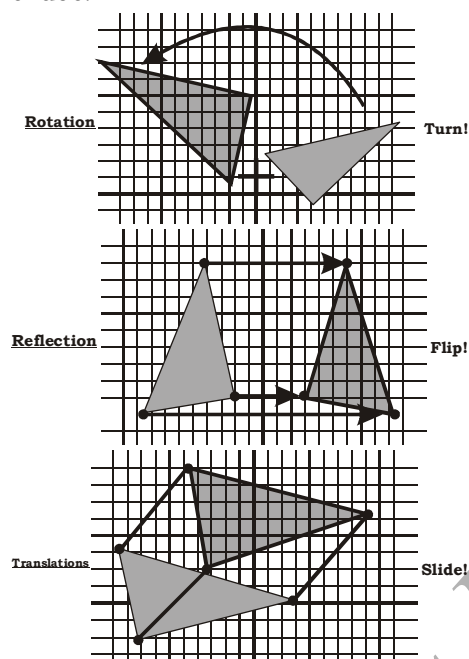


TRIANGLES CONGRUENCE & SIMILARITY

Congruent Triangles

Triangles are congruent when they have exactly the **same three sides** and exactly the **same three angles**.
What is "Congruent" ?

It means that one shape can become another using turns, flips and/or slides:

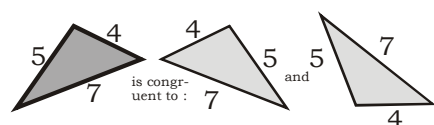


The equal sides and angles may not be in the same position (if there is a turn or a flip), but they are there.

Same sides

When the sides are the same then the triangles are congruent.

For example :

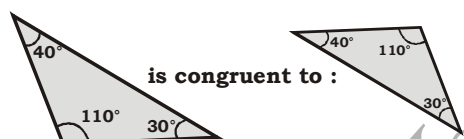


Because the two triangles do not have exactly the same sides.

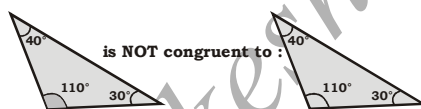
Same Angles

Does this also work with angles? Not always!

Two triangles with the same angles **might be** congruent;



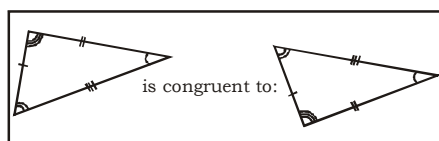
Only because they are the same size
But they might **NOT** be congruent because of **different sizes**:



because, even though all angles match, **one is larger than the other**.
So just having the same angles is no guarantee they are congruent.

Marking

When two triangles are congruent we often mark corresponding sides and angles like this;



These sides marked one are equal in length. Similarly for the sides marked with two lines. Also for the sides marked with three lines.

The angles marked with one arc are equal in size. Similarly for the angles marked with two arcs. Also for the angles marked with three arcs.

Congruence Tests for Triangles

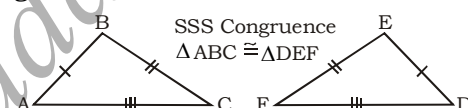
SSS, SAS, ASA, AAS and HL. These tests describe combinations of congru-

ent sides and/or angles that are used to determine if two triangles are congruent.

1. SSS Congruence →

Side-Side-Side congruence.

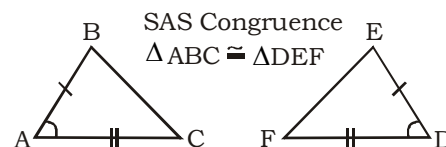
When two triangles have corresponding sides equal that are congruent as shown below, the triangles are congruent.



2. SAS Congruence →

Side-Angle-Side congruence.

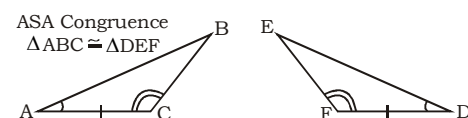
When two triangles have corresponding angles and sides equal that are congruent as shown below, the triangles are congruent.



3. ASA Congruence →

Angle-side-angle congruence.

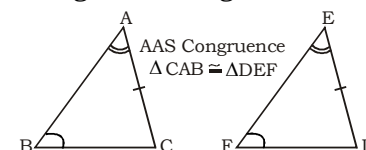
When two triangles have corresponding angles and sides equal that are congruent as shown below, the triangles themselves are congruent.



4. AAS Congruence OR SAA Congruence →

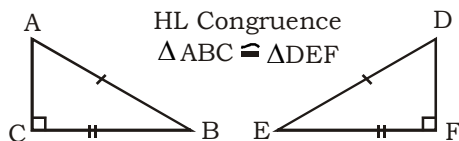
Angle-Angle-Side congruence.

When two triangles have corresponding angles and sides equal that are congruent as shown below, the triangles are congruent.



5. HL Congruence →

Hypotenuse-leg congruence. When two right triangles have corresponding sides equal that are congruent as shown below, the triangles are congruent.



Similarity of Triangles

Two triangles are **similar** if and only if the corresponding sides are in proportion and the corresponding angles are congruent.

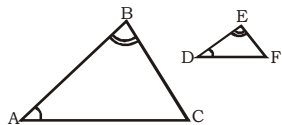
There are three accepted methods of proving triangles similar:

AA

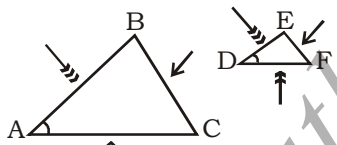
To show two triangles are similar, it is sufficient to show that two angles of one triangle are congruent (equal) to two angles of the other triangle.

Theorem

If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.



If $\angle A = \angle D$
 $\angle B = \angle E$
 Then: $\triangle ABC \sim \triangle DEF$



If $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$
 Then: $\triangle ABC \sim \triangle DEF$

SSS for Similarity

BE CAREFUL!! SSS for similar triangles is NOT the same theorem as we used for congruent triangles. To show triangles are similar, it is sufficient to show that the three sets of corresponding sides are in proportion.

Theorem

If the three sets of corresponding sides

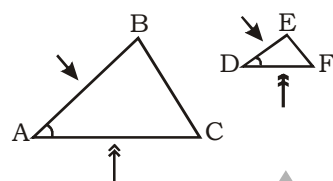
of two triangles are in proportion, the triangles are similar.

SAS for similarity

BE CAREFUL!! SAS for similar triangles is NOT the same theorem as we used for congruent triangles. To show triangles are similar, it is sufficient to show that two sets of corresponding sides are in proportion and the angles they include are congruent.

Theorem

If an angle of one triangle is congruent to the corresponding angle of another triangle and the lengths of the sides including these angles are in proportion, the triangles are similar.

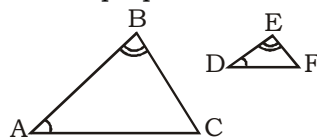


If $\angle A = \angle D$
 $\frac{AB}{DE} = \frac{AC}{DF}$
 Then: $\triangle ABC \sim \triangle DEF$

Once the triangles are similar

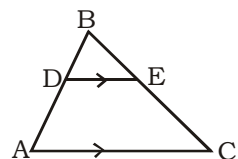
Theorem :

The corresponding sides of similar triangles are in proportion.



Then: $\triangle ABC \sim \triangle DEF$
 Then: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Dealing with overlapping triangles : Many problems involving similar triangles have one triangle ON TOP of (overlapping) another triangle. Since DE is marked to be parallel to AC, we know that we have $\angle BDE$ congruent to $\angle DAC$ by corresponding angles. $\angle B$ is shared by both triangles, so the two triangles are similar by AA



There is an additional theorem that can be used when working with overlapping triangles.

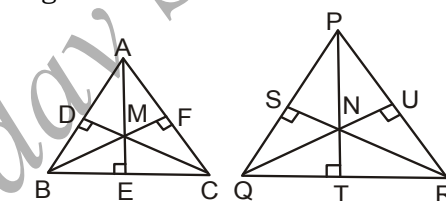
Additional Theorem :

Basic proportionality theorem (BPT) or Thales theorem

If a line is parallel to one side of a triangle and intersects the other two sides of the triangle, the line divides these two sides proportionally.

Properties of Similar triangles

If the two triangles are similar, then for the proportional/corresponding sides we have the following results.



1.

$\left\{ \begin{array}{l} \text{Ratio of} \\ \text{sides} \end{array} \right\} = \left\{ \begin{array}{l} \text{Ratio of height} \\ \text{(altitudes)} \end{array} \right\}$
 = Ratio of medians
 = Ratio of angle bisectors

tors

= Ratio of inradii
 = Ratio of circumradii

2. Ratio of areas = Ratio of squares of corresponding sides.

i.e. If $\triangle ABC \sim \triangle PQR$,

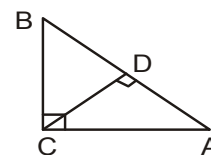
Then,

$$\frac{\text{Area}(\triangle ABC)}{\text{Area}(\triangle PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

* Perimeter of triangle is:-

$$\frac{\text{Perimeter}(\triangle ABC)}{\text{Perimeter}(\triangle PQR)} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

3. In a right angled triangle, the triangles on each side of the altitude drawn from the vertex of the right angle to the hypotenuse are similar to the original triangle and to each other too.



i.e., $\triangle BCA \sim \triangle BDC \sim \triangle CDA$.

Some facts on Right angle triangle

Some facts on Right angle triangle	
(A)	$CD^2 = BD \times DA$
(B)	$BC \times CA = BA \times CD$
(C)	$BC^2 = BD \times BA$
(D)	$AC^2 = AD \times BA$
(E)	$\frac{BD}{DA} = \frac{BC^2}{AC^2}$
(F)	$\frac{1}{CD^2} = \frac{1}{BC^2} + \frac{1}{CA^2}$

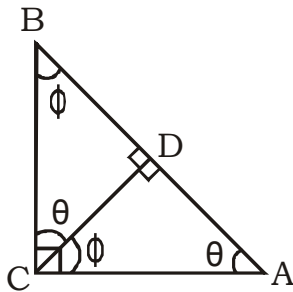
Explanation of these facts

$$\angle DCB + \angle DCA = 90^\circ$$

$$\angle DCB + \angle CBD = 90^\circ$$

$$\therefore \angle DCA = \angle DBC$$

$$\therefore \triangle BCA \sim \triangle BDC \sim \triangle CDA$$



A. If $\triangle BDC \sim \triangle CDA$

$$\frac{BD}{CD} = \frac{DC}{DA}$$

$$CD^2 = BD \times DA$$

B. Area of $\triangle ABC$

$$\Rightarrow \frac{1}{2} \times BC \times AC = \frac{1}{2} \times AB \times CD$$

$$BC \times AC = AB \times CD$$

$$CD = \frac{BC \times AC}{AB}$$

C. $\triangle BCA \sim \triangle BDC$

$$\frac{BC}{BD} = \frac{BA}{CB}$$

$$BC^2 = BA \times BD$$

D. $\triangle BCA \sim \triangle CDA$

$$\frac{BA}{CA} = \frac{CA}{DA}$$

$$AC^2 = AB \times AD$$

E. In below equation C divided by equation D

$$\frac{BC^2}{AC^2} = \frac{BD \times BA}{AD \times BA}$$

$$\frac{BD}{DA} = \frac{BC^2}{AC^2}$$

F. In eq. (ii) put the value of $AB = \sqrt{BC^2 + AC^2}$

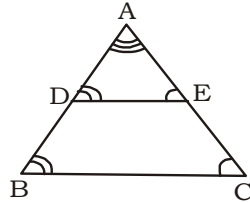
$$BC \times CA = CD (\sqrt{BC^2 + AC^2})$$

$$\text{Squaring both sides, } BC^2 \times CA^2 = CD^2 \times (BC^2 + AC^2)$$

$$CD^2 = \frac{BC^2 \times CA^2}{BC^2 + AC^2}$$

$$\frac{1}{CD^2} = \frac{1}{BC^2} + \frac{1}{AC^2}$$

4. If D and E are the mid-points of AB and AC & $DE \parallel BC$



$$\text{then, } DE = \frac{1}{2} BC$$

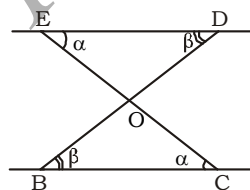
□ Vice versa is also true i.e. If a line divides any two sides in the same ratio

$$\left(\text{i.e. } \frac{AD}{DB} = \frac{AE}{EC} \right) \text{ then the line}$$

is parallel to third line

i.e. $DE \parallel BC$

5. Two triangles b/w the two parallel lines will always be similar.

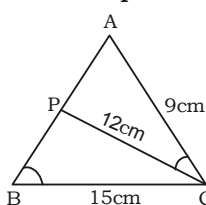


If $ED \parallel BC$

Then, $\triangle EOD \sim \triangle COB$

EXAMPLES

1. ABC is a triangle and P is any point on AB such that $\angle ACP = \angle ABC$, if $AC = 9\text{cm}$, $CP = 12\text{cm}$ and $BC = 15\text{cm}$, then AP is equal to :



In $\triangle APC$ and $\triangle ABC$,

$$\angle ACP = \angle ABC \text{ (Given)}$$

$$\angle A = \angle A \text{ (common)}$$

$$\therefore \triangle APC \sim \triangle ACB$$

$$\therefore \frac{AP}{AC} = \frac{PC}{BC} \Rightarrow \frac{AP}{9} = \frac{12}{15}$$

$$\Rightarrow AP = 7.2\text{cm}$$

2. If the three side of one triangle are equal to the corresponding sides of the other triangle then the triangle are :

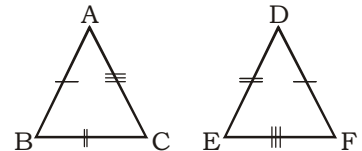
Sol. Here, triangles are congruent. And congruent triangles are always similar.

3. If $\triangle ABC$ and $\triangle DEF$ are so related that

$$\frac{AB}{FD} = \frac{BC}{DE} = \frac{CA}{EF}, \text{ then which of the following is true?}$$

- (a) $\angle A = \angle F$ and $\angle B = \angle D$
- (b) $\angle C = \angle F$ and $\angle A = \angle D$
- (c) $\angle B = \angle F$ and $\angle C = \angle D$
- (d) $\angle A = \angle E$ and $\angle B = \angle D$

Sol. (a) According to the question,



$$\frac{AB}{FD} = \frac{BC}{DE} = \frac{CA}{EF}$$

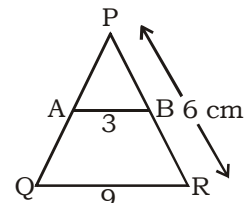
$$\angle C = \angle E$$

$$\angle B = \angle D$$

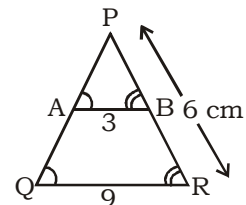
$$\angle A = \angle F$$

So, option (a) is true.

4. In the figure $AB \parallel QR$. Find the length of PB :



Sol. According to the question,



$$\triangle PQR \sim \triangle PAB$$

$$\frac{AB}{QR} = \frac{PB}{PR}$$

$$\frac{3}{9} = \frac{PB}{6}$$

$$PB = 2 \text{ cm}$$

5. In $\triangle ABC$ and $\triangle DEF$, If $\angle A = 50^\circ$, $\angle B = 70^\circ$, $\angle C = 60^\circ$, $\angle E = 70^\circ$, $\angle F = 50^\circ$, $\angle D = 60^\circ$ then :

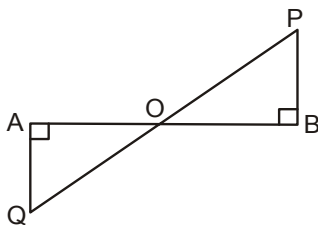
Sol. In $\triangle ABC$,
 $\angle A = 50^\circ$, $\angle B = 70^\circ$,
 $\angle C = 60^\circ$

and In $\triangle DEF$,
 $\angle E = 70^\circ$, $\angle F = 50^\circ$ and
 $\angle D = 60^\circ$

$\therefore$ In $\triangle FED$,
 $\angle F = 50^\circ$, $\angle E = 70^\circ$ and
 $\angle D = 60^\circ$

$\therefore \triangle ABC \sim \triangle FED$.

6. In the given figure, QA and PB are perpendiculars to AB. If AO = 9cm, BO = 6cm and BP = 8cm. Find AQ :

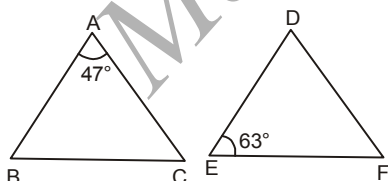


Sol. $\frac{AO}{BO} = \frac{AQ}{BP}$
 $\frac{9}{6} = \frac{AQ}{8}$

$$AQ = 12 \text{ cm}$$

7. If $\triangle ABC$ is similar to $\triangle DEF$, such that $\angle A = 47^\circ$ and $\angle E = 63^\circ$, then $\angle C$ is equal to :

Sol.



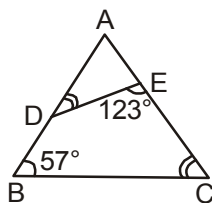
$\triangle ABC \sim \triangle DEF$

$\therefore \angle A = 47^\circ = \angle D$

$\angle B = \angle E = 63^\circ$

$\therefore \angle C = 180^\circ - 47^\circ - 63^\circ$
 $= 70^\circ$

8. In the given figure, AD = 11cm, AB = 18 cm and AE = 9 cm. Find EC :



Sol. $\triangle ADE \sim \triangle ACB$ (A-A property)
 $\therefore \angle A = \angle A$, $\angle AED = \angle ABC$ Then
 $\angle ADE = \angle ACB$

$$\therefore \frac{AE}{AB} = \frac{AD}{AC} \Rightarrow \frac{9}{18} = \frac{11}{AC}$$

$$\Rightarrow AC = 22 \text{ cm}$$

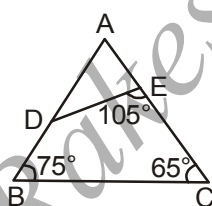
$$\therefore EC = AC - AE$$

$$= 22 - 9$$

$$= 13 \text{ cm}$$

9. In the given figure, if

$$\frac{DE}{BC} = \frac{2}{3} \text{ and } AE = 12. \text{ Find } AB:$$



Sol. In $\triangle ABC$ and $\triangle ADE$,
 $\angle BAC = \angle DAE$
 $= 180^\circ - (75^\circ + 65^\circ)$
 $= 40^\circ$

$\angle AED = 75^\circ = \angle ABC$

$\therefore \triangle AED \sim \triangle ABC$ (by AA)

$$\therefore \frac{DE}{BC} = \frac{AE}{AB} \Rightarrow \frac{2}{3} = \frac{12}{AB}$$

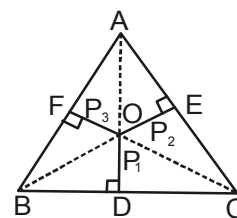
$$\Rightarrow AB = 18 \text{ cm}$$

10. The corresponding sides of two similar triangles are in the ratio 1 : 3. Their altitude will be in the ratio:

Sol. Ratio of altitude = Ratio of corresponding sides = 1 : 3

11. The lengths of perpendiculars drawn from any point in the interior of an equilateral triangle to the respective side of the triangle are P_1 , P_2 and P_3 , then the side of triangle is :

Sol.



Let the side of $\triangle ABC$ be a . O is the point in the interior of $\triangle ABC$.

OD, OE, OF are perpendiculars of BC, AC and AB

$\text{ar}(\triangle OAB) + \text{ar}(\triangle OBC) + \text{ar}(\triangle OAC) = \text{ar}(\triangle ABC)$

$$\Rightarrow \frac{1}{2}a \times P_3 + \frac{1}{2}a \times P_1 + \frac{1}{2}a \times P_2 = \frac{\sqrt{3}}{4}a^2$$

$$\Rightarrow \frac{1}{2}a(P_1 + P_2 + P_3) = \frac{\sqrt{3}}{4}a^2$$

$$\Rightarrow a = \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)$$

12. The lengths of perpendiculars drawn from any point in the interior of an equilateral triangle to the respective sides are 6cm, 8cm, and 10cm. The length of each side of the triangle is :

Sol. $a = \frac{2}{\sqrt{3}}(P_1 + P_2 + P_3)$

$$= \frac{2}{\sqrt{3}}(6 + 8 + 10)$$

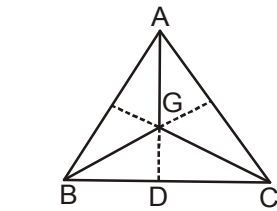
$$= \frac{48}{\sqrt{3}} = 16\sqrt{3} \text{ cm}$$

13. Two triangles ABC and DEF are similar to each other in which AB = 10cm, DE = 8cm. Then the ratio of the areas of triangles ABC and DEF is :

Sol. $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AB^2}{DE^2} = \frac{100}{64} = \frac{25}{16}$

14. If G be the centroid of $\triangle ABC$ and the area of $\triangle GBD$ is 6 sq.cm, where D is the mid-point of side BC, then the area of $\triangle ABC$ is :

Sol.



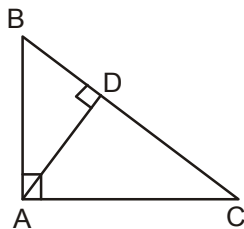
$$\begin{aligned}\text{Area of } \triangle ABC &= 6 \times \text{ar}(\triangle BGD) \\ &= 6 \times 6 = 36 \text{ sq. cm}\end{aligned}$$

- 15.** The perimeters of two similar triangles $\triangle ABC$ and $\triangle PQR$ are 36cm and 24cm respectively. If $PQ = 10$ cm, then AB is :

Sol. $\frac{AB}{PQ} = \frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle PQR} = \frac{36}{24}$

$$\Rightarrow AB = \frac{36}{24} \times 10 = 15 \text{ cm}$$

- 16.** Which of the following is true in the given figure, where AD is the altitude to the hypotenuse of a right angle triangle ABC ?



- (i) $\triangle ABD \sim \triangle CAD$
(ii) $\triangle ABD \cong \triangle CDA$
(iii) $\triangle ADB \sim \triangle CAB$

Of these statements, the correct ones are :

Sol. In $\triangle ABD$ and $\triangle CAD$
 $\angle ADB = \angle ADC = 90^\circ$ each
 $\angle BAD = \angle ACD = 90^\circ - \angle B$
and
 $AD = AD$ (common)
 $\therefore \triangle ADB \sim \triangle CAD$ and
 $\triangle ABD \cong \triangle CAD$

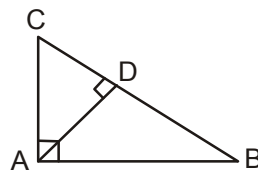
In $\triangle ADB$ and $\triangle CAB$
 $\angle ADB = \angle BAC = 90^\circ$ each
and $\angle ABC = \angle ABD$ (common)

$\therefore \triangle ADB \sim \triangle CAB$
Here, (i) and (iii) are correct statements.

- 17.** In a triangle ABC , $\angle BAC = 90^\circ$ and AD is perpendicular

to BC . If $AD = 6$ cm and $BD = 4$ cm, then the length of BC is:

Sol.



$$\begin{aligned}AB &= \sqrt{AD^2 + BD^2} = \sqrt{36 + 16} = \sqrt{52} \text{ cm} \\ \triangle ABC &\sim \triangle DBA\end{aligned}$$

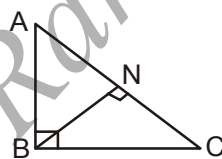
$$\begin{aligned}\therefore \frac{AB}{BC} &= \frac{BD}{AB} \\ \Rightarrow AB^2 &= BC \times BD \\ \Rightarrow 52 &= BC \times 4 \\ \Rightarrow BC &= 13 \text{ cm}\end{aligned}$$

Alternate

$$\begin{aligned}AD^2 &= BD \times CD \\ 6^2 &= 4 \times CD \\ CD &= 9 \text{ cm} \\ \therefore BC &= 9 + 4 = 13 \text{ cm}\end{aligned}$$

- 18.** In a right angled $\triangle ABC$, $\angle ABC = 90^\circ$; $BN \perp AC$, $AB = 6$ cm, $AC = 10$ cm. Then $AN : NC$ is :

Sol.



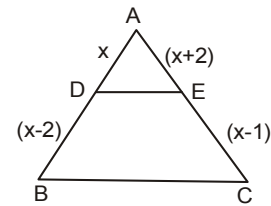
In $\triangle ABC$ & $\triangle BNC$,
 $\angle ABC = \angle BNC = 90^\circ$
and $\angle C = \angle C$ (common)
 $\therefore \triangle ABC \sim \triangle BNC$
and $BC = \sqrt{10^2 - 6^2} = 8$ cm

$$\begin{aligned}\therefore \frac{AC}{BC} &= \frac{BC}{NC} \\ \Rightarrow \frac{10}{8} &= \frac{8}{NC} \\ \Rightarrow NC &= 6.4 \\ \therefore AN &= 10 - 6.4 = 3.6 \\ \therefore AN : NC &= 3.6 : 6.4 = 9 : 16\end{aligned}$$

Alternate

$$\begin{aligned}\frac{AN}{NC} &= \frac{AB^2}{BC^2} = \frac{6^2}{8^2} \\ &= \frac{36}{64} = \frac{9}{16} \\ \therefore AN : NC &= 9 : 16\end{aligned}$$

- 19.** In the given figure, $DE \parallel BC$ then the value of x is :

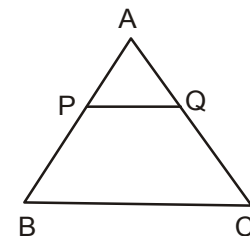


Sol. $\frac{AD}{DB} = \frac{AE}{EC}$ (by basic proportionality theorem: $\because DE \parallel BC$)

$$\begin{aligned}\Rightarrow \frac{x}{x-2} &= \frac{x+2}{x-1} \\ \Rightarrow x^2 - x &= x^2 - 4 \Rightarrow x = 4\end{aligned}$$

- 20.** In $\triangle ABC$, $PQ \parallel BC$, If $AP : PB = 1 : 2$ and $AQ = 3$ cm, AC is:

Sol.



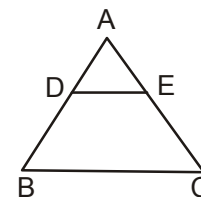
$$\begin{aligned}\frac{AP}{BP} &= \frac{AQ}{QC} = \frac{1}{2} \\ \Rightarrow QC &= 2AQ \\ \Rightarrow QC &= 2 \times 3 = 6 \\ \Rightarrow AC &= AQ + QC \\ &= 3 + 6 = 9 \text{ cm}\end{aligned}$$

- 21.** The points D and E are taken on the sides AB and AC

of $\triangle ABC$ such that $AD = \frac{1}{3} AB$,

$AE = \frac{1}{3} AC$. If the length of BC is 15 cm, then the length of DE is :

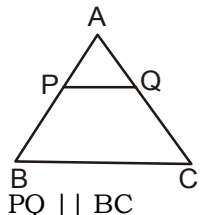
Sol.



$$\begin{aligned}\frac{AD}{AB} &= \frac{AE}{AC} = \frac{1}{3} \\ \therefore \triangle ABC &\sim \triangle ADE \\ \therefore \frac{DE}{BC} &= \frac{1}{3} \Rightarrow DE = \frac{15}{3} = 5 \text{ cm}\end{aligned}$$

- 22.** ABC is an equilateral triangle. P and Q are two points on $\overline{AB}$ and $\overline{AC}$ respectively such that $\overline{PQ} \parallel \overline{BC}$. If $\overline{PQ} = 5$ cm then the area of $\triangle APQ$ is:

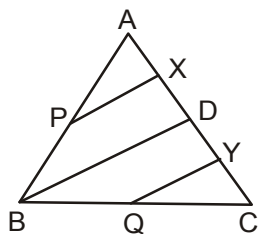
Sol.



$$\begin{aligned} \therefore \angle APQ &= \angle ABC = 60^\circ \\ \&\angle AQP &= \angle ACB = 60^\circ \\ \therefore \text{Area of } \triangle APQ &= \\ \frac{\sqrt{3}}{4} \times (PQ)^2 &= \\ = \frac{\sqrt{3}}{4} \times 25 &= \frac{25\sqrt{3}}{4} \text{ sq.cm} \end{aligned}$$

- 23.** D is any point on side AC of $\triangle ABC$. If P, Q, X & Y are the mid-points of AB, BC, AD and DC respectively, then the ratio of PX and QY is :

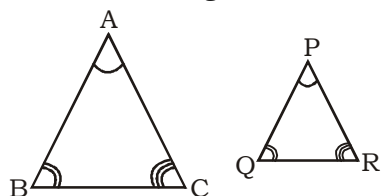
Sol. (By mid-point theorem)



$$\begin{aligned} PX \parallel BD \text{ and } PX &= \frac{1}{2} BD \\ QY \parallel BD \text{ and } QY &= \frac{1}{2} BD \\ \therefore PX : QY &= 1 : 1 \end{aligned}$$

Area Based question when two triangles are similar →

Condition for two triangle to be similar angle of two triangles are equal then the two triangles are similar



If $\triangle ABC \sim \triangle PQR$ Then

* The Ratio of their area is the square of their sides, Medians, Altitudes, Perimeters, Angle Bisectors.

$$\begin{aligned} \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} &= \left(\frac{AB}{PQ} \right)^2 = \left(\frac{AC}{PR} \right)^2 \\ &= \left(\frac{BC}{QR} \right)^2 = \left(\frac{\text{Angle Bisector}_1}{\text{Angle Bisector}_2} \right)^2 \\ &= \left(\frac{\text{Altitude}_1}{\text{Altitude}_2} \right)^2 = \left(\frac{\text{Median}_1}{\text{Median}_2} \right)^2 \\ &= \left(\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle PQR} \right)^2 \end{aligned}$$

- 24.** The corresponding sides of two similar triangle in the ratio 5 : 6. Their area will be in the ratio.

Sol. When triangle similar, Area of triangle = (side of triangle)²

$$\left(\frac{\text{Area of } \triangle_1}{\text{Area of } \triangle_2} \right) = \left(\frac{5}{6} \right)^2 = \frac{25}{36}$$

- 25.** Area of two similar triangle are respectively 196 cm² and 289 cm². If shortest side of the shorter triangle 7 cm. Then find the shorter side of the larger triangle.

Sol. In two similar triangle the ratio of their area is the square of the ratio of their sides.

- 26.** Area of two similar triangles are respectively 81 cm² & 121 cm². If altitude of first triangle is 4.5 cm. Find the corresponding altitude of the second triangle.

$$\begin{aligned} \frac{\text{Area of first triangle}}{\text{Area of second triangle}} &= \left(\frac{\text{Altitude of first } \triangle}{\text{Altitude of second } \triangle} \right)^2 \\ \left(\frac{81}{121} \right) &= \left(\frac{4.5}{M} \right)^2 \\ \frac{9}{11} &= \frac{4.5}{M} \\ M &= \frac{11}{2} \end{aligned}$$

Here M = (Altitude of second triangle)

- 27.** The corresponding medians of two similar triangle are respectively 12 cm & 15 cm. If area of first triangle is 288 cm². Find the corresponding area of second triangle.

Sol.

$$\begin{aligned} \left(\frac{\text{Area of first triangle}}{\text{Area of second triangle}} \right) &= \\ \left(\frac{\text{Median of first triangle}}{\text{Median of second triangle}} \right)^2 &= \\ \frac{288}{A_2} &= \left(\frac{12}{15} \right)^2 \end{aligned}$$

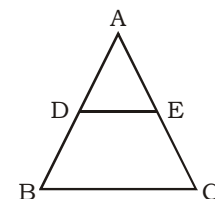
$$\frac{288}{A_2} = \frac{16}{25}$$

$$A_2 = 450 \text{ cm}^2$$

∴ (A₂ = Area of second triangle.)

- 28.** In a $\triangle ABC$ a line DE is drawn parallel to BC. If $\frac{AD}{DB} = \frac{2}{3}$ then find the ratio of area of $\triangle ADE$ & Area of $\square DECB$.

Sol.



DE $\parallel$ BC (Given)

So $\triangle ADE \sim \triangle ABC$

AD : DB = 2 : 3

Then AB = AD + DB = 5

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle ABC} = \left(\frac{AD}{AB} \right)^2$$

$$= \left(\frac{2}{5} \right)^2 = \frac{4}{25}$$

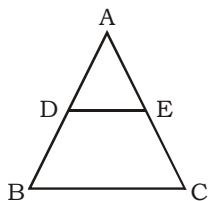
$$\begin{aligned} \text{Area of } \square DECB &= \text{Area of } \\ \triangle ABC - \triangle ADE &= 25 - 4 = 21 \end{aligned}$$

Area of $\triangle ADE$: Area of $\square DECB$ = 4 : 21

- 29.** In a $\triangle ABC$ a line DE is Drawn parallel to BC. If $\frac{AD}{DB} = \frac{6}{7}$,

Find the ratio of area of $\triangle ADE$
and Area of $\square DEBC$

Sol.



$DE \parallel BC$ (Given)

$\triangle ADE \sim \triangle ABC$

$$\frac{AD}{DB} = \frac{6}{7}$$

$$AB = AD + DB = 6 + 7 = 13$$

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle ADE} = \left(\frac{AB}{AD}\right)^2 =$$

$$\left(\frac{13}{6}\right)^2 = \frac{169}{36}$$

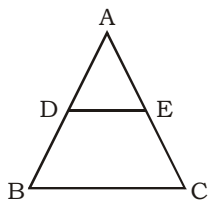
$$\text{Area of } \square DECB = 169 - 36 = 133$$

$$\text{Area of } \triangle ADE : \text{Area of } \square DECB = 36 : 133$$

- 30.** In a $\triangle ABC$ a line DE is drawn $\parallel$ to BC . If ratio of areas of $\triangle ADE$ & $\square DECB$ is $9 : 16$

find the value of $\frac{AD}{DB}$

Sol.



$DE \parallel BC$ (Given)

So, $\triangle ADE \sim \triangle ABC$

$$\begin{aligned} \text{Area of } \triangle ABC &= \text{Area of } \triangle ADE + \text{Area of } \square DECB \\ &= 9 + 16 = 25 \end{aligned}$$

$$\triangle ADE : \triangle ABC$$

$$\text{Area} \rightarrow 9 : 25$$

$$\text{Side} \rightarrow 3 : 5$$

The ratio of their sides is the square root of the ratio of their area

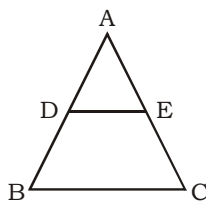
$$\text{So, } \frac{AB}{AD} = \frac{5}{3}$$

$$BD = AB - AD = 5 - 3 = 2$$

$$\text{Then, } \frac{AD}{DB} = \frac{3}{2}$$

- 31.** In a $\triangle ABC$ a line DE draw $\parallel$ to BC . If area of $\square DECB$ is 238 cm^2 and length of AD & AB respectively 5 cm & 12 cm find the area of $\triangle ABC$.

Sol.



$DE \parallel BC$ (Given)

So, $\triangle ABC \sim \triangle ADE$

$$AD = 5 \text{ cm, } AB = 12 \text{ cm}$$

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle ABC} = \left(\frac{AD}{AB}\right)^2$$

$$= \left(\frac{5}{12}\right)^2 = \frac{25}{144}$$

$$\text{Area of } \square DECB = 144 - 25 = 119$$

$$119 \text{ units} = 238$$

$$1 \text{ unit} = 2$$

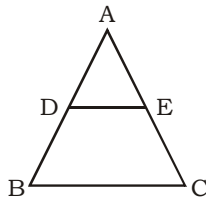
$$\text{So, Area of } \triangle ABC = 144 \text{ units}$$

$$= 144 \times 2 = 288 \text{ cm}^2$$

- 32.** In a $\triangle ABC$ a line DE is Drawn $\parallel$ to BC . Which Divide the triangle In equal Area. Then find

$$\frac{AD}{DB}$$

Sol.



$$\text{Area of } \triangle ADE : \text{Area of } \square DECB = 1 : 1$$

$$\text{So, Area of } \triangle ABC = 1 + 1 = 2 \text{ units}$$

$$\triangle ADE : \triangle ABC$$

$$\text{Area} \rightarrow 1 : 2$$

$$\text{Side} \rightarrow 1 : \sqrt{2}$$

$$\frac{AD}{AB} = \frac{1}{\sqrt{2}}$$

$$BD = AB - AD = \sqrt{2} - 1$$

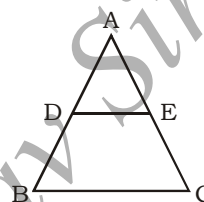
Then,

$$\frac{AD}{DB} = \frac{1}{\sqrt{2}-1} = 1 : \sqrt{2}-1$$

- 33.** In a $\triangle ABC$, D and E are the mid point of AB and AC , find

$$\frac{\text{Ar. of } \triangle ADE}{\text{Ar. of } \square DECB}$$

Sol.



$$AD = DB = \frac{AB}{2} \text{ (D mid point of AB)}$$

$$AE = EC = \frac{AC}{2} \text{ (E mid point of AC)}$$

$$\text{Let } AB = 2 \text{ cm}$$

$$\text{Then } AD = 1 \text{ cm}$$

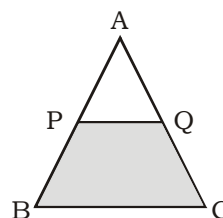
$$\begin{array}{ccc} \triangle ABC & : & \triangle ADE \\ \text{Side} \rightarrow 2 & : & 1 \\ \text{Area} \rightarrow 4 & : & 1 \end{array}$$

$$\begin{aligned} \text{Area of } \square DECB &= \text{Area of } \triangle ABC - \text{Area of } \triangle ADE \\ &= 4 - 1 = 3 \text{ cm}^2 \end{aligned}$$

$$\text{Now, } \frac{\text{Ar. of } \triangle ADE}{\text{Ar. of } \square DECB} = \frac{1}{3}$$

- 34.** In a $\triangle ABC$ a line PQ is Draw parallel to side BC , if $AB = 9 \text{ cm}$ and $AP = 4 \text{ cm}$, find the ratio of shaded and unshaded portion

Sol.



$PQ \parallel BC$ (Given)

So $\triangle APQ \sim \triangle ABC$

$$AP = 4 \text{ cm, } AB = 9 \text{ cm}$$

$$\frac{\text{Ar. of } \triangle APQ}{\text{Ar. of } \triangle ABC} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

$$\text{Ar. of } \square PQCB = \text{Ar. of } \triangle ABC - \text{Ar. of } \triangle APQ$$

$$= 81 - 16 = 65$$

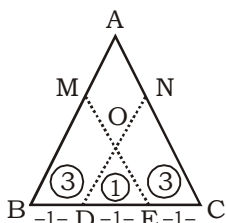
$$= \text{Area of shaded part} : \text{Area of unshaded part}$$

$$= 65 : 16$$

35. In a $\triangle ABC$, D and E are two points on the side BC such that they trisect the line BC. In M and N are the two point on line AB & AC. $ME \parallel AC$ and $ND \parallel AB$. The line ME and ND intersect at O. Find

$$\frac{\text{Ar. of } \triangle DOE + \text{Ar. of } \square AMON}{\text{Ar. of } \triangle ABC}$$

Sol.



Let $BC = 3 \text{ cm}$

Then $BD = DE = EC = 1 \text{ cm}$

In $\triangle MEB$

$OD \parallel BM$

($\therefore DN \parallel AB$)

$\triangle ODE \sim \triangle EMB$

$$\frac{\text{Ar. of } \triangle ODE}{\text{Ar. of } \triangle EMB} = \left(\frac{ED}{BE}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Ar. of $\square ODBM = \text{Ar. of } \triangle EBM - \text{Ar.}$

of $\triangle ODE$

$$= 4 - 1 = 3 \text{ cm}^2$$

Same as

In $\triangle DNC$

$$\frac{\text{Ar. of } \triangle ODE}{\text{Ar. of } \triangle DNC} = \left(\frac{DE}{DC}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Ar. of $\square OECD = 4 - 1 = 3 \text{ cm}^2$

Area of $\triangle ABC = (BC)^2 = (3)^2 = 9 \text{ cm}^2$

Area of $\square AMON = 9 - (3 + 1 + 3) = 2 \text{ cm}^2$

Now,

$$\frac{\text{Ar. of } \triangle ODE + \text{Ar. of } \square AMON}{\text{Ar. of } \triangle ABC} =$$

$$\frac{1+2}{9} = \frac{1}{3}$$

36. In a $\triangle ABC$, D and E are two points on the side BC.

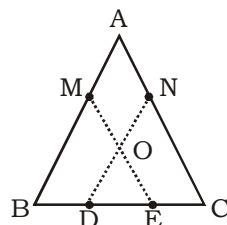
Such that $\frac{BD}{DE} = \frac{2}{3}$ and

$\frac{DE}{EC} = \frac{6}{5}$. M and N are the

two points on line AB & AC. $ME \parallel AC$, $ND \parallel AB$. The line ME and ND intersect at O. Find

$$\frac{\text{Ar. of } \triangle DOE + \text{Ar. of } \square AMON}{\text{Ar. of } \triangle ABC}$$

Sol.



$$\frac{BD}{DE} = \frac{2}{3}, \quad \frac{DE}{EC} = \frac{6}{5}$$

$$BD : DE : EC$$

$$2 : 3$$

$$6 : 5$$

(To make equal ratio)

$$BD : DE : EC$$

$$4 : 6 : 5$$

$$BE = BD + DE = 4 + 6 = 10 \text{ units}$$

$$DC = DE + EC = 6 + 5 = 11 \text{ units}$$

$$BC = BD + DC = 4 + 11 = 15 \text{ units}$$

In $\triangle BME$, $OD \parallel MB$ ($\therefore AB \parallel ND$)

$\triangle ODE \sim \triangle BME$

$$\frac{\text{Ar. of } \triangle ODE}{\text{Ar. of } \triangle BME} = \left(\frac{DE}{BE}\right)^2 = \left(\frac{6}{10}\right)^2 = \frac{36}{100}$$

$$\text{Ar. of } \square ODBM = \text{Ar. of } \triangle BME -$$

$$\text{Ar. of } \triangle ODE$$

$$= 100 - 36 = 64 \text{ units}$$

Same as In $\triangle DNC$

$$\frac{\text{Ar. of } \triangle ODE}{\text{Ar. of } \triangle DNC} = \left(\frac{DE}{DC}\right)^2 = \left(\frac{6}{11}\right)^2 = \frac{36}{121}$$

$$\text{Ar. of } \square ONCE = \text{Area of } \triangle DNC -$$

$$\text{Area of } \triangle DOE$$

$$= 121 - 36 = 85 \text{ units}$$

$$\text{Area of } \triangle ABC = (BC)^2 = (15)^2 = 225 \text{ units}$$

$$\text{Area of } \square AMON = 225 - (64 + 36 + 85)$$

$$= 40 \text{ units}$$

Now,

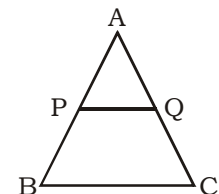
$$\frac{\text{Ar. of } \triangle DOE + \text{Ar. of } \square AMON}{\text{Ar. of } \triangle ABC}$$

$$= \frac{36+40}{225} = \frac{76}{225}$$

37. In a $\triangle ABC$ Line PQ is draw parallel to side BC where P and Q are respectively lie on side

AB and AC. If $AB = 3AP$ what is percentage of Area $\square PQCB$ in the respect of $\triangle ABC$.

Sol.



$PQ \parallel BC$

$\triangle APQ \sim \triangle ABC$

$$AB = 3AP$$

$$\frac{AB}{3} : \frac{AP}{1}$$

$$\frac{\text{Ar. of } \triangle APQ}{\text{Ar. of } \triangle ABC} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

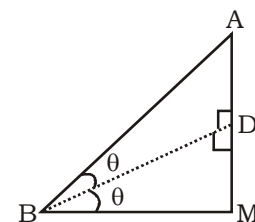
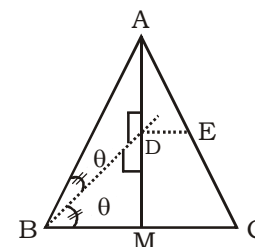
$$\text{Area of } \square PQCB = 9 - 1 = 8 \text{ cm}^2$$

Percentage of $\square PQCB$ in the respect of $\triangle ABC$

$$= \frac{8}{9} \times 100 = 88\frac{8}{9} \%$$

38. In $\triangle ABC$, AD is perpendicular to the angle bisector of angle B. line DE is drawn parallel to BC. E is a point on line AC. If $AC = 12 \text{ cm}$. Find the length of AE.

Sol.



$\triangle ABD$ and $\triangle BDM$

$$\angle ADB = \angle BDM \quad (90^\circ)$$

$$BD = BD \text{ (Common)}$$

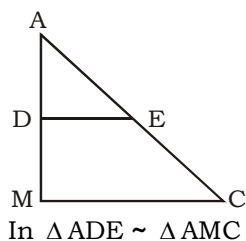
$$\angle ABD = \angle DBM \quad (\text{BD Angle Bisector})$$

$$\triangle ABD \cong \triangle BDM$$

$$AD = DM = \frac{AM}{2}$$

We can say,

D is the mid point of AM



$$\frac{AD}{AM} = \frac{AE}{AN}$$

$$\frac{AM/2}{AM} = \frac{AN}{AN}$$

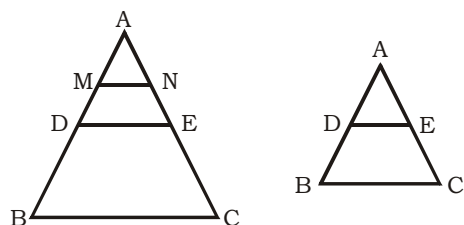
$$\frac{1}{2} = \frac{AN}{AN}$$

$$AN = 6 \text{ cm}$$

39. In the $\triangle ABC$, D and E are the mid points of line AB & AC. M and N are the mid point of AD & AE find the ratio of

Area of $\triangle AMN$: Area of $\triangle ADE$:
Area of $\triangle ABC$.

Sol.

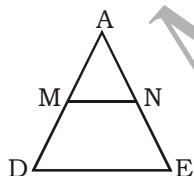


We know that when D & E mid point AB and AC then $DE \parallel BC$ and

$$DE = \frac{1}{2} BC.$$

$$\text{Now, } \frac{DE}{BC} = \frac{1}{2} = \frac{AD}{AB} = \frac{AE}{AC}$$

$$\frac{\text{Ar. of } \triangle ADE}{\text{Ar. of } \triangle ABC} = \left(\frac{AD}{AB}\right)^2 = \frac{1}{4}$$



M & N mid points of AD & AE (Given)

Then, $MN \parallel DE$ and $MN = \frac{1}{2} DE$

$$\frac{MN}{DE} = \frac{1}{2} = \frac{AM}{AD} = \frac{AN}{AE}$$

$$\frac{\text{Ar. of } \triangle AMN}{\text{Ar. of } \triangle ADE} = \left(\frac{AM}{AD}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

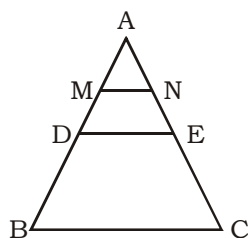
$$\begin{array}{ccc} \text{Ar. of } \triangle AMN & : & \text{Ar. of } \triangle ADE & : & \text{Ar. of } \triangle ABC \\ 1 & : & 4 & : & 16 \end{array}$$

So,

$$\begin{array}{ccc} \text{Ar. of } \triangle AMN & : & \text{Ar. of } \triangle ADE & : & \text{Ar. of } \triangle ABC \\ 1 & : & 4 & : & 16 \end{array}$$

(To make equal ratio)

Alternate



Let, $AM = 1$

Then, $AD = 2 AM = 2 \times 1 = 2$

and $AB = 2 \times AD = 2 \times 2 = 4$

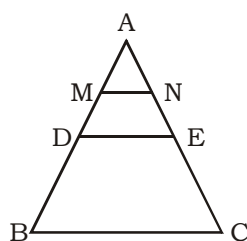
$$\begin{array}{ccc} \text{Ar. of } \triangle AMN & : & \text{Ar. of } \triangle ADE & : & \text{Ar. of } \triangle ABC \\ 1 & : & 4 & : & 16 \end{array}$$

Side $\rightarrow 1 : 2 : 4$

Area $\rightarrow 1 : 4 : 16$

40. In a triangle ABC, D and E are the mid points of line AB & AC. M and N are the mid points of AD & AE. Then find the ratio of Area of $\triangle AMN$, Area of $\square DENM$ & Area of $\square BCED$.

Sol.



$$\begin{array}{ccc} \text{Ar. of } \triangle AMN & : & \text{Ar. of } \triangle ADE & : & \text{Ar. of } \triangle ABC \\ 1 & : & 4 & : & 16 \end{array}$$

(Discuss in above question)

Area of $\square DENM$ = Area of $\triangle ADE$ -

Area of $\triangle AMN$ = $4 - 1 = 3$

Area of $\square BCED$ = Area of $\triangle ABC$

- Area of $\triangle ADE$

$$= 16 - 4 = 12$$

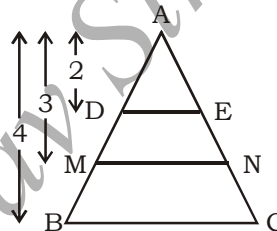
So,

$$\begin{array}{ccc} \text{Ar. of } \triangle AMN & : & \text{Ar. of } \square DENM & : & \text{Ar. of } \square BCED \\ 1 & : & 3 & : & 12 \end{array}$$

41. In a $\triangle ABC$, D and E are the mid points of AB & AC. M and N are the mid point BD & EC. Then find the ratio of

$$\text{Ar. of } \triangle ADE : \text{Ar. of } \triangle AMN : \text{Ar. of } \triangle ABC$$

Sol.



Let, $AD = 2$

$$AD = BD = \frac{AB}{2} = 2$$

($\because$ D is mid point AB)

Then, $AB = 2 \times 2 = 4$

M is mid point BD

$$\text{Then, } BM = DM = \frac{BD}{2} = \frac{2}{2} = 1$$

$$AM = AD + DM = 2 + 1 = 3$$

$$\begin{array}{ccc} \text{Ar. of } \triangle ADE & : & \text{Ar. of } \triangle AMN & : & \text{Ar. of } \triangle ABC \\ 4 & : & 9 & : & 16 \end{array}$$

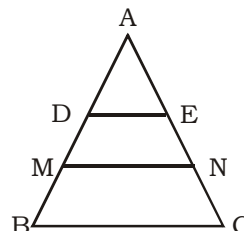
Side $\rightarrow 2 : 3 : 4$

Area $\rightarrow 4 : 9 : 16$

42. In a $\triangle ABC$, D and E are the mid point of AB & AC. M and N are the mid points of BD & EC then find the ratio of

$$\text{Ar. of } \triangle ADE : \text{Ar. of } \square DENM : \text{Ar. of } \square MNCB$$

Sol.



$$\begin{array}{ccc} \text{Ar. of } \triangle ADE & : & \text{Ar. of } \triangle AMN & : & \text{Ar. of } \triangle ABC \\ 4 & : & 9 & : & 16 \end{array}$$

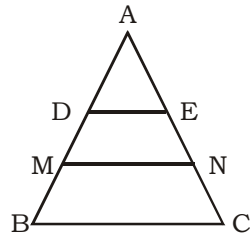
(Discuss in above solution)
 $\Rightarrow$ Area of $\triangle DENM$ = Area of $\triangle AMN$ - Area of $\triangle ADE$
 $= 9 - 4 = 5$ units
 $\Rightarrow$ Area of $\triangle MNCB$ = Area of $\triangle ABC$ - Area of $\triangle AMN$
 $= 16 - 9 = 7$ units

Now,

$$\begin{array}{ccc} \text{Ar. of} & \text{Ar. of} & \text{Ar. of} \\ \triangle ADE & \triangle DENM & \triangle MNCB \\ 4 & 5 & 7 \end{array}$$

- 43.** In a $\triangle ABC$, D and E are points lie on AB and AC. M and N are points lie on BD & EC. If $\frac{AD}{DM} = \frac{2}{3}$ and $\frac{DM}{MB} = \frac{4}{5}$ and Area of $\triangle AMN$ 800 cm^2 . Find the area of $\triangle MNCB$.

Sol.



$$\frac{AD}{DM} = \frac{2}{3}, \frac{DM}{MB} = \frac{4}{5}$$

$$AD : DM : MB$$

$$2 : 3 : 4$$

(To make equal ratio)

$$AD : DM : MB$$

$$8 : 12 : 15$$

$$AM = AD + DM = 8 + 12 = 20$$

$$AB = AM + MB = 20 + 15 = 35$$

$$\begin{array}{ccc} \text{Ar. of} & \text{Ar. of} & \text{Ar. of} \\ \triangle ADE & \triangle AMN & \triangle ABC \\ \text{Side} \rightarrow & 8 & 20 & 35 \\ \text{Area} \rightarrow & 64 & 400 & 1225 \end{array}$$

$$400 \text{ units} = 800 \text{ cm}^2$$

$$1 \text{ unit} = 2 \text{ cm}^2$$

$$\text{Area of } \triangle ABC = 1225 \times 2 = 2450$$

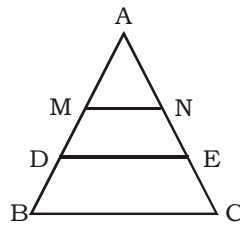
$$\begin{aligned} \text{Area of } \triangle MNCB &= \text{Area of } \triangle ABC \\ &- \text{Area of } \triangle AMN \\ &= 2450 - 800 = 1650 \text{ cm}^2 \end{aligned}$$

- 44.** In a $\triangle ABC$, D and E are point lies on AB & AC, and M and N are points lies on AD & AE. If

$$\frac{AM}{DM} = \frac{1}{3} \text{ and } \frac{DM}{DB} = \frac{6}{5} \text{ and}$$

Area of $\triangle DECB$ is 210 cm^2 find the area of $\triangle DENM$.

Sol.



$$\frac{AM}{DM} = \frac{1}{3} \text{ and } \frac{DM}{DB} = \frac{6}{5}$$

$$AM : MD : DB$$

$$1 : 3 : 6$$

(to make equal ratio)

$$AM : MD : DB$$

$$2 : 6 : 5$$

$$AD = AM + MD = 2 + 6 = 8$$

$$AB = AD + DB = 8 + 5 = 13$$

$$\begin{array}{ccc} \text{Ar. of} & \text{Ar. of} & \text{Ar. of} \\ \triangle AMN & \triangle ADE & \triangle ABC \\ \text{Side} \rightarrow & 2 & 8 & 13 \\ \text{Area} \rightarrow & 4 & 64 & 169 \end{array}$$

$$\begin{aligned} \text{Area of } \triangle DECB &= \text{Area of } \triangle ABC - \text{Area of } \triangle ADE \\ &= 169 - 64 = 105 \\ 105 \text{ units} &= 210 \text{ cm}^2 \\ 1 \text{ unit} &= 2 \text{ cm}^2 \end{aligned}$$

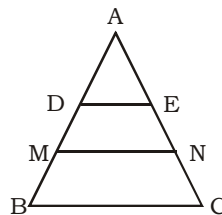
$$\begin{aligned} \text{Area of } \triangle MNED &= \text{Area of } \triangle ADE - \text{Area of } \triangle AMN \\ &= 64 - 4 = 60 \text{ units} \\ &= 60 \times 2 = 120 \text{ cm}^2 \end{aligned}$$

- 45.** In a $\triangle ABC$, D and E are points lie on AB & AC. M and N are point lie on BD & EC. If

$$\frac{AD}{DM} = \frac{3}{2}, \frac{DM}{MB} = \frac{6}{7} \text{ and Area of}$$

$\triangle DENM$ is 432 cm^2 Find the area of $\triangle MNCB$.

Sol.



$$\frac{AD}{DM} = \frac{3}{2}, \frac{DM}{MB} = \frac{6}{7}$$

$$AD : DM : MB$$

$$3 : 2 : 6$$

(To make equal ratio)

$$AD : DM : MB$$

$$9 : 6 : 7$$

$$AM = AD + DM = 9 + 6 = 15$$

$$AB = AM + MB = 15 + 7 = 22$$

$$\begin{array}{ccc} \text{Ar. of} & \text{Ar. of} & \text{Ar. of} \\ \triangle ADE & \triangle AMN & \triangle ABC \\ \text{Side} \rightarrow & 9 & 15 & 22 \\ \text{Area} \rightarrow & 81 & 225 & 484 \end{array}$$

$$\begin{aligned} \text{Area of } \triangle DENM &= \text{Area of } \triangle AMN - \text{Area of } \triangle ADE \\ &= 225 - 81 = 144 \text{ units} \\ 144 \text{ units} &= 432 \text{ cm}^2 \\ 1 \text{ unit} &= 3 \text{ cm}^2 \end{aligned}$$

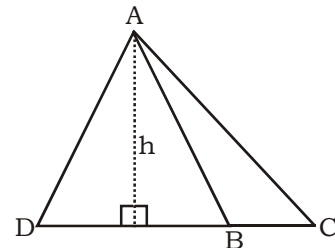
$$\begin{aligned} \text{Area of } \triangle MNCB &= \text{Area of } \triangle ABC - \text{Area of } \triangle AMN \\ &= 484 - 225 = 259 \text{ units} \end{aligned}$$

$$\text{Now, Area of } \triangle MNCB = 259 \times 3 = 777 \text{ cm}^2$$

* (Area of triangles based on a same straight line.)

- If two or more than two triangles are based on a same straight line. Then the ratio of their area are same as the ratio of length of their base these triangles have common vertex.

(i)

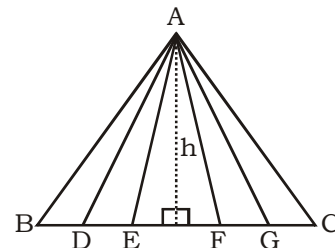


Proof:- Area of $\triangle ABC$: Area of $\triangle ADB$

$$\frac{1}{2} \times BC \times h : \frac{1}{2} \times BD \times h$$

$$BC : BD$$

(ii)

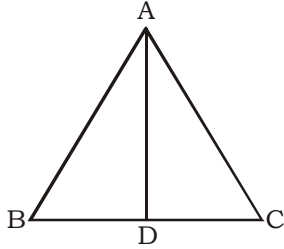


$$\begin{array}{ccc} \text{Ar. of} & \text{Ar. of} & \text{Ar. of} & \text{Ar. of} \\ \triangle ABD & \triangle ADE & \triangle AEF & \triangle AGC \\ \text{BD} & \text{DE} & \text{EF} & \text{GC} \end{array}$$

- 46.** In a triangle ABC, D is a point on BC. If area of $\triangle ABD$ is 80

cm² and length of BD & DC are respectively 4 cm & 5 cm. Find the area of $\triangle ABC$.

Sol.



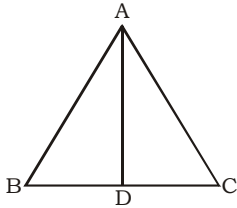
$$\frac{\text{Area of } \triangle ABD}{\text{Area of } \triangle ADC} = \frac{BD}{DC} = \frac{4}{5} = \frac{A_1}{A_2}$$

$$A_2 = 100 \text{ cm}^2$$

(here $A_2 \rightarrow$ Area of $\triangle ABD$)

- 47.** In $\triangle ABC$, D is a point on BC. Area of $\triangle ABD$ 60 cm² and Area of $\triangle ADC$ 75 cm². Find the ratio of BD and DC.

Sol.

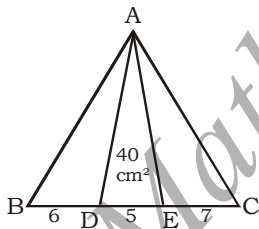


$$\frac{\text{Area of } \triangle ABD}{\text{Area of } \triangle ADC} = \frac{\text{Base of } \triangle ABD}{\text{Base of } \triangle ADC} = \frac{BD}{DC}$$

$$\frac{60}{75} = \frac{BD}{DC}$$

$$BD : DC = 4 : 5$$

- 48.** In figure find the area of $\triangle ABC$? If $BD = 6$ cm, $DE = 5$ cm, $EC = 7$ cm

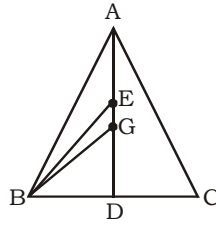


- Sol.** In $\triangle ADE$,
5 units = 40 cm²
1 unit = 8 cm²

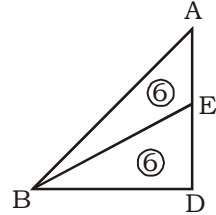
Area of $\triangle ABC$,
 $BC = BD + DE + EC = 5 + 6 + 7$
 $= 18$ units
 $18 \text{ units} = 18 \times 8 = 144 \text{ cm}^2$

- 49.** In $\triangle ABC$, AD is median, E is the mid-point of line AD and G is centroid find the ratio of area of $\triangle BEG$ and $\triangle ABC$.

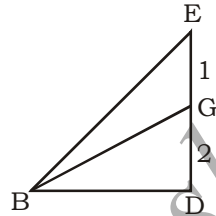
Sol.



Let area of $\triangle ABC = 24$ units
then area of $\triangle ABD = 12$ units



Area of $\triangle ABE = \text{Area } \triangle BED = 6$ units
(E is mid point of AD)



Let $AD = 6$ units

$$ED = \frac{AD}{2} = \frac{6}{2} = 3 \text{ (E is mid point of AD)}$$

$$AG = \frac{2}{3} AD = \frac{2}{3} \times 6 = 4 \text{ units}$$

$$GD = \frac{1}{3} AD = 6 \times \frac{1}{3} = 2 \text{ units}$$

$$EG = AG - AE = 4 - 3 = 1 \text{ unit}$$

$$\frac{\text{Ar. of } \triangle BEG}{\text{Ar. of } \triangle BGD} = \frac{EG}{GD} = \frac{1}{2}$$

***** (When two triangles are based on a same straight line then the ratio of their area are same as the ratio of there base)

Area of $\triangle BED = 6$ units

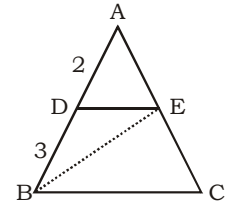
$$\text{Then, Area of } \triangle BEG = 6 \times \frac{1}{3} = 2 \text{ units}$$

$$\frac{\text{Ar. of } \triangle BEG}{\text{Ar. of } \triangle ABC} = \frac{2}{24} = \frac{1}{12}$$

$$\text{Ar. of } \triangle BEG : \text{Ar. of } \triangle ABC = 1 : 12$$

- 50.** In a $\triangle ABC$ a line DE is draw parallel to BC. If area of $\triangle ABC$ 150 cm² and $AD : DB = 2 : 3$ find the area of $\triangle DBE$.

Sol.



$DE \parallel BC$ (Given)

$\triangle ADE \sim \triangle ABC$

$$AD : DB = 2 : 3$$

$$AB = AD + DB = 5 \text{ units}$$

Area of $\triangle ADE : \text{Area of } \triangle ABC$

$$\text{Side} \rightarrow 2 : 5$$

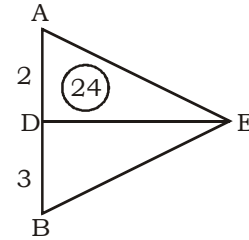
$$\text{Area} \rightarrow 4 : 25$$

$$25 \text{ units} = 150 \text{ cm}^2$$

$$1 \text{ unit} = 6 \text{ cm}^2$$

Area of $\triangle ADE = 4 \text{ units} = 4 \times 6 = 24 \text{ cm}^2$ In $\triangle AEB$,

***** When two triangle are based on a same straight line (AB). Then the ratio of their area are same as the ratio of length of their base have triangles have common vertex.



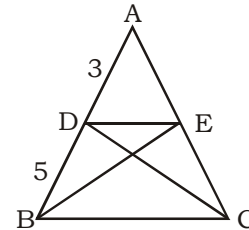
$$2 \text{ units} = 24 \text{ cm}^2$$

$$1 \text{ unit} = 12 \text{ cm}^2$$

$$\text{Area of } \triangle BDE = 3 \text{ units} = 3 \times 12 = 36 \text{ cm}^2$$

- 51.** In a $\triangle ABC$ a line DE is parallel to BC. If Area of $\triangle ABC$ 192 cm² and $AD : DB = 3 : 5$ find the area of $\triangle DEB$.

Sol.



$DE \parallel BC$ (Given)

$\triangle ADE \sim \triangle ABC$

$$\frac{AD}{DB} = \frac{3}{5}$$

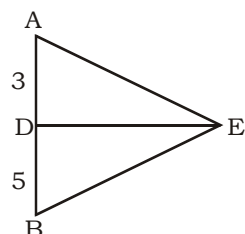
$$AB = 3 + 5 = 8 \text{ units}$$

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle ABC} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{3}{8}\right)^2 = \frac{9}{64}$$

$$64 \text{ units} = 192 \text{ cm}^2$$

$$1 \text{ unit} = 3 \text{ cm}^2$$

$$\text{Area of } \triangle ADE = 9 \times 3 = 27 \text{ cm}^2$$



$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle DEB} = \frac{AD}{DB} = \frac{3}{5}$$

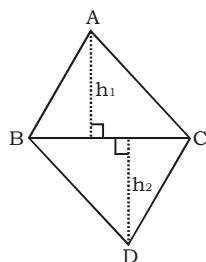
$$3 \text{ unit} = 27$$

$$1 \text{ unit} = 9$$

$$\text{Area of } \triangle DBE = 5 \text{ unit} = 5 \times 9 = 45 \text{ cm}^2$$

* Area of triangle which have common base

If base of two or more triangles have common base then the ratio of their area are same as the ratio of length of altitude drawn on same base.



Proof

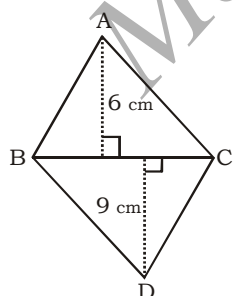
$$\text{Area of } \triangle ABC : \text{Area of } \triangle BCD$$

$$\frac{1}{2} \times BC \times h_1 : \frac{1}{2} \times BC \times h_2$$

$$h_1 : h_2$$

- 52.** $\triangle ABC$ and $\triangle BCD$ have common base BC. If the length of their altitude 6 cm and 9 cm. Find the ratio of their areas.

Sol.



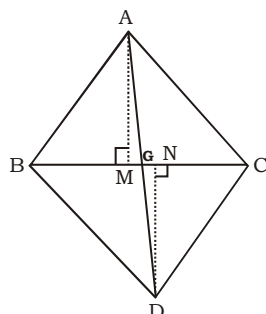
$$\frac{\text{Ar. of } \triangle ABC}{\text{Ar. of } \triangle BCD} = \frac{\text{Altitude of } \triangle ABC}{\text{Altitude of } \triangle BCD} =$$

$$\frac{6}{9} = \frac{2}{3}$$

$$\text{Ar. of } \triangle ABC : \text{Area of } \triangle BCD = 2 : 3$$

- 53.** $\triangle ABC$ and $\triangle BCD$ have common base. If the length of their median 7 cm and 11 cm. Find the ratio of their areas.

Sol.



Draw AM & DN are altitude on BC.

AG and GD are medians

$\triangle AMG$ and $\triangle DGN$

$$\angle AMG = \angle DNG \quad (90^\circ)$$

$$\angle DGN = \angle AGM$$

(Vertically opposite)

$$\triangle AMG \sim \triangle DGN$$

$$\frac{AG}{GD} = \frac{AM}{DN}$$

$$\text{So, } \frac{AM}{DN} = \frac{7}{11}$$

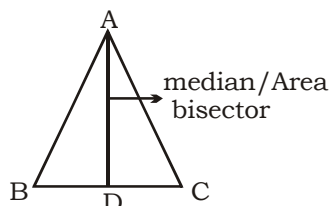
We know that when two triangle have common base. Then the ratio of their area are same as the ratio of length of altitude.

So,

$$\begin{aligned} \text{Area of } \triangle ABC : \text{Area of } \triangle BDC &= \\ AM : DN &= \\ 7 : 11 \end{aligned}$$

Median

A line from corner to the midpoint of the opposite side. It is called median. It is also called Area Bisector of triangle



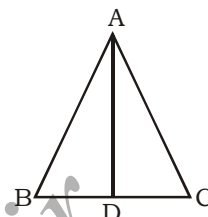
When AD is median, $BD = DC$

$$\text{ar } \triangle ABD = \text{ar of } \triangle ADC =$$

$$\frac{1}{2} \text{ Ar. of } \triangle ABC$$

- 54.** In a triangle ABC, AD is median. If area of $\triangle ABC$ 120 cm^2 find the Area of $\triangle ABD$.

Sol. According to the Question



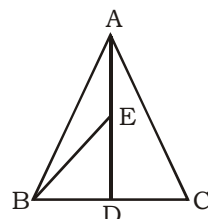
AD is median then ,

$$\text{ar of } \triangle ABD = \frac{1}{2} \text{ Ar. of } \triangle ABC$$

$$= \frac{1}{2} \times 120 = 60 \text{ cm}^2$$

- 55.** In a $\triangle ABC$, D is the midpoint of line BC and E is the mid point of AD. then find the ratio of area of $\triangle BEA$ and $\triangle ABC$.

Sol.



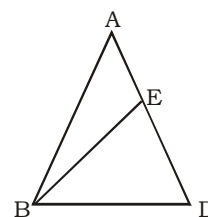
Let the area of $\triangle ABC = 100$ units

AD is median

Then,

$$\text{Area of } \triangle ABD = \frac{1}{2} \times \text{area of } \triangle ABC$$

$$= \frac{1}{2} \times 100 = 50 \text{ units}$$



E is mid point of AD

So,

BE is median.

$$\text{Ar of } \triangle BED = \text{Ar of } \triangle BEA = \frac{1}{2} \times$$

ABD

$$\text{Ar of } \triangle BEA = \frac{1}{2} \times 50 = 25 \text{ units}$$

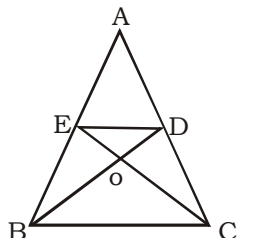
$$\text{Ar of } \triangle BEA : \text{Ar of } \triangle ABC = 25 : 100$$

$$= 1 : 4$$

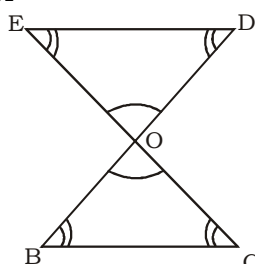
In a $\triangle ABC$, BD and CE are two median which intersect each other at 'O'. $\Rightarrow$

Some Important results

- (i) Ar of $\triangle DOE$: Ar of $\triangle ABC = 1 : 12$
- (ii) Ar of $\triangle DOE$: Ar of $\triangle DOC = 1 : 2$
- (iii) Ar of $\triangle DOE$: Ar of $\triangle BOC = 1 : 4$



Proof



E and D mid point
Then,

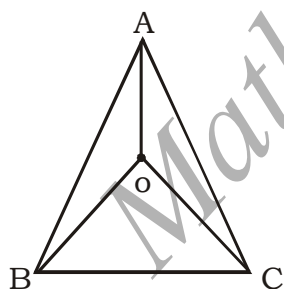
$$ED \parallel BC \text{ and } ED = \frac{1}{2} BC$$

So,

$$\frac{Ar \triangle OED}{Ar \triangle BOC} = \left(\frac{ED}{BC} \right)^2 = \left(\frac{BC/2}{BC} \right)^2 = \frac{1}{4}$$

Ar of $\triangle OED$: Ar of $\triangle BOC = 1 : 4$

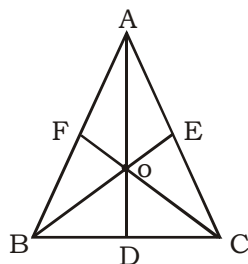
(iv). In a $\triangle ABC$, O is centroid then



Ar. of $\triangle AOB = \text{Ar. of } \triangle BOC = \text{Ar. of } \triangle AOC$

$$= \frac{1}{3} (Ar. of \triangle ABC)$$

(v). In a $\triangle ABC$, AD, BE and CF are medians and then all triangles have same Areas.

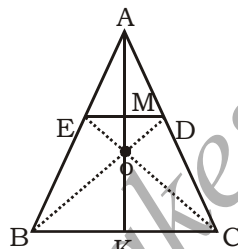


Ar. of $\triangle BOD = \text{Ar. of } \triangle DOC =$
Ar. of $\triangle OEC = \text{Ar. of } \triangle OAE =$
Ar. of $\triangle AOF = \text{Ar. of } \triangle OBF.$

$$= \frac{1}{6} (Ar. of \triangle ABC)$$

56. In a $\triangle ABC$, BD and CE are two medians which intersect each other at 'O'. AO intersect the line ED at M. find the ratio of AM : MO

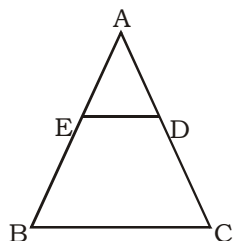
Sol.



Let AO = 4 unit (Large median)

$$\text{then } AK = \frac{3}{2} AO$$

$$= \frac{3}{2} \times 4 = 6 \text{ units}$$



E and D are mid points
(Because CE and BD are median)
Then $ED \parallel BC$

$$ED = \frac{1}{2} BC$$

$$\frac{AE}{AB} = \frac{\text{median of } \triangle AED}{\text{median of } \triangle ABC} = \frac{AM}{AK}$$

$$\frac{1}{2} = \frac{AM}{6}$$

$$AM = 3 \text{ units}$$

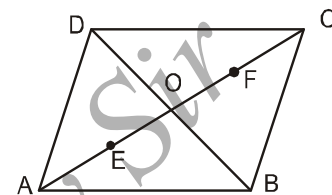
$$OM = AO - AM = 4 - 3 = 1$$

So,

$$AM : MO$$

$$3 : 1$$

57. In the adjoining figure ABCD is a ||gm and E, F are the centroids of $\triangle ABD$ and $\triangle BCD$ respectively, then EF equals :



- (a) AE
- (b) BE
- (c) CE
- (d) DE

Sol. AE : EO = 2 : 1 and CF : FO = 2 : 1

$$\therefore OE = \frac{1}{3} AO \text{ and } OF = \frac{1}{3} OC$$

$$\therefore EF = OE + OF$$

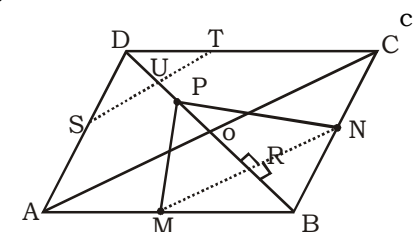
$$= \frac{1}{3} (AO + OC) = \frac{1}{3} AC = AE$$

58. A, B, C and D are the vertex of a Rhombus M and N are the mid-points of AB and BC. P is a point on diagonal BD In such a way BP

$$= \frac{3}{5} BD$$

Find $\frac{Ar \text{ of } \triangle BMN}{Ar \text{ of } \triangle MNP}$

Sol.



We know

BR : RO : OU : UD (By property)

$$1 : 1 : 1 : 1$$

(when M, N, S and T are mid-points)

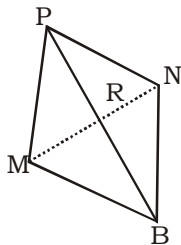
$$BP = \frac{3}{5} BD \text{ (given)}$$

$$BR = \frac{1}{4} BD$$

Let BD = 20 cm

then BR = OR = OU = UD = 5 cm

and $BP = 20 \times \frac{3}{5} = 12$ cm
 $RP = BP - BR = 12 - 5 = 7$ cm



$$\frac{\text{Ar of } \triangle BMN}{\text{Ar of } \triangle MNP} = \frac{BR}{RP} = \frac{5}{7}$$

When two triangles have common base then the ratio of their Area is same as the ratio length of their Altitude.

So,

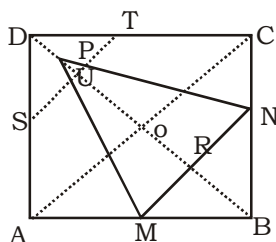
Ar of $\triangle BMN$: Ar of $\triangle MNP = 5 : 7$

59. A, B, C and D are the vertex of a square M and N are the mid point of line AB and BC. P is a point on

the Diagonal BD, if $BP = \frac{5}{6}BD$.

Find $\frac{\text{Ar. of } \triangle BMN}{\text{Ar. of } \triangle MNP}$

Sol.



$BR : OR : OU : UD$ (When M, N, S, and T are mid points) $1 : 1 : 1 : 1$

So,

$$BR = \frac{1}{4}BD$$

$$BP = \frac{5}{6}BD$$

L.C.M. of 4 and 6 = 12

Let $BD = 12$ cm.

$$BR = \frac{1}{4} \times 12 = 3$$

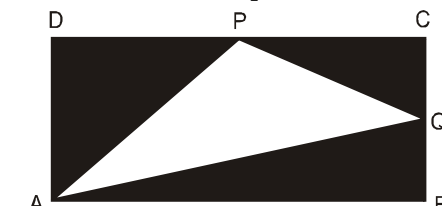
$$BP = \frac{5}{6} \times 12 = 10$$

$$RP = BP - BR = 10 - 3 = 7$$
 cm

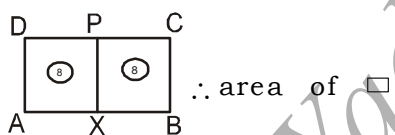
$$\frac{\text{Ar of } \triangle BMN}{\text{Ar of } \triangle MNP} = \frac{BR}{RP} = \frac{3}{7}$$

Ar. of $\triangle BMN$: Ar. of $\triangle MNP = 3 : 7$

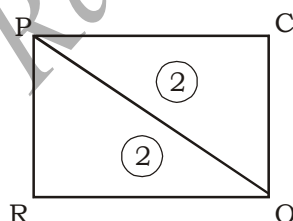
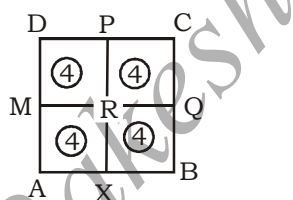
60. In the given figure, ABCD is a rectangle. P and Q are the mid-points of sides CD and BC respectively. Then the ratio of area of shaded portion : area of unshaded portion is :



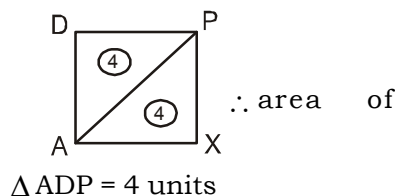
Sol. (c) Let total area of rectangle ABCD = 16 units



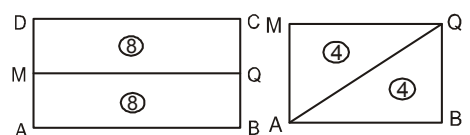
$$AXPD = \frac{16}{2} = 8 \text{ units}$$



$$\therefore \text{area of } \triangle PCQ = 2 \text{ units}$$



$$\triangle ADP = 4 \text{ units}$$



$$\therefore \text{area of } \triangle ABQ = 4 \text{ units}$$

$$\therefore \text{total area of shaded portion} = \text{area of } \triangle ADP + \text{area of } \triangle ABQ + \text{area of } \triangle PCQ$$

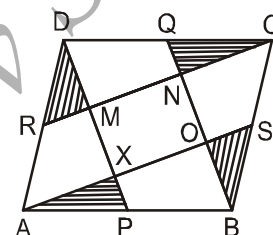
$$= 4 + 2 + 4 = 10 \text{ units}$$

$$\therefore \text{area of unshaded portion} = \text{area of } \triangle APQ$$

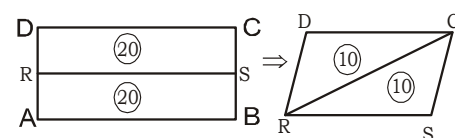
$$= 16 - \text{Area of shaded portion} = 16 - 10 = 6 \text{ units}$$

$$\therefore \text{Required ratio} = 10 : 6 = 5 : 3$$

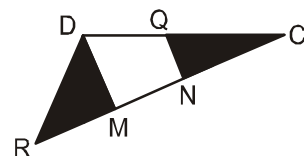
61. In the $\parallel$ gm ABCD, P, Q, R and S are mid-points of sides AB, CD, DA and BC respectively. AS, BQ, CR and DP are joined. Find the ratio of the area of the shaded region to the area of the $\parallel$ gm ABCD.



Sol. (a)



Let total area of $\parallel$ ABCD = 40 units



In $\triangle DMC$, Q is the mid point of DC and $QN \parallel DM$

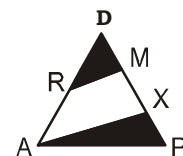
$\therefore DMNQ$ is also a $\parallel$ gm

$\Rightarrow N$ is the mid-point of MC

$$\therefore \text{ar}(\triangle QCN) : \text{ar}(\triangle DMC) = 1 : 4$$

$$\text{Let ar}(\triangle QCN) = 1 \text{ unit}$$

$$\Rightarrow \text{ar}(\text{quadrilateral DMNQ}) = 4 - 1 = 3 \text{ units}$$



$\triangle DAX$

$$\text{ar}(\triangle DRM) : \text{ar}(\triangle DAX) = 1 : 4$$

$$\text{ar}(\triangle DRM) = 1 \text{ unit}$$

$$\Rightarrow \text{ar}(\triangle DAX) = 4 \text{ units}$$

$$\Rightarrow \text{ar}(\square RMXA) = 4 - 1 = 3 \text{ units}$$

$$\therefore \text{ar}(\triangle DRM) + \text{ar}(\triangle QNC)$$

$$= 1 + 1$$

$$= 2 \text{ units}$$

$$\text{and ar}(\triangle DRC) = 4 + 1 = 5 \text{ units}$$

$$\text{but from (i) ar}(\triangle DRC) = 10 \text{ unit (2 times)}$$

$$\therefore \text{ar}(\triangle DRM) + \text{ar}(\triangle QNC)$$

$$= 2 \times (2 \text{ times}) = 4 \text{ units}$$

$$\text{Similarly, ar}(\triangle APX) + \text{ar}(\triangle BOS)$$

$$= 2 \times 2$$

$$= 4 \text{ units}$$

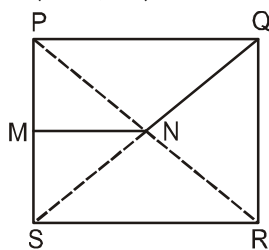
$$\therefore \text{total shaded area} = 4 + 4$$

$$= 8 \text{ units}$$

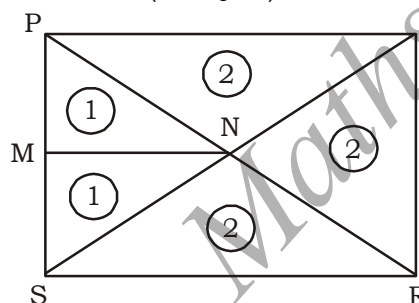
$$\& \text{ area of } \square \text{ ABCD} = 40 \text{ units}$$

$$\therefore \text{Required ratio} = \frac{8}{40} = \frac{1}{5}$$

62. PQRS is a square, M is the mid-point of side PS and N is the intersecting point of its diagonals. Then the ratio Area ($\square PQNM$) : Area ($\square PQRS$) is :



Sol. Let ar. ($\square PQRS$) = 8 units



$$\text{Area of } PQNM = \text{area of } \triangle PNQ$$

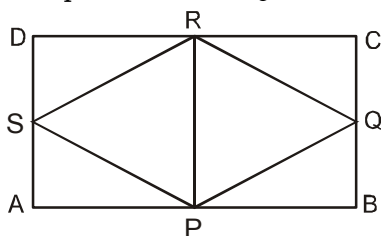
$$+ \text{area of } \triangle PNM$$

$$= 2 + 1 = 3 \text{ units}$$

$$\therefore \text{area}(\square PQNM) : \text{area}(\square PQRS) = 3 : 8$$

63. ABCD is a $\square$. P, Q, R and S are points on sides AB, BC, CD and DA respectively such that AP = DR. If the area of the $\square$ is

ABCD is 20cm^2 , then the area of quadrilateral PQRS is :



Sol. Area of ($\triangle PRS + \triangle PQR$) = $\frac{1}{2}$

(area of $\square APRD$)

$$+ \frac{1}{2}(\text{area of } \square BPRC)$$

$$= \frac{1}{2}(AP \times AD) + \frac{1}{2}(PB \times BC)$$

$$= \frac{1}{2}(AP \times AD) + \frac{1}{2}(PB \times AD)$$

$$(\because BC = AD)$$

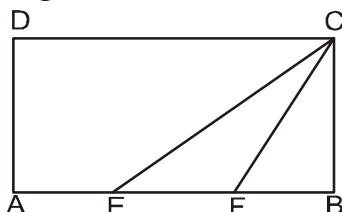
$$= \frac{1}{2}AD(AP + PB) =$$

$$\frac{1}{2}(AD \times AB)$$

$$= \frac{1}{2}(\text{area of } \square ABCD)$$

$$= \frac{1}{2} \times 20 = 10\text{cm}^2$$

64. In the given figure, ABCD is a rectangle with AE = EF = FB. What is the ratio of the area of the $\triangle CEF$ to that of the rectangle ?



Sol. Let BC = x and FB = EF = AE = y

$$\therefore AB = CD = 3y$$

$$\text{Now, Area of } \triangle CBF = \frac{1}{2}xy$$

$$\text{and area of } \triangle CBE =$$

$$\frac{1}{2}x \times 2y = xy$$

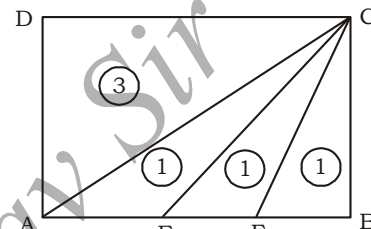
$$\therefore \text{area of } \triangle CEF$$

$$= xy - \frac{1}{2}xy = \frac{1}{2}xy$$

$$\text{and area of rectangle ABCD} = 3xy$$

$$\therefore \text{Required ratio} = \frac{1}{2}xy : 3xy = 1 : 6$$

Alternative:



Let ar($\square ABCD$) = 6 units

Base and height are same

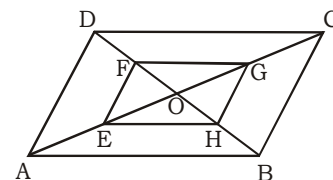
$$\text{ar}(\triangle CAE) = \text{ar}(\triangle CEF) =$$

$$\text{ar}(\triangle CFB) = 1 \text{ unit}$$

$$\therefore \text{required ratio} = 1 : 6$$

65. ABCD is a parallelogram in which diagonals AC and BD intersect at O. If E, F, G and H are the mid points of AO, DO, CO and BO respectively, then the ratio of the perimeter of the quadrilateral EFGH to the perimeter of parallelogram ABCD is:

Sol.



$\therefore$ E and H are mid points of OA and OB respectively

$$\Rightarrow EH = \frac{1}{2}AB$$

$$\text{similarly, } GH = \frac{1}{2}BC,$$

$$FG = \frac{1}{2}CD, EF = \frac{1}{2}AD$$

$$\text{Perimeter of parallelogram} = (AB + BC + CD + DA)$$

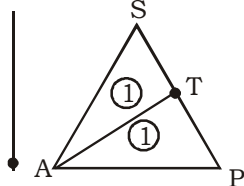
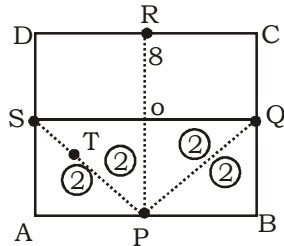
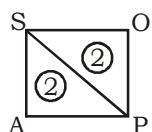
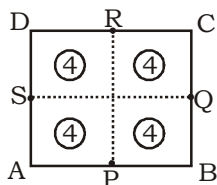
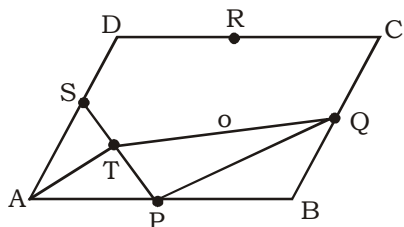
$$\text{Perimeter of quadrilateral EFGH} = EH + HG + FG + EF$$

$$= \frac{1}{2}(AB + BC + CD + DA)$$

$$\therefore \frac{\text{Perimeter of EFGH}}{\text{Perimeter of ABCD}} = 1 : 2$$

- 66.** A, B, C and D are the vertex of a $\parallel$ gram. P, Q, R and S are the mid points of side AB, BC, CD and DA. T is the midpoint of line PS. then find the ratio of Areas of $\triangle ATS$ and $\triangle PTQ$

Sol. Let the area of parallelogram = 16 units



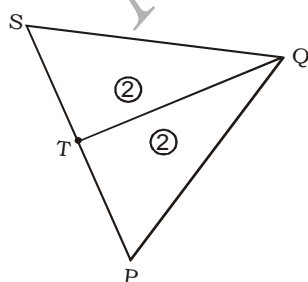
In $\triangle ASP$

Area of $\triangle ASP = 2$ units

T is the mid point of PS

The

Area of $\triangle ATS = 1$ units



Area of $\triangle PSQ = 4$ units

T is midpoint of SP

Then, Ar of $\triangle PTQ =$

$$\frac{1}{2} \times \text{Ar. of } \triangle PSQ$$

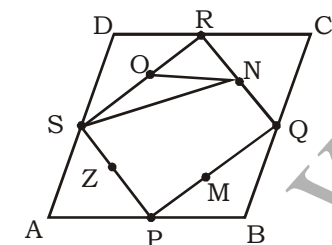
$$= \frac{1}{2} \times 4 = 2 \text{ units}$$

Now,

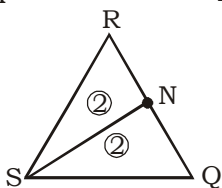
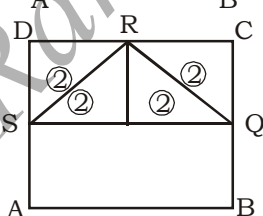
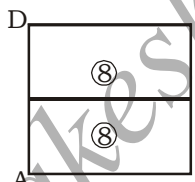
Ar of $\triangle ATS : \text{Ar of } \triangle PTQ = 1 : 2$

- 67.** A, B, C and D are the vertex of $\parallel$ gram P, Q, R and S are the mid-point AB, BC, CD and DA. M, N, O and Z are the mid points PQ, QR, RS, and SP. find ratio of area of $\triangle SON$ and $\square ABCD$.

Sol.



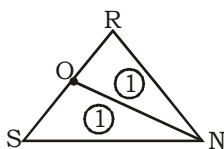
Let Area of $\square ABCD = 16$ units



Ar of $\triangle SRQ = 4$ units

N is the mid point on RQ

Then



Ar of $\triangle SRN = 2$ units

O is midpoint on RS

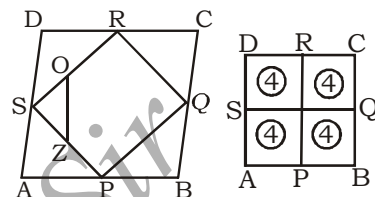
So,

Ar of $\triangle SON = 1$ unit

area of $\triangle SON : \text{area of } \square ABCD = 1 : 16$

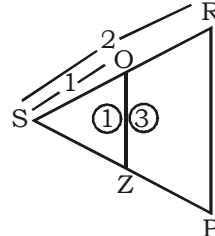
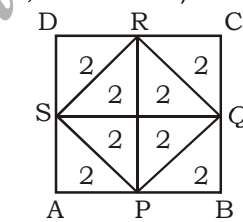
- 68.** A, B, C and D are the vertex of $\parallel$ gram P, Q, R and S are the mid point AB, BC, CD and DA. O and Z are the mid points RS and SP. Find ratio of area of $\triangle SOZ$ and $\square ABCD$

Sol.



Let the ar. of Parallelogram ABCD = 16 units.

(P, Q, R and S are the mid points of AB, BC, CD and DA)



Let SR = 2 cm

SO = 1 cm

O and Z are mid points SR and SP. Then,

$$OZ \parallel RP \text{ and } OZ = \frac{1}{2} RP$$

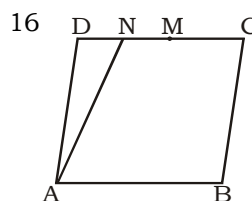
$$\frac{\text{Area of } \triangle SOZ}{\text{Area of } \triangle SRP} = \left(\frac{OS}{SR}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Now,

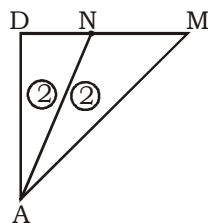
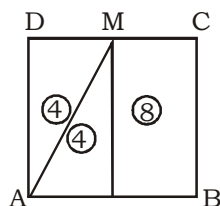
Area of $\triangle SOZ : \text{area of } \square ABCD = 1 : 16$

- 69.** A, B, C and D are the vertex of a $\parallel$ gram. M and N are the mid points of CD and MD respectively. Find the ratio of area of $\triangle ADN$ and $\square ABCD$

Sol. Let area of $\square ABCD =$



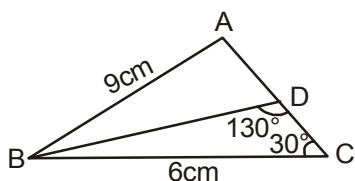
M is the mid point of CD
Ar. of $\triangle ADM = 4$ units



N is mid point of DM
Then area of $\triangle ADN = 2$ unit
So,
area of $\triangle ADN$: area of $\square ABCD$
 $= 2 : 16$
 $= 1 : 8$

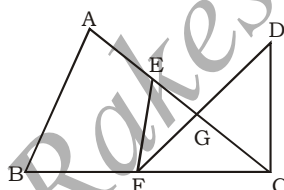
EXERCISE

1. In the given figure, $AD : DC = 3 : 2$, then $\angle ABC$:

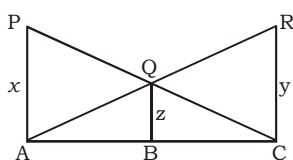


- (a) 30° (b) 40°
(c) 45° (d) 50°
2. Triangle ABC is such that $AB = 9$ cm, $BC = 6$ cm, $AC = 7.5$ cm. Triangle DEF is similar to $\triangle ABC$. If $EF = 12$ cm then DE is:
(a) 6 cm (b) 16 cm
(c) 18 cm (d) 15 cm
3. ABCD is a rhombus. A straight line through C cuts AD produced at P and AB produced at Q. If $DP = \frac{1}{2} AB$, then the ratio of the length of BQ and AB is
(a) $2 : 1$ (b) $1 : 2$
(c) $1 : 1$ (d) $3 : 1$
4. D and E are two points on the sides AC and BC respectively of $\triangle ABC$ such that $DE = 18$ cm, $CE = 5$ cm and $\angle DEC = 90^\circ$. If $\tan \angle ABC = 3.6$, then $AC : CD$ is
(a) $BC : 2 CE$ (b) $2CE : BC$
(c) $2BC : CE$ (d) $CE : 2BC$
5. In $\triangle ABC$ and $\triangle DEF$, $AB = DE$ and $BC = EF$, then one can infer that $\triangle ABC \cong \triangle DEF$, when
(a) $\angle BAC = \angle EDF$
(b) $\angle ACB = \angle EDF$
(c) $\angle ACB = \angle DFE$
(d) $\angle ABC = \angle DEF$

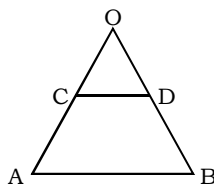
6. For a triangle ABC, D, E, F are the mid - points of its sides. if area of $\triangle ABC = 24$ sq. units, then area of $\triangle DEF$ is
(a) 4 sq. units (b) 6 sq. units
(c) 8 sq. units (d) 12 sq. units
7. In the adjoining figure (not drawn to scale) AB, EF and CD are parallel lines. Given that $EG = 5$ cm, $GC = 10$ cm and $DC = 18$ cm. Calculate AC, if $AB = 15$ cm:



- (a) 21 cm (b) 25 cm
(c) 18 cm (d) 28 cm
8. In the adjoining figure PA, QB and RC are each perpendicular to AC. Which one of the following is true :



- (a) $x + y = z$ (b) $xy = 2z$
(c) $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ (d) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$
9. In the given diagram $AB \parallel CD$, then which one of the following is true ?



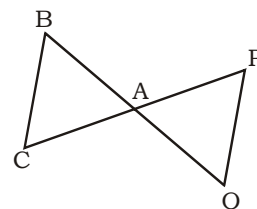
(a) $\frac{AB}{CD} = \frac{AO}{OC}$

(b) $\frac{AB}{CD} = \frac{BO}{OD}$

(c) $\triangle AOB \sim \triangle COD$

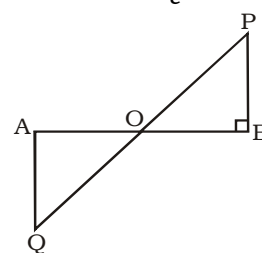
(d) All of these

10. In the figure $\triangle ACB \sim \triangle APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $AP = 2.8$ cm, find CA :



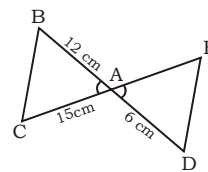
- (a) 8 cm (b) 6.5 cm
(c) 5.6 cm (d) None of these

11. In the figure QA and PB are perpendicular to AB. If $AO = 10$ cm, $BO = 6$ cm and $PB = 9$ cm. Find AQ :

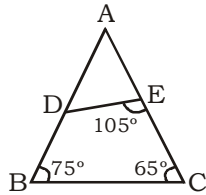


- (a) 8 cm (b) 9 cm
(c) 15 cm (d) 12 cm

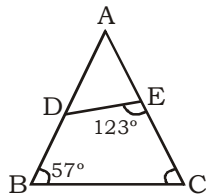
12. In the given figure $AB = 12$ cm, $AC = 15$ cm and $AD = 6$ cm. $BC \parallel DE$, Find the length of AE:



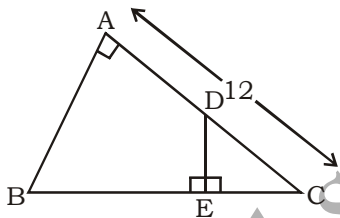
- (a) 6 cm (b) 7.5 cm
(c) 9 cm (d) 10 cm
13. In the given figure, if $\frac{DE}{BC} = \frac{2}{3}$ and $AE = 10$ cm. Find AB:



- (a) 10 cm (b) 15 cm
(c) 8 cm (d) 9 cm
14. In the figure $AD = 12$ cm, $AB = 20$ cm and $AE = 10$ cm. Find EC:

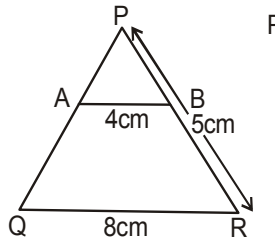


- (a) 14 cm (b) 10 cm
(c) 8 cm (d) 15 cm
15. In a right angle triangle $\triangle BAC$ in which $AB = BE = 5$ cm and $AC = 12$ cm. DE is perpendicular on BC. Then find DE?



- (a) 3.3 cm (b) 4.3 cm
(c) 5.3 cm (d) 6.3 cm
16. The side AB of a parallelogram ABCD is produced to E in such way that $BE = AB$, DE intersects BC at Q. The point Q divides BC in the ratio
(a) 1 : 2 (b) 1 : 1
(c) 2 : 3 (d) 2 : 1
17. ABCD is a trapezium whose side AD is parallel to BC, Diagonals AC and BD intersect at O. If $AO = 3$, $CO = x - 3$, $BO = 3x - 19$ and $DO = x - 5$, then the value(s) of x will be :
(a) 7, 6 (b) 12, 6
(c) 7, 10 (d) 8, 9

18. In the given figure, $AB \parallel QR$. Find the length of PB :



- (a) 2.5 cm (b) 2 cm
(c) 3 cm (d) 3.5 cm
19. The areas of two similar triangles are in the ratio 9 : 16. Their corresponding sides will be in the ratio :
(a) 3 : 5 (b) 3 : 4
(c) 4 : 5 (d) 4 : 3

20. If G is centroid and AD, BE, CF are three medians of $\triangle ABC$ with area 72cm^2 , then the area of $\triangle BDG$ is :
(a) 12cm^2 (b) 16cm^2
(c) 24cm^2 (d) 8cm^2

21. The three medians AD, BE and CF of $\triangle ABC$ intersect at G. If the area of $\triangle ABC$ is 60sq.cm then the area of the quadrilateral BDGF is :
(a) 10sq.cm (b) 15sq.cm
(c) 20sq.cm (d) 30sq.cm

22. The areas of the similar triangles are in the ratio of 25 : 36. What is the ratio of their respective heights ?
(a) 5 : 6 (b) 6 : 5
(c) 1 : 11 (d) 2 : 3

23. Given that : $\triangle ABC \sim \triangle PQR$, if $\frac{\text{area}(\triangle PQR)}{\text{area}(\triangle ABC)} = \frac{256}{441}$ and $PR = 12$ cm, then AC is equal to?
(a) $12\sqrt{2}$ cm (b) 15.5 cm
(c) 16 cm (d) 15.75 cm

24. Two isosceles triangles have equal vertical angles and their areas are in the ratio 9 : 16. then the ratio of their corresponding heights is
(a) 4.5 : 8 (b) 3 : 4
(c) 4 : 3 (d) 8 : 4.5

25. Given that the ratio of altitudes of two triangles is 4:5, ratio of their areas is 3 : 2, The ratio of their corresponding bases is

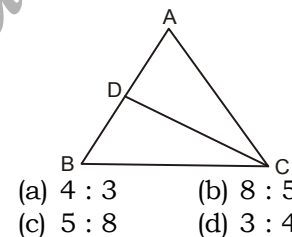
- (a) 5 : 8 (b) 15 : 8
(c) 8 : 5 (d) 8 : 15

26. ABCD is a square. Draw a triangle QBC on side BC considering BC as base and draw a triangle PAC on AC as its base such that $\triangle QBC \sim \triangle PAC$ then,

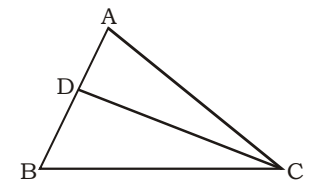
$\frac{\text{Area of } \triangle QBC}{\text{Area of } \triangle PAC}$ is equal to :

- (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{1}$

27. In the given figure, $\angle BAC = \angle BCD$, $AB = 32\text{cm}$ and $BD = 18\text{cm}$, then the ratio of perimeter of $\triangle BCD$ and $\triangle ABC$ is :



- (a) 4 : 3 (b) 8 : 5
(c) 5 : 8 (d) 3 : 4
28. In the given figure In $\triangle ABC$, $\angle BAC = \angle BCD$ and $AD = 14$ cm, $AB = 32$ cm,



Find $\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle BDC}$?

- (a) $\frac{4}{3}$ (b) $\frac{5}{3}$ (c) $\frac{6}{3}$ (d) $\frac{7}{3}$

29. ABC is a right-angled triangle. AD is perpendicular to the hypotenuse BC. If $AC = 2AB$, then the value of BD is :

- (a) $\frac{BC}{2}$ (b) $\frac{BC}{3}$
(c) $\frac{BC}{4}$ (d) $\frac{BC}{5}$

30. The difference between altitude and base of a right angled triangle is 17 cm and its hypotenuse is 25 cm. What is the sum of the base and altitude of the triangle is :
(a) 30 cm (b) 31 cm
(c) 32 cm
(d) can't be determined

31. One side other than the hypotenuse of right angle isosceles triangle is 6 cm. The length of the perpendicular on the hypotenuse from the opposite vertex is :

(a) 6 cm (b) $6\sqrt{2}$ cm
(c) 4 cm (d) $3\sqrt{2}$ cm

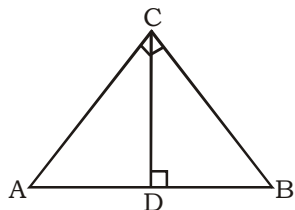
32. In right angled $\triangle ABC$, $\angle B = 90^\circ$, If P and Q are points on the sides AB and BC respectively, then :

(a) $AQ^2 + CP^2 = PQ^2 + AC^2$
(b) $AB^2 + CP^2 = PQ^2 + AC^2$
(c) $2(AQ^2 + CP^2) = PQ^2 + AC^2$
(d) $AQ^2 + CP^2 = 2(PQ^2 + AC^2)$

33. If ABC is right angled triangle at B and M, N are the mid-points of AB and BC, then $4(AN^2 + CM^2)$ is equal to:

(a) $4AC^2$ (b) $6AC^2$
(c) $5AC^2$ (d) $\frac{5}{4}AC^2$

34. In a right angled $\triangle ABC$, $\angle C = 90^\circ$ and CD is the perpendicular on hypotenuse AB if BC = 15 cm and AC = 20 cm then CD is equal to :



(a) 18 cm (b) 12 cm
(c) 17.5 cm
(d) can't be determined

35. ABC is a right angle triangle at A and AD is perpendicular to the hypotenuse. Then $\frac{BD}{CD}$ is equal to :

(a) $\left(\frac{AB}{AC}\right)^2$ (b) $\left(\frac{AB}{AD}\right)^2$
(c) $\frac{AB}{AC}$ (d) $\frac{AB}{AD}$

36. Suppose $\triangle ABC$ be a right-angled triangle where $\angle A = 90^\circ$ and $AD \perp BC$. If $\text{ar}(\triangle ABC) = 40\text{cm}^2$, $\text{ar}(\triangle ACD) = 10\text{cm}^2$ and AC = 9 cm, then the length of BC is

(a) 12 cm (b) 18 cm
(c) 4 cm (d) 6 cm

37. In a triangle ABC, $\angle BAC = 90^\circ$ and AD is perpendicular to BC. If AD = 6 cm and BD = 4 cm then the length of BC is :

(a) 8 cm (b) 10 cm
(c) 9 cm (d) 13 cm

38. In a right angled $\triangle ABC$, $\angle ABC = 90^\circ$, AB = 3, BC = 4, CA = 5; BN is perpendicular to AC, AN : NC is

(a) 3 : 4 (b) 9 : 16
(c) 3 : 16 (d) 1 : 4

39. In $\triangle ABC$ $\angle A = 90^\circ$ and $AD \perp BC$ where D lies on BC. If BC = 8 cm, AC = 6 cm, then area of $\triangle ABC$: area of $\triangle ACD$ = ?

(a) 4 : 3 (b) 25 : 16
(c) 16 : 9 (d) 25 : 9

40. A point D is taken from the side BC of a right-angled triangle ABC, where AB is hypotenuse. Then

(a) $AB^2 + CD^2 = BC^2 + AD^2$
(b) $CD^2 + BD^2 = 2AD^2$
(c) $AB^2 + AC^2 = 2AD^2$
(d) $AB^2 = AD^2 + BC^2$

41. BL and CM are medians of $\triangle ABC$ right-angled at A and BC

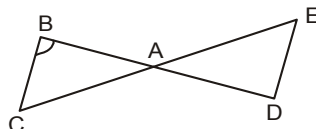
= 5 cm. If $BL = \frac{3\sqrt{5}}{2}$ cm, then the length of CM is

(a) $2\sqrt{5}$ cm (b) $5\sqrt{2}$ cm
(c) $10\sqrt{2}$ cm (d) $4\sqrt{5}$ cm

42. Q is a point in the interior of a rectangle ABCD, if QA = 3 cm, QB = 4 cm and QC = 5 cm then the length of QD (in cm) is

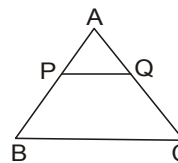
(a) $3\sqrt{2}$ (b) $5\sqrt{2}$
(c) $\sqrt{34}$ (d) $\sqrt{41}$

43. In the given figure, AB = 18cm, AC = 11cm and AD = 9 cm and BC || DE, find the length of AE



(a) 6 cm (b) 7 cm
(c) 5.5 cm (d) 7.5 cm

44. In the given triangle ABC, BP = 3 AP, QC = 3AQ and BC = 36cm. Find the value of PQ ?



(a) 9cm (b) 8cm
(c) 6cm (d) 7cm

45. A straight line parallel to base BC of the triangle ABC intersects AB and AC at the points D and E respectively. If the area of the $\triangle ACD$ is 36sq cm, then the area of $\triangle ABE$ is :

(a) 36 sq.cm (b) 18 sq.cm
(c) 12 sq.cm (d) None of these

46. In $\triangle ABC$, P and Q are the mid points of the sides AB and AC respectively. R is a point on the segment PQ such that PR : RQ = 1 : 2, If PR = 2cm, then BC =

(a) 4 cm (b) 2 cm
(c) 12 cm (d) 6 cm

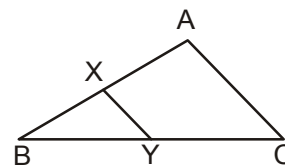
47. Find the maximum area that can be enclosed in a triangle of perimeter 24cm :

(a) 32 cm^2 (b) $16\sqrt{3}\text{ cm}^2$

(c) $16\sqrt{2}\text{ cm}^2$ (d) 27 cm^2

48. In the given figure, the line segment XY || AC and it divides the triangle into two parts of equal areas. Find

ratio $\frac{AX}{AB}$:



(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{2}}{2}$ (d) $\frac{\sqrt{2}}{2}$

49. D and E are the mid-points of AB and AC of $\triangle ABC$, BC is produced to any point P; DE, DP and EP are joined. then, area of :

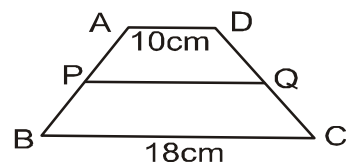
(a) $\Delta PED = \frac{1}{4} \Delta ABC$

(b) $\Delta PED = \Delta BEC$

(c) $\Delta ADE = \Delta BEC$

(d) $\Delta BDE = \Delta BEC$

50. In the given figure, ABCD is a trapezium such that $AD \parallel BC$ and P, Q are the points on AB and CD respectively such that $PQ \parallel AD$ and $AP : PB = 5 : 3$. Then PQ is :

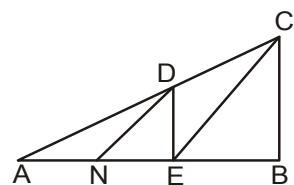


- (a) 12.5cm (b) 15cm
(c) 17.5cm (d) 20cm

51. In a ΔABC , D is the mid-point of BC and E is the mid-point of AD. The line BE is extended and it intersects AC at T. If $AB = 18$ cm, $BC = 17$ cm and $AC = 15$ cm. Find TC ?

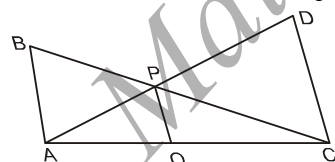
- (a) 8cm (b) 9cm
(c) 10cm (d) 7cm

52. In the given figure, $DE \parallel BC$ and $EC \parallel ND$, $AE : EB = 4 : 5$, then $EN : EB$ is :



- (a) 5 : 9 (b) 9 : 4
(c) 4 : 5 (d) 4 : 9

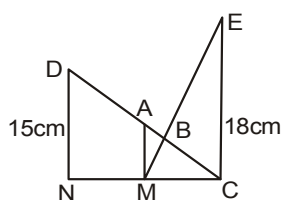
53. In the given figure, $AB \parallel CD \parallel PQ$, $AB = 12$ cm, $CD = 18$ cm and $AC = 6$ cm. Then PQ is:



- (a) $\frac{36}{5}$ cm (b) $\frac{18}{5}$ cm

- (c) 9cm (d) $\frac{14}{5}$ cm

54. In the given figure, $EC \parallel AM \parallel DN$ and $AB = 5$ cm, $BC = 10$ cm. Find DC:



- (a) 19 cm (b) 20cm
(c) 25 cm (d) 17.5cm

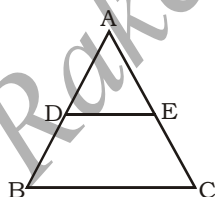
55. The triangle is formed by joining the mid-points of the sides AB, BC and CA of ΔABC and the area of ΔPQR is 6 cm^2 , then the area of ΔABC is

- (a) 36 cm^2 (b) 12 cm^2
(c) 18 cm^2 (d) 24 cm^2

56. ABC is a triangle and DE is drawn parallel to BC cutting the other side at D and E. If $AB = 3.6$ cm, $AC = 2.4$ cm and $AD = 2.1$ cm, then AE is equal to:

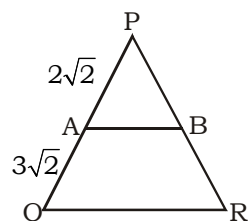
- (a) 1.4 cm (b) 1.8 cm
(c) 1.2 cm (d) 1.05 cm

57. In ΔABC , $AC = 5$ cm. Calculate the length of AE where $DE \parallel BC$. Given that $AD = 3$ cm and $BD = 7$ cm :



- (a) 2 cm (b) 1 cm
(c) 1.5 cm (d) 2.5 cm

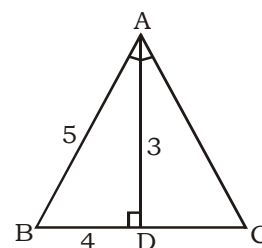
58. In ΔPQR , $AP = 2\sqrt{2}$ cm, $AQ = 3\sqrt{2}$ cm and $PR = 10$ cm, $AB \parallel QR$. Find the length of BR :



- (a) $6\sqrt{2}$ cm (b) 6 cm

- (c) $5\sqrt{2}$ cm
(d) None of these

59. In the adjoining figure the $\angle BAC$ and $\angle ADB$ are right angles. $BA = 5$ cm, $AD = 3$ cm and $BD = 4$ cm, what is the length of DC ?



- (a) 2.5 (b) 3
(c) 2.25 (d) 2

60. ABC is a triangle in which $\angle A = 90^\circ$, AN is perpendicular to BC, $AC = 12$ cm and $AB = 5$ cm. Find the ratio of the areas of ΔANB and ΔANC :

- (a) 125 : 44 (b) 25 : 144
(c) 144 : 25 (d) 12 : 5

61. Area of $\Delta ABC = 30 \text{ cm}^2$. D and E are the mid-points of BC and AB respectively. Find Ar (ΔBDE) :

- (a) 10 cm (b) 7.5 cm
(c) 15 cm
(d) None of these

62. ABCD is a trapezium in which $AB \parallel CD$ and E and F are the point on DA & BC, $DC = 40$ cm, $AB = 105$ cm. The ratio of

$$\frac{DE}{EA} = \frac{2}{3}.$$
 Then find EF.

- (a) 66 cm (b) 70 cm
(c) 72 cm (d) 73 cm

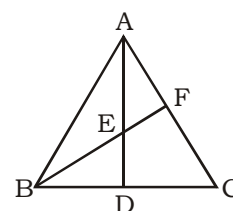
63. In ΔABC , D and E are mid-point of side AB and BC. P is point on line AC in such a way that R and S are the mid-point of AP & PC find DR : ES

- (a) 1 : 1 (b) 2 : 1
(c) 4 : 1 (d) 3 : 1

64. In the given figure

$$\frac{AE}{ED} = \frac{BD}{DC} = \frac{2}{3}, \quad AC = 760 \text{ cm}$$

Then find AE = ?



- (a) 80 cm (b) 90 cm
(c) 100 cm (d) 120 cm

65. The length of the diagonal BD of the parallelogram ABCD is 18 cm. If P and Q are the centroid of the $\triangle ABC$ and $\triangle ADC$ respectively then the length of the line segment PQ is
(a) 4 cm (b) 6 cm
(c) 9 cm (d) 12 cm
66. A straight line parallel to BC of $\triangle ABC$ intersects AB and AC at points P and Q respectively. $AP = QC$, $PB = 4$ units and $AQ = 9$ units, then the length of AP is :
(a) 25 units (b) 3 units
(c) 6 units (d) 6.5 units
67. D is any point on side AC of $\triangle ABC$. If P, Q, X, Y are the mid-points of AB, BC, AD and DC respectively, then the ratio of PX and QY is
(a) 1 : 2 (b) 1 : 1
(c) 2 : 1 (d) 2 : 3
68. In $\triangle ABC$, PQ is parallel to BC. If $AP : PB = 1 : 2$ and $AQ = 3$ cm; AC is equal to
(a) 6 cm (b) 9 cm
(c) 12 cm (d) 8 cm
69. In $\triangle ABC$, D and E are points on AB and AC respectively such that $DE \parallel BC$ and DE divides the $\triangle ABC$ into two parts of equal areas. Then ratio of AD and BD is
(a) 1 : 1 (b) $1 : \sqrt{2} - 1$
(c) $1 : \sqrt{2}$ (d) $1 : \sqrt{2} + 1$
70. ABCD is a rectangle where the ratio of the length of AB and BC is 3 : 2. If P is the mid-point of AB, then the value of $\sin \angle CPB$ is
(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{5}$
71. Inside a triangle ABC, a straight line parallel to BC intersects AB and AC at the point P and Q respectively. If $AB = 3$ PB, then PQ : BC is
(a) 1 : 3 (b) 3 : 4
(c) 1 : 2 (d) 2 : 3
72. In $\triangle ABC$, $DE \parallel AC$, D and E are two points on AB and CB respectively. If $AB = 10$ cm and $AD = 4$ cm, then BE : CE is
(a) 2 : 3 (b) 2 : 5
(c) 5 : 2 (d) 3 : 2
73. For a triangle ABC, D and E are two points on AB and AC such that $AD = \frac{1}{4} AB$, $AE = \frac{1}{4} AC$. If BC = 12 cm, then DE is
(a) 5 cm (b) 4 cm
(c) 3 cm (d) 6 cm
74. In $\triangle PQR$, S and T are point on sides PR and PQ respectively such that $\angle PQR = \angle PST$, If PT = 5 cm, PS = 3 cm and TQ = 3 cm, then length of SR is
(a) 5 cm (b) 6 cm
(c) $\frac{31}{3}$ cm (d) $\frac{41}{3}$ cm
75. In $\triangle ABC$, two points D and E are taken on the lines AB and BC respectively in such a way that AC is parallel to DE. Then $\triangle ABC$ and $\triangle DBE$ are
(a) similar only If D lies outside the line segment AB
(b) congruent only If D lies outside the line segment AB
(c) always similar
(d) always congruent
76. In triangle ABC a straight line parallel to BC intersects AB and AC at D and E respectively. If $AB = 2AD$, then DE : BC is
(a) 2 : 3 (b) 2 : 1
(c) 1 : 2 (d) 1 : 3
77. In a $\triangle ABC$, D and E are two points on AB and AC respectively such that $DE \parallel BC$. DE bisects the $\triangle ABC$ in two equal areas. Then the ratio BD : AB is
(a) $1 : \sqrt{2}$ (b) 1 : 2
(c) $(\sqrt{2} - 1) : \sqrt{2}$ (d) $\sqrt{2} : 1$
78. If in a triangle ABC, D and E are on the sides AB and AC, such that, DE is parallel to BC and $\frac{AD}{BD} = \frac{3}{5}$. If AC = 4 cm, then AE is
(a) 1.5 cm (b) 2.0 cm
(c) 1.8 cm (d) 2.4 cm
79. ABC is a triangle in which $DE \parallel BC$ and $AD : DB = 5 : 4$. Then DE : BC is
(a) 4 : 5 (b) 9 : 5
(c) 4 : 9 (d) 5 : 9
80. D and E are mid-points of sides AB and AC respectively of the $\triangle ABC$. A line drawn from A meets BC at H and DE at K. $AK : KH = ?$
(a) 2 : 1 (b) 1 : 1
(c) 1 : 3 (d) 1 : 2
81. If ABC be an equilateral triangle and AD perpendicular to BC, Then $AB^2 + BC^2 + CA^2 = ?$
(a) $3AD^2$ (b) $5AD^2$
(c) $2AD^2$ (d) $4AD^2$

ANSWER KEY

1. (b)	9. (d)	17. (d)	25. (b)	33. (c)	41. (a)	49. (a)	57. (c)	65. (b)	73. (c)
2. (c)	10. (c)	18. (b)	26. (c)	34. (b)	42. (a)	50. (b)	58. (b)	66. (c)	74. (c)
3. (a)	11. (c)	19. (b)	27. (d)	35. (a)	43. (c)	51. (c)	59. (c)	67. (b)	75. (c)
4. (a)	12. (b)	20. (a)	28. (a)	36. (b)	44. (a)	52. (d)	60. (b)	68. (b)	76. (c)
5. (d)	13. (b)	21. (c)	29. (d)	37. (d)	45. (a)	53. (a)	61. (b)	69. (b)	77. (c)
6. (b)	14. (a)	22. (a)	30. (b)	38. (b)	46. (c)	54. (c)	62. (a)	70. (d)	78. (a)
7. (b)	15. (a)	23. (d)	31. (d)	39. (c)	47. (b)	55. (d)	63. (a)	71. (d)	79. (d)
8. (c)	16. (b)	24. (b)	32. (a)	40. (a)	48. (d)	56. (a)	64. (a)	72. (d)	80. (b)
									81. (d)

SOLUTION

1. (b) $\frac{AD}{DC} = \frac{3}{2}$ and $\frac{AB}{BC} = \frac{3}{2}$,

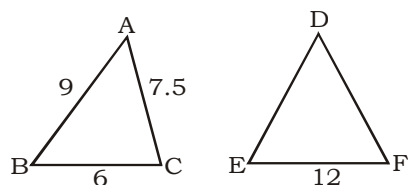
$$\therefore \frac{AD}{DC} = \frac{AB}{BC}$$

$\therefore$ BD is the bisector of $\angle B$

$$\text{Now, } \angle CBD = 180^\circ - (130^\circ + 30^\circ) = 20^\circ$$

$$\therefore \angle B = 2(\angle CBD) = 40^\circ$$

2. (c) According to the question,



$\triangle ABC \sim \triangle DEF$

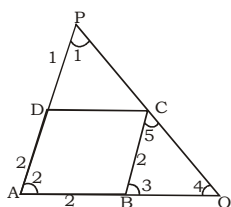
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{9}{DE} = \frac{6}{12}$$

$$DE = 18 \text{ cm}$$

3. (a) According to question,

Given:



ABCD is a rhombus
 $AB = BC = CD = DA$

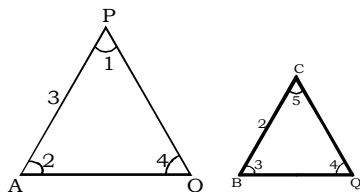
$$DP = \frac{1}{2} AB$$

$$\frac{DP}{AB} = \frac{1}{2}$$

In a rhombus $\angle 2 = \angle 3$

$\therefore \triangle APQ \sim \triangle BCQ$

($\because \angle Q$ is common angle $\angle 2 = \angle 3$)



$$\frac{AP}{BC} = \frac{AQ}{BQ} \quad \frac{AP}{BC} = \frac{3}{2}$$

$$\frac{AB+BQ}{BQ} = \frac{3}{2}$$

$$(\because AQ = AB + BQ)$$

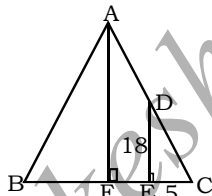
$$\frac{AB}{BQ} + 1 = \frac{3}{2}$$

$$\frac{AB}{BQ} = \frac{3}{2} - 1$$

$$\frac{AB}{BQ} = \frac{1}{2}$$

$$\therefore \frac{BQ}{AB} = \frac{2}{1}$$

4. (a) According to question



Given: $DE = 18 \text{ cm}$

$EC = 5 \text{ cm}$

$$\tan \angle ABC = 3.6$$

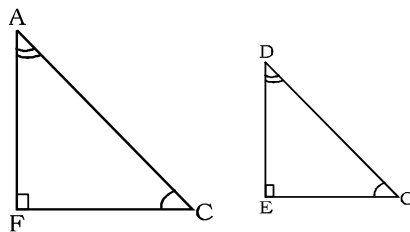
$$\tan C = \frac{DE}{EC}$$

$$\tan C = \frac{18}{5} = 3.6$$

$$\therefore \tan \angle ABC = \tan \angle ACB$$

Note:- In a isosceles triangle perpendicular bisects the opposite sides

$$\therefore \triangle AFC \sim \triangle DEC$$



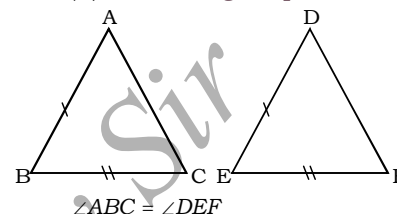
$$\frac{AF}{DE} = \frac{AC}{DC} = \frac{FC}{EC}$$

$$\therefore \frac{AC}{CD} = \frac{FC}{EC} (\because FC = \frac{BC}{2})$$

$$\frac{AC}{CD} = \frac{BC}{2EC}$$

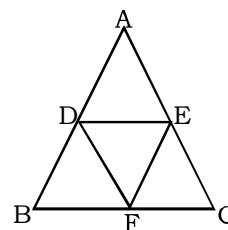
$$\Rightarrow AC : CD = BC : 2EC$$

5. (d) According to question



Note: Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the included angles of the other triangle (SAS criterion).

6. (b) According to question



As we know that

Given: area of $\triangle ABC = 24$ square units

D, E and F are the midpoints of AB, AC and BC

$\therefore$ Area of $\triangle ADE =$ area of $\triangle DBF$
 $=$ area of $\triangle DEF =$ area of $\triangle EFC$

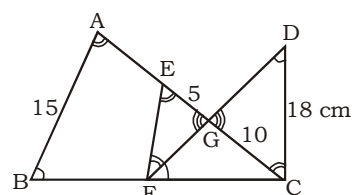
$\therefore$ Area of $\triangle DEF$

$$= \frac{1}{4} \text{ area of } \triangle ABC$$

$$\text{Area of } \triangle DEF = \frac{1}{4} \times 24$$

$$= 6 \text{ sq. units}$$

7. (b) According to the question,



$$\angle E = \angle C$$

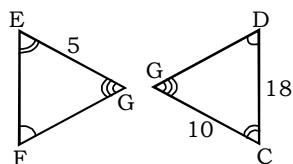
$$\angle EFG = \angle D$$

$$\therefore \triangle GEF \sim \triangle GCD$$

$$\frac{EF}{CD} = \frac{EG}{CG}$$

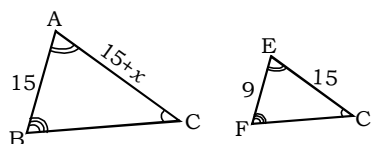
$$\frac{EF}{18} = \frac{5}{10}$$

$$EF = 9 \text{ cm}$$



$$\angle A = \angle E$$

$$\angle B = \angle F$$



$$\therefore \triangle CAB \sim \triangle CEF$$

$$\frac{AB}{EF} = \frac{AC}{EC}$$

$$\frac{15}{9} = \frac{15+x}{15}$$

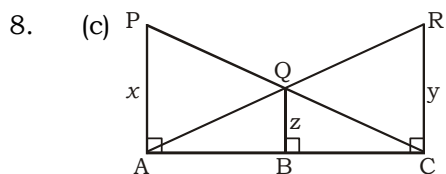
$$135 + 9x = 225$$

$$9x = 90$$

$$x = 10 \text{ cm}$$

$$\therefore AC = 15 + x$$

$$AC = 15 + 10 = 25 \text{ cm}$$



In $\triangle APC$ & $\triangle BQC$

$$\angle PAC = \angle QBC \text{ (both } 90^\circ)$$

$$\angle C = \angle C \text{ (common angle)}$$

$$\therefore \triangle APC \sim \triangle BQC \text{ (AA similarity)}$$

So,

$$\frac{AC}{BC} = \frac{x}{z} \text{ (sides are proportional)}$$

$$\text{or } \frac{AB+BC}{BC} = \frac{x}{z}$$

$$\frac{AB}{BC} + 1 = \frac{x}{z}$$

$$\frac{AB}{BC} = \frac{x}{z} - 1 = \frac{x-z}{z} \quad \dots(i)$$

Similarly,

$$\triangle CRA \sim \triangle BQA$$

$$\text{So, } \frac{AC}{AB} = \frac{y}{z}$$

$$\frac{AB+BC}{AB} = \frac{y}{z}$$

$$1 + \frac{BC}{AB} = \frac{y}{z}$$

$$\frac{BC}{AB} = \frac{y}{z} - 1$$

$$\frac{BC}{AB} = \frac{y-z}{z}$$

$$\text{or } \frac{AB}{BC} = \frac{z}{y-z}$$

.....(ii)

From (i) & (ii)

$$\frac{x-z}{z} = \frac{z}{y-z}$$

$$\therefore z^2 = xy - xz - zy + z^2$$

$$xy = zx + zy$$

(divide xyz both sides)

$$\frac{1}{z} = \frac{1}{y} + \frac{1}{x}$$

9. (d) According to the question,

$$\angle DOC = \angle AOB \text{ (Common)}$$

$$\angle OCD = \angle OAB$$

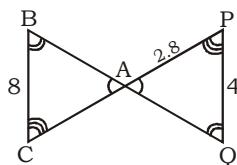
$$\angle ODC = \angle OBA$$

$$\therefore \triangle COD \sim \triangle AOB$$

$$\frac{CD}{AB} = \frac{OC}{AO} = \frac{OD}{OB}$$

10. (c) According to the question,

$$\triangle ACB \sim \triangle APQ$$

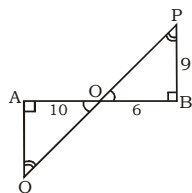


$$\frac{BC}{PQ} = \frac{CA}{AP}$$

$$\frac{8}{4} = \frac{CA}{2.8}$$

$$CA = 5.6 \text{ cm}$$

11. (c) According to the question,



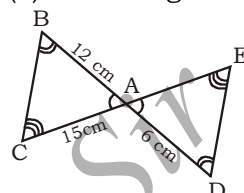
$$\triangle OAQ \sim \triangle OBP$$

$$\frac{AQ}{BP} = \frac{OA}{OB}$$

$$\frac{AQ}{9} = \frac{10}{6}$$

$$AQ = 15 \text{ cm}$$

12. (b) According to the question,



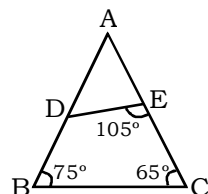
$$\triangle ACB \sim \triangle AED$$

$$\frac{AC}{AE} = \frac{AB}{AD}$$

$$\frac{15}{AE} = \frac{12}{6}$$

$$AE = 7.5 \text{ cm}$$

13. (b) According to the question,



$$\angle AED = 180^\circ - 105^\circ = 75^\circ$$

$$\angle A = 180^\circ - 75^\circ - 65^\circ$$

$$\angle A = 40^\circ$$

$$\angle AED = \angle ABC$$

$$\angle ADE = \angle ACB$$

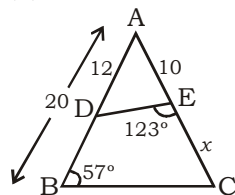
$$\therefore \triangle ABC \sim \triangle ADE$$

$$\frac{BC}{DE} = \frac{AB}{AE}$$

$$\frac{3}{2} = \frac{AB}{10}$$

$$AB = 15 \text{ cm}$$

14. (a) According to the question,



$$\angle AED = 180^\circ - 123^\circ = 57^\circ$$

$$\angle D = \angle C$$

$$\angle E = \angle B$$

$$\therefore \triangle ABC \sim \triangle AED$$

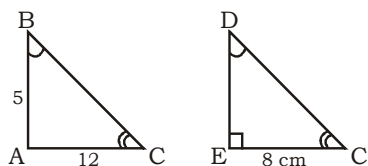
$$\frac{AB}{AE} = \frac{AC}{AD}$$

$$\frac{20}{10} = \frac{10+x}{12}$$

$$24 = 10 + x$$

$$x = 14 \text{ cm}$$

15. (a) According to the question,
 $AB = BE = 5 \text{ cm}$
 $AC = 12 \text{ cm}$
 $\angle BAC = \angle DEC$ [each 90°]
 $\angle ECD = \angle ACB$ [common]
 $\therefore \triangle ABC \sim \triangle EDC$
 In $\triangle ABC$ & $\triangle EDC$
 By using pythagoras theorem.



$$BC^2 = AB^2 + AC^2$$

$$BC^2 = (5)^2 + (12)^2$$

$$BC^2 = 25 + 144$$

$$BC = 13 \text{ cm}$$

$$\therefore BC = BE + EC$$

$$EC = 13 - 5$$

$$EC = 8 \text{ cm}$$

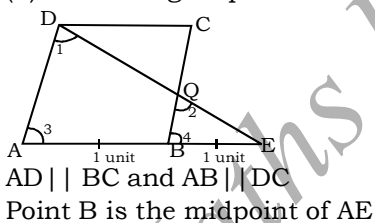
$$\therefore \frac{AB}{DE} = \frac{AC}{EC}$$

$$\frac{5}{DE} = \frac{12}{8}$$

$$DE = \frac{10}{3}$$

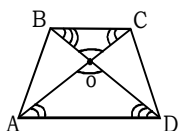
$$DE = 3.3 \text{ cm}$$

16. (b) According to question

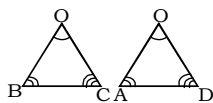


$\angle 1 = \angle 2$
 (Corresponding Angle)
 $\angle 3 = \angle 4$
 (Corresponding Angle)
 $\therefore \triangle EQB \sim \triangle EDA$
 $\therefore \frac{EB}{EA} = \frac{EQ}{ED} = \frac{QB}{AD} = \frac{1}{2}$
 $QC : QB = 1 : 1$

17. (d) According to question,



$$\triangle AOD \sim \triangle COB$$



$$\therefore \frac{OB}{OD} = \frac{OC}{OA}$$

$$\frac{3x-19}{x-5} = \frac{x-3}{3}$$

$$9x-57 = x^2-8x+15$$

$$x^2-17x+72 = 0$$

$$x(x-8)-9(x-8) = 0$$

$$(x-8)(x-9) = 0$$

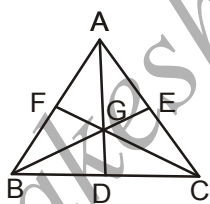
$$x = 8 \text{ or } 9$$

18. (a) $\frac{AB}{QR} = \frac{PB}{PR}$

$$\frac{4}{8} = \frac{PB}{5} \Rightarrow PB = 2.5 \text{ cm}$$

19. (b) Required ratio = $\sqrt{\frac{9}{16}} = \frac{3}{4}$
 $= 3 : 4$

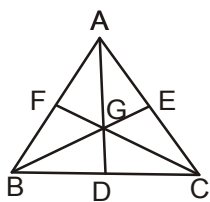
20. (a)



Area of $\triangle BDG = \frac{1}{6} \times$ Area of $\triangle ABC$

$$= \frac{1}{6} \times 72 = 12 \text{ cm}^2$$

21. (c)

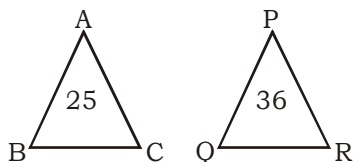


Required area of $\square BDGF$

$$= \frac{1}{3} \times 60 = 20 \text{ sq.cm.}$$

22. (a) According to the question,

As we know that

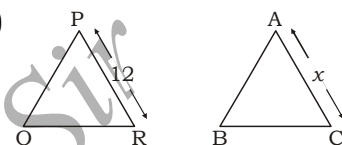


$$\frac{AB^2}{PQ^2} = \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR}$$

$$\frac{AB}{PQ} = \sqrt{\frac{25}{36}}$$

$$\frac{AB}{PQ} = \frac{5}{6}$$

23. (d)



$$\therefore \triangle PQR \sim \triangle ABC$$

$\Rightarrow$ We know that in similar triangle

$$\frac{\text{Area of triangle}_1}{\text{Area of triangle}_2} = \left(\frac{\text{Corresponding side}_1}{\text{Corresponding side}_2} \right)^2$$

$$\Rightarrow \frac{\triangle PQR}{\triangle ABC} = \frac{256}{441}$$

$$\Rightarrow \frac{\triangle PQR}{\triangle ABC} = \left(\frac{PR}{AC} \right)^2$$

$$\Rightarrow \frac{256}{441} = \left(\frac{12}{AC} \right)^2$$

$$\Rightarrow \left(\frac{16}{21} \right)^2 = \left(\frac{12}{AC} \right)^2$$

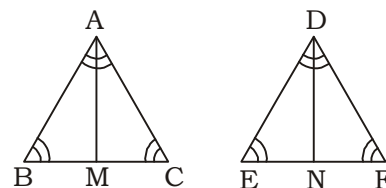
$$\Rightarrow \frac{16}{21} = \frac{12}{AC}$$

$$\Rightarrow \frac{4}{21} = \frac{3}{AC}$$

$$\Rightarrow AC = \frac{63}{4}$$

$$\Rightarrow AC = 15.75 \text{ cm.}$$

24. (b)



If two isosceles triangles have equal vertical angles then both triangle is similar

So, $\triangle ABC \sim \triangle DEF$

We know,

In similarity case

$$\Rightarrow \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{(AB)^2}{(DE)^2} \text{ corresponding sides square}$$

$$= \frac{(AM)^2}{(DN)^2} \text{ height}$$

$$\Rightarrow \frac{9}{16} = \frac{(AM)^2}{(DN)^2}$$

$$= \sqrt{\frac{9}{16}} = \text{Ratio of their height}$$

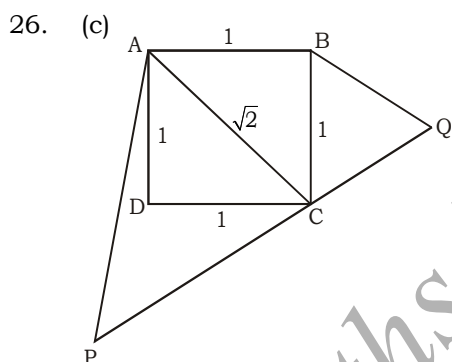
$$\Rightarrow \text{Ratio of height} = 3:4$$

$$25. (b) \frac{\text{area of triangle}_1}{\text{area of triangle}_2} = \frac{3}{2}$$

$$\frac{\frac{1}{2} \times B_1 \times 4}{\frac{1}{2} \times B_2 \times 5} = \frac{3}{2}$$

$$\frac{B_1}{B_2} = \frac{3}{2} \times \frac{5}{4} = \frac{15}{8}$$

$$B_1 : B_2 = 15 : 8 \text{ Ans.}$$



Given

$$\therefore \triangle QBC \sim \triangle PAC$$

$$\Rightarrow \text{Let each side of square} = 1$$

$$\Rightarrow \text{then diagonal of square} = \sqrt{2}$$

$$\Rightarrow \therefore \triangle QBC \sim \triangle PAC$$

$$\Rightarrow \frac{\text{Area of } \triangle QBC}{\text{Area of } \triangle PAC} = \frac{(BC)^2}{(AC)^2}$$

$$= \frac{(QC)^2}{(PC)^2} = \frac{(QB)^2}{(PA)^2} = \frac{1^2}{(\sqrt{2})^2} = \frac{1}{2}$$

27. (d) In $\triangle ABC$ and $\triangle BDC$
 $\angle BAC = \angle BCD$ (given)
and $\angle B = \angle B$ (common)
 $\therefore \triangle ABC \sim \triangle CBD$

$$\frac{AB}{BC} = \frac{BC}{BD} \Rightarrow \frac{32}{BC} = \frac{BC}{18}$$

$$\Rightarrow BC^2 = 18 \times 32$$

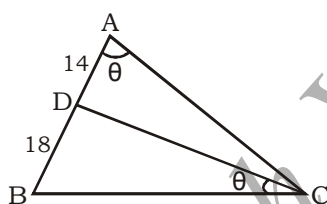
$$\Rightarrow BC = 24 \text{ cm}$$

$$\therefore \text{perimeter of } \triangle BCD : \text{perimeter of } \triangle ABC = BC : AB$$

$$= 24 : 32$$

$$= 3 : 4$$

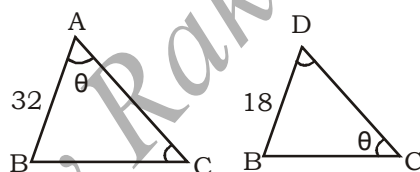
28. (a) According to the question,



$$\text{Let } \angle BAC = \angle BCD = \theta$$

$$\angle B \text{ is common in both triangle}$$

$$\therefore \triangle ABC \sim \triangle CBD$$



$$\frac{BC}{AB} = \frac{BD}{BC}$$

$$BC^2 = AB \times BD$$

$$BC^2 = 32 \times 18$$

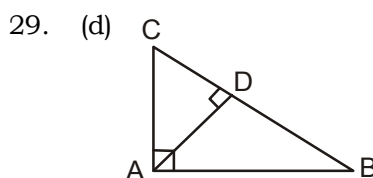
$$BC^2 = 576$$

$$BC = 24 \text{ cm}$$

$$\Rightarrow \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle BDC} = \frac{BC}{BD}$$

$$= \frac{AB}{BC} = \frac{AC}{CD}$$

$$\Rightarrow = \frac{24}{18} = \frac{4}{3}$$



In $\triangle ABC$

By using pythagoras theorem

$$AC^2 + AB^2 = BC^2$$

$$(2AB)^2 + AB^2 = BC^2$$

$$5AB^2 = BC^2$$

$$BC = \sqrt{5} AB$$

$$AB = \frac{BC}{\sqrt{5}}$$

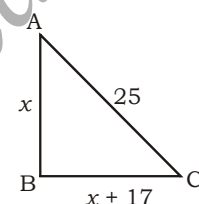
$$AB^2 = BD \times BC$$

$$\left(\frac{BC}{\sqrt{5}}\right)^2 = BD \times BC$$

$$BD = \frac{BC}{5}$$

30. (b) Let the altitude of a triangle = x

According to the question,



$$AC^2 = AB^2 + BC^2$$

$$(25)^2 = x^2 + (x+17)^2$$

$$625 = x^2 + x^2 + 289 + 34x$$

$$2x^2 + 34x - 336 = 0$$

$$x^2 + 17x - 168 = 0$$

$$x^2 + 24x - 7x - 168 = 0$$

$$x(x+24) - 7(x+24) = 0$$

$$(x+24)(x-7) = 0$$

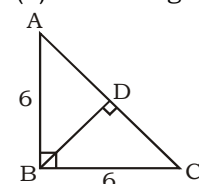
$$x = 7$$

$$\therefore \text{Sum of altitude and base}$$

$$= x + x + 17$$

$$= 7 + 7 + 17 = 31 \text{ cm}$$

31. (d) According to the question,



$$AB = BC = 6 \text{ cm}$$

By using pythagoras theorem.

$$AC^2 = BC^2 + AB^2$$

$$AC^2 = 6^2 + 6^2$$

$$AC^2 = 72$$

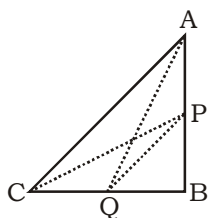
$$AC = 6\sqrt{2}$$

$$\therefore BD = \frac{AB \times BC}{AC}$$

$$BD = \frac{6 \times 6}{6\sqrt{2}}$$

$$BD = 3\sqrt{2} \text{ cm}$$

32. (a) According to the question,
By using Pythagoras theorem,



In $\triangle AQB$,
 $AQ^2 = BQ^2 + AB^2$... (i)

In $\triangle PCB$
 $PC^2 = BC^2 + BP^2$... (ii)

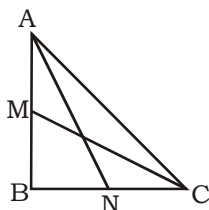
In $\triangle ACB$
 $AC^2 = BC^2 + AB^2$... (iii)

In $\triangle PQB$
 $PQ^2 = BQ^2 + BP^2$... (iv)

Adding eq. (i), (ii), (iii) & (iv)
 $AQ^2 + PC^2 + AC^2 + PQ^2 = BQ^2 + AB^2 + BC^2 + BP^2 + BC^2 + AB^2 + BQ^2 + BP^2$
 $AQ^2 + PC^2 + AC^2 + PQ^2 = 2(BQ^2 + AB^2 + BC^2 + BP^2)$
 $AQ^2 + PC^2 + AC^2 + PQ^2 = 2(PQ^2 + AC^2)$

$$\boxed{AQ^2 + CP^2 = PQ^2 + AC^2}$$

33. (c) According to the question,
By using Pythagoras theorem,



In $\triangle ABN$,
 $AN^2 = AB^2 + BN^2$... (i)

In $\triangle MBC$,
 $MC^2 = BM^2 + BC^2$... (ii)

In $\triangle ABC$,
 $AC^2 = AB^2 + BC^2$... (iii)

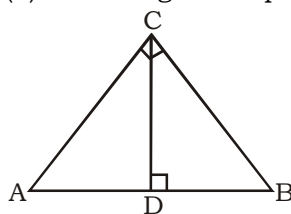
Adding eq (i) and (ii)
 $AN^2 + CM^2 = AB^2 + BN^2 + BM^2 + BC^2$
 $AN^2 + CM^2 = AC^2 + BN^2 + BM^2$

$$AN^2 + CM^2 = AC^2 + \frac{BC^2}{4} + \frac{AB^2}{4}$$

$$AN^2 + CM^2 = \frac{4AC^2 + AC^2}{4}$$

$$\boxed{4(AN^2 + CM^2) = 5AC^2}$$

34. (b) According to the question,



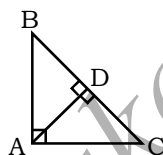
$\angle C = 90^\circ$
 $BC = 15 \text{ cm}$
 $AC = 20 \text{ cm}$
By using pythagoras theorem,
 $AB^2 = AC^2 + BC^2$
 $AB^2 = (20)^2 + (15)^2$
 $AB^2 = 400 + 225$
 $AB = 25 \text{ cm}$
As we know that,

$$CD = \frac{BC \times AC}{AB}$$

$$CD = \frac{20 \times 15}{25}$$

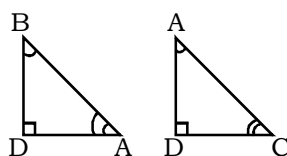
$$CD = 12 \text{ cm}$$

35. (a) According to the question,



$\therefore \angle DAB = \angle DCA$
 $\angle DBA = \angle DAC$
 $\therefore \triangle ADB \sim \triangle CDA$

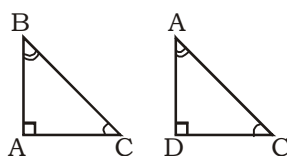
$$\frac{AD}{CD} = \frac{AB}{AC} = \frac{BD}{AD}$$



$$AD^2 = BD \times CD \quad \dots (i)$$

$$\triangle BAC \sim \triangle ADC$$

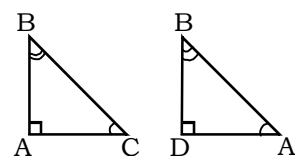
$$\frac{AB}{AD} = \frac{BC}{AC} = \frac{AC}{CD}$$



$$AC^2 = BC \times CD \quad \dots (ii)$$

$$\triangle BAC \sim \triangle BDA$$

$$\frac{BA}{BD} = \frac{BC}{BA} = \frac{AC}{AD}$$

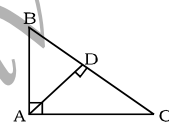


$AB^2 = BC \times BD$... (iii)
eq. (iii) divide by eq. (ii)

$$\frac{AB^2}{AC^2} = \frac{BC \times BD}{BC \times CD}$$

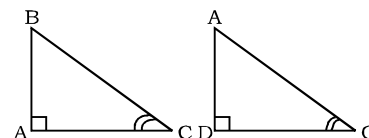
$$\boxed{\frac{BD}{CD} = \left(\frac{AB}{AC}\right)^2}$$

36. (b) According to Question
Given: $AC = 9 \text{ cm}$



area of $\triangle ABC = 40 \text{ cm}^2$
area of $\triangle ADC = 10 \text{ cm}^2$

$$\triangle ABC \sim \triangle DAC$$



$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle ADC} = \frac{AB^2}{AD^2} = \frac{BC^2}{AC^2} = \frac{AC^2}{DC^2}$$

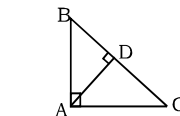
(In similar $\triangle$ ratio of their area is square of ratio of corresponding sides)

$$\frac{40}{10} = \frac{BC^2}{(9)^2}$$

$$\frac{40}{10} \times 81 = BC^2$$

$$BC = 18 \text{ cm}$$

37. (d) According to Question



Given: $\angle BAC$ is a right angle
triangled

$$AD \perp BC$$

$$AD = 6 \text{ cm}$$

$$BD = 4 \text{ cm}$$

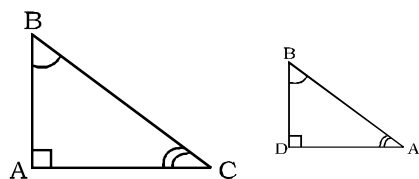
$$BC = ?$$

In $\triangle BAD$

$$AB = \sqrt{BD^2 + AD^2}$$

$$AB = \sqrt{4^2 + 6^2} = \sqrt{52} \text{ cm}$$

$$\triangle BAC \sim \triangle BDA$$



$$\therefore \frac{BC}{AB} = \frac{AB}{BD}$$

$$\therefore \frac{BC}{\sqrt{52}} = \frac{\sqrt{52}}{4}$$

$$BC = \frac{52}{4}$$

$$BC = 13 \text{ cm}$$

Alternate:-

$$AB^2 = BD \cdot BC$$

$$(\sqrt{BD^2 + AD^2})^2 = BD \cdot BC$$

$$(\sqrt{4^2 + 6^2})^2 = 4 \cdot BC$$

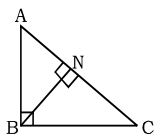
$$\frac{52}{4} = BC,$$

$$\therefore BC = 13 \text{ cm}$$

38. (b) According to question

Given: $\angle ABC = 90^\circ$

$$\frac{AN}{NC} = ?$$

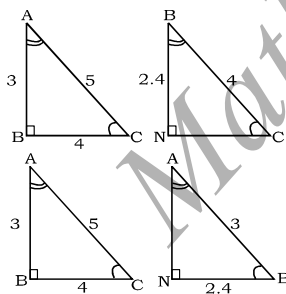


$$\triangle ABC \sim \triangle BNC$$

$$\triangle ABC \sim \triangle ANB$$

$$\therefore \triangle ABC \sim \triangle BNC \sim \triangle ANB$$

$$AB = 3, BC = 4, AC = 5$$



$$BN = \frac{AB \times BC}{AC} = \frac{3 \times 4}{5} = 2.4$$

$$\frac{BC}{NC} = \frac{AB}{NB}$$

$$\frac{4}{NC} = \frac{3}{2.4}$$

$$NC = 3.2$$

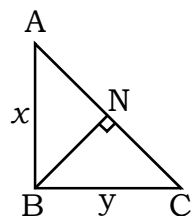
$$\frac{AB}{AN} = \frac{BC}{NB} \quad \frac{3}{AN} = \frac{4}{2.4}$$

$$AN = 1.8$$

$$\therefore \frac{AN}{NC} = \frac{1.8}{3.2} = \frac{9}{16}$$

Alternate:-

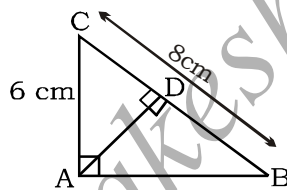
In such cases use the following method to save your valuable time.



$$\frac{AN}{NC} = \frac{x^2}{y^2} = \frac{3^2}{4^2} = \frac{9}{16}$$

$$\Rightarrow AN : NC = 9 : 16$$

39. (c) According to question



$$\triangle CAB \sim \triangle CDA$$

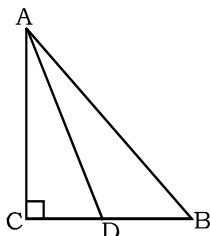
$$\therefore \frac{\text{area of } \triangle CAB}{\text{area of } \triangle CDA} = \frac{BC^2}{AC^2}$$

$$\frac{\text{area of } \triangle CAB}{\text{area of } \triangle CDA} = \frac{8^2}{6^2}$$

$$\frac{\text{area of } \triangle CAB}{\text{area of } \triangle CDA} = \frac{64}{36}$$

$$\frac{\text{area of } \triangle CAB}{\text{area of } \triangle CDA} = \frac{16}{9}$$

40. (a) According to question



In $\triangle ABC$

$$AB^2 = AC^2 + BC^2$$

.....(i)

In $\triangle ACD$

$$AD^2 = AC^2 + CD^2$$

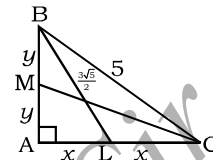
$$AC^2 = AD^2 - CD^2 \quad \text{.....(ii)}$$

Put the value of AC^2 in equation (i)

$$AB^2 = AD^2 - CD^2 + BC^2$$

$$AB^2 + CD^2 = AD^2 + BC^2$$

41. (a) According to question



According to figure, when two medians intersect each other in a right angled triangle then we use this equation.

$$\Rightarrow 4(BL^2 + CM^2) = 5BC^2$$

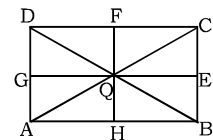
$$\Rightarrow 4 \times \left(\frac{3\sqrt{5}}{2} \right)^2 + 4CM^2 = 5(5)^2$$

$$\Rightarrow 45 + 4CM^2 = 125$$

$$\Rightarrow CM^2 = \frac{125 - 45}{4} = 20$$

$$\Rightarrow CM = 2\sqrt{5} \text{ cm}$$

42. (a) According to question



Given:

$$QA = 3 \text{ cm}$$

$$QB = 4 \text{ cm}$$

$$QC = 5 \text{ cm}$$

$$QD = ?$$

As we know that

$$QD^2 + QB^2 = QA^2 + QC^2$$

(By using Pythagoras theorem)

$$QD^2 + (4)^2 = (3)^2 + (5)^2$$

$$QD^2 + 16 = 9 + 25$$

$$QD^2 = 34 - 16$$

$$QD^2 = 18$$

$$QD = \sqrt{18},$$

$$QD = 3\sqrt{2}$$

43. (c) $BC \parallel DE$

$$\therefore \angle ABC = \angle ADE$$

$$\angle ACB = \angle AED \text{ and } \angle BAC = \angle EAD$$

$$\therefore \triangle ABC \sim \triangle AED$$

$$\frac{AB}{AD} = \frac{AC}{AE} \Rightarrow \frac{18}{9} = \frac{11}{AE}$$

$$\Rightarrow AE = 5.5 \text{ cm}$$

44. (a) $AB = AP + BP$
 $\Rightarrow AP + 3AP = 4AP$

$$\Rightarrow \frac{AB}{AP} = \frac{4}{1} \&$$

$$AC = AQ + QC$$

$$= AQ + 3AQ$$

$$= 4AQ$$

$$\Rightarrow \frac{AC}{AQ} = \frac{4}{1}$$

In $\triangle ABC$ and $\triangle APQ$

$\angle BAC = \angle PAQ$ (common) and

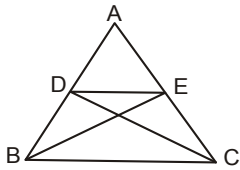
$$\frac{AB}{AP} = \frac{AC}{AQ}$$

$\therefore \triangle ABC \sim \triangle APQ$

$$\therefore \frac{AB}{AP} = \frac{AC}{AQ} = \frac{BC}{PQ} = 4$$

$$\Rightarrow \frac{BC}{PQ} = \frac{4}{1} \Rightarrow \frac{36}{PQ} = 4 \Rightarrow PQ = 9cm$$

45. (a)



$\triangle DBC$ & $\triangle EBC$ lie on the same base BC and between same parallel lines.

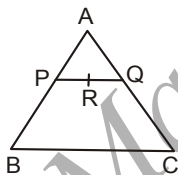
$$\therefore \text{ar}(\triangle DBC) = \text{ar}(\triangle EBC)$$

$$\Rightarrow \text{ar}(\triangle ABE) = \text{ar}(\triangle ACD)$$

$$\Rightarrow \text{ar}(\triangle ACD) = \text{ar}(\triangle ABE)$$

$$= 36 \text{sq.cm}$$

46. (c)



$$\frac{PR}{RQ} = \frac{1}{2} \Rightarrow \frac{2}{RQ} = \frac{1}{2} \Rightarrow RQ = 4$$

$$\therefore PQ = PR + RQ$$

$$= 2 + 4 = 6cm$$

The line joining the mid-points of two sides of a triangle is parallel to and half of the third side.

$$BC = 2PQ$$

$$= 2 \times 6$$

$$= 12cm.$$

47. (b) For the given perimeter of a triangle the maximum area is enclosed by an equilateral triangle.

$$\therefore 3a = 24 \text{ cm} \Rightarrow a = 8cm$$

$$\therefore \text{Area} = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times (8)^2$$

$$= 16\sqrt{3}cm^2$$

48. (d) $XY \parallel AC$ (given)

$$\therefore \angle BXY = \angle A \text{ and}$$

$$\angle BYX = \angle C$$

$$\therefore \triangle ABC \sim \triangle XBY$$

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle XBY)} = \left(\frac{AB}{XB}\right)^2 \dots\dots(i)$$

Also,

$$\text{ar}(\triangle ABC) = 2 \text{ ar}(\triangle XBY)$$

(given)

$$\Rightarrow \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle XBY)} = 2 \dots\dots(ii)$$

therefore from (i) & (ii)

$$\left(\frac{AB}{XB}\right)^2 = \frac{2}{1}$$

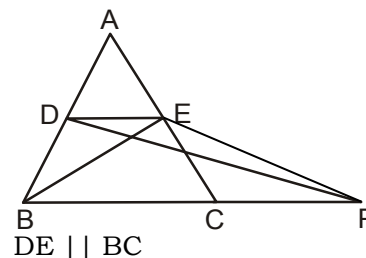
$$\frac{AB}{XB} = \frac{\sqrt{2}}{1}$$

$$\text{or } \frac{XB}{AB} = \frac{1}{\sqrt{2}} \text{ or } 1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{AB - XB}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

$$\Rightarrow \frac{AX}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

49. (a) (By mid-point theorem)



$$DE = \frac{1}{2} BC$$

$$\Rightarrow \text{ar}(\triangle BDE)$$

$$= \frac{1}{4} \times \text{ar}(\triangle ABC)$$

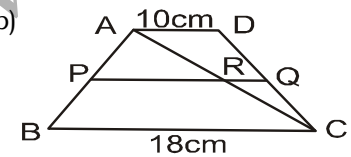
and $\triangle BDE = \triangle PED$

[$\because$ both triangles lie on the same base DE and between two parallel lines DE and BP]

$$\therefore \text{ar}(\triangle PED)$$

$$= \frac{1}{4} \times \text{ar}(\triangle ABC)$$

50. (b)



Now In $\triangle APR$ and $\triangle ABC$

$$\angle APR = \angle ABC (\because PQ \parallel AD \parallel BC)$$

$$\text{and } \angle ARP = \angle ACB (\because PQ \parallel BC)$$

$$\therefore \triangle APR \sim \triangle ABC$$

$\therefore$

$$\frac{AP}{AB} = \frac{PR}{BC} \Rightarrow PR = \frac{AP}{AB} \times BC$$

$$= \frac{AP}{AP + PB} \times BC$$

$$\Rightarrow PR = \frac{5}{8} \times 18 = \frac{45}{4} cm$$

$$\text{and } \frac{AP}{PB} = \frac{AR}{RC} = \frac{5}{3}$$

similarly, $\triangle RCQ \sim \triangle CAD$

$$\therefore \frac{RQ}{AD} = \frac{RC}{AC}$$

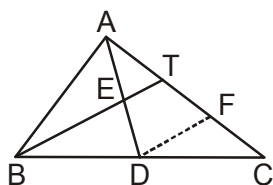
$$\Rightarrow RQ = \frac{RC}{AR + RC} \times AD$$

$$= \frac{3}{8} \times 10 = \frac{15}{4} cm$$

$$\therefore PQ = PR + RQ = \frac{45}{4} + \frac{15}{4}$$

$$= 15cm$$

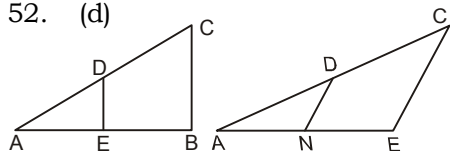
51. (c)



Draw $DF \parallel ET$
 In $\triangle ADF$, E is the mid-point of AD and $DF \parallel ET$
 $\therefore$ T will be the mid-point of AF.
 i.e. $AT = TF$ (i)
 Now, In $\triangle BTC$,
 D is the mid-point of BC & $DF \parallel ET \parallel BT$
 $\therefore$ F will be the mid-point of TC
 i.e. $TF = FC$ (ii)
 $\therefore$ from (i) and (ii)
 $AT = TF = FC$

$\therefore AC = 15\text{cm}, AT = TF = FC = \frac{AC}{3}$
 $= 5\text{cm}$
 $\therefore TC = TF + FC$
 $= 5 + 5$
 $= 10\text{cm}$

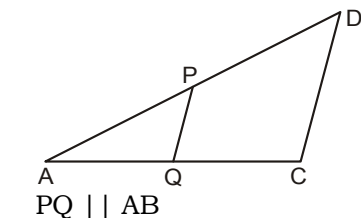
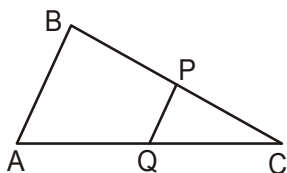
52. (d)



In $\triangle ABC$,
 $\therefore DE \parallel BC$
 $\therefore \frac{AD}{DC} = \frac{AE}{EB} = \frac{4}{5} = 4:5$ (i)

In $\triangle AEC$
 $EC \parallel ND$
 $\therefore \frac{AN}{NE} = \frac{AD}{DC} = \frac{4}{5} = 4:5$ (ii)
 Let $AE = 40$ $\therefore EB = 50$ and
 $\therefore EN = 40 \times \frac{5}{9} = \frac{200}{9}$
 $EN : EB = \frac{200}{9} : 50 = 4:9$

53. (a)



$PQ \parallel AB$
 $\therefore \triangle ABC \sim \triangle PQC$
 $\frac{AB}{PQ} = \frac{AC}{QC}$
 $\Rightarrow PQ = \frac{AB}{AC} \times QC$ (i)

$\therefore PQ \parallel CD$
 $\therefore \triangle ACD \sim \triangle AQP$
 $\therefore \frac{CD}{PQ} = \frac{AC}{AQ}$
 $\Rightarrow PQ = \frac{AQ}{AC} \times CD$ (ii)
 from (i) & (ii) -
 $\frac{AQ}{AC} \times CD = \frac{AB}{AC} \times QC$
 $\Rightarrow AQ \times CD = AB \times QC$
 $\Rightarrow (AC - QC) \times 18 = 12 \times QC$
 $\Rightarrow (6 - QC) \times 18 = 12 \times QC$
 $\Rightarrow (6 - QC) \times 3 = 2QC$
 $\Rightarrow 5QC = 18 \Rightarrow QC = \frac{18}{5}$

$\therefore$ from (i)
 $PQ = \frac{12}{6} \times \frac{18}{5} = \frac{36}{5}\text{cm}$

Alternatively

Let $QC = x \Rightarrow AQ = 6 - x$
 In $\triangle ABC$ and triangle PQC

$\frac{PQ}{AB} = \frac{QC}{AC}$
 $\frac{PQ}{12} = \frac{x}{6} \Rightarrow PQ = 2x$ (i)

In $\triangle ACD$ and Triangle AQP

$\frac{PQ}{CD} = \frac{AQ}{AC}$
 $\frac{PQ}{18} = \frac{6 - x}{6}$
 $\Rightarrow PQ = 18 - 3x$ (ii)
 From (i) and (ii),
 $2x = 18 - 3x$
 $x = \frac{18}{5}$

$\therefore$ From (i), $PQ = 2x = 2 \times \frac{18}{5} =$

$$\frac{36}{5}\text{cm}$$

Alternate

When $AB \parallel PQ \parallel CD$

$$\frac{1}{PQ} = \frac{1}{AB} + \frac{1}{CD}$$

$$= \frac{1}{12} + \frac{1}{18} = \frac{5}{36}$$

$$PQ = \frac{36}{5}$$

54. (c) In $\triangle ABM$ and $\triangle BEC$

$\angle BAM = \angle BCE$
 $\angle BMA = \angle BEC$
 $(\because AM \parallel EC)$
 $\therefore \triangle ABM \sim \triangle BEC$

$$\therefore \frac{AB}{BC} = \frac{AM}{EC} \Rightarrow \frac{5}{10} = \frac{AM}{18}$$

$$\Rightarrow AM = 9\text{cm}$$

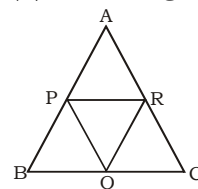
$$\therefore AM \parallel DN$$

$$\therefore \triangle AMC \sim \triangle DNC$$

$$\therefore \frac{DN}{AM} = \frac{DC}{AC} \Rightarrow \frac{15}{9} = \frac{DC}{15}$$

$$\Rightarrow DC = \frac{15 \times 15}{9} = 25\text{cm}$$

55. (d) According to the question,

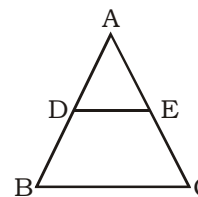


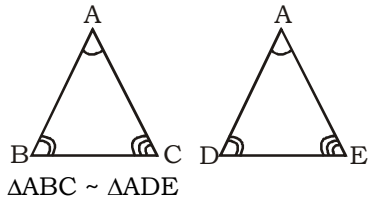
Area of $\triangle PQR = 6\text{cm}^2$
 There are 4 congruent triangles.

$\triangle APR \sim \triangle BPQ \sim \triangle PQR \sim \triangle RQC$
 Hence area of $\triangle ABC = 4 \times$ area of $\triangle PQR = 4 \times 6 = 24\text{cm}^2$

56. (a) According to the question,

$AB = 3.6\text{cm}$
 $AC = 2.4\text{cm}$
 $AD = 2.1\text{cm}$
 $AE = ?$





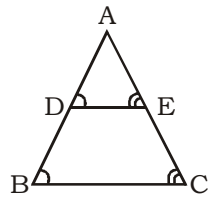
$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$\frac{3.6}{2.1} = \frac{2.4}{AE}$$

$$AE = \frac{2.1 \times 2.4}{3.6}$$

$$AE = 1.4 \text{ cm}$$

57. (c) According to the question,



$$AC = 5 \text{ cm}$$

$$AD = 3 \text{ cm}$$

$$BD = 7 \text{ cm}$$

By using midpoint theorem,

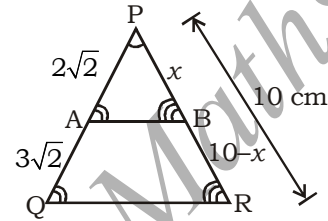
$$\frac{AD}{BD} = \frac{AE}{EC}$$

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\frac{3}{10} = \frac{AE}{5}$$

$$AE = 1.5 \text{ cm}$$

58. (b) According to the question,



$$\angle PAB = \angle PQR$$

$$\angle PBA = \angle PRQ$$

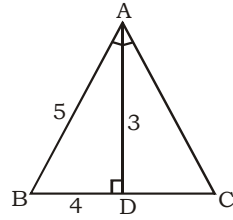
$$\therefore \Delta PAB \sim \Delta PQR$$

$$\frac{x}{10} = \frac{2\sqrt{2}}{5\sqrt{2}}$$

$$x = 4 \text{ cm}$$

$$\therefore BR = 10 - x = 10 - 4 = 6 \text{ cm}$$

59. (c) According to the question,
As we know that,



$$\frac{1}{AD^2} = \frac{1}{AB^2} + \frac{1}{AC^2}$$

$$\frac{1}{(3)^2} = \frac{1}{(5)^2} + \frac{1}{(AC)^2}$$

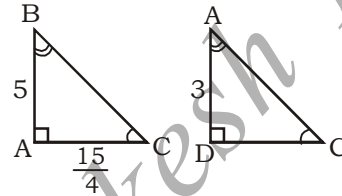
$$\frac{1}{AC^2} = \frac{1}{9} - \frac{1}{25}$$

$$AC = \frac{15}{4}$$

In ΔBAC and ΔADC

$$\angle B = \angle A$$

$$\angle A = \angle D$$



$$\therefore \Delta CAB \sim \Delta CDA$$

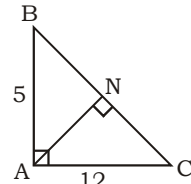
$$\frac{AC}{DC} = \frac{AB}{AD}$$

$$\frac{15}{4DC} = \frac{5}{3}$$

$$DC = \frac{9}{4}$$

$$DC = 2.25$$

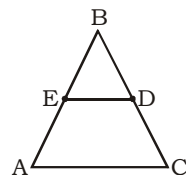
60. (b) According to the question,



$$\frac{\text{Area of } \Delta ANB}{\text{Area of } \Delta ANC} = \frac{AB^2}{AC^2} = \frac{(5)^2}{(12)^2}$$

$$= \frac{25}{144}$$

61. (b) According to the question,
E & D are the midpoint of AB & BC.



$$\therefore \frac{BE}{BA} = \frac{BD}{BC} = \frac{ED}{AC} = \frac{1}{2}$$

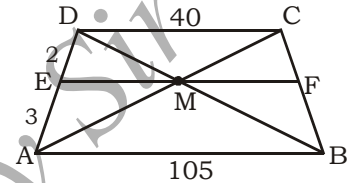
$$\therefore \frac{\text{Area of } \Delta BED}{\text{Area of } \Delta BAC} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$4 \text{ units} = 30$$

$$1 \text{ unit} = 7.5$$

$$\therefore \text{Area of } \Delta BED = 7.5 \text{ cm}^2$$

62. (a) According to the question,



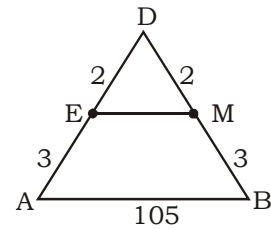
$$AB = 105 \text{ cm}$$

$$DC = 40 \text{ cm}$$

$$\frac{DE}{EA} = \frac{2}{3}$$

In ΔADB , E & M are the mid points of AD & BD.

By using B.P.T. theorem,



$$\frac{DE}{EA} = \frac{DM}{MB} = \frac{2}{3}$$

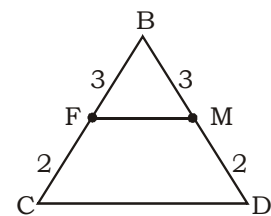
$$\frac{DE}{DA} = \frac{DM}{DB} = \frac{EM}{AB}$$

$$\frac{2}{5} = \frac{EM}{105}$$

$$EM = 42 \text{ cm}$$

In ΔBDC , M & F are the mid points of BC & BD

By using B.P.T theorem



$$\frac{BF}{BC} = \frac{FM}{CD}$$

$$\frac{3}{5} = \frac{FM}{40}$$

$$FM = 24 \text{ cm}$$

$$\therefore EF = EM + FM$$

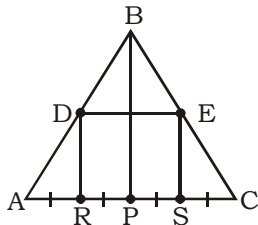
$$EF = 24 + 42 = 66 \text{ cm}$$

63. (a) Let $ES = x$

According to the question,

In $\triangle BPC$,

E is the midpoint of BC and S is the midpoint of PC.



- By using midpoint theorem.

$$ES = \frac{1}{2}BP$$

$$BP = 2x$$

In $\triangle BPA$,

D is the midpoint of AB and R is the midpoint of AP

By using midpoint theorem,

$$DR = \frac{1}{2} \times BP$$

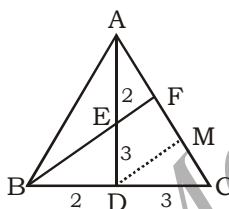
$$DR = \frac{1}{2} \times 2x$$

$$DR = x$$

$$\therefore ES = DR$$

$$\frac{ES}{DR} = \frac{x}{x} = \frac{1}{1}$$

64. (a) According to the question, Let draw a imaginary line DM which is parallel to EF.



In $\triangle ADM$.

By using B.P.T.

$$\frac{AE}{ED} = \frac{AF}{FM} = \frac{2}{3}$$

In $\triangle CBF$

By using B.P.T.

$$\frac{CD}{DB} = \frac{CM}{FM} = \frac{3}{2}$$

$\therefore$ to make FM same

$$\begin{array}{ccc} AF & : & FM & : & MC \\ 2 \times \frac{2}{3} = 4 & & 3 \times \frac{2}{3} = 6 & & 3 \times \frac{3}{3} = 9 \\ \hline 4 & : & 6 & : & 9 \end{array}$$

$$\therefore \text{Length of AC} = AF + FM + MC = 4 + 6 + 9 = 19 \text{ units}$$

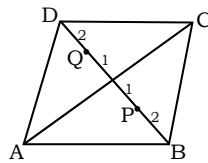
$$19 \text{ units} = 760$$

$$1 \text{ unit} = \frac{760}{19}$$

$$2 \text{ units} = \frac{760}{19} \times 2 = 80 \text{ cm}$$

$$\therefore AE = 2 \text{ units} = 80 \text{ cm}$$

65. (b) According to question,



Given: $BD = 18 \text{ cm}$

Note: Centroid is the point where medians intersect and it divides median in 2 : 1

$$BD = 6 \text{ units}, PQ = 2 \text{ units}$$

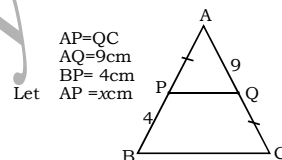
$$6 \text{ units} = 18 \text{ cm}$$

$$1 \text{ unit} = \frac{18}{6} = 3$$

$$2 \text{ units} = 3 \times 2 = 6$$

$$\therefore PQ = 6 \text{ cm}$$

66. (c) According to Question Given:



$$\triangle APQ \sim \triangle ABC$$

To apply similarity property

$$\frac{AP}{BP} = \frac{AQ}{QC}$$

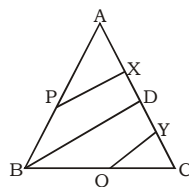
$$\frac{x}{4} = \frac{9}{x}$$

$$x^2 = 36$$

$$x = 6$$

$$\therefore AP = 6 \text{ cm}$$

67. (b) According to question



$PX \parallel BD$ [mid point theorem]

$$\therefore PX = \frac{1}{2}BD$$

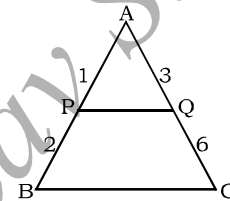
Similarly, $QY \parallel BD$

$$\therefore QY = \frac{1}{2}BD$$

$$\therefore PX : QY = \frac{1}{2}BD : \frac{1}{2}BD$$

$$PX : QY = 1 : 1$$

68. (b) According to question Given:



$$\frac{AP}{PB} = \frac{1}{2}$$

$$AQ = 3$$

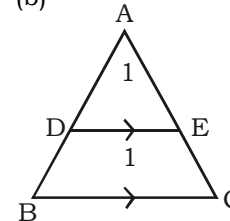
To apply similar triangle property.

$$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$$

$$\frac{1}{3} = \frac{3}{AC}$$

$$AC = 9 \text{ cm}$$

69. (b)



$$\text{ar}(\triangle ADE) = \text{ar} \triangle ECB$$

So, $\text{ar} \triangle ADE = 1 \text{ unit}^2$ and $\text{ar} \triangle ABC = 2 \text{ unit}^2$

$$\frac{\text{ar} \triangle ADE}{\text{ar} \triangle ABC} = \frac{AD^2}{AB^2}$$

$$\frac{1}{2} = \left(\frac{AD}{AB}\right)^2$$

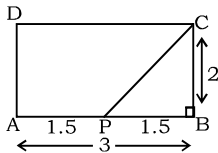
$$\frac{1}{\sqrt{2}} = \frac{AD}{AB}$$

$$\therefore \frac{AD}{DB} = \frac{1}{\sqrt{2} - 1}$$

$$(\therefore DB = AB - AD = \sqrt{2} - 1)$$

$$\text{So, } AD : BD = 1 : \sqrt{2} - 1$$

70. (d) According to question



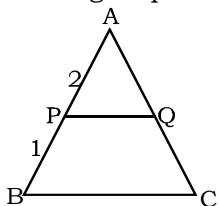
$$\begin{aligned} \text{In } \triangle CBP \\ CP^2 &= BP^2 + BC^2 \\ CP^2 &= (1.5)^2 + (2)^2 \\ CP^2 &= 2.25 + 4 \\ CP^2 &= 6.25 \\ CP &= \sqrt{6.25} \\ CP &= 2.5 \end{aligned}$$

$$\therefore \sin \angle CPB = \frac{BC}{CP}$$

$$\sin \angle CPB = \frac{2}{2.5}$$

$$\sin \angle CPB = \frac{4}{5}$$

71. (d) According to question



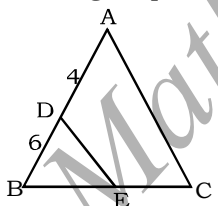
$$\text{Given: } \frac{AP}{PB} = \frac{3}{1}$$

To apply B.P.T

$$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$$

$$\frac{PQ}{BC} = \frac{2}{3}$$

72. (d) According to question



$$\text{Given: } AB = 10 \text{ cm}$$

$$AD = 4 \text{ cm}$$

$$DE \parallel AC$$

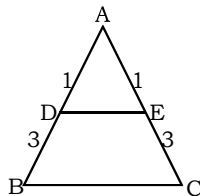
$$\triangle ABC \sim \triangle DBE$$

$$\therefore \frac{BD}{AD} = \frac{BE}{CE}$$

$$\frac{BE}{CE} = \frac{6}{4}$$

$$\frac{BE}{CE} = \frac{3}{2}$$

73. (c) According to question



By using B.P.T

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

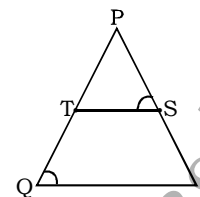
$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{1}{4} = \frac{DE}{12}$$

$$DE = 3 \text{ cm}$$

74. (c) According to question

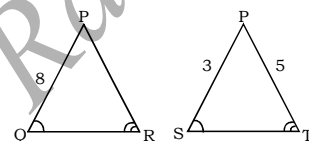
Given:



$$PT = 5 \text{ cm}, \quad PS = 3 \text{ cm}$$

$$TQ = 3 \text{ cm}, \quad SR = ?$$

$$\triangle PQR \sim \triangle PST$$



$$\frac{PR}{PT} = \frac{PQ}{PS} \Rightarrow \frac{PR}{5} = \frac{8}{3}$$

$$PR = \frac{40}{3}$$

$$\therefore SR = PR - PS$$

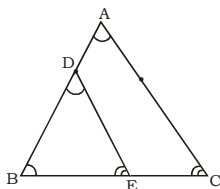
$$SR = \frac{40}{3} - 3$$

$$SR = \frac{40 - 9}{3}, \quad SR = \frac{31}{3} \text{ cm}$$

75. (c) According to question

Given:

'D' and 'E' are the points on AB and BC



$$AC \parallel DE$$

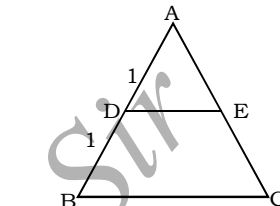
$$\angle D = \angle A$$

$$\angle E = \angle C$$

$$\therefore \triangle BDE \sim \triangle BAC$$

76. (c) According to question

Given:



$$AB = 2AD$$

$$\frac{AB}{AD} = \frac{2}{1}$$

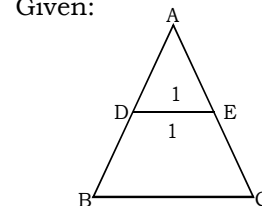
By applying B.P.T

$$\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$

$$\frac{DE}{BC} = \frac{1}{2}$$

77. (c) According to question

Given:



$$DE \parallel BC$$

$$\text{Area of } \triangle ABC = 2 \text{ (Area of } \triangle ADE)$$

$$\therefore \angle E = \angle C$$

$$\angle D = \angle B$$

$$\therefore \triangle ABC \sim \triangle ADE$$

$$\therefore \frac{\text{area of } \triangle ABC}{\text{area of } \triangle ADE}$$

$$= \frac{AB^2}{AD^2} = \frac{AC^2}{AE^2} = \frac{BC^2}{DE^2}$$

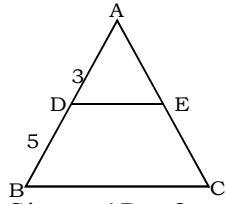
$$\Rightarrow \frac{2}{1} = \frac{AB^2}{AD^2}$$

$$\therefore \frac{AB}{AD} = \frac{\sqrt{2}}{1}$$

$$\therefore BD = \sqrt{2} - 1$$

$$\therefore \frac{BD}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

78. (a) According to question



Given: $AD = 3$, $BD = 5$
 $AB = 8$, $AC = 4$
 $AE = ?$

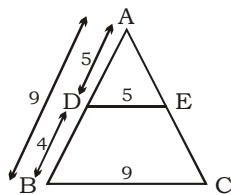
By applying B.P.T

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\frac{3}{8} = \frac{AE}{4}$$

$$AE = \frac{3}{2} = 1.5 \text{ cm}$$

79. (d) In $\triangle ABC$, $DE \parallel BC$



$$\frac{AE}{AC} = \frac{AD}{AB} = \frac{DE}{BC}$$

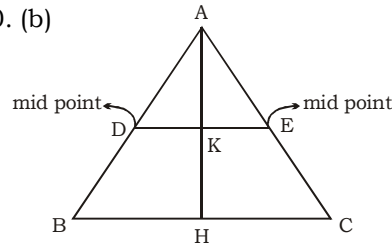
(Basic Prop. theorem)

$$\text{Here, } \frac{AD}{DB} = \frac{5}{4}$$

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC} = \frac{5}{9}$$

$$\Rightarrow DE : BC = 5 : 9$$

80. (b)



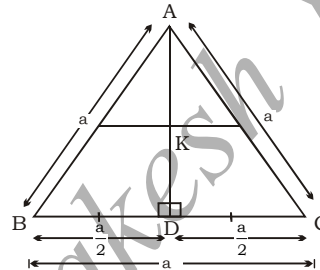
$\therefore$ Point D and E are mid points of sides AB and AC respectively

Then DE will be parallel to BC [by thales theorem]

$\Rightarrow$ And DE, always cuts the two equal part

$\Rightarrow$ Therefore $AK : KH = 1 : 1$

81. (d)



Let, the side of equilateral triangle is = a

Then $AB = BC = CA = a$

$$BD = DC = \frac{a}{2}$$

$$AB^2 = AD^2 + BD^2 \quad \dots(i)$$

$$AC^2 = AD^2 + CD^2 \quad \dots(ii)$$

Add eq. (i) & (ii)

$$AB^2 + AC^2 = 2AD^2 + BD^2 + CD^2$$

Add BC^2 both sides

$$AB^2 + AC^2 + BC^2 = 2AD^2 + BD^2 + CD^2 + BC^2$$

Put the value of side in R.H.S

$$2AD^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + a^2$$

$$2AD^2 + \frac{3}{2}a^2 = 4AD^2$$

$$\left(a^2 - \frac{a^2}{4} = AD^2 = \frac{3a^2}{4}\right)$$

CENTRES OF TRIANGLES

In geometry, a triangle centre is a point in the plane that is in some sense a centre of a triangle and can be obtained by simple constructions. Centre of triangle always occupy the same position (relative to the vertices) under the operations of rotation, reflection and dilation.

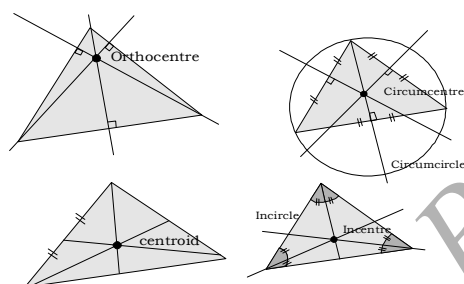
Ancient Greeks discovered the classic centres of triangle (centroid, circumcentre, incentre and orthocentre)

Triangle Centres

Where is the centre of triangle ?

There are actually thousands of centres !

Here are the 4 most popular ones:



Centroid, Circumcentre, Incentre and Orthocentre

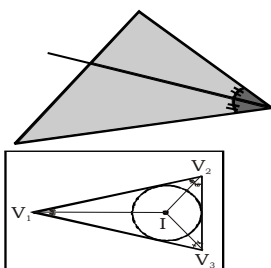
For each of those, the "centre" is where special lines cross, so **it all depends on those lines!**

INCENTRE

The incentre is the point of intersection of the three angle bisectors. The angle bisectors of a triangle are each one of the lines that divide an angle into two equal parts.

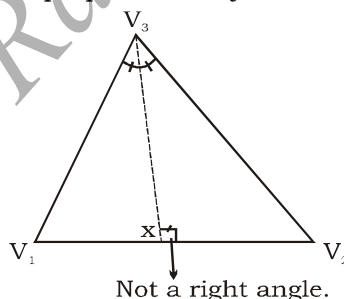
The Incentre is also the centre of the circle inscribed in the

triangle.



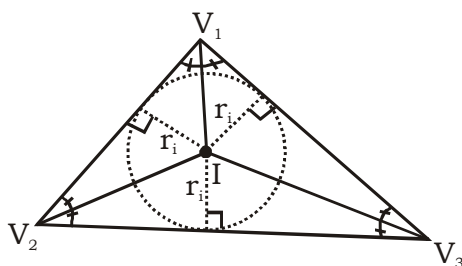
Properties

1. Incentre of any triangle lies inside that triangle.
2. Incentre is the only centre which is equiperpendicular distance from all the sides of a triangle.
3. Generally angle bisector doesn't intersect the opposite side perpendicularly.

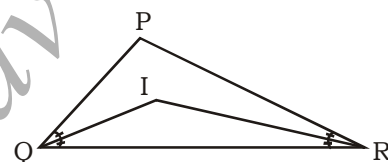


Note:- The angle x as given in the figure above is a right angle only in the case of a isosceles and equilateral triangle.

4. The radius of the circle inscribed in a triangle is known as inradius (r_i) of the triangle.



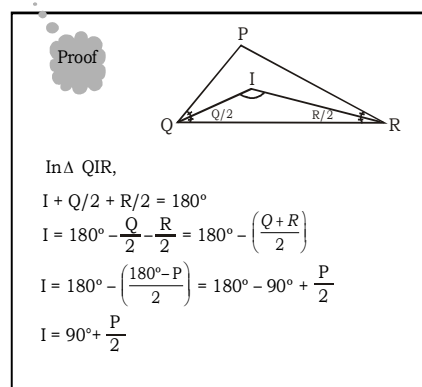
5. The angle between line segments drawn from the two vertices to the incentre is equal to the sum of a right angle (90°) and half of third vertex.



$$\angle QIR = 90^\circ + \frac{\angle P}{2}$$

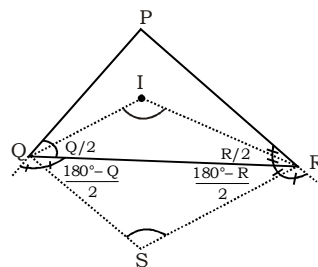
$$\angle PIQ = 90^\circ + \frac{\angle R}{2}$$

$$\angle PIR = 90^\circ + \frac{\angle Q}{2}$$



6. The angle between the external bisectors of two angles of a triangle is difference between right angle and half of the third angle.

$$\angle QSR = 90^\circ - \frac{\angle P}{2}$$



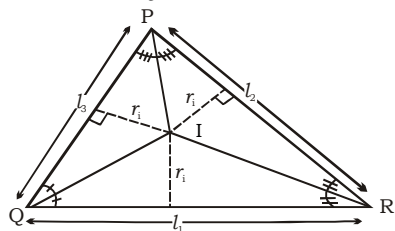
Proof

$$\begin{aligned}\angle IQS &= \frac{Q}{2} + \frac{180^\circ - Q}{2} = 90^\circ \\ \angle IRS &= \frac{R}{2} + \frac{180^\circ - R}{2} = 90^\circ \\ \text{In Quadrilateral QIRS,} \\ \angle IRS + \angle IQS &= 180^\circ \\ \text{Then } \angle QSR + \angle QIR &= 180^\circ \\ \angle QSR &= 180^\circ - \angle QIR \\ &= 180^\circ - \left(90^\circ + \frac{P}{2}\right) \\ &= 90^\circ - \frac{P}{2}\end{aligned}$$

7. The ratio of sides of a triangle is equal to the area of triangle formed by corresponding sides and incentre.

$$l_1 : l_2 : l_3 = \text{Area } \triangle QIR : \text{Area } \triangle PIR : \text{Area } \triangle PIQ$$

$$\begin{aligned}\Rightarrow \frac{l_1}{\text{Area } \triangle QIR} : \frac{l_2}{\text{Area } \triangle PIR} : \frac{l_3}{\text{Area } \triangle PIQ} &= 1 : 1 : 1 \\ \Rightarrow \frac{l_1}{\text{Area } \triangle QIR} = \frac{l_2}{\text{Area } \triangle PIR} = \frac{l_3}{\text{Area } \triangle PIQ}\end{aligned}$$



Proof

$$\begin{aligned}\text{Ar } \triangle QIR : \text{Ar } \triangle PIR : \text{Ar } \triangle PIQ \\ = \frac{1}{2} \times l_1 \times r_i : \frac{1}{2} \times l_2 \times r_i : \frac{1}{2} \times l_3 \times r_i \\ = l_1 : l_2 : l_3\end{aligned}$$

[Because altitude of all triangles is same and equal to inradius (r)]

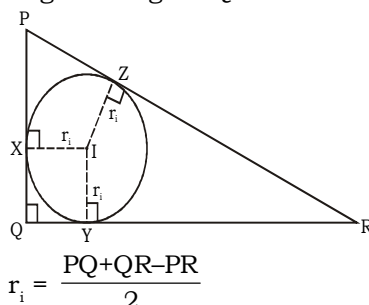
8. Area of a triangle can be obtained by multiplying inradius (r_i) of triangle to the semiperimeter of that triangle.

$$\text{Area } \triangle = r_i \times s$$

Proof

$$\begin{aligned}\text{Area of triangle PQR} \\ \Rightarrow \text{Ar } \triangle QIR + \text{Ar } \triangle PIR + \text{Ar } \triangle PIQ \\ \Rightarrow \frac{1}{2} \times l_1 \times r_i + \frac{1}{2} \times l_2 \times r_i + \frac{1}{2} \times l_3 \times r_i \\ \Rightarrow r_i \times \left[\frac{1}{2} (l_1 + l_2 + l_3) \right] \\ \Rightarrow r_i \times s\end{aligned}$$

9. Inradius of a right angled triangle PQR.



$$r_i = \frac{PQ + QR - PR}{2}$$

Proof

PXQ is tangent at X and QYR is tangent at Y
So that IX $\perp$ PQ and IY $\perp$ QR

PX = PZ, QX = QY = r_i , RY = RZ
(Tangents from external points)

$$\begin{aligned}\Rightarrow PQ + QR - PR &= (PX + r_i) + (r_i + YR) - (PZ + ZR) \\ &= 2 \times r_i \\ (\because PX = PZ, ZR = YR)\end{aligned}$$

$$r_i = \frac{PQ + QR - PR}{2}$$

10. ANGLE BISECTOR THEOREM:-

The angle bisector theorem states that an angle bisector divides the opposite side of a triangle into two segments that are proportional to the triangle's other two sides.

$$\text{In other words } \Rightarrow \frac{PQ}{PR} = \frac{QX}{XR}$$

Proof

Given that PX is the bisector of $\angle QPR$.

$\Rightarrow$ Draw a line parallel to PX from R. Which intersects extended QP at Y.
Now,

$$\begin{aligned}\angle QPX &= \angle XPR = \theta \leftarrow \text{bisected angles} \\ \angle QPX &= \angle PYR = \theta \leftarrow \text{corresponding angles} \\ \angle XPR &= \angle PRY = \theta \leftarrow \text{alternative interior angles}\end{aligned}$$

$\triangle PRY$ is isosceles and PY = PR
(as $\angle PRY = \angle PYR = \theta$)

Now as $\triangle QPX \sim \triangle QYR$

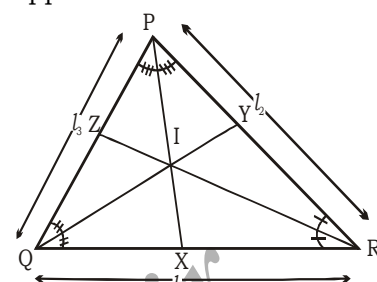
$$\text{So, } \frac{PQ}{PY} = \frac{QX}{XR}$$

Since PY = PR, we have

$$\frac{PQ}{PR} = \frac{QX}{XR}$$

11. Each angle bisector divided by Incentre is divided in the ratio equal to the ratio of length of

sum of two adjacent sides and opposite side.



$$\begin{aligned}PI : IX &= l_2 + l_3 : l_1 \\ QI : IY &= l_1 + l_3 : l_2 \\ RI : IZ &= l_1 + l_2 : l_3\end{aligned}$$

Proof

By Interior angle bisector theorem if PX is angle bisector of $\angle P$, then

$$\frac{QX}{XR} = \frac{l_2}{l_3} \Rightarrow \frac{QX}{XR} + 1 = \frac{l_2}{l_3} + 1$$

$$\Rightarrow \frac{QX + XR}{XR} = \frac{l_2 + l_3}{l_3} \Rightarrow \frac{QR}{XR} = \frac{l_2 + l_3}{l_3}$$

$$XR = \frac{l_3 \times QR}{l_2 + l_3} = \frac{l_3 \times l_1}{l_2 + l_3}$$

In $\triangle PXR$

$$\frac{PI}{IX} = \frac{PR}{XR} = \frac{l_2}{XR} = \frac{l_2 \times (l_2 + l_3)}{l_3 \times l_1}$$

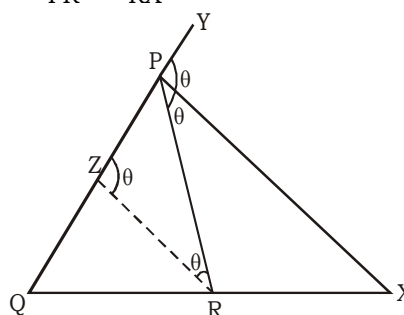
$$\frac{l_2 + l_3}{l_1} \left[\because XR = \frac{l_3 \times l_1}{l_2 + l_3} \right]$$

12. EXTERNAL ANGLE BISECTOR THEOREM.

The external angle bisector theorem states that the external bisector of an angle of triangle divides the opposite side externally in the ratio of the sides containing the angle.

In other words.

$$\frac{PQ}{PR} = \frac{QX}{RX}$$



- $\Rightarrow$ Given that PX is the external bisector of $\angle RPQ$
 $\Rightarrow$ Draw a line parallel to PX from R, which intersect PQ at Z.

Now

$$\angle RPX = \angle XPY = \theta \leftarrow \text{bisected angle}$$

$\angle ZRP = \angle RPX = \theta \Leftarrow$ alternative interior angles

$\angle RZP = \angle XPY = \theta =$ corresponding angles

$\triangle PRZ$ is isosceles and $PZ = PR$ as $\angle PZR = \angle PRZ$

Now as $\triangle QPX \sim \triangle QZR$

So,

$$\frac{PQ}{PZ} = \frac{QX}{RX}$$

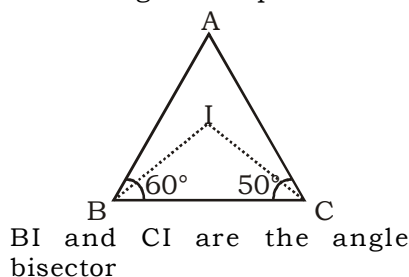
Since $PZ = PR$, we have

$$\frac{PQ}{PR} = \frac{QX}{RX}$$

Examples

1. I is the incentre of $\triangle ABC$, $\angle ABC = 60^\circ$ and $\angle ACB = 50^\circ$. Then $\angle BIC$ is

Sol. According to the question



$$\therefore \angle CBI = 30^\circ$$

$$\angle BCI = 25^\circ$$

In $\triangle BIC$

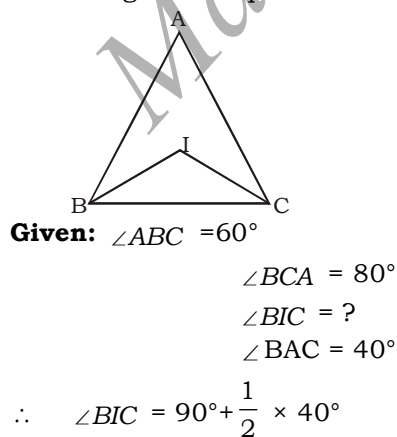
$$\angle CBI + \angle BCI + \angle BIC = 180^\circ$$

$$30^\circ + 25^\circ + \angle BIC = 180^\circ$$

$$\angle BIC = 125^\circ$$

2. I is the incentre of $\triangle ABC$. If $\angle ABC = 60^\circ$, $\angle BCA = 80^\circ$, then the $\angle BIC$ is

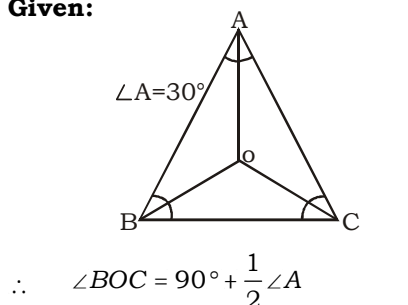
Sol. According to the question



$$\angle BIC = 110^\circ$$

3. O is the incentre of $\triangle ABC$ and $\angle A = 30^\circ$, then $\angle BOC$ is

Sol. According to the Question,
Given:

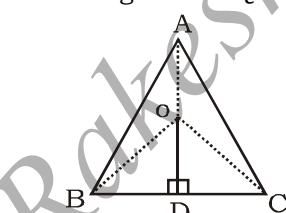


$$= 90^\circ + \frac{1}{2} \times 30^\circ = 90^\circ + 15^\circ$$

$$\angle BOC = 105^\circ$$

4. Let O be the in-centre of a triangle ABC and D be a point on the side BC of $\triangle ABC$, such that $OD \perp BC$. If $\angle BOD = 15^\circ$, then $\angle ABC =$

Sol. According to the Question,



Given: $\angle BOD = 15^\circ$

In $\triangle BOD$

$$\therefore \angle BDO + \angle DOB + \angle DBO = 180^\circ$$

$$\angle DBO = 180 - (90 + 15) = 75^\circ$$

$$\angle ABC = 2 \times \angle DBO$$

$$\angle ABC = 2 \times 75^\circ$$

$$\angle ABC = 150^\circ$$

5. I is the incentre of a triangle ABC. If $\angle ACB = 55^\circ$, $\angle ABC = 65^\circ$ then the value of $\angle BIC$ is

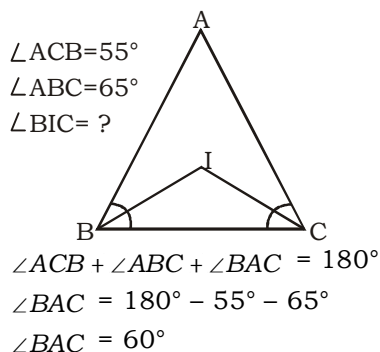
Sol. According to the question

Given:

$$\angle ACB = 55^\circ$$

$$\angle ABC = 65^\circ$$

$$\angle BIC = ?$$



We know that

$$\angle BIC = 90^\circ + \frac{1}{2} \angle A$$

$$\angle BIC = 90^\circ + \frac{1}{2} \times 60^\circ = 90^\circ + 30^\circ$$

$$\angle BIC = 120^\circ$$

Alternate

In $\triangle BIC$,

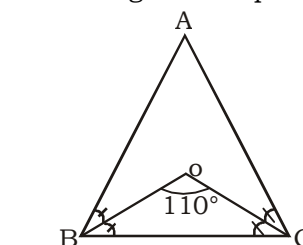
$$\frac{1}{2} \angle B + \frac{1}{2} \angle C + \angle BIC = 180^\circ$$

$$\frac{1}{2} (65^\circ + 55^\circ) + \angle BIC = 180^\circ$$

$$\angle BIC = 180^\circ - 60^\circ = 120^\circ$$

6. The internal bisectors of $\angle ABC$ and $\angle ACB$ of $\triangle ABC$ meet each other at O. If $\angle BOC = 110^\circ$, then $\angle BAC$ is equal to

Sol. According to the question



Given: $\angle BOC = 110^\circ$

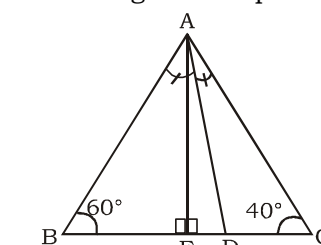
$$\angle BOC = 90^\circ + \frac{\angle A}{2}$$

$$110^\circ = 90^\circ + \frac{\angle A}{2}$$

$$20^\circ = \frac{\angle A}{2}, \quad \angle A = 40^\circ$$

7. In $\triangle ABC$, $\angle B = 60^\circ$ and $\angle C = 40^\circ$. If AD and AE be respectively the internal bisector of $\angle A$ and perpendicular on BC, then the measure of $\angle DAE$ is

Sol. According to the question,



Given: $\angle B = 60^\circ$

$$\angle C = 40^\circ$$

As we know that

$$\begin{aligned}\angle A + \angle B + \angle C &= 180^\circ \\ \angle A &= 180^\circ - 60^\circ - 40^\circ \\ \angle A &= 80^\circ\end{aligned}$$

$$\therefore \angle BAD = \frac{80^\circ}{2} = 40^\circ$$

In $\triangle AEB$

$$\begin{aligned}\angle A + \angle B + \angle E &= 180^\circ \\ \angle A &= 180^\circ - 60^\circ - 90^\circ \\ \angle A &= 30^\circ\end{aligned}$$

Then,

$$\begin{aligned}\angle DAE &= \angle DAB - \angle EAB \\ &= 40^\circ - 30^\circ\end{aligned}$$

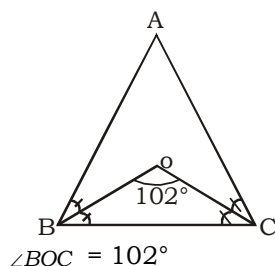
$$\angle DAE = 10^\circ$$

Alternate

$$\angle DAE = \frac{\angle B - \angle C}{2} = \frac{60^\circ - 40^\circ}{2} = 10^\circ$$

8. Internal bisectors of $\angle B$ and $\angle C$ of $\triangle ABC$ intersect at O. If $\angle BOC = 102^\circ$, then the value of $\angle BAC$ is

Sol. According to the Question,
Given:

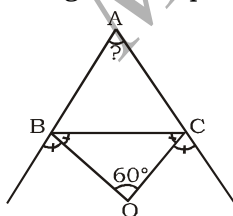


$$102^\circ = 90^\circ + \frac{1}{2}\angle A$$

$$\frac{\angle A}{2} = 12^\circ, \angle A = 24^\circ$$

9. The angle between the external bisectors of two angles of a triangle is 60° . Then the third angle of the triangle is

Sol. According to the question



Given: $\angle BOC = 60^\circ$

As we know that

$$\therefore \angle O = 90^\circ - \frac{1}{2}\angle A$$

$$\frac{1}{2}\angle A = 90^\circ - 60^\circ$$

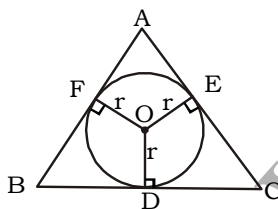
$$\frac{1}{2}\angle A = 30^\circ$$

$$\angle A = 60^\circ$$

10. The point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle is :

- circumcentre
- centroid
- incentre
- orthocentre

Sol.



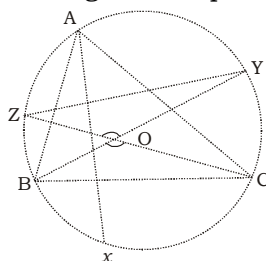
Incentre is a point in the plane of a triangle which is at equal perpendicular distance from the sides of the triangle

Here,

$OD \perp BC, OE \perp AC, OF \perp AB$
and $OD = OE = OF$ (Inradius)

11. In $\triangle ABC$, the internal bisector of the $\angle A, \angle B$ and $\angle C$ intersect the circumcircle at x, y and z respectively. If $\angle A = 50^\circ, \angle CZY = 30^\circ$ then $\angle BYZ$ will be

Sol. According to the question



$$\angle ZOY = \angle BOC = 90^\circ + \frac{A}{2}$$

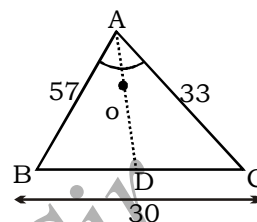
$$= 90^\circ + \frac{50^\circ}{2} = 115^\circ$$

In $\triangle ZOY, \angle ZOY = 180^\circ - (30^\circ + 115^\circ) = 35^\circ$

12. Three sides of a $\triangle ABC$ are, $a = 30$ cm, $b = 33$ cm, $c = 57$ cm. The internal bisector of $\angle A$

meets BC at D, and the bisector passes through incentre O. The ratio $AO : OD$ is-

Sol. By 7 property of incentre

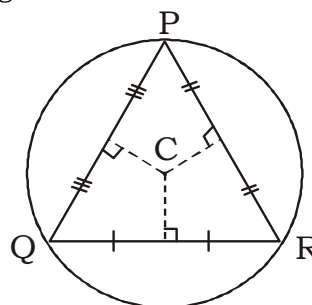


$$\frac{AO}{OD} = \frac{33 + 57}{30} = \frac{90}{30} = \frac{3}{1}$$

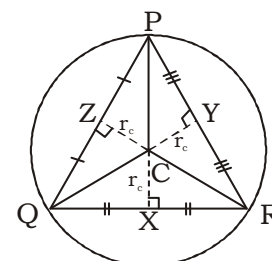
CIRCUMCENTRE

The circumcentre is the point where the perpendicular bisectors of all 3 sides of a triangle intersect.

It is also the centre of the triangle's circumcircle.



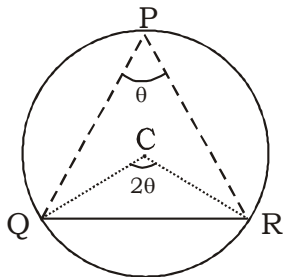
PROPERTIES



- The length from all 3 vertices to the circumcentre is equal. and it is called circumradius.
 $PC = QC = RC = R_c$
- If we draw a circle joining all 3 vertices of the triangle, it will be called the circumcircle of that triangle. And circumcentre of that triangle will also be the centre of its circumcircle.
- The angle between line segments joining the circumcentre and two vertices is double the angle of the third vertex.

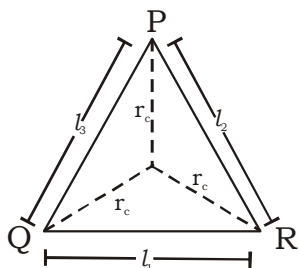
$$\begin{aligned}\angle QCR &= 2\angle P \\ \angle PCQ &= 2\angle R \\ \angle PCR &= 2\angle Q\end{aligned}$$

Proof:-



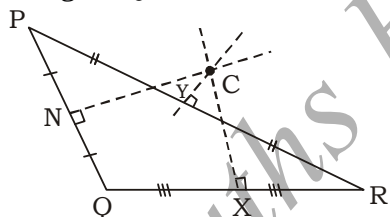
Because of the property of chord of a circle, an angle subtended by a chord at the centre is twice the angle subtended by the chord at any point on major arc of the circle.

4. Circumradius of a triangle (r_c) is

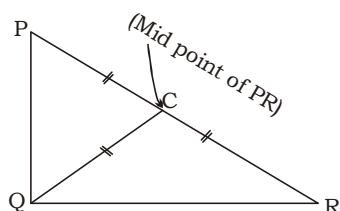


$$r_c = \frac{l_1 \times l_2 \times l_3}{4 \times (\text{Area} \Delta PQR)}$$

5. If ΔPQR is an obtuse angled triangle its circumcentre lies outside the triangle PQR.



6. If ΔPQR is a right angled triangle, then its circumcentre lies on the mid-point of its hypotenuse and circumradius (r_c) is half of its hypotenuse.



$PC = QC = RC = r_c$ (radius of circumcentre)

= half of hypotenuse

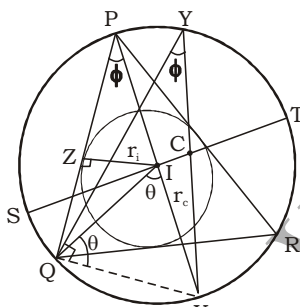
(Note:- QC is also the median of ΔPQR)

7. The distance (d) between the circumcentre (r_c) and Incentre (r_i) of a triangle can be expressed as

$$d = \sqrt{r_c^2 - 2r_c r_i}$$

or

$$\frac{1}{r_c + d} + \frac{1}{r_c - d} = \frac{1}{r_i}$$



Let C be the circumcentre of triangle PQR and I be the incentre

$\Rightarrow$ The extension of PI meets circumcircle at X, then X is the midpoint of arc QR

$\Rightarrow$ Join XC and extend it so that it intersects the circumcircle at Y.

$\Rightarrow$ From I construct a perpendicular to PQ and let Z be its foot. so $IZ = r_i$

Now $\Delta PZI \sim YQX$

or

$$\angle PZI = \angle YQX = 90^\circ$$

($\therefore \angle YQX = 90^\circ$, an angle in a semicircle)

$$\angle ZPI = \angle QYX = \phi$$

($\therefore$ angles made by a chord)

$$\text{So } \frac{IZ}{XQ} = \frac{PI}{YX}$$

$$\Rightarrow IZ \times YX = PI \times XQ$$

$$PI \times XQ = 2r_c r_i \quad \left[\begin{array}{l} \therefore IZ = r_i \\ XY = 2 \times r_c \end{array} \right]$$

Now join QI.

$$\Rightarrow \angle QIX = \angle QPI + \angle PQI = \angle P/2 + \angle PQR/2$$

$$\Rightarrow \angle IQX = \angle IQR + \angle RQX = \angle PQR/2 + \angle P/2$$

[$\therefore \angle RQX = \angle P/2$ angles from same chord XR]

We have $\angle QIX = \angle IQX$

So $IX = QX$

Now extend IC, so that it intersects the circumcircle at S and T.

Now,

$$TI \times SI = PI \times IX$$

$$(r_c + d) \times (r_c - d) = 2r_c r_i$$

$$r_c^2 - d^2 = 2r_c r_i$$

$$d^2 = r_c^2 - 2r_c r_i$$

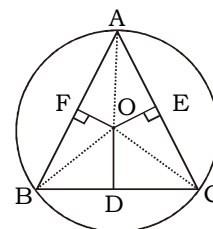
$$d = \sqrt{r_c^2 - 2r_c r_i}$$

Examples

13. The equidistant point from the vertices of a triangle is called its :

- centroid
- incentre
- circumcentre
- orthocentre

Sol. The Perpendicular bisector of sides meet at a point called 'circumcentre'.



$OA = OB = OC$ (Circumradius)

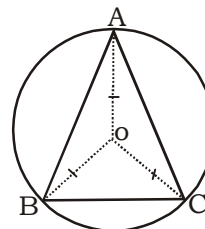
14. The circumcentre of a triangle ABC is O. If $\angle BAC = 85^\circ$ and $\angle BCA = 75^\circ$, then the value of $\angle OAC$ is

Sol. According to the Question Given:

$$\angle BAC = 85^\circ$$

$$\angle BCA = 75^\circ$$

$$\angle OAC = ?$$



In ΔABC

$$\angle ABC + \angle BCA + \angle CAB = 180^\circ$$

$$\angle ABC = 20^\circ$$

$$\therefore \angle COA = 2 \times \angle ABC$$

$$\angle COA = 2 \times 20 = 40^\circ$$

In ΔAOC

We know $OC = OA$
(Circum Radius)

$$\therefore \angle OAC = \angle OCA$$

$$\therefore \angle OAC + \angle OCA + \angle COA = 180^\circ$$

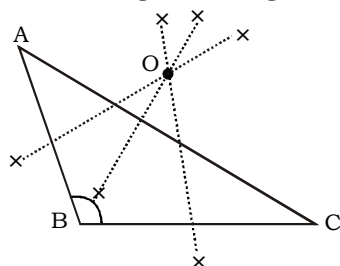
$$2\angle OAC = 180^\circ - 40^\circ$$

$$2\angle OAC = 140^\circ$$

$$\angle OAC = 70^\circ$$

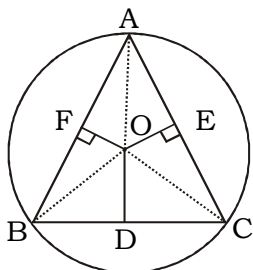
- 15.** If the circumcentre of a triangle lies outside it, then the triangle is

Sol. Circumcentre of a triangle lies outside it, then triangle is obtuse angled triangle.



- 16.** The equidistant point from the vertices of a triangle is called its:

Sol. The equidistant point from the vertices of a triangle is called circumcentre

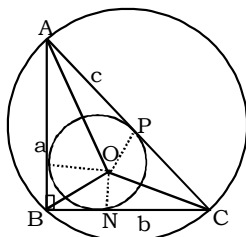


Here $OA = OB = OC$ (Circum Radius)

- 17.** The radius of the circumcircle of a right angled triangle is 15 cm and the radius of its incircle is 6 cm. Find the sides of the triangle.

- (a) 30, 40, 41
(b) 18, 24, 30
(c) 30, 24, 25
(d) 24, 36, 20

Sol. According to question



Given:

$$PC = 15 \text{ cm} = I_R \text{ (circumradius)}$$

$$ON = 6 \text{ cm} = I_r \text{ (Inradius)}$$

As we know that

$$I_R = \frac{AC}{2},$$

$$AC = 2 \times I_R = 2 \times 15 = 30 \text{ cm}$$

$$\text{and } I_r = \frac{a+b-c}{2}$$

$$a+b-c = 12$$

$$a+b = 12+c$$

$$a+b = 12+30$$

$$a+b = 42 \text{ cm}$$

Now check the option, any one option is satisfied

option: (b) Here $a = 18$

$$b = 24$$

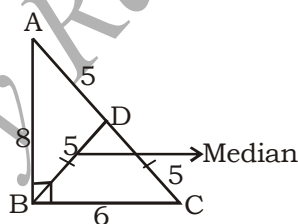
$$c = 30$$

$$a+b = 18+24 = 42 \text{ cm}$$

- 18.** If the length of the three sides of a triangle are 6 cm, 8 cm and 10 cm, then the length of the median to its greatest side is

Sol. According to question

Length of the three sides of a triangle are 6 cm, 8 cm and 10 cm, this is right angle triangle.



In right angle triangle median divides the hypotenuse in two equal parts

$$\therefore BD = \frac{H}{2}$$

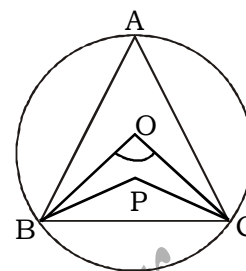
$$BD = \frac{10}{2}$$

$$BD = 5 \text{ cm}$$

- 19.** O is the circumcentre of a triangle ABC whose $\angle A = 50^\circ$. If bisector of $\angle OBC$ and $\angle OCB$ intersect at P then what is the measure of $\angle BPC$?

Sol. Since angle subtended at the centre of the circle is double the angle subtended at circumfer-

ence



$$\therefore \angle BOC = 50^\circ \times 2 = 100^\circ$$

$$\therefore OB = OC$$

$$\therefore \angle BOC = \angle OCB$$

$$= \frac{180^\circ - 100^\circ}{2} = 40^\circ$$

In $\triangle BPC$,

$$\angle BPC + \angle PBC + \angle PCB = 180^\circ$$

$$\text{or, } \angle BPC + \frac{1}{2} \times 40^\circ + \frac{1}{2} \times 40^\circ = 180^\circ$$

$$\therefore \angle BPC = 180^\circ - 20^\circ - 20^\circ = 140^\circ$$

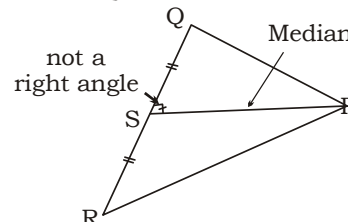
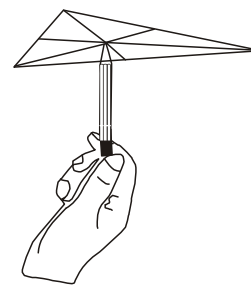
Alternate

$$\therefore \angle BPC = 90^\circ + \frac{\angle BOC}{2} = 90^\circ + A =$$

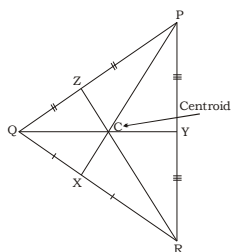
$$90^\circ + 50^\circ = 140^\circ$$

CENTROID

Draw a line (called a "median") from the vertices to the midpoint of the opposite sides. When all three medians intersect is called the centroid which is also the centre of mass.



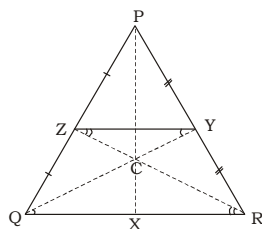
In the above figure the $\angle PSR$ is 90° only in the case of an equilateral triangle or if P is the top vertex of isosceles triangle.



Properties

- Each median is divided by the centroid in the ratio 2 : 1
i.e. $PC : CX = QC : CY = RC : CZ = 2 : 1$

Proof



$$\triangle CZY \sim \triangle CQR$$

$$\angle CZY = \angle CRQ$$

$$\angle CYZ = \angle CQR$$

alternate interior angle.

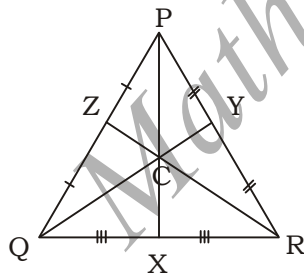
$$\frac{CY}{CQ} = \frac{ZY}{QR}$$

$$\left[\because \frac{ZY}{QR} = \frac{1}{2} \text{ (by mid point theorem)} \right]$$

So

$$\frac{CY}{CQ} = \frac{1}{2}$$

- When we draw all 3 medians in a triangle, then it divides the triangle into six small triangles of same area.



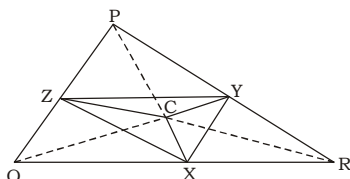
$$\begin{aligned} \text{Area } \triangle PCZ &= \text{Area } \triangle ZCQ = \text{Area } \triangle QCX \\ &= \text{Area } \triangle XCR \\ &= \text{Area } \triangle RCY = \text{Area } \triangle CPY = \\ &= \frac{1}{6} \text{ Area } \triangle PQR \end{aligned}$$

- Area of triangle formed by joining the centroid to mid points

of 2 sides is $\frac{1}{12}$ th of area of triangle

$$\begin{aligned} \text{Area } \triangle XYZ &= \text{Area } \triangle YZC = \\ &= \text{Area } \triangle ZXC \end{aligned}$$

$$= \frac{1}{12} \text{ Area } \triangle PQR$$



Proof:-

$$\triangle CZY \sim \triangle CRQ$$

$$[ZY \parallel QR = \text{Mid-point theorem}]$$

So,

$$\angle YZC = \angle CRQ \text{ and}$$

$$\angle ZYC = \angle CQR$$

$$\begin{aligned} \therefore \frac{\text{Area } \triangle CZY}{\text{Area } \triangle CQR} &= \frac{(ZY)^2}{(QR)^2} \\ &= \left(\frac{\frac{1}{2} QR}{QR} \right)^2 = \frac{1}{4} \end{aligned}$$

$$\text{Area } \triangle CZY = \frac{1}{4} \times \text{Area } \triangle CQR = \frac{1}{4}$$

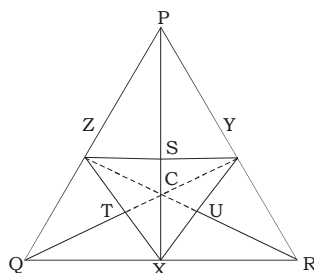
$$\times \left[\frac{1}{3} \text{ Area } \triangle PQR \right]$$

$$\left[\because \text{Area } \triangle CQR = \frac{1}{3} \text{ Area } \triangle PQR \right]$$

So,

$$\text{Area } \triangle CZY = \frac{1}{12} \text{ Area } \triangle PQR$$

- C is also the centroid of $\triangle XYZ$
- The line segment drawn via joining the mid points of two sides divides the line joining from centroid to the common vertex of two sides in the ratio 1 : 3



$$\begin{aligned} CS : SP &= CT : TQ = CU : UR = 1 : 3 \\ PS : SC &= QT : TC = RU : UC = 3 : 1 \end{aligned}$$

Proof

Let $PX = 6n$ then $PC = 4n$ and $CX = 2n$

$$\triangle PZS \sim \triangle PQX$$

$$\Rightarrow \angle PZS = \angle PQX$$

$$[\because ZS \parallel QX \text{ corresponding angle}]$$

$$\Rightarrow \angle ZPS = \angle QPX, [\because \text{common angle}]$$

$$\frac{PS}{PX} = \frac{ZS}{QX} = \frac{1}{2}$$

[S is the mid point of PX]

$$\text{So, } PS = \frac{PX}{2} = \frac{6n}{2} = 3n$$

$$\text{then } SC = PC - PS = 4n - 3n = n$$

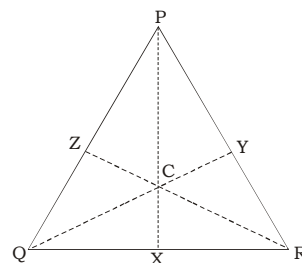
$$\therefore \frac{PS}{SC} = \frac{3n}{n} = \frac{3}{1}$$

- In a triangle the ratio of sum of medians to the perimeter is

always smaller than $\frac{3}{2}$, or

$$\text{vice-versa } \frac{PX+QY+RZ}{PQ+QR+RP} < \frac{3}{2} \text{ or}$$

$$\frac{PQ+QR+RP}{PX+QY+RZ} > \frac{2}{3}$$



Proof:-

Apply triangle property in $\triangle PQX$

$$PQ + QX > PX$$

$$PQ + \frac{QR}{2} > PX \quad \dots\dots (i)$$

Similarly in $\triangle QYR$

$$QR + \frac{PR}{2} > QY \quad \dots\dots (ii)$$

Similarly in $\triangle RZP$

$$PR + \frac{PQ}{2} > RZ \quad \dots\dots (iii)$$

adding (i), (ii) and (iii), we get

$$\frac{3}{2} (PQ + QR + RP) > (PX + QY + RZ)$$

$$\frac{(PX+QY+RZ)}{(PQ+QR+RP)} < \frac{3}{2}$$

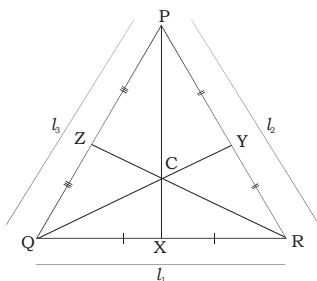
$$\text{or } \frac{(PQ+QR+RP)}{(PX+QY+RZ)} > \frac{2}{3}$$

7. The length of medians can be obtained from Apollonius theorem

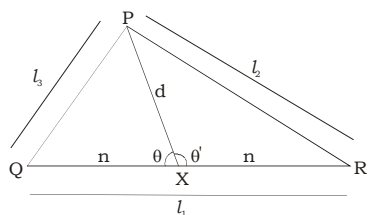
$$PX = M_P = \frac{1}{2} \sqrt{2l_2^2 + 2l_3^2 - l_1^2}$$

$$QY = M_Q = \frac{1}{2} \sqrt{2l_1^2 + 2l_3^2 - l_2^2}$$

$$RZ = M_R = \frac{1}{2} \sqrt{2l_1^2 + 2l_2^2 - l_3^2}$$



Proof:-



Let $PX = d$
and $QX = XR = n$
So

$$n = \frac{l_1}{2}$$

Now In $\triangle PQX$

$$l_3^2 = n^2 + d^2 - 2dn \cos \theta \dots\dots (i)$$

[$\therefore$ using cosine formula]

In $\triangle PXR$,

$$l_2^2 = n^2 + d^2 - 2dn \cos \theta'$$

$$= n^2 + d^2 - 2dn \cos (180^\circ - \theta)$$

$$[\theta + \theta' = 180^\circ]$$

$$= n^2 + d^2 + 2dn \cos \theta \dots\dots (ii)$$

$$[\cos(180^\circ - \theta) = -\cos \theta]$$

adding (i) and (ii)

$$l_2^2 + l_3^2 = 2(n^2 + d^2) \dots\dots (iii)$$

Multiply (iii) by 2.

$$2l_2^2 + 2l_3^2 = 4n^2 + 4d^2$$

$$2l_2^2 + 2l_3^2 = l_1^2 + 4d^2$$

$$[4n^2 = (2n)^2 = l_1^2]$$

$$4d^2 = 2l_2^2 + 2l_3^2 - l_1^2$$

$$2d = \sqrt{2l_2^2 + 2l_3^2 - l_1^2}$$

$$d = \frac{1}{2} \sqrt{2l_2^2 + 2l_3^2 - l_1^2}$$

$$PX = \frac{1}{2} \sqrt{2l_2^2 + 2l_3^2 - l_1^2}$$

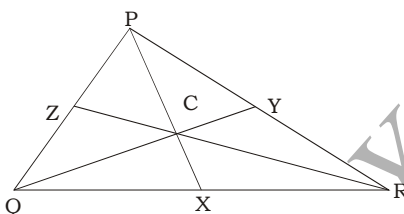
Note:- We can also find the length of sides if medians are given

$$l_1 = \frac{2}{3} \sqrt{2M_Q^2 + 2M_R^2 - M_P^2}$$

$$l_2 = \frac{2}{3} \sqrt{2M_P^2 + 2M_R^2 - M_Q^2}$$

$$l_3 = \frac{2}{3} \sqrt{2M_P^2 + 2M_Q^2 - M_R^2}$$

8. In a triangle four times of sum of squares of median is equal to three times the sum of squares of sides.



$$4(PX^2 + QY^2 + RZ^2) = 3(PQ^2 + QR^2 + RP^2)$$

Proof:

Apply apollonius theorem for all three medians

$$\Rightarrow 4M_P^2 + 4M_Q^2 + 4M_R^2 = (2l_2^2 + 2l_3^2 - l_1^2) + (2l_1^2 + 2l_3^2 - l_2^2) + (2l_1^2 + 2l_2^2 - l_3^2)$$

$$4(M_P^2 + M_Q^2 + M_R^2) = 3(l_1^2 + l_2^2 + l_3^2)$$

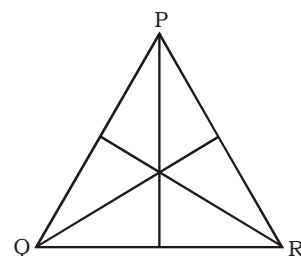
9. Determining the area of a triangle by Heron's formula for medians, providing medians of a triangles are given

let M_P , M_Q and M_R are the three medians of a triangle, and

$$S_M = \frac{M_P + M_Q + M_R}{2}$$

$$\text{Area } \triangle PQR =$$

$$\frac{4}{3} \sqrt{S_M(S_M - M_P)(S_M - M_Q)(S_M - M_R)}$$

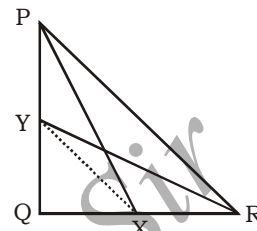


Note:- If $M_P^2 + M_Q^2 = M_R^2$ then area of triangle

$$= \frac{4}{3} \times \left(\frac{1}{2} \times M_P \times M_Q \right) = \frac{2}{3} \times M_P$$

$$\times M_Q$$

10. In a right angle triangle five times square of hypotenuse is equal to four times of sum of square of two medians (not right angle vertex median)



$$5(PR)^2 = 4(M_P^2 + M_R^2)$$

$\Rightarrow$ Points To Remember:-

$$M_P^2 + M_R^2 = (XY)^2 + (PR)^2 \dots(i)$$

$$4(M_P^2 + M_R^2) = 5(PR)^2 \dots(ii)$$

$$M_P^2 + M_R^2 = 5(XY)^2 \dots(iii)$$

$$M_P^2 + M_R^2 = 5M_Q^2 \dots(iv)$$

Proof:-

$$M_P^2 + M_R^2 = (PQ^2 + QX^2) + (YQ^2 + QR^2)$$

$$M_P^2 + M_R^2 = XY^2 + PR^2$$

$$[PQ^2 + QR^2 = PR^2 \text{ \& } YQ^2 + QX^2 = XY^2]$$

According to mid-point theorem.

$$XY = \frac{PR}{2} \text{ So, } XY^2 = \frac{PR^2}{4}$$

$$M_P^2 + M_R^2 = \frac{PR^2}{4} + PR^2$$

$$4(M_P^2 + M_R^2) = 5PR^2$$

$$\text{Now as } PR = 2XY, PR^2 = 4XY^2$$

$$4(M_P^2 + M_R^2) = 5 \times (4XY^2)$$

$$M_P^2 + M_R^2 = 5XY^2$$

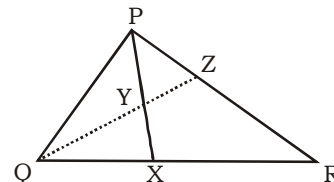
According to property of circumcentre of right angled triangle

$$M_Q = \frac{PR}{2}$$

$$M_P^2 + M_R^2 = 5M_Q^2$$

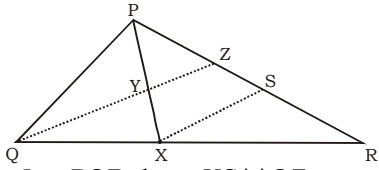
11. The line segment joining the mid-point of a median to vertex divides opposite side in the ratio 1 : 2.

Y is the mid-point of PX, then



$$PZ : ZR = 1 : 2$$

Proof:-



In ΔPQR draw $XS \parallel QZ$

as PX is a median then X is the mid-point of QR , then S is also the mid-point of ZR .

(According to mid point theorem)

So,

$$ZS : SR = 1 : 1$$

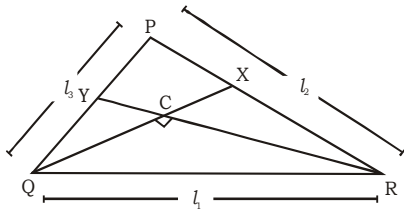
In ΔPXS ,

$YZ \parallel XS$ and Y is the mid point of PX , then Z is also the mid-point of PS .

$$PZ : ZS = 1 : 1$$

$$PZ : ZR = 1 : 2$$

12. If 2 medians are perpendicular to each other then five times the square of common side is equal to sum of square of other two sides



Proof

If $M_Q \perp M_R$ then

In ΔQCR ,

$$QC^2 + CR^2 = QR^2$$

$$\left(\frac{2}{3}M_Q\right)^2 + \left(\frac{2}{3}M_R\right)^2 = l_1^2$$

$$M_Q^2 + M_R^2 = \frac{9}{4}l_1^2$$

by appolonius theorem

$$\frac{2l_1^2 + 2l_3^2 - l_2^2}{4} + \frac{2l_1^2 + 2l_2^2 - l_3^2}{4}$$

$$= \frac{9}{4}l_1^2$$

$$l_2^2 + l_3^2 = 5l_1^2$$

Note:- If median form pythagoras triplet with each other, i.e.,

$$M_Q^2 + M_R^2 = M_P^2$$

then also result will be the same

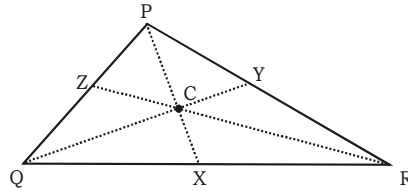
i.e.,

$$l_2^2 + l_3^2 = 5l_1^2$$

$\Rightarrow$ If two medians are perpendicular then all medians will form

pythagorean triplets.

13. The sum of any two sides of a triangle is greater than twice the median drawn to the third side.



$$PQ + PR > 2PX \quad \dots(i)$$

$$PQ + QR > 2QY \quad \dots(ii)$$

$$PR + QR > 2RZ \quad \dots(iii)$$

Adding (i), (ii) and (iii)

$$2(PQ + QR + RP) > 2(PX + QY + RZ)$$

$$PQ + QR + RP > PX + QY + RZ$$

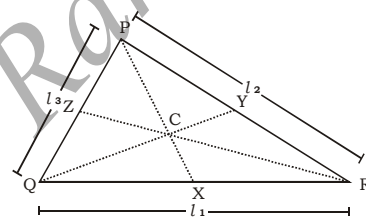
14. Sum of three medians of a triangle is always greater

than $\frac{3}{4}$ of perimeter

i.e.,

$$M_P + M_Q + M_R > \frac{3}{4}(l_1 + l_2 + l_3)$$

Proof:-



$$PC = \frac{2}{3}M_P \quad [\because PX = M_P]$$

$$QC = \frac{2}{3}M_Q \quad [\because QY = M_Q]$$

$$RC = \frac{2}{3}M_R \quad [\because RZ = M_R]$$

Apply inequality of triangle

In ΔPCQ

$$\frac{2}{3}M_P + \frac{2}{3}M_Q > l_3 \quad \dots(i)$$

In ΔPCR

$$\frac{2}{3}M_P + \frac{2}{3}M_R > l_2 \quad \dots(ii)$$

In ΔQCR

$$\frac{2}{3}M_Q + \frac{2}{3}M_R > l_1 \quad \dots(iii)$$

Adding (i), (ii) & (iii),

We get,

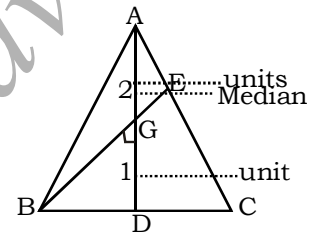
$$\frac{4}{3}(M_P + M_Q + M_R) > l_1 + l_2 + l_3$$

$$M_P + M_Q + M_R > \frac{3}{4}(l_1 + l_2 + l_3)$$

Examples

20. Two medians AD and BE of ΔABC intersect at G at right angles. If $AD = 9$ cm and $BE = 6$ cm, then the length of BD (in cm) is

Sol. According to question



G is the centroid which divides the median in $2 : 1$

$$\therefore AD = 3 \text{ units} = 9 \text{ cm}$$

$$3 \text{ units} = 9 \text{ cm}$$

$$1 \text{ unit} = \frac{9}{3} = 3 \text{ cm}$$

$$\therefore GD = 3 \text{ cm}$$

$$BE = 3 \text{ units} = 6 \text{ cm}$$

$$3 \text{ units} = 6 \text{ cm}$$

$$1 \text{ unit} = \frac{6}{3}$$

$$2 \text{ units} = \frac{6}{3} \times 2 = 4 \text{ cm}$$

$$\therefore BG = 4 \text{ cm}$$

ΔBGD is a right angle triangle

$$BD^2 = BG^2 + GD^2$$

$$BD^2 = (4)^2 + (3)^2$$

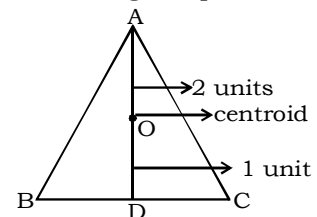
$$BD^2 = 16 + 9$$

$$BD = \sqrt{25}$$

$$BD = 5 \text{ cm}$$

21. AD is the median of a triangle ABC and O is the centroid such that $AO = 10$ cm. The length of OD (in cm) is

Sol. According to question

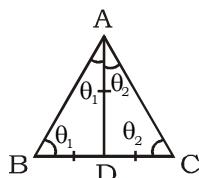


AD is the median and 'O' is the centroid

- ∴ AO = 10 cm
2 units = 10
1 units = 5
∴ OD = 5 cm

- 22.** If the median drawn on the base of a triangle is half its base the triangle will be

Sol. According to question
If the median drawn on the base of a triangle is half its base of the triangle will be right angled triangle.



In $\triangle ABC$, AD is median on base BC

According to the question,

$$AD = \frac{BC}{2}$$

AD = BD = DC (AD is median)

In $\triangle ABD$,
BD = DA

$$\angle DBA = \angle DAB = \theta_1$$

In $\triangle ADC$

DC = DA

$$\angle DAC = \angle DCA = \theta_2$$

Now,

In $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\theta_1 + \theta_2 + \theta_1 + \theta_2 = 180^\circ$$

$$2(\theta_1 + \theta_2) = 90^\circ$$

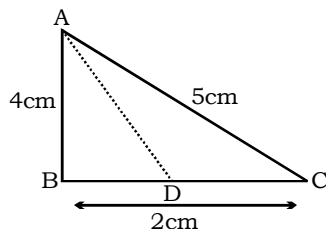
$$\theta_1 + \theta_2 = 90^\circ$$

$$\therefore \angle A = \theta_1 + \theta_2 = 90^\circ$$

So, Triangle will be Right Angle triangle

- 24.** In a $\triangle ABC$, three sides are 5cm, 4cm, and 2 cm. Find the length of median from smallest angle vertex.

Sol. Smallest angle is opposite to smallest side. By Apollonius theorem.



$$AD = \sqrt{\frac{2AB^2 + 2AC^2 - BC^2}{4}}$$

$$= \sqrt{\frac{32 + 50 - 4}{4}} = \sqrt{\frac{78}{4}}$$

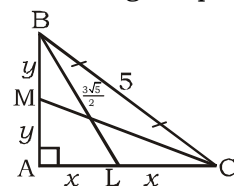
$$AD = \sqrt{\frac{39}{2}}$$

- 25..** BL and CM are medians of $\triangle ABC$ right-angled at A and

BC = 5 cm. If BL = $\frac{3\sqrt{5}}{2}$ cm, then

the length of CM is

Sol. According to question



According to figure, when two medians intersect each other in a right angled triangle then we use, this equation.

$$\Rightarrow 4(BL^2 + CM^2) = 5BC^2$$

$$\Rightarrow 4 \times \left(\frac{3\sqrt{5}}{2}\right)^2 + 4CM^2 = 5 \times (5)^2$$

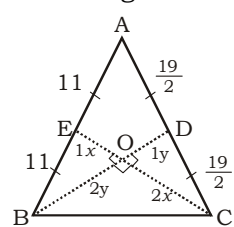
$$\Rightarrow 45 + 4CM^2 = 125$$

$$\Rightarrow CM^2 = \frac{125 - 45}{4} = 20$$

$$\Rightarrow CM = 2\sqrt{5} \text{ cm}$$

- 26.** In a $\triangle ABC$, BD & CE are the two medians which intersect each other at right angle. AB = 22, AC = 19, find BC = ?

Sol. According to the question,



In right angle $\triangle COD$,
 $OC^2 + OD^2 = CD^2$

$$4x^2 + y^2 = \frac{361}{4} \quad \dots(i)$$

In right angle $\triangle BOE$,
 $OB^2 + OE^2 = BE^2$

$$x^2 + 4y^2 = 121 \quad \dots(ii)$$

Add Both Eq. (i) & (ii)

$$5x^2 + 5y^2 = \frac{361}{4} + 121$$

$$x^2 + y^2 = \frac{169}{4} \quad \dots(iii)$$

In right angle $\triangle BOC$,

$$OB^2 + OC^2 = BC^2$$

$$4y^2 + 4x^2 = BC^2$$

$$4(y^2 + x^2) = BC^2$$

From Eq. (iii)

$$BC^2 = 169$$

$$BC = 13$$

Alternate:

In this type of question we use direct formula.

$$5BC^2 = AB^2 + AC^2$$

$$BC = \sqrt{\frac{AB^2 + AC^2}{5}}$$

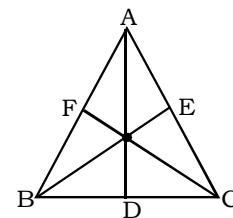
$$= \sqrt{\frac{(22)^2 + (19)^2}{5}} = \sqrt{\frac{484 + 361}{5}}$$

$$= \sqrt{\frac{845}{5}} = \sqrt{169} = 13 \text{ cm}$$

- 27.** If AD, BE and CF are medians of $\triangle ABC$, then which one of the following statements is correct?

- (a) $AB + BC + CA < AD + BE + CF$
(b) $AB + BC + CA > AD + BE + CF$
(c) $AB + BC + CA = AD + BE + CF$
(d) None of these

Sol. According to question



Points D, E & F are midpoint of BC, CA and AB.

$$\therefore AB + AC > 2AD \quad \dots(i)$$

$$AB + BC > 2BE \quad \dots(ii)$$

$$BC + CA > 2CF \quad \dots(iii)$$

Adding to equation (i), (ii) and (iii) we get

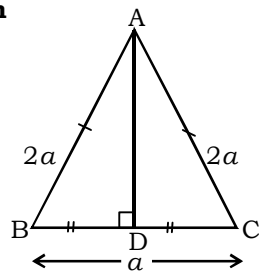
$$2(AB + BC + CA) > 2(AD + BE + CF)$$

$$\therefore (AB + BC + CA) > (AD + BE + CF)$$

- 28.** $\triangle ABC$ is an isosceles triangle and $\overline{AB} = \overline{AC} = 2a$ unit, $\overline{BC} = a$ unit. Draw $\overline{AD} \perp \overline{BC}$, and find the length of $\overline{AD}$.

Sol. According to the question

Given



$$AB = AC = 2a$$

$$BC = a$$

$$AD \perp BC$$

In isosceles triangle perpendicular sides bisects the opposite side of the length

$$\therefore BD = \frac{BC}{2}$$

$$BD = \frac{a}{2}$$

In $\triangle ADB$ using pythagoras theorem .

$$AB^2 = BD^2 + AD^2$$

$$\Rightarrow (2a)^2 = \left(\frac{a}{2}\right)^2 + AD^2$$

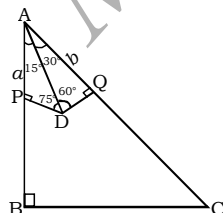
$$\Rightarrow 4a^2 = \frac{a^2}{4} + AD^2$$

$$\Rightarrow AD^2 = 4a^2 - \frac{a^2}{4}$$

$$\Rightarrow AD = \frac{\sqrt{15}a}{2} \text{ units}$$

- 29.** An isosceles triangle ABC is right-angled at B. D is a point inside the triangle ABC. P and Q are the feet of the perpendiculars drawn from D on the side AB and AC respectively of $\triangle ABC$. If $AP = a$ cm, $AQ = b$ cm and $\angle BAD = 15^\circ$, Find the value of $\sin 75^\circ$?

Sol. According to the question,



$\triangle ABC$ is a isosceles triangle

So, $\angle B = 90^\circ$, $\angle C = \angle A = 45^\circ$

In $\triangle AQD$

$$\angle DAC = \angle BAC - \angle BAD = 45^\circ - 15^\circ = 30^\circ$$

$$\text{and } \angle QDA = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$$

From $\triangle AQD$

$$\sin 60^\circ = \frac{AQ}{AD}$$

$$\frac{\sqrt{3}}{2} = \frac{b}{AD}$$

$$AD = \frac{2b}{\sqrt{3}}$$

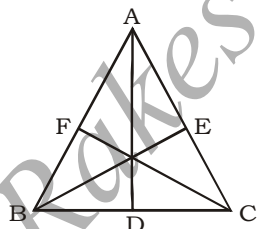
From $\triangle APD$

$$\sin 75^\circ = \frac{AP}{AD}$$

$$\sin 75^\circ = \frac{a}{\frac{2b}{\sqrt{3}}} \times \sqrt{3} = \frac{\sqrt{3}a}{2b}$$

- 30.** In $\triangle ABC$, BE, AD, CF is a median on AC, BC and AB respectively. $AD = 10$, $BE = 6$ and $CF = 8$ cm. Then find the area of $\triangle ABC$?

Sol. According to the question,



By using this formula,

We can calculate the area of $\triangle ABC$.

$$\text{Area of } \triangle ABC = \frac{4}{3} \sqrt{S(S-a)(S-b)(S-c)}$$

Here, a, b & c are the length of the median.

$$\therefore S = \frac{a+b+c}{2} = \frac{8+6+10}{2} = 12$$

$$\therefore \text{Area} = \frac{4}{3} \sqrt{12(12-6)(12-8)(12-10)}$$

$$= \frac{4}{3} \sqrt{12 \times 6 \times 4 \times 2}$$

$$= \frac{4}{3} \times 24 = 32 \text{ cm}^2$$

Alternate:

When $m_1^2 + m_2^2 = m_3^2$

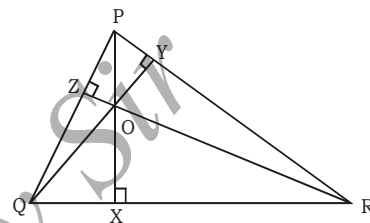
Then

$$\text{Area of } \triangle = \frac{2}{3} \times m_1 m_2$$

$$= \frac{2}{3} \times 6 \times 8 = 32 \text{ cm}^2$$

Orthocentre

A point where all 3 altitudes of a triangle meet is called the orthocentre.



Here,

PX, QY & RZ are the altitudes of $\triangle PQR$ & O is the orthocentre.

Properties

The sum of angle between line segments joining the orthocentre and two vertices and the third angle is always a supplementary angle.

$$\angle QOR + \angle QPR = 180^\circ = \angle ZOY + \angle QPR$$

$$[\angle QOR = \angle ZOY, \text{ Opposite angle}]$$

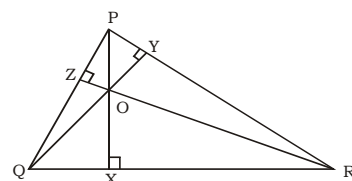
$$\Rightarrow \angle POR + \angle PQR = 180^\circ = \angle ZOX + \angle PQR$$

$$[\angle POR = \angle ZOX, \text{ Opposite angles}]$$

$$\Rightarrow \angle POQ + \angle PRQ = 180^\circ \Rightarrow \angle YOX + \angle PRQ$$

$$[\angle POQ = \angle YOX, \text{ Opposite angles}]$$

Proof:-



In Quadrilateral PYOZ

$$\angle Z = \angle Y = 90^\circ$$

So,

$$\angle P + \angle O = 180^\circ$$

$$\angle QPR + \angle ZOY = 180^\circ$$

$$\angle QPR + \angle QOR = 180^\circ$$

2. Orthocentre of triangles

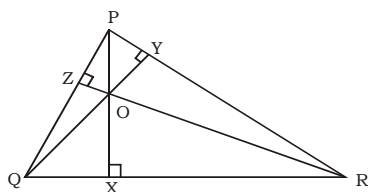
Orthocentre of $\triangle PQR = O$

Orthocentre of $\triangle QOR = P$

Orthocentre of $\triangle POQ = R$

Orthocentre of $\triangle POR = Q$

3. Pair of similar triangles when all three altitudes are drawn

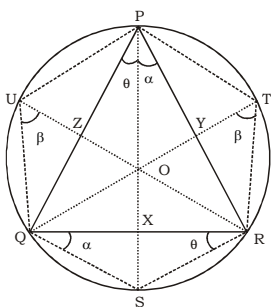


$$\triangle OYR \sim \triangle OZQ$$

$$\triangle OXR \sim \triangle OZP$$

$$\triangle OXQ \sim \triangle OYP$$

4. It must be noted that, In triangles PQR $\triangle OXQ$, $\triangle OXR$, $\triangle OYP$, $\triangle OYR$, $\triangle OZP$ & $\triangle OZQ$ are right angle triangles.
5. O is the orthocentre of $\triangle PQR$. Draw a circumcircle to triangle PQR. Since angles in the same segments of a circle are equal.

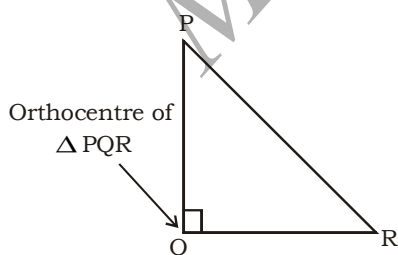


On Base, QS $\angle QPS = \angle QRS$

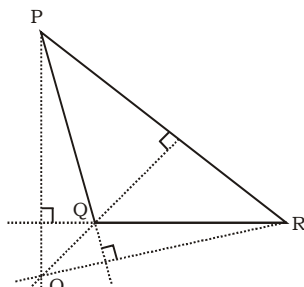
On Base, RS $\angle RPS = \angle RQS$

On Base, QR $\angle QTR = \angle QUR$ etc.

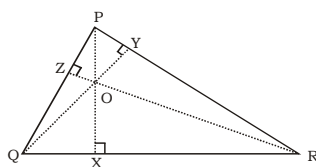
6. The orthocentre of a right angled triangle is that point where triangle forms the right angle.



7. The orthocentre of an obtuse angled triangle lies outside the triangle.



8. In this figure Orthocentre (O) lies outside the triangle PQR. Sum of three altitudes of a triangle is less than sum of three sides of the triangle.



In any right angled triangle Hypotenuse > side (altitude)

In $\triangle PXQ$, $PQ > PX$... (i)

In $\triangle QYR$, $QR > QY$... (ii)

In $\triangle PRZ$, $RP > RZ$... (iii)

adding (i), (ii) & (iii)

We get,

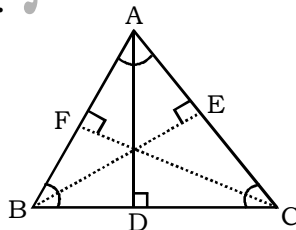
$$PQ + QR + RP > PX + QY + RZ$$

Examples

31. In $\triangle ABC$, AD, BE and CF are the altitudes in the ratio 1:2:3 respectively, then the ratio of AB : BC : CA is:

- (a) 3 : 2 : 1 (b) 1 : 2 : 3
(c) 1 : 4 : 9 (d) 2 : 6 : 3

Sol.



$$\text{Area of } \triangle ABC = \frac{1}{2} BC \times AD$$

$$= \frac{1}{2} AC \times BE = \frac{1}{2} AB \times CF$$

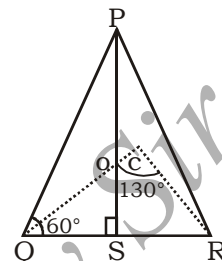
$$\therefore AB : BC : AC = \frac{1}{CF} : \frac{1}{AD} : \frac{1}{BE}$$

$$= \frac{1}{3} : \frac{1}{1} : \frac{1}{2} = 2 : 6 : 3$$

32. O and C are respectively the orthocentre and the circumcentre of an acute-

angled triangle PQR. The points P and O are joined and produced to meet the side QR at S. If $\angle PQS = 60^\circ$ and $\angle QCR = 130^\circ$, then $\angle RPS = ?$

Sol. According to question



Given $\angle PQS = 60^\circ$

$\angle QCR = 130^\circ$

$$\therefore \angle QPR = \frac{1}{2} \angle QCR$$

$$\angle QPR = \frac{1}{2} \times 130^\circ = 65^\circ$$

Now,

$$\angle PQS + \angle PSQ + \angle QPS = 180^\circ$$

$$60^\circ + 90^\circ + \angle QPS = 180^\circ$$

$$\angle QPS = 30^\circ$$

$$\angle RPS = \angle QPR - \angle QPS$$

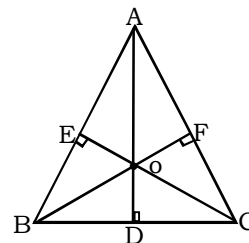
$$= 65^\circ - 30^\circ$$

$$\angle RPS = 35^\circ$$

33. The perpendiculars drawn from the vertices to the opposite sides of a triangle, meet at the point whose name is

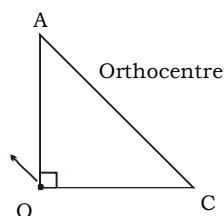
Sol. According to question

Draw a line (called the "altitude") at right angles to a side and going through the opposite corner. Where all three lines intersect is the "orthocentre" O is Orthocentre.



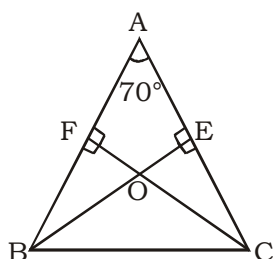
34. The orthocentre of a right angled triangle lies

Sol. The orthocentre of a right angled triangle lies at the right angular vertex



- 35.** In $\triangle ABC$, draw $BE \perp AC$ and $CF \perp AB$ and the perpendicular BE and CF intersect at the point O . If $\angle BAC = 70^\circ$, then the value of $\angle BOC$ is

Sol. According to question



Given: $\angle A = 70^\circ$

AEOF is a quadrilateral

$\therefore$ In a quadrilateral sum of all angles are 360°

$$\angle A + \angle F + \angle O + \angle E = 360^\circ$$

$$70^\circ + 90^\circ + \angle O + 90^\circ = 360^\circ$$

$$\angle O = 360^\circ - 250^\circ$$

$$\angle O = 110^\circ$$

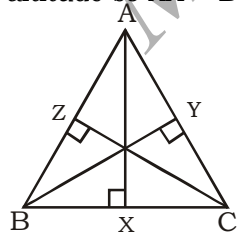
$$\angle BOC = 110^\circ$$

(Vertically Opposite angle)

- 36.** Let ABC be an equilateral triangle and AX, BY, CZ be the altitudes. Then the right statement out of the four given responses is

- $AX = BY = CZ$
- $AX \neq BY \neq CZ$
- $AX = BY \neq CZ$
- None of these

Sol. ABC is an equilateral triangle and AX, BY and CZ be the altitude so $AX = BY = CZ$



$$\text{Area of } \triangle ABC = \frac{1}{2} \times \text{Base} \times \text{height}$$

$$= \frac{1}{2} \times AB \times CZ = \frac{1}{2} \times BC \times AX$$

$$= \frac{1}{2} \times AC \times BY$$

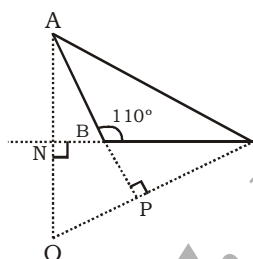
$$CZ = AX = BY$$

$\therefore AB = BC = CA$ (Side of equilateral)

so, we can say if three altitudes are equal then the triangle is equilateral triangle

- 37.** In obtuse angle $\triangle$, obtuse angle is 110° . Find the angle made on its orthocentre:

Sol. According to the question,



$$\angle ABC = 110^\circ = \angle NBP$$

(Vertically opposite)

In quadrilateral BNOP

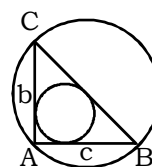
$$\therefore \angle N + \angle B + \angle P + \angle O = 360^\circ$$

$$\angle B = 360^\circ - 90^\circ - 90^\circ - 110^\circ$$

$$\angle B = 70^\circ$$

Mixed properties of centres of a triangle

- In an equilateral triangle all the four centres are coincident i.e., centroid, incentre, circumcentre and orthocentre of an equilateral triangle lie at the same point.
- Centroid (G), orthocentre (P) and circumcentre (O) of a triangle are always collinear (i.e., lie in a straight line) and $PG:GO = 2:1$.
- The orthocentre of a right angled triangle lies at the right angled vertex while its circumcentre is mid point of hypotenuse.
- Circumcentre and orthocentre of an obtuse angled triangle always lie outside the triangle.
- The sum of diameters of circumcircle and incircle of right angled triangle is equal to the sum of its perpendicular sides.

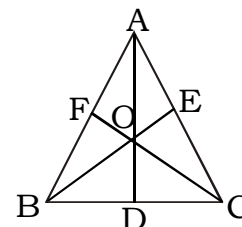


In the given figure

ABC is a right angled triangle with $\angle A = 90^\circ$. If radius of circumcircle and incircle of the triangle be respectively R and r then $2(R+r) = b+c$

- 6.** The distance between incentre and circumcentre of a triangle is $\sqrt{R^2 - 2rR}$ where R is circumradius and r is inradius.
- 7.** In an equilateral triangle, length or radius of the circumcircle is equal to twice the radius of its incircle i.e., if $\triangle ABC$ is equilateral then $R = 2r$
- 8. Ceva Theorem:-** If O is any point inside the triangle ABC and AO, BO, CO meet sides BC, CA, AB respectively at point D, E, F then

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$$

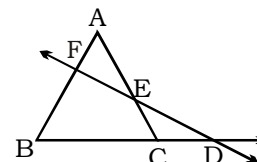


Since Ceva Theorem is true for any point inside the triangle, it is therefore also true for centroid, incentre, orthocentre and circumcentre of the triangle.

Converse of Ceva Theorem is also true.

- 9. Menelaus Theorem:-** If a transversal cuts the sides BC, CA and AB (or its produced part) of a triangle at D, E, F then

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = -1$$



Converse of the Theorem is also true.

$$\Rightarrow \frac{4}{3} = \frac{6}{AC} \Rightarrow 4AC = 18$$

$$\Rightarrow AC = \frac{18}{4} = 4.5 \text{ cm.}$$

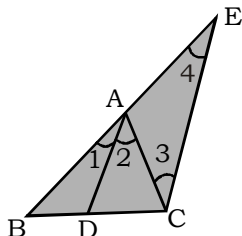
Interior Angle Bisector Theorem

Interior Angle Bisector Theorem :

The angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

Given: In $\triangle ABC$ in which AD is the internal bisector of $\angle A$ and meets BC at D.

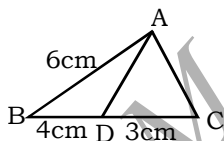
Prove that : $\frac{BD}{DC} = \frac{AB}{AC}$



Statements	Reasons
1) $CE \parallel DA$	1) By Construction
2) $\angle 2 = \angle 3$	2) Alternate interior angles
3) $\angle 1 = \angle 4$	3) Corresponding angles
4) AD is the bisector	4) Given
5) $\angle 1 = \angle 2$	5) Definition of angle bisector
6) $\angle 3 = \angle 4$	6) From (2),(3) and (5)
7) $AE = AC$	7) In $\triangle ACE$, side opposite to equal angles are equal
8) $\frac{BD}{DC} = \frac{BA}{AE}$	8) In $\triangle BCE$ $DA \parallel CE$ and by BPT theorem
9) $\frac{BD}{DC} = \frac{AB}{AC}$	9) From (7)

Examples

38. In the given figure, AD is the Internal bisector of $\angle A$. If $BD = 4\text{cm}$, $DC = 3\text{cm}$ and $AB = 6\text{cm}$, find AC.



Sol. In $\triangle ABC$, AD is the bisector of $\angle A$

$$\therefore \frac{BD}{DC} = \frac{AB}{AC} \text{ (Angle bisector theorem)}$$

Exterior Angle Bisector Theorem

Exterior angle bisector theorem:

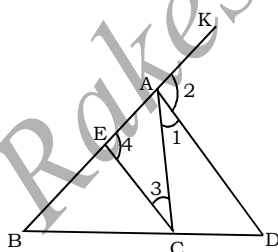
The external bisector of an angle of a triangle divides the opposite side externally in the ratio of the sides containing the angle.

Given

In $\triangle ABC$, in which AD is the bisector of the exterior $\angle A$ and intersects BC produced at D.

Prove that : $\frac{BD}{CD} = \frac{AB}{AC}$

Construction : Draw $CE \parallel DA$ meeting AB at E.

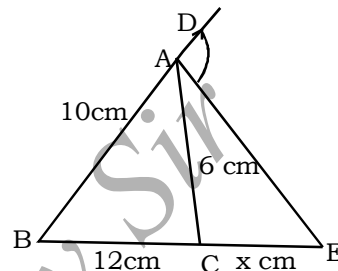


Statements	Reasons
1) $CE \parallel DA$	1) By Construction
2) $\angle 1 = \angle 3$	2) Alternate interior angles
3) $\angle 2 = \angle 4$	3) Corresponding angles ($CE \parallel DA$ and BK is a transversal)
4) AD is a bisector of A	4) Given
5) $\angle 1 = \angle 2$	5) Definition of angle bisector
6) $\angle 3 = \angle 4$	6) From (2),(3) and (5)
7) $AE = AC$	7) If angles are equal then side opposite to them are also equal
8) $\frac{BD}{CD} = \frac{BA}{EA}$	8) By Basic proportionality theorem ($EC \parallel AD$)
9) $\frac{BD}{CD} = \frac{AB}{AC}$	9) $BA = AB$ and $EA = AE$
10) $\frac{BD}{CD} = \frac{AB}{AC}$	10) $AE = EC$ and from (7)

Examples

39. In the given figure, AE is the bisector of the exterior $\angle CAD$ meeting BC produced in E. If $AB = 10\text{ cm}$, $AC = 6\text{ cm}$ and $BC = 12\text{ cm}$, Find CE.

Sol.



Given : $AB = 10\text{ cm}$, $AC = 6\text{ cm}$ and $BC = 12\text{ cm}$,

By exterior angle bisector theorem

$CE = x$ (Let)

$$\frac{BE}{CE} = \frac{AB}{AC}$$

$$\frac{12 + x}{x} = \frac{10}{6}$$

$6(12 + x) = 10x$ [by cross multiplication]

$$72 + 6x = 10x$$

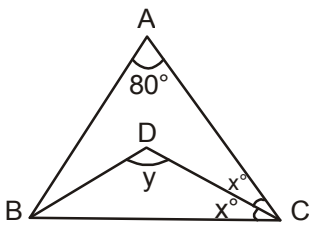
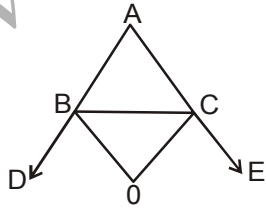
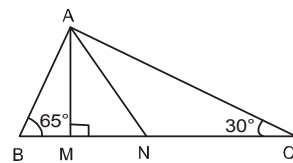
$$72 = 10x - 6x$$

$$72 = 4x$$

$$x = 18$$

$$CE = 18\text{ cm}$$

EXERCISE

1. Internal bisectors of angles $\angle B$ and $\angle C$ of a triangle ABC meet at O . If $\angle BAC = 80^\circ$, then the value of $\angle BOC$ is
(a) 120° (b) 140°
(c) 110° (d) 130°
2. The external bisector of $\angle B$ and $\angle C$ of $\triangle ABC$ (where AB and AC extended to E and F respectively) meet at point P . If $\angle BAC = 100^\circ$, then the measure of $\angle BPC$ is
(a) 50° (b) 80° (c) 40° (d) 52°
3. Internal bisectors of angles $\angle B$ and $\angle C$ of a triangle ABC meet at O . If $\angle BAC = 70^\circ$, then the value of $\angle BOC$ is
(a) 125° (b) 140°
(c) 110° (d) 130°
4. In $\triangle ABC$, the internal bisectors of $\angle ABC$ and $\angle ACB$ meet at I and $\angle BAC = 50^\circ$. The measure of $\angle BIC$ is
(a) 105° (b) 115°
(c) 125° (d) 130°
5. If I be the incentre of $\triangle ABC$ and $\angle B = 70^\circ$ and $\angle C = 50^\circ$, then the magnitude of $\angle BIC$ is
(a) 130° (b) 60°
(c) 120° (d) 105°
6. ABC is an equilateral triangle and CD is the internal bisector of $\angle C$. If DC is produced to E such that $AC = CE$, then $\angle CAE$ is equal to
(a) 45° (b) 75° (c) 30° (d) 15°
7. The radius of the incircle of the equilateral triangle having each side 6 cm is
(a) $2\sqrt{3}$ cm (b) $\sqrt{3}$ cm
(c) $6\sqrt{3}$ cm (d) 2 cm
8. The internal bisectors of the angles B and C of a triangle ABC meet at I . If $\angle BIC = \frac{\angle A}{2} + X$, then X is equal to
(a) 60° (b) 30° (c) 90° (d) 45°
9. Internal bisectors of $\angle Q$ and $\angle R$ of $\triangle PQR$ intersect at O . If $\angle ROQ = 96^\circ$ then the value of $\angle RPQ$ is :
(a) 12° (b) 24° (c) 36° (d) 6°
10. O is the incentre of $\triangle PQR$ and $\angle QPR = 50^\circ$, then the measure of $\angle QOR$ is:
(a) 125° (b) 100°
(c) 130° (d) 115°
11. AD is perpendicular to the internal bisector of $\angle ABC$ of $\triangle ABC$. DE is drawn through D and parallel to BC to meet AC at E . If the length of AC is 12 cm, then the length of AE (in cm.) is
(a) 8 (b) 3 (c) 4 (d) 6
12. If any two sides of a triangle are produced beyond its base and the exterior angles thus obtained are bisected, then these bisectors will include :
(a) half the sum of the base angles
(b) sum of the base angles
(c) half the difference of the base angles
(d) difference of the base angles
13. If I is the in-centre of $\triangle ABC$ and $\angle A = 60^\circ$, then the value of $\angle BIC$ is:
(a) 100° (b) 120°
(c) 150° (d) 110°
14. In the given figure, $\angle A = 80^\circ$, $\angle B = 60^\circ$, $\angle C = 2x$ and $\angle BDC = y^\circ$, BD and CD bisect angles B and C respectively. The value of x and y respectively are :

(a) 15° and 70° (b) 10° and 160°
(c) 20° and 130° (d) 20° and 125°
15. The sides of a right angle triangle containing the right angle measure 3 cm and 4 cm. The radius of the incircle of the triangle is :
(a) 3.5 cm (b) 1.75 cm
(c) 1 cm (d) 0.875 cm
16. In the given figure, BO and CO are the bisector of $\angle CBD$ and $\angle BCE$ respectively and $\angle A = 40^\circ$, then $\angle BOC$ is equal to :

(a) 60° (b) 65° (c) 75° (d) 70°
17. In the given figure, $AM \perp BC$ and AN is the bisector of $\angle A$. What is the measure of $\angle MAN$?

(a) 17.5° (b) 15.5°
(c) 16° (d) 20°
18. If the bisector of an angle of $\triangle$ bisects the opposite side, then $\triangle$ is :
(a) Scalene
(b) Isosceles
(c) Right triangle
(d) None of these.
19. Incentre of a triangle lies in the interior of :
(a) an isosceles triangle only
(b) an equilateral triangle only
(c) every triangle
(d) a Right-triangle only.
20. In $\triangle ABC$, the bisectors of $\angle B$ and $\angle C$ intersect each other at a point O , then $\angle BOC$ is equal to :

- (a) $90^\circ - \frac{1}{2} \angle A$
- (b) $90^\circ + \frac{1}{2} \angle A$
- (c) $120^\circ + \frac{1}{2} \angle A$
- (d) $120^\circ - \frac{1}{2} \angle A$
21. In $\triangle ABC$, the sides AB and AC are produced to P and Q respectively. The bisectors of $\angle PBC$ and $\angle QCB$ intersect at a point O, then $\angle BOC$ is equal to :
- (a) $90^\circ - \frac{1}{2} \angle A$
- (b) $90^\circ + \frac{1}{2} \angle A$
- (c) $120^\circ + \frac{1}{2} \angle A$
- (d) $120^\circ - \frac{1}{2} \angle A$
22. O is the incentre of $\triangle ABC$ and $\angle BOC = 130^\circ$. Find $\angle BAC$:
(a) 80° (b) 40° (c) 150° (d) 50°
23. The internal bisector of $\angle ABC$ and $\angle ACB$ of $\triangle ABC$ meet each other at O. If $\angle BOC = 120^\circ$, then $\angle BAC$ is equal to :
(a) 80° (b) 50° (c) 60° (d) 90°
24. O is the incentre of $\triangle ABC$ and $\angle A = 30^\circ$, then $\angle BOC$ is :
(a) 100° (b) 105° (c) 110° (d) 90°
25. In $\triangle ABC$, AD is the internal bisector of $\angle A$, meeting the side BC at D. If $BD = 5\text{cm}$, $BC = 7.5\text{cm}$, then $AB : AC$ is :
(a) $2 : 1$ (b) $1 : 2$ (c) $4 : 5$ (d) $3 : 5$
26. The external bisector of $\angle B$ and $\angle C$ of $\triangle ABC$ meet at point P. If $\angle BAC = 80^\circ$, then $\angle BPC$ is:
(a) 50° (b) 40° (c) 80° (d) 100°
27. Internal bisectors of $\angle B$ and $\angle C$ of a $\triangle ABC$ intersect at O, if $\angle BOC = 102^\circ$, then the value of $\angle BAC$ is :
(a) 12° (b) 24° (c) 48° (d) 60°
28. The in-radius of an equilateral triangle is of length 3cm, Then the length of each of its medians is :
(a) 12 cm (b) $\frac{9}{2}$ cm (c) 4cm (d) 9 cm
29. O is any point on the bisector of the acute angle $\angle ABC$. From O a line parallel to CB meets AB at P. Then $\triangle BPO$ is:
(a) right angled isosceles triangle
(b) isosceles but not a right angled
(c) equilateral triangle
(d) None of these
30. If O be the circumcentre of a triangle PQR and $\angle QOR = 110^\circ$, $\angle OPR = 25^\circ$, then the measure of $\angle PRQ$ is
(a) 65° (b) 50° (c) 55° (d) 60°
31. For a triangle circumcentre lies on one of its sides. The triangle is
(a) right angled
(b) obtused angled
(c) isosceles
(d) equilateral
32. In $\triangle ABC$, $\angle ABC = 70^\circ$, $\angle BCA = 40^\circ$, O is the point of intersection of the perpendicular bisectors of the sides, then the angle $\angle BOC$ is
(a) 100° (b) 120° (c) 130° (d) 140°
33. ABC is an equilateral triangle and O is its circumcentre, then the $\angle BOC$ is
(a) 100° (b) 110° (c) 120° (d) 130°
34. O is the circumcentre of $\triangle ABC$. If $\angle BAC = 85^\circ$, $\angle BCA = 75^\circ$, the $\angle OAC$ is equal to:
(a) 70° (b) 60° (c) 50° (d) 40°
35. If O is the circumcentre of a triangle ABC lying inside the triangle, the $\angle OBC + \angle BAC$ is equal to
(a) 120° (b) 110° (c) 90° (d) 60°
36. The circum-centre of a triangle is always the point of intersection of the:
(a) Medians
(b) bisectors
(c) Perpendiculars
(d) Perpendiculars bisector of the sides
37. The radius of circum-circle of an equilateral triangle of side 12cm is :
(a) $(4/3)\sqrt{3}$ (b) $4\sqrt{2}$ (c) $4\sqrt{3}$ (d) 4
38. In $\triangle ABC$, $\angle B$ is a right angle, $AC = 6\text{cm}$, and D is the mid-point of AC. The length of BD is:
(a) 4 cm (b) $\sqrt{6}$ cm (c) 3 cm (d) 3.5 cm
39. If P and Q are the mid-points of the sides CA and CB respectively of a triangle ABC, right-angled at C, then the value of $4(AQ^2 + BP^2)$ is equal to :
(a) $4BC^2$ (b) $2AC^2$ (c) $2BC^2$ (d) $5AB^2$
40. The length of the two sides forming the right angle of a right angled triangle are 6cm and 8cm. The length of its circum-radius :
(a) 5 cm (b) 7 cm (c) 6 cm (d) 10 cm
41. In $\triangle ABC$, $\angle B = 90^\circ$, $\angle C = 45^\circ$ and D is the mid-point of AC. If $AC = 4\sqrt{2}$ units, then BD is:
(a) $2\sqrt{2}$ units (b) $4\sqrt{2}$ (c) $\frac{5}{2}$ units (d) 2 units
42. In a triangle ABC, median is AD and centroid is O, $AO = 10\text{ cm}$. The length of OD (in cm) is
(a) 6 (b) 4 (c) 5 (d) 3.3
43. G is the centroid of the equilateral $\triangle ABC$. If $AB = 10\text{ cm}$ then length of AG is
(a) $\frac{5\sqrt{3}}{3}$ cm (b) $\frac{10\sqrt{3}}{3}$ cm (c) $5\sqrt{3}$ cm (d) $10\sqrt{3}$ cm

44. If the three medians of a triangle are same, then the triangle is
 (a) equilateral
 (b) isosceles
 (c) right-angled
 (d) obtuse-angle

45. In $\triangle ABC$, D is the mid-point of BC. Length AD is 27 cm. N is a point in AD such that the length of DN is 12 cm. The distance of N from the centroid of $\triangle ABC$ is equal to

(a) 3 cm (b) 6 cm
 (c) 9 cm (d) 15 cm

46. G is the centroid of $\triangle ABC$. The medians AD and BE intersect at right angles. If the lengths of AD and BE are 9 cm and 12 cm respectively; then the length of AB (in cm) is?

(a) 11 (b) 10
 (c) 10.5 (d) 85

47. The centroid of a $\triangle ABC$ is G. The area of $\triangle ABC$ is 60 cm^2 . The area of $\triangle GBC$ is

(a) 30 cm^2 (b) 40 cm^2
 (c) 10 cm^2 (d) 20 cm^2

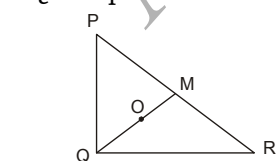
48. The medians CD and BE of a triangle ABC intersect each other at O. The ratio of $\triangle ODE$: $\triangle ABC$ is equal to

(a) 1 : 12 (b) 12 : 1
 (c) 4 : 3 (d) 3 : 4

49. In $\triangle ABC$, the median BE intersects AC at E, if BG = 6 cm, where G is the centroid, then BE is equal to :

(a) 8 cm (b) 10 cm
 (c) 7 cm (d) 9 cm

50. If in the given figure, $\angle PQR = 90^\circ$, O is the centroid of $\triangle PQR$, PQ = 5 cm and QR = 12 cm, then OQ is equal to:



(a) $3\frac{1}{2}$ (b) $4\frac{1}{3}$
 (c) $4\frac{1}{2}$ (d) $5\frac{1}{3}$

51. The medians AD, BE, CF of a triangle ABC intersect in G. Which of the following is true for any $\triangle ABC$?

(a) $GB + GC = 3GA$
 (b) $GB + GC < GA$
 (c) $GB + GC > GA$
 (d) $GB + GC = GA$

52. If G is the centroid and AD be a median with length 12 cm of $\triangle ABC$, then the value of AG is:

(a) 4 cm (b) 6 cm
 (c) 10 cm (d) 8 cm

53. Two medians AD and BE of $\triangle ABC$ intersect at G at right angles. If AD = 9 cm and BE = 6 cm, then the length of BD, in cm is :

(a) 10 (b) 6 (c) 5 (d) 3

54. In $\triangle ABC$, AD is the median and

$AD = \frac{1}{2} BC$. If $\angle BAD = 30^\circ$ is,

then $\angle ACB$ is :
 (a) 90° (b) 45° (c) 30° (d) 60°

55. In $\triangle ABC$, G is the centroid, AB = 15 cm, BC = 18 cm, and AC = 25 cm. Find GD, where D is the mid-point of BC :

(a) $\frac{1}{2}\sqrt{86}$ (b) $\frac{1}{3}\sqrt{86}$
 (c) $\frac{7}{3}\sqrt{86}$ cm (d) $\frac{2}{3}\sqrt{86}$ cm

56. If G is the centroid of $\triangle ABC$ and AG = BC, then $\angle BGC$ is :

(a) 75° (b) 45° (c) 90° (d) 60°

57. I and O are respectively the incentre and circumcentre of a triangle ABC. The line AI produced intersects the circumcircle of $\triangle ABC$ at the point D. If $\angle ABC = x^\circ$, $\angle BID = y^\circ$

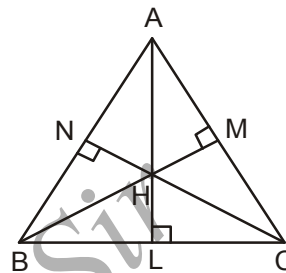
and $\angle BOD = z^\circ$, then $\frac{z+x}{y} = ?$

(a) 3 (b) 1 (c) 2 (d) 4

58. In an obtuse-angled triangle ABC, $\angle A$ is the obtuse angle and O is the orthocentre. If $\angle BOC = 54^\circ$, then $\angle BAC$ is :

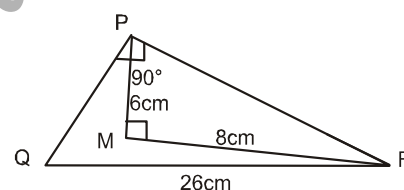
(a) 108° (b) 126°
 (c) 136° (d) 116°

59. If H is the orthocentre of $\triangle ABC$, then the orthocentre of $\triangle HBC$ is (figure given) :



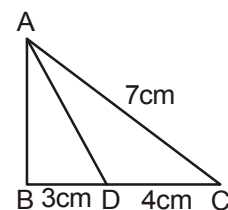
(a) N (b) A (c) L (d) M

60. In the given figure $\angle QPR = 90^\circ$, QR = 26 cm, PM = 6 cm, MR = 8 cm and $\angle PMR = 90^\circ$, find the area of $\triangle PQR$.



(a) 180 cm^2 (b) 240 cm^2
 (c) 120 cm^2 (d) 150 cm^2

61. In the given figure, if AD is bisector of $\angle BAC$ then AB is:



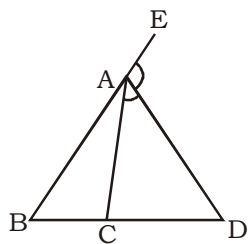
(a) 6 cm (b) 5 cm
 (c) 5.25 cm (d) 5.75 cm

62. In $\triangle ABC$, D is a point on BC

such that $\frac{AB}{AC} = \frac{BD}{DC}$. If $\angle B = 70^\circ$, $\angle C = 50^\circ$, then the value of $\angle BAD$:

(a) 30° (b) 60°
 (c) 40° (d) 50°

63. In the figure AD is the external bisector of $\angle EAC$, intersects BC produced to D. If AB = 12 cm, AC = 8 cm and BC = 4 cm, find CD:



- (a) 10cm (b) 6 cm
(c) 8 cm (d) 9 cm

64. In triangle ABC, $DE \parallel BC$ where D is a point on AB and E is point on AC. DE divides the area of $\triangle ABC$ into two equal parts. Then $DB : AB$ is equal to
(a) $\sqrt{2} : (\sqrt{2} + 1)$ (b) $(\sqrt{2} - 1) : \sqrt{2}$
(c) $\sqrt{2} : (\sqrt{2} - 1)$ (d) $(\sqrt{2} + 1) : \sqrt{2}$

65. Let ABC be a triangle with $AB = 3\text{cm}$. and $AC = 5\text{ cm}$. If AD is a median drawn from the vertex A to the side BC, then which one of the following is correct?
(a) AD is always greater than 4cm. but less than 5 cm
(b) AD is always greater than 5 cm.
(c) AD is always less than 4 cm
(d) None of the Above
66. In $\triangle ABC$, the internal bisector of the $\angle A$, $\angle B$ and $\angle C$ intersect the circumcircle at x, y and z respectively. If $\angle A = 50^\circ$, $\angle cyz = 30^\circ$ then $\angle Byz$ will be

- (a) 10° (b) 35°
(c) 100° (d) 115°

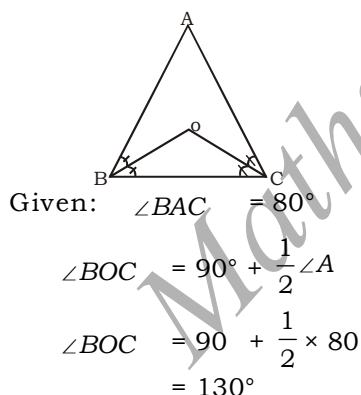
67. In an isosceles $\triangle ABC$, $AB = AC$ and $\angle A$ is two times of $\angle B$. If $AB = 3\text{ cm}$. then ratio of inradius to the circumradius is-
(a) $1 : 2$ (b) $\sqrt{2} - 1 : 1$
(c) $2\sqrt{2} - 1 : 1$ (d) $1 : 2\sqrt{2} - 1$
68. If O be the orthocentre of ABC, $OF \perp AB$ and $OE \perp AC$. If $OE = 2\text{cm}$ and $BE = 5\text{ cm}$ then find the value of $OF \times OC$.
(a) 10 (b) 3 (c) 6 (d) 3

ANSWER KEY

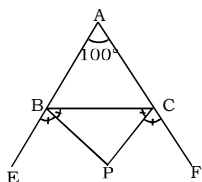
1. (d)	8. (c)	15. (c)	22. (a)	29. (b)	36. (d)	43. (b)	50. (b)	57. (c)	64. (b)
2. (c)	9. (a)	16. (d)	23. (c)	30. (d)	37. (c)	44. (a)	51. (c)	58. (b)	65. (c)
3. (a)	10. (d)	17. (a)	24. (b)	31. (a)	38. (c)	45. (a)	52. (d)	59. (b)	66. (b)
4. (b)	11. (d)	18. (b)	25. (a)	32. (d)	39. (d)	46. (b)	53. (c)	60. (c)	67. (b)
5. (c)	12. (a)	19. (c)	26. (a)	33. (c)	40. (a)	47. (d)	54. (d)	61. (c)	68. (c)
6. (d)	13. (b)	20. (b)	27. (b)	34. (a)	41. (a)	48. (a)	55. (d)	62. (a)	
7. (b)	14. (c)	21. (a)	28. (d)	35. (c)	42. (c)	49. (d)	56. (c)	63. (c)	

SOLUTION

1. (d) According to question



2. (c) According to question
Given:

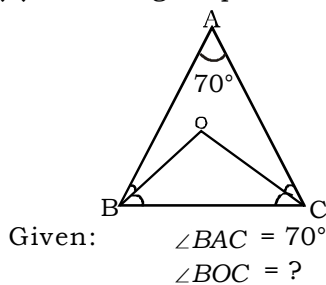


$$\therefore \angle BPC = 90^\circ - \frac{1}{2} \angle A$$

$$= 90^\circ - \frac{1}{2} \times 100^\circ$$

$$\angle BPC = 40^\circ$$

3. (a) According to question.

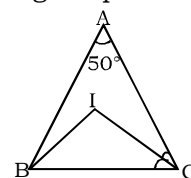


$$\therefore \angle BOC = 90^\circ + \frac{1}{2} \angle A$$

$$\angle BOC = 90^\circ + \frac{1}{2} \times 70^\circ = 90^\circ + 35^\circ$$

$$\angle BOC = 125^\circ$$

4. (b) According to question



Given: $\angle A = 50^\circ$
I is the incentre

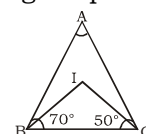
$$\angle BIC = 90^\circ + \frac{1}{2} \angle A$$

$$\angle BIC = 90^\circ + \frac{1}{2} \times 50^\circ$$

$$\angle BIC = 90^\circ + 25^\circ$$

$$\angle BIC = 115^\circ$$

5. (c) According to question



As we know that

$$\begin{aligned}\angle A + \angle B + \angle C &= 180^\circ \\ \therefore \angle A &= 180^\circ - 70^\circ - 50^\circ \\ \angle A &= 60^\circ\end{aligned}$$

$$\therefore \angle BIC = 90^\circ + \frac{1}{2} \times 60^\circ$$

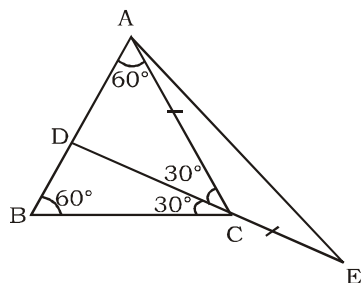
$$\angle BIC = 120^\circ$$

6. (d) According to question

Given:

ABC is an equilateral triangle

CD is the angle bisector of $\angle C$



$$AC = CE$$

$$\therefore \angle CAE = \angle CEA$$

$$\angle ACD = 30^\circ$$

$$\begin{aligned}\therefore \angle ECA &= 180^\circ - 30^\circ \\ &= 150^\circ\end{aligned}$$

In $\triangle CAE$

$$\angle CAE + \angle CEA + \angle ECA = 180^\circ$$

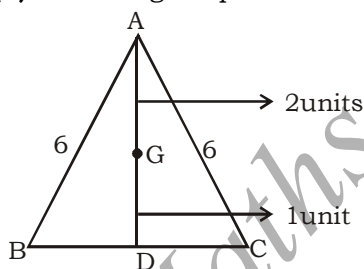
(CE = CA)

$$\therefore 2\angle CAE = 180^\circ - 150^\circ$$

$$2\angle CAE = 30^\circ$$

$$\angle CAE = 15^\circ$$

7. (b) According to question



Given

$$AB = BC = CA = 6 \text{ cm}$$

$$AG = I_R = \text{Circumradius} = 2 \text{ units}$$

$$GD = I_r = \text{Inradius} = 1 \text{ unit}$$

$$AD = \text{height} = 3 \text{ units}$$

As we know that height of the

$$\text{equilateral triangle is } h = \frac{\sqrt{3}}{2} a,$$

where 'a' is the sides of a triangle

$$AD = \frac{\sqrt{3}}{2} \times 6 = 3\sqrt{3} \text{ cm}$$

$$\therefore 3 \text{ units} = 3\sqrt{3}$$

$$1 \text{ unit} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

$$\therefore GD = I_r = \sqrt{3}$$

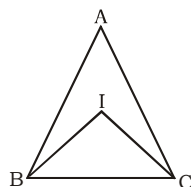
Alternate

$$r_{in} = \frac{a}{2\sqrt{3}}$$

$$r_{in} = \frac{6}{2\sqrt{3}} = \sqrt{3} \text{ cm}$$

8. (c) According to question

Given:



$$\angle BIC = \frac{\angle A}{2} + X \dots\dots(i)$$

As we know that

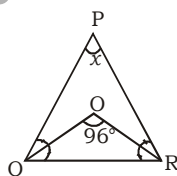
$$\angle BIC = 90^\circ + \frac{\angle A}{2} \dots\dots(ii)$$

Compare equation (i) and (ii)

$$X = 90^\circ$$

9. (a) According to the question,

$$\text{Value of } \angle ROQ = 90 + \frac{\angle P}{2}$$



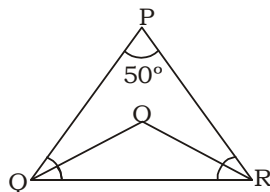
$$\Rightarrow 96 = 90 + \frac{\angle P}{2}$$

$$\Rightarrow 6 = \frac{\angle P}{2}$$

$$\Rightarrow \angle P = 12^\circ$$

Therefore $\angle RPQ = 12^\circ$

10. (d) According to the question,

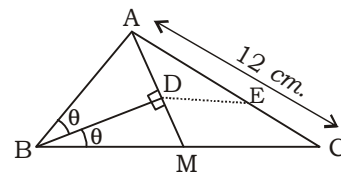


$$\Rightarrow \angle QOR = 90^\circ + \frac{\angle A}{2}$$

$$\Rightarrow \angle QOR = 90^\circ + \frac{50^\circ}{2}$$

$$\Rightarrow \angle QOR = 90^\circ + 25^\circ = 115^\circ$$

11. (d) According to the question,



$$\angle ABD = \angle MBD = \theta$$

(angle bisector)

$$\therefore BD \perp AM$$

$$\angle BDA = \angle BDM = 90^\circ$$

It happens only in equilateral and isosceles triangle

$$\therefore AD = DM$$

$$\text{i.e. } AD = AM/2$$

Given $DE \parallel BC$

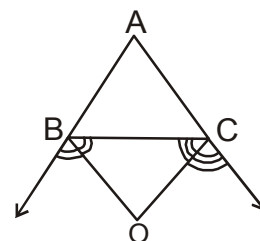
From thales theorem

E will be mid point of AC.

$$\therefore AC = 12 \text{ cm.}$$

$$\text{So, } AE = 6 \text{ cm.}$$

$$12. (a) \angle BOC = 90^\circ - \frac{1}{2} \angle A \dots\dots(i)$$



$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A = 180^\circ - (\angle B + \angle C)$$

multiply both sides by $\frac{1}{2}$

$$\Rightarrow \frac{1}{2} \angle A = 90^\circ - \frac{1}{2} (\angle B + \angle C)$$

$$\Rightarrow \frac{1}{2} (\angle B + \angle C) = 90^\circ - \frac{1}{2} \angle A$$

....(ii)

From Eq. (i) = (ii)

$$\therefore \angle BOC = \frac{1}{2} (\angle B + \angle C)$$

$$13. (b) \angle BIC = 90^\circ + \frac{A}{2} = 90^\circ + 30^\circ = 120^\circ$$

$$14. (c) \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - (\angle A + \angle B)$$

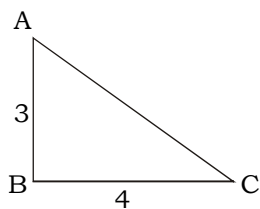
$$\Rightarrow \angle C = 180^\circ - (80^\circ + 60^\circ) = 40^\circ$$

$$\Rightarrow 2x = 40 \Rightarrow x = 20^\circ$$

$$(\because C = 2x)$$

$$y = 90^\circ + \frac{1}{2} \angle A = 90^\circ + \frac{1}{2} (80) \\ = 130^\circ$$

15.(c) According to the question



$$AC = \sqrt{3^2 + 4^2} = 5\text{cm}$$

$$A = \text{area of } \triangle ABC = \frac{1}{2} \times 4 \times 3 \\ = 6\text{cm}$$

$$S = \frac{3+4+5}{2} = 6\text{cm}$$

$$\therefore r = A/S = 6/6 = 1\text{cm}$$

Alternate:-

For right angle triangle

$$r = \frac{AB + BC - CA}{2}$$

$$r = \frac{3+4-5}{2} = 1$$

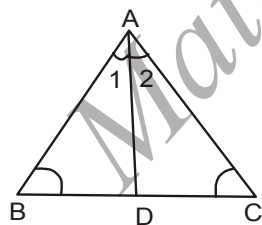
16.(d) According to the question

$$\angle BOC = 90^\circ - \frac{1}{2} \angle A = 90^\circ - \frac{1}{2} (40) = 70^\circ$$

$$\mathbf{17.(a)} \angle MAN = \frac{1}{2} (\angle B - \angle C)$$

$$= \frac{1}{2} (65^\circ - 30^\circ) = \frac{1}{2} (35^\circ) \\ = 17.5^\circ$$

18.(b)



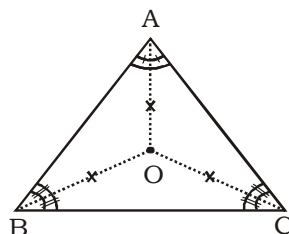
$$\text{Since } \angle 1 = \angle 2$$

$$\therefore \frac{AB}{AC} = \frac{BD}{CD} \Rightarrow \frac{AB}{AC} = 1 \quad (\because BD \\ = CD \text{ given}) \\ \Rightarrow AB = AC \\ (\because \text{the given } \triangle \text{ is isosceles})$$

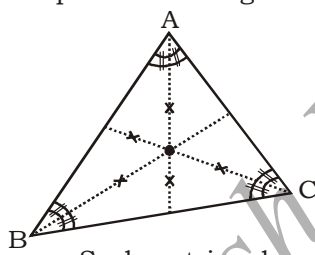
* This theorem apply for equilateral triangle because equilateral has also all sides equal ($AB = BC = CA$)

19.(c) Incentre:- meeting point of angle bisector.

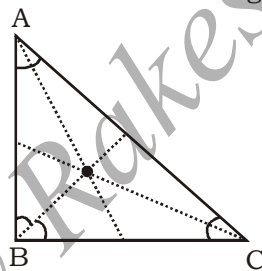
Every triangle:- lies incentre of a triangle in the interior



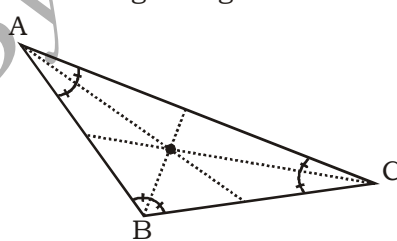
Equilateral triangle



Scalene triangle

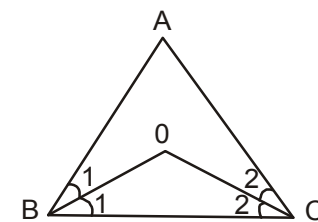


Right Angle



Obtuse angle triangle

20.(b)



$$A + B + C = 180^\circ \\ \Rightarrow A + 2\angle 1 + 2\angle 2 = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = \frac{1}{2} (180^\circ - A)$$

$$= 90^\circ - \frac{1}{2} A \quad \dots(i)$$

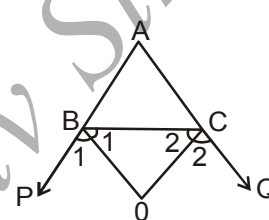
$$\therefore \angle BOC = 180^\circ - (\angle 1 + \angle 2)$$

From Eq. (i)

$$= 180^\circ - (90^\circ - \frac{1}{2} \angle A)$$

$$= 90^\circ + \frac{1}{2} \angle A$$

21.(a)



$$2\angle 1 + \angle B = 180^\circ$$

$$\Rightarrow \angle 1 = 90^\circ - \frac{1}{2} \angle B \quad \dots(i)$$

similarly,

$$\angle 2 = 90^\circ - \frac{1}{2} \angle C \quad \dots(ii)$$

Add both (i) and (ii)

$$\angle 1 + \angle 2 = 180^\circ - \frac{1}{2} (B + C)$$

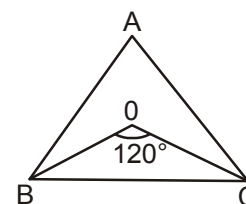
$$\therefore \angle BOC = 180^\circ - (\angle 1 + \angle 2) \\ = 180^\circ - \left(180^\circ - \frac{1}{2} (\angle B + \angle C) \right)$$

$$= \frac{1}{2} [\angle B + \angle C] = \frac{1}{2} (180^\circ - \angle A) \\ = 90^\circ - \frac{1}{2} \angle A$$

$$\mathbf{22.(a)} \angle BOC = 90^\circ + \frac{1}{2} (\angle BAC)$$

$$\Rightarrow \angle BAC = (130 - 90) \times 2 \\ = 80^\circ$$

23.(c)



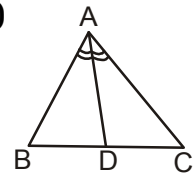
$$\angle BOC = 90^\circ + \frac{1}{2} (\angle BAC)$$

$$\Rightarrow \angle BAC = (120^\circ - 90^\circ) \times 2 = 60^\circ$$

$$24.(b) \angle BOC = 90^\circ + \frac{1}{2} (\angle A)$$

$$= 90^\circ + \frac{1}{2} \times 30^\circ = 105^\circ$$

25.(a)



$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$DC = BC - BD = 7.5 - 5 = 2.5$$

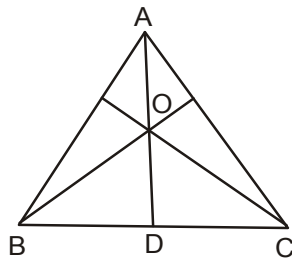
$$= \frac{AB}{AC} = \frac{5}{2.5} = 2 : 1$$

$$26.(a) \angle BPC = 90^\circ - \frac{A}{2} = 90^\circ - 40^\circ = 50^\circ$$

$$27.(b) \angle BOC = 90^\circ + \frac{1}{2} (\angle BAC)$$

$$\Rightarrow \angle BAC = (102^\circ - 90^\circ) \times 2 = 24^\circ$$

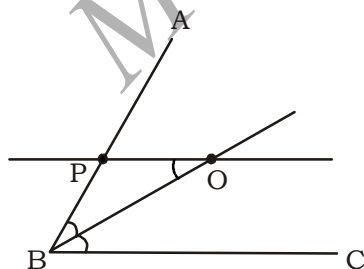
28.(d)



In equilateral triangle centroid, incentre, orthocentre coincide at the same point.
 $\therefore$ in-radius = $OD = 3$ cm

$$\therefore \frac{AD}{3} = 3 \text{ cm} \Rightarrow AD = 9 \text{ cm} = \text{median.}$$

29.(b)



$$\therefore OP \parallel BC$$

$$\Rightarrow \angle POB = \angle OBC \quad \dots(i)$$

(Alternate angle)

$$\Rightarrow \angle PBO = \angle POB \because OB \text{ is the bisector of } \angle B.$$

$$\angle PBO = \angle OBC \quad \dots(ii)$$

From (i) and (ii)

$$\therefore PB = PO$$

$$\therefore \angle ABC \text{ is an acute angle or } \angle ABC < 90^\circ$$

$$\therefore \frac{1}{2} \angle ABC < 45^\circ$$

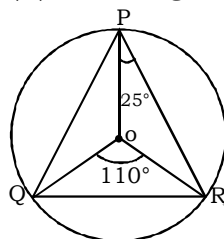
$$\Rightarrow \angle POB = \angle PBO < 45^\circ$$

$$\therefore \angle BPO > 90^\circ$$

Hence, $\triangle PBO$ is isosceles

$\triangle$ but not a right-angled triangle.

30. (d) According to question



$$\text{Given: } \angle QOR = 110^\circ$$

$$\angle OPR = 25^\circ$$

'O' is the circumcentre then

$$OP = OR = OQ$$

$$\therefore \angle OPR = \angle ORP = 25^\circ$$

In $\triangle OQR$

$$\Rightarrow \angle OQR + \angle ORQ + \angle QOR = 180^\circ$$

$$\Rightarrow 2\angle ORQ = 180^\circ - 110^\circ \text{ (OQ = OR)}$$

$$\Rightarrow 2\angle ORQ = 70^\circ$$

$$\Rightarrow \angle ORQ = \frac{70^\circ}{2}$$

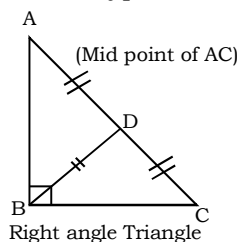
$$\Rightarrow \angle ORQ = 35^\circ$$

$$\therefore \angle PRQ = \angle PRO + \angle ORQ$$

$$\angle PRQ = 60^\circ$$

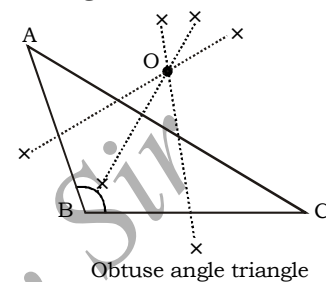
31. (a) Only right angled triangle where circumcentre lies on hypotenuse (side)

For a right angled triangle, circumcentre is **midpoint of hypotenuse** and circumradius is half of hypotenuse.

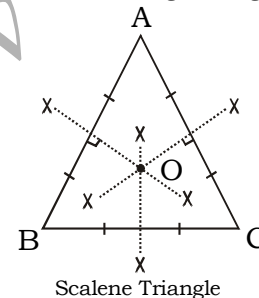


BD = Median at hypotenuse
 = Same as circumradius
 = Half of hypotenuse

For an obtuse angled triangle, circumcentre lies outside the triangle



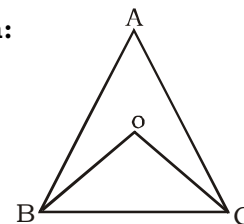
Obtuse angle triangle



Scalene Triangle

32. (d) According to question

Given:



$$\angle C = 40^\circ \quad \angle B = 70^\circ$$

$\therefore$ In $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A = 180^\circ - 40^\circ - 70^\circ$$

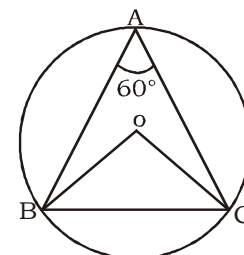
$$\angle A = 70^\circ \text{ As we know that}$$

$$\therefore \angle BOC = 2\angle A$$

(O is a circumcentre)

$$\angle BOC = 2 \times 70^\circ = 140^\circ$$

33. (c) According to question



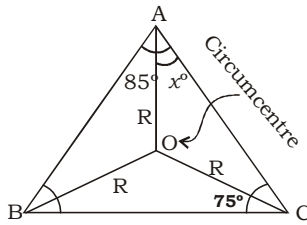
$$\angle A = \angle B = \angle C = 60^\circ$$

$$\angle BOC = 2\angle A$$

$$\angle BOC = 2 \times 60^\circ$$

$$\angle BOC = 120^\circ$$

34. (a)



∴ O is circumcentre
So, $OA = OB = OC = R$ (Radius)
∴ $\angle BAC = 85^\circ$, $\angle BCA = 75^\circ$
Then, $[\angle ABC = 180^\circ - (\angle BAC + \angle BCA)]$
 $\angle ABC = 180^\circ - (85^\circ + 75^\circ)$
 $\Rightarrow \angle ABC = 180^\circ - 160^\circ$
 $\Rightarrow \angle ABC = 20^\circ$

[Angle made by same chord at the centre is doubled than that of any other part of the circumference at same sector]

Then, $\angle AOC = 20^\circ \times 2 = 40^\circ$

∴ $OA = OC = R$

So, $\triangle AOC =$ Isosceles triangle

Then, $\angle OCA + \angle OAC + \angle AOC = 180^\circ$

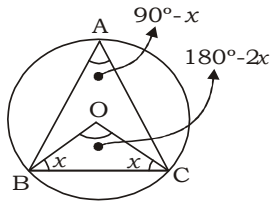
$$x + x + \angle AOC = 180^\circ$$

$$2x + 40^\circ = 180^\circ$$

$$x = 70^\circ$$

Therefore $\angle OAC = 70^\circ$

35. (c)



In $\triangle BOC$

$OB = OC$ (circum radius)

$\angle OBC = \angle OCB = x$ (Let)

then,

$$\Rightarrow \angle BOC = 180^\circ - 2x^\circ$$

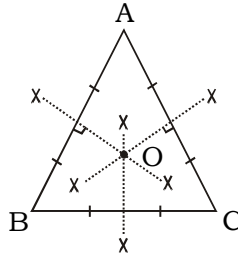
$$\Rightarrow \angle BAC = \frac{\angle BOC}{2} = \frac{180 - 2x}{2}$$

$$\Rightarrow \angle BAC = 90 - x$$

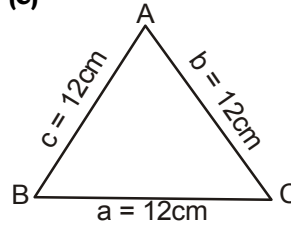
$$\Rightarrow \angle OBC + \angle BAC = 90^\circ - x + x = 90^\circ$$

36.(d) Perpendicular bisectors

Draw a line (called a "perpendicular bisector") at right angles to the midpoint of each side. Where all three lines intersect is the centre of a triangle's "circumcircle", called the "circumcentre":

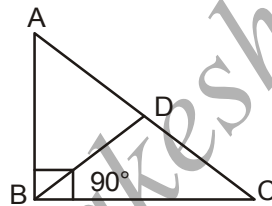


37. (c)



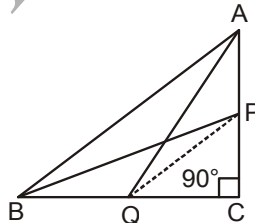
$$R = \frac{a}{\sqrt{3}} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

38.(c) In the right-angled triangle the length of median to the hypotenuse is half the length of the hypotenuse.



$$\text{Hence, } BD = \frac{1}{2} AC = 3\text{cm}$$

39.(d)



$$\Rightarrow AQ^2 = AC^2 + QC^2 \dots\dots(i)$$

$$\Rightarrow BP^2 = BC^2 + CP^2 \dots\dots(ii)$$

add (i) and (ii)

$$\Rightarrow AQ^2 + BP^2 = (AC^2 + BC^2) + (QC^2 + CP^2)$$

$$\Rightarrow AB^2 + \left(\frac{BC}{2}\right)^2 + \left(\frac{AC}{2}\right)^2$$

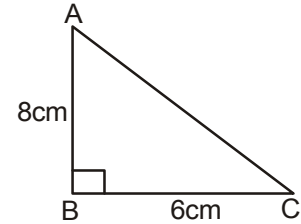
$$\Rightarrow AB^2 + \frac{1}{4}(BC^2 + AC^2)$$

$$\Rightarrow (AQ^2 + BP^2)$$

$$= AB^2 + \frac{1}{4} AB^2 = \frac{5}{4} AB^2$$

$$\Rightarrow 4(AQ^2 + BP^2) = 5 AB^2$$

40.(a)

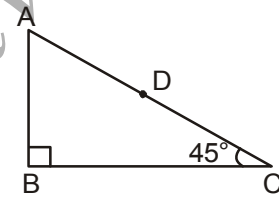


$$AC = \sqrt{6^2 + 8^2} = 10\text{cm.}$$

$$\therefore \text{circum radius} = \frac{10}{2} = 5\text{cm.}$$

i.e. mid-point of hypotenuse.

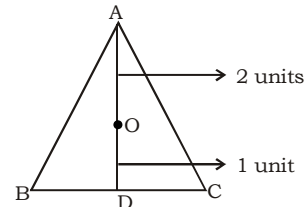
41.(a)



$BD = AD = CD$ (mid-point of hypotenuse is circumcentre.)

$$\therefore BD = \frac{1}{2} (4\sqrt{2}) = 2\sqrt{2} \text{ units.}$$

42. (c) According to question



Let $AO = 2$ units

$OD = 1$ unit

Given:-

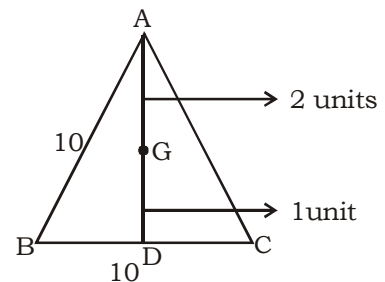
$AO = 10$ cm

$$\therefore 2 \text{ units} = 10 \text{ cm}$$

1 unit = 5 cm

$$\therefore OD = 5 \text{ cm}$$

43. (b) According to question



Given:

$$AB = BC = CA = 10 \text{ cm}$$

G = Centroid

$$AG = 2 \text{ units}$$

$$GD = 1 \text{ unit}$$

$$\therefore AD = 3 \text{ units} = \text{Height}$$

As we know that the height of

$$\text{the equilateral triangle is} = \frac{\sqrt{3}}{2}$$

$$\times 10 = 5\sqrt{3}$$

$$\therefore 3 \text{ units} = 5\sqrt{3}$$

$$1 \text{ unit} = \frac{5\sqrt{3}}{3}$$

$$2 \text{ units} = \frac{5\sqrt{3}}{3} \times 2 = \frac{10\sqrt{3}}{3}$$

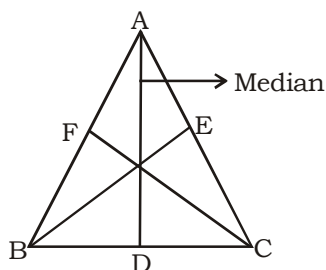
$$\therefore AG = \frac{10\sqrt{3}}{3} \text{ cm}$$

Alternate

$$AG (r_c) = \frac{AB(a)}{\sqrt{3}}$$

$$AG = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3} \text{ cm}$$

44. (a) According to question



Given: $AD = BE = CF = \text{median}$

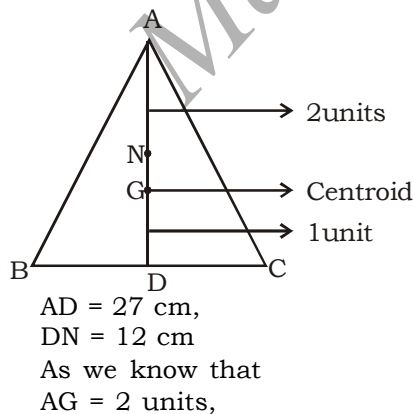
Then

$$AB = BC = CA$$

$\therefore$ The triangle is an equilateral triangle.

45. (a) According to question

Given:-



$$GD = 1 \text{ unit}$$

$$\therefore AD = 3 \text{ units} = 27 \text{ cm}$$

$$3 \text{ units} = 27 \text{ cm}$$

$$1 \text{ unit} = 9 \text{ cm}$$

$$\therefore GD = 9 \text{ cm}$$

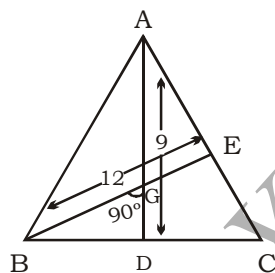
$$\therefore GN = DN - GD$$

$$= 12 - 9 = 3 \text{ cm}$$

46. (b) Medians AD and BE intersect at G on 90°

i.e. $\angle AGB = 90^\circ$ and $\triangle AGB$ will be a right angled triangle

We know, In a triangle centroid divides the medians in 2 : 1 Ratio



$$\Rightarrow \text{Then } BG = \frac{2}{3} \times BE$$

$$BG = \frac{2}{3} \times 12$$

$$BG = 8$$

$$\Rightarrow AG = \frac{2}{3} \times AD$$

$$\Rightarrow AG = \frac{2}{3} \times 9$$

$$\Rightarrow AG = 6 \text{ cm}$$

In right angled triangle AB will be a hypotenuse

Using pythagoras theorem

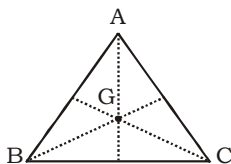
$$\Rightarrow (AB)^2 = (AG)^2 + (BG)^2$$

$$\Rightarrow (AB)^2 = 6^2 + 8^2$$

$$\Rightarrow AB = 10 \text{ cm}$$

Therefore, length of AB = 10 cm.

47. (d)



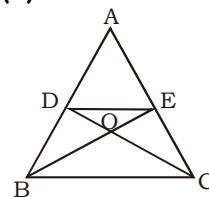
$$\text{Area of } \triangle ABC = 60 \text{ cm}^2$$

$$\text{Area of } \triangle GBC = 2 \times \left(\frac{1}{6} \text{ of } \triangle ABC \right)$$

[A Median divides a triangle in two equal parts]

$$\Rightarrow 2 \times \frac{1}{6} \times 60 = 20 \text{ cm}^2$$

48. (a) In $\triangle ODE$ & $\triangle BCO$



$$\frac{(OE)^2}{(OB)^2} = \frac{\text{Area of } \triangle ODE}{\text{Area of } \triangle BCO}$$

$$\frac{1}{4} = \frac{\text{Area of } \triangle ODE}{\text{Area of } \triangle BCO}$$

$$\text{Area of } \triangle BCO = \frac{1}{3} \text{ Area of } \triangle ABC$$

$$4 \text{ Area of } \triangle ODE = \frac{1}{3} \text{ of } \triangle ABC$$

$$\text{Area of } \triangle ABC = 12 \times \text{area of } \triangle ODE$$

$$\triangle ODE : \triangle ABC$$

$$1 : 12$$

49. (d) We know that the centroid of a triangle divides each median in the ratio of 2 : 1

$$\therefore BG : BE = 2 : 3$$

$$\Rightarrow BE = \frac{3}{2} BG = \frac{3}{2} \times 6 = 9 \text{ cm.}$$

50. (b) By Pythagoras theorem,

$$PR = \sqrt{PQ^2 + QR^2} = \sqrt{5^2 + 12^2} = 13 \text{ cm}$$

$\therefore$ O is the centroid $\Rightarrow$ QM is median and M is the mid-point of PR.

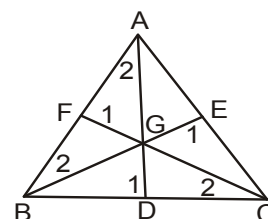
$$\therefore QM = PM = \frac{13}{2}$$

$\therefore$ centroid divides median in ratio 2 : 1

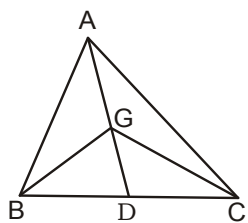
$$\therefore OQ = \frac{2}{3} QM$$

$$= \frac{2}{3} \times \frac{13}{2} = \frac{13}{3} = 4 \frac{1}{3} \text{ cm}$$

51. (c)

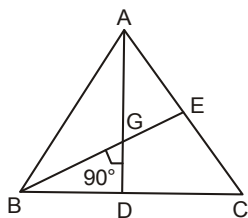


52.(d)



$$AG = \frac{2}{3} AD = \frac{2}{3} \times 12 = 8\text{cm.}$$

53.(c)



$$AD = 9\text{cm}$$

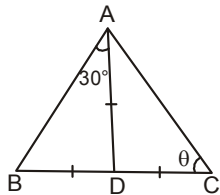
$$\Rightarrow GD = \frac{1}{3} \times 9 = 3\text{cm}$$

$$BE = 6\text{cm}$$

$$\Rightarrow BG = \frac{2}{3} \times 6 = 4\text{cm}$$

$$\therefore BD = \sqrt{3^2 + 4^2} = 5\text{cm.}$$

54.(d)



$$BD = CD = AD$$

$$\therefore \angle BAD = 30^\circ$$

$$\text{Now in } \triangle ABD$$

$$\angle ABD = 30^\circ (\because BD = AD)$$

$$\therefore AD = DC$$

$$\therefore \angle DAC = \angle DCA = \theta \text{ (let)}$$

Now,

$$\text{In } \triangle ABC$$

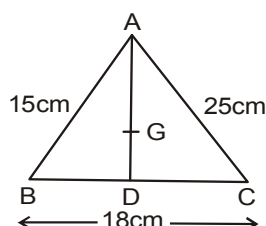
$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 30^\circ + \theta + 30^\circ + \theta = 180^\circ$$

$$\Rightarrow 2\theta = 120^\circ$$

$$\Rightarrow \theta = \angle ACB = 60^\circ$$

55.(d)



$$AB^2 + AC^2 = 2(AD^2 + BD^2)$$

$$\Rightarrow 225 + 625 = 2(AD^2 + 81)$$

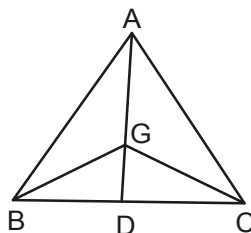
$$\Rightarrow AD^2 = 344$$

$$AD = 2\sqrt{86} \text{ and}$$

$$GD = \frac{1}{3} AD$$

$$\Rightarrow GD = \frac{2}{3} \sqrt{86} \text{cm}$$

56.(c)



$$AG = BC = 2x \text{ (let)}$$

$$\therefore GD = x (\because \text{centroid divides median in } 2:1)$$

$$\text{Now in } \triangle BDG, BD = GD = x$$

$$\therefore \angle DBG = \angle BGD = \theta_1 \text{ (let)}$$

$$\text{Similarly in } \triangle DGC, CD = GD = x$$

$$\therefore \angle DCG = \angle DGC = \theta_2 \text{ (let)}$$

$$\therefore \angle BGC = \theta_1 + \theta_2$$

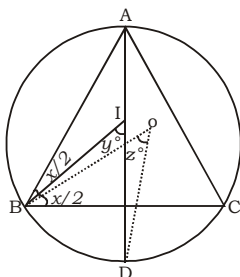
$$\text{Now in } \triangle BGC = \theta_1 + \theta_2 +$$

$$(\theta_1 + \theta_2) = 180^\circ$$

$$\Rightarrow \theta_1 + \theta_2 = 90^\circ$$

$$\Rightarrow \angle BGC = 90^\circ$$

57. (c) According to question



$$\text{Given: } \angle ABC = x^\circ$$

$$\angle BID = y^\circ,$$

$$\angle BOD = z^\circ$$

$$\therefore \text{'I' is the incentre}$$

$$\therefore \angle ABI = \frac{1}{2} \angle ABC$$

$$\angle ABI = \frac{1}{2} x^\circ = \frac{x^\circ}{2}$$

$$\angle BAD = \frac{1}{2} \angle BOD$$

$\therefore$ Angle subtended on the circumcircle is half the angle subtended on the centre of circle

$$\angle BAD = \frac{1}{2} \angle BOD$$

$$\angle BAD = \frac{z^\circ}{2}$$

$$\therefore y^\circ = \frac{x^\circ}{2} + \frac{z^\circ}{2} \text{ (Exterior angle)}$$

$$\therefore y^\circ = \frac{x^\circ + z^\circ}{2}$$

$$2y^\circ = x^\circ + z^\circ$$

Now,

$$\frac{z^\circ + x^\circ}{y^\circ} = \frac{2y^\circ}{y^\circ} = 2$$

$$58.(b) \angle BAC = 180^\circ - \angle BOC$$

$$= 180^\circ - 54^\circ = 126^\circ$$

59.(b) In $\triangle ABC$, $HL \perp BC$ and $BN \perp HN$

Thus, the two altitudes HL and BN of $\triangle HBC$, intersect at A.

$$60.(c) PR = \sqrt{PM^2 + MR^2}$$

$$= \sqrt{36 + 64} = 10\text{cm}$$

$$\Rightarrow PQ = \sqrt{QR^2 - PR^2}$$

$$= \sqrt{26^2 - 10^2} = 24\text{cm}$$

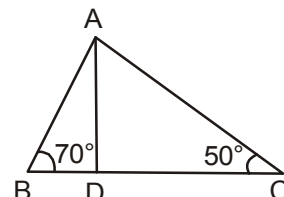
$$\therefore \text{ar}(\triangle PQR) = \frac{1}{2} (PR) (PQ)$$

$$= \frac{1}{2} \times 10 \times 24 = 120\text{cm}^2$$

$$61.(c) \frac{AB}{AC} = \frac{BD}{DC} \Rightarrow AB = \frac{3}{4} \times 7$$

$$= \frac{21}{4} = 5.25\text{cm}$$

62.(a)



$$\angle A + 70^\circ + 50^\circ = 180^\circ$$

$$\Rightarrow \angle A = 60^\circ$$

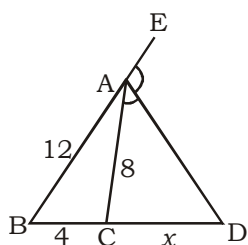
$$\therefore \frac{AB}{AC} = \frac{BD}{DC}$$

$\therefore$ AD is the bisector of $\angle BAC$.

$\therefore \angle BAD$

$$= \frac{1}{2} \angle BAC = \frac{1}{2} \times 60^\circ = 30^\circ$$

- 63. (c)** According to the question, AD is the external bisector of $\angle CAE$ By using external bisector theorem.



$$\frac{AB}{AC} = \frac{BD}{CD}$$

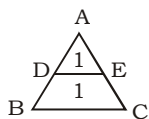
$$\frac{12}{8} = \frac{4+x}{x}$$

$$12x = 32 + 8x$$

$$4x = 32$$

$$x = 8$$

- 64. (b)**



ar of $\triangle ADE = 1$,

ar of $\triangle ABC = 2$

$$\frac{\text{ar of } \triangle ABC}{\text{ar of } \triangle ADE} = \frac{2}{1}$$

$$= \frac{\text{Side of } \triangle ABC (AB)^2}{\text{Side of } \triangle ADE (AD)^2}$$

By square root $\frac{\sqrt{2}}{1} = \frac{AB}{AD}$

$$DB = AB - AD$$

$$= \sqrt{2} - 1$$

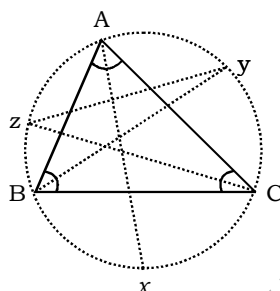
$$DB : AB = (\sqrt{2} - 1) : \sqrt{2}$$

- 65. (c)** By property 12.

$$AB + AC > 2AD$$

$$3 + 5 > 2AD \Rightarrow AD < 4$$

- 66. (b)**



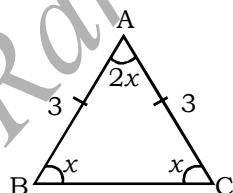
$$\angle ZOY = \angle BOC = 90 + \frac{50}{2} = 115^\circ$$

(Property 2)

In $\triangle ZOY$,

$$\angle BYZ = 180 - (30 + 115) = 35^\circ$$

- 67. (b)**



$$x + x + 2x = 180^\circ$$

$$x = 45^\circ$$

$$\therefore \angle A = 2 \times 45 = 90^\circ$$

$$BC = \sqrt{3^2 + 3^2} = 3\sqrt{2} \text{ cm.}$$

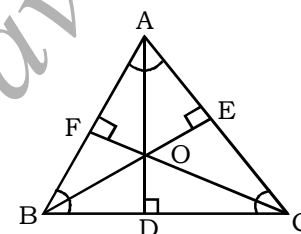
$$\text{Inradius}(r) = \frac{3 + 3 - 3\sqrt{2}}{2}$$

$$= \frac{3(2 - \sqrt{2})}{2}$$

$$\text{Circumradius}(R) = \frac{3\sqrt{2}}{2}$$

$$\frac{r}{R} = \frac{2 - \sqrt{2}}{\sqrt{2}} = \sqrt{2} - 1 : 1$$

- 68. (c)** In $\triangle OBF$ and $\triangle OCE$,



$$\angle OFB = \angle OEC = 90^\circ$$

$$\text{and } \angle BOF = \angle EOC$$

$$\therefore \triangle OBF \sim \triangle OCE$$

$$\text{Hence, } \frac{OB}{OC} = \frac{OF}{OE} \text{ or,}$$

$$\frac{OB}{OF} = \frac{OC}{OE}$$

$$\text{or, } OB \times OE = OF \times OC$$

$$OF \times OC = OB \times OE$$

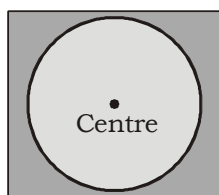
$$= (BE - OE)(OE)$$

$$= (5 - 2) \times 2 = 6$$

CIRCLES, CHORDS & TANGENTS

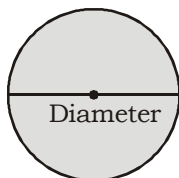
Circle

A circle is a shape with all points the same distance from its center. Some real world examples of a circle are a wheel, a dinner plate and the surface of a coin.



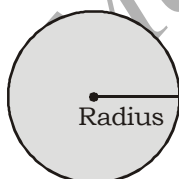
Diameter

The distance across a circle through the center is called the diameter. A real-world example of diameter is a 9-inch plate.



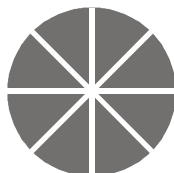
Radius

The distance between the centre and a point lies on the circumference of a circle is called radius. A real-world example of radius of a 9-inch plate.



We can look at a pizza pie to find real-world examples of diameter and radius. Look at the pizza to the right which has been sliced into 8 equal parts through its center. A radius is formed by making a straight cut from the

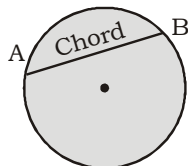
center to a point lies on the circumference of a circle.



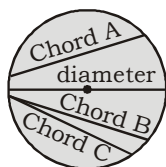
A straight cut made from a point on the circle, continuing through its center to another point on the circle, is a diameter. As you can see, a circle has many different radii and diameters, each passing through its center.

Chord

A chord is a line segment that joins two points on a curve. In geometry, a chord is often used to describe a line segment joining two endpoints that lie on a circle. The circle to the right contains chord AB. If this circle was a pizza pie, you could cut off a piece of pizza along chord AB. By cutting along chord AB, you are cutting off a segment of pizza that includes this chord.

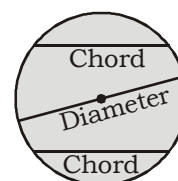


A circle has many different chords. Some chords pass through the center and some do not. A chord that passes through the center is called a diameter.

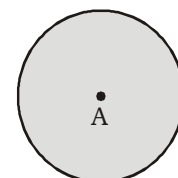


It turns out that a diameter of a circle is the longest chord of that

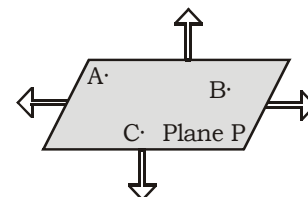
circle since it passes through the center. A diameter satisfies the definition of a chord, however, a chord is not necessarily a diameter. This is because every diameter passes through the center of a circle, but some chords do not pass through the center. Thus, it can be stated, every diameter is a chord, but not every chord is a diameter.



Let's revisit the definition of a circle. A circle is the set of points that are equidistant from a special point in the plane. The special point is the center. In the circle to the right, the center is point A. Thus we have circle A.

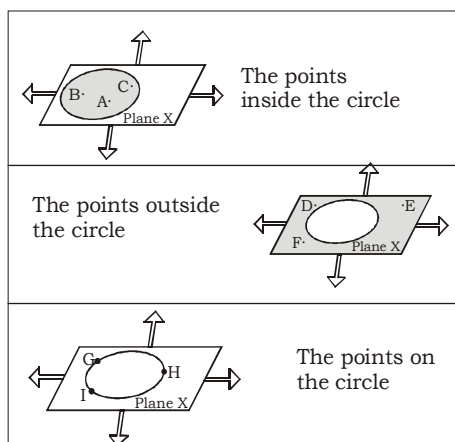


A plane is flat surface that extends without end in all directions. In the diagram to the right, Plane P contains points A, B and C.



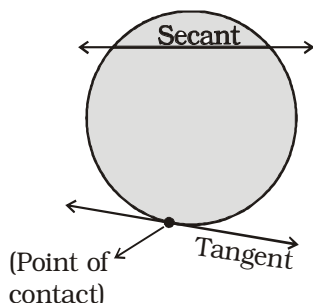
A circle divides the plane into three parts:

1. The points INSIDE the circle
2. The points OUTSIDE the circle
3. The points ON the circle



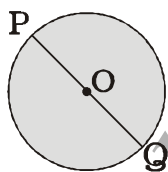
Secant

A line which intersects a circle in two distinct points is called a "Secant".

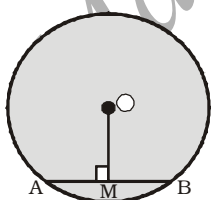


PROPERTIES OF CIRCLE

1. A chord which passes through the centre is called the diameter of the circle. It is a largest chord of the circle.

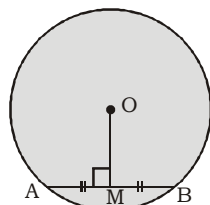


2. The perpendicular from the centre of a circle to a chord bisects the chord.



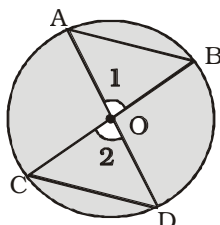
i.e. if $OM \perp AB$, then $AM = BM$

3. **Converse of the above theorem:-** The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.



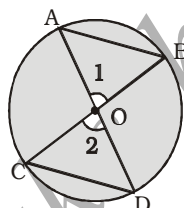
i.e. $AM = MB$, then $OM \perp AB$.

4. Equal chords of a circle subtend equal angles at the centre.



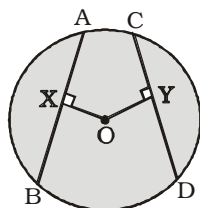
i.e. if $AB = CD$, then $\angle 1 = \angle 2$.

5. **Converse of the above theorem:** Angles subtended by two chords at the centre of a circle are equal then the chords are equal.



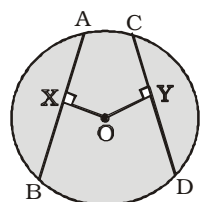
i.e. if $\angle 1 = \angle 2$, then $AB = CD$.

6. Equal chords of a circle are equidistant from the centre.



i.e. if $AB = CD$, $OX \perp AB$ and $OY \perp CD$, then $OX = OY$.

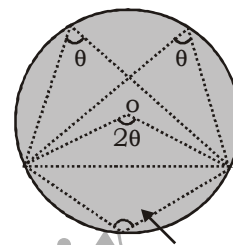
7. **Converse of the above theorem:** chords equidistant from the centre of the circle are equal.



i.e. If $OX \perp AB$, $OY \perp CD$ and $OX = OY$ then $AB = CD$.

8. **Degree Measure Theorem:-** The angle subtended by a chord at the centre of a circle is twice

the angle subtended by the chord at any point on the major arc.



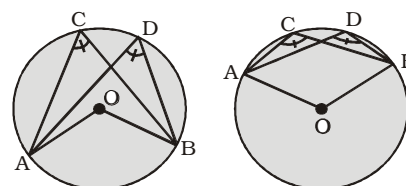
In a Minor Arc it is $180 - \theta$

i.e. $\angle x$ at the centre and

$\angle y$ at the circumference made by the same arc AB, then

$$\angle x = 2\angle y$$

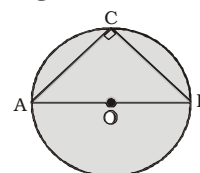
9. Angles in the same segment of a circle are equal.



i.e. $\angle ACB = \angle ADB$

(angles in same arc) or (angles in same segment)

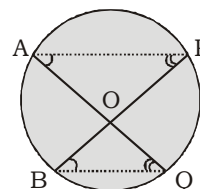
10. The angles in a semi circle is a right angle.



i.e. $\angle ACB = 90^\circ$.

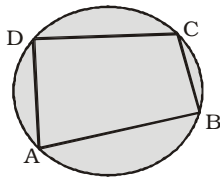
11. **Converse of the above theorem:** The circle, drawn with hypotenuse of a right triangle as diameter, passes through its opposite vertex.

12. If $\angle APB = \angle AQB$, and if P, Q are on the same side of AB, then A, B, Q, O & P are concyclic i.e. lie on the same circle.



13. The sum of the either pair of the

opposite angles of a cyclic quadrilateral is 180° .

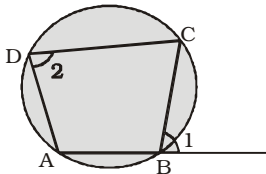


i.e. $\angle A + \angle C = \angle B + \angle D = 180^\circ$

14. Converse of the above theorem

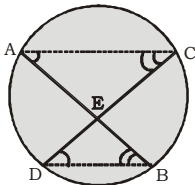
: If the two angles of a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is 'cyclic'.

15. If a side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.



i.e. $\angle 1 = \angle 2$.

16. If two chords AB and CD intersect internally or externally at a point E, then



Then, $AE \times EB = CE \times ED$

PROOF

In $\triangle ACE$ & $\triangle DBE$,

$\angle EAC = \angle EDB$ (Angle subtended by same chord)

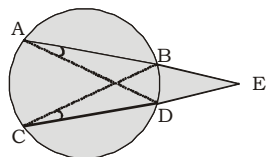
$\angle ECA = \angle EBD$ (Angle subtended by same chord)

Then, $\triangle ACE \sim \triangle DBE$

$$\therefore \frac{AE}{ED} = \frac{CE}{EB} \\ \Rightarrow AE \times EB = CE \times ED$$

(b)

Then, $EA \times EB = EC \times ED$



PROOF

In $\triangle EAD$ & $\triangle ECB$

$\angle BCD = \angle BAD$

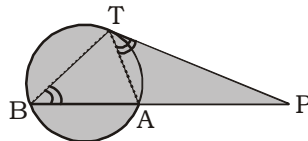
(Angle subtended by Same Chord)

$\angle AED = \angle BED$ (common)

Then, $\triangle EAD \sim \triangle ECB$

$$\therefore \frac{EA}{EC} = \frac{ED}{EB} \Rightarrow EA \times EB = CE \times ED$$

17. If a secant and a tangent externally intersect each other.



Then, $PT^2 = PA \times PB$

PROOF

$\angle ATP = \angle PBT$ (Alternate segment theorem)

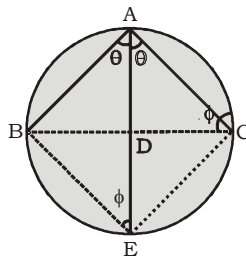
$\angle BPT = \angle APT$ (common angle)

Then, $\triangle BTP \sim \triangle ATP$

$$\therefore \frac{PT}{PA} = \frac{PB}{PT} \Rightarrow PT^2 = PA \times PB$$

18. AE is angle bisector of $\angle BAC$ then

$AB \cdot AC + DE \cdot AE = AE^2$



PROOF

$\angle AEB = \angle ACB = \phi$ (say)

[Angles by same chord]

$\angle BAD = \angle DAC = \theta$ (given)

$\therefore \triangle AEB \sim \triangle ACD$

$$\Rightarrow \frac{AE}{AC} = \frac{AB}{AD} \Rightarrow AD = \frac{AB \cdot AC}{AE}$$

$\therefore AE = AD + DE$, **AD = AE - DE.....(i)**

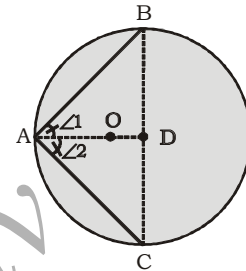
Put the value of AD

$$AE - DE = \frac{AB \cdot AC}{AE}$$

$$AE = \frac{AB \cdot AC}{AE} + DE$$

$$\therefore AE^2 = AB \cdot AC + DE \cdot AE$$

19. If two chords AB and AC of a circle are equal, then the bisector of $\angle BAC$ passes through the centre O of the circle. $\angle 1 = \angle 2$



PROOF :-

Let, AD is angle bisector of $\angle BAC$ then $\triangle BAD \cong \triangle CAD$

$$\left[\begin{array}{l} BA = CA \\ \angle BAD = \angle CAD \\ AD = AD \end{array} \right] \text{ SAS}$$

$\therefore \angle BDA = \angle ADC = 90^\circ$ (Angle at a straight line)

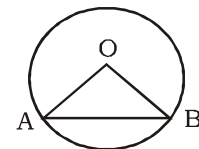
So, AD is the perpendicular bisector of the chord BC.

$\Rightarrow$ AD passes through the centre O.

TYPE I

Ex1. O is the centre of the circle. A chord is drawn equal to the radius. Find the centre angle.

Sol.

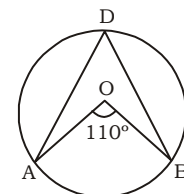


$OA = OB = AB$ (given)

$\therefore \triangle OAB$ is an equilateral triangle.

Hence, $\angle O = \angle A = \angle B = 60^\circ$

Ex2. In the given figure, O is the centre of the circle and $\angle AOB = 110^\circ$, then $\angle ADB$ will be:

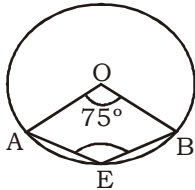


Sol. $\angle AOB = 2 \times \angle ADB$ [Centre angle of a circle is twice the angle of the major arc]

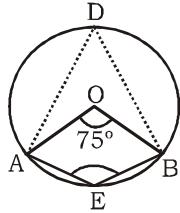
$$110^\circ = 2 \times \angle ADB$$

$$\therefore \angle ADB = 55^\circ$$

Ex3. In the given figure, O is the centre of the circle and $\angle AOB = 75^\circ$, then $\angle AEB$ will be:



Sol.



$$\angle AOB = 75^\circ$$

$\angle ADB = \frac{\angle AOB}{2}$ [Center angle of a circle is twice the angle of the major arc]

$$= \frac{75^\circ}{2} = 37.5^\circ$$

AEBD is a cyclic quadrilateral then,

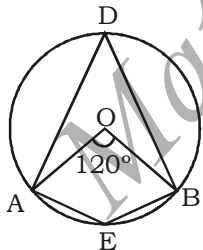
$$\angle E + \angle D = 180^\circ$$

$$\angle E + 37.5^\circ = 180^\circ$$

$$\therefore \angle E = 142.5^\circ$$

Ex4. In a circle, center angle is 120° . Find the ratio of major angle and minor angle.

Sol.



$$\angle ADB = \frac{\angle O}{2} = \frac{120^\circ}{2} = 60^\circ$$

[Center angle of a circle is twice the angle of the major arc]

AEBD is a cyclic quadrilateral then,

$$\angle AEB + \angle ADB = 180^\circ$$

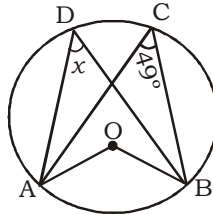
$$\angle AEB = 180^\circ - 60^\circ$$

$$\angle AEB = 120^\circ$$

Required ratio = Major angle : Minor angle
 $= 120^\circ : 60^\circ = 2 : 1$

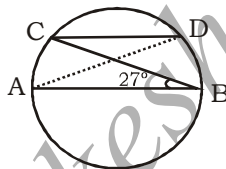
Ex5. In the given figure, O is the centre of the circle, then x is equal to:

Sol.



$\angle ADB = \angle ACB = 49^\circ$ [Angle in the same segment of a circle are equal]

Ex6. AB is the diameter of the circle. AB is parallel to CD. If $\angle ABC = 27^\circ$, then find $\angle BCD$ & $\angle CDA$?



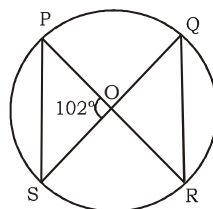
Sol. $\angle ABC = \angle BCD = 27^\circ$

[Alternate angle]

$$\angle CDA = \angle CBA = 27^\circ$$

[Angle in the same segment of a circle are equal]

Ex7. In the given figure O is the centre of the circle. If $\angle POS = 102^\circ$, then $\angle PRQ = ?$



Sol. In $\triangle OPS$,

$$\angle O + \angle P + \angle S = 180^\circ$$

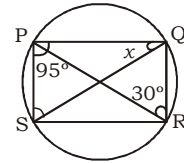
$$102^\circ + \angle S + \angle S = 180^\circ \text{ [OP = OS = OQ = OR = Radius]}$$

$$2\angle S = 78^\circ$$

$$\angle S = 39^\circ$$

$\angle PSQ = \angle PRQ = 39^\circ$ [Angle in the same segment of a circle are equal]

Ex8. The value of x will be :



Sol. $\angle PRQ = \angle PSQ = 30^\circ$ [Angle in the same segment of a circle are equal]

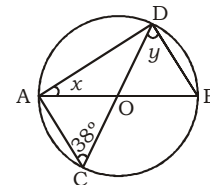
In $\triangle QPS$,

$$\angle Q + \angle P + \angle S = 180^\circ$$

$$\Rightarrow x + 95^\circ + 30^\circ = 180^\circ$$

$$\therefore x = 55^\circ$$

Ex9. In the given figure, AB & CD is a diameter of circle, $\angle ACD = 38^\circ$. Find x & y .



Sol. In $\triangle ACD$,

$\angle A + \angle C + \angle D = 180^\circ$ [$\angle A = 90^\circ$, It is a semicircle]

$$90^\circ + 38^\circ + \angle D = 180^\circ$$

$$\angle D = 52^\circ$$

$\angle ADB = 90^\circ$ [Semicircle]

$$\therefore y = 90^\circ - 52^\circ = 38^\circ$$

$\angle DBA = \angle ACD = 38^\circ$ [Angle in the same segment of a circle are equal]

In $\triangle ADB$

$$\angle A + \angle D + \angle B = 180^\circ$$

$$x + 90^\circ + 38^\circ = 180^\circ$$

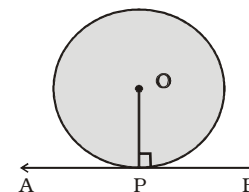
$$\therefore x = 52^\circ$$

THEOREM ON TANGENTS

Tangent

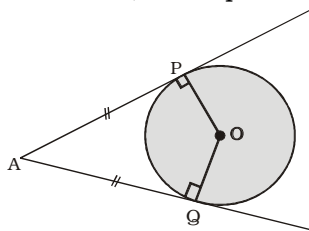
A line meeting a circle in only one point is called a tangent.

1. A tangent at any point of a circle is perpendicular to the radius through the point of contact.



i.e. If AB is a tangent at P, then $OP \perp AB$. (converse of this theorem is also true)

2. The lengths of two tangents, drawn from an external point to a circle, are equal.



i.e. $AP = AQ$.

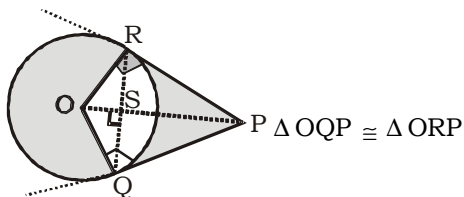
3. When tangents drawn from an external point to a circle
(a) Tangents are equal in length

(b) $\angle QPO = \angle OPR$

(c) $\angle QOP = \angle POR$

(d) $\frac{PQ}{QO} = \frac{QS}{SO}$

Proof:-



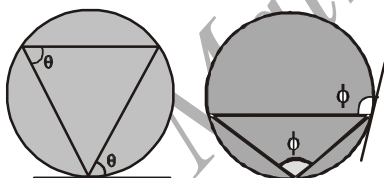
$OQ = OR = \text{Radius}$

$OP = OP$

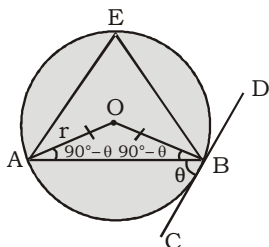
$\therefore \angle PQO = \angle ORP = 90^\circ$

4. **Alternate Segment Theorem**

The angle between a chord and a tangent drawn at end point of chord is equal respectively to the angle formed in the corresponding alternate segments.



Proof:-



$\angle OBC = 90^\circ$

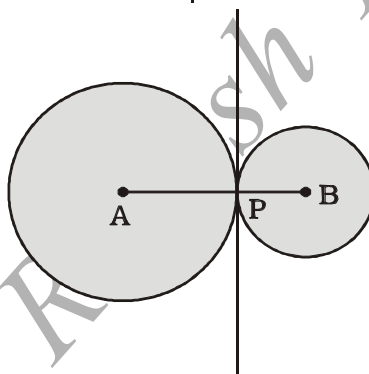
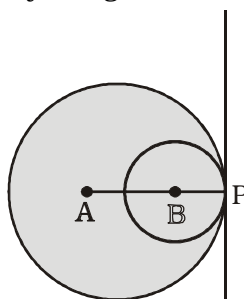
Let $\angle ABC = \theta$ then $\angle OBA = 90^\circ - \theta$

$\angle OAB = \angle OBA = 90^\circ - \theta$ (OA = OB, Radius)

$\angle AOB = 180^\circ - (90^\circ - \theta + 90^\circ - \theta) = 2\theta$

$\therefore \angle AEB = \theta$

5. If two circles touch each other internally or externally the point of contact lies on the line joining their centres.



i.e. A, B and P are collinear.

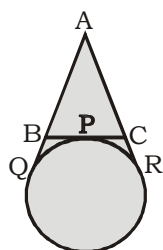
Distance between their centres (d)

- (i) When touch internally, $d = AP - BP$

- (ii) When touch externally, $d = AP + BP$

6. A circle externally touch side BC of a ΔABC at P, AB produced at Q and AC produced at R. If QA

= a cm, then **perimeter**



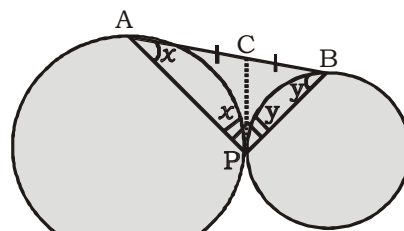
of ΔABC is = 2a

PROOF:-

Perimeter = $2 \times AQ = 2a$

$AQ = AR = a$
 $AQ = AB + BQ = AB + BP$
($\because BQ = BP$)
 $AR = AC + CR = AC + CP$
($\because CR = CP$)
Perimeter = $(AB + BP) + (AC + CP)$
 $= a + a = 2a$

- 7.



Two circles externally touch each other at P. AB is direct common tangent (DCT) of the circles. If $\angle BAP = x$ then find $\angle ABP = ?$

Remember, $\angle APB$ is always right angle

$\angle ABP = 90^\circ - x$

PROOF:-

$CA = CP$ (tangent of circle).....(i)

$CP = CB$ (tangent of circle).....(ii)

From (i) & (ii)

So, $CA = CP = CB$

In ΔACP ,

$\angle CAP = \angle CPA = x$

In ΔCPB ,

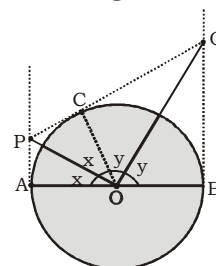
$\angle CBP = \angle CPB = y$

In ΔAPB

$\therefore x + (x + y) + y = 180^\circ$

$\Rightarrow x + y = 90^\circ$

8. AB is a diameter of a circle. Two tangents drawn at A & B. Tangent drawn at any point C of the circle meet both tangents at P & Q. Find angle $\angle POQ = ?$



PROOF:-

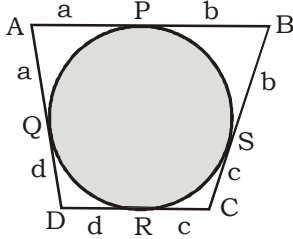
Remember, $\angle POQ = 90^\circ$

$\Delta POA \cong \Delta POC$

$\Delta QCO \cong \Delta QBO$

$\therefore x + x + y + y = 180^\circ \Rightarrow x + y = 90^\circ$

9. If a quadrilateral circumscribed a circle. (Not general case/only for some specific quadrilateral)



then, sum of opposite sides are equal

$$AB + CD = AD + BC$$

PROOF

Length of tangents drawn from an external point to a circle are equal.

$$AP = AQ = a$$

$$BP = BS = b$$

$$DQ = DR = d$$

$$CR = CS = c$$

L.H.S.

$$AB + DC = (a+b) + (d+c) \dots (i)$$

R.H.S.

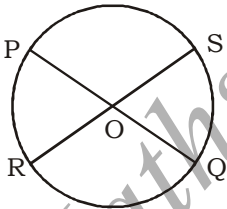
$$AD + BC = (a+d) + (b+c) \dots (ii)$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$$AB + CD = AD + BC$$

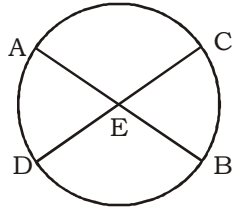
TYPE II

- Ex1.** In the given figure, $PQ = 25$ cm, $OS = 12$ cm, if $OS = OR$ then $OP = ?$



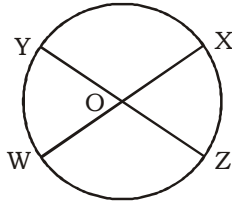
- Sol.** Let, $OP = x$
 $\therefore OQ = 25 - x$
 $\therefore OP \times OQ = OR \times OS$
 $\therefore x \times (25 - x) = 12 \times 12$
 $\Rightarrow 25x - x^2 = 144$
 $\Rightarrow x^2 - 16x - 9x + 144 = 0$
 $\Rightarrow x(x - 16) - 9(x - 16) = 0$
 $\Rightarrow (x - 16)(x - 9) = 0$
 $\Rightarrow x = 16, x = 9$
 $\Rightarrow OP = 16, 9$

- Ex2.** In the given figure, $AE = 7$, $EB = 6$ cm, $EC = 14$ cm. Find $DE = ?$



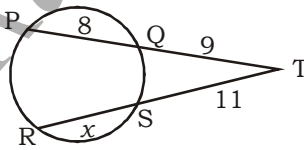
- Sol.** $\therefore AE \times EB = DE \times EC$
 $\therefore 7 \times 6 = DE \times 14$
 $\Rightarrow DE = 3$ cm

- Ex3.** In the given figure, $OY : OX = 5 : 4$, $OZ = 40$ cm, $YZ = 60$ cm. Find WX ?



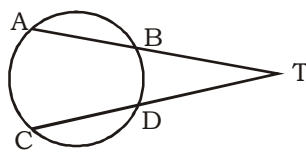
- Sol.** $OY = YZ - OZ = 60 - 40 = 20$ cm
 $\Rightarrow \frac{OY}{OX} = \frac{5}{4} = \frac{20}{OX}$
 $\Rightarrow OX = 16$ cm
 $\therefore OX \times OW = OY \times OZ$
 $\Rightarrow 16 \times OW = 20 \times 40$
 $\Rightarrow OW = 50$ cm
 $\therefore WX = OW + OX = 50 + 16 = 66$ cm

- Ex4.** In the given figure, the value of x



- Sol.** $\therefore QT \times PT = ST \times RT$
 $\therefore 9 \times (9 + 8) = 11 \times (11 + x)$
 $\Rightarrow \frac{9 \times 17}{11} = 11 + x$
 $\Rightarrow x = \frac{153 - 121}{11} = \frac{32}{11} = 2\frac{10}{11}$

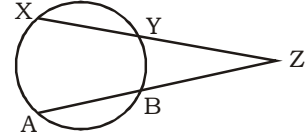
- Ex5.** In the given figure, $BT = CD$, $BT = 7$ cm, $DT = 21$ cm, find $AB = ?$



- $\therefore AT \times BT = DT \times CT$
 $\Rightarrow AT \times 7 = 21 \times 28$ ($\because CT = CD + DT = 7 + 21 = 28$)
 $\Rightarrow AT = 28 \times 3 = 84$ cm
 $\Rightarrow AB = AT - BT$
 $\Rightarrow AB = 84 - 7$

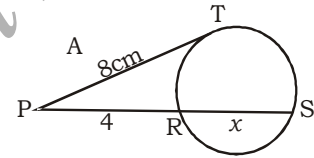
$$\Rightarrow AB = 77 \text{ cm}$$

- Ex6.** In the given figure, $XY : YZ = 3 : 4$, $AB = 15$ cm, $BZ = 20$ cm. Find XZ .



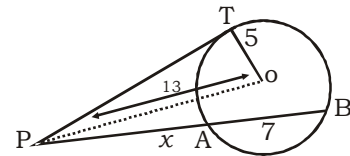
- Sol.** $XZ \times YZ = BZ \times AZ$
 $\Rightarrow 7x \times 4x = 20 \times 35$
 $\Rightarrow x^2 = 25$
 $\Rightarrow x = 5$
 $\Rightarrow XZ = 3x + 4x$
 $\Rightarrow XZ = 7x = 7 \times 5$
 $\Rightarrow XZ = 35$

- Ex7.** The value of x is



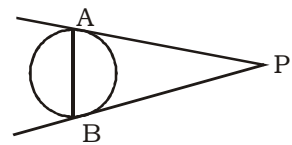
- Sol.** $PT^2 = PR \times PS$
 $\Rightarrow 8^2 = 4 \times (x + 4)$
 $\Rightarrow x + 4 = 16$
 $\Rightarrow x = 12$ cm

- Ex8.** The value of x is



- In $\triangle PTO$,
 $\Rightarrow PT^2 + OT^2 = OP^2$
 $\Rightarrow PT^2 = 169 - 25$
 $\Rightarrow PT = 12$ cm
 $\Rightarrow PT^2 = PA \times PB$
 $\Rightarrow 144 = x(x + 7)$
 $\Rightarrow x = 9$

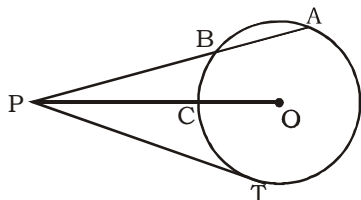
- Ex9.** In the given figure, PA and PB are tangents from a point P to a circle such that $PA = 16$ cm and $\angle APB = 60^\circ$. What is the length of the chord AB ?



- Sol.** $AP = PB$ [Tangent of the same exterior point]
 $(\angle PAB = \angle PBA)$
In $\triangle APB$
 $\angle A + \angle P + \angle B = 180^\circ$
 $\Rightarrow \angle A + 60^\circ + \angle A = 180^\circ$

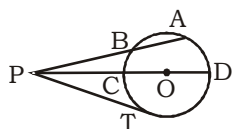
$$\begin{aligned} \Rightarrow 2\angle A &= 120^\circ \\ \Rightarrow \angle A &= 60^\circ \\ \Rightarrow \angle B &= 60^\circ \\ \therefore \triangle APB &\text{ is an equilateral } \triangle. \\ \text{Hence, } AP &= PB = AB = 16 \text{ cm.} \end{aligned}$$

Ex10.



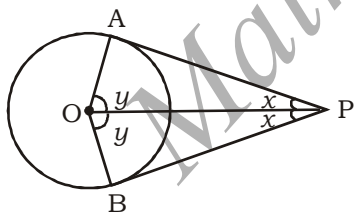
In the figure, PT is a tangent to a circle of radius 6 cm. If P is at a distance of 10 cm from the centre O and PB = 5 cm, then what is the length of the chord BA?

Sol.



$$\begin{aligned} PD &= PO + OD, 10 + 6 = 16 \text{ cm} \\ PC &= PO - OC, 10 - 6 = 4 \text{ cm} \\ &\quad (OD = OC = 6 \text{ Radius}) \\ \therefore PB \times PA &= PC \times PD \\ \therefore 5 \times PA &= 4 \times 16 \\ \Rightarrow PA &= \frac{64}{5} \\ \therefore AB &= PA - PB \\ &= \frac{64}{5} - 5 \\ &= \frac{39}{5} = 7.8 \text{ cm} \end{aligned}$$

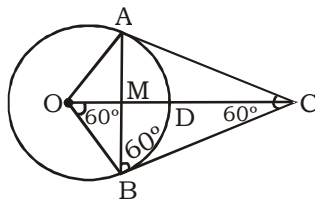
Ex11. If $x = 30^\circ$, then $y = ?$



Sol. $\angle A = \angle B = 90^\circ$ [Tangent of the circle]
In $\triangle AOP$,
 $\angle A + \angle O + \angle P = 180^\circ$
 $\Rightarrow 90^\circ + y + 30^\circ = 180^\circ$
 $\therefore y = 60^\circ$

Ex12. O is the centre of the circle and OA and OB is radius. $\angle AOB = 120^\circ$. Two tangents A & B are drawn intersect each other at point C. If D is a point divided OC is equally, then the ratio of D divides OC?

Sol.



$$\begin{aligned} \triangle OAM &\cong \triangle OBM \\ \angle AOC &= \angle BOC = \frac{120^\circ}{2} = 60^\circ \\ \angle A &= \angle B = 90^\circ \text{ [tangents]} \\ \angle CAB &= \angle CBA = 90^\circ - \angle OAM \\ \text{In } \triangle ABC, \\ \angle ACB &= 180^\circ - \angle AOB = 180^\circ - 120^\circ = 60^\circ \\ \therefore CA &= CB \text{ (tangent of circle)} \\ \therefore \angle CAB &= \angle CBA \\ \text{Now, } \angle A + \angle B + \angle C &= 180^\circ \\ \therefore \angle A + \angle A + \angle C &= 180^\circ \\ \therefore 2\angle A &= 180^\circ - 60^\circ \\ \therefore 2\angle A &= 120^\circ \\ \therefore \angle A &= 60^\circ \\ \text{So,} \\ \angle A &= \angle B = \angle C = 60^\circ \\ \triangle ABC &\text{ is an equilateral } \triangle. \\ \text{In } \triangle AMC, \\ \cos 30^\circ &= \frac{CM}{CA} \end{aligned}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{CM}{\sqrt{3}r}$$

$$\Rightarrow CM = \frac{3}{2}r$$

In $\triangle OAM$

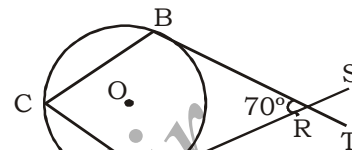
$$\Rightarrow \cos 60^\circ = \frac{OM}{OA}$$

$$\Rightarrow \frac{1}{2} = \frac{OM}{r}$$

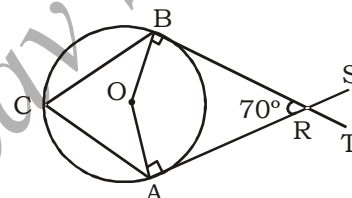
$$\Rightarrow OM = \frac{r}{2}$$

$$\begin{aligned} OC &= OM + CM = 2r \\ \therefore OD : DC &= 1 : 1 \end{aligned}$$

Ex13. In the given figure, BT & AS are two tangents. Then $\angle C$ is:

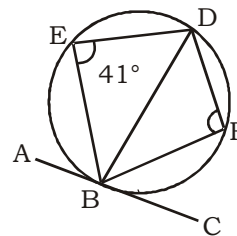


Sol.



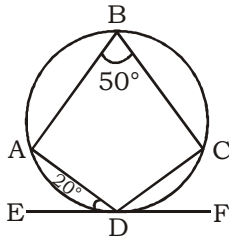
In $\square AOB$,
 $\angle A + \angle O + \angle B + \angle R = 360^\circ$
 $90^\circ + \angle O + 90^\circ + 70^\circ = 360^\circ$
[$\therefore \angle A = \angle B = 90^\circ$ tangents]
 $\angle O = 110^\circ$
 $\therefore \angle BCA = \frac{\angle BOA}{2} = 55^\circ$

Ex14. Find $\angle ABD$

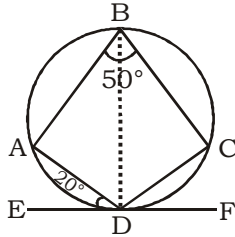


Sol. $\square DEBF$ is a cyclic quadrilateral,
 $\therefore \angle E + \angle F = 180^\circ$
($\therefore$ Sum of opposite angles of cyclic quadrilateral is 180°)
 $\Rightarrow 41^\circ + \angle F = 180^\circ$
 $\Rightarrow \angle F = 139^\circ$
 $\angle DFB = \angle ABD = 139^\circ$
(alternate segment angle)

Ex15. In the given figure, $\angle ABC = 50^\circ$, $\angle ADE = 20^\circ$, Find $\angle CDF = ?$

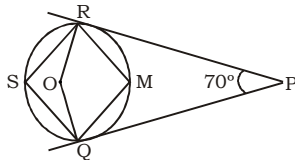


Sol.



$\angle ABD = \angle ADE = 20^\circ$ [Alternate segment]
 $\angle DBC = 50^\circ - 20^\circ = 30^\circ$
 $\angle DBC = \angle CDF = 30^\circ$ [Alternate segment]

Ex-16 In the given figure, Find the value of $\angle m$?



Sol. In $\triangle PQR$,
 $\therefore \angle O + \angle R + \angle P + \angle Q = 360^\circ$
 $\Rightarrow \angle O + 90^\circ + 70^\circ + 90^\circ = 360^\circ$
 $\Rightarrow \angle O = 110^\circ$

$\therefore \angle QSR = \frac{\angle O}{2} = 55^\circ$

$\therefore \angle S + \angle M = 180^\circ$

$\therefore \angle M = 125^\circ$

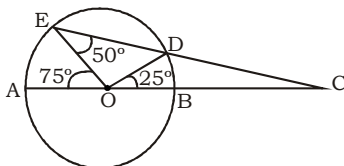
Alternate

$$\angle m = 90^\circ + \frac{\angle P}{2}$$

$$= 90^\circ + \frac{70^\circ}{2}$$

$$\angle m = 125^\circ$$

Ex-17 In the given circle, AB is a diameter. $\angle BOD = 25^\circ$ and $\angle EOA = 75^\circ$, then $\angle ECA = ?$



Sol. $\angle AOE + \angle EOD + \angle DOB = 180^\circ$

[$\because$ AOC is a Straight Line]

$$\Rightarrow 75^\circ + \angle EOD + 25^\circ = 180^\circ$$

$$\Rightarrow \angle EOD = 80^\circ$$

$$\therefore \angle E + \angle EOD = \angle ODC$$

[Sum of two interior angle is equal to opposite of exterior angle]

$$\therefore 50^\circ + 80^\circ = \angle ODC$$

$$\Rightarrow \angle ODC = 130^\circ$$

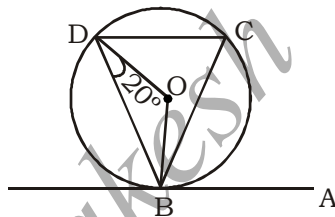
In $\triangle ODC$,

$$\angle O + \angle D + \angle C = 180^\circ$$

$$\Rightarrow 25^\circ + 130^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 25^\circ$$

Ex-18 In the given figure, O is the centre of the circle and AB is tangent $\angle ODB = 20^\circ$ and $\angle BDC$ and $\angle ABD$ are supplementary to each other then find $\angle OBC$



Sol. $\angle ODB = \angle OBD = 20^\circ$ [$\because$ OD = OB = radius]

$$\angle ABD = \angle ABO + \angle OBD$$

$$\angle ABD = 90^\circ + 20^\circ = 110^\circ$$

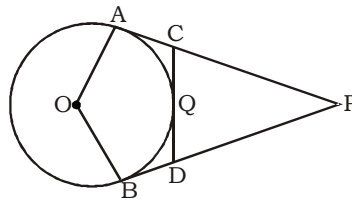
$$\therefore \angle BDC + \angle ABD = 180^\circ \quad (\text{supplementary angle})$$

$$\Rightarrow \angle BDC = 180^\circ - 110^\circ = 70^\circ$$

$$\therefore \angle ABC = \angle BDC = 70^\circ \quad [\text{Alternate segment}]$$

$$\therefore \angle OBC = 90^\circ - 70^\circ = 20^\circ$$

Ex-19



In the given, circle with centre O have two tangents PA, PB. CD is also tangent, then

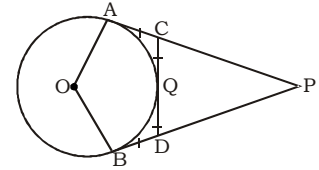
$$(a) PC + PD = CD$$

$$(b) 3PB = PD + DC + PC$$

$$(c) 4PA = PD + CD + PC$$

$$(d) 2PB = PD + CD + PC$$

Sol.



As template question (4)

$$PA = PC + CA = PC + CQ$$

$$(\because CA = CQ)$$

$$PB = PD + DB = PD + DQ$$

$$(\because DB = DQ)$$

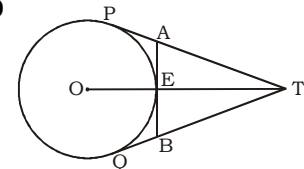
$$\& PA = PB$$

$$\therefore 2PB = (PC + CQ) + (PD + DQ)$$

$$= PC + CD + PD$$

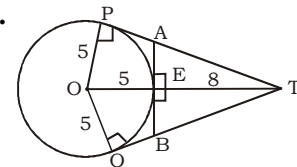
$$(\because CD = CQ + DQ)$$

Ex-20



In the figure given above, from a point T, 13cm. away from the centre O of a circle of radius 5cm., tangents PT and QT are drawn. What is the length of AB?

Sol.



$$ET = OT - OE = 13 - 5 = 8 \text{ cm}$$

In $\triangle OPT$,

By pythagoras theorem,

$$TP = \sqrt{OT^2 - OP^2} = \sqrt{13^2 - 5^2}$$

$$= 12 \text{ cm}$$

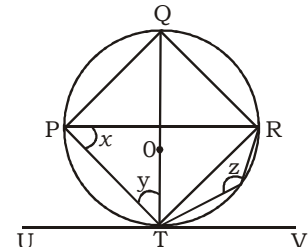
$$\triangle TPO \sim \triangle TEA$$

$$\frac{TP}{TE} = \frac{PO}{EA}$$

$$\frac{12}{8} = \frac{5}{EA} \Rightarrow EA = \frac{10}{3}$$

$$\text{Hence, } AB = 2EA = \frac{20}{3} \text{ cm.}$$

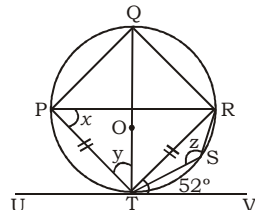
Ex-21



In the figure given above, O is the centre of the circle. The line UTV is a tangent to the

circle at T, $\angle VTR = 52^\circ$ and $\triangle PTR$ is an isosceles triangle such that $TP = TR$. What is $\angle x + \angle y + \angle z$ equal to?

Sol.



$x = \angle VTR = 52^\circ$ (By alternate segment theorem)

$z = 180^\circ - x = 180^\circ - 52^\circ = 128^\circ$ (Since PTRS is a cyclic Quadrilateral)

In $\triangle PTR$,

$PT = TR$ (given)

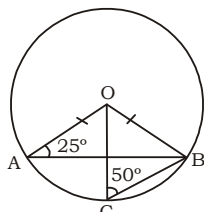
$\therefore \angle PRT = x = 52^\circ$

$\angle PTU = \angle PRT = 52^\circ$ (By alternate Segment theorem)

$\therefore y = 90^\circ - \angle PTU = 90^\circ - 52^\circ = 38^\circ$

$\Rightarrow x + y + z = 52^\circ + 38^\circ + 128^\circ = 218^\circ$

Ex-22 In the given figure, $\angle OCB = 50^\circ$ and $\angle OAB = 25^\circ$. Then the value of the $\angle AOC$ is :



Sol. $\angle OAB = \angle OBA = 25^\circ$

[OA = OB = radius]

$\angle OCB = \angle OBC = 50^\circ$

[OB = OC = radius]

In $\triangle COB$,

$\angle COB + \angle OCB + \angle OBC = 180^\circ$

$\angle COB + 50^\circ + 50^\circ = 180^\circ$

$\angle COB = 80^\circ$

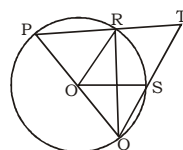
In $\triangle AOB$,

$\angle AOB = 180^\circ - 25^\circ - 25^\circ = 130^\circ$

$\therefore \angle AOC = \angle AOB - \angle COB$

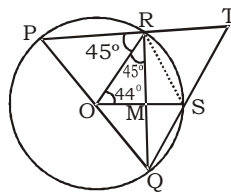
$= 130^\circ - 80^\circ = 50^\circ$

Ex-23



In the figure given below, PQ is a diameter of the circle whose centre is at O. If $\angle ROS = 44^\circ$ and OR a bisector of $\angle PRQ$ then what is the value of $\angle PTQ$?

Sol.



In $\triangle ORS$, $OR = OS = \text{radius}$

$\Rightarrow \angle ORS = \angle OSR = \frac{180^\circ - 44^\circ}{2} = 68^\circ$

$\angle PRS = 45^\circ + 68^\circ = 113^\circ$

$\angle PQS = 180^\circ - \angle PRS = 180^\circ - 113^\circ = 67^\circ$ (By property of cyclic Quadrilateral)

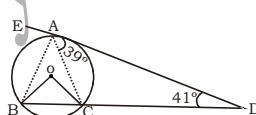
In $\triangle OPR$, $OP = OR \Rightarrow \angle PRO = 45^\circ$

In $\triangle PQT$, $\angle PTQ = 180^\circ - \angle TPQ - \angle PQT$

$= 180^\circ - 45^\circ - 67^\circ = 68^\circ$

Ex-24 A, B & C are three points on a circle such that a tangent touches the circle at A and intersects the extended part of chord BC at D. Find the central angle made by chord BC. If $\angle CAD = 39^\circ$, $\angle CDA = 41^\circ$

Sol.



$\angle ACB = \angle CAD + \angle CDA$ [Sum of two Interior angle is equal to opposite of exterior angle]

$\angle ACB = 39^\circ + 41^\circ = 80^\circ$

$\angle BAE = \angle BCA = 80^\circ$

[Alternate segment]

$\angle EAB + \angle BAC + \angle CAD = 180^\circ$

[Linear angle]

$80^\circ + \angle BAC + 39^\circ = 180^\circ$

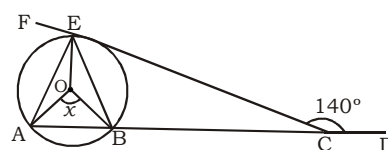
$\angle BAC = 61^\circ$

$\therefore \angle BOC = 2 \times \angle BAC$

[Center angle is twice the angle subtended by the major arc]

$= 2 \times 61^\circ = 122^\circ$

Ex-25 In the given figure, $BE = BC$, $\angle ECD = 140^\circ$, find $\angle AOB = ?$



Sol. $\angle BEC = \angle BCE$ [$\because BE = BC$]

$\angle BCE = 180^\circ - 140^\circ$

$\angle BCE = 40^\circ$

$\angle EBA = \angle BEC + \angle BCE$

$\angle EBA = 40^\circ + 40^\circ$

$\angle EBA = 80^\circ$

$\angle OEB = 90^\circ - \angle BEC$

$= 90^\circ - 40^\circ$

$= 50^\circ$

$\angle OEB = \angle OBE = 50^\circ$ [$\because OB = OE = \text{radius}$]

$\angle OBA = \angle ABE - \angle OBE$

$\angle OBA = 80^\circ - 50^\circ$

$\angle OBA = 30^\circ = \angle OAB$ [$\because OA = OB = \text{radius}$]

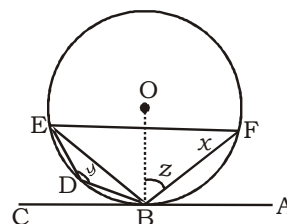
In $\triangle AOB$,

$\angle A + \angle O + \angle B = 180^\circ$

$30^\circ + \angle O + 30^\circ = 180^\circ$

$\therefore \angle O = 120^\circ$

Ex-26. In the given figure, if $BF = BE$, $\angle EBC = 44^\circ$, then $x + y + z$ is:



Sol. $\angle EBC = \angle EFB = x = 44^\circ$ (alternate segment)

$\Rightarrow \angle BEF = \angle BFE = 44^\circ$ [$\because BF = BE$]

In $\triangle BEF$,

$\Rightarrow \angle B = 180^\circ - 44^\circ - 44^\circ = 92^\circ$

$\Rightarrow \angle CBF = \angle CBO + \angle OBF$

$\Rightarrow = 90^\circ + z \dots \dots \dots (i)$

$\Rightarrow \angle CBF = \angle CBE + \angle EBF \dots (ii)$

(i) = (ii)

$\Rightarrow 90^\circ + z = \angle CBE + \angle EBF$

$\Rightarrow \angle Z = \angle CBE + \angle EBF - 90^\circ$

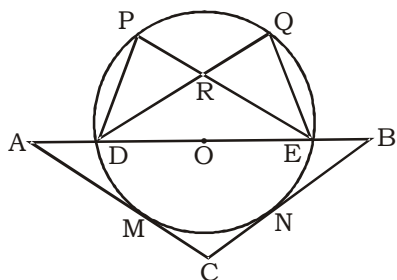
$\Rightarrow \angle Z = 44^\circ + 92^\circ - 90^\circ$

$\Rightarrow \therefore \angle Z = 46^\circ$

In $\square BDEF$,

$$\begin{aligned}\Rightarrow \angle D + \angle F &= 180^\circ \\ \Rightarrow \angle D &= 180^\circ - 44^\circ \\ \Rightarrow \angle D &= 136^\circ = y \\ \Rightarrow \therefore x + y + z &= 44^\circ + 136^\circ + 46^\circ = 226^\circ\end{aligned}$$

Ex-27. ABC is an isosceles triangle and AC, BC are the tangents at M & N respectively. DE is the diameter of the circle. $\angle ADP = \angle BEQ = 100^\circ$. What is the value of $\angle PRD$?



Sol. $\angle PDB = 180^\circ - \angle PDA$ (linear angle)
 $= 180^\circ - 100^\circ = 80^\circ$
(i)

$$\begin{aligned}\angle QEA &= 180^\circ - \angle QEB \\ &= 180^\circ - 100^\circ = 80^\circ \text{.....(ii)}\end{aligned}$$

From (i) & (ii)

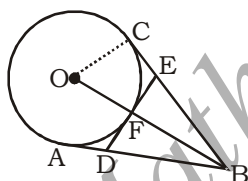
$$\angle PDB = \angle QEA = 80^\circ$$

$$(\because \angle DPE = \angle DQE = 90^\circ)$$

Angle in the semicircle

$$\begin{aligned}\therefore \angle PED &= \angle QDE = 10^\circ \\ \therefore \angle DRE &= 180^\circ - (10^\circ + 10^\circ) = 160^\circ \\ \therefore \angle PRD &= 180^\circ - \angle DRE \text{ (Linear angle)} \\ &= 180^\circ - 160^\circ = 20^\circ\end{aligned}$$

Ex-28.



In the given figure, $BE = 9$ cm, $BF = 6$ cm, Find OB.

Sol. In $\triangle BEF$,
 $EF^2 = BE^2 - BF^2$

$$\Rightarrow EF^2 = 9^2 - 6^2$$

$$\Rightarrow EF^2 = 45$$

$$\Rightarrow EF = 3\sqrt{5}$$

$$CE = EF = 3\sqrt{5} \text{ [tangents of the circle]}$$

$$BC = BE + EC$$

$$\Rightarrow BC = 9 + 3\sqrt{5}$$

In $\triangle OBC$,

$$OB^2 = OC^2 + BC^2$$

$$\begin{aligned}(r + 6)^2 &= r^2 + (9 + 3\sqrt{5})^2 \\ r^2 + 36 + 12r &= r^2 + 81 + 45 \\ &+ 54\sqrt{5}\end{aligned}$$

$$12r = 90 + 54\sqrt{5}$$

$$2r = 15 + 9\sqrt{5}$$

$$r = \frac{15 + 9\sqrt{5}}{2}$$

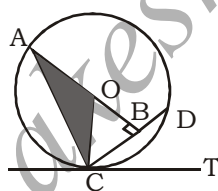
$$\therefore OB = OF + BF$$

$$OB = \frac{15 + 9\sqrt{5}}{2} + 6$$

$$\Rightarrow OB = \frac{27 + 9\sqrt{5}}{2} \text{ cm}$$

Ex-29. In the given diagram CT is tangent at C, making an

angle of $\frac{\pi}{4}$ with CD, O is the centre of the circle. $CD = 10$ cm. What is the perimeter of the shaded region ($\triangle AOC$) approximately



Sol. $\angle OCT = 90^\circ$ (tangent angle)

$$\angle DCT = 45^\circ \text{ (given)}$$

$$\therefore \angle OCB = \angle OCT - \angle DCT = 90^\circ - 45^\circ = 45^\circ$$

$$\therefore \angle COB = 45^\circ \text{ (BOC is a right angled triangle)}$$

In $\triangle BOC$ isosceles right angle triangle

$$\therefore \angle AOC = 180^\circ - 45^\circ = 135^\circ$$

$$\text{Now, } CD = 10 \Rightarrow BC = 5 \text{ cm} = OB$$

$$OC^2 = OB^2 + BC^2 = (5)^2 + (5)^2 = 50$$

$$\Rightarrow OC = 5\sqrt{2} \text{ cm} = OA \text{ (radius)}$$

$$\text{Again, } AC^2 = OA^2 + OC^2 - 2OA \cdot$$

$$OC \cos 135^\circ = 2(OA)^2 - 2(OA)^2 \cdot \cos 135^\circ$$

$$= 2(5\sqrt{2})^2 - 2(5\sqrt{2})^2 \times \left(-\frac{1}{\sqrt{2}}\right)$$

$$= 100 + \frac{100}{\sqrt{2}}$$

$$AC^2 \approx 170.70$$

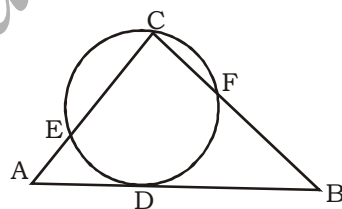
$$\Rightarrow AC \approx 13 \text{ cm}$$

$$\therefore \text{Perimeter of } \triangle OAC = OA + OC + AC$$

$$\begin{aligned}&= 5\sqrt{2} + 5\sqrt{2} + 13 = 10\sqrt{2} + 13 \\ &= 10 \times 1.414 + 13 \approx 27\end{aligned}$$

Ex-30. ABC is an isosceles triangle a circle is such that it passes through vertex C and AB acts as a tangent at D for the same circle. AC and BC intersects the circle at E and F respectively $AC = BC = 4$ cm and $AB = 6$ cm. Also, D is the mid-point of AB. What is the ratio of $EC : (AE + AD)$?

Sol.



Here, AC and BC are the secants of the circle and AB is tangent at D

$$AD = DB = \frac{6}{2}$$

$$\therefore AE \times AC = AD^2$$

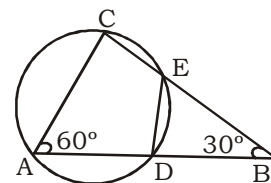
$$AE \times 4 = (3)^2 \Rightarrow AE = \frac{9}{4}$$

$$\therefore CE = 4 - \frac{9}{4} = \frac{7}{4}$$

$$\therefore CE : (AE + AD)$$

$$= \frac{7}{4} : \left(\frac{9}{4} + 3\right) = \frac{7}{4} : \frac{21}{4} = 1 : 3$$

Ex-31. In the given figure ADEC is cyclic quadrilateral, CE and AD are extended to meet at B. $\angle CAD = 60^\circ$ and $\angle CBA = 30^\circ$. $BD = 6$ cm and $CE = 5\sqrt{3}$ cm, What is the ratio of $AC : AD$?



Sol. $\angle CED = 120^\circ$ [$\square ACED$ is cyclic]

In $\triangle BED$,

$$\therefore \angle BED = 180^\circ - \angle CED = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore \angle EDB = 180^\circ - (30^\circ + 60^\circ) = 90^\circ$$

$$\frac{BD}{BE} = \cos 30^\circ$$

$$\frac{6}{BE} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow BE = 4\sqrt{3} \text{ cm}$$

$$\therefore BC = BE + EC$$

$$= 4\sqrt{3} + 5\sqrt{3} = 9\sqrt{3} \text{ cm}$$

Now, since AB and CB are the secants of the circle.

$$\therefore BD \times BA = BE \times BC$$

$$6 \times BA = 4\sqrt{3} \times 9\sqrt{3}$$

$$\Rightarrow BA = 18 \text{ cm}$$

In $\triangle ACB$,

$$\angle ACB = 180^\circ - (60^\circ + 30^\circ) = 90^\circ$$

Again, $\triangle ACB$ is a right angled triangle

$$(\because \angle C = 90^\circ)$$

$$\therefore AD = AB - BD = 12 \text{ cm}$$

$$AC^2 = AB^2 - BC^2$$

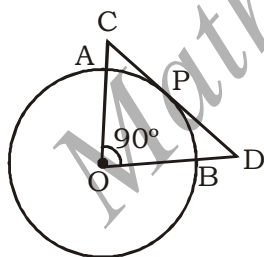
$$= (18)^2 - (9\sqrt{3})^2$$

$$= 324 - 243$$

$$AC = \sqrt{81} = 9$$

$$\therefore \frac{AC}{AD} = \frac{9}{12} = \frac{3}{4}$$

Ex-32. In a circle O is the centre and COD is right angle. $AC = BD$ and CD is the tangent at P. What is the value of $AC + CP$, if the radius of the circle is 1 metre?



Sol. $OC = OA + AC$ (i)

$OD = OB + BD$ (ii)

$AC = BD$ (given)

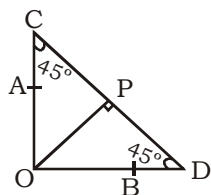
$OA = OB$ (radius)

then,

$OC = OD$ and $OA = OP = OB$

(radius)

In $\triangle OCP$,



$$\angle OPC = 90^\circ \text{ (OP} \perp \text{CD)}$$

$$\angle COP = 180^\circ - (90^\circ + 45^\circ) = 45^\circ$$

$$OP = PC = 1$$

Now,

$$OC^2 = OP^2 + PC^2 = (1)^2 + (1)^2 = 2$$

$$OC = \sqrt{2} \text{ m}$$

$$AC = OC - OA$$

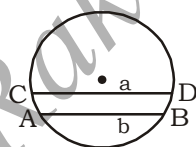
$$= (\sqrt{2} - 1) \text{ m}$$

$$\text{and } AC + CP = (\sqrt{2} - 1) + 1$$

$$= \sqrt{2} \text{ m} = 1.414 \text{ m} = 141.4 \text{ cm}$$

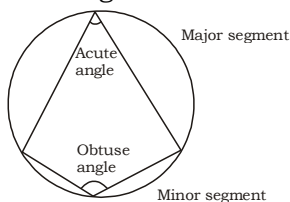
TWO CIRCLE

- Two circles are congruent if and only if they have equal radii.
- Of any two chords of a circle, the one which is greater is nearer to the centre.

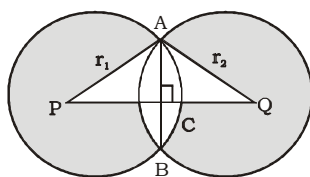


Here, $CD > AB = a > b$

- Angle in a major segment of a circle is acute and angle in a minor segment is obtuse.



- If circles intersect each other



AB is common chord of the circles,

Distance between centres,

$$PQ = PC + CQ$$

$$\Rightarrow PQ = \sqrt{r_1^2 - \left(\frac{AB}{2}\right)^2} + \sqrt{r_2^2 - \left(\frac{AB}{2}\right)^2}$$

Case - 1

If AQ is tangent of circle₁ or PA is tangent of circle₂ then $\triangle PAQ$ is a right angled triangle and $AC \perp PQ$

$$\text{then } PA \times AQ = PQ \times AC$$

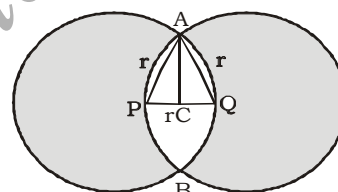
$$\Rightarrow AC = \frac{PA \times AQ}{PQ}$$

Length of common chord AB = 2AC

$$= \frac{2PA \times AQ}{PQ}$$

Case - 2

If each circle passes through the centre of the other then length of common chord is $\sqrt{3} r$.



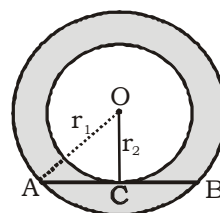
$\triangle PAQ$ is an equilateral triangle

$$\& \text{ Height } AC = \frac{\sqrt{3}}{2} r$$

So, **length of common chord =**

$$2AC = \sqrt{3} r$$

- If circles are concentric

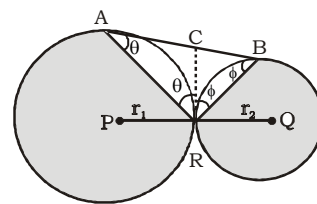


AB is chord of the greater circle
be a tangent to smaller circle

$$\text{Length of } AB = 2AC =$$

$$2\sqrt{r_1^2 - r_2^2}$$

- If circles externally touch each other



Distance between centre = $r_1 + r_2$

R is common point of circles AB
is direct common tangent (DCT)

$$\text{then } \angle ARB = 90^\circ$$

Proof

Tangent at R meet AB at C. $AC = CR$ & $CB = CR$ (Tangents from external point)

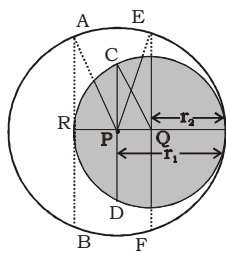
So, $\angle CAR = \angle CRA = \theta$

& $\angle CBR = \angle BRC = \phi$

Sum of three interior angle of $\Delta = 180^\circ$

$$\therefore \theta + (\theta + \phi) + \phi = 180^\circ \Rightarrow \theta + \phi = 90^\circ$$

7. If circles internally touch each other



P & Q are centre of greater and smaller circle respectively. then **distance between centre = $r_1 - r_2$**

- (a) AB - The biggest chord of the greater circle which is outside the inner circle

$$AB = 2\sqrt{AP^2 - RP^2}$$

$$\text{Where } AP = r_1, RP = (r_2 + r_2) - r_1 = 2r_2 - r_1$$

$$AB = 2\sqrt{r_1^2 - (2r_2 - r_1)^2} = 2\sqrt{4r_2(r_1 - r_2)} = 4\sqrt{r_2(r_1 - r_2)}$$

- (b) CD - The smallest chord of the smaller circle which passes through the centre of greater circle.

CD =

$$2\sqrt{CQ^2 - PQ^2} = 2\sqrt{r_2^2 - (r_1 - r_2)^2}$$

$$CD = 2\sqrt{r_1(2r_2 - r_1)}$$

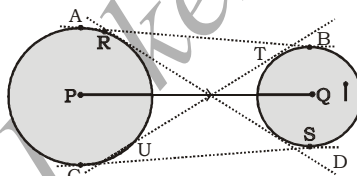
- (c) EF - The smallest chord of greater circle which passes through centre smaller circle.

$$EF = 2\sqrt{PE^2 - PQ^2} =$$

$$2\sqrt{r_1^2 - (r_1 - r_2)^2}$$

$$EF = 2\sqrt{r_2(2r_2 - r_1)}$$

8. If circles place at some distance

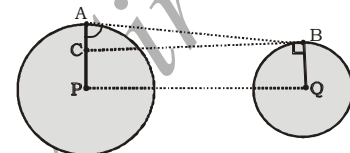


$\Rightarrow$ **Direct common tangent (DCT) AB and CD =**

$$\sqrt{(\text{Distance between centres})^2 - (r_1 - r_2)^2}$$

$\Rightarrow$ **Transverse common tangent (TCT) RS and TU**

$$= \sqrt{(\text{Distance between centres})^2 - (r_1 + r_2)^2}$$



Proof,

$PA \perp AB$ and $QB \perp AB$ then

$PA \parallel BQ$

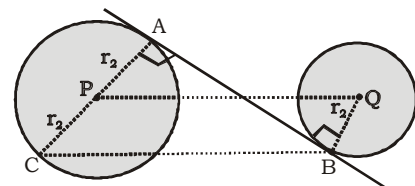
$$\therefore AC = PA - PC = r_1 - r_2$$

In ΔCAB ,

$$\text{Length of } AB = \sqrt{CB^2 - AC^2}$$

$\downarrow$

$$DCT = \sqrt{(\text{Distance between centres})^2 - (r_1 - r_2)^2}$$



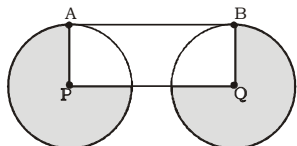
Minimum and Maximum Possible Common Tangents in two Circles

Circle are	Minimum Common Tangent	Maximum Common Tangent
(1) Intersecting each other	2	
(2) Neither touch nor Intersecting	0	
(3) Touch each other	1 (Internally)	3 (externally)

$$\begin{aligned} \text{In } \triangle ACB, AB &= \sqrt{CB^2 - AC^2} \\ &= \sqrt{\left(\text{Distance between centres}\right)^2 - (r_1 - r_2)^2} \end{aligned}$$

Case : 1

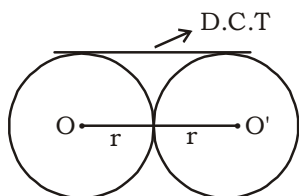
If radius of circles are same
 $r_1 = r_2 = r$



$$\text{DCT, } AB = \sqrt{(PQ)^2 - (r - r)^2}$$

$AB = PQ$ i.e. length of direct common tangent equal to distance between circle.

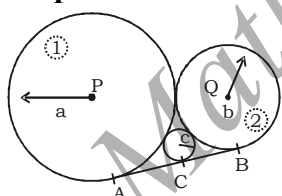
Case : 2



If circle externally touch each other i.e. distance between centre
 $= r_1 + r_2$

$$\begin{aligned} \Rightarrow \text{TCT} &= \sqrt{(r_1 + r_2)^2 - (r_1 + r_2)^2} = 0 \\ \Rightarrow \text{D} & \quad \text{C} \quad \text{T} \\ &= \sqrt{(r_1 + r_2)^2 - (r_1 - r_2)^2} = 2\sqrt{r_1 r_2} \end{aligned}$$

Template question



Two circles of radius a cm. & b cm. externally touch each other. AB is DCT of circles. A third circle with radius C cm. externally two circles and DCT. Then relation between a, b & c is:

According formula of Case 2

$$AB = 2\sqrt{ab}$$

$$AC = 2\sqrt{ac} \text{ [It is DCT of (1) \& (3)]}$$

circles]

$$CB = 2\sqrt{bc} \text{ [It is DCT of (2) \& (3)]}$$

$$AB = AC + CB$$

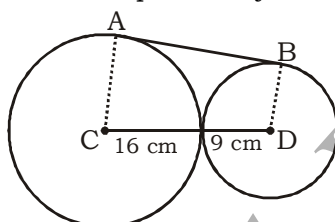
$$2\sqrt{ab} = 2\sqrt{ac} + 2\sqrt{bc}$$

$$\Rightarrow \frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}$$

Type IV

Ex-1. Find the length of the common tangent of two externally touch circle with radius 16 cm & 9 cm respectively.

Sol.



Length of common tangent =
 (Distance between two circle)² - (Radius₁ - Radius₂)²

$$AB^2 = CD^2 - (16 - 9)^2$$

$$AB^2 = (16 + 9)^2 - (7)^2$$

$$AB^2 = 625 - 49$$

$$AB^2 = 576$$

$$AB = 24\text{cm}$$

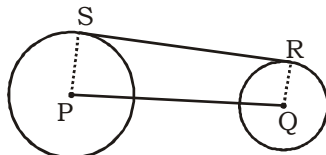
$$\text{Alternate} \rightarrow AB^2 = (R_1 + R_2)^2 - (R_1 - R_2)^2$$

$$\begin{aligned} AB &= 2\sqrt{R_1 R_2} = 2\sqrt{16 \times 9} \\ &= 2 \times 12 = 24\text{cm} \end{aligned}$$

Note: Use this formula, two circle intersect each other externally.

Ex-2. Find the length of the common tangent of two circle with radius 5 cm & 12 cm. Distance between two centre of the circle is 25 cm.

Sol.



$$RS^2 = \sqrt{PQ^2 - (R_1 - R_2)^2}$$

$$RS^2 = \sqrt{(25)^2 - (12 - 5)^2}$$

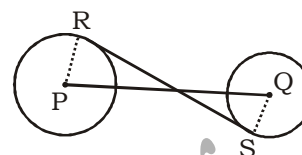
$$RS^2 = \sqrt{625 - 49}$$

$$RS^2 = 576$$

$$RS = 24\text{cm}$$

Ex-3. Find the length of common transversal tangent of two circle with radius 5 cm & 4 cm. Distance between two centre of the circle is 15 cm.

Sol.



$$RS^2 = \sqrt{(PQ)^2 - (R_1 + R_2)^2}$$

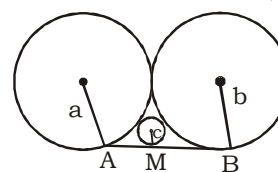
$$RS^2 = \sqrt{(15)^2 - (5 + 4)^2}$$

$$RS^2 = \sqrt{225 - 81}$$

$$RS^2 = 144$$

$$RS = 12\text{cm}$$

Ex-4. Two circle of radius a cm & b cm. Externally touch each other. AB is a direct common tangent of circles. A third circle with radius c cm externally touch two circle & direct common tangent. The relation between a, b & c is:



$$\Rightarrow AB = 2\sqrt{ab}$$

$$\Rightarrow AM = 2\sqrt{ac}$$

$$\Rightarrow BM = 2\sqrt{bc}$$

$$\Rightarrow AB = AM + MB$$

$$\Rightarrow 2\sqrt{ab} = 2\sqrt{ac} + 2\sqrt{bc}$$

$$\Rightarrow \sqrt{ab} = \sqrt{ac} + \sqrt{bc}$$

$$\text{Divide by } \sqrt{abc}$$

$$\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{a}}$$

Ex-5. If direct common tangent and line joining of centre of circle of extended and meet each other. Find the distance between external point & smaller circle & larger circle:

Sol. 'T' externally divides QS in ratio of radius two circles

$$\Rightarrow \frac{QT}{ST} = \frac{R}{r}$$

$$\Rightarrow \frac{QS}{ST} = \frac{R-r}{r}$$

$$\Rightarrow \text{QS} = \left(\frac{\text{R-r}}{\text{r}} \right) \text{ST}$$

$$ST = \left(\frac{r}{R-r} \right) QS \text{ [Smaller circle]}$$

$$\Rightarrow \frac{QT}{ST} = \frac{R}{r}$$

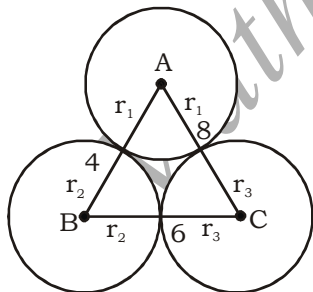
$$\Rightarrow \frac{QT}{QT-OS} = \frac{R}{r}$$

$$\Rightarrow 1 - \frac{QS}{QT} = \frac{r}{R}$$

$$\Rightarrow \frac{QS}{QT} = \left(\frac{R-r}{R} \right)$$

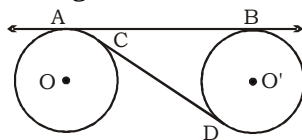
$$Q_T = \left(\frac{R}{R-r} \right) Q_S \quad \text{[Larger circle]}$$

Sol.

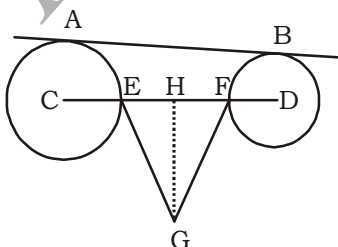


Solve the equations

Ex-7. Two circle having same radius 12cm have two common tangent AB and CD. Which touch the circle on A, C & B, D respectively and if $CD = 32$ cm. Find the length of AB.


$$\therefore AB = OO' = 40$$

Ex-8. In the given figure, $CE = 15$ cm, $FD = 8$ cm, $AB = 24$ cm, $EF = EG = GF$, Find $GH = ?$


$$D.C.T = \sqrt{\text{Distance b/w centres}^2 - \text{diff.b/w radii}^2}$$

$$AB^2=CD^2-\left[(CE)-(DF)\right]^2$$

$$(24)^2 = (\text{CD})^2 - [(15) - (8)]^2$$

$$CD^2 = 576 + 49$$

$$\Rightarrow CD = 25 \text{ cm}$$

$$= 2 \text{ cm}$$

$$EF = GF = EG = 2 \text{ cm.}$$

$$GH = \frac{\sqrt{3}}{2} EF = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3} \text{ cm}.$$

Ex-9. AB is a line segment of length 12 cm & M is the mid point. Two semicircle are drawn with AM, MB & AB is diameter on the same side of the line AB. A circle with centre 'O' and radius 'r' is drawn so that it touches all the three semicircle. Find the value of 'r' if L & N are midpoint of AM & MB respectively & the point O, P, L & the point O, Q, N & the point R, O, M are collinear?

$$\therefore r = 2$$

Ex-10. The radii of two concentric circle are 13 cm & 8 cm. AB is diameter of bigger circle. BD is tangent of smaller circle touching it at D. Find the length of AD?

$$\Rightarrow DB = \sqrt{105} = DE$$

In Δ BDO & Δ BEA,

$$\frac{OD}{AE} = \frac{BD}{BE}$$

$$\frac{8}{AE} = \frac{\sqrt{105}}{2\sqrt{105}}$$

$$\Rightarrow AE = 16 \text{ cm}$$

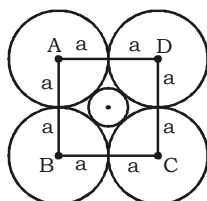
In $\triangle AED$,
 $AD^2 = AE^2 + ED^2$

$$AD^2 = (16)^2 + (\sqrt{105})^2$$

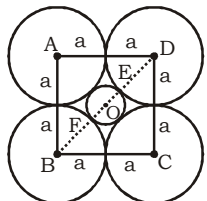
$$AD^2 = 256 + 105$$

$$\therefore AD = 19 \text{ cm}$$

Ex-11. In the given figure, radius of smaller circle is:



Sol.



In $\triangle BCD$,
 $BD^2 = CD^2 + BC^2$

$$\Rightarrow BD = 2\sqrt{2}a$$

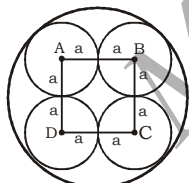
$$EF = 2\sqrt{2}a - a - a$$

$$EF = 2(\sqrt{2}a - a)$$

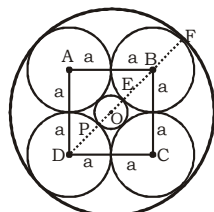
$$\therefore OE = \frac{1}{2} \times EF$$

$$\therefore OE = \frac{1}{2} \times [2a(\sqrt{2} - 1)] = (\sqrt{2} - 1)a$$

Ex-12. In the given figure, find the radius of bigger circle is



Sol. In $\triangle BCD$,



$$BD^2 = BC^2 + CD^2$$

$$\Rightarrow BD = 2\sqrt{2}a$$

$$\therefore OB = \frac{1}{2} \times BD$$

$$\therefore OB = \frac{1}{2} \times 2\sqrt{2}a = \sqrt{2}a$$

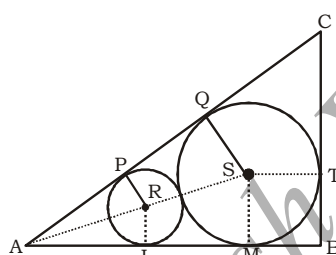
$$\therefore OF = OB + BF$$

$$OF = \sqrt{2}a + a$$

$$\therefore OF = (\sqrt{2} + 1)a$$

Ex-13. Two circles inscribed in a right angle triangle radius is 12 cm & 3 cm. Find length of AP.

Sol.



$$PQ = \sqrt{(RS)^2 - (QS - PR)^2}$$

$$PQ = \sqrt{(15)^2 - (12 - 3)^2}$$

$$PQ = 12 \text{ cm} = LM$$

$$\therefore \text{STBM is a square}$$

$$MB = BT = ST = SM = 12 \text{ cm.}$$

In $\triangle APR$ & $\triangle ASQ$,

$$\frac{AP}{AQ} = \frac{PR}{SQ}$$

$$\frac{AP}{AP + PQ} = \frac{3}{12}$$

$$\frac{AP}{AP + 12} = \frac{1}{4}$$

$$4AP = AP + 12$$

$$3AP = 12$$

$$\therefore AP = 4 \text{ cm.}$$

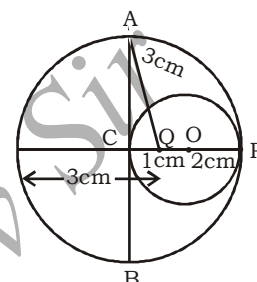
Ex-14. Two circles with R & r cm, one circle is inscribed in another circle. If shortest distance between the circles is S cm. Then the distance between the centres is:

Sol. $OA = R$
 $O'B = r$
 $AB = S$

$$OO' = OA - O'B - AB$$

$$\therefore OO' = R - r - S$$

Ex-15. Two circles internally connect each other. It's radius are 2 cm & 3 cm respectively. Find the maximum chord in the outer circle that Joint incircle.



Sol. In $\triangle ACQ$,

$$AQ^2 = AC^2 + CQ^2$$

$$3^2 = AC^2 + 1^2$$

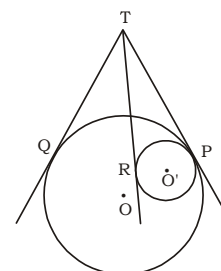
$$AC^2 = 8$$

$$AC = 2\sqrt{2}$$

$$\therefore AB = 2 \times AC$$

$$\Rightarrow AB = 2 \times 2\sqrt{2} = 4\sqrt{2} \text{ cm}$$

Ex-16. In the given figure, there are two circles with the centres O and O' touching each other internally at P. Tangents TQ and TP are drawn to the larger circle and tangents TP and TR are drawn to the smaller circle. Find TQ : TR



Sol. $TQ = TP$ (i) (tangent of larger circle)

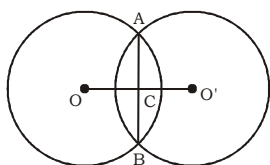
$TR = TP$ (ii) (tangent of smaller circle)

From (i) and (ii)

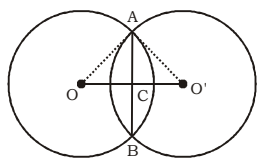
$TQ = TP = TR$ [Tangents of the circle]

Required ratio = $TQ : TR = 1 : 1$

Ex-17. O and O' the centres of circle of radii 20 cm and 37 cm. $AB = 24 \text{ cm}$. What is the distance OO'?



Sol. $AC = \frac{AB}{2} = 12 \text{ cm}$



$OA = 20 \text{ cm}, O'A = 37 \text{ cm}.$

In $\triangle OAC$,

$OA^2 = AC^2 + OC^2$

$\Rightarrow (20)^2 = (12)^2 + OC^2$

$\Rightarrow OC = 16 \text{ cm}$

In $\triangle ACO'$,

$(AO')^2 = (AC)^2 + (O'C)^2$

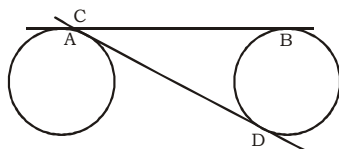
$\Rightarrow (37)^2 = (12)^2 + (O'C)^2$

$\Rightarrow O'C = 35 \text{ cm}$

$\therefore OO' = OC + O'C$

$= 16 + 35 = 51 \text{ cm}$

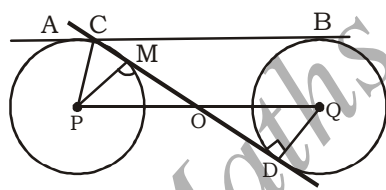
Ex-18. There are two circles each with radius 5 cm. Tangent AB is 26 cm. The length of tangent CD is:



Sol. $AB = PQ = 26 \text{ cm}$

$PO = OQ = 13 \text{ cm}$

$CO = \sqrt{(PO)^2 - (PC)^2}$



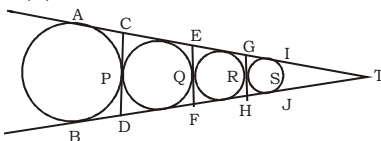
$CO = \sqrt{(13)^2 - (5)^2}$

$CO = 12 \text{ cm}$

$\therefore CD = 2 CO = 24 \text{ cm}$

Ex-19. In the adjoining figure AT and BT are the two tangents at A and B respectively CD is also a tangent at P. There are some more circles touching each other and the tangents AT and BT also. Which one of the following is true?

- (a) $PC + CT = PD + DT$
- (b) $AE + ET = BF + FT$
- (c) $AQ + ET = BQ + FT$
- (d) $FH + HT = EG + GT$



Sol. (c) $AT = BT$

($\because$ Tangents on the same circle from a fixed point is equal)

$AC = PC$ and $BD = PD$

$\therefore AT = BT$

$\Rightarrow AC + CT = BD + DT$

$PC + CT = PD + DT$

Similarly, all the relation

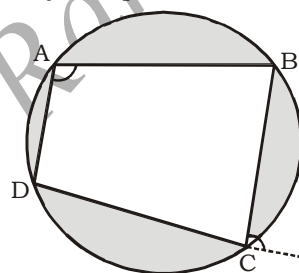
(i), (ii) & (iv) are true.

Cyclic Quadrilateral

If all vertices of a quadrilateral lie on the circle, it is called a cyclic quadrilateral.

Properties

- (i) The sum of opposite angles of a cyclic quadrilateral is 180° .



$\angle A + \angle C = \angle B + \angle D = 180^\circ$

- (ii) The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

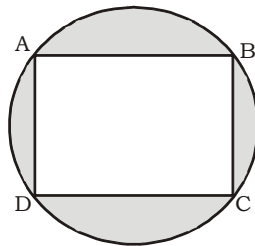
$\angle DAB = \angle BCE$

$\angle DAB + \angle BCD = 180^\circ$

& $\angle BCD + \angle BCE = 180^\circ$

$\therefore \angle DAB = \angle BCE$

- (iii) All angles of cyclic parallelogram is 90° .



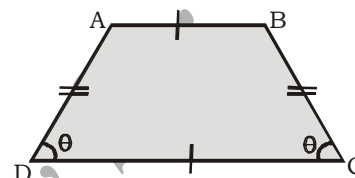
$\angle A + \angle C = 180^\circ$

but according to parallelogram property $\angle A = \angle C = \theta$

$\therefore 2\theta = 180 \Rightarrow \theta = 90^\circ$

then, $\angle D = \angle B = 90^\circ$

- (iv) If non parallel sides of a trapezium are equal then it will be a cyclic quadrilateral.



$\therefore AB \parallel DC$

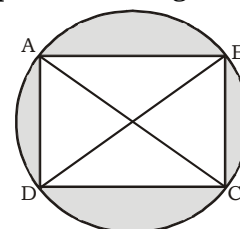
So, $\angle A = 180^\circ - \theta$ & $\angle B = 180^\circ - \theta$

If sum of opposite angles is 180°

($\angle A + \angle C = \angle B + \angle D = 180^\circ$)

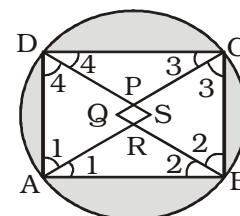
then it is cyclic quadrilateral.

- v. **Ptolemy's theorem** : In a cyclic quadrilateral the sum of products of the measures of the pairs of opposite sides is equal to its diagonals.



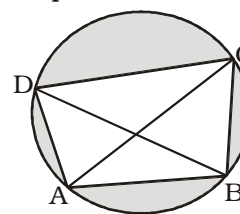
$AC \cdot BD = AB \cdot DC + AD \cdot BC$

- VI. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.



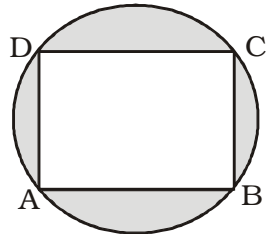
i.e. If ABCD is a cyclic quadrilateral, then $\square PQRS$ is also a cyclic.

- VII. If A cyclic trapezium is isosceles and its diagonals are equal

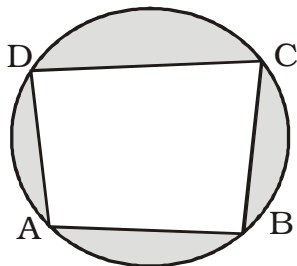


i.e. If ABCD is a cyclic trapezium s.t. $AB \parallel DC$, then $AD = BC$ and $AC = BD$.

- VIII.** If two opposite sides of a cyclic quadrilateral are equal, then the other two sides are parallel.

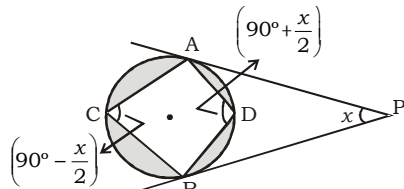


i.e. If $AD = BC$, then $AB \parallel CD$.
IX. An isosceles trapezium is always cyclic.



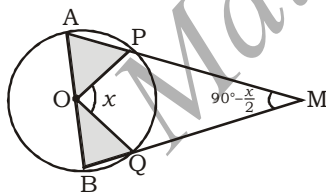
i.e. If $AB \parallel DC$ and $AD = BC$. Then, ABCD is a cyclic trapezium.

X.



- XI.** If AB is a diameter of the circle (centre O) and APM & BQM are its two secants. If $\angle POQ = x$ then

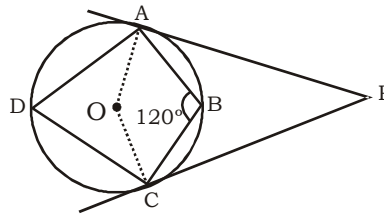
$$\angle PMQ = 90^\circ - \frac{x}{2}$$



Type -VI

- Ex-1.** ABCD is a concyclic quadrilateral. The tangents at A & C intersect each other at P. If $\angle ABC = 120^\circ$, then what is $\angle APC$?

Sol.



$\angle ABC + \angle ADC = 180^\circ$ ($\because$ ABCD is a cyclic quadrilateral)

$$\therefore \angle ADC = 180^\circ - 120^\circ = 60^\circ$$

$\angle AOC = 2 \times \angle ADC$ ($\because$ Centre angle is double at the angle of major arc)

$$\angle AOC = 2 \times 60^\circ = 120^\circ$$

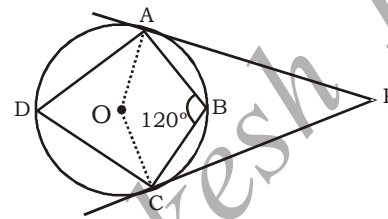
In $\square$ AOCP,

$$\angle A + \angle O + \angle C + \angle P = 360^\circ$$

$$90^\circ + 120^\circ + 90^\circ + \angle P = 360^\circ$$

$$\therefore \angle P = 60^\circ$$

Alternate



$\angle ADC = 180^\circ - \angle ABC$
($\because$ ABCD is a cyclic quadrilateral)
 $= 180 - 120^\circ = 60^\circ$

$\angle CAP = \angle ACP = \angle ADC = 60^\circ$
[Alternate segments]

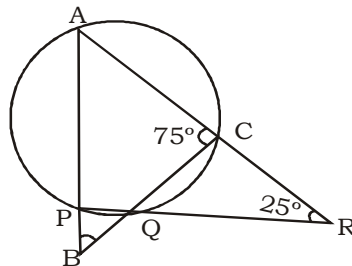
In $\triangle$ APC,

$$\angle A + \angle P + \angle C = 180^\circ$$

$$60^\circ + \angle P + 60^\circ = 180^\circ$$

$$\angle P = 60^\circ$$

Ex-2.



In the given figure, find $\angle CBA$ = ?

- Sol.** $\angle APQ + \angle ACQ = 180^\circ$ ($\because$ APQC is a cyclic quadrilateral)

$$\angle APQ = 180^\circ - 75^\circ$$

$$\therefore \angle APQ = 105^\circ$$

$$\angle BPQ = 180^\circ - 105^\circ = 75^\circ$$

$\angle ACQ = \angle CQR + \angle CRQ$ [Sum of Interior angle]

$$75^\circ = \angle CQR + 25^\circ$$

$\angle CQR = 50^\circ = \angle PQB$ [Vertically opposite angle]

In $\triangle$ PBQ,

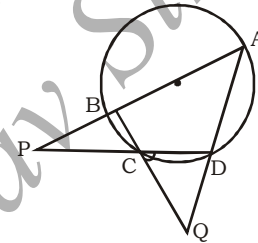
$$\angle P + \angle B + \angle Q = 180^\circ$$

$$75^\circ + \angle B + 50^\circ = 180^\circ$$

$$\angle PBQ = 55^\circ$$

$$\therefore \angle CBA = 55^\circ$$

Ex-3.

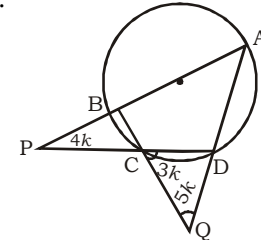


In the given figure, if

$$\frac{x}{3} = \frac{y}{4} = \frac{z}{5}, \text{ where } \angle DCQ = x,$$

$\angle BPC = y$ and $\angle DQC = z$, then what are the values of x , y and z respectively?

Sol.



$$\frac{x}{3} = \frac{y}{4} = \frac{z}{5} = k \text{ (say)}$$

$$x = 3k, y = 4k, z = 5k$$

In $\triangle$ CDQ, $\angle CDQ = 180^\circ - (3k + 5k)$
 $= 180^\circ - 8k$

$\angle ABC = \angle CDQ$ (exterior angle of cyclic quadrilateral equal to opposite interior angle)
 $= 180^\circ - 8k$

In $\triangle$ PBC,

$\angle BCP = \angle DCQ = 3k$ (vertically opposite angle)

$$\angle ABC = \angle P + \angle C$$

$$180^\circ - 8k = 4k + 3k \Rightarrow k$$

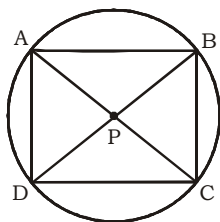
$$= \frac{180^\circ}{15} = 12^\circ$$

$$\therefore x = 36^\circ, y = 48^\circ, z = 60^\circ$$

Ex-4. The diagonals AC and BD of a cyclic quadrilateral ABCD intersect each other at the point P.

AP = 3, CP = 6, DP = 9 and find BP = ?

Sol.



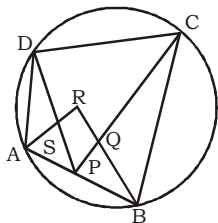
By property of chords of circle
Then, it is always true that:

$$AP \times PC = BP \times PD$$

$$3 \times 6 = BP \times 9$$

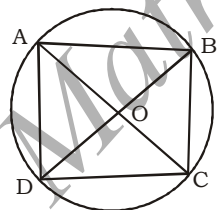
$$BP = 2$$

Ex-5. ABCD is a cyclic quadrilateral. The angle bisector of A, B, C and D intersect at P, Q, R and S as shown in the figure. These four points form a quadrilateral PQRS. Quadrilateral PQRS is a:



Sol. PQRS is the rectangle

Ex-6. In the given figure ABCD is a cyclic quadrilateral DO = 8 cm and CO = 4 cm. AC is the angle bisector of BAD. The length of AD is equal to the length of AB. DB intersects diagonal AC at O, then what is the length of the diagonal AC?



Sol.

$$\frac{AD}{AB} = \frac{DO}{BO} = 1$$

$$OB = OD = 8 \text{ cm}$$

Also, $\triangle ADB$ is isosceles $\triangle$, and Ao is the angle bisector

$\therefore$ AO will also be the median

$\therefore$ ABCD is a cyclic quadrilateral

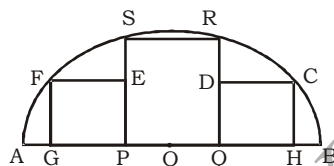
$$\therefore DO \times BO = CO \times AO$$

$$8 \times 8 = 4 \times AO$$

$$\Rightarrow AO = 16 \text{ cm}$$

$$\therefore AC = 16 + 4 = 20 \text{ cm}$$

Ex-7. In the following figure there is semicircle with centre 'O' and diameter AB ($= 2r$). PQRS is a square of maximum possible area. P and Q lie on the diameter AB and R, S lie on the arc of semicircle, there are two more squares of maximum possible area EFGP and CDQH. What is the sum of lengths of RC and FS?



Sol. In $\triangle ORQ$,

$$\Rightarrow OR^2 = (OQ)^2 + (RQ)^2$$

$$\Rightarrow OR^2 = 5OQ^2 \quad (\because RQ = 2(OQ))$$

$$\text{radius } (r) = OQ \sqrt{5}$$

Again,

In $\triangle OCH$,

$$OC^2 = OH^2 + HC^2$$

$$r^2 = (OQ + QH)^2 + (QH)^2$$

$$(\because HQ = HC)$$

$$\Rightarrow r^2 = \left(\frac{r}{\sqrt{5}} + QH \right)^2 + (QH)^2$$

$$\Rightarrow QH = \frac{r}{\sqrt{5}}$$

$$\therefore HC = \frac{r}{\sqrt{5}} = \frac{RQ}{2}$$

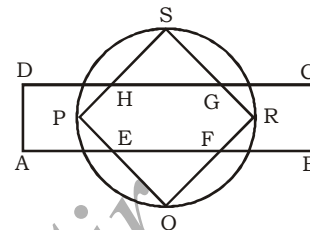
$$RC = \sqrt{(RD)^2 + (DC)^2}$$

$$= \sqrt{\left(\frac{r}{\sqrt{5}} \right)^2 + \left(\frac{r}{\sqrt{5}} \right)^2} = r \sqrt{\frac{2}{5}}$$

$$\therefore RC + FS = 2r \sqrt{\frac{2}{5}}$$

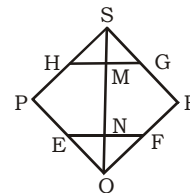
Ex-8. In the adjoining figure ABCD is a rectangle in which length is twice of breadth. H and G divide the line CD into three equal parts. Similarly points E and F trisect the line AB. A circle PQRS is circumscribed a square PQRS

which passes through the points E, F, G and H. What is the ratio of areas of circle to that of rectangle?



Sol. Let AD = 3a and DC = 6a

$$DH = HG = GC = \frac{6a}{3} = 2a$$



$$HM = MG = \frac{2a}{2} = a = SM$$

$$NQ = a$$

$$SQ = SM + MN + NQ$$

$$= a + 3a + a = 5a$$

Since diagonal of square SQ = 5a

But, diameter of circle SQ = diagonal of square SQ

$$\text{Radius of the circle} = \frac{5a}{2}$$

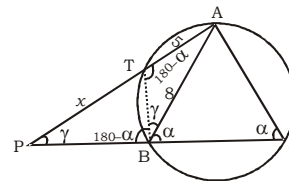
$$\text{Area of the circle} = \pi \times \left(\frac{5a}{2} \right)^2$$

Here ,

$$\frac{\text{Area of circle}}{\text{Area of rectangle}} = \frac{\frac{25}{4}(a^2\pi)}{3a \times 6a} = \frac{25\pi}{72}$$

Ex-9. There are two chords AB and AC of equal length 8 cm. CB is produced to P. AP cuts circle at T such that AT = 5 cm. Find length of PT.

Sol.



$$\therefore AB = AC$$

$$\Rightarrow \angle ABC = \angle ACB = \alpha \text{ (Let)}$$

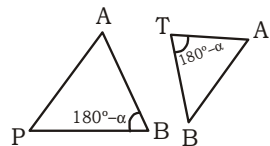
$$\angle ABP = 180^\circ - \angle ABC = 180^\circ - \alpha$$

$\therefore$ ATBC is a cyclic quadrilateral

$$\Rightarrow \angle ATB + \angle ACB = 180^\circ$$

$$\Rightarrow \angle ATB = 180^\circ - \alpha$$

Now,



In $\triangle ABP$ and $\triangle BTA$

$$\angle ABP = \angle BTA = 180^\circ - \alpha$$

$$\angle BAP = \angle BAT \text{ (common)}$$

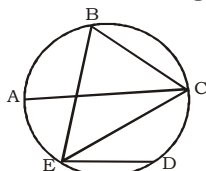
$$\angle BPA = \angle TBA = \gamma \text{ (Let)}$$

$$\therefore \triangle ABP \sim \triangle BTA$$

$$\frac{AP}{AB} = \frac{AB}{AT} = \frac{x+5}{8} = \frac{8}{5} = x = \frac{39}{5}$$

$$PT = \frac{39}{5}$$

Ex-10. In the figure, chord ED is parallel to the diameter AC of the circle. If angle CBE = 65° , then, what is the value of angle DEC.



Sol. $\angle ABC = 90^\circ$

$$\angle ABE = \angle ABC - \angle CBE$$

$$\angle ABE = 90^\circ - 65^\circ = 25^\circ,$$

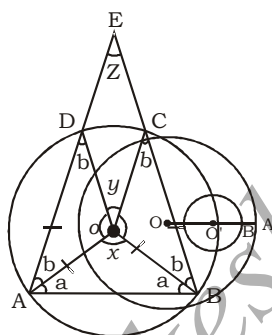
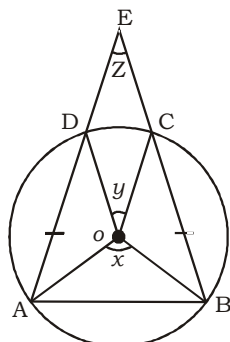
$$\angle ABE = \angle ACE = 25^\circ$$

$$\angle ACE = \angle CED = 25^\circ$$

(alternate angles)

Ex-11. In the given figure, $AD = CB$,

$$\text{find } \frac{x-y}{z}$$



Sol.

$$\angle OAB = \angle OBA = a$$

[$\because OA = OB = \text{radius}$]

In $\triangle ODA$ and $\triangle OCB$,

$$AD = CB \text{ (given)}$$

$$OA = OB \text{ (radius)}$$

$$\triangle ODA \sim \triangle OCB \text{ [by S \& S]}$$

$$\angle ADO = \angle DAO = \angle OCB =$$

$$\angle OBC = b \text{ [by C.P.C.T]}$$

$$\angle DOA = \angle BOC = 180^\circ - 2b$$

$$\angle AOB = x = 180^\circ - 2a$$

$$\angle DOC = y = 360^\circ - [180^\circ - 2b + 180^\circ - 2b + 180^\circ - 2a]$$

$$y = 360^\circ - 180^\circ + 2b - 180^\circ + 2b - 180^\circ + 2a$$

$$y = 4b + 2a - 180^\circ \dots (i)$$

both sides subtract by x

$$x - y = 180^\circ - 2a - [4b + 2a - 180^\circ]$$

$$x - y = 180^\circ - 2a - 4b - 2a + 180^\circ$$

$$x - y = 360^\circ - 4a - 4b$$

$$\angle ODE = \angle OCE = 180^\circ - b$$

In $\square CODE$,

$$\angle C + \angle O + \angle D + \angle E = 360^\circ$$

$$180^\circ - b + 4b + 2a - 180^\circ + 180^\circ$$

$$- b + \angle E = 360^\circ$$

$$\angle E = Z = 180^\circ - 2a - 2b$$

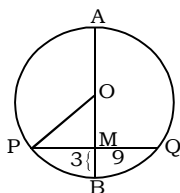
$$\therefore \frac{x-y}{z} = \frac{360^\circ - 4a - 4b}{180^\circ - 2a - 2b} = \frac{2}{1}$$

EXERCISE

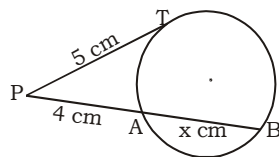
- The length of tangent drawn from an external point P to a circle of radius 5 cm. is 12 cm. The distance of P from the centre of the circle is:
(a) 12 cm. (b) 9 cm.
(c) 7 cm. (d) 13 cm.
- The centres of three circles of equal radii, touching each other externally form a/an
(a) Scalene triangle
(b) Equilateral triangle
(c) Isosceles triangle
(d) Right angle triangle
- The length of two parallel chords of a circle of radius 5 cm are 6 cm and 8 cm in the same side of the centre. The distance between them is
(a) 1 cm (b) 2 cm
(c) 3 cm (d) 1.5 cm
- AB is a diameter of a circle having centre at O. P is a point on the circumference of the circle. If $\angle POA = 120^\circ$, then measure of $\angle PBO$ is
(a) 75° (b) 60° (c) 68° (d) 70°
- AB is the diameter of a circle with centre O. P be a point on it. If $\angle POA = 120^\circ$. Then, $\angle PBO = ?$
(a) 60° (b) 50° (c) 120° (d) 45°
- AB and AC are tangents to a circle with centre O. A is the external point of the circle. The line AO intersect the chord BC at D. The measure of the $\angle BDO$ is:
(a) 45° (b) 75° (c) 90° (d) 60° .
- In a circle with centre at O and radius 5 cm, AB is a chord of length 8 cm. If OM is perpendicular to AB, then the length of OM is:
(a) 3 cm (b) 4 cm
(c) 1 cm (d) 2.5 cm
- AB is a diameter of a circle having centre at O. PQ is a chord which does not intersect AB. Join AP and BQ. If $\angle PAB = \angle ABQ$, then ABQP is a:
(a) Cyclic rhombus
(b) Cyclic rectangle
(c) Cyclic trapezium
(d) Cyclic square
- The distance between centres of two circles of radii 3 cm and

8 cm is 13 cm. If the points of contact of a direct common tangent the circles are P and Q, then the length of the line segment PQ is:

- 11.9 cm (b) 12 cm
 - 11.5 cm (d) 11.58 cm
- Two circles of radii 5 cm and 3 cm touch externally, then the ratio in which the direct common tangent to the circles divides externally the line joining the centres of the circles is:
(a) 5 : 3 (b) 3 : 5
(c) 1.5 : 2.5 (d) 2.5 : 1.5
 - The diameter of a circle is 10 cm. If the distance of a chord from the centre of the circle be 4 cm, then the length of chord:
(a) 5 cm. (b) 6 cm.
(c) 4 cm. (d) 3 cm.
 - Two circles of radii 10 cm and 8 cm intersect and the length of the common chord is 12 cm. Then the distance between their centres is :
(a) 13.3 (b) 15 (c) 10 (d) 8
 - In a given circle, the chord PQ is of length 18 cm. AB is the perpendicular bisector of PQ at M. If MB = 3. find the length of AB



- 25 cm (b) 30 cm
 - 28 cm (d) 27 cm
- The chord of a circle is equal to its radius. The angle subtended by this chord at the minor arc of the circle is
(a) 150° (b) 60° (c) 75° (d) 120°
 - In the given figure, PAB is a secant and PT is a tangent to the circle from P. If PT = 5 cm, PA = 4 cm and AB = x cm, then x is



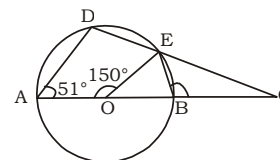
- 4/9 cm (b) 2/3 cm
- 9/4 cm (d) 5 cm

- Two circles with their centres at O and P and radii 8 cm and 4 cm respectively touch each other externally. The length of their common tangent is
(a) 8 cm (b) 8.5 cm
(c) $8\sqrt{2}$ cm (d) $8\sqrt{3}$ cm
- Two circles of diameters 10 cm and 6 cm have the same centre. A chord of the larger circle is a tangent of the smaller one. The length of the chord is
(a) 8 cm (b) 10 cm
(c) 6 cm (d) 4 cm
- A and B are centres of two circles of radii 11 cm and 6 cm, respectively, PQ is a direct common tangent to the circle. If $\overline{AB} = 13$ cm, then length of $\overline{PQ}$ will be
(a) 12 cm (b) 13 cm
(c) 8.5 cm (d) 17 cm
- The radii of two concentric circles are 17 cm and 25 cm. a straight line PQRS intersects the larger circle at the points P and S and intersects the smaller circle at the points Q and R. If QR = 16 cm, then the length (in cm.) of PS is
(a) 41 (b) 33 (c) 32 (d) 40
- AB is a diameter of a circle with centre O. The tangents at C meets AB produced at Q. If $\angle CAB = 34^\circ$, then measure of $\angle CBA$ is
(a) 56° (b) 68° (c) 34° (d) 124°
- Let P and Q be two points on a circle with centre O. If two tangents of the circle through P and Q meet at A with $\angle PAQ = 48^\circ$, then $\angle APQ$ is
(a) 96° (b) 66° (c) 48° (d) 60°
- The radius of a circle is 6 cm. The distance of a point lying outside the circle from the centre is 10 cm. The length of the tangent drawn from the outside point to the circle is
(a) 5 cm (b) 6 cm
(c) 7 cm (d) 8 cm
- DE is a tangent to the circum-circle of $\triangle ABC$ at the vertex A such that $DE \parallel BC$. If AB = 17

- cm, then the length of AC is equal to
 (a) 16.0 cm (b) 16.8 cm
 (c) 17.3 cm (d) 17 cm
24. ST is a tangent to the circle at P and QR is a diameter of the circle. If $\angle RPT = 50^\circ$, then the value of $\angle SPQ$ is
 (a) 40° (b) 60° (c) 80° (d) 100°
25. If PA and PB are two tangents to a circle with centre O such that $\angle AOB = 110^\circ$, then $\angle APB$ is
 (a) 90° (b) 70° (c) 60° (d) 55°
26. AC is a transverse common tangent to two circle with centres P and Q and radii 6 cm and 3 cm at the point A and C respectively. If AC cuts PQ at the point B and AB = 8 cm, then the length of PQ is:
 (a) 12 cm (b) 15 cm
 (c) 13 cm (d) 10 cm
27. AB and CD are two parallel chords of a circle lying on the opposite side of the centre and the distance between them is 17 cm. The length of AB and CD are 10 cm and 24 cm respectively. The radius(in cm) of the circle is:
 (a) 13 (b) 18 (c) 9 (d) 15
28. A tangent is drawn to a circle of radius 6 cm from a point situated at a distance of 10 cm from the centre of the circle. The length of tangent will be (a) 4 cm (b) 5 cm
 (c) 8 cm (d) 7 cm
29. Two chords of length a unit and b unit of a circle make angles 60° and 90° at the centre of a circle respectively, then the correct relation is:
 (a) $b = \sqrt{2} a$ (b) $b = 2a$
 (c) $b = \sqrt{3} a$ (d) $b = 3/2a$
30. Two circles touch externally. The sum of their areas is 130π sq cm and the distance between their centres is 14 cm. The radius of the smaller circle is:
 (a) 2cm (b) 3 cm
 (c) 4 cm (d) 5 cm
31. XY and XZ are tangents to a circle. ST is another tangent to the circle at the point R on the circle which intersects XY and XZ at S and T respectively, If XY = 9 cm and TX = 15 cm, then RT is :
 (a) 4.5 cm (b) 3 cm
 (c) 7.5 cm (d) 6 cm
32. A circle touches the four sides of a quadrilateral ABCD. The value of $\frac{(AB+CD)}{CB+DA}$ is equal to:
 (a) $\frac{1}{3}$ (b) 1 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$
33. Two chords AB and CD of a circle with centre O, intersect each other at P. If $\angle AOD = 100^\circ$ and $\angle BOC = 70^\circ$, then the value of $\angle APC$ is
 (a) 80° (b) 75° (c) 85° (d) 95°
34. Chords AC and BD of a circle with centre O intersect at right angles at E. If $\angle OAB = 25^\circ$, then the value of $\angle EBC$ is
 (a) 30° (b) 25° (c) 20° (d) 15°
35. Two circles touch externally at P. QR is a common tangent of the circles touching the circles at Q and R. Then measure of $\angle QPR$ is
 (a) 120° (b) 60° (c) 90° (d) 45°
36. Two circles intersect each other at the points A and B. A straight line parallel to AB intersects the circles at C, D, E and F. If CD = 4.5 cm, then the measure of EF is
 (a) 1.50 cm (b) 2.25 cm
 (c) 4.50 cm (d) 9.00 cm
37. Two circles C_1 and C_2 touch each other internally at P. Two lines PCA and PDB meet the circles C_1 in C, D and C_2 in A, B respectively. If $\angle BDC = 120^\circ$, then the value of $\angle ABP$ is equal to
 (a) 60° (b) 80°
 (c) 100° (d) 120°
38. Two circles having radii r units intersect each other in such a way that each of them passes through the centre of the other. Then the length of their common chord is
 (a) $\sqrt{2}r$ units (b) $\sqrt{3}r$ units
 (c) $\sqrt{5}r$ units (d) r units
39. Two circles with centres A and B of radii 5 cm and 3 cm respectively touch each other internally. If the perpendicular bisector of AB meets the bigger circle in P and Q, then the value of PQ is
 (a) $\sqrt{6}$ cm (b) $2\sqrt{6}$ cm
 (c) $3\sqrt{6}$ cm (d) $4\sqrt{6}$ cm
40. The length of a tangent from an external point to a circle is $5\sqrt{3}$ unit. If radius of the circle is 5 units, then the distance of the point from the circle is
 (a) 5 units (b) 15 units
 (c) -5 units (d) -15 units
41. Two circles are of radii 7 cm and 2 cm their centres being 13cm apart. Then the length of direct common tangent to the circles between the points of contact is
 (a) 12 cm (b) 15 cm
 (c) 10 cm (d) 5 cm
42. The length of chord of a circle is equal to the radius of the circle. The angle which this chord subtends in the major segment of the circle is equal to
 (a) 30° (b) 45° (c) 60° (d) 90°
43. AB = 8 cm and CD = 6 cm are two parallel chords on the same side of the centre of a circle. The distance between them is 1 cm. The radius of the circle is
 (a) 5 cm (b) 4 cm
 (c) 3 cm (d) 2 cm
44. The length of two chords AB and AC of a circle are 8 cm and 6 cm and $\angle BAC = 90^\circ$, then the radius of circle is
 (a) 25 cm (b) 20 cm
 (c) 4 cm (d) 5 cm
45. The distance between two parallel chords of length 8 cm each in a circle of diameter 10 cm is
 (a) 6 cm (b) 7cm
 (c) 8 cm (d) 5.5 cm
46. The radius of two concentric circles are 9 cm and 15 cm. If the chord of the greater circle be a tangent to the smaller circle, then the length of that chord is
 (a) 24 cm (b) 12 cm
 (c) 30 cm (d) 18 cm
47. If chord of a circle of radius 5 cm is a tangent to another circle of radius 3 cm, both the circles being concentric, then the length of the chord is

- (a) 10 cm (b) 12.5 cm
(c) 8 cm (d) 7 cm
48. The two tangents are drawn at the extremities of diameter AB of a circle with centre P. If a tangent to the circle at the point C intersects the other two tangents at Q and R, then the measure of the $\angle QPR$ is
(a) 45° (b) 60°
(c) 90° (d) 180°
49. AB is a chord to a circle and PAT is the tangent to the circle at A. If $\angle BAT = 75^\circ$ and $\angle BAC = 45^\circ$ and C being a point on the circle, then $\angle ABC$ is equal to
(a) 40° (b) 45° (c) 60° (d) 70°
50. Two circles touch each other externally at point A and PQ is a direct common tangent which touches the circles at P and Q respectively. Then $\angle PAQ = ?$
(a) 45° (b) 90° (c) 80° (d) 100°
51. PR is tangent to a circle, with centre O and radius 4 cm, at point Q. If $\angle POR = 90^\circ$, OR = 5 cm and $OP = \frac{20}{3}$ cm, then (in cm) the length of PR is :
(a) 3 (b) $\frac{16}{3}$ (c) $\frac{23}{3}$ (d) $\frac{25}{3}$
52. Two chords AB and CD of circle whose centre is O, meet at the point P and $\angle AOC = 50^\circ$, $\angle BOD = 40^\circ$. Then the value of $\angle BPD$ is
(a) 60° (b) 40° (c) 45° (d) 75°
53. Two equal circles of radius 4 cm intersect each other such that each passes through the centre of the other. The length of the common chord is
(a) $2\sqrt{3}$ cm (b) $4\sqrt{3}$ cm
(c) $2\sqrt{2}$ cm (d) 8 cm
54. One chord of a circle is known to be 10.1 cm. The radius of this circle must be ;
(a) 5 cm
(b) greater than 5 cm
(c) greater than or equal to 5 cm
(d) less than 5 cm
55. The length of the chord of a circle is 8 cm and perpendicular distance between centre and the chord is 3 cm. Then the radius of the circle is equal to :
(a) 4 cm (b) 5 cm
(c) 6 cm (d) 8 cm
56. The length of the common chord of two intersecting circles is 24 cm. If the diameter of the circles are 30 cm and 26 cm, then the distance between the centre (in cm) is
(a) 13 (b) 14 (c) 15 (d) 16
57. In a circle of radius 21 cm an arc subtends an angle of 72° at the centre. The length of the arc is
(a) 21.6 cm (b) 26.4 cm
(d) 13.2 cm (d) 198.8 cm
58. A unique circle can always be drawn through x number of given non-collinear points, then x must be
(a) 2 (b) 3 (c) 4 (d) 1
59. Two parallel chords are drawn in a circle of diameter 30 cm. The length of one chord is 24 cm and the distance between the two chords is 21 cm. The length of the other chord is
(a) 10 cm (b) 18 cm
(c) 12 cm (d) 16 cm
60. If two equal circles whose centres are O and O' intersect each other at the point A and B, $OO' = 12$ cm and AB = 16 cm, then the radius of the circle is
(a) 10 cm (b) 8 cm
(c) 12 cm (d) 14 cm
61. Chords AB and CD of a circle intersect externally at P. If AB = 6 cm, CD = 3 cm and PD = 5 cm, then the length of PB is
(a) 5 cm (b) 7.35 cm
(c) 6 cm (d) 4 cm
62. AB and CD are two parallel chords on the opposite sides of the centre of the circle. If AB = 10 cm, CD = 24 cm and the radius of the circle is 13 cm, the distance between the chords is
(a) 17 cm (b) 15 cm
(c) 16 cm (d) 18 cm
63. Two circles touch each other externally at P. AB is a direct common tangent to the two circles, A and B are point of contact and $\angle PAB = 35^\circ$. Then $\angle ABP$ is
(a) 35° (b) 55° (c) 65° (d) 75°
64. If the radii of two circles be 6 cm and 3 cm and the length the transverse common tangent be 8 cm, then the distance between the two centres is
(a) $\sqrt{145}$ cm (b) $\sqrt{140}$ cm
(c) $\sqrt{150}$ cm (d) $\sqrt{135}$ cm
65. The distance between the centre of two equal circles each of radius 3 cm, is 10 cm. The length of a transverse common tangent is
(a) 8 cm (b) 10 cm
(c) 4 cm (d) 6 cm
66. The radii of two circles are 5 cm and 3 cm, the distance between their centre is 24 cm. Then the length of the transverse common tangent is
(a) 16 cm (b) $15\sqrt{2}$
(c) $16\sqrt{2}$ (d) 15 cm
67. P and Q are centre of two circles with radii 9 cm and 2 cm respectively, where PQ = 17 cm. R is the centre of another circle of radius x cm, which touches each of the above two circles externally. If $\angle PRQ = 90^\circ$, then the value of x is
(a) 4 cm (b) 6 cm
(c) 7 cm (d) 8 cm
68. Two chords AB and CD of a circle with centre O intersect each other at the point P. If $\angle AOD = 20^\circ$ and $\angle BOC = 30^\circ$, then $\angle BPC$ is equal to :
(a) 50° (b) 20° (c) 25° (d) 30°
69. AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm. If the chords are on the opposite sides of the centre and distance between them is 17 cm, then the radius of the circle is :
(a) 11 cm (b) 12 cm
(c) 13 cm (d) 10 cm
70. A chord AB of a circle C_1 of radius $(\sqrt{3}+1)$ cm touches a circle C_2 which is concentric to C_1 . If the radius of C_2 is $(\sqrt{3}-1)$ cm. The length of AB is :
(a) $2\sqrt{3}$ cm (b) $8\sqrt{3}$ cm
(c) $4\sqrt{3}$ cm (d) $4\sqrt{3}$ cm
71. The length of the common chord of two circles of radii 30 cm and 40 cm whose centres are 50 cm apart is (in cm)
(a) 12 (b) 24 (c) 36 (d) 48

72. Chords AB and CD of a circle intersect at E and are perpendicular to each other. Segments AE, EB and ED are of lengths 2 cm, 6 cm and 3 cm respectively. Then the length of the diameter of the circle (in cm) is
 (a) $\sqrt{65}$ (b) $\frac{1}{2}\sqrt{65}$
 (c) 65 (d) $\frac{65}{2}$
73. Two circles of same radius 5 cm, intersect each other at A and B. If AB = 8 cm, then the distance between the centre is ;
 (a) 6 cm (b) 8 cm
 (c) 10 cm (d) 4 cm
74. AD is the chord of a circle with centre O and DOC is a line segment originating from a point D on the circle and intersecting AB produced at C such that BC = OD. If $\angle BCD = 20^\circ$, then $\angle AOD = ?$
 (a) 20° (b) 30° (c) 40° (d) 60°
75. In a circle of radius 17 cm, two parallel chords of length 30 cm and 16 cm are drawn. If both chords are on the same side of the centre, then the distance between the chords is
 (a) 9 cm (b) 7 cm
 (c) 23 cm (d) 11 cm
76. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the greater circle which is outside the inner circle is of length
 (a) $2\sqrt{2}$ cm (b) $3\sqrt{2}$ cm
 (c) $2\sqrt{3}$ cm (d) $4\sqrt{2}$ cm
77. Two circles touch each other externally. The distance between their centre is 7 cm. If the radius of one circle is 4 cm, then the radius of the other circle is
 (a) 3.5 cm (b) 3 cm
 (c) 4 cm (d) 2 cm
78. A, B and C are the three points on a circle such that the angles subtended by the chords AB and AC at the centre O are 90° and 110° respectively. $\angle BAC$ is equal to
 (a) 70° (b) 80° (c) 90° (d) 100°
79. N is the foot of the perpendicular from a point P of a circle with radius 7 cm, on a diameter AB of the circle. If the length of the chord PB is 12 cm, the distance of the point N from the point B is
 (a) $6\frac{5}{7}$ cm (b) $12\frac{2}{7}$ cm
 (c) $3\frac{5}{7}$ cm (d) $10\frac{2}{7}$ cm
80. A, B, C, D are four points on a circle, AC and BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$. $\angle BAC$ is
 (a) 120° (b) 90° (c) 100° (d) 110°
81. If two concentric circles are of radii 5 cm and 3 cm, then the length of the chord of the larger circle which touches the smaller circle is :
 (a) 6 cm (b) 7 cm
 (c) 10 cm (d) 8 cm
82. A chord 12 cm long is drawn in a circle of diameter 20 cm. The distance of the chord from the centre is
 (a) 8 cm (b) 6 cm
 (c) 10 cm (d) 16 cm
83. If the chord of a circle is equal to the radius of the circle, then the angle subtended by the chord on centre is
 (a) 150° (b) 60° (c) 120° (d) 30°
84. In a right angled triangle, the circumcentre of the triangle lies
 (a) inside the triangle
 (b) outside the triangle
 (c) on midpoint of the hypotenuse
 (d) on one vertex
85. P and Q are two points on a circle with centre at O. R is a point on the minor arc of the circle, between the points P and Q. The tangents to the circle at the points P and Q meet each other at the point S. If $\angle PSQ = 20^\circ$, then $\angle PRQ = ?$
 (a) 80° (b) 200°
 (c) 160° (d) 100°
86. Two circles intersect at A and B, P is a point on produced BA. PT and PQ are tangents to the circles. The relation of PT and PQ is
 (a) $PT = 2PQ$ (b) $PT < PQ$
 (c) $PT > PQ$ (d) $PT = PQ$
87. The length of the tangent drawn to a circle of radius 4 cm from a point 5 cm away from the centre of the circle is
 (a) 3 cm (b) $4\sqrt{2}$ cm
 (c) $5\sqrt{2}$ cm (d) $3\sqrt{2}$ cm
88. From a point P, two tangents PA and PB are drawn to a circle with centre O. If OP is equal to diameter of the circle, then $\angle APB$ is
 (a) 45° (b) 90° (c) 30° (d) 60°
89. The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle and BD is a tangent to the smaller circle touching it at D and the bigger circle at E. Point A is joined to D. The length of AD is
 (a) 20 cm (b) 19 cm
 (c) 18 cm (d) 17 cm
90. PQ is a chord of length 8 cm of a circle with centre O and radius 5 cm. The tangents at P and Q intersect at a point T. The length of TP is
 (a) $\frac{20}{3}$ cm (b) $\frac{21}{4}$ cm
 (c) $\frac{10}{3}$ cm (d) $\frac{15}{4}$ cm
91. The maximum number of common tangents drawn to two circles when both the circles touch each other externally is
 (a) 1 (b) 2 (c) 3 (d) 0
92. The radius of two concentric circles are 17 cm and 10 cm. A straight line ABCD intersects the larger circle at the point A and D and intersects the smaller circle at the points B and C. If BC = 12 cm, then the length of AD (in cm) is
 (a) 20 (b) 24 (c) 30 (d) 34
93. Two chords AB, CD of a circle with centre O intersect each other at P. $\angle ADP = 23^\circ$ and $\angle APC = 70^\circ$, then the $\angle BCD$ is
 (a) 45° (b) 47° (c) 57° (d) 67°
94. In the following figure, AB is the diameter of a circle whose centre is O. If $\angle AOE = 150^\circ$, $\angle DAO = 51^\circ$ then the measure of $\angle CBE$ is :



- (a) 115° (b) 110°
 (c) 105° (d) 120°

95. The angle in a semi-circle is
 (a) a reflex angle
 (b) an obtuse angle
 (c) an acute angle
 (d) a right angle

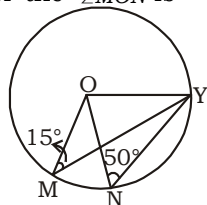
96. The angle subtended by a chord at its centre is 60° , then ratio between chord and radius is
 (a) 1 : 2 (b) 1 : 1
 (c) $\sqrt{2} : 1$ (d) 2 : 1

97. Each of the circles of equal radii with centres A and B pass through the centre of one another circle they cut at C and D then $\angle DBC$ is equal to
 (a) 60° (b) 100° (c) 120° (d) 140°

98. The three equal circles touch each other externally. If the centres of these circles are A, B, C, then ABC is
 (a) a right angle triangle
 (b) an equilateral triangle
 (c) an isosceles triangle
 (d) a scalene triangle

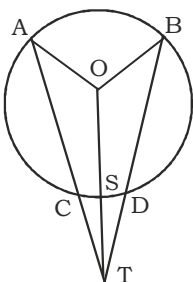
99. 'O' is the centre of the circle, AB is a chord of the circle, $OM \perp AB$. If $AB = 20$ cm, $OM = 2\sqrt{11}$ cm, then radius of the circle is
 (a) 15 cm (b) 12 cm
 (c) 10 cm (d) 11 cm

100. In the given figure, $\angle ONY = 50^\circ$ and $\angle OMY = 15^\circ$. Then the value of the $\angle MON$ is



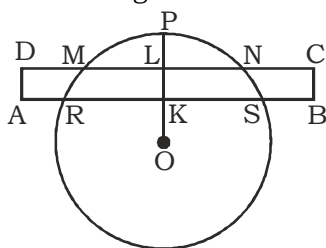
- (a) 30° (b) 40° (c) 20° (d) 70°

101. In the adjoining figure 'O' is the centre of circle AC and BD are the two chords of circle which meet at T outside the circle. OT bisects CD, $OA = OB = 8$ cm and $OT = 17$ cm. What is the ratio of distance of AC and BD from the centre of the circle?



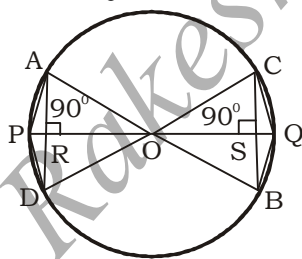
- (a) 15 : 17 (b) 8 : 15
 (c) 8 : 9
 (d) none of these

102. In the adjoining figure O is the centre of the circle. The radius OP bisects a rectangle ABCD, at right angle. $DM = NC = 2$ cm and $AR = SB = 1$ cm and $KS = 4$ cm and $OP = 5$ cm. What is the area of the rectangle?



- (a) 8 cm^2 (b) 10 cm^2
 (c) 12 cm^2
 (d) none of these

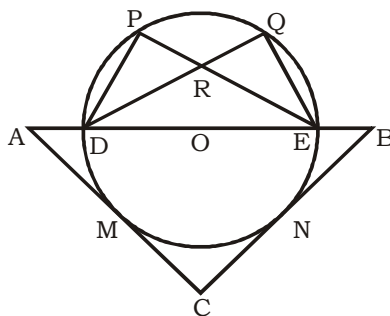
103. In the adjoining figure O is the centre of the circle. $\angle AOD = 120^\circ$. If the radius of the circle be 'r', then find the sum of the areas of quadrilaterals AODP and OBQC:



- (a) $\frac{\sqrt{3}}{2} r^2$ (b) $3\sqrt{3} r^2$
 (c) $\sqrt{3} r^2$ (d) none of these

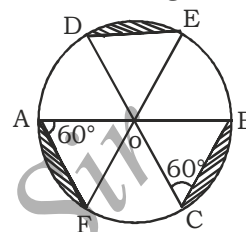
104. ABC is an isosceles triangle and AC, BC are the tangents at M and N respectively. DE is the diameter of the circle. $\angle ADP = \angle BEQ = 100^\circ$.

What is the value of $\angle PRD$?



- (a) 60° (b) 50°
 (c) 20°
 (d) Can't be determined.

105. In the adjoining figure O is the centre of the circle with radius 'r'. AB, CD and EF are the diameters of the circle. $\angle OAF = \angle OCB = 60^\circ$. What is the area of the shaded region?



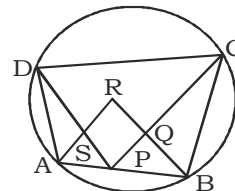
(a) $\frac{r^2}{2} \left(\pi - \frac{3\sqrt{3}}{2} \right)$

(b) $\frac{r^2}{2} \left(\pi - \frac{3\sqrt{3}}{4} \right)$

(c) $\frac{r^2}{3} \left(\pi - \frac{2\sqrt{3}}{4} \right)$

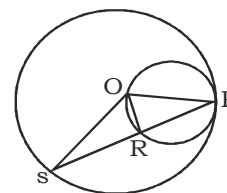
- (d) data insufficient

106. ABCD is a cyclic quadrilateral. The angle bisector of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ intersect at P, Q, R and S as shown in the figure. These four points form a quadrilateral PQRS. Quadrilateral PQRS is a :



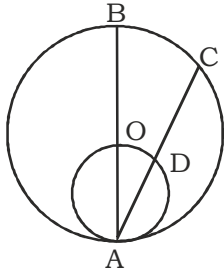
- (a) Square
 (b) rhombus
 (c) rectangle
 (d) cyclic quadrilateral

107. In the adjoining figure the diameter of the larger circle is 10 cm and the smaller circle touches internally the larger circle at P and passes through O, the centre of the larger circle at R and OR is equal to 4 cm. What is the length of the chord SP?

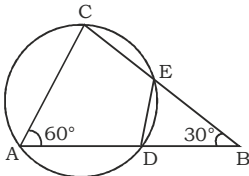


- (a) 9 cm (b) 12 cm
 (c) 6 cm (d) $8\sqrt{2}$ cm

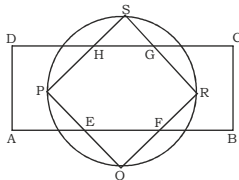
108. A smaller circle touches internally to a larger circle at A and passes through the centre of the larger circle. O is the centre of the larger circle and BA, OA are of the diameters of the larger and smaller circles respectively. Chord AC intersects the smaller circle at a point D. If AC = 12 cm, then AD is :



- (a) 4 cm (b) 6 cm
(c) 5.6 cm
(d) data insufficient
109. In the given figure ADEC is a cyclic quadrilateral, CE and AD are extended to meet at B. $\angle CAD = 60^\circ$ and $\angle CBA = 30^\circ$. BD = 6 cm and CE = $5\sqrt{3}$ cm, What is the ratio of AC : AD?



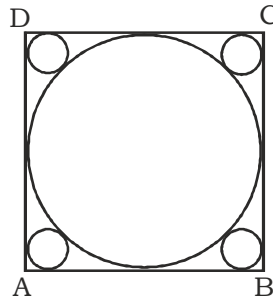
- (a) $\frac{3}{4}$ (b) $\frac{4}{5}$
(c) $\frac{2\sqrt{3}}{5}$
(d) can't be determined
110. In the adjoining figure ABCD is a rectangle in which length is twice of breadth. H and G divide the line CD into three equal parts. Similarly points E and F trisect the line AB. A circle PQRS is circumscribed by a square PQRS which passes through the points E, F, G and H. What is the ratio of area of circle to that of area of rectangle?



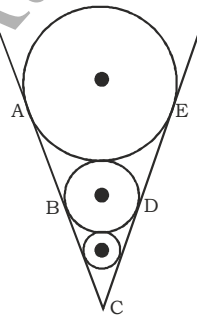
- (a) $3\pi : 7$ (b) $3 : 4$
(c) $25\pi : 72$ (d) $32\pi : 115$

111. ABCD is a square, in which a circle is inscribed touching all the sides of square. In the four corners of square 4 smaller circles of equal radii are drawn, containing maximum possible area.

What is the ratio of the area of larger circle to that of sum of the areas of four smaller circles?

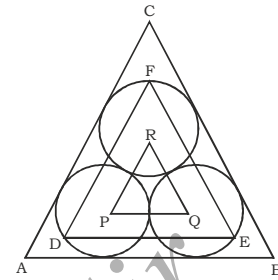


- (a) $1 : (68 - 48\sqrt{2})$
(b) $1 : (17 - 12\sqrt{2})$
(c) $12 : 17\sqrt{2}$
(d) None of these
112. In the adjoining figure $\angle ACE$ is a right angle there are three circles which just touch each other and AC and EC are the tangents to all the three circles. What is the ratio of the smallest circle?

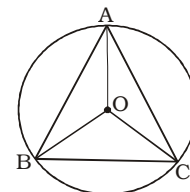
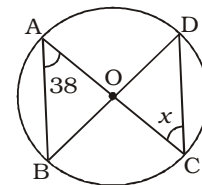


- (a) $17 : 12\sqrt{2}$
(b) $1 : (17 - 12\sqrt{2})$
(c) $12 : 17\sqrt{2}$
(d) none of these
113. In the adjoining figure three congruent circles are touching each other. Triangle ABC circumscribes all the three circles. Triangles PQR is formed by joining the centres of the circle. There is a third triangle DEF. Points A, D, P and B, E, Q and C, F, R lie in the same straight respectively.

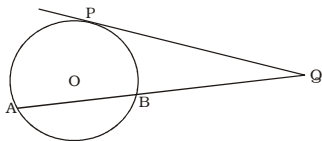
What is the ratio of perimeters of $\triangle ABC : \triangle DEF : \triangle PQR$



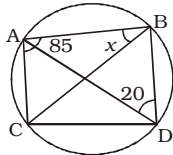
- (a) $3\sqrt{2} : 2\sqrt{2} : 1$
(b) $2(4 + \sqrt{3}) : (2 + \sqrt{3}) : \sqrt{3}$
(c) $2(1 + \sqrt{3}) : (2 + \sqrt{3}) : 2$
(d) $2(1 + \sqrt{3}) : 2\sqrt{3} : \sqrt{3}$
114. Through any given set of four points A, B, C, D it is possible to draw:-
(a) atmost one circle
(b) exactly one circle
(c) exactly two circles
(d) exactly three circles
115. The number of common tangents that can be drawn to two given circles intersect each other is :-
(a) one (b) two
(c) three (d) four
116. The radius of a circle is 13 cm and the length of one of its chords is 10 cm. Find the distance of chord from the centre.
(a) 8 cm (b) 10 cm
(c) 9 cm (d) 12 cm
117. In the given figure O is the centre of the circle. If $\angle BAC = 38^\circ$, then $\angle OCD$ is
(a) 76° (b) 52°
(c) 38° (d) 19°



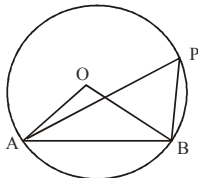
- (a) 80° (b) 70°
(c) 100° (d) 140°
119. In the given figure $PQ = 12$ cm, $BQ = 8$ cm, then the length of chord AB :-



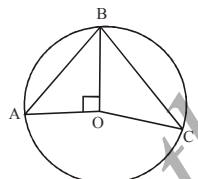
- (a) 10 cm (b) $4\sqrt{5}$ cm
(c) 4 cm (d) 18 cm
120. The value of x will be:-



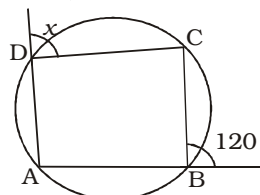
- (a) 70° (b) 90°
(c) 60° (d) 75°
121. In the given figure, O is the centre of the circle and $\angle AOB = 90^\circ$, then $\angle APB$ will be:-



- (a) 90° (b) 60°
(c) 45° (d) 30°
122. In the given figure, O is the centre of the circle $\angle AOB = 90^\circ$, $\angle BOC = 110^\circ$, then $\angle ABC$ is

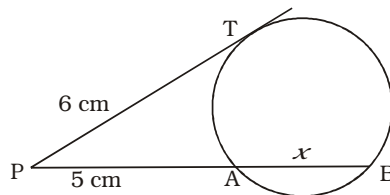


- (a) 75° (b) 60°
(c) 80° (d) 70°
123. ABCD is a cyclic quadrilateral, then the value of x will be :-

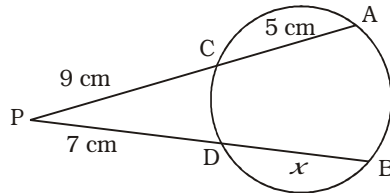


- (a) 50° (b) 60°
(c) 120° (d) 70°

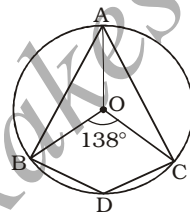
124. The value of x :-



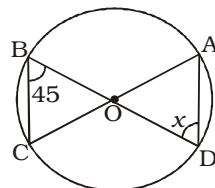
- (a) 2.2 cm (b) 1.6 cm
(c) 3 cm (d) 2.6 cm
125. The value of x :-



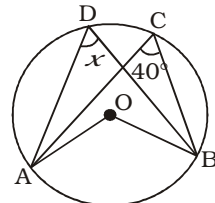
- (a) 10 cm (b) 9 cm
(c) 7.5 cm (d) 11 cm
126. In the given figure ABCD is a cyclic quadrilateral and O is the centre of the circle. If $\angle BOC = 138^\circ$, then $\angle BDC$ will be



- (a) 112° (b) 111°
(c) 109° (d) None of these
127. In the given figure, O is the centre of the circle, then the value of x will be:-

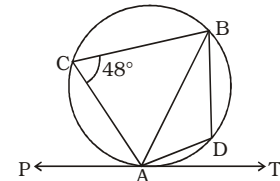


- (a) 40° (b) 90°
(c) 45° (d) 30°
128. If O is centre of the circle, then x is equal to

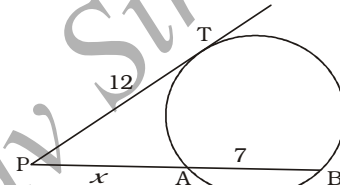


- (a) 40° (b) 45°
(c) 39° (d) 35°

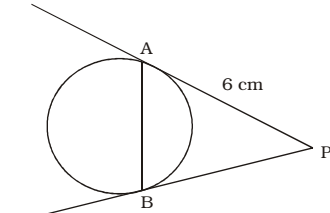
129. In the given figure, $\angle ADB$:-



- (a) 144° (b) 132°
(c) 48° (d) 96°
130. Find the value of x in the given figure:

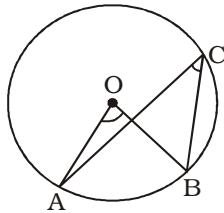


- (a) 16 cm (b) 9 cm
(c) 12 cm (d) 7 cm
131. In the given figure, PA and PB are tangents from a point P to a circle such that $PA = 6$ cm and $\angle APB = 60^\circ$. What is the length of the chord AB?

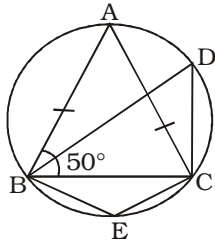


- (a) 12 cm (b) 8 cm
(c) 9 cm (d) 6 cm
132. ABC is a right angled triangle $AB = 3$ cm, $BC = 5$ cm and $AC = 4$ cm, then the inradius of the circle is
(a) 1 cm (b) 1.25 cm
(c) 1.5 cm (d) None of these
133. The number of common tangents that can be drawn to two given circles is at the most
(a) 1 (b) 2
(c) 3 (d) 4
134. Two circle of radii 12 cm and 7 cm touch each other internally. Find the distance between their centres.
(a) 6 cm (b) 13 cm
(c) 9 cm (d) 5 cm
135. Three circles touch each other externally. The distance between their centres is 5 cm, 6 cm and 7 cm. Find radii of the circles:-

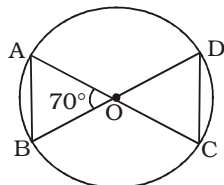
- (a) 2 cm, 3 cm, 4 cm
 (b) 3 cm, 4 cm, 1 cm
 (d) 1 cm, 2 cm, 4 cm
 (d) None of these
136. In the given figure, O is the centre of the circle and $\angle ACB = 30^\circ$. Find $\angle AOB$.



- (a) 30° (b) 90°
 (c) 60° (d) 50°
137. In the given figure, $AB = AC$ and $\angle ABC = 50^\circ$. Find $\angle BDC$:



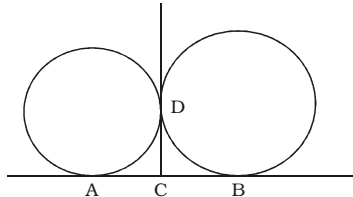
- (a) 60° (b) 80°
 (c) 100° (d) 90°
138. In the given figure, O is the centre of the circle. $\angle AOB = 70^\circ$, find $\angle OCD$.



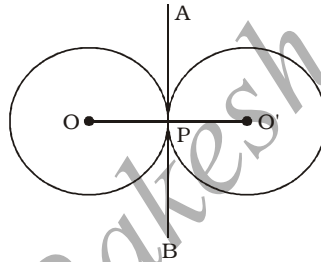
- (a) 70° (b) 55°
 (c) 65° (d) 110°
139. If the diagonals of a cyclic quadrilateral are equal, then the quadrilateral is
 (a) rhombus (b) square
 (c) rectangle (d) None of these
140. The quadrilateral formed by angle bisectors of cyclic quadrilateral is a
 (a) rectangle
 (b) square
 (c) parallelogram
 (d) cyclic quadrilateral
141. The distance between the centres of equal circles each of radius 3 cm is 10 cm. The length of a transverse tangent is

- gent is
 (a) 4 cm (b) 6 cm
 (c) 8 cm (d) 10 cm

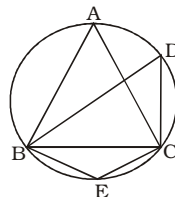
142. In the given figure, AB and CD are two common tangents to the two touching circle. If $CD = 7$ cm, then AB is equal to



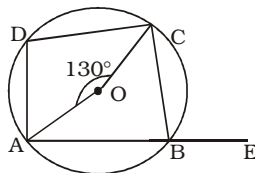
- (a) 14 cm (b) 10.5 cm
 (c) 12 cm (d) None of these
143. O and O' are the centres of two circles which touch each other externally at P. If AB is a common tangent. Find $\angle APO$.



- (a) 90° (b) 120°
 (c) 60° (d) data insufficient
144. In the given figure, $\triangle ABC$ is an equilateral triangle. Find $\angle BEC$.

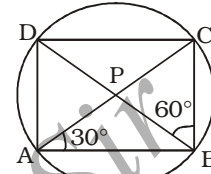


- (a) 60° (b) 120°
 (c) 80° (d) 90°
145. In the given figure, $\angle AOC = 130^\circ$. Find $\angle CBE$, where O is the centre.

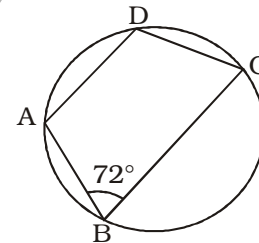


- (a) 130° (b) 100°
 (c) 115° (d) 105°

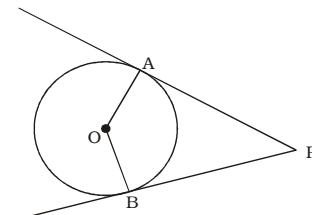
146. In the given figure, ABCD is a cyclic quadrilateral and diagonals bisect each other at P. If $\angle DBC = 60^\circ$, and $\angle BAC = 30^\circ$ then $\angle BCD$ is



- (a) 90° (b) 60°
 (c) 80° (d) None of these
147. In the given figure, $AD \parallel BC$, if $\angle ABC = 72^\circ$, then $\angle BCD = ?$

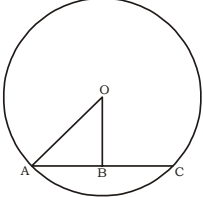


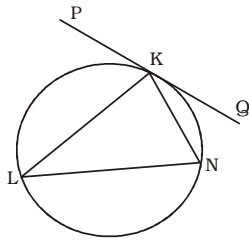
- (a) 108° (b) 36°
 (c) 90° (d) 72°
148. In the given figure, O is the centre of the circle. PA and PB are tangents if $\angle AOB : \angle APB = 5 : 1$, then $\angle APB$



- (a) 150° (b) 30°
 (c) 60° (d) 90°
149. R and r are the radius of two circles ($R > r$). If the distance between the centre of the two circles be d, then length of common tangent of two circles is :

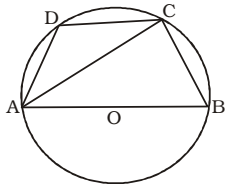
- (a) $\sqrt{r^2 - d^2}$
 (b) $\sqrt{d^2 - (R - r)^2}$
 (c) $\sqrt{(R - r)^2 - d^2}$
 (d) $\sqrt{R^2 - d^2}$

150. Two circles of radii 8 cm and 2 cm respectively touch each other externally at the point A. PQ is the direct common tangent of those two circles of centres O_1 and O_2 respectively. Then length of QP is equal to :
 (a) 2 cm (b) 3 cm
 (c) 4 cm (d) 8 cm
151. PQ is a direct common tangent of two circles of radii r_1 and r_2 touching each other externally at A. Then the value of PQ^2 is :
 (a) $r_1 r_2$ (b) $2r_1 r_2$
 (c) $3r_1 r_2$ (d) $4r_1 r_2$
152. Two circles with radii 5 cm and 8 cm touch each other externally at a point A. If a straight line through the point A cuts the circles at points P and Q respectively, then AP : AQ is :
 (a) 8 : 5 (b) 5 : 8
 (c) 3 : 4 (d) 4 : 5
153. The radius of a circle is 6 cm. An external point is at a distance of 10 cm from the centre. Then the length of the tangent drawn to the circle from the external point upto the point of contact is :
 (a) 8 cm (b) 10 cm
 (c) 6 cm (d) 12 cm
154. A triangle is inscribed in a circle and the diameter of the circle is its one side. Then the triangle will be :
 (a) right-angled
 (b) obtuse-angled
 (c) equilateral
 (d) a square
155. Two circles of radii 4 cm and 9 cm respectively touch each other externally at a point and a common tangent touches them at the points P and Q respectively. Then the area of square with one side PQ, is :
 (a) 97 sq.cm (b) 194 sq.cm
 (c) 72 sq.cm (d) 144 sq.cm
156. The length of the chord of a circle is 8 cm and perpendicular distance between centre and the chord is 3 cm. Then the radius of the circle is equal to :
 (a) 4 cm (b) 5 cm
 (c) 6 cm (d) 8 cm
157. The radius of a circle is 13 cm and XY is a chord which is at a distance of 12 cm from the centre. The length of the chord is :
 (a) 15 cm (b) 12 cm
 (c) 10 cm (d) 20 cm
158. SR is a direct common tangent to the circles of radii 8 cm and 3 cm respectively, their centres being 13 cm apart. If the points S and R are the respective points of intersect, then the length of SR is :
 (a) 12 cm (b) 11 cm
 (c) 17 cm (d) 10 cm
159. In the following figure, if OA = 10 and AC = 16, then OB must be :
- 
- (a) 5 (b) 6
 (c) 3 (d) 4
160. One chord of a circle is known to be 10.1 cm. The radius of this circle must be :
 (a) 5 cm
 (b) greater than 5 cm
 (c) greater than or equal to 5 cm
 (d) less than 5 cm
161. The length of two chords AB and AC of a circle are 8 cm and 6 cm and $\angle BAC = 90^\circ$, then the radius of circle is :
 (a) 25 cm (b) 20 cm
 (c) 30 cm (d) 5 cm
162. If a chord of length 16 cm is at a distance of 15 cm from the centre of the circle, then the length of the chord of the same circle which is at distance of 8 cm from the centre is equal to :
 (a) 10 cm (b) 20 cm
 (c) 30 cm (d) 40 cm
163. PR is tangent to circle, with centre O and radius 4 cm, at point Q. If $\angle POR = 90^\circ$, OR = 5 cm and $OQ = \frac{20}{3}$ cm, then, in cm, the length of PR is :
 (a) 3 (b) $\frac{16}{3}$
 (c) $\frac{23}{3}$ (d) $\frac{25}{3}$
164. Circumcentre of $\triangle ABC$ is O. If $\angle BAC = 85^\circ$, $\angle BCA = 80^\circ$, then $\angle AOC$ is :
 (a) 80° (b) 30°
 (c) 60° (d) 75°
165. If O is the circumcentre of $\triangle ABC$ and $\angle OBC = 35^\circ$, then the $\angle BAC$ is equal to :
 (a) 55° (b) 110°
 (c) 70° (d) 35°
166. If I is the incentre of $\triangle ABC$ and $\angle BIC = 135^\circ$, then the $\triangle ABC$ is :
 (a) Acute angled
 (b) equilateral
 (c) right angled
 (d) obtuse angled
167. If S is the circumcentre of $\triangle ABC$ and $\angle A = 50^\circ$, then the value of $\angle BCS$ is :
 (a) 20° (b) 40°
 (c) 60° (d) 80°
168. The distance between the centres of two equal circles, each of radius 3 cm, is 10 cm. The length of a transverse common tangent is :
 (a) 8 cm (b) 10 cm
 (c) 4 cm (d) 6 cm
169. A unique circle can always be drawn through x number of given non-collinear points, then x must be :
 (a) 2 (b) 3 (c) 4 (d) 1
170. The length of radius of a circumcircle of a triangle having sides 3 cm, 4 cm and 5 cm is :
 (a) 2 cm (b) 2.5 cm
 (c) 3 cm (d) 1.5 cm
171. AB and CD are two parallel chords of a circle such that AB = 6 cm and CD = 8 cm. If the chords lie on the same side of centre O and radius 5 cm the distance between AB and CD is :
 (a) 2 cm (b) 1 cm
 (c) 2.5 cm (d) 3 cm
172. In the given figure PKQ is a tangent and LN is the diameter of the circles. If $\angle KLN = 30^\circ$ then $\angle PKL$ will be :



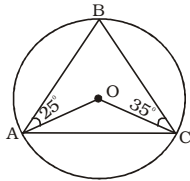
- (a) 30° (b) 50°
(c) 60° (d) 70°

173. In the given figure $\angle ADC = 120^\circ$ and AOB is the diameter of the circle, then $\angle BAC$:



- (a) 30° (b) 40°
(c) 50° (d) 60°

174. $\angle OAB = 25^\circ$, $\angle OCB = 35^\circ$ then $\angle AOC$ will be :

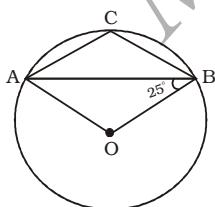


- (a) 60° (b) 80°
(c) 100° (d) 120°

175. AB and CD are two parallel chords of a circle such that AB = 10cm and CD = 24cm, If the chords are on the opposite sides of the centre and the distance between them is 17cm, then the radius of the circle is :

- (a) 8 cm (b) 15 cm
(c) 11 cm (d) 13 cm

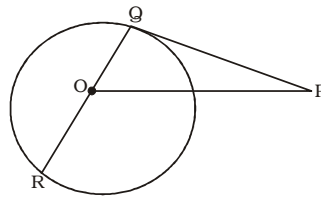
176. In the given figure, O is the centre of the circle then $\angle ACB$ will be :



- (a) 105° (b) 230°
(c) 115° (d) 100°

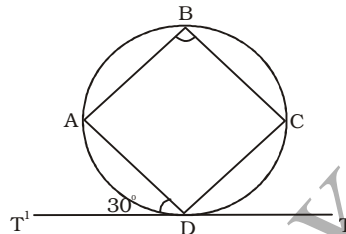
177. In the given figure, ROQ is the diameter of the circle. If

$\angle POR = 120^\circ$ then $\angle QPO$ will be:



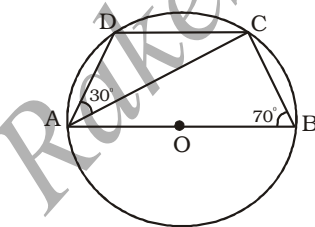
- (a) 40° (b) 30°
(c) 60° (d) 50°

178. In the given figure $\angle ABC = 55^\circ$, the $\angle CDT$ is:



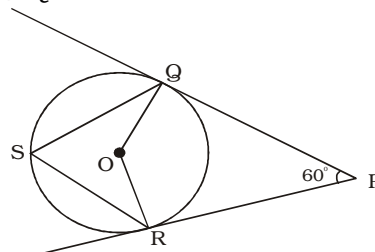
- (a) 15° (b) 20°
(c) 25° (d) 30°

179. In the given figure if AB is the diameter of the circle, then $\angle ACD$ will be :



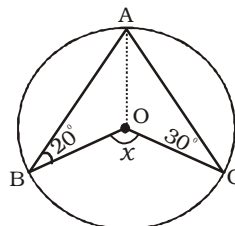
- (a) 40° (b) 50° (c) 35° (d) 90°

180. $\angle QSR$ is :-



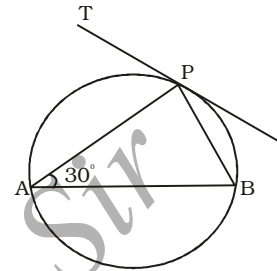
- (a) 55° (b) 90°
(c) 70° (d) 60°

181. Find the value of $\angle x$ in the given figure :-



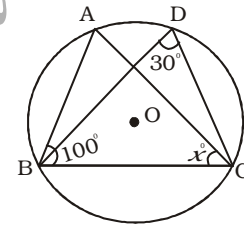
- (a) 120° (b) 130°
(c) 110° (d) 100°

182. In the given figure AB is the diameter of the circle and $\angle PAB = 30^\circ$, Find $\angle TPA$



- (a) 30° (b) 60°
(c) 50° (d) 70°

183. In the following figure, find the value of x

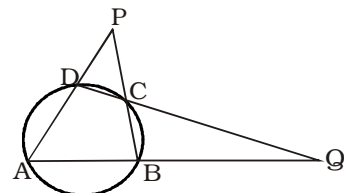


- (a) 40° (b) 45°
(c) 50° (d) 60°

184. If a circle is provided with a measure of 19° on centre, is it possible to divide the circle into 360 equal parts ?

- (a) Never
(b) Possible when one measure of 20° is given
(c) Always
(d) Possible when one measure of 21° is given

185. In the adjoining figure $\angle A = 60^\circ$ and $\angle ABC = 80^\circ$, $\angle BQC = ?$



- (a) 40° (b) 80° (c) 20° (d) 30°

186. Two circles of radius 37cm and 20cm intersect each other at A and B. O and O' are the centres of the circles. If the length of AB is 24cm, then OO' :-

- (a) 50cm (b) 51cm
(c) 40cm (d) 57cm

187. Two circles of radius 4cm and 6cm touch each other internally. Find the longest chord of the bigger circle which is outside of the smaller circle?

(a) $8\sqrt{2}$ cm (b) $4\sqrt{2}$ cm
(c) $6\sqrt{2}$ cm (d) $3\sqrt{2}$ cm

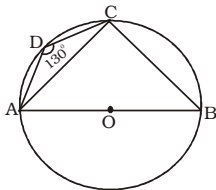
188. In a circle of radius 17cm two parallel chords are present on the opposite side of the diameter. If the distance between them is 23cm and the length of one chord is 16cm then the length of other chord is:-

(a) 15 cm (b) 20cm
(c) 18 cm (d) 30cm

189. AB is a chord of the circle (centre O). P is a point on the circle such that $OP \perp AB$ and OP intersect AB at point M. If $AB = 8$ cm, $MP = 2$ cm then radius (r):-

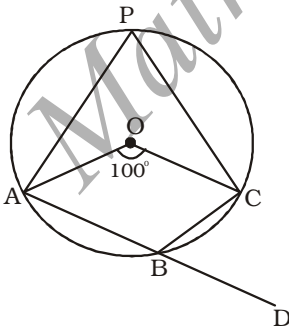
(a) 7 cm (b) 5cm
(c) 6 cm (d) 4cm

190. In the given figure, ABCD is a cyclic quadrilateral whose side AB is a diameter of the circle through A, B and C. If $\angle ADC = 130^\circ$ find $\angle CAB$.



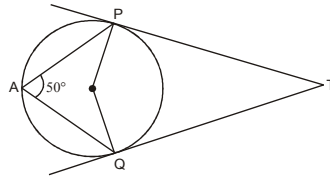
(a) 40° (b) 50°
(c) 30° (d) 130°

191. In the given figure, O is the centre of the circle find $\angle CBD$



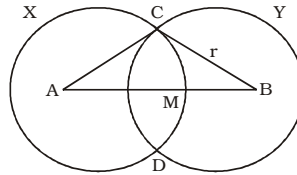
(a) 140° (b) 50°
(c) 40° (d) 130°

192. In the given figure, TP and TQ are tangents to the circle. If $\angle PAQ = 50^\circ$, what is $\angle PTQ$?



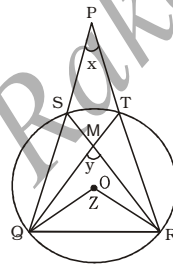
(a) 80° (b) 70°
(c) 100° (d) 90°

193. Two circles X and Y with centres A and B intersect at C and D. If Area of circle X is 4 times area of circle Y, then $AB = ?$



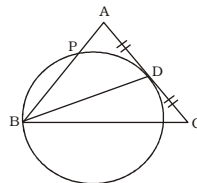
(a) $5r$ (b) $\sqrt{5}r$
(c) $3r$ (d) $\frac{\sqrt{5}}{2}r$

194. In the given figure, O is the centre of the circle. Then $\angle x + \angle y$ is equal to-



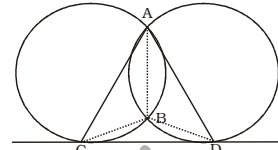
(a) $2Z$ (b) $\frac{Z}{2}$
(c) Z
(d) None of these

195. In the figure, ABC is a triangle in which $AB = AC$. A circle through B touches AC at D and intersects AB at P. If D is the mid-point of AC, Find the value of $\angle AB$:-



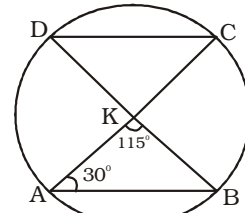
(a) $2AP$ (b) $3AP$
(c) $4AP$
(d) None of these

196. In the given figure, CD is a direct common tangent to two circles intersecting each other at A and B, then $\angle CAD + \angle CBD = ?$



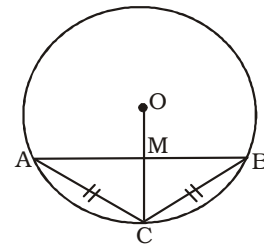
(a) 120° (b) 90°
(c) 360°
(d) None of these

197. In the given figure, $\angle CAB = 30^\circ$ and $\angle AKB = 115^\circ$, find $\angle KCD$:-



(a) 65° (b) 35°
(c) 40° (d) 72°

198. In the given figure, the chords AC and BC are equal. The radius OC intersect AB at M then $AM : BM$:-



(a) 1:1 (b) $\sqrt{2} : 3$
(c) $3 : \sqrt{2}$
(d) None of these

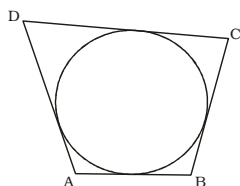
199. If two circles are such that the centre of one lies on the circumference of the other, then the ratio of the common chord of two circles to the radius of any of the circles is :-

(a) $\sqrt{3} : 2$ (b) $\sqrt{3} : 1$
(c) $\sqrt{5} : 1$
(d) None of these

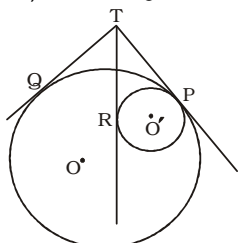
200. ABCD is a cyclic trapezium whose sides AD and BC are parallel to each other; if $\angle ABC = 75^\circ$ then the measure of $\angle BCD$ is:

(a) 75° (b) 95° (c) 45° (d) 105°

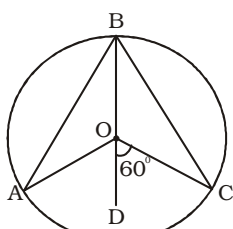
201. A circle touches a quadrilateral ABCD. Find the true statement :-



- (a) $BD = AC$
 (b) $AB + BC = CD + AD$
 (c) $AB + BC = AC$
 (d) $AB + CD = BC + AD$
202. In the given figure, Tangents TQ and TP are drawn to the larger circle centre O and tangents TP and TR are drawn to the smaller circle (centre O'). Find TQ : TR :-



- (a) 1 : 1 (b) 5 : 4
 (c) 8 : 7 (d) 7 : 8
203. 'O' is the centre of the circle, line segment $\angle BOD$ is the angle bisector of $\angle AOC$, $\angle COD = 60^\circ$. Find $\angle ABC$:-



- (a) 120° (b) 60°
 (c) 30° (d) 90°
204. If O is the centre of the circle and PA and PB are two tangents drawn from a point P on the circumference of the circle. If $\angle APB = 68^\circ$, then $\angle POA = ?$
 (a) 68° (b) 34°
 (c) 56° (d) 90°
205. In a circle, AB is the diameter of the circle, and CD is a chord such that $CD \parallel AB$. P is any point on the circle such that $\angle BPC = 48^\circ$, then $\angle BCD = ?$
 (a) 48° (b) 42° (c) 24° (d) 96°

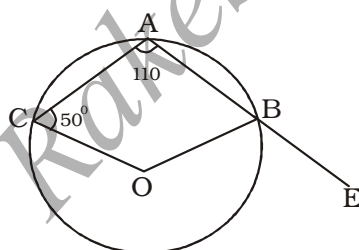
206. AB and CD are two chords of a circle intersect at a point P. If $\angle APC = 80^\circ$ and $\angle ADP = 30^\circ$, then $\angle BCD = ?$

- (a) 30° (b) 80°
 (c) 100° (d) 50°
207. ABCD is a cyclic quadrilateral. Side AB and DC when produced meet at P and side AD and BC when Produced meet at Q. If $\angle APD = 40^\circ$, $\angle ADC = 85^\circ$, then $\angle AQB$ is equal to :-

- (a) 30° (b) 40°
 (c) 50° (d) 55°
208. AB is the diameter of the circle with centre O. DC is a chord such that $DC \parallel AB$. If $\angle BAC = 20^\circ$, then $\angle ADC$ is equal to :-

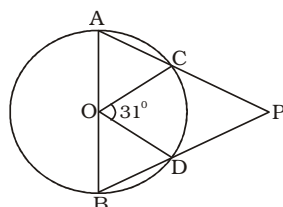
- (a) 100° (b) 90°
 (c) 110° (d) 120°
209. In question above, find $\angle COD$?

- (a) 50° (b) 100°
 (c) 25° (d) 90°
210. Find $\angle OBE$?



- (a) 120° (b) 100°
 (c) 115° (d) None of these
211. AB is the diameter of a circle whose center is O and CD is a chord in the circle and $CD = \frac{1}{2} AB$. AC and BD on producing meet at P. Find $\angle APB$?

- (a) 30° (b) 40°
 (c) 50° (d) 60°
212. In the given figure, AB is the diameter of the circle and O is the centre, Find $\angle APB$?



- (a) 149° (b) 74.5°
 (c) 62°
 (d) None of these

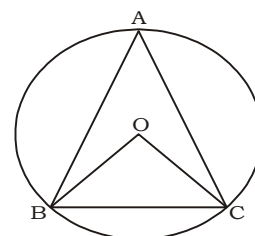
213. O is the circum centre of the triangle ABC with circumradius 13 cm. Let $BC = 24$ cm and OD is perpendicular to BC. Then the length of OD is:

- (a) 7 cm (b) 3 cm
 (c) 4 cm (d) 5 cm

214. A, B, C are three points on a circle. The tangent at A meets BC produced at T, $\angle BTA = 40^\circ$ and $\angle CAT = 44^\circ$. The angle subtended by BC at the centre of the circle is:

- (a) 84° (b) 92°
 (c) 96° (d) 104°

215. BC is the chord of a circle with centre O. A is a point on major arc BC as shown in the figure. What is the value of $\angle BAC + \angle OBC$?



- (a) 120° (b) 60°
 (c) 90° (d) 180°

216. AB and CD are two parallel chords drawn on two opposite sides of their parallel diameter such that $AB = 6$ cm, $CD = 8$ cm. If the radius of the circle is 5 cm, the distance between the chords, in cm, is :

- (a) 2 (b) 7 (c) 5 (d) 3

217. A chord AB of length $3\sqrt{2}$ unit subtends a right angle at the centre O of a circle. Area of the sector AOB (in sq. units) is:

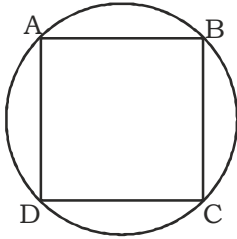
- (a) $\frac{9}{4}$ sq. units
 (b) 5sq. units
 (c) 9sq. units
 (d) $\frac{9}{2}$ sq. units

218. AB and BC are two chords of a circle with centre O. If P and

Q are the mid-points of AB and BC respectively, then the quadrilateral OQBP must be:

- (a) a rhombus (b) concyclic
(c) a rectangle (d) a square

219. If the area of the circle in the figure is 36 sq. cm, and ABCD is a square, then the area of $\triangle ACD$, in sq. cm, is :



- (a) 12π (b) $\frac{36}{\pi}$
(c) 12 (d) 18

220. Two tangents are drawn from a point P to a circle at A and B. O is the centre of the circle. If $\angle AOP = 60^\circ$, then $\angle APB$ is :
(a) 120° (b) 90°
(c) 60° (d) 30°

221. If the length of a chord of a circle, which makes an angle 45° with the tangent drawn at one end point of the chord, is 6 cm, then the radius of the circle is :

- (a) $6\sqrt{2}$ cm (b) 5 cm
(c) $3\sqrt{2}$ cm (d) 6 cm

222. Two equal circles pass through each other's centre. If the radius of each circle is 5 cm, what is the length of the common chord?

- (a) 5 (b) $5\sqrt{3}$
(c) $10\sqrt{3}$ (d) $\frac{5\sqrt{3}}{2}$

223. PA and PB are two tangents drawn from external point P to a circle with centre O where the points A and B are the points of contact. The quadrilateral OAPB must be:
(a) a rectangle (b) a rhombus
(c) a square (d) concyclic

224. The radius of two concentric circles are 9 cm and 15 cm. If the chord of the greater circle

be a tangent to the smaller circle, then the length of that chord is :

- (a) 24 cm (b) 12 cm
(c) 30 cm (d) 18 cm

225. The length of a chord of a circle is equal to the radius of the circle. The angle which this chord subtends in the major segment of the circle is equal to :
(a) 30° (b) 45° (c) 60° (d) 90°

226. The ratio of the areas of the circumcircle and the incircle of an equilateral triangle is:
(a) 2 : 1 (b) 4 : 1
(c) 8 : 1 (d) 3 : 2

227. AB = 8 cm and CD = 6 cm are two parallel chords on the same side of the centre of a circle. The distance between them is 1 cm. The radius of the circle is :

- (a) 5 cm (b) 4 cm
(c) 3 cm (d) 2 cm

228. Two equal circles of radius 4 cm intersect each other such that each passes through the centre of the other. The length of the common chord is :

- (a) $2\sqrt{2}$ cm (b) $4\sqrt{3}$ cm
(c) $2\sqrt{3}$ cm (d) 8 cm

229. ABCD is a cyclic parallelogram. The $\angle B$ is equal to :
(a) 30° (b) 60° (c) 45° (d) 90°

230. From four corners of a square sheet of side 4 cm, four pieces, each in the shape of arc of a circle with radius 2 cm, are cut out. The area of the remaining portion is :

- (a) $(8 - \pi)$ sq. cm.
(b) $(16 - 4\pi)$ sq. cm.
(c) $(16 - 8\pi)$ sq. cm.
(d) $(4 - 2\pi)$ sq. cm.

231. If a chord of a circle of radius 5 cm is a tangent to a circle of radius 3 cm, both the circles being concentric, then the length of the chord is :

- (a) 10 cm (b) 12.5 cm
(c) 8 cm (d) 7 cm

232. Two circles touch each other

externally at point A and PQ is a direct common tangent which touches at P and Q respectively. Then $\angle PAQ = ?$

- (a) 45° (b) 90°
(c) 80° (d) 100°

233. AB is a chord to a circle and PAT is the tangent to the circle at A. If $\angle BAT = 75^\circ$ and $\angle BAC = 45^\circ$, C being a point on the circle, then $\angle ABC$ is equal to :

- (a) 40° (b) 45° (c) 60° (d) 70°

234. O is the centre of a circle and arc ABC subtends an angle of 130° at O. AB is extended to P. Then $\angle PBC$ is :

- (a) 75° (b) 70° (c) 65° (d) 80°

235. The circumcentre of a triangle ABC is O. If $\angle BAC = 85^\circ$ and $\angle BCA = 75^\circ$, then the value of $\angle OAC$ is:

- (a) 40° (b) 60° (c) 70° (d) 90°

236. The length of each side of an equilateral triangle is $14\sqrt{3}$ cm. The area of the incircle, in cm^2 , is :

- (a) 450 (b) 308
(c) 154 (d) 77

237. If the radii of two circles be 6 cm and 3 cm and the length of the transverse common tangent be 8 cm, then the distance between the two centres is :

- (a) $\sqrt{154}$ cm (b) $\sqrt{140}$ cm
(c) $\sqrt{145}$ cm (d) $\sqrt{135}$ cm

238. Two parallel chords are drawn in a circle of diameter 30 cm. The length of one chord is 24 cm and the distance between the two chords is 21 cm. The length of the other chord is :

- (a) 10 cm (b) 18 cm
(c) 12 cm (d) 16 cm

239. If two equal circles whose centres are O and O', intersect each other at the points A and B, $OO' = 12$ cm and $AB = 16$ cm, then the radius of the circles is :

- (a) 10 cm (b) 8 cm
(c) 12 cm (d) 14 cm

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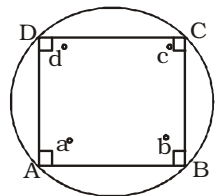
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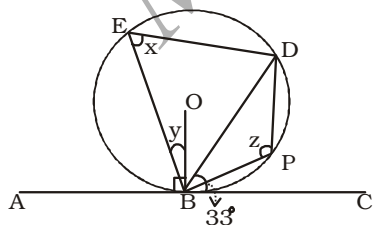
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254. In the given figure, $AB \parallel CD$ if a, b, c and d are integers, what is the number of possible value of $(a+b-c-d)$?



- (a) 179 (b) 89
(c) 357 (d) 358
255. Three equal circle of unit radius touch each other. Then, the area of the circle circumscribing the three circle is :-
(a) $6\pi(2 + \sqrt{3})^2$
(b) $\frac{\pi}{6}(2 + \sqrt{3})^2$
(c) $\frac{\pi}{3}(2 + \sqrt{3})^2$
(d) $3\pi(2 + \sqrt{3})^2$
256. In $\triangle ABC$, $AB=4\text{cm}$, $BC=3.4\text{cm}$ and $AC=2.2\text{cm}$. Three circles are drawn with centre A, B and C in such a way that each circle touches the other two. Then the diameter of the bigger circle is:
(a) 5.2 cm (b) 2.6 cm
(c) 2.8 cm
(d) None of these
257. The angle bisectors of angle A, B and C of a $\triangle ABC$ intersect the circumference of the circumcircle at X, Y and Z respectively. If $\angle A = 50^\circ$, $\angle CZY = 42^\circ$, then $\angle BYZ$ is equal to :-
(a) 46° (b) 42° (c) 23° (d) 21°
258. In the given figure, chord $BE = BD$, $\angle CBD = 33^\circ$, & $OB \perp AC$ then $x+y+z$ is equal to



- (a) 230° (b) 237°
(c) 337°
(d) None of these

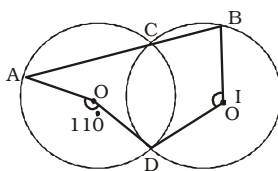
259. ABC and MNC are two secants of a circle whose centre is O. AN is the diameter of the circle if $\angle BAN = 38^\circ$ and $\angle ACM = 20^\circ$ then $\angle MBN$:-

- (a) 38° (b) 42°
(c) 28° (d) 32°

260. PT is a tangent of a circle at T and AB is a chord. If $AB = 18\text{cm}$ and $PT = 2AP$ then find PT ?

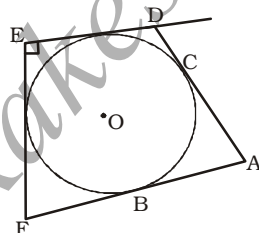
- (a) 12cm (b) 18cm
(c) 6 cm (d) 9cm

261. Find $\angle BO'D$?



- (a) 220° (b) 110°
(c) 55° (d) 70°

262. In the given figure, $AB = 27$, $AD = 38\text{cm}$, $ED = 24\text{cm}$ and $\angle E = 90^\circ$, then radius of the circle is equal to :-



- (a) 11 cm (b) 15 cm
(c) 13 cm (d) 17 cm

263. Two circles having radius 'a'cm and 'b'cm touch each other externally. another circle whose radius is 'c'cm, touches both the circles and also their common tangent. Then which statement will be true :-

- (a) $\sqrt{a} + \sqrt{b} = \sqrt{c}$
(b) $\sqrt{a} = \sqrt{b} + \sqrt{c}$
(c) $\sqrt{ab} + \sqrt{bc} = \sqrt{ac}$
(d) $\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$

264. In a $\triangle ABC$, I and O are the in-centre and circum-centre respectively. The line AI is produced to a point D on the circumcircle. If $\angle BOD = Z$,

$\angle BID = Y$ and $\angle ABC = X$, then

$\frac{x+z}{3y}$ is equal to :-

- (a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{4}{3}$ (d) 1

265. P and Q are the middle points of two chords (not diameters) AB and AC respectively of a circle with centre at a point O. The lines OP and OQ are produced to meet the circle respectively at the points R and S. T is any point on the major arc between the points R and S of the circle. If $\angle BAC = 32^\circ$, $\angle RTS = ?$

- (a) 32° (b) 74°
(c) 106° (d) 64°

266. O and C are respectively the orthocentre and circumcentre of an acute-angled triangle PQR. The points P and O are joined and produced to meet the side QR at S. If $\angle PQS = 60^\circ$ and $\angle QCR = 130^\circ$, then $\angle RPS = ?$

- (a) 30° (b) 35° (c) 100° (d) 60°

267. Two chords AB and CD of circle whose centre is O, meet at the point P and $\angle AOC = 50^\circ$, $\angle BOD = 40^\circ$. Then the value of $\angle BPD$ is :

- (a) 60° (b) 40° (c) 45° (d) 75°

268. The tangents are drawn at the extremities of a diameter AB of a circle with centre P. If a tangent to the circle at the point C intersects the other two tangents at Q and R, then the measure of the $\angle QPR$ is :

- (a) 45° (b) 60° (c) 90° (d) 180°

269. Two circles of radii 9 cm and 2 cm respectively have centres X and Y and $\overline{XY} = 17\text{cm}$. Circle of radius r cm with centre Z touches two given circles externally. If $\angle XZY = 90^\circ$, find r :

- (a) 18 cm (b) 3 cm
(c) 12 cm (d) 6 cm

270. A circle (with centre at O) is touching two intersecting lines AX and BY. The two points of contact A and B subtend an angle of 65° at any point C on the circumference of the

- circle. If P is the point of intersection of the two lines, then the measure of $\angle APO$ is:
 (a) 25° (b) 65° (c) 90° (d) 40°
271. ABCD is a cyclic trapezium such that $AD \parallel BC$, if $\angle ABC = 70^\circ$, then the value of $\angle BCD$ is :
 (a) 60° (b) 70° (c) 40° (d) 80°
272. ABCD is a cyclic trapezium whose sides AD and BC are parallel to each other. If $\angle ABC = 72^\circ$, then the measure of the $\angle BCD$ is
 (a) 162° (b) 18°
 (c) 108° (d) 72°
273. If an exterior angle of a cyclic quadrilateral be 50° , then the interior opposite angle is :
 (a) 130° (b) 40° (c) 50° (d) 90°
274. ABCD is cyclic parallelogram. The angle $\angle B$ is equal to :
 (a) 30° (b) 60° (c) 45° (d) 90°
275. ABCD is a cyclic quadrilateral and O is the centre of the circle. If $\angle COD = 140^\circ$ and $\angle BAC = 40^\circ$, then the value of $\angle BCD$ is equal to
 (a) 70° (b) 90° (c) 60° (d) 80°
276. ABCD is a quadrilateral inscribed in a circle with centre O. If $\angle COD = 120^\circ$ and $\angle BAC = 30^\circ$, then $\angle BCD$ is :
 (a) 75° (b) 90° (c) 120° (d) 60°
277. ABCD is a cyclic trapezium with $AB \parallel DC$ and AB is a diameter of the circle. If $\angle CAB = 30^\circ$, then $\angle ADC$ is
 (a) 60° (b) 120° (c) 150° (d) 30°
278. ABCD is a cyclic quadrilateral. AB and DC are produced to meet at P. If $\angle ADC = 70^\circ$ and $\angle DAB = 60^\circ$, then the $\angle PBC + \angle PCB$ is
 (a) 130° (b) 150°
 (c) 155° (d) 180°

279. A cyclic quadrilateral ABCD is such that $AB = BC$, $AD = DC$, $AC \perp BD$, $\angle CAD = \theta$, then the angle $\angle ABC = ?$

(a) θ (b) $\frac{\theta}{2}$ (c) 2θ (d) 3θ

280. A quadrilateral ABCD circumscribes a circle and $AB = 6$ cm, $CD = 5$ cm and $AD = 7$ cm. The length of side BC is

(a) 4 cm (b) 5 cm
 (c) 3 cm (d) 6 cm

281. In a cyclic quadrilateral ABCD, $\angle A + \angle B + \angle C + \angle D = ?$

(a) 90° (b) 360°
 (c) 180° (d) 120°

282. A square ABCD is inscribed in a circle of 1 unit radius. Semi-circles are inscribed on each side of the square. The area of the region bounded by the four semi-circles and the circle is

(a) 1 sq. unit (b) 2 sq. unit
 (c) 1.5 sq. unit (d) 2.5 sq. unit

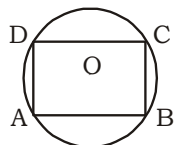
283. All sides of a quadrilateral ABCD touch a circle, If $AB = 6$ cm, $BC = 7.5$ cm, $CD = 3$ cm, then DA is

(a) 3.5 cm (b) 4.5 cm
 (c) 2.5 cm (d) 1.5 cm

284. ABCD is a cyclic quadrilateral. The side AB is extended to E in such a way that $BE = BC$, If $\angle ADC = 70^\circ$, $\angle BAD = 95^\circ$, then $\angle DCE$ is equal to

(a) 140° (b) 120°
 (c) 165° (d) 110°

285. In a cyclic quadrilateral $\angle A + \angle C = \angle B + \angle D = ?$



- (a) 270° (b) 360° (c) 90° (d) 180°
286. If ABCD be a cyclic quadrilateral in which $\angle A = 4x^\circ$, $\angle B = 7x^\circ$, $\angle C = 5y^\circ$, $\angle D = y^\circ$, then $x : y$ is

- (a) 3 : 4 (b) 4 : 3
 (c) 5 : 4 (d) 4 : 5

287. ABCD is a cyclic quadrilateral and AD is a diameter. If $\angle DAC = 55^\circ$, then value of $\angle ABC$ is

(a) 55° (b) 35°
 (c) 145° (d) 125°

288. A square is inscribed in a quarter-circle in such a manner that two of its adjacent vertices lie on the two radii at an equal distance from the centre, while the other two vertices lie on the circular arc. If the square has sides of length x . then the radius of the circle is:

(a) $\frac{16x}{\pi + 4}$ (b) $\frac{2x}{\sqrt{x}}$

(c) $\frac{\sqrt{5}x}{\sqrt{2}}$ (d) $\sqrt{2}x$

289. ABC is a cyclic triangle and the bisectors of $\angle BAC$, $\angle ABC$ and $\angle BCA$ meet the circle at P, Q and R respectively. Then the angle $\angle RQP$ is :

(a) $90^\circ - \frac{B}{2}$ (b) $90^\circ + \frac{C}{2}$

(c) $90^\circ - \frac{A}{2}$ (d) $90^\circ + \frac{B}{2}$

290. ABCD is a cyclic quadrilateral. AB and DC when produced meet at P, If $PA = 8$ cm, $PB = 6$, $PC = 4$ cm, then the length (in cm) of PD is

(a) 10 cm (b) 6 cm
 (c) 12 cm (d) 8 cm

291. AB is a diameter of a circle with centre O. The tangents at C meets AB produced at Q. If $\angle CAB = 34^\circ$, then measure of $\angle CBA$ is

(a) 56° (b) 68° (c) 34° (d) 124°

ANSWER KEY

1. (d)	31. (d)	61. (d)	91. (c)	121.(c)	151.(d)	181.(d)	211.(d)	241.(b)	271.(b)
2. (b)	32. (b)	62. (a)	92. (c)	122.(c)	152.(b)	182.(b)	212.(b)	242.(c)	272.(d)
3. (a)	33. (d)	63. (b)	93. (b)	123.(b)	153.(a)	183.(c)	213.(d)	243.(a)	273.(c)
4. (b)	34. (b)	64. (a)	94. (c)	124.(a)	154.(a)	184.(c)	214.(d)	244.(d)	274.(d)
5. (a)	35. (c)	65. (a)	95. (d)	125.(d)	155.(d)	185.(d)	215.(c)	245.(b)	275.(a)
6. (c)	36. (c)	66. (c)	96. (b)	126.(b)	156.(b)	186.(b)	216.(b)	246.(c)	276.(b)
7. (a)	37. (a)	67. (b)	97. (c)	127.(c)	157.(c)	187.(a)	217.(a)	247.(a)	277.(b)
8. (c)	38. (b)	68. (c)	98. (b)	128.(a)	158.(a)	188.(d)	218.(b)	248.(d)	278.(a)
9. (b)	39. (d)	69. (c)	99. (b)	129.(b)	159.(b)	189.(b)	219.(b)	249.(b)	279.(c)
10. (a)	40. (a)	70. (c)	100.(d)	130.(b)	160.(b)	190.(a)	220.(c)	250.(c)	280.(a)
11. (b)	41. (a)	71. (d)	101.(d)	131.(d)	161.(d)	191.(b)	221.(c)	251.(a)	281.(b)
12. (a)	42. (a)	72. (a)	102.(b)	132.(a)	162.(c)	192.(a)	222.(b)	252.(b)	282.(b)
13. (b)	43. (a)	73. (a)	103.(c)	133.(d)	163.(d)	193.(b)	223.(d)	253.(d)	283.(d)
14. (a)	44. (d)	74. (d)	104.(c)	134.(d)	164.(b)	194.(c)	224.(a)	254.(a)	284.(a)
15. (c)	45. (a)	75. (b)	105.(a)	135.(a)	165.(a)	195.(c)	225.(a)	255.(c)	285.(d)
16. (c)	46. (a)	76. (d)	106.(d)	136.(c)	166.(c)	196.(d)	226.(a)	256.(a)	286.(b)
17. (a)	47. (c)	77. (b)	107.(c)	137.(b)	167.(b)	197.(b)	227.(a)	257.(c)	287.(c)
18. (a)	48. (c)	78. (b)	108.(b)	138.(b)	168.(a)	198.(a)	228.(b)	258.(b)	288.(c)
19. (d)	49. (c)	79. (d)	109.(a)	139.(c)	169.(b)	199.(b)	229.(d)	259.(d)	289.(a)
20. (a)	50. (b)	80. (d)	110.(c)	140.(d)	170.(b)	200.(a)	230.(b)	260.(a)	290.(c)
21. (b)	51. (d)	81. (d)	111.(a)	141.(c)	171.(b)	201.(d)	231.(c)	261.(b)	291.(a)
22. (d)	52. (c)	82. (a)	112.(*)	142.(a)	172.(c)	202.(a)	232.(b)	262.(c)	
23. (d)	53. (b)	83. (b)	113.(c)	143.(a)	173.(a)	203.(b)	233.(c)	263.(d)	
24. (a)	54. (b)	84. (c)	114.(a)	144.(b)	174.(d)	204.(c)	234.(c)	264.(a)	
25. (b)	55. (b)	85. (d)	115.(b)	145.(c)	175.(d)	205.(b)	235.(c)	265.(b)	
26. (b)	56. (b)	86. (d)	116.(d)	146.(a)	176.(c)	206.(d)	236.(c)	266.(b)	
27. (a)	57. (b)	87. (a)	117.(c)	147.(d)	177.(b)	207.(a)	237.(c)	267.(c)	
28. (c)	58. (b)	88. (d)	118.(b)	148.(b)	178.(c)	208.(c)	238.(b)	268.(c)	
29. (a)	59. (b)	89. (b)	119.(a)	149.(b)	179.(a)	209.(b)	239.(a)	269.(d)	
30. (b)	60. (a)	90. (a)	120.(d)	150.(d)	180.(d)	210.(a)	240.(b)	270.(a)	

SOLUTION

- $$\angle POA = 120^\circ$$

- $$OM^2 = 9, \quad OM = 3 \text{ cm}$$

-

$$\begin{aligned} OO' &= 8 + \sqrt{28} \\ &= 8 + 5.29 \\ &= 13.3 \text{ cm} \end{aligned}$$

Note: $\therefore \Delta AMO =$ Right angled triangle

In, ΔAMO

$$\Rightarrow AM = 6, AO = 10$$

then,

$$OM = 8$$

In, $\Delta AMO'$

$$\Rightarrow AM = 6, AO' = 8$$

then,

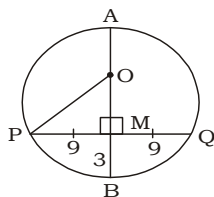
$$O'M = \sqrt{28}$$

$$\Rightarrow OO' = OM + O'M$$

$$= 8 + \sqrt{28}$$

$$\Rightarrow 13.3 \text{ cm}$$

13. (b)



According to the question

$$\text{Let } OA = x = OP$$

$$AB = 2x$$

$$OM = x - 3$$

In ΔOMP ,

$$x^2 = (9)^2 + (x - 3)^2$$

$$x^2 = 81 + x^2 + 9 - 6x$$

$$90 = 6x$$

$$x = 15$$

$$\therefore AB = 2 \times 15 = 30 \text{ cm.}$$

Alternate:-

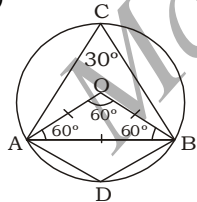
$$PM \times MQ = MB \times AM$$

$$9 \times 9 = 3 \times x$$

$$x = 27$$

$$\therefore AB = AM + BM = 27 + 3 = 30 \text{ cm}$$

14. (a)



Angle subtended by an arc at the centre is twice the angle subtended by the arc on the circle.

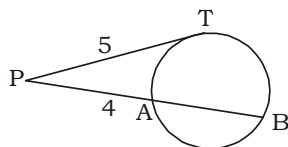
$$\therefore \angle C = 30^\circ$$

$$\angle D = 150^\circ$$

[Because in a cyclic quadrilateral sum of opposite

angle is 180°]

15. (c)



According to the question

$$PT = 5 \text{ cm.}$$

$$PA = 4 \text{ cm.}$$

$$PB = (4 + x) \text{ cm.}$$

As we know that

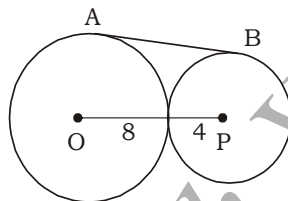
$$PT^2 = PA \times PB$$

$$25 = 4(4 + x)$$

$$25 = 16 + 4x$$

$$x = \frac{9}{4} \text{ cm.}$$

16. (c)



We know that,

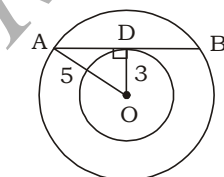
$$AB = 2\sqrt{r_1 r_2}$$

$$AB = 2\sqrt{8 \times 4}$$

$$AB = 2 \times \sqrt{32}$$

$$AB = 8\sqrt{2} \text{ cm}$$

17. (a)



According to the question

$$\text{Let } AD = DB = x$$

$$OA = 5$$

$$OD = 3$$

$$AB = 2x$$

In ΔODA ,

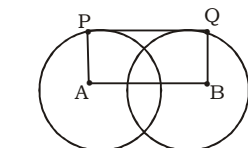
$$OA^2 = OD^2 + AD^2$$

$$(5)^2 = (3)^2 + (x)^2$$

$$x = 4$$

$$\therefore AB = 2 \times 4 = 8 \text{ cm.}$$

18. (a)



$$AB = 13 \text{ cm, } r_1 = 11 \text{ cm}$$

$$r_2 = 6 \text{ cm}$$

$$PQ = \sqrt{(\text{Distance between centres})^2}$$

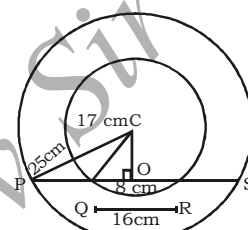
$$= \sqrt{(\text{radius}_1 - \text{radius}_2)^2}$$

$$PQ = \sqrt{(13)^2 - (11 - 6)^2}$$

$$PQ = \sqrt{169 - 25}$$

$$PQ = \sqrt{144}, \quad PQ = 12 \text{ cm}$$

19. (d) In right ΔCOQ ,



$$QC^2 = OQ^2 + OC^2 \text{ (By pt)}$$

$$17^2 = 8^2 + OC^2$$

$$OC = 15 \text{ cm}$$

In right ΔCOP

$$CP^2 = OP^2 + CO^2$$

$$25^2 = OP^2 + 15^2$$

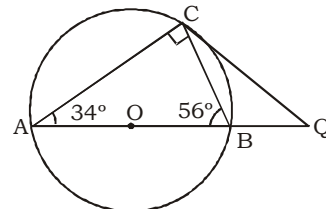
$$OP = 20 \text{ cm}$$

$$PS = 2 \times OP$$

$$= 2 \times 20$$

$$= 40 \text{ cm}$$

20. (a) In ΔACB



$$\angle ACB = 90^\circ$$

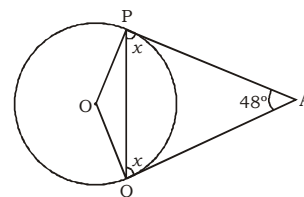
(Angle formed by semicircle is 90°)

$$\angle ACB + \angle CAB + \angle CBA = 180^\circ$$

$$90^\circ + 34^\circ + \angle CBA = 180^\circ$$

$$\angle CBA = 56^\circ$$

21. (b) In ΔAPQ , $\angle P = \angle Q = x$



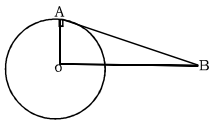
$$x^\circ + x^\circ + 48 = 180^\circ$$

$$2x = 132^\circ$$

$$x = 66^\circ$$

$$\therefore \angle APQ = 66^\circ$$

22. (d) According to question
Given:



OB = 10 cm, OA = radius = 6 cm

AB = tangent

In right angle $\triangle OAB$

By using pythagoras theorem

$$OB^2 = OA^2 + AB^2$$

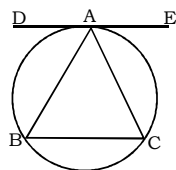
$$(10)^2 = (6)^2 + (AB)^2$$

$$(AB)^2 = 100 - 36$$

$$(AB)^2 = 64$$

$$AB = 8$$

23. (d) According to question
Given:



DE || BC, AB = 17 cm
AC = ?

$$\angle DAB = \angle ACB$$

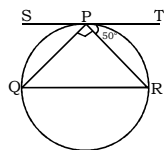
(By alternate segment theorem)

$$\angle DAB = \angle ABC \text{ (Alternate angle)}$$

$$\therefore \angle ABC = \angle ACB$$

$$AB = AC = 17 \text{ cm}$$

24. (a) According to question
Given:



$$\angle RPT = 50^\circ$$

$\angle QPR = 90^\circ$ (Angle in Semicircle)

$$\therefore \angle PQR = 50^\circ$$

(Alternate segment theorem)

In $\triangle PQR$

$$\angle P + \angle Q + \angle R = 180^\circ$$

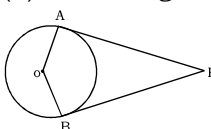
$$\angle R = 180^\circ - 90^\circ - 50^\circ$$

$$\angle R = 40^\circ$$

$$\therefore \angle SPQ = 40^\circ$$

(Alternate segment theorem)

25. (b) According to question



$$\angle AOB = 110^\circ$$

$$\angle APB = ?$$

AOPB is a quadrilateral

$$\angle O + \angle A + \angle P + \angle B = 360^\circ$$

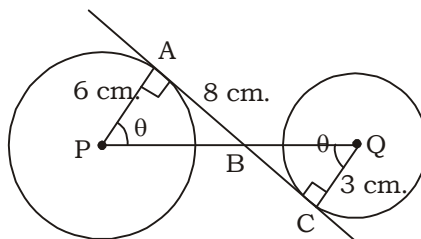
$$110^\circ + 90^\circ + 90^\circ + \angle P = 360^\circ$$

$$\angle P = 360^\circ - 290^\circ$$

$$\angle APB = 70^\circ$$

Therefore $\angle APB = 70^\circ$

26. (b)



According to the question

$$\Rightarrow AP = 6 \text{ cm (Radius}_1\text{)}$$

$$\Rightarrow QC = 3 \text{ cm (Radius}_2\text{)}$$

As we know, any line draw from centre to the tangent be perpendicular

$$\Rightarrow \text{So, } \angle PAB = \angle QCB = 90^\circ$$

$$\Rightarrow \angle APB = \angle CQB = \theta$$

[same alternative angle]

$$\Rightarrow \text{So, } \triangle APB \sim \triangle CQB$$

$$\Rightarrow \frac{AP}{CQ} = \frac{AB}{CB}$$

$$\Rightarrow \frac{6}{3} = \frac{8}{CB}$$

$$\Rightarrow CB = 4 \text{ cm.}$$

In right angled triangle $\triangle PAB$

$$\Rightarrow (PB)^2 = (PA)^2 + (AB)^2$$

$$\Rightarrow (PB)^2 = 6^2 + 8^2$$

$$\Rightarrow PB = 10 \text{ cm.}$$

Again, right angled triangle $\triangle CQB$

$$\Rightarrow BQ^2 = (BC)^2 + (CQ)^2$$

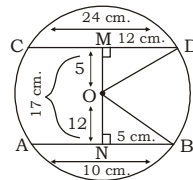
$$\Rightarrow (BQ)^2 = 3^2 + 4^2$$

$$\Rightarrow BQ = 5 \text{ cm.}$$

$$\Rightarrow \text{Therefore } PQ = PB + BQ$$

$$\Rightarrow 10 + 5 = 15 \text{ cm.}$$

27. (a)



According to the question

$\therefore \triangle OMD$ and $\triangle ONB$ are right angled triangle

And, $OD = OB = R$

In $\triangle OMD$, from triplets

5, 12, 13

$OM = 5 \text{ cm, } OD = 13 \text{ cm, } MD = 12 \text{ cm.}$

Again, In $\triangle ONB$, from triplets

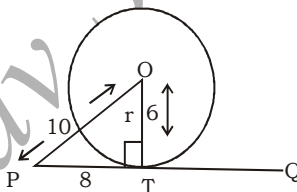
5, 12, 13

$OB = 13 \text{ cm, } ON = 12 \text{ cm}$

$\therefore OB = OD = 13 \text{ cm}$

Radius = 13 cm.

28. (c)



Let PTQ is the

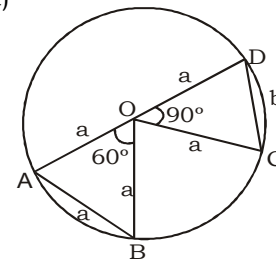
Tangent of circle having centre O and Radius = r and point T, touches the circle

$\Rightarrow$ We know that any line draw on tangent's touching point from centre, always makes a perpendicular

So $\angle OTP = 90^\circ$ length of PT = 8 cm

$$\left\{ \begin{array}{ccc} 3, & 4, & 5 \\ 2 \times \downarrow & \times \downarrow 2 & \times \downarrow 2 \\ 6 & 8 & 10 \end{array} \right\}$$

29. (a)



$$\angle AOB = 60^\circ$$

$$\angle COD = 90^\circ$$

$\Rightarrow$ length of chord AB = a

$\Rightarrow$ length of chord CD = b

$\Rightarrow AO = OB = AB = OD = OC = a$

$\Rightarrow$ In $\triangle ODC$

$$\Rightarrow OD^2 + OC^2 = CD^2$$

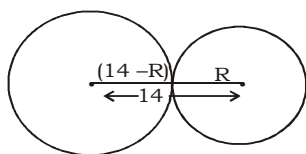
$$\Rightarrow a^2 + a^2 = b^2$$

$$\Rightarrow b = \sqrt{2}a$$

30. (b) Let smallest circle radius = R

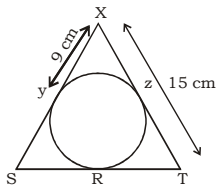
Then biggest circle radius

$$= (14 - R)$$



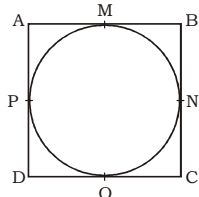
- $\Rightarrow$ According to the question
 $\Rightarrow \pi(14-R)^2 + \pi R^2 = 130\pi$
 $\Rightarrow (14-R)^2 + R^2 = 130$
 $\Rightarrow 196 + R^2 - 28R + R^2 = 130$
 $\Rightarrow R = 3 \text{ cm}$
 $\Rightarrow$ Radius of smallest circle R = 3 cm

31. (d)



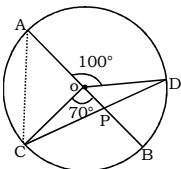
- $xy = 9 \text{ cm}, \quad tx = 15 \text{ cm}$
 $\Rightarrow$ We know
 Length of tangents drawn from a point to the circle are equal
 Therefore
 $xy = xz = 9 \text{ cm}, \quad tz = rt$
 $tx = 15 \text{ cm}$
 $xz + zt = 15$
 $zt = 15 - 9 = 6$
 $RT = ZT = 6 \text{ cm}$

32. (a) (b)



- $\Rightarrow$ According to figure
 $\Rightarrow PA = AM$
 (equal tangent drawn from a external point)
 $\Rightarrow PD = OD$
 $\Rightarrow MB = BN$
 $\Rightarrow OC = CN$
 $\Rightarrow \frac{(AB+CD)}{(CB+AD)}$
 $= \frac{(AM+BM)+(OD+OC)}{(CN+NB)+(AP+DP)} = 1$

33. (d) According to question
Given :



$$\angle AOD = 100^\circ \quad \angle BOC = 70^\circ$$

$$\therefore \angle ACD = \angle ACP = \frac{100^\circ}{2} = 50^\circ$$

$\therefore$ The angle subtended at the centre is twice to that of angle at the circumference by the same arc

$$\angle BOC = 70^\circ$$

$$\therefore \angle BDC = \angle BAC = \frac{70^\circ}{2} = 35^\circ$$

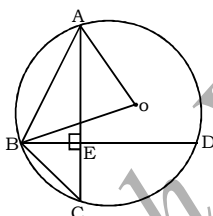
In $\triangle APC$,

$$\angle PAC + \angle ACP + \angle APC = 180^\circ$$

$$\angle APC = 180^\circ - 50^\circ - 35^\circ$$

$$\angle APC = 95^\circ$$

34. (b) According to question
Given:



$$\angle OAB = 25^\circ \quad OA = OB = r$$

$$\therefore \angle OAB = \angle OBA = 25^\circ$$

$$\therefore \angle AOB = 180^\circ - 25^\circ - 25^\circ$$

$$\angle AOB = 130^\circ$$

$$\therefore \angle ACB = \frac{1}{2} \angle AOB$$

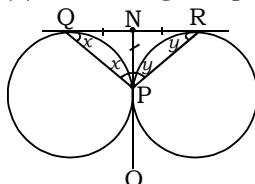
$$= \frac{130^\circ}{2} = 65^\circ$$

In right angle $\triangle BEC$

$$\angle BEC + \angle CBE + \angle ECB = 180^\circ$$

$$\angle CBE = 180^\circ - 65^\circ - 90^\circ = 25^\circ$$

35. (c) According to question



QR is the common tangent and NO is also the common tangent.

$$\therefore QN = NP = NR$$

In $\triangle QPN$

$$\angle NQP = \angle NPQ$$

$$\angle NRP = \angle NPR$$

In $\triangle PQR$

$$\angle P + \angle Q + \angle R = 180^\circ$$

$$x + y + x + y = 180^\circ$$

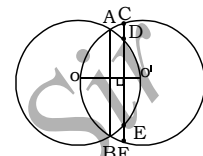
$$2x + 2y = 180^\circ$$

$$x + y = 90^\circ$$

As shown in the figure

$$x + y = \angle P = 90^\circ$$

36. (c) According to question
Given :



$$CD = 4.5 \text{ cm}$$

$$EF = ?$$

Let O and O' be the centre of a circle

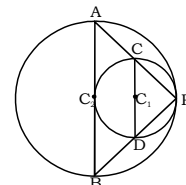
$$\therefore OC = OF = \text{radius of circle B}$$

$$\therefore OO' \perp O'E$$

$$\therefore O'D = O'E$$

$$\text{Clearly } CD = EF = 4.5 \text{ cm}$$

37. (a) According to question
Given:



$$\angle BDC = 120^\circ \quad \angle ABP = ?$$

$$\therefore \angle CDP = 180^\circ - \angle BDC$$

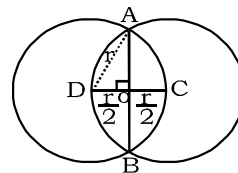
$$\angle CDP = 180^\circ - 120^\circ$$

$$\angle CDP = 60^\circ$$

$$CD \parallel AB$$

$$\therefore \angle CDP = \angle ABP = 60^\circ$$

38. (b) According to question



Let the radius of the circle be = r

$$\therefore DO = OC = \frac{r}{2}$$

In right angle $\triangle AOD$

By using pythagoras theorem

$$AD^2 = OD^2 + AO^2$$

$$r^2 = \frac{r^2}{4} + AO^2$$

$$AO^2 = r^2 - \frac{r^2}{4}$$

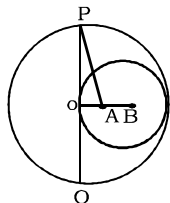
$$AO^2 = \frac{3r^2}{4}$$

$$AO = \frac{\sqrt{3}r}{2} \quad AB = 2 \times AO$$

$$AB = \frac{\sqrt{3}}{2} r \times 2,$$

$$AB = \sqrt{3}r \text{ units}$$

39. (d) According to question



$$AP = 5 \text{ cm}, \quad OB = 3 \text{ cm}$$

PQ is $\perp$ bisector

$$\therefore AO = 1, \quad PO = OQ$$

In right angle $\triangle POA$

$$AP^2 = OA^2 + OP^2$$

$$(5)^2 = (1)^2 + (OP)^2$$

$$(OP)^2 = 25 - 1$$

$$(OP)^2 = 24$$

$$(OP) = 2\sqrt{6} \text{ cm}$$

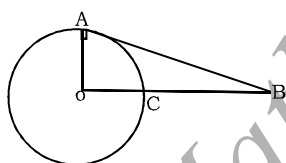
$$\therefore PQ = 2 \times OP$$

$$PQ = 2 \times 2\sqrt{6}$$

$$PQ = 4\sqrt{6} \text{ cm}$$

40. (a) According to question

Given:



$$OA = \text{radius} = 5 \text{ units}$$

$$AB = 5\sqrt{3} \text{ units}$$

$$BC = ?$$

In right angle $\triangle OAB$

$$OB^2 = AB^2 + OA^2$$

$$OB^2 = (5\sqrt{3})^2 + (5)^2$$

$$OB^2 = 75 + 25$$

$$OB^2 = 100$$

$$OB = 10 \text{ units}$$

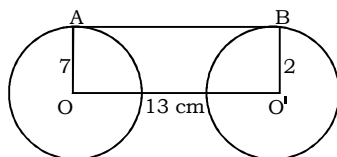
$$\therefore BC = OB - OC$$

$$BC = 10 - 5$$

$$BC = 5 \text{ units}$$

41. (a) According to question

Given:



$$OO' = 13 \text{ cm}, \quad OA = 7 \text{ cm}$$

$$O'B = 2 \text{ cm}$$

$\therefore$ Length of direct common tangent

$$AB = \sqrt{(OO')^2 - (R - r)^2}$$

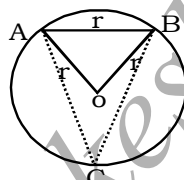
$$AB = \sqrt{(13)^2 - (7 - 2)^2}$$

$$AB = \sqrt{169 - 25}$$

$$AB = \sqrt{144}$$

$$AB = 12 \text{ cm}$$

42. (a) According to question



Let AB is the chord and 'O' is the centre of circle

Given : The length of AB is equal to radius

$$\therefore OA = OB = AB = r$$

$\therefore \triangle AOB$ is an equilateral triangle

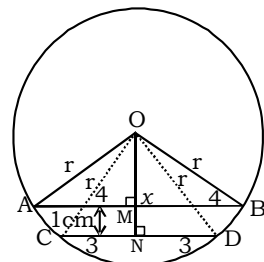
$$\angle AOB = 60^\circ$$

$\therefore \angle ACB$ which chord subtends in the major segment

$$\text{is } = \frac{60^\circ}{2} = 30^\circ$$

43. (a) According to question,

Given:



AB and CD are chord

$$AB = 8 \text{ cm}$$

$$\text{Let } ON = x \text{ cm}$$

$\therefore$ In, $\triangle OMA$

$$OA^2 = OM^2 + AM^2$$

$$r^2 = (x-1)^2 + (4)^2$$

$$r^2 = (x-1)^2 + 16 \quad \dots\dots\dots(i)$$

In $\triangle OND$

$$OD^2 = ON^2 + ND^2$$

$$r^2 = x^2 + (3)^2$$

$$r^2 = x^2 + 9 \quad \dots\dots\dots(ii)$$

Comparing equations (i) and (ii)

$$(x-1)^2 + 16 = x^2 + 9$$

$$x^2 + 1 - 2x + 16 = x^2 + 9$$

$$17 - 2x = 9$$

$$2x = 8$$

$$x = 4$$

Put the value of 'x' in equations(ii)

$$r^2 = (4)^2 + 9$$

$$r^2 = 16 + 9$$

$$r^2 = 25$$

$$r = 5 \text{ cm}$$

Alternate:-

$$AM = 4 \text{ cm}$$

$$CN = 3 \text{ cm}$$

{3,4,5} = formed a triplet

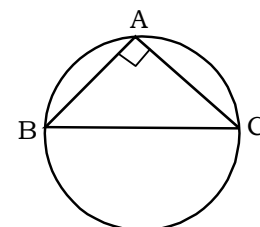
$\therefore$ Radius = 5 cm

44. (d) According to question,

AB and AC is a chord

$$AB = 8$$

$$AC = 6$$



In $\triangle BAC$

$$\angle A = 90^\circ$$

$$\therefore BC^2 = AB^2 + AC^2$$

$$BC^2 = 8^2 + 6^2$$

$$BC^2 = 64 + 36$$

$$BC^2 = 100$$

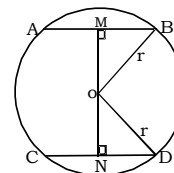
$$BC = 10 \text{ cm}$$

Here BC is the diameter of a circle because only subtended on the arc of semi circle is 90°

$$\therefore \frac{BC}{2} = \text{radius} = \frac{10}{2} = 5 \text{ cm}$$

45. (a) According to question

Given:



$$AB = CD = 8 \text{ cm}$$

$$r = 5 \text{ cm}$$

∴ In $\triangle OMB$

$$OB^2 = OM^2 + MB^2$$

$$r^2 = OM^2 + (4)^2$$

$$(5)^2 = OM^2 + 16$$

$$25 - 16 = OM^2$$

$$OM^2 = 25 - 16$$

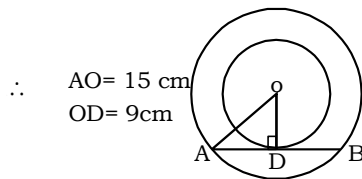
$$OM^2 = 9$$

$$OM = 3$$

$$\therefore MN = 2 \times OM$$

$$MN = 2 \times 3 = 6 \text{ cm}$$

46. (a) According to questions,
Let 'O' be the centre of a circle and 'AB' is the chord of the biggest circle



∴ In $\triangle ODA$

$$OA^2 = OD^2 + AD^2$$

$$(15)^2 = (9)^2 + AD^2$$

$$AD^2 = 225 - 81$$

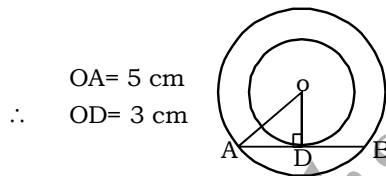
$$AD^2 = 144$$

$$AD = 12 \text{ cm}$$

$$\therefore AB = 2 \times AD$$

$$AB = 2 \times 12 = 24 \text{ cm}$$

47. (c) According to question
Let 'AB' is the chord of biggest circle and 'O' be the centre of a circle



∴ In $\triangle ODA$

$$OA^2 = OD^2 + AD^2$$

$$(5)^2 = (3)^2 + AD^2$$

$$AD^2 = 25 - 9$$

$$AD^2 = 16$$

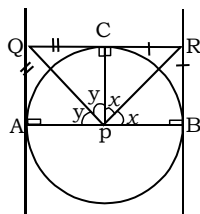
$$AD = 4 \text{ cm}$$

$$\therefore AB = 2 \times AD$$

$$= 2 \times 4$$

$$AB = 8 \text{ cm}$$

48. (c) According to question



In $\triangle PCR$ and $\triangle RBP$

PC = PB (radius)

RC = RB

PR is common

$$\therefore \triangle PCR \cong \triangle RBP$$

$$\angle CPR = \angle RPB = x$$

(CPCT)

Similarly,

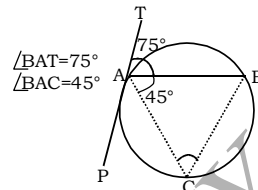
$$\angle APQ = \angle CPQ = y \quad (\text{CPCT})$$

$$\therefore 2y + 2x = 180^\circ$$

$$x + y = 90^\circ$$

$$\therefore \angle QPR = 90^\circ$$

49. (c) According to question,
Given:



$$\angle BAT = \angle BCA$$

(∵ Due to Alternate Segment theorem)

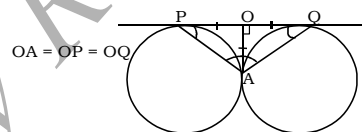
$$\therefore \angle BCA = 75^\circ$$

$$\text{Then } \angle BAC + \angle BCA + \angle ABC = 180^\circ$$

$$45^\circ + 75^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 60^\circ$$

50. (b) According to Question,
AO is perpendicular to PQ

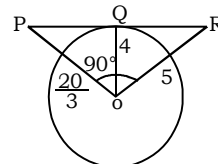


$$\angle OPA = \angle OAP = \angle OQA = \angle OAQ$$

$$\angle OAQ = 45^\circ$$

$$\therefore \angle PAQ = 90^\circ$$

51. (d) According to Question,
Given:



$$\angle POR = 90^\circ$$

$$OR = 5 \text{ cm}$$

$$OQ = 4 \text{ cm}$$

$$OP = \frac{20}{3} \text{ cm}$$

∴ In $\triangle POR$

$$PR^2 = PO^2 + OR^2$$

$$PR^2 = \left(\frac{20}{3}\right)^2 + (5)^2$$

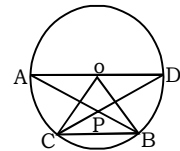
$$PR^2 = \frac{400}{9} + 25$$

$$PR^2 = \frac{400 + 225}{9}$$

$$PR^2 = \frac{625}{9}$$

$$PR = \frac{25}{3} \text{ cm}$$

52. (c) According to Question



$$\angle AOC + \angle BOD = 2\angle ABC + 2\angle BCD$$

(Angle formed on major arc is half the angle formed on centre)

$$= 2\angle ABC + 2\angle BCD$$

$$= 2\angle BPD$$

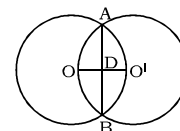
[Exterior angles of triangles

$$\angle AOC + \angle BOD = 2\angle BPD$$

$$2\angle BPD = 50^\circ + 40^\circ$$

$$\angle BPD = \frac{1}{2} \times 90^\circ = 45^\circ$$

53. (b) According to question
AB is a common chord
O and O' is the centre of the circle.



In $\triangle ODA$

$$AO^2 = AD^2 + OD^2$$

$$(4)^2 = (AD)^2 + (2)^2$$

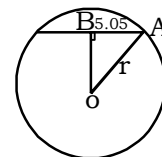
$$AD^2 = 12$$

$$AD = 2\sqrt{3}$$

$$\therefore AB = 2 \times AD$$

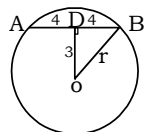
$$AB = 2 \times 2\sqrt{3} = 4\sqrt{3}$$

54. (b) According to question
OBA is a right angle triangle



- ∴ OA is a hypotenuse
 ∴ Hypotenuse is always greater than other two sides
 ∴ Radius is always greater than 5 cm

55. (b) According to question.



In $\triangle BDO$, using pythagoras

$$BO^2 = OD^2 + BD^2$$

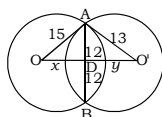
$$r^2 = (4)^2 + (3)^2$$

$$r^2 = 16 + 9$$

$$r^2 = 25$$

$$r = 5$$

56. (b) Let $OD = x$ and $DO' = y$
 According to question



In $\triangle ADO$

$$AO^2 = OD^2 + AD^2$$

$$(15)^2 = x^2 + (12)^2$$

$$x^2 = 225 - 144$$

$$x^2 = 81$$

$$x = 9$$

In $\triangle ADO'$

$$(AO')^2 = AD^2 + DO'^2$$

$$(13)^2 = (12)^2 + y^2$$

$$169 = 144 + y^2$$

$$y^2 = 169 - 144$$

$$y^2 = 25$$

$$y = 5$$

$$\therefore x + y = 9 + 5 = 14$$

57. (b) According to question

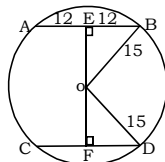
$$\text{length of arc} = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{72^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$$

$$= 26.4 \text{ cm}$$

58. (b) one and only one circle can pass through 3 non-collinear points.

59. (b) According to question



$$AB = 24 \text{ cm}$$

$$AE = EB = 12 \text{ cm}$$

$$OE = \sqrt{OB^2 - EB^2}$$

$$= \sqrt{15^2 - 12^2}$$

$$= \sqrt{225 - 144} = \sqrt{81}$$

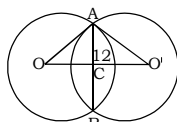
$$= 9 \text{ cm}$$

$$\therefore OF = 21 - 9 = 12 \text{ cm}$$

$$\text{also } FD = \sqrt{15^2 - 12^2} = 9 \text{ cm}$$

$$\therefore CD = 2 \times 9 = 18 \text{ cm}$$

60. (a) According to Question



$$AB = 16$$

$$AC = BC = 8 \text{ cm}$$

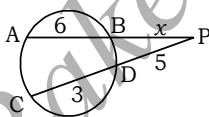
$$OC = OC' = 6 \text{ cm}$$

$$OA = \sqrt{OC^2 + CA^2}$$

$$OA = \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$$

$$OA = \sqrt{100} = 10 \text{ cm}$$

61. (d) According to Question



Given: $AB = 6$, $CD = 3$.

$$PD = 5$$

Let $PB = x$

Note: If chords AB and CD intersect

externally at point, p then

$$PB \times PA = PD \times PC$$

$$x \times (6 + x) = 5 \times 8$$

$$x^2 + 6x - 40 = 0$$

$$x^2 + 10x - 4x - 40 = 0$$

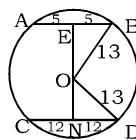
$$x(x + 10) - 4(x + 10) = 0$$

$$(x + 10)(x - 4) = 0$$

$$x = 4, -10 \quad (-10 \text{ neglected})$$

$$\therefore PB = 4 \text{ cm}$$

62. (a) According to question.



$$AE = EB = 5 \text{ cm}$$

$$CN = DN = 12 \text{ cm}$$

$$\triangle EOB$$

$$r = \sqrt{OE^2 + EB^2}$$

$$(13)^2 = (OE)^2 + (5)^2$$

$$169 = (OE)^2 + 25$$

$$OE^2 = 169 - 25$$

$$OE^2 = 144$$

$$OE = 12 \text{ cm}$$

In $\triangle OND$

$$OD^2 = ON^2 + ND^2$$

$$(13)^2 = ON^2 + (12)^2$$

$$169 = ON^2 + 144$$

$$ON^2 = 169 - 144$$

$$ON^2 = 25$$

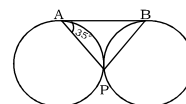
$$ON = 5$$

$$\therefore EN = OE + ON$$

$$EN = 12 + 5$$

$$= 17 \text{ cm}$$

63. (b) According to question



Given: $\angle PAB = 35^\circ$

As we know that

$$\angle APB = 90^\circ$$

Therefore,

$$\therefore \angle PAB + \angle APB + \angle ABP = 180^\circ$$

$$\angle ABP = 180^\circ - 90^\circ - 35^\circ$$

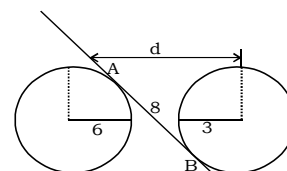
$$\angle ABP = 55^\circ$$

64. (a) According to question

Let length of transverse

common tangent = $AB = 8 \text{ cm}$

Distance between them = d



$$AB = \sqrt{d^2 - (R_1 + R_2)^2}$$

$$AB^2 = d^2 - (R_1 + R_2)^2$$

$$(8)^2 = d^2 - (6 + 3)^2$$

$$64 = d^2 - 81$$

$$d^2 = 145$$

$$d = \sqrt{145}$$

65. (a) According to Question

Let length of transverse

common tangent be AB

Distance between them = 10 cm

$$AB = \sqrt{d^2 - (R_1 + R_2)^2}$$

$$AB = \sqrt{(10)^2 - (3 + 3)^2}$$

$$AB = \sqrt{100-36}$$

$$AB = \sqrt{64}$$

$$AB = 8 \text{ cm}$$

66. (c) According to question
Length of the transverse common tangent

$$AB = \sqrt{d^2 - (R_1 + R_2)^2}$$

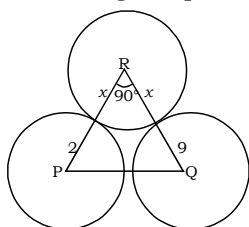
$$AB = \sqrt{(24)^2 - (5+3)^2}$$

$$AB = \sqrt{576-64}$$

$$AB = \sqrt{512}$$

$$AB = 16\sqrt{2}$$

67. (b) According to question



$$\angle PRQ = 90^\circ$$

$$PR = 2 + x$$

$$PQ = 17$$

$$RQ = 9 + x$$

By using pythagoras theorem

$$PQ^2 = PR^2 + RQ^2$$

$$(17)^2 = (2+x)^2 + (9+x)^2$$

$$289 = 4 + x^2 + 4x + 81 + x^2 + 18x$$

$$x^2 + 11x - 102 = 0$$

$$x^2 + 17x - 6x - 102 = 0$$

$$x(x+17) - 6(x+17) = 0$$

$$(x+17)(x-6) = 0$$

$$x = 6 \text{ as } x \neq -17$$

$$\therefore x = 6 \text{ cm}$$

Alternate

$\Delta PRQ = \text{Right angle } \Delta$

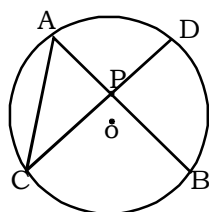
PQ(H) QR(B) PR(P)

17cm (9+x)cm (2+x)cm

Triplet = (17, 15, 8)

$$\therefore x = 6 \text{ cm}$$

68. (c) According to question



$$\angle BOC = 2\angle BAC$$

$$\therefore \angle AOD = 2\angle ACD$$

$$\therefore \angle BOC + \angle AOD = 2(\angle BAC + \angle ACD)$$

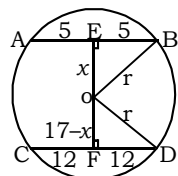
$$= 2\angle BPC$$

$$30^\circ + 20^\circ = 2\angle BPC$$

$$\angle BPC = \frac{50^\circ}{2}$$

$$\angle BPC = 25^\circ$$

69. (c) According to question



$$AE = EB = 5 \text{ cm}$$

$$CF = FD = 12 \text{ cm}$$

$$BO = OD = r \text{ cm}$$

$$\therefore \text{In } \Delta BOE$$

$$r^2 = x^2 + 5^2$$

$$\dots (i)$$

$$\text{In } \Delta DOF$$

$$r^2 = (17-x)^2 + (12)^2$$

$$\dots (ii)$$

Compare equation (i) and (ii)

$$x^2 + 25 = 289 + x^2 - 34x + 144$$

$$25 = 433 - 34x$$

$$34x = 408$$

$$x = 12 \dots (iii)$$

Put the value of x in equation (i)

$$r^2 = (12)^2 + (5)^2$$

$$r^2 = 144 + 25$$

$$r^2 = 169$$

$$r = 13 \text{ cm}$$

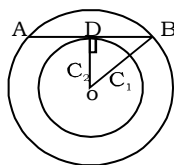
Alternate:-

Apply triplet

$$5, 12, 13$$

$$r = 13 \text{ cm}$$

70. (c) According to question



$$AD = DB = x$$

$$C_2 = (\sqrt{3}-1) \text{ cm}$$

$$C_1 = (\sqrt{3}+1) \text{ cm}$$

$$\text{In } \Delta BOD$$

$$C_1^2 = C_2^2 + BD^2$$

$$(\sqrt{3}+1)^2 = (\sqrt{3}-1)^2 + x^2$$

$$4 + 2\sqrt{3} = 4 - 2\sqrt{3} + x^2$$

$$x^2 = 4\sqrt{3}$$

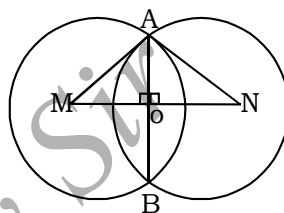
$$x = (4\sqrt{3})^{1/2} = 2\sqrt{3}$$

$$\therefore AB = 2 \times BD$$

$$AB = 2 \times 2\sqrt{3}$$

$$AB = 4\sqrt{3} \text{ cm}$$

71. (d) According to question



$$\text{Let } AO = OB = x$$

$$MO = y$$

$$ON = 50 - y$$

$$AM = 30 \text{ cm}$$

$$AN = 40 \text{ cm}$$

$$\text{In } \Delta AOM$$

$$AM^2 = OA^2 + OM^2$$

$$(30)^2 = OA^2 + y^2$$

$$x^2 = 900 - y^2 \dots (i)$$

$$\text{In } \Delta AON$$

$$AN^2 = ON^2 + OA^2$$

$$(40)^2 = (50 - y)^2 + x^2$$

$$(x)^2 = 1600 - (50 - y)^2 \dots (ii)$$

Compare equation (i) and (ii)

$$900 - y^2 = 1600 - (50 - y)^2$$

$$900 - y^2 = 1600 - (2500 + y^2 - 100y)$$

$$900 - y^2 = 1600 - 2500 - y^2 + 100y$$

$$y = 18 \dots (iii)$$

put the value of y in equation (i)

$$x^2 = 900 - 324$$

$$x^2 = 576$$

$$x = 24 \text{ cm}$$

$$OA = 24 \text{ cm}$$

$$AB = 2 \times 24$$

$$AB = 48 \text{ cm}$$

Alternate:-

$$30, 40, 50 \text{ (triplet)}$$

$\Delta AMN = \text{Right triangle}$

$$\angle MAN = 90^\circ$$

$$\Delta AMN = \frac{1}{2} \times b \times h$$

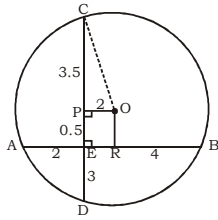
$$\frac{1}{2} \times 30 \times 40 = \frac{1}{2} \times 50 \times AO$$

$$AO = 24$$

$$AB = 2AO$$

$$\therefore AB = 48 \text{ cm}$$

72. (a) According to questions



Given:-

$$AE = 2 \text{ cm}$$

$$EB = 6 \text{ cm}$$

$$ED = 3 \text{ cm}$$

As we know that

$$AE \times EB = EC \times ED$$

$$2 \times 6 = EC \times 3$$

$$EC = 4 \text{ cm}$$

$\therefore$ In $\triangle OPC$

$$OC^2 = CP^2 + PO^2$$

$$r^2 = (2)^2 + \left(\frac{7}{2}\right)^2$$

$$r^2 = 4 + \frac{49}{4}$$

$$r^2 = \frac{65}{4}$$

$$r = \frac{\sqrt{65}}{2}$$

$$\therefore \text{Diameter} = 2r$$

$$D = 2 \times \frac{\sqrt{65}}{2}$$

$$D = \sqrt{65}$$

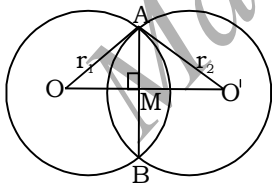
Alternate:-

$$\text{Diameter} = \sqrt{AE^2 + EB^2 + EC^2 + ED^2}$$

$$= \sqrt{2^2 + 6^2 + 3^2 + 4^2} = \sqrt{65}$$

$$\text{radius is equal to } \frac{\sqrt{65}}{2}$$

73. (a) According to question



$$r_1 = r_2 = 5 \text{ cm}$$

$$AM = MB = 4 \text{ cm}$$

$\therefore$ In $\triangle AMO$

$$OA^2 = OM^2 + AM^2$$

$$25 = OM^2 + 16$$

$$OM^2 = 25 - 16$$

$$OM^2 = 9$$

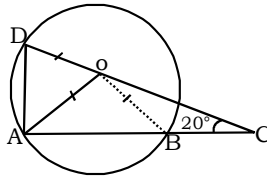
$$OM = 3 \text{ cm}$$

$$\therefore OO' = 2 \times OM$$

$$OO' = 2 \times 3$$

$$OO' = 6 \text{ cm}$$

74. (d) According to question



$$BC = DO = OA = OB = r$$

In $\triangle OBC$

$$\angle OCB = \angle COB = 20^\circ$$

In $\triangle AOB$

$$\angle OBA = 20^\circ + 20^\circ$$

$$\angle OBA = 40^\circ$$

$$\angle OBA = \angle OAB = 40^\circ$$

In $\triangle AOB$

$$\angle A + \angle O + \angle B = 180^\circ$$

$$40^\circ + \angle O + 40^\circ = 180^\circ$$

$$\angle O = 100^\circ$$

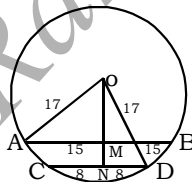
DOC is a line

$$\angle COB + \angle AOB + \angle DOA = 180^\circ$$

$$20^\circ + 100^\circ + \angle DOA = 180^\circ$$

$$\angle DOA = 60^\circ$$

75. (b) According to question



$$OA = OD = 17 \text{ cm}$$

$$AM = MB = 15 \text{ cm}$$

$$CN = ND = 8 \text{ cm}$$

In $\triangle OMA$

$$OA^2 = AM^2 + OM^2$$

$$(17)^2 = (15)^2 + OM^2$$

$$289 = 225 + OM^2$$

$$OM^2 = 289 - 225$$

$$OM^2 = 64$$

$$OM = 8$$

In $\triangle OND$

$$OD^2 = ON^2 + ND^2$$

$$(17)^2 = (8)^2 + ON^2$$

$$289 = 64 + ON^2$$

$$ON^2 = 289 - 64$$

$$ON^2 = 225$$

$$ON = 15$$

$$\therefore MN = ON - OM$$

$$MN = 15 - 8$$

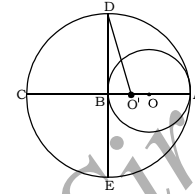
$$MN = 7 \text{ cm}$$

Alternate:-

$$17, 15, 8 \text{ (triplet)}$$

$$\text{distance on same side between chords} = (15 - 8) = 7 \text{ cm}$$

76. (d) According to question



$$O'A = 3 \text{ cm}$$

$$OA = 2 \text{ cm}$$

$$O'D = 3 \text{ cm}$$

$$O'B = 1 \text{ cm}$$

In $\triangle BDO$

$$O'D^2 = DB^2 + BO^2$$

$$BD^2 = (3)^2 - (1)^2$$

$$BD^2 = 9 - 1$$

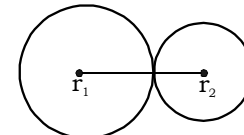
$$BD^2 = 8, \quad BD = 2\sqrt{2}$$

$$\therefore DE = 2 \times BD$$

$$DE = 2 \times 2\sqrt{2}$$

$$DE = 4\sqrt{2} \text{ cm}$$

77. (b) According to question



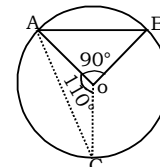
$$\text{Given: } r_1 + r_2 = 7 \text{ cm}$$

$$r_1 = 4 \text{ cm}$$

$$\therefore r_2 = 7 - 4$$

$$r_2 = 3 \text{ cm}$$

78. (b) According to question



$$OA = OB = OC$$

$\therefore$ In $\triangle OAB$

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$2\angle OAB = 180^\circ - 90$$

$$2\angle OAB = 90^\circ$$

$$\angle OAB = 45^\circ$$

In $\triangle OAC$

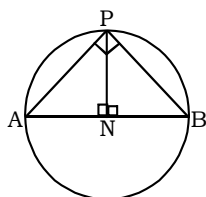
$$\angle OAC + \angle OCA + \angle AOC = 180^\circ$$

$$2\angle OAC = 180^\circ - 110^\circ$$

$$\angle OAC = 35^\circ$$

$$\therefore \angle BAC = 45^\circ + 35^\circ = 80^\circ$$

79. (d) According to question



$$AB = 2r = 14 \text{ cm}$$

$$PB = 12 \text{ cm}$$

$\angle APB = 90^\circ$ (angle in the semicircle)

$$\text{Let } AN = x \text{ and } NB = (14 - x)$$

$\therefore$ In $\triangle APB$

$$AB^2 = PB^2 + AP^2$$

$$(14)^2 = (12)^2 + (AP)^2$$

$$196 = 144 + (AP)^2$$

$$(AP)^2 = 196 - 144$$

$$(AP)^2 = 52$$

$$AP = \sqrt{52}$$

In $\triangle APN$

$$AP^2 = PN^2 + AN^2$$

$$(\sqrt{52})^2 = x^2 + PN^2$$

$$PN^2 = 52 - x^2 \dots\dots\dots(i)$$

In $\triangle PNB$

$$PB^2 = PN^2 + NB^2$$

$$(12)^2 = PN^2 + (14 - x)^2$$

$$PN^2 = 144 - (14 - x)^2 \dots\dots\dots(ii)$$

$$52 - x^2 = 144 - 196 - x^2 + 28x$$

$$28x = 104$$

$$x = \frac{104}{28}$$

$$x = \frac{26}{7}$$

$$NB = 14 - x$$

$$NB = 14 - \frac{26}{7}$$

$$NB = \frac{72}{7}$$

$$NB = 10\frac{2}{7} \text{ cm}$$

Alternate:-

$$PB^2 = PN \times AB$$

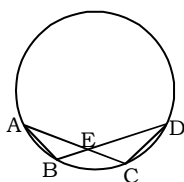
$$12^2 = 14 \times BN$$

$$BN = 12 \times 12 \div 14$$

$$BN = 72 \div 7$$

$$BN = 10\frac{2}{7}$$

80. (d) According to question



$$\text{Given: } \angle BEC = 130^\circ$$

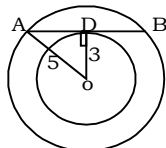
$$\Rightarrow \angle DEC = 180^\circ - 130^\circ = 50^\circ$$

$$\therefore \angle EDC = 180^\circ - 50^\circ - 20^\circ = 110^\circ$$

$$\therefore \angle BAC = \angle EDC = 110^\circ$$

(Angle on the same arc are equal)

81. (d) According to question



$$\text{Let } AD = DB = x$$

$$OA = 5 \text{ cm}$$

$$OD = 3 \text{ cm}$$

In $\triangle ODA$

$$OA^2 = OD^2 + AD^2$$

$$(5)^2 = (3)^2 + (AD)^2$$

$$25 = 9 + (AD)^2$$

$$(AD)^2 = 25 - 9$$

$$(AD)^2 = 16$$

$$AD = 4$$

$$\therefore AB = 2 \times AD$$

$$AB = 2 \times 4 = 8 \text{ cm}$$

Alternate:-

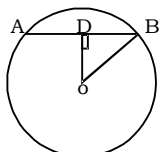
$$\text{In } \triangle AOD, \quad 3, 4, 5 \text{ (triplet)}$$

$$AD = 4 \text{ cm}$$

$$AB = 2AD$$

$$= 2 \times 4 = 8 \text{ cm}$$

82. (a) According to question



$$\text{Let } OD = x, \quad AD = DB = 6 \text{ cm}$$

$$OB = 10 \text{ cm}$$

In $\triangle ODB$

$$OB^2 = DB^2 + OD^2$$

$$OD^2 = OB^2 - DB^2$$

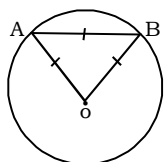
$$OD^2 = (10)^2 - (6)^2$$

$$OD^2 = 100 - 36$$

$$OD^2 = 64$$

$$OD = 8 \text{ cm}$$

83. (b) According to question



Let AB is the chord and 'O' is the centre of a circle

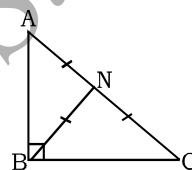
Given:-

$$OA = OB = AB$$

$\therefore$ All sides are equal then triangle is equilateral triangle.

$\therefore$ Then the angle subtended by the chord is 60°

84. (c) In a right angled triangle the circumcentre of the triangle lies on mid point of the hypotenuse



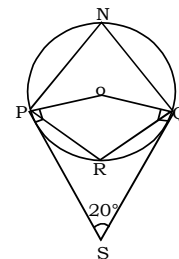
$$BN = \text{circumradius}$$

$$N = \text{circumcentre}$$

$$\therefore BN = AN = NC$$

$$BN = \frac{AC}{2} = \frac{H}{2}$$

85. (d) According to question,



$$\text{Given: } \angle PSQ = 20^\circ$$

$$\angle PRQ = ?$$

OPSQ is a quadrilateral

$$\angle OPS = \angle OQS = 90^\circ$$

$$\therefore \angle OPS + \angle OQS + \angle POQ + \angle QSP = 360^\circ$$

$$\angle OPS + \angle OQS + \angle POQ + \angle QSP = 360^\circ$$

$$\angle POQ = 360^\circ - 90^\circ - 90^\circ - 20^\circ$$

$$\angle POQ = 160^\circ$$

$$\therefore \angle PNQ = \frac{1}{2} \angle POQ$$

$$\angle PNQ = \frac{1}{2} \times 160^\circ = 80^\circ$$

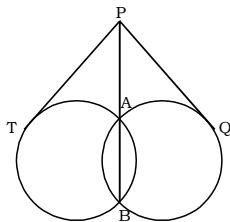
$\therefore$ NPRQ is a cyclic quadrilateral
sum of opposite angles of cyclic quadrilateral is 180°

$$\therefore \angle PNQ + \angle PRQ = 180^\circ$$

$$\angle PRQ = 180^\circ - 80^\circ$$

$$\angle PRQ = 100^\circ$$

86. (d) According to question



As shown in the figure Tangent are equal

$\therefore PT = PQ$

Alternate:-

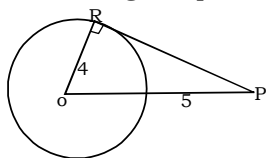
$$PQ^2 = PA \times PB \quad \dots (i)$$

$$PT^2 = PA \times PB \quad \dots (ii)$$

From both equation,

$$PT^2 = PQ^2 \quad PT = PQ$$

87. (a) According to question



ΔORP is a right angle triangle

$\therefore$ By using pythagoras theorem.

$$OP^2 = OR^2 + RP^2$$

$$(5)^2 = (4)^2 + (RP)^2$$

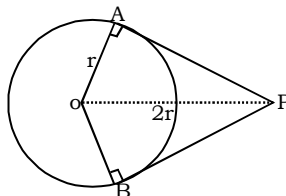
$$25 = 16 + (RP)^2$$

$$(RP)^2 = 25 - 16$$

$$(RP)^2 = 9$$

$$RP = 3 \text{ cm}$$

88. (d) According to question



Given: $OA = OB = r$ (radius)

$OP = 2r$ (diameter)

In ΔOAP

$$OP^2 = OA^2 + AP^2$$

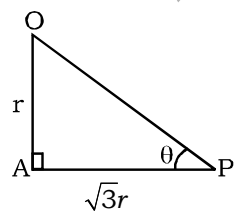
$$(2r)^2 = r^2 + AP^2$$

$$AP^2 = 4r^2 - r^2$$

$$AP^2 = 3r^2$$

$$AP = \sqrt{3}r$$

In ΔOAP



$$\tan \theta = \frac{OA}{AP} \quad \tan \theta = \frac{r}{\sqrt{3}r}$$

$$\tan \theta = \frac{1}{\sqrt{3}} \quad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\therefore \angle OPA = 30^\circ$$

Similarly in $\angle OPB$

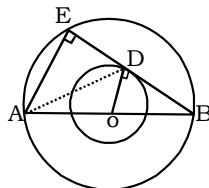
$$\therefore \angle OPB = 30^\circ$$

$$\therefore \angle APB = \angle OPA + \angle OPB$$

$$\angle APB = 30^\circ + 30^\circ$$

$$\angle APB = 60^\circ$$

89. (b) According to question



Given: $OA = OB = 13 \text{ cm}$

$OD = 8 \text{ cm}$

$$\therefore AE = 2 \times OD$$

$$AE = 2 \times 8 = 16 \text{ cm}$$

In ΔODB

$$OB^2 = OD^2 + BD^2$$

$$BD^2 = OB^2 - OD^2$$

$$BD^2 = (13)^2 - (8)^2$$

$$BD^2 = 169 - 64$$

$$BD^2 = 105$$

$$BD = \sqrt{105} \text{ cm}$$

$$\therefore DE = BD = \sqrt{105} \text{ cm}$$

$\therefore$ In ΔAED

$$AD^2 = DE^2 + AE^2$$

$$AD^2 = (\sqrt{105})^2 + (16)^2$$

$$AD^2 = 105 + 256$$

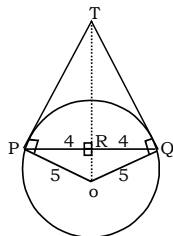
$$AD^2 = 361$$

$$AD = 19 \text{ cm}$$

90. (a) According to question

OT is the perpendicular bisector of chord PQ.

let $TR = y$



In right angle ΔPRO

$$PO^2 = PR^2 + RO^2$$

$$(5)^2 = (4)^2 + RO^2$$

$$(RO)^2 = 25 - 16$$

$$(RO)^2 = 9, \quad RO = 3 \text{ cm}$$

Right angle ΔTPO and ΔTRP

$$TO^2 = PT^2 + OP^2 \quad \dots (i)$$

$$PT^2 = TR^2 + PR^2 \quad \dots (ii)$$

Put the value of PT^2 in equation (i)

$$TO^2 = TR^2 + PR^2 + OP^2$$

$$(y + 3)^2 = y^2 + (4)^2 + (5)^2$$

$$y^2 + 9 + 6y = y^2 + 16 + 25$$

$$9 + 6y = 41, \quad 6y = 32$$

$$y = \frac{32}{6} = \frac{16}{3} \text{ cm}$$

In right angle ΔTRP

$$PT^2 = TR^2 + PR^2$$

$$PT^2 = \left(\frac{16}{3}\right)^2 + (4)^2$$

$$PT^2 = \frac{256}{9} + 16$$

$$PT^2 = \frac{400}{9}, \quad PT = \frac{20}{3} \text{ cm}$$

Alternate:-

In ΔPOR ,

$$OP^2 = OR^2 + PR^2$$

$$5^2 = OR^2 + 4^2$$

$$OR^2 = 25 - 16 = 9$$

$$\Rightarrow OR = 3 \text{ cm}$$

In ΔPOR and ΔPOT

$$\angle PRO = \angle TOP$$

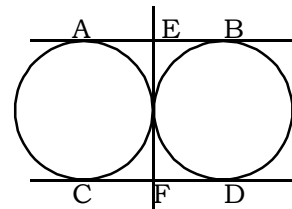
$$\Rightarrow \Delta POR \sim \Delta POT$$

$$\Rightarrow \frac{PR}{PT} = \frac{OR}{OP}$$

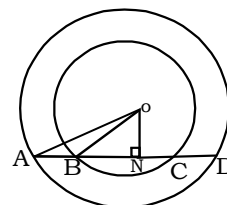
$$\Rightarrow \frac{4}{PT} = \frac{3}{5}$$

$$\Rightarrow PT = \frac{20}{3} \text{ cm}$$

91. (c) Maximum no. of tangent are



92. (c) According to question



Given: $BC = 12\text{ cm}$, $OA = 17\text{ cm}$

$$OB = 10\text{ cm}$$

$$\therefore BN = NC = 6\text{ cm}$$

$\therefore$ In right angle $\triangle ONB$

$$OB^2 = ON^2 + BN^2$$

$$(10)^2 = ON^2 + (6)^2$$

$$ON^2 = 100 - 36$$

$$ON^2 = 64$$

$$ON = 8\text{ cm}$$

In right angle $\triangle ONA$

$$OA^2 = ON^2 + AN^2$$

$$(17)^2 = (8)^2 + AN^2$$

$$AN^2 = 289 - 64$$

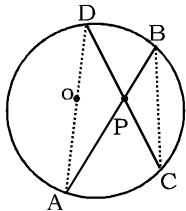
$$AN^2 = 225$$

$$AN = 15\text{ cm}$$

$$AD = 2 \times AN$$

$$\therefore AD = 15 \times 2 = 30\text{ cm}$$

93. (b) According to question



$$\angle ADP = \angle ABC = 23^\circ$$

$$\angle APC = 70^\circ = \angle DPB$$

$$\therefore \angle APD = 180^\circ - 70^\circ = 110^\circ$$

$$\angle APD = \angle BPC \text{ (Vertically opposite angle)}$$

$$\text{Also } \angle BCD = 180^\circ - 23^\circ - 110^\circ = 47^\circ$$

94. (c) According to figure

$$\angle DAC = 51^\circ$$

$$\angle EOB = 180^\circ - 150^\circ = 30^\circ$$

$$OB = OE = \text{radius}$$

$$\therefore \angle OEB = \angle OBE$$

then

$$\angle OEB + \angle OBE + \angle EOB = 180^\circ$$

$$2\angle OBE = 180^\circ - 30^\circ$$

$$\angle OBE = 75^\circ$$

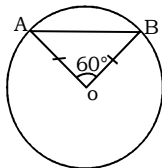
$$\therefore \angle CBE = 180^\circ - 75^\circ$$

$$\angle CBE = 105^\circ$$

95. (d) The angle in a semi-circle is a right angle

96. (b) According to question

Given :



$$\angle BOA = 60^\circ$$

$$OA = OB = r$$

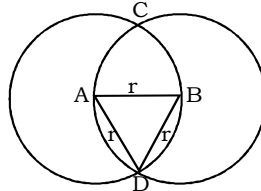
$$\therefore \angle OAB = \angle OBA = 60^\circ$$

$\therefore \triangle OAB$ is an equilateral triangle

$$\therefore OA = OB = AB$$

$$\frac{AB}{OB} = \frac{1}{1}$$

97. (c) According to question



$$AB = AD = DB = r$$

$\therefore \triangle ADB$ is an equilateral triangle

$$\angle DBA = 60^\circ$$

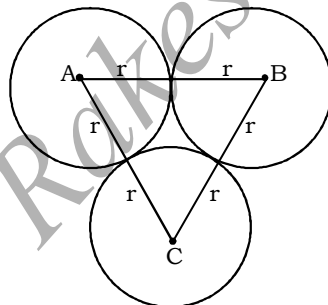
Similar In $\triangle ABC$

$$\angle ABC = 60^\circ$$

$$\angle DBC = 60^\circ + 60^\circ$$

$$\therefore \angle DBC = 120^\circ$$

98. (b) According to the question



Let radius of the circle be $= r$

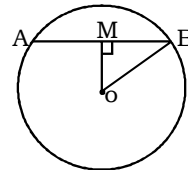
$$\therefore AB = 2r, BC = 2r, CA = 2r$$

All these sides are equal

$\therefore$ Triangle ABC is an equilateral $\triangle$

99. (b) According to the question

Given:



$$AB = 20\text{ cm}$$

$$AM = MB = 10\text{ cm}$$

$$OM = 2\sqrt{11}\text{ cm}$$

$$OM \perp AB$$

$$OB = \text{radius}$$

$\therefore$ In right angle $\triangle OMB$

By using pythagoras theorem

$$OB^2 = OM^2 + MB^2$$

$$OB^2 = (2\sqrt{11})^2 + (10)^2$$

$$OB^2 = 44 + 100$$

$$OB^2 = 144$$

$$OB = 12\text{ cm}$$

100. (d) According to the figure.

$$OM = OY = ON$$

$\therefore$ In $\triangle OMY$

$$\angle OMY = \angle OYM = 15^\circ$$

$$\therefore \angle MOY = 180^\circ - 15^\circ - 15^\circ$$

$$\angle MOY = 150^\circ$$

In $\triangle ONY$

$$\angle ONY = \angle OYN = 50^\circ$$

$$\therefore \angle NOY = 180^\circ - 50^\circ - 50^\circ$$

$$\angle NOY = 80^\circ$$

$$\therefore \angle MON = 150^\circ - 80^\circ$$

$$\angle MON = 70^\circ$$

101. (d) Since $CS = SD$

The two chords must be equidistant from the centre 'O'.

Thus, the required ratio is $1 : 1$

$$102. (b) (OS)^2 = (OK)^2 + (KS)^2$$

$$25 = OK^2 + 16 \Rightarrow OK = 3$$

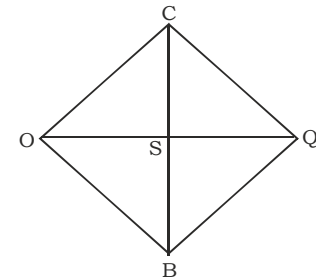
$$\text{and } (OS)^2 = (OL)^2 + (LN)^2$$

$$\Rightarrow OL = 4\text{ cm}$$

$$KL = OL - OK = 1\text{ cm}$$

$$\text{Area of rectangle} = 1 \times 10 = 10\text{ cm}^2$$

103. (c) $OQ = OB = OC = r$ (say)



$$\angle AOD = \angle BOC = 120^\circ$$

$$\angle BOQ = \angle COQ = 60^\circ$$

$$\therefore \frac{SB}{OB} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\Rightarrow SB = \frac{r\sqrt{3}}{2}$$

$$BC = 2SB = r\sqrt{3}$$

Area of quadrilateral BQCO

$$= \frac{1}{2} \times BC \times OQ$$

Again,

$$\frac{PM}{AM} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\frac{r}{AM} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AM = r\sqrt{3} = BN$$

$$AB = AM + MN + NB$$

$$= r\sqrt{3} + 2r + r\sqrt{3}$$

$$= 2r(1 + \sqrt{3})$$

$$AB = BC = AC = 2r(1 + \sqrt{3})$$

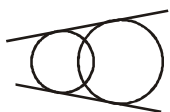
Ratio of perimeter of equilateral triangle = ratio of their sides

Ratio of perimeter of $\triangle ABC$: $\triangle DEF$: $\triangle PQR$

$$= 2(1 + \sqrt{3}) : (2 + \sqrt{3}) : 2$$

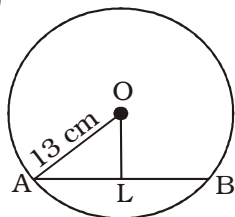
114. (a)

115. (b)



If two circles intersect each other then maximum two tangents will be drawn.

116. (d)



The perpendicular from the centre of a circle to a chord bisects the chord.

$$\therefore AL = LB = \frac{1}{2}AB = 5 \text{ cm}$$

In $\triangle OAL$

$$\therefore OA^2 = OL^2 + AL^2$$

$$\Rightarrow 13^2 = OL^2 + 5^2$$

$$\Rightarrow OL^2 = 13^2 - 5^2$$

$$\Rightarrow OL = 12 \text{ cm}$$

117. (c) $\angle ODC = \angle BAC = 38^\circ$ (Angle made by same arc BC)
and $OC = OD = \text{Radius}$

$$\therefore \angle OCD = \angle ODC = 38^\circ$$

118. (b) $OB = OC$

$$\Rightarrow \angle OCB = \angle OBC = 20^\circ$$

$$\therefore \angle BOC = 180^\circ - (20 + 20) = 140^\circ$$

$$\therefore \angle BAC = \frac{1}{2} \angle BOC = 70^\circ$$

$$119. (a) PQ^2 = BQ \times AQ$$

$$\Rightarrow (12)^2 = AQ \times 8$$

$$\Rightarrow AQ = 18 \text{ cm}$$

$$\therefore AB = AQ - BQ$$

$$= 18 - 8 = 10 \text{ cm}$$

120. (d) $\angle ACB = \angle ADB = 20^\circ$
(made by same arc AB)

$\therefore$ In $\triangle ACB$,

$$\angle x^\circ = 180^\circ - 85^\circ - 20^\circ = 75^\circ$$

$$121. (c) \angle APB = \frac{1}{2} \angle AOB$$

$$\frac{1}{2} \angle AOB = \frac{1}{2} \times 90^\circ$$

$$= 45^\circ$$

$$122. (c) \angle AOC = 360^\circ - (90^\circ + 110^\circ) = 160^\circ$$

$$\therefore \angle ABC = \frac{1}{2} \angle AOC = 80^\circ$$

123. (b) $\angle ADC = \text{Opposite exterior angle} = 120^\circ$

$$\therefore x^\circ = 180^\circ - 120^\circ = 60^\circ$$

$$124. (a) PT^2 = PA \times PB$$

$$\Rightarrow 36 = 5(5 + x)$$

$$\Rightarrow 5 + x = \frac{36}{5} = 7.2$$

$$\Rightarrow x = 2.2 \text{ cm}$$

$$125. (d) PA \times PC = PB \times PD$$

$$\Rightarrow 14 \times 9 = (7 + x) \times 7$$

$$\Rightarrow 18 = 7 + x \Rightarrow x = 11 \text{ cm}$$

$$126. (b) \angle BAC = \frac{1}{2} \times 138^\circ = 69^\circ$$

$$\therefore \angle BDC = 180^\circ - 69^\circ = 111^\circ$$

($\therefore$ ABCD is a cyclic quadrilateral)

127. (c) In $\triangle OBC$

$$OB = OC$$

$$\therefore \angle B = \angle C = 45^\circ$$

$$\therefore \angle D = \angle C$$

($\therefore$ made by same arc AB)

$$\therefore \angle D = x = 45^\circ$$

$$128. (a) x = 40^\circ$$

($\therefore$ made by same arc AB)

129. (b) $\therefore$ ABCD is a cyclic quadrilateral.

$$\therefore \angle C + \angle D = 180^\circ$$

$$\Rightarrow \angle D = 180^\circ - 48^\circ = 132^\circ$$

$$130. (b) (PT)^2 = PA \times PB$$

$$\Rightarrow 144 = x \times (7 + x)$$

$$\Rightarrow x^2 + 7x - 144 = 0$$

$$\Rightarrow (x + 16)(x - 9) = 0$$

$$\Rightarrow x = 9 \text{ or } -16$$

-16 cannot be the length, hence this value is discarded thus, $x = 9 \text{ cm}$.

$$131. (d) PA = PB$$

$$\therefore \angle PAB = \angle PBA$$

In $\triangle APB$

$$\text{Also, } \angle PAB + \angle PBA + \angle APB = 180^\circ$$

$$\Rightarrow \angle PAB + \angle PBA = 120^\circ$$

$$\therefore \angle PAB = \angle PBA = 60^\circ$$

i.e. $\triangle PAB$ is an equilateral triangle.

$$\therefore AB = 6 \text{ cm}$$

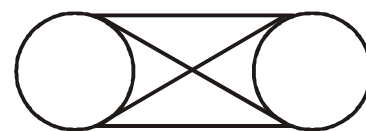
$$132. (a) A = \text{Area of } \triangle ABC = \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$$

$$S = \text{Semiperimeter of } \triangle ABC =$$

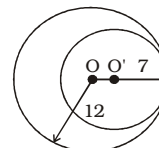
$$\frac{3+5+4}{2} = 6 \text{ cm}$$

$$\therefore \text{inradius} = \frac{A}{S} = \frac{6}{6} = 1 \text{ cm}$$

133. (d)

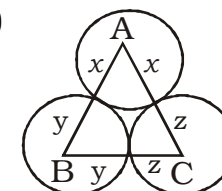


134. (d)



$$\therefore OO' = 12 - 7 = 5 \text{ cm}$$

135. (a)



$$AB = 5 \text{ cm} = x + y$$

$$\begin{aligned}
 BC &= 6 \text{ cm} = y + z \\
 AC &= 7 \text{ cm} = z + x \\
 \therefore 2(x + y + z) &= 5 + 6 + 7 = 18 \\
 \Rightarrow x + y + z &= 9 \\
 \Rightarrow 5 + z &= 9 \Rightarrow z = 4 \text{ cm} \\
 \therefore x &= 7 - z = 3 \text{ cm and} \\
 y &= 6 - z = 2 \text{ cm} \\
 \therefore x &= 3 \text{ cm, } y = 2 \text{ cm, } z = 4 \text{ cm}
 \end{aligned}$$

$$136.(c) \angle AOB = 2 \angle ACB = 2 \times 30^\circ = 60^\circ$$

$$137.(b) AB = AC$$

$$\Rightarrow \angle ACB = \angle ABC = 50^\circ$$

$$\therefore \angle BAC = 180^\circ - (50 + 50) = 80^\circ$$

$$\therefore \angle BDC = \angle BAC = 80^\circ$$

(angle by same arc BC)

$$138.(b) \angle DOC = \angle AOB = 70^\circ$$

$$\therefore OD = OC = \text{radius}$$

$$\therefore \angle OCD = \angle ODC = \frac{1}{2} (180^\circ - 70^\circ) = 55^\circ$$

$$139.(c) \text{ It is necessarily a rectangle.}$$

$$140.(d)$$

$$141.(c) \text{ Length of transverse tangent}$$

$$= \sqrt{d^2 - (R_1 + R_2)^2}$$

$$\text{here } d = 10 \text{ cm, } R_1 = R_2 = 3 \text{ cm}$$

$$\therefore \text{length} = \sqrt{(10)^2 - (6)^2}$$

$$= 8 \text{ cm}$$

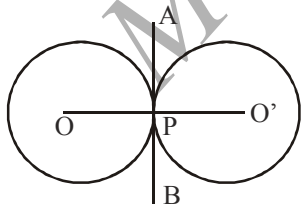
$$142.(a) CD = 7 \text{ cm}$$

$$\therefore AC = 7 \text{ cm and } BC = 7 \text{ cm}$$

$$(\because AC = CD \text{ and } BC = CD. \text{ Two tangents from the same point are always equal})$$

$$\therefore AB = 7 + 7 = 14 \text{ cm}$$

$$143.(a) \text{ Tangent is always perpendicular to the radius.}$$



$$144.(b) \angle BAC = 60^\circ$$

$$\therefore \angle BEC = 180^\circ - 60^\circ = 120^\circ$$

$$145.(c) \angle CBA = \frac{1}{2} \angle AOC = 65^\circ$$

$$\therefore \angle CBE = 180^\circ - 65^\circ = 115^\circ$$

$$146.(a) \angle BDC = \angle BAC = 30^\circ$$

$$\text{In } \triangle BCD$$

$$\therefore \angle BCD + \angle BDC + \angle DBC = 180^\circ$$

$$\therefore \angle BCD = 180^\circ - (30^\circ + 60^\circ) = 90^\circ$$

$$147.(d) \angle D = 180^\circ - 72^\circ = 108^\circ$$

$$\therefore \angle BCD = 180^\circ - 108^\circ = 72^\circ (\because AD \parallel BC)$$

$$148.(b) \angle OAP = \angle OBP = 90^\circ$$

$$\text{In } \square AOBP, \angle O + \angle P = 180^\circ$$

$$\Rightarrow \angle AOB + \angle APB = 180^\circ$$

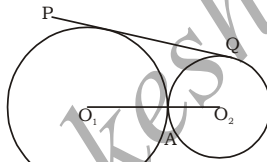
$$\therefore \angle APB = \frac{1}{6} \times 180^\circ = 30^\circ$$

$$[\angle AOB : \angle APB = 5 : 1]$$

$$149.(b) \text{ Length of common tangent}$$

$$= \sqrt{d^2 - (R - r)^2}$$

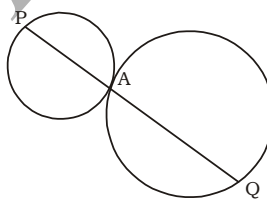
$$150.(d)$$



$$\therefore PQ = 2\sqrt{r_1 r_2} = 2\sqrt{8 \times 2} = 8 \text{ cm}$$

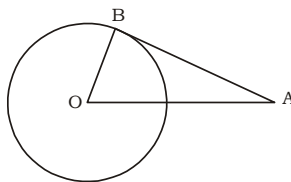
$$151.(d) PQ^2 = (r_1 + r_2)^2 - (r_1 - r_2)^2 = 4r_1 r_2$$

$$152.(b)$$



$$\therefore AP : AQ = 5 : 8$$

$$153.(a)$$



$$\angle OBA = 90^\circ;$$

$$OA = 10 \text{ cm, } OB = 6 \text{ cm}$$

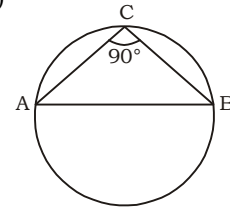
$$\text{From } \triangle OAB,$$

$$AB = \sqrt{OA^2 - OB^2}$$

$$= \sqrt{10^2 - 6^2}$$

$$= \sqrt{100 - 36} = \sqrt{64} = 8 \text{ cm}$$

$$154.(a)$$

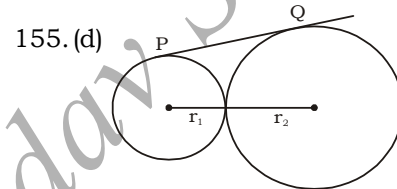


$$AB = \text{diameter of circle.}$$

$$\text{Angle of a semi-circle is a right angle.}$$

$$\text{i.e. } \angle ACB = 90^\circ$$

$$\therefore ABC \text{ is a right angled triangle.}$$



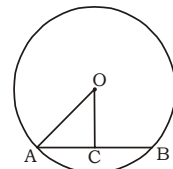
$$155.(d)$$

$$PQ = 2\sqrt{r_1 r_2} = 2\sqrt{4 \times 9} = 12 \text{ cm}$$

$$\text{Required area (side PQ)}$$

$$= (12)^2 = 144$$

$$156.(b)$$



$$AC = CB = 4 \text{ cm}$$

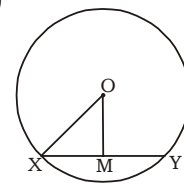
$$OC = 3 \text{ cm}$$

$$\therefore OA = \sqrt{OC^2 + CA^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16} = \sqrt{25} = 5 \text{ cm}$$

$$157.(c)$$



$$XM = MY$$

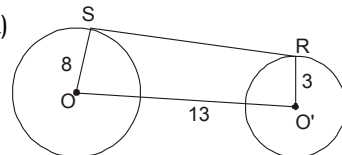
$$OM = 12 \text{ cm}$$

$$OX = 13 \text{ cm}$$

$$\therefore XM = \sqrt{OX^2 - OM^2} = \sqrt{13^2 - 12^2} = 5$$

$$\therefore XY = 2XM = 10 \text{ cm}$$

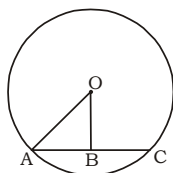
$$158.(a)$$



$$SR = \sqrt{(OO')^2 - (R_1 - R_2)^2}$$

$$= \sqrt{(13)^2 - (5)^2} = \sqrt{18 \times 8} = 12 \text{ cm}$$

159.(b)



$$AB = BC = 8$$

$$OA = 10$$

In $\triangle OAB$

$$\therefore OB = \sqrt{OA^2 - AB^2}$$

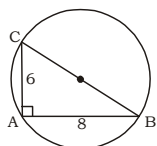
$$= \sqrt{10^2 - 8^2} = \sqrt{36} = 6$$

160.(b) The largest chord of circle is its diameter.

$$R = \frac{10.1}{2} = 5.05$$

(Greater than 5)

161.(d)



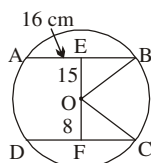
$$\angle BAC = 90^\circ$$

$\therefore$ BC is the diameter of the circle.

$$\therefore BC = \sqrt{64 + 36} = \sqrt{100} = 10 \text{ cm}$$

$\therefore$ Radius of the circle = 5 cm

162.(c)



In $\triangle BEO$

$$OB^2 = BE^2 + OE^2$$

$$OB^2 = 15^2 + 8^2$$

$$OB = 17 \text{ cm}$$

In $\triangle OFC$

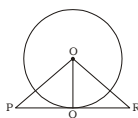
$$OC^2 = OF^2 + FC^2$$

$$(17)^2 = 8^2 + FC^2$$

$$FC = 15 \text{ cm}$$

$$DC = 2 \times FC = 30 \text{ cm.}$$

163.(d)



$$OR = 5 \text{ cm}$$

$$OP = \frac{20}{3}$$

$$\angle POR = 90^\circ$$

In triangle POR

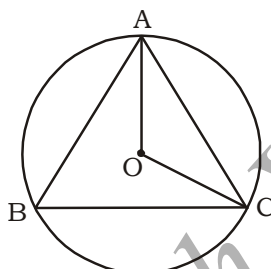
$$PR^2 = PO^2 + OR^2$$

$$PR = \sqrt{\left(\frac{20}{3}\right)^2 + 5^2}$$

$$\sqrt{\frac{400}{9} + 25}$$

$$\sqrt{\frac{625}{9}} = \frac{25}{3}$$

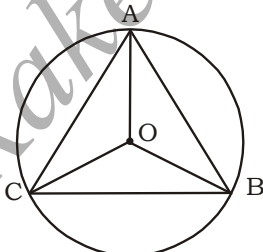
164.(b)



$$\angle ABC = 180^\circ - 80^\circ - 85^\circ = 15^\circ$$

$$\Rightarrow \angle AOC = 2 \angle ABC = 2 \times 15^\circ = 30^\circ$$

165.(a)



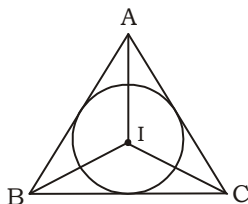
$$OB = OC = \text{radius}$$

$$\therefore \angle OBC = \angle OCB = 35^\circ$$

$$\angle BOC = 180^\circ - 35^\circ - 35^\circ = 110^\circ$$

$$\therefore \angle BAC = \frac{1}{2} \times \angle BOC = 55^\circ$$

166.(c)



$$\angle BIC = 135^\circ$$

$$\angle BIC = 90^\circ + \frac{1}{2} \angle A$$

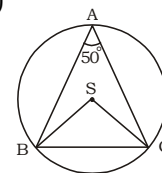
$$\frac{1}{2} \angle A = 135 - 90$$

$$\angle A = 45 \times 2 = 90^\circ$$

$\angle B = \angle C =$ Incentre is angle bisector

So, $\triangle ABC$ is a right angled $\triangle$

167.(b)



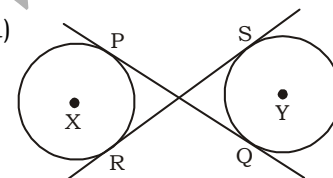
$$\angle BAC = 50^\circ$$

$$\therefore \angle BSC = 100^\circ$$

$$BS = SC = \text{radius}$$

$$\therefore \angle BCS = \frac{1}{2}(180 - 100) = 40^\circ$$

168.(a)



Transverse common tangent

$$= \sqrt{(\text{Distance between centres})^2 - (r_1 + r_2)^2}$$

$$= \sqrt{10^2 - 6^2} = \sqrt{16 \times 4} = 8 \text{ cm}$$

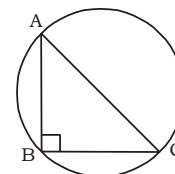
169.(b) One and only one circle can pass through three non-col-linear points.

$$170.(b) 3^2 + 4^2 = 5^2$$

$\triangle ABC$ is a right angled triangle.

$\angle B = 90^\circ =$ angle at the circumference

$\therefore$ Diameter of circle = 5 cm



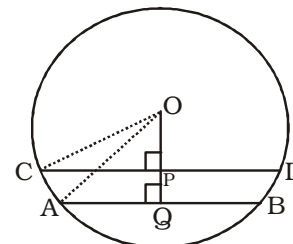
$\therefore$ Circum-radius = 2.5 cm

171.(b) In $\triangle OPC$

$$OC^2 = OP^2 + CP^2$$

$$\Rightarrow 5^2 = OP^2 + \left(\frac{8}{2}\right)^2 \Rightarrow OP^2 = 5^2 - 4^2$$

$$\Rightarrow OP^2 = 9 \Rightarrow OP = 3 \text{ cm}$$



In $\triangle OQA$

$$OA^2 = OQ^2 + AQ^2$$

$$\Rightarrow 5^2 = OQ^2 + \left(\frac{6}{2}\right)^2 \Rightarrow OQ^2 = 5^2 - 3^2$$

$$\Rightarrow OQ = 4\text{cm}$$

$\therefore$ Distance between chords AB and CD =

$$PQ = OQ - OP = 4 - 3 = 1\text{cm}$$

172.(c) $\angle LKN = 90^\circ$ (angle in semicircle)

$$\therefore \angle LNK = 180^\circ - (90 + 30) = 60^\circ$$

$$\therefore \angle PKL = \angle LNK = 60^\circ$$

(angle in alternate segment)

173.(a) $\angle ADC = 120^\circ$

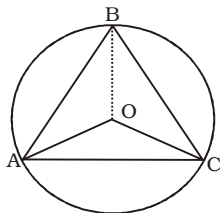
$$\angle ABC = 180^\circ - 120 = 60^\circ$$

and $\angle ACB = 90^\circ$ (angle in semicircle)

$$\therefore \angle BAC = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$$

174.(d) In $\triangle AOB$,

$AO = OB$ (radius)



$$\therefore \angle OBA = \angle OAB = 25^\circ$$

similarly In $\triangle OBC$,

$$\therefore \angle OBC = \angle OCB = 35^\circ$$

$$\therefore \angle ABC = 25 + 35 = 60^\circ$$

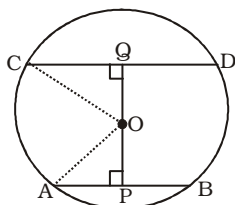
$$\therefore \angle AOC = 2 \times \angle ABC = 2 \times 60^\circ = 120^\circ$$

175.(d) Let $OP = x$ cm

$\therefore OQ = (17 - x)$ cm and radius = r cm

In $\triangle OQC$,
 $(OC)^2 = (OQ)^2 + (QC)^2$

$$\Rightarrow r^2 = (17 - x)^2 + (12)^2 \dots\dots(i)$$



and In $\triangle OAP$,
 $(OA)^2 = (OP)^2 + (AP)^2$

$$\Rightarrow r^2 = x^2 + (5)^2 \dots\dots(ii)$$

$$\therefore (17 - x)^2 + (12)^2 = x^2 + 5^2$$

$$\Rightarrow 289 - 34x + x^2 + 144 = x^2 + 25$$

$$\Rightarrow 34x = 408 \Rightarrow x = 12\text{cm}$$

$\therefore$ from(ii)

$$r^2 = (12)^2 + (5)^2 = (13)^2$$

$$\Rightarrow r = 13\text{ cm}$$

176. (C) $OA = OB = \text{radius}$

$$\Rightarrow \angle OAB = \angle OBA = 25^\circ$$

$$\therefore \angle AOB = 180^\circ - 50^\circ = 130^\circ$$

$\therefore$ Major

$$\angle AOB = 360^\circ - 130^\circ = 230^\circ$$

$$\Rightarrow \angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 230 = 115^\circ$$

177.(b) $\angle QOP = 180^\circ - 120^\circ = 60^\circ$

$$\text{and } \angle PQO = 90^\circ$$

$$\therefore \angle QPO = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$$

178.(c) $\angle ADC = 180^\circ - 55 = 125^\circ$

$$\therefore \angle CDT = 180^\circ - (125 + 30^\circ) = 25^\circ$$

179.(a) In cyclic $\square ABCD$,

$$\angle ADC + \angle ABC = 180^\circ$$

$$\Rightarrow \angle ADC = 180^\circ - 70^\circ = 110^\circ$$

now In $\triangle ADC$,

$$\angle ACD = 180^\circ - (30^\circ + 110^\circ) = 40^\circ$$

180.(d) $\angle PQO = \angle PRO = 90^\circ$

($\therefore PQ$ and PR are tangents)

$\therefore$ In $\square PQOR$

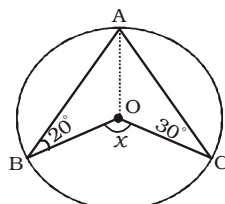
$$\angle ROQ = 360^\circ - (90^\circ + 90^\circ + 60^\circ) = 120^\circ$$

$$\therefore \angle QSR = \frac{1}{2} \angle QOR = \frac{1}{2} \times 120^\circ = 60^\circ$$

181.(d) In $\triangle AOB$

$OA = OB = \text{radius}$

$$\therefore \angle OAB = \angle OBA = 20^\circ$$



similarly In $\triangle AOC$,

$$\angle OAC = \angle OCA = 30^\circ$$

$$\therefore \angle BAC = 20^\circ + 30^\circ = 50^\circ$$

$$\therefore \angle BOC = 2 \times \angle BAC \Rightarrow \angle X = 100^\circ$$

182.(b) $\angle APB = 90^\circ$

(angle in a semicircle = 90°)

$$\therefore \angle PBA = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$$

$$\therefore \angle TPA = \angle PBA = 60^\circ$$

(by alternate segment theorem)

183.(c) $\angle BAC = \angle BDC = 30^\circ$

($\therefore$ made by same arc BC)

In $\triangle ABC$,

$$\angle x = 180^\circ - (100 + 30^\circ) = 50^\circ$$

184.(c) It will always be possible to divide a circle into 360 equal parts, because the sum of angle that can be subtended at the centre = 360°

185.(d) ABCD is a cyclic quadrilateral. Therefore

$$\angle DCB = 180^\circ - \angle A = 180^\circ - 60^\circ = 120^\circ$$

$$\therefore \angle BCQ = 180^\circ - 120^\circ$$

$$= 60^\circ \angle ABC = 80;$$

$$\therefore \angle CBQ = 180^\circ - 80^\circ = 100^\circ$$

In $\triangle BCQ$,

$$\angle Q = 180^\circ - (100 + 60^\circ) = 20^\circ$$

186.(b) $AB = 24\text{cm}$

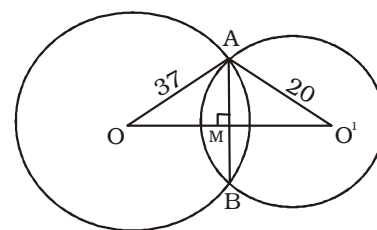
$$\therefore AM = MB = 12\text{cm}$$

In $\triangle AMO$,

$$(OM)^2 = (AO)^2 - (AM)^2$$

$$(OM)^2 = (37)^2 - (12)^2$$

$$\Rightarrow OM = 35\text{cm}$$



In $\triangle AMO$;

$$(O'M)^2 = (AO')^2 - (AM)^2$$

$$(O'M)^2 = (20)^2 - (12)^2$$

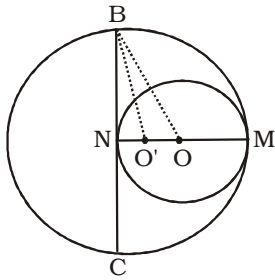
$$\Rightarrow O'M = 16\text{cm}$$

$$\therefore OO' = OM + O'M = 35 + 16 = 51\text{cm}$$

187. (a) $OM = 4\text{cm}$ = radius of smaller circle and $O'M = 6\text{cm}$ = radius of bigger circle

$$\therefore O'N = 8 - 6 = 2\text{cm}$$

In $\triangle O'NB$,



$$(O'B)^2 = (O'N)^2 + (BN)^2$$

$$\Rightarrow (BN)^2 = 36 - 4 = 32$$

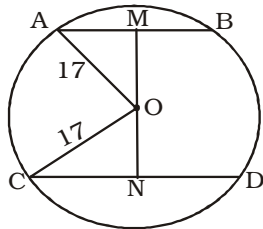
$$\Rightarrow BN = 4\sqrt{2}$$

$$\therefore NC = BN = 4\sqrt{2}$$

$$\therefore BC = 4\sqrt{2} + 4\sqrt{2} = 8\sqrt{2} \text{ cm}$$

$$188.(d) MN = 23 \text{ cm}$$

$$AM = MB = \frac{16}{2} = 8 \text{ cm}$$



$$\therefore \text{In } \triangle AMO,$$

$$(OM)^2 = (17)^2 - (8)^2$$

$$\Rightarrow OM = 15 \text{ cm}$$

$$\therefore ON = 23 - 15 = 8 \text{ cm}$$

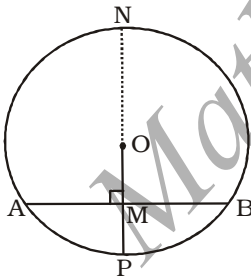
$$\text{In } \triangle ONC,$$

$$(CN)^2 = (17)^2 - (8)^2 \Rightarrow CN = 15 \text{ cm}$$

$$\therefore CD = 2CN = 30 \text{ cm}$$

$$189.(b) AB = 8 \text{ cm}$$

$$\therefore AM = MB = 4 \text{ cm}$$



$$AM \times MB = PM \times MN$$

$$\Rightarrow 4 \times 4 = 2 \times (2r - 2)$$

$$\Rightarrow 4 = r - 1 \Rightarrow r = 5 \text{ cm}$$

$$190.(a) ABCD \text{ is a cyclic quadrilateral}$$

$$\therefore \angle ADC + \angle ABC = 180^\circ$$

$$\Rightarrow \therefore \angle ABC = 180^\circ - 130^\circ = 50^\circ \quad 197.(b)$$

$$\text{also, } \angle ACB = 90^\circ$$

$$\therefore \text{In } \triangle ABC,$$

$$\angle CAB = 180^\circ - (90^\circ + 50^\circ) = 40^\circ$$

$$191.(b) \angle AOC = 2\angle APC$$

$$\Rightarrow \angle APC = 50^\circ$$

$$\text{Also, } ABCP \text{ is a cyclic quadrilateral}$$

$$\therefore \angle ABC + \angle APC = 180^\circ$$

$$\Rightarrow \angle ABC = 180^\circ - 50^\circ = 130^\circ$$

$$\therefore \angle CBD = 180^\circ - 130^\circ = 50^\circ$$

$$192.(a) \angle TPQ = \angle PAQ = 50^\circ$$

$$(\angle \text{s in the alternate segment})$$

$$\therefore TP = TQ \Rightarrow \angle TQP = \angle TPQ = 50^\circ$$

$$\therefore \angle PTQ = 180^\circ - 50^\circ - 50^\circ = 80^\circ$$

$$193.(b) \angle ACB = 90^\circ \text{ [angle at the point of intersection to the centres of the circles.]}$$

$$BC = r$$

$$AC = 2r \text{ (as area of } x=4 \text{ area of } y)$$

$$\therefore AB = \sqrt{r^2 + 4r^2} = \sqrt{5}r$$

$$194.(c) \angle QSR = \angle QTR = \frac{Z}{2}$$

$$\therefore \angle PSR = \angle PTQ = 180^\circ - \frac{Z}{2}$$

$$\text{Also, } \angle QMR = \angle SMT = y$$

$$\therefore \text{In quadrilateral PSMT}$$

$$180^\circ - \frac{Z}{2} + 180^\circ - \frac{Z}{2} + x + y = 360^\circ$$

$$\Rightarrow x + y = Z$$

$$195.(c) AB = AC \text{ and } AD = CD$$

$$\therefore AB = 2AD$$

$$\text{Now, since } AD \text{ is a tangent}$$

$$\therefore AD^2 = AP \times AB$$

$$\Rightarrow \left(\frac{AB}{2}\right)^2 = AP \times AB \Rightarrow AB = 4AP$$

$$196.(d) \angle CAB = \angle BCD$$

$$\text{and } \angle DAB = \angle BDC$$

$$(\text{alternate segment theorem})$$

$$\therefore \angle CAD = \angle CAB + \angle DAB$$

$$= \angle BCD + \angle BDC$$

$$\therefore \angle CAD + \angle CBD = \angle BCD +$$

$$\angle BDC + \angle CBD = 180^\circ$$

$$\angle ABK = 180^\circ - (115 + 30) = 35^\circ$$

$$\therefore \angle KCD = \angle ABK = 35^\circ$$

$$(\angle \text{s made by same arc } AD)$$

$$198.(a) \angle AOC = \angle BOC$$

$$(\because AC = BC)$$

$$\therefore OC \text{ is the perpendicular bisector of } AB$$

$$\therefore AM = BM$$

Ind -Method

$$\text{In } \triangle AOM \text{ and } \triangle BOM$$

$$OM = OM \text{ (common)}$$

$$\angle AOM = \angle BOM \quad (\because AC = BC)$$

$$OA = OB = \text{radius}$$

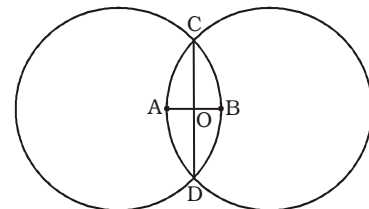
$$\therefore \triangle AOM \cong \triangle BOM$$

$$\therefore AM = BM \Rightarrow AM : BM = 1 : 1$$

$$199.(b) AB = r \text{ (say)}$$

$$\text{then } AC = BC = r, \text{ also}$$

$$\therefore OA = OB = \frac{r}{2}$$



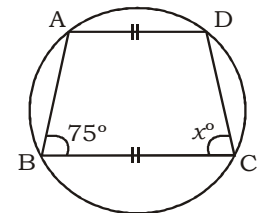
$$\therefore OC = \sqrt{(AC)^2 - (OA)^2} =$$

$$\sqrt{r^2 - \left(\frac{r}{2}\right)^2} = \frac{\sqrt{3}}{2}r$$

$$\therefore CD = 2CO = \sqrt{3}r$$

$$\therefore \frac{CD}{AC} = \frac{\sqrt{3}r}{r} = \frac{\sqrt{3}}{1}$$

$$200. (a)$$



$$\text{According to figure}$$

$$\Rightarrow AD \parallel BC$$

$$\Rightarrow \angle ABC = 75^\circ$$

$$\text{Then}$$

$$\Rightarrow \angle ABC + \angle ADC = 180^\circ$$

$$\Rightarrow 75^\circ + \angle ADC = 180^\circ$$

$$\Rightarrow \angle ADC = 180^\circ - 75^\circ$$

$$\Rightarrow \angle ADC = 105^\circ$$

$\Rightarrow$ As we know in a cyclic quadrilateral

$$\angle ADC + \angle DCB = 180^\circ$$

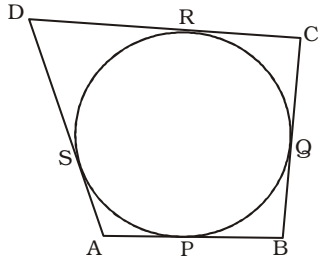
(AD || BC, corresponding angle)

$$\Rightarrow 105 + \angle DCB = 180^\circ$$

$$\Rightarrow \angle DCB = 75^\circ$$

201.(d) AP = AS, BP = BQ, CQ = CR and DR = DS

$$\therefore AB = AP + BP = AS + BQ$$



$$CD = CR + DR = CQ + DS$$

$$\therefore AB + CD = (AS + DS) + (BQ + CQ) = BC + AD$$

202.(a) TQ = TP and TP = TR

$$\therefore TQ = TP = TR$$

$$\Rightarrow TQ : TR = 1 : 1$$

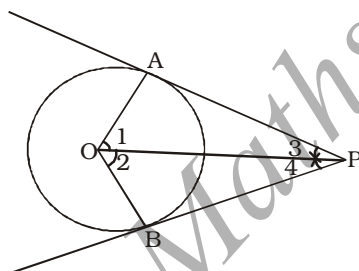
203.(b) $\angle AOC = 2 \times 60^\circ = 120^\circ$

$$\angle ABC = \frac{120}{2} = 60^\circ$$

204.(c) $\angle APB = \angle 3 + \angle 4 = 68^\circ$

In $\triangle AOP$ and $\triangle BOP$

$$PA = PB$$



$$OP = OP \text{ (common)}$$

$$\text{and } OA = OB = \text{radius}$$

$$\therefore \triangle AOP \cong \triangle BOP$$

$$\therefore \angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4$$

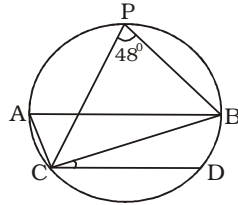
$$\therefore \angle 3 = \frac{68}{2} = 34^\circ$$

$$\therefore \angle 1 = \angle POA = 180^\circ - (90^\circ + 34^\circ) = 56^\circ$$

205.(b) $\angle BAC = \angle BPC = 48^\circ$ (by

same arc BC) and $\angle ACB = 90^\circ$

($\because$ AB is diameter)

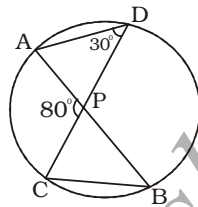


$$\therefore \angle ABC = 180^\circ - (90^\circ + 48^\circ) = 42^\circ$$

$$\therefore \angle BCD = \angle ABC = 42^\circ$$

($\because$ AB || CD)

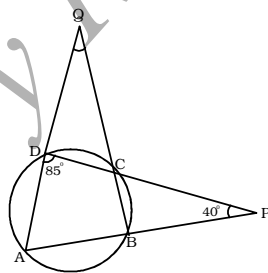
206. (d) $\angle APD = 180^\circ - 80^\circ = 100^\circ$



$$\therefore \angle PAD = 180^\circ - (100^\circ + 30^\circ) = 50^\circ$$

$$\therefore \angle BCD = \angle BAD = 50^\circ \text{ [}\angle \text{s by same arc BD]}$$

207.(a) $\angle ABC = 180^\circ - 85^\circ = 95^\circ$



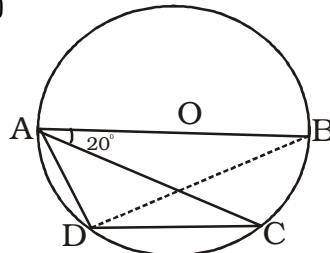
In $\triangle ADP$,

$$\angle DAP = 180^\circ - (85 + 40) = 55^\circ$$

$\therefore$ In $\triangle AQB$,

$$\angle AQB = 180^\circ - (95^\circ + 55^\circ) = 30^\circ$$

208. (c)



$$\angle ADB = 90^\circ \text{ [semi circle]}$$

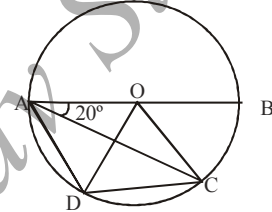
$$\angle BDC = \angle BAC = 20^\circ \text{ [}\angle \text{s by same arc BC]}$$

$$\therefore \angle ADC = 90^\circ + 20^\circ = 110^\circ$$

209. (b) $\angle ACD = \angle BAC = 20^\circ$

($\because$ AB || DC) and

$$\angle ADC = 110^\circ \text{ (from Q above)}$$

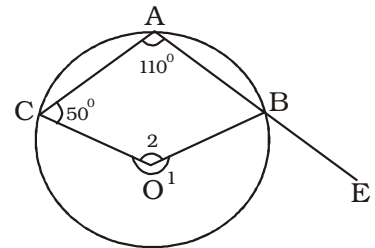


$$\angle DAC = 180^\circ - (110^\circ + 20^\circ) = 50^\circ$$

$$\therefore \angle COD = 2 \times \angle DAC = 2 \times 50^\circ = 100^\circ$$

210. (a) $\angle 1 = 2\angle A = 220^\circ$

$$\therefore \angle 2 = 360^\circ - 220^\circ = 140^\circ$$

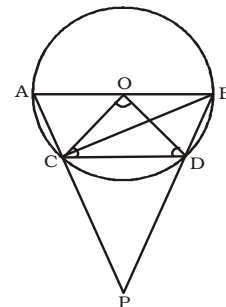


$$\therefore \angle ABO = 360^\circ - (110^\circ + 50^\circ + 140^\circ)$$

$$= 60^\circ$$

$$\therefore \angle OBE = 180^\circ - 60^\circ = 120^\circ$$

211. (d) $CD = \frac{1}{2} AB = AO = OB = \text{radius}$



$$\therefore CD = OC = OD$$

$\therefore \triangle OCD$ is an equilateral triangle

$$\therefore \angle COD = \angle OCD = \angle ODC = 60^\circ$$

618

$$\therefore \triangle OAP \cong \triangle OBP$$

$$\therefore \angle AOP = \angle POB$$

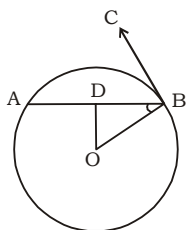
$$\text{and } \angle APO = \angle OPB$$

In $\triangle AOP$,

$$\angle APO = 180^\circ - 90^\circ - 60^\circ = 30^\circ$$

$$\therefore \angle APB = 2 \times 30 = 60^\circ$$

221.(c)



$$\angle ABC = 45^\circ$$

$$\Rightarrow \angle ABO = 45^\circ (\because \angle OBC = 90^\circ)$$

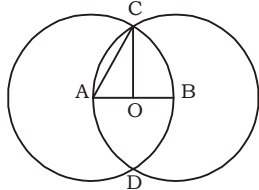
$$BD = 3 \text{ cm}$$

$\therefore$ In $\triangle OBD$

$$\cos 45^\circ = \frac{3}{OB} \Rightarrow \frac{1}{\sqrt{2}} = \frac{3}{OB}$$

$$\Rightarrow OB = 3\sqrt{2} \text{ cm}$$

222.(b)



$$AO = OB = \frac{5}{2}$$

$$AC = 5$$

In $\triangle OAC$

$$\therefore OC = \sqrt{5^2 - \left(\frac{5}{2}\right)^2} = \sqrt{25 - \frac{25}{4}}$$

$$= \sqrt{\frac{100-25}{4}} = \sqrt{\frac{75}{4}} = \frac{5\sqrt{3}}{2}$$

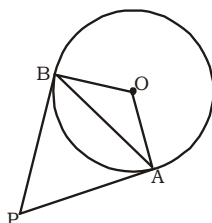
$$\therefore CD = 2 \times OC = 2 \times \frac{5\sqrt{3}}{2} = 5\sqrt{3} \text{ cm}$$

223.(d) $OA \perp AP$ and $OB \perp BP$

$$\angle OAP = 90^\circ \text{ and }$$

$$\angle OBP = 90^\circ$$

$$\therefore \angle OAP + \angle OBP = 90^\circ + 90^\circ = 180^\circ$$



In quadrilateral OAPB,

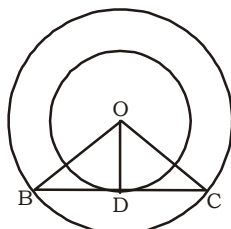
$$\angle AOB + \angle OAP + \angle APB$$

$$+ \angle OBP = 360^\circ$$

$$\Rightarrow \angle APB + \angle AOB = 180^\circ$$

$\therefore$ The quadrilateral will be concyclic.

224.(a)



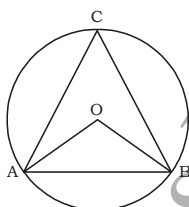
$$BO = OC = 15 \text{ cm.}$$

$$OD = 9 \text{ cm}$$

$$BD = \sqrt{15^2 - 9^2} = \sqrt{24 \times 6} = 12 \text{ cm}$$

$$\therefore BC = 2 \times 12 = 24 \text{ cm.}$$

225.(a)



$$AO = OB = AB$$

$$\therefore \angle AOB = 60^\circ$$

$$\therefore \angle ACB = 30^\circ$$

226.(b) Side of the equilateral triangle a,

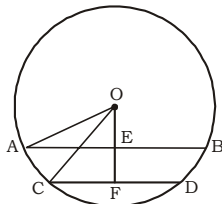
$$\text{In radius} = \frac{a}{2\sqrt{3}}$$

$$\text{Circum-radius} = \frac{a}{\sqrt{3}}$$

Required ratio

$$= \pi \left(\frac{a}{\sqrt{3}} \right)^2 : \pi \left(\frac{a}{2\sqrt{3}} \right)^2 = \frac{1}{3} : \frac{1}{12} = 4 : 1$$

227.(a)



$$\text{Let } OE = x \text{ cm}$$

$$\therefore OF = (x + 1) \text{ cm}$$

$$OA = OC = r \text{ cm}$$

$$AE = 4 \text{ cm, } CF = 3 \text{ cm}$$

In $\triangle OAE$,

$$OA^2 = AE^2 + OE^2$$

$$\Rightarrow r^2 = 16 + x^2$$

$$\Rightarrow x^2 = r^2 - 16$$

In $\triangle OCF$,

$$(x + 1)^2 = r^2 - 9$$

By equation (ii) - (i)

$$(x + 1)^2 - x^2 = r^2 - 9 - r^2 + 16$$

$$\Rightarrow 2x + 1 = 7$$

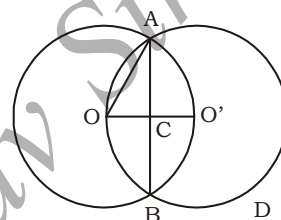
$$\Rightarrow x = 3 \text{ cm}$$

$\therefore$ From equation (i),

$$9 = r^2 - 16 \Rightarrow r^2 = 25$$

$$\Rightarrow r = 5$$

228.(b)



$$OC = 2 \text{ cm}$$

$$OA = 4 \text{ cm}$$

$$\therefore AC = \sqrt{4^2 - 2^2} = \sqrt{16 - 4} = \sqrt{12} = 2\sqrt{3}$$

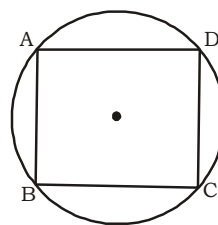
$$\therefore AB = 4\sqrt{3} \text{ cm}$$

229.(d) ABCD is cyclic parallelogram.

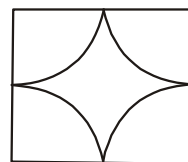
$$\therefore \angle B + \angle D = 180^\circ$$

$$\Rightarrow 2\angle B = 180^\circ$$

$$\Rightarrow \angle B = 90^\circ$$



230.(b)



Area of sectors =

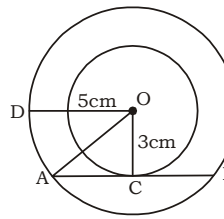
$$\pi r^2 = 4\pi \text{ sq.cm.}$$

$$\text{Area of square} = 4 \times 4 = 16 \text{ cm.}$$

$$\text{Area of the remaining portion}$$

$$= (16 - 4\pi) \text{ sq.cm.}$$

231.(c)

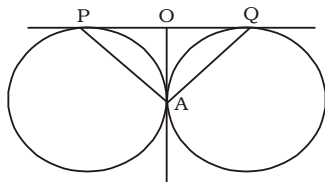


$$AC = \sqrt{AO^2 - OC^2} = \sqrt{5^2 - 3^2}$$

$$= \sqrt{25-9} = \sqrt{16} = 4 \text{ cm}$$

$$\therefore AB = 2 \times 4 = 8 \text{ cm}$$

232.(b)

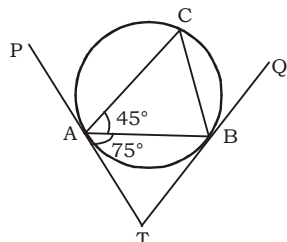


OA = OP
and OA = OQ
 $\therefore$ OA = OP = OQ
Let $\angle OPA = \angle OAP = \alpha$
 $\angle OQA = \angle OAQ = \beta$
Now In $\triangle PAQ$,
 $\beta + \alpha + (\alpha + \beta) = 180^\circ$

$$\Rightarrow \alpha + \beta = 90^\circ$$

$$\therefore \angle PAQ = 90^\circ$$

233.(c)

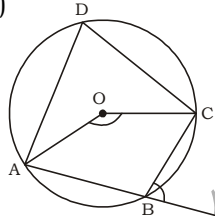


$\angle ACB = \angle BAT = 75^\circ$
(angles in the alternate segment)

In $\triangle ABC$,

$$\angle ABC = 180^\circ - 45^\circ - 75^\circ = 60^\circ$$

234.(c)

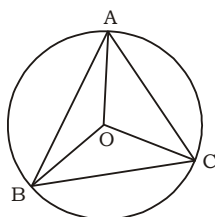


$$\angle AOC = 130^\circ$$

$$\angle ADC = \frac{1}{2} \times 130^\circ = 65^\circ$$

$\angle PBC = \angle ADC = 65^\circ$
(exterior angle is equal to the opposite interior angle)

235.(c)



$$\angle ABC = 180^\circ - 85^\circ - 75^\circ$$

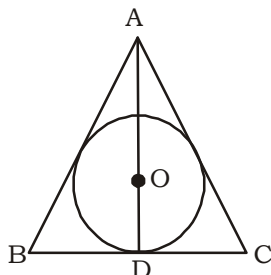
$$= 20^\circ$$

$$\therefore \angle AOC = 2 \times 20^\circ$$

$$= 40^\circ$$

$$\angle OAC = \frac{180^\circ - 40^\circ}{2} = 70^\circ$$

236.(c)



$$BD = DC = 7\sqrt{3} \text{ cm}$$

$$AD = \sqrt{AB^2 - BD^2} = \sqrt{(14\sqrt{3})^2 - (7\sqrt{3})^2}$$

$$= \sqrt{(14\sqrt{3} + 7\sqrt{3})(14\sqrt{3} - 7\sqrt{3})}$$

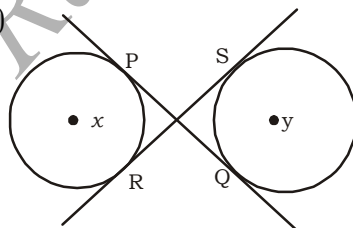
$$= \sqrt{21\sqrt{3} \times 7\sqrt{3}} = 21 \text{ cm}$$

$$\therefore OD = \frac{1}{3} \times 21 = 7 \text{ cm}$$

$$\therefore \text{Area of circle} = \pi r^2$$

$$= \frac{22}{7} \times 7 \times 7 = 154 \text{ sq.cm}$$

237.(c)



Length of common transverse tangent

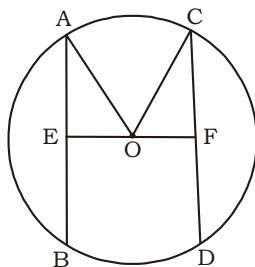
$$= \sqrt{XY^2 - (r_1 + r_2)^2}$$

$$\Rightarrow 8 = \sqrt{XY^2 - 9^2}$$

$$\Rightarrow XY^2 = 64 + 81 = 145$$

$$\Rightarrow XY = \sqrt{145}$$

238.(b)



$$AO = OC \Rightarrow 5 \text{ cm (radius)}$$

$$AB = 24 \text{ cm}$$

$$AE = EB = 12 \text{ cm}$$

In $\triangle AEO$

$$OE = \sqrt{15^2 - 12^2}$$

$$= \sqrt{225 - 144} = \sqrt{81} = 9 \text{ cm}$$

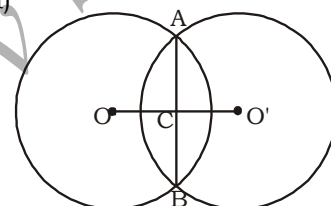
$$\therefore OF = 21 - 9 = 12 \text{ cm}$$

In $\triangle OCF$

$$\therefore CF = \sqrt{15^2 - 12^2} = 9 \text{ cm}$$

$$\therefore CD = 2 \times 9 = 18 \text{ cm}$$

239.(a)



$$AB = 16, AC = BC = 8 \text{ cm}$$

$$OC = CO' = 6 \text{ cm}$$

$$\therefore OA = \sqrt{OC^2 + CA^2}$$

$$= \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$$

$$= \sqrt{100} = 10 \text{ cm}$$

240.(b) Tangents drawn from any external point are of same length

$$\therefore AD = AE, BD = BF \text{ and}$$

$$CE = CF$$

$$AD = AB + BD = AB + BF$$

$$\text{and } AD = AE = AC + CE$$

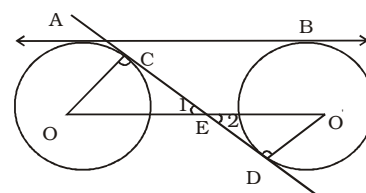
$$= AC + CF$$

$$\therefore 2AD = AB + AC + BF + CF$$

$$= AB + BC + CA$$

241.(b)

242.(c)



$$OC = O'D = 5 \text{ cm (radius)}$$

$$CE = ED \text{ (CD = 24 cm)}$$

$$\triangle COE \cong \triangle EO'D$$

$$(\because \angle C = \angle D = 90^\circ \text{ and } \angle 1 = \angle 2)$$

$$\therefore OE = O'E$$

$$\text{and } CE = ED = 12 \text{ cm}$$

In $\triangle COE$;

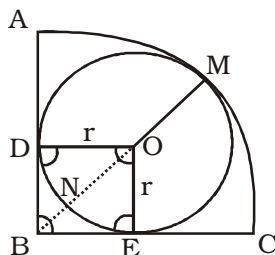
$$OE^2 = CE^2 + OC^2$$

$$OE^2 = 12^2 + 5^2 = 169$$

$$\Rightarrow OE = 13$$

$$\therefore OO' = OE + EO' = 13 + 13 = 26\text{cm} = AB$$

243.(a) Let radius of smaller circle = r



$\therefore$ AB and BC are tangents to smaller circle

$\therefore OD \perp AB$ and $OE \perp BC$

$\therefore \angle DOE = 90^\circ$

& $OD = OE = r = \text{radius}$

$BM = 1\text{cm}$

$\therefore OB = (1-r)\text{cm}$

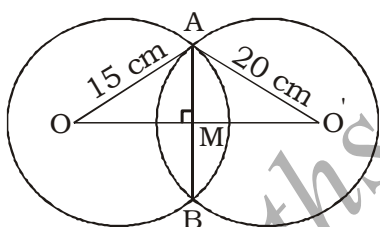
$\therefore$ ODBE is a square

$$\therefore OB = \sqrt{2}r$$

$$\therefore \sqrt{2}r = 1 - r \Rightarrow r(\sqrt{2} + 1) = 1$$

$$\Rightarrow r = \frac{1}{\sqrt{2} + 1} \times \frac{(\sqrt{2} - 1)}{(\sqrt{2} - 1)} = (\sqrt{2} - 1)\text{cm}$$

244.(d)



$$OO' = 25\text{cm}$$

$$\therefore (25)^2 = (15)^2 + (20)^2$$

$\therefore \triangle OAO'$ is a right angle triangle

AB is common chord

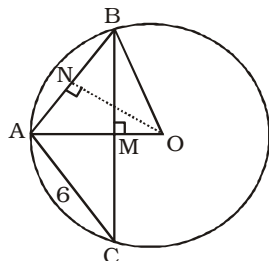
$$\triangle OMA \cong \triangle O'AM$$

$\{\therefore \angle OAO' = \angle AMO = 90^\circ \text{ and } \angle O = \angle O' \text{ (common)}\}$

$$\therefore \frac{OA}{OO'} = \frac{AM}{O'A} \Rightarrow AM = \frac{15 \times 20}{25} = 12\text{cm}$$

$$\therefore AB = 2AM = 24\text{cm}$$

245.(b) Let $BM = x\text{ cm}$



$$\therefore \text{Area of } \triangle AOB = \frac{1}{2} \times OA \times BM$$

$$= \frac{1}{2} \times 5 \times x = \frac{5x}{2}\text{cm}$$

$$ON \perp AB$$

$$\therefore AN = BN = \frac{6}{2} = 3\text{ cm}$$

In $\triangle ANO$,

$$ON = \sqrt{(5)^2 - (3)^2} = 4\text{cm}$$

$\therefore$ Again Area of $\triangle AOB$

$$= \frac{1}{2} \times AB \times ON = \frac{1}{2} \times 6 \times 4 = 12\text{cm}$$

$$\therefore \frac{5x}{2} = 12 \Rightarrow x = \frac{24}{5} = 4.8\text{cm}$$

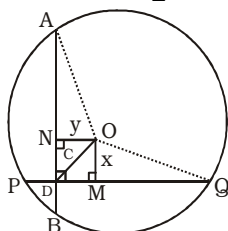
$$\therefore BC = 2x = 9.6\text{ cm}$$

246.(c) Let $AB = 2a$ and $PQ = 2b$ & 'O' is the centre

$OM \perp PQ$ and $ON \perp AB$

$$\therefore PM = MQ = \frac{2b}{2} = b$$

$$\text{and } AN = NB = \frac{2a}{2} = a$$



In quadrilateral DMON,

$$\angle D = \angle M = \angle N = 90^\circ$$

$$\therefore \angle NOM = 90^\circ$$

it mean DMON is a rectangle

$$\therefore \text{let } OM = DN = x$$

then $ON = DM = y$

Let radius = $OQ = r\text{ cm}$

$$\text{In } \triangle OMQ, r^2 = x^2 + b^2 \text{ (i)}$$

$$\text{In } \triangle ONA, r^2 = y^2 + a^2 \text{ (ii)}$$

(i)+(ii)

$$2r^2 = a^2 + b^2 + (x^2 + y^2)$$

$$2r^2 = a^2 + b^2 + c^2$$

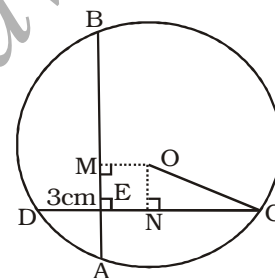
$$\Rightarrow r = \sqrt{\frac{a^2 + b^2 + c^2}{2}}$$

247.(a) $AE \times EB = DE \times CE$

$$\Rightarrow CE = \frac{2 \times 6}{3} = 4\text{cm}$$

Let O is the centre

$OM \perp AB$



$$\therefore AM = BM = \frac{8}{2} = 4\text{cm}$$

$$\therefore EM = 4 - 2 = 2\text{cm}$$

$$\therefore ON = EM = 2\text{cm}$$

($\therefore$ ONEM is a rectangle)

Now In $\triangle ONC$,

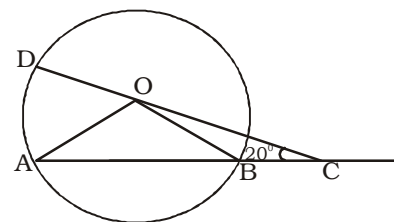
$$(OC)^2 = (ON)^2 + (NC)^2$$

$$\Rightarrow r^2 = (2)^2 + \left(\frac{7}{2}\right)^2 = 4 + \frac{49}{4} = \frac{65}{4}$$

$$\Rightarrow r = \frac{\sqrt{65}}{2}\text{cm}$$

248.(d) $BC = OD$ (given)

$\therefore BC = OD = OB = BA = \text{radius}$



In $\triangle BOC$,

$BC = OB$

$$\therefore \angle BOC = \angle OCB = 20^\circ$$

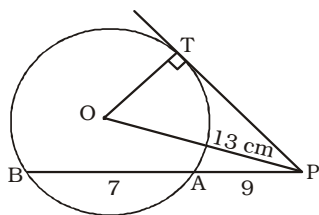
$$\therefore \angle ABO = 20^\circ + 20^\circ = 40^\circ$$

In $\triangle OAB$,

$AO = OB$

$$\begin{aligned}\therefore \angle OAB &= \angle ABO = 40^\circ \\ \therefore \angle AOB &= 180^\circ - (40^\circ + 40^\circ) = 100^\circ \\ \therefore \angle AOD &= 180^\circ - (100^\circ + 20^\circ) = 60^\circ\end{aligned}$$

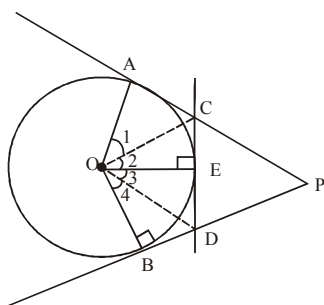
249.(b) Draw a tangent (PT) from P-



$$\begin{aligned}\therefore PT^2 &= PA \times PB \\ \Rightarrow PT^2 &= 9 \times 16 \Rightarrow PT = 12\text{cm} \\ \text{In } \triangle OTP, \angle T &= 90^\circ \\ \therefore (OT)^2 &= (13)^2 - (12)^2 = 25 \\ \Rightarrow OT &= r = 5\text{cm}\end{aligned}$$

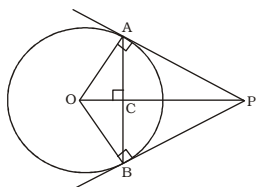
250.(c) In $\square AOBP$

$$\begin{aligned}(\because \angle B &= \angle A = 90^\circ) \\ \therefore \angle AOB &= 180^\circ - 34 = 146^\circ \\ \text{In } \triangle OAC \text{ and } \triangle OEC \\ OC &= OC \text{ (common)} \\ OA &= OE = \text{radius} \\ \text{and } CA &= CE\end{aligned}$$



$$\begin{aligned}\therefore \triangle OAC &\cong \triangle OEC \\ \therefore \angle AOC &= \angle COE \Rightarrow \angle 1 = \angle 2 \\ \text{Similarly } \triangle OBD &\cong \triangle OED \\ \therefore \angle 3 &= \angle 4 \\ \triangle \text{ In } AOB \\ \angle 1 + \angle 2 + \angle 3 + \angle 4 &= 146^\circ \\ \Rightarrow \angle 2 + \angle 3 &= 73^\circ \\ \Rightarrow \angle COD &= 73^\circ\end{aligned}$$

251.(a)



$$AB = 6\text{cm}$$

$$\therefore AC = BC = \frac{6}{2} = 3\text{cm}$$

$$\begin{aligned}\text{In } \triangle OAP \text{ and } \triangle OCA \\ \angle OAP &= \angle OCA = 90^\circ \\ \angle AOP &= \angle AOC \\ \therefore \triangle OAP &\cong \triangle OCA\end{aligned}$$

$$\frac{PO}{AO} = \frac{OA}{OC} \Rightarrow OP = \frac{OA^2}{OC} = \frac{(5)^2}{4}$$

$$\Rightarrow OP = \frac{25}{4}\text{cm}$$

$$\begin{aligned}252.(b) BC &= 2(OB) = \sqrt{a^2 + 4^2} \\ &= \sqrt{a^2 + 16}\end{aligned}$$

$$(\because \angle A = 90^\circ)$$

$$\therefore \triangle ABD \cong \triangle CBA$$

$$\therefore \frac{BD}{AB} = \frac{AB}{BC} \Rightarrow BD \cdot BC = a^2$$

$$\Rightarrow BD = \frac{a^2}{BC} = \frac{a^2}{\sqrt{a^2 + 16}}$$

$$\therefore OD = OB - BD = \frac{\sqrt{a^2 + 16}}{2} -$$

$$\frac{a^2}{\sqrt{a^2 + 16}} = \frac{16 - a^2}{2\sqrt{a^2 + 16}}$$

$$253.(d) \triangle PQS \cong \triangle PMN \cong \triangle PRT$$

$$\begin{aligned}\therefore N \text{ is the mid-point of } ST \\ \text{Also In } \triangle PQS, PS^2 &= (24)^2 + (7)^2 \\ &= 625\end{aligned}$$

$$\Rightarrow PS = 25\text{cm}$$

$$\text{As } \triangle PQS \cong \triangle PRT$$

$$\Rightarrow \frac{QS}{RT} = \frac{PQ}{PR} = \frac{PS}{PT} = \frac{7}{21} = \frac{1}{3}$$

$$\therefore PR = 3 \times PQ = 72\text{cm}$$

$$\text{and } PT = 3 \times PS = 75\text{cm}$$

$$\therefore ST = PT - PS = 50\text{cm}$$

$$\therefore SN = 25\text{cm}$$

254.(a) If a pair of sides of a cyclic quadrilateral are parallel, it become an isosceles

trapezium.

$$\text{Here, } a + c = b + d = 180^\circ$$

(cyclic quadrilateral) $a = b$ and $c = d$

(Isosceles trapezium)

$$\therefore a + b - c - d = (a + b + c + d) - 2(c + d) = 360^\circ - 4c (\because c = d)$$

Since, no angle of the quadrilateral ABCD is reflex i.e., $> 180^\circ$

$$\therefore C \text{ can take any value 1 to 179}$$

$\therefore (a + b - c - d)$ cm take one value for each value of C, i.e., 179 values.

$$255.(c) AB = BC = AC = 2\text{cm}$$

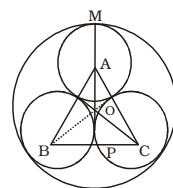
($\because$ radius of each circle = 1cm)

$$\therefore AP = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}\text{cm}$$

Let O is the centroid, then

$$OA = \frac{2}{3} \times \sqrt{3} = \frac{2}{\sqrt{3}}\text{cm}$$

$$\begin{aligned}\therefore OM &= OA + AM = \frac{2}{\sqrt{3}} + 1 \\ &= \frac{2 + \sqrt{3}}{\sqrt{3}}\text{cm}\end{aligned}$$



OM is the radius of the larger circle.

$\therefore$ Area of the circumscribing

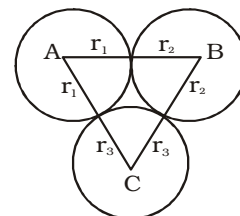
$$\text{circle} = \pi R^2$$

$$= \pi \left(\frac{2 + \sqrt{3}}{\sqrt{3}} \right)^2$$

$$= \frac{\pi}{3} (2 + \sqrt{3})^2$$

$$256.(a) \therefore r_1 + r_2 = 4$$

$$r_2 + r_3 = 3.4, r_1 + r_3 = 2.2$$



$$\Rightarrow 2(r_1 + r_2 + r_3) = 4 + 3.4 + 2.2 = 9.6$$

$$\Rightarrow r_1 + r_2 + r_3 = 4.8$$

$$\therefore r_1 = 1.4\text{cm}, r_2 = 2.6\text{cm}, \text{ and}$$

$$r_3 = 0.8\text{cm},$$

$\therefore$ diameters

$$d_1 = 2r_1 = 2.8\text{cm}$$

$$d_2 = 2r_2 = 5.2\text{cm}$$

$$d_3 = 2r_3 = 1.6\text{cm}$$

$\therefore d_2$ is bigger circle &

$$d_2 = 5.2\text{cm}$$

$$257.(c) \angle CBY = \angle CZY = 42^\circ$$

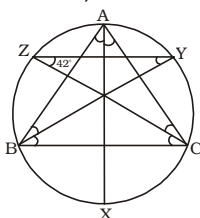
($\angle$ S by same arc YC)

$$\therefore \angle ABC = 42 + 42 = 84$$

$$\therefore \angle C = 180^\circ - (84 + 50^\circ) = 46^\circ$$

$$\therefore \angle BCZ = \frac{\angle C}{2} = 23^\circ$$

$\therefore \angle BYZ = \angle BCZ = 23^\circ$ ($\angle$ S by same arc BZ)



$$258.(b) \angle x = \angle CBD = 33^\circ$$

(by alternate segment theorem)

$$\therefore \angle Z = 180^\circ - \angle x = 147^\circ$$

(BPDE is a cyclic quadrilateral)

$$\text{and } \angle BDE = \angle BED = x = 33^\circ$$

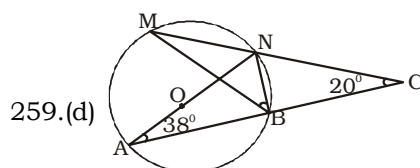
{ $\because$ BE=BD}

$$\therefore \angle ABE = \angle BDE = 33^\circ$$

{by alternate segment theorem}

$$\therefore \angle y = 90^\circ - 33^\circ = 57^\circ$$

$$x + y + z = 33 + 57 + 147 = 237^\circ$$



$$\angle ABN = 90^\circ \text{ { angle in$$

semicircle}

$$\therefore \angle CBN = 90^\circ$$

$\therefore$ In $\triangle CBN$

$$\therefore \angle BNC = 180^\circ - 90^\circ - 20^\circ = 70^\circ$$

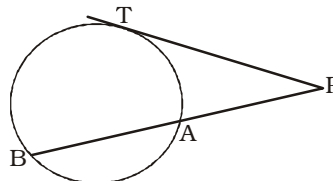
$$\therefore \angle MNB = 180^\circ - 70^\circ = 110^\circ$$

$$\text{and } \angle BMN = \therefore \angle BAN = 38^\circ$$

$\therefore$ In $\triangle BMN$

$$\angle MBN = 180^\circ - 38^\circ - 110^\circ = 32^\circ$$

260.(a)



$$(PT)^2 = PA \times PB$$

$$\Rightarrow (2AP)^2 = PA \times PB$$

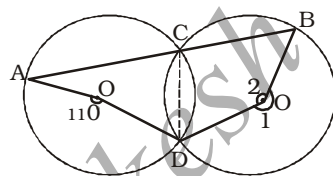
$$\Rightarrow 4AP^2 = AP \times BP \Rightarrow 4AP = BP$$

$$\Rightarrow 4AP = (18 + AP) \Rightarrow 3AP = 18$$

$$\Rightarrow AP = 6\text{cm}$$

$$\therefore PT = 2AP = 12\text{ cm}$$

261.(b)



$$\angle ACD = \frac{1}{2} \angle AOD = 55^\circ$$

$$\therefore \angle BCD = 180^\circ - 55^\circ$$

$$= 125^\circ$$

$$\therefore \angle 1 = 2 \angle BCD = 250^\circ$$

$$\therefore \angle 2 = \angle BOD = 360^\circ - 250^\circ = 110^\circ$$

$$262.(c) AB = 27\text{ cm}$$

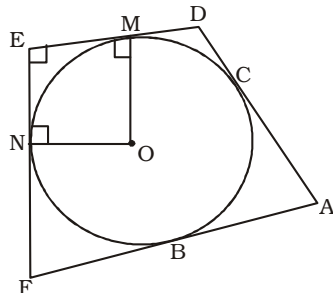
$$\therefore AC = 27\text{ cm}$$

$$\therefore CD = 38 - 27 = 11\text{ cm}$$

$$\therefore DM = CD = 11\text{ cm}$$

$$\therefore EM = 24 - 11 = 13\text{ cm}$$

$$\therefore EN = EM = 13\text{ cm}$$



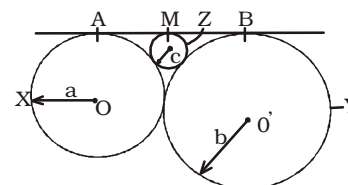
$$OM \perp DE \text{ and } ON \perp EF$$

$$\therefore ONEM \text{ is a square}$$

$$(\therefore EM = EN)$$

$$\therefore OM = ON = \text{radius} = 13\text{cm}$$

263.(d)



AB is a common tangent of X and Y

$$\therefore AB = 2\sqrt{ab}$$

Similarly $AM = 2\sqrt{ac}$ and

$$BM = 2\sqrt{bc}$$

$$\therefore 2\sqrt{ab} = 2\sqrt{ac} + 2\sqrt{bc}$$

$$[\because AB = AM + BM]$$

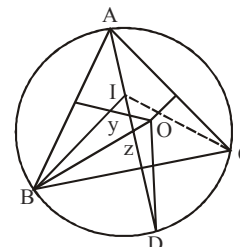
$$\Rightarrow \sqrt{ab} = \sqrt{ac} + \sqrt{bc}$$

on dividing both sides by $\sqrt{abc}$

$$\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{a}}$$

$$\Rightarrow \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$$

264.(a)



$$\therefore \angle BOD = Z$$

$$\therefore \angle BAI = \frac{Z}{2}$$

[$\because$ angle made by arc are at the circumferences is half of the angle made by the arc at the centre.]

$$\angle ABI = \angle IBC = \frac{X}{2}$$

[$\because$ I is the incentre]

Now, In $\triangle ABI$,

$$\angle BID = y \text{ (exterior angle)}$$

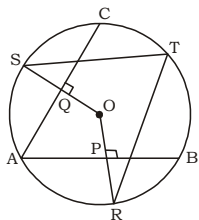
$$\therefore y = \angle BAI + \angle ABI$$

$$= \frac{Z}{2} + \frac{X}{2} = \frac{X+Z}{2}$$

$$\Rightarrow x + z = 2y$$

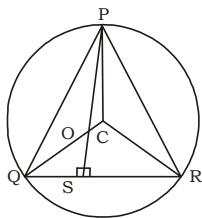
$$\therefore \frac{x+z}{3y} = \frac{2y}{3y} = \frac{2}{3}$$

265.(b)



$$\begin{aligned}\angle OQA &= \angle OPA = 90^\circ \\ \angle QOP + \angle QAP &= 180^\circ \\ \Rightarrow \angle QOP &= 180^\circ - 32^\circ = 148^\circ \\ \angle QOP &= \angle SOR = 2\angle STR \\ \therefore \angle RTS &= \frac{148}{2} = 74^\circ\end{aligned}$$

266.(b)



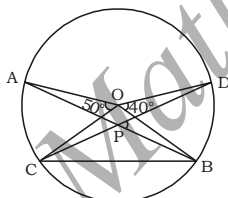
$$\begin{aligned}\angle PQS &= 60^\circ \\ \angle QCR &= 130^\circ \\ \therefore \angle QPR &= \frac{1}{2} \times 130^\circ = 65^\circ\end{aligned}$$

In $\triangle QSP$,
 $\angle PQS = 60^\circ$
 $\angle PSQ = 90^\circ$
 $\angle QPS = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$
 $\angle RPS = \angle QPR - \angle QPS = 65^\circ - 30^\circ = 35^\circ$

267.(c) Join BC

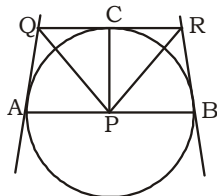
$$\therefore \angle BCP = \frac{1}{2} \angle BOD = 20^\circ$$

$$\text{and } \angle CBP = \frac{1}{2} \angle AOC = 25^\circ$$



In $\triangle PBC$,
 $\angle BPD = \angle BCP + \angle CBP$
 $= 20^\circ + 25^\circ = 45^\circ$

268.(c)



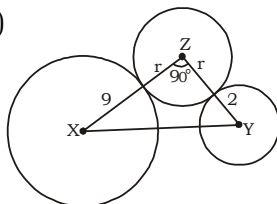
In $\triangle PCR$ and $\triangle RBP$,
 $PC = PB$ (radii)
 $RC = RB$
 PR is common.

$\therefore \triangle PCR \cong \triangle RPB$
 Similarly,

$$\begin{aligned}\angle CPQ &\cong \angle QPA \\ \therefore \angle QPR &= 90^\circ\end{aligned}$$

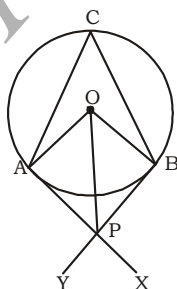
Because $\angle APB = 180^\circ$

269.(d)



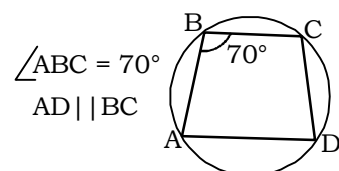
$$\begin{aligned}\angle XZY &= 90^\circ \\ XZ &= (9 + r) \text{ cm}, \\ YZ &= (r + 2) \text{ cm} \\ XY &= 17 \text{ cm} \\ \therefore XY^2 &= XZ^2 + ZY^2 \\ \Rightarrow 17^2 &= (9 + r)^2 + (r + 2)^2 \\ \Rightarrow 289 &= 81 + 18r + r^2 + r^2 + 4r + 4 \\ \Rightarrow 2r^2 + 22r - 204 &= 0 \\ \Rightarrow r^2 + 11r - 102 &= 0 \\ \Rightarrow r^2 + 17r - 6r - 102 &= 0 \\ \Rightarrow r(r + 17) - 6(r + 17) &= 0 \\ \Rightarrow (r - 6)(r + 17) &= 0 \\ \Rightarrow r &= 6 \text{ cm}\end{aligned}$$

270.(a)



$$\begin{aligned}\angle ACB &= 65^\circ \\ \angle AOB &= 2 \times 65^\circ = 130^\circ \\ \angle OAP &= 90^\circ, \angle AOP = 65^\circ \\ \angle APO &= 180^\circ - 90^\circ - 65^\circ = 25^\circ\end{aligned}$$

271.(b) According to question
 Given:



$$\therefore \angle BAD = 180 - 70^\circ = 110^\circ$$

($\therefore$ Corresponding angles between two parallel line sums to 180°)

$$\angle BCD = 180 - 110^\circ = 70^\circ$$

Note: In trapezium sum of opposite angles are 180° :

Alternate

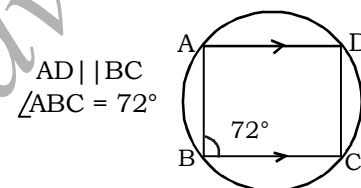
In cyclic trapezium

$$\angle A = \angle D$$

$$\text{and } \angle B = \angle C$$

$$\therefore \angle BCD = \angle ABC = 70^\circ$$

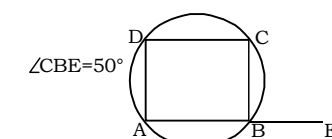
272.(d) According to question
 Given:



$$\begin{aligned}\angle ABC + \angle CDA &= 180^\circ \\ \angle CDA &= 180 - 72 = 108^\circ \\ AD &\parallel BC\end{aligned}$$

$$\begin{aligned}\therefore \angle ADC + \angle BCD &= 180^\circ \\ (\therefore \text{Sum of corresponding angle of parallel line is } 180^\circ) \\ \angle BCD &= 180^\circ - 108^\circ \\ \angle BCD &= 72^\circ\end{aligned}$$

273.(c) According to question,
 Given:

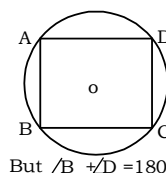


$$\begin{aligned}\angle ABC + \angle CBE &= 180^\circ \\ \angle ABC &= 180^\circ - 50^\circ \\ \angle ABC &= 130^\circ\end{aligned}$$

In cyclic quadrilateral, sum of opposite angles is 180°

$$\begin{aligned}\angle CDA &= 180^\circ - 130^\circ \\ \angle CDA &= 50^\circ\end{aligned}$$

274.(d) According to question
 ABCD is a cyclic parallelogram
 In a cyclic quadrilateral, sum of opposite angle is 180°
 But In cyclic parallelogram opposite angle is same



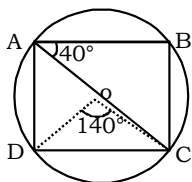
$$\angle B + \angle D = 180^\circ$$

$$2\angle B = 180^\circ$$

$$\angle B = \frac{180^\circ}{2}$$

$$\angle B = 90^\circ$$

275. (a) According to question
ABCD is a cyclic quadrilateral



$$\angle CAD = \frac{1}{2} \angle COD$$

(The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle)

$$\angle CAD = \frac{1}{2} \times 140^\circ$$

$$\angle CAD = 70^\circ$$

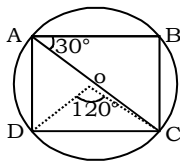
$$\therefore \angle DAB = 70 + 40 = 110^\circ$$

In cyclic quadrilateral sum of opposite angles are 180°

$$\angle A + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 110^\circ = 70^\circ$$

276. (b) According to question, ABCD is cyclic quadrilateral with centre 'O'.



Given:

$$\angle COD = 120^\circ$$

$$\angle BAC = 30^\circ$$

$$\angle BCD = ?$$

$$\angle CAD = \frac{1}{2} \angle COD$$

$$\angle CAD = \frac{1}{2} \times 120^\circ = 60^\circ$$

$$\therefore \angle BAD = 30^\circ + 60^\circ = 90^\circ$$

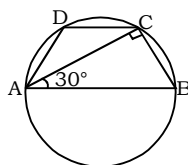
Note: In cyclic quadrilateral sum of opposite angle is 180°

$$\angle BAD + \angle BCD = 180^\circ$$

$$\angle BCD = 180^\circ - 90^\circ$$

$$\angle BCD = 90^\circ$$

277. (b) According to question



Given: AB is a diameter

$$\angle CAB = 30^\circ$$

As we know that

$$\angle ACB = 90^\circ$$

$$\therefore \angle ACB + \angle CAB + \angle CBA = 180^\circ$$

$$\angle CBA = 180^\circ - 90^\circ - 30^\circ$$

$$\angle CBA = 60^\circ$$

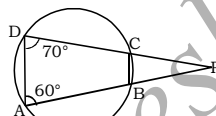
Note: In a cyclic trapezium sum of opposite angle is 180°

$$\therefore \angle D + \angle B = 180^\circ$$

$$\angle D = 180^\circ - 60^\circ$$

$$\angle D = 120^\circ$$

278. (a) According to question



$$\angle ADC = 70^\circ$$

$$\angle ABC = 180^\circ - 70^\circ = 110^\circ$$

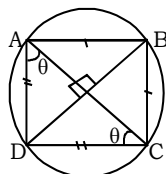
$$\Rightarrow \angle PBC = 70^\circ$$

$$\angle BCD = 180^\circ - 60^\circ = 120^\circ$$

$$\Rightarrow \angle PCB = 60^\circ$$

$$\therefore \angle PBC + \angle PCB = 70^\circ + 60^\circ = 130^\circ$$

279. (c) According to question



In $\triangle ADC$

$$\angle A + \angle D + \angle C = 180^\circ$$

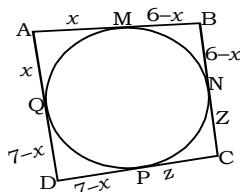
$$\angle D = 180^\circ - 2\theta$$

$$\angle B + \angle D = 180^\circ$$

$$180^\circ - 2\theta + \angle B = 180^\circ$$

$$\angle B = 2\theta$$

280. (a) According to question



We know tangents drawn to circle from same external point are equal

$$\Rightarrow AM = AQ = x$$

$$\therefore MB = BN = 6 - x$$

$$QD = DP = 7 - x$$

$$\text{Let } NC = PC = z$$

Now $7 - x + z = 5$ (consider side DC)

$$-x + z = -2 \quad \dots(i)$$

$$BC = 6 - x + z \quad \dots(ii)$$

Put the value of equation (i) in equation (ii)

$$BC = 6 - 2$$

$$BC = 4 \text{ cm}$$

Alternate:-

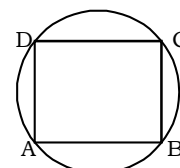
$$AB + CD = BC + AD$$

$$6 + 5 = BC + 7$$

$$11 - 7 = BC$$

$$4 \text{ cm} = BC$$

281. (b) According to question



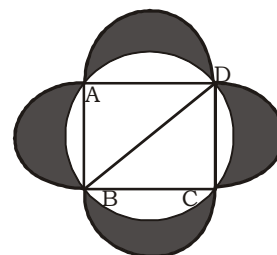
ABCD is a cyclic quadrilateral

$$\therefore \angle A + \angle C = 180^\circ$$

$$\angle B + \angle D = 180^\circ$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

282. (b) According to question



$$BD = 2 \text{ units}$$

$$\therefore AB = \sqrt{2} \text{ units}$$

$$\text{Area of square} = 2 \text{ units}$$

$$\text{Area of four semicircles}$$

$$= 4 \times \frac{\pi r^2}{2} = 2\pi r^2$$

$$= 2\pi \left(\frac{\sqrt{2}}{2} \right)^2 = 2\pi \left(\frac{2}{4} \right) = \pi \text{ units}$$

$$\text{Area of circle} = \pi r^2$$

$$= \pi 1^2 = \pi$$

Required area of shaded portion

$$= \text{Area of square} - \text{Area of circle}$$

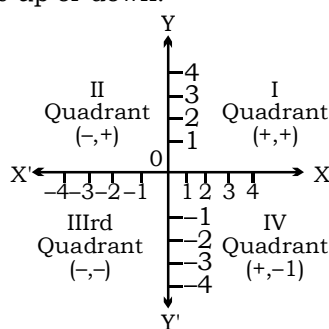
$$+ \text{area of semicircle}$$

$$= 2 + \pi - \pi$$

$$= 2 \text{ sq. units}$$

CO-ORDINATE GEOMETRY

From the SSC point of view, Co-Ordinate Geometry is an important Chapter. From which generally 3 to 4 questions are asked in every exam. Co-ordinates are a set of values that show an exact position. On maps and graphs it is common to have a pair of numbers to show where a point is; the first number shows the distance along and the second number shows the distance up or down.



Region	Quadrant	Nature of X & Y	Signs of co-ordinate
XOY	I	$x > 0, y > 0$	(+, +)
YOX'	II	$x < 0, y > 0$	(-, +)
X'OY'	III	$x < 0, y < 0$	(-, -)
Y'OX'	IV	$x > 0, y < 0$	(+, -)

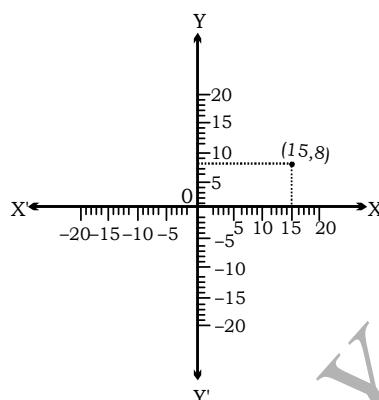
CARTESIAN CO-ORDINATE SYSTEM

Rectangular Co-ordinate System : Let $X'OX$ and $Y'OY$ be two mutually perpendicular lines through any point O in the plane of the paper. Point O is known as the origin. The line $X'OX$ is called the x -axis or axis of x ; the line $Y'OY$ is known as the y -axis or axis of y , and the two lines taken together are called the co-ordinates axes or the axes of co-ordinates.

Ex.1 The point $(9,6)$ is 9 units along, 6 units up if it is $(9, -6)$ then - 6 unit down.

Ex.2 Mark the point $(15,8)$ on the Graph?

Sol.



$\therefore (15,8)$ lies in 1st quadrant. In first quadrant x and y is always greater than '0'.

Ex.3 In which quadrants do the given points lie ?

- (a) $(5,-2)$ (b) $(-5,6)$
(c) $(-1,-4)$ (d) $(3,6)$

Sol. (a) Point of the type $(+, -)$ lie in the 4th quadrant.

Hence, the point $(5, -2)$ lies in quadrant IV.

(b) Point of the type $(-, +)$ lie in the 2nd quadrant.

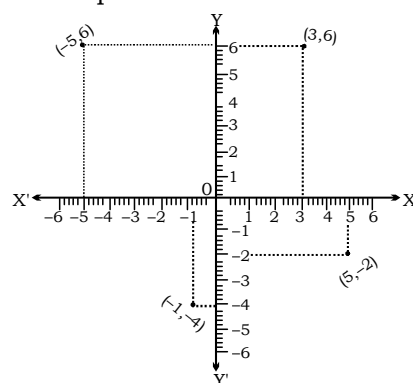
Hence, the point $(-5,6)$ lies in quadrant II.

(c) Point of the type $(-, -)$ lie in the 3rd quadrant.

Hence, the point $(-1,-4)$ lies in quadrant III.

(d) Points of the type $(+, +)$ lie in the 1st quadrant.

Hence, the point $(3,6)$ lies in quadrant I.



Cartesian Co-ordinates

Using cartesian Co-ordinates you marks a point on a graph how far along and how far up it is.

X axis and Y Axis:-

→ The X axis runs horizontally through Zero

→ The Y axis runs vertically through zero

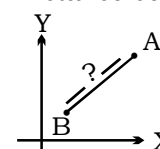
→ Axis is the reference line from which distance are measured.

→ The horizontal "x" value in a pair at coordinates is known as Abscissa.

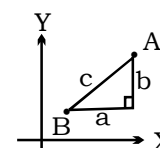
→ The vertical "Y" value in a pair of Co-ordinates is known as ordinate.

1. Distance Formula

Distance between 2 Points:-



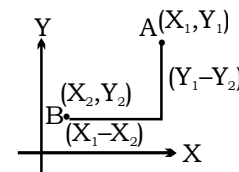
How to calculate the distance between A and B



We can extend Line down from A and along from B to make a right angle.

Now with the help from phythagoras theorem

$$a^2 + b^2 = c^2$$



X_1 means the X - Co-ordinate of Point A

Y_1 means the Y- Co-ordinate of point A

Horizontal distance a is $(X_1 - X_2)$

Vertical distance b is $(Y_1 - Y_2)$

Now we can solve for C the distance between the points is

$$C^2 = a^2 + b^2$$

$$C^2 = (X_1 - X_2)^2 + (Y_1 - Y_2)^2$$

$$C = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Ex. 4 Find the distance between the point (9,7) and (3,2) ?

$$(X_1, Y_1) = (9, 7)$$

$$(X_2, Y_2) = (3, 2)$$

$$= \sqrt{(9-3)^2 + (7-2)^2}$$

$$\Rightarrow \sqrt{36+25} = \sqrt{61}$$

Ex. 5 Find the distance between the point (5,2) and (3,4)

Sol. $C = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$= \sqrt{(5-3)^2 + (2-4)^2}$$

$$= \sqrt{4+4} = 2\sqrt{2}$$

Ex. 6 Find the distance between points (-2,5) and (6, -1)

Sol. $C = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$= \sqrt{(-2-6)^2 + (5-(-1))^2}$$

$$= \sqrt{100} = 10$$

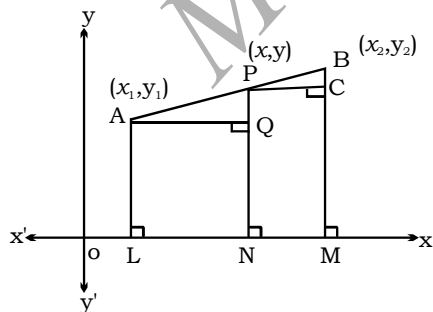
Sectional Formula:- This formula tells us the coordinate of a point P(x,y) which divides a line Segment $\overline{AB}$ where $A=(x_1, y_1)$ and $B=(x_2, y_2)$ in the Ratio m : n internally

$$x = \frac{(mx_2 + nx_1)}{(m+n)};$$

$$y = \frac{(my_2 + ny_1)}{(m+n)}$$

Proof of Internal division:-

Consider Line Segment $\overline{AB}$ joining two points $A = (x_1, y_1)$ and $B = (x_2, y_2)$ and assume that point $P = (x, y)$ divide $\overline{AB}$ internally in the Ratio of m : n.



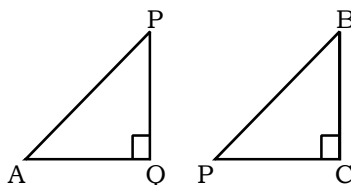
From the figure we get
 $\angle PAQ = \angle BPC$

$$\angle AQP = \angle BCP = 90^\circ$$

$$\angle APQ = \angle PBC$$

So, $\triangle APQ \sim \triangle PBC$.

Since the triangle are Similar, the ratio of their sides are also equal



$$\frac{AQ}{PC} = \frac{AP}{PB}$$

$$\frac{x - x_1}{x_2 - x} = \frac{m}{n}$$

$$nx - nx_1 = mx_2 - mx$$

$$nx + mx = mx_2 + nx_1$$

$$x(n+m) = nx_1 + mx_2$$

$$x = \frac{mx_2 + nx_1}{m+n} \dots\dots(i)$$

Similarly for y

$$\frac{PQ}{BC} = \frac{m}{n}$$

$$\frac{y - y_1}{y_2 - y} = \frac{m}{n}$$

$$y = \frac{my_2 + ny_1}{m+n} \dots\dots(ii)$$

From (1) and (2)

$$P = (x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

Ex. 7 Find the point which divides the line Segment joining (2,5) and (1,2) in the ratio 2 : 1 Internally

Sol. $P(x) = \frac{mx_2 + nx_1}{m+n} = \frac{2 \cdot 1 + 1 \cdot 2}{2+1} = \frac{4}{3}$

$$P(y) = \frac{my_2 + ny_1}{m+n} = \frac{2 \cdot 2 + 1 \cdot 5}{2+1} = 3$$

$$P(x, y) = \left(\frac{4}{3}, 3 \right)$$

Ex. 8 Find the point which divides the line Segment joining (3,7) and (1,-2) in the ratio 3 : 2 Internally

Sol. $P(x) = \frac{mx_2 + nx_1}{m+n}$

$$= \frac{3 \times 1 + 2 \times 3}{3+2}$$

$$= \frac{3+6}{5} = \frac{9}{5}$$

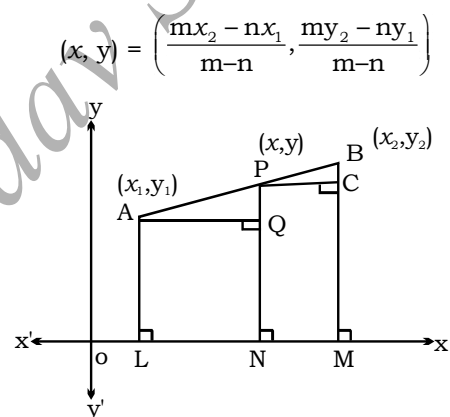
$$P(y) = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{-3 \times 2 + 2 \times 7}{3+2} = \frac{8}{5}$$

$$P(x, y) = \left(\frac{9}{5}, \frac{8}{5} \right)$$

Proof of External Division

Consider Line Segment $\overline{AB}$ joining two points $A = (x_1, y_1)$ and $B = (x_2, y_2)$ and assume that point $P = (x, y)$ divide $\overline{AB}$ externally in the Ratio of m : n.



From the figure we get

$$\angle PAQ = \angle BPC$$

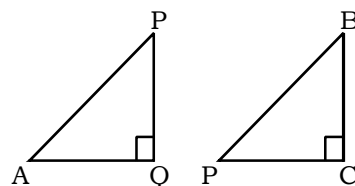
$$\angle AQP = \angle BCP = 90^\circ$$

$$\angle APQ = \angle PBC$$

So,

$$\triangle APQ \sim \triangle PBC.$$

Since the triangle are Similar, the ratio of their sides are also equal



Since the $\triangle$ are Similar, the ratio of the Sides

$$\frac{AQ}{PC} = \frac{AP}{PB}$$

$$\frac{x - x_1}{x - x_2} = \frac{m}{n}$$

$$nx - nx_1 = mx - mx_2$$

$$nx - mx = nx_1 - mx_2$$

$$x(m - n) = mx_2 - nx_1$$

$$x = \frac{mx_2 - nx_1}{m-n} \dots\dots(iii)$$

Similarly for y

$$\frac{PQ}{BC} = \frac{m}{n}; \frac{y - y_1}{y - y_2} = \frac{m}{n}$$

$$y = \frac{my_2 - ny_1}{m-n} \quad \dots (iv)$$

From (iii) and (iv)

$$P = (x, y)$$

$$= \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right)$$

Ex9. Find the coordinate of the point P when it divides the line AB Externally in the ratio of 3:1 where A = (9, 4) and B = (5, 2)

$$\text{Sol. } P(x) = \frac{mx_2 - nx_1}{m-n}$$

$$= \frac{3 \times 5 - 1 \times 9}{3-1} = \frac{6}{2} = 3$$

$$P(y) = \frac{my_2 - ny_1}{m-n} = \frac{3 \times 2 - 1 \times 4}{3-1}$$

$$= \frac{2}{2} = 1$$

$$P(x, y) = (3, 1)$$

Ex10. Find the co-ordinates of the point which divides the joint of the points (2, 4) and (6, 8) externally in the ratio 5 : 3.

$$\text{Sol. } P(x) = \frac{5 \times 6 - 3 \times 2}{5-3} = \frac{24}{2} = 12$$

$$P(y) = \frac{5 \times 8 - 3 \times 4}{5-3} = \frac{28}{2} = 14$$

Mid-Point:- The Co-ordinates of the mid-point of the Line Segment joining the two points A(x₁, y₁) and B(x₂, y₂) are given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$(x_1, y_1) \quad \text{P} \quad (x_2, y_2)$$

The midpoint is halfway between the two end points

Ex11. What is the mid-point of the points (-3, 5), (8, -1)

$$\text{Sol. M} = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

$$= \frac{(-3) + 8}{2}, \frac{5 + (-1)}{2}$$

$$\Rightarrow \frac{5}{2}, 2 \Rightarrow 2.5, 2$$

Ex12. What is the mid-point of the points (6, -8), (10, -4)

$$\text{Sol. M} = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

$$= \frac{6+10}{2}, \frac{-8-4}{2}$$

$$\Rightarrow 8, -6$$

Division by Axes : If P (x₁, y₁) and Q (x₂, y₂), then PQ is divided by

$$(i) \text{ x - axis in the ratio } = \frac{-y_1}{y_2}$$

$$(ii) \text{ y - axis in the ratio } = -\frac{x_1}{x_2}$$

Division by a Line : A line ax + by + c = 0 divides PQ in the ratio

$$= -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$$

Area of a triangle : The area of a triangle ABC whose vertices are A(x₁, y₁), B(x₂, y₂) and C(x₃, y₃) is denoted by Δ.

$$\therefore \Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Area of Polygon : The area of the polygon whose vertices are (x₁, y₁), (x₂, y₂), ..., (x_n, y_n) is -

$$= \frac{1}{2} [(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + \dots + (x_n y_1 - x_1 y_n)]$$

Ex.13. The area of the triangle whose vertices are P (3, 5), Q(-2, 8) and R(4, 7), (in square units) is :

Sol.

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3$$

$$(y_1 - y_2)]$$

$$\frac{1}{2} [3(8-7) + (-2)(7-5) + 4(5-8)]$$

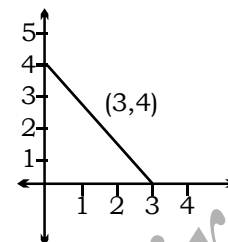
$$= \frac{1}{2} (-13) = \frac{13}{2} \text{ Square units.}$$

Ex14. Find the area of triangle which formed by 4x + 3y = 12 on x and y Axis.

$$\text{Sol. } 4x + 3y = 12$$

$$\text{If } x = 0; y = 4$$

$$x = 3; y = 0$$



Area of Triangle

$$= \frac{1}{2} \times 4 \times 3 = 6 \text{ Square units.}$$

• **Centroid :** If (x₁, y₁), (x₂, y₂) and (x₃, y₃) are the vertices of a triangle, then the co-ordinates of its centroid are -

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Ex15:- Find the centroid of a triangle whose vertices are (10, 7), (9, 5) and (5, 6)

$$\text{Sol. } \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$$= \frac{10+9+5}{3}, \frac{7+5+6}{3} = 8, 6$$

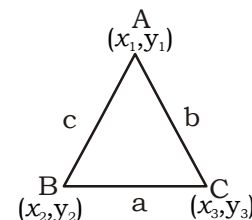
Ex16:- Find the centroid of a triangle whose vertices are (5, 3), (4, 6) and (8, 2)

Sol. Centroid of Δ (x, y)

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$$= \frac{5+4+8}{3}, \frac{3+6+2}{3} = \frac{17}{3}, \frac{11}{3}$$

• **Incentre :** If A (x₁, y₁), B(x₂, y₂) and C(x₃, y₃) are the vertices of a ΔABC, BC = a, CA = b and AB = c, then the co-ordinates of its incentre are

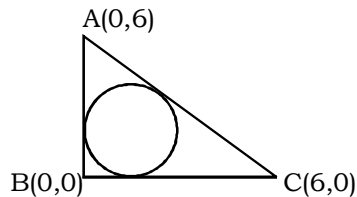


$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

Ex.17 Find the in-centre of the right angled isosceles triangle having one vertex at the origin and having the other two vertices at (6,0) and (0,6).

Sol. AS it is an isosceles triangle the length of the two sides AB and BC of the triangle is 6 units and the length of the third side is $(6^2 + 6^2)^{1/2}$.

Hence $a = c = 6$, $b = 6\sqrt{2}$



In- centre will be at

$$\frac{(6.0 + 6\sqrt{2}.0 + 6.6)}{(6 + 6 + 6\sqrt{2})} \cdot \frac{(6.6 + 6\sqrt{2}.0 + 6.0)}{(6 + 6 + 6\sqrt{2})}$$

$$= \frac{36}{12 + 6\sqrt{2}} \cdot \frac{36}{12 + 6\sqrt{2}}$$

Note:-

- If the triangle is equilateral, then centroid, incentre, orthocentre, circumcentre coincides.
- Orthocentre, centroid and circumcentre are always collinear and centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1.
- In an isosceles triangle centroid, orthocentre, incentre, circumcentre lies on the same line.
- Incentre divides the angles bisectors in the ratio $(b + c) : a$, $(c + a) : b$, $(a + b) : c$.
- Area of the triangle formed by co- ordinate axes and the line $ax + by + c = 0$ is $\frac{c^2}{2ab}$

Straight Line : A straight line is a curve such that every point on the line segment joining any two points on it lies on it.

Different Forms of the Equations of a Straight Line :

(a) General Form : The general Form of the equation of a straight line is $ax + by + c = 0$ (First degree equation in x and y). Where a , b and c are real constants and a , b are not simultaneously equal to zero.

In this equation, slope of the line is given

by $\left(-\frac{a}{b}\right)$

The general form is also given by $y = mx + c$; where m is the slope and c is the intercept on y -axis.

(b) Line Parallel to the X-axis : The equation of a straight line to the x -axis and at a distance b from it, is given by $y = b$

Obviously, the equation of the x -axis is $y = 0$

(c) Line Parallel to Y-axis : The equation of a straight line parallel to the y -axis and at a distance a from it given by $x = a$ obviously, the equation of y -axis is $x = 0$

(d) Slope Intercept Form : The equation of a straight line passing through the point $A(x_1, y_1)$ and having a slope m is given by $(y - y_1) = m(x - x_1)$

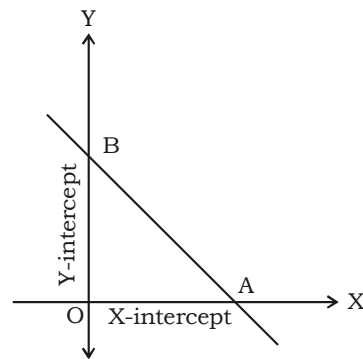
(e) Two Points Form : The equation of a straight line passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Its slope $(m) = \frac{y_2 - y_1}{x_2 - x_1}$

(f) Intercept Form : The equation of a straight line making intercepts a and b on the axes of x and y respectively is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$



Ex 18. Find the point where the straight line $2x - 3y = 12$ cuts x -axis and y -axis. Also find the length intercepted by the line between the axis.

Sol. $2x - 3y = 12$

$$\text{or } \frac{2x}{12} - \frac{3y}{12} = 1$$

$$\text{or, } \frac{x}{6} + \frac{y}{(-4)} = 1$$

Thus straight line cuts x -axis at (6,0) and y -axis at (0,-4)

Length intercepted between axis

$$= \sqrt{6^2 + (-4)^2} = \sqrt{36 + 16}$$

$$= \sqrt{52} = 2\sqrt{13}$$

Ex19. Find the equation of line whose ending points are (4,6) and (10,8)

Sol. $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$$(x_1, y_1) = 4, 6$$

$$(x_2, y_2) = 10, 8$$

$$y - 6 = \left(\frac{8 - 6}{10 - 4}\right)(x - 4)$$

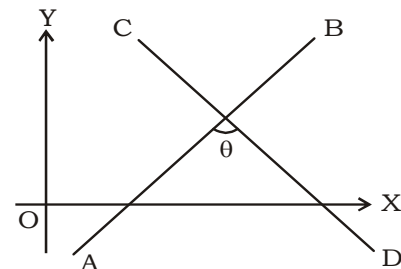
$$y - 6 = \left(\frac{2}{6}\right)(x - 4)$$

$$6y - 36 = 2x - 8$$

$$\text{Equation of Given} = 2x - 6y + 28 = 0$$

Angle between two lines

$$\tan \theta = \pm \left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$$



- **Condition of Parallellism of lines:** If the slopes of two lines is m_1 and m_2 and if they are parallel, then,

$$m_1 = m_2$$

- **Length of Perpendicular it y or Distance of a Point from a Line:** The length of perpendicular from a given point (x_1, y_1) to a line $ax + by + c = 0$ is :

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Note: The length of Perpendicular from the origin to the line $ax + by + c = 0$ is given by

$$\frac{|c|}{\sqrt{a^2+b^2}}$$

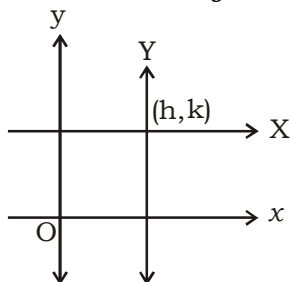
- **Distance between two Parallel Lines :** If two lines are parallel, the distance between them will always be the same.

When two straight lines are parallel whose equations are $ax_1 + by_1 + c_1 = 0$ and $ax_2 + by_2 + c_2 = 0$, then the distance between them

is given by $\frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}$.

- **Changes of Axes :** If origin (0, 0) is shifted to (h, k) then the coordinates of the point (x, y) referred to the old axes and (X, Y) referred to the new axes can be related with the relation

$$x = X + h \text{ and } y = Y + k$$



- **Point of Intersection of Two Lines:** Point of intersection of two lines can be obtained by solving the equations as simultaneous equations.

- If the given equations of straight line are

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0, \text{ then}$$

- (i) The angle between the lines 'θ' is given by

$$\tan \theta = \frac{a_2b_1 - a_1b_2}{a_1a_2 + b_1b_2}$$

- (ii) If the lines are parallel, then

$$a_2b_1 - a_1b_2 = 0 \text{ or } \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

- (iii) If the lines are perpendicular, then

$$a_1a_2 + b_1b_2 = 0$$

- (iv) Coincident : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- Angle between lines
 $x \cos \alpha + y \sin \alpha = P_1$ and
 $x \cos \beta + y \sin \beta = P_2$ is
 $|\alpha - \beta|$

Ex20. Find the perpendicular distance from a point (2,3) to the line $3x + 4y + 7 = 0$

Sol. $P = \frac{|3(2) + 4(3) + 7|}{\sqrt{3^2 + 4^2}}$
 $= \frac{|6 + 12 + 7|}{5} = \frac{25}{5} = 5$

Ex21. Find the angle between two lines $x - 3y + 13 = 0$ and $x + 2y - 11 = 0$.

Sol. $3y = x + 13 \Rightarrow y = \frac{x}{3} + \frac{13}{3}$ then m_1

$$= \frac{1}{3}$$

$$2y = -x + 11$$

$$\Rightarrow y = -\frac{1}{2}x + \frac{11}{2} \text{ then } m_2 = -\frac{1}{2}$$

$$\tan \alpha = \left| \frac{\frac{1}{3} - (-\frac{1}{2})}{1 + \frac{1}{3}(-\frac{1}{2})} \right| = \left| \frac{\frac{5}{6}}{1 - \frac{1}{6}} \right| = 1$$

$$\therefore \alpha = 45^\circ$$

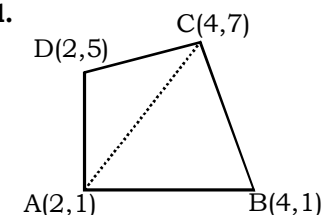
Quadrilateral

There is a quadrilateral whose vertices are A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) and D (x_4, y_4) then quadrilateral will be Rhombus, Square, Rectangle and Parallelogram.

	Name of the figure	Conditions
1.	Square	Four sides are equal and the diagonals are also equal
2.	Rhombus	Four sides are equal.
3.	Rectangle	Opposite sides are equal and diagonals are also equal
4.	Parallelogram	Opposite sides are equal
5.	Parallelogram but not a rectangle	Opposite sides are equal but the diagonals are not equal
6.	Rhombus but not a square	All sides are equal but the diagonals are not equal

Ex.22 Find the area quadrilateral formed by joining points (2,1), (4,1), (2,5) and (4,7)

Sol.



Area of triangle ACD

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [2(7 - 5) + 4(5 - 1) + 4(1 - 7)] = 4$$

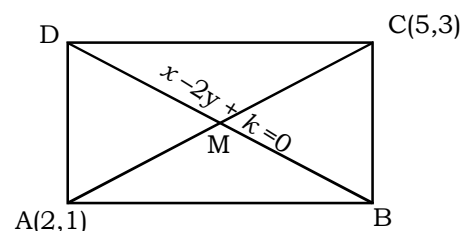
Area of triangle ABC

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [2(1 - 7) + 4(7 - 1) + 4(1 - 1)] = 6$$

Hence, Area of quadrilateral ABCD = Area of triangle ADC + Area of triangle ABC = 4 + 6 = 10 sq. unit

Ex.23. If two opposite vertex of a rectangle are (2,1) and (5,3) and the equation of other diagonal is $x - 2y + k = 0$ find k.



M is the mid-point of diagonal AC & BD.

Co-ordinate of M

$$= \left(\frac{2+5}{2}, \frac{3+1}{2} \right) = (3.5, 2)$$

Point M is also situated on line $x - 2y + k = 0$

So, $3.5 - 2 \times 2 + k = 0$

$$\Rightarrow k = \frac{1}{2}$$

Conditions for Solvability

The system of equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ has :

- (i) a unique solution, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

- (ii) an infinite number of solutions,

$$\text{if } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

- (iii) no solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Homogenous System of Equations:

The system of equations $a_1x + b_1y = 0$; $a_2x + b_2y = 0$ has

- (i) only solution $x = 0, y = 0$, when

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

- (ii) an infinite number of solutions

$$\text{when } \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

- The graphs of $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ will be :
 - Parallel, if the system has no Solution;
 - Coincident, if the system has infinite number of solutions ;
 - Intersecting, if the system has a unique solution.

Ex25. If $9x + 4y = 15$ and $kx + 12y = 45$ are coincide lines find value of k .

Sol. $\frac{9}{k} = \frac{4}{12} = \frac{15}{45}$

$$\Rightarrow \frac{1}{3} = \frac{9}{k}$$

$$\Rightarrow k = 27$$

Ex26. If $2x + 3y = 122$ and $4x + ky = 119$ have unique solution.

Sol. For unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{2}{4} \neq \frac{3}{k}$$

$$\Rightarrow k \neq 6$$

- Ordered Pair :** A pair of numbers a and b listed in a specific order with a at the first place and b at the second place is called an ordered pair (a, b)
Note that $(a, b) \neq (b, a)$
Thus $(3, 5)$ is one ordered pair and $(5, 3)$ is another ordered pair.

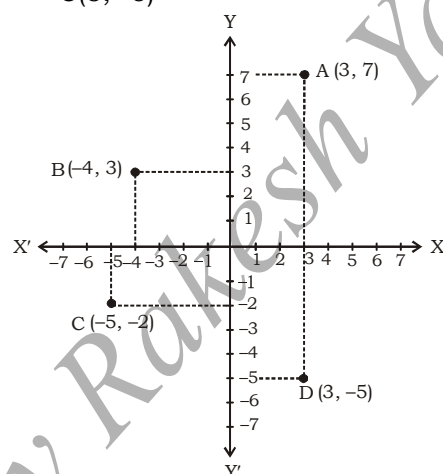
Example: Draw the lines XOX' and YOY' as axes on the plane of a paper and plot the points given below :

- $A(3, 7)$
- $B(-4, 3)$
- $C(-5, -2)$
- $D(3, -5)$

Sol: Let XOX' and YOY' be the co-ordinate axes.

Fix a convenient unit of length and starting from O , mark equal distances on OX, OX', OY and OY' . Use the convention of signs.

- Starting from O , take $+3$ units on the x -axis and then $+7$ units on the y -axis to obtain the point $A(3, 7)$.
- Starting from O , take -4 units on the x -axis and then $+3$ units on the y -axis to obtain the point $B(-4, 3)$.
- Starting from O , take -5 units on the x -axis and then -2 units on the y -axis to obtain the point $C(-5, -2)$.
- Starting from O , take $+3$ units on the x -axis and then -5 units on the y -axis to obtain the point $D(3, -5)$.



Graph of $y = mx + c$

Example: Draw the graph of the equation $y = 3x + 2$:

Sol. Construct a table and choose simple x values.

x	-2	-1	0	1	2
y					

In order to find the y - values for the table, substitute each x - values into the rule $y = 3x + 2$.

$$\text{when } x = -2, y = 3(-2) + 2$$

$$= -6 + 2 = -4$$

$$\text{when } x = -1, y = 3(-1) + 2$$

$$= -3 + 2 = -1$$

$$\text{when } x = 0, y = 3(0) + 2$$

$$= 0 + 2 = 2$$

$$\text{when } x = 1, y = 3 \times 1 + 2$$

$$= 3 + 2 = 5$$

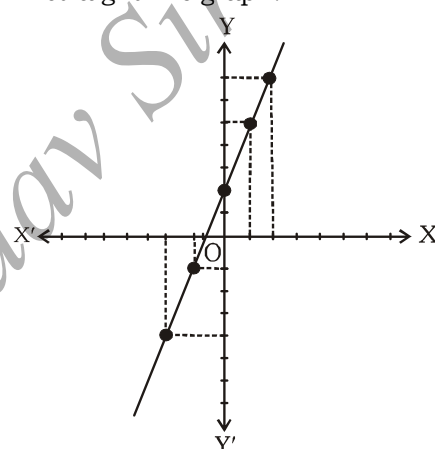
$$\text{when } x = 2, y = 3 \times 2 + 2$$

$$= 6 + 2 = 8$$

The table of values obtained after entering the values of y is as follows:

x	-2	-1	0	1	2
y	-4	-1	2	5	8

Now, Draw a Cartesian plane and plot the points. Then join the points with a ruler to obtain a straight line graph.



Solving Simultaneous Linear Equation (Graphical Method) :

Let the given system of linear equations be

$$a_1x + b_1y + c_1 = 0 \quad \dots(i)$$

$$a_2x + b_2y + c_2 = 0 \quad \dots(ii)$$

On the same graph paper, we draw the graph of each one of the given linear equations.

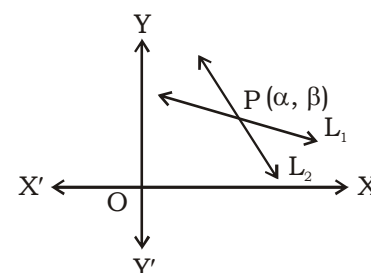
Each such graph is always a straight line.

Let the lines L_1 and L_2 represent the graph of (i) and (ii) respectively.

Now, the following cases arise:

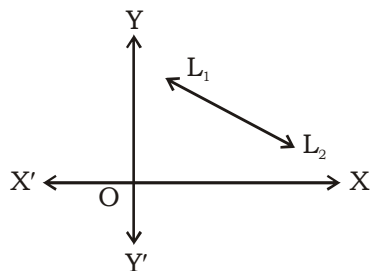
Case-1. When the lines L_1 and L_2 intersect at a point $P(\alpha, \beta)$

Then, $x = \alpha, y = \beta$ is the unique solution of the given system of equations.



Case-2 . When the lines L_1 and L_2 are coincident.

Then, given system of equations has infinitely many solutions.



Case-3. When the line L_1 and L_2 are parallel. In this case, there is no common solution of the given system of equations.

Thus, in this case, the given system is inconsistent.

- **Point of Intersection of Two Lines :** Point of intersection of two lines can be obtained by solving the equations as simultaneous equations.

Inequalities

Graph : Let us consider an inequality $ax + by \leq c$.

Step 1. Consider the equation $ax + by = c$. Draw the graph of this equation, which is a line.

In case of strict inequalities draw the line dotted, otherwise mark it thick.

Step 2. Choose a point [if possible (0, 0)], not lying on this line. Substitute its coordinates in the given in equations. If this point satisfies the given in equation, then shade the portion of the plane which contains the chosen point, otherwise shade the portion which does not contain this point.

The shaded portion represents the solution set. The dotted line is not a part of the solution set, while thick line is a part of it.

Example: Graph of the inequality $2x - y \geq 1$?

Sol : Consider the equation

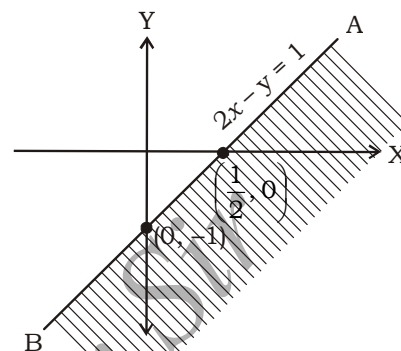
$$2x - y = 1$$

$$\text{or } \frac{x}{1/2} + \frac{y}{-1} = 1$$

i.e. x -intercept = $1/2$ and y -intercept = -1

i.e. it meets x -axis at $(1/2, 0)$ and y -axis at $(0, -1)$

Join these two point with a thick line AB.



Now, consider (0, 0) put (0, 0) in the given inequality.

$$\text{L.H.S} = 2 \times 0 - 0 \neq \text{R.H.S}$$

i.e. it does not satisfy $2x - y \geq 1$.

So, shade the portion of the plane not containing (0, 0).

Shaded portion constitutes the solution set.

Note : The area bounded by $|x| + |y| = k$ is $2k^2$.

EXERCISE

- Find the value of 'A' if the distance between the points (8,A) and (4,3) is 5.
(a) 6 (b) 0
(c) Both (a) and (b)
(d) Not
- Find the value of C if the distance between the point (C,4) and the origin is 5 units
(a) 3 (b) -3
(c) Both a & b (d) Not.
- Point A(-3, 2) and B(5,4) are the end points of a line segment, find the Co-Ordinates of the mid points of the line Segment.
(a) $(\frac{3}{2}, 1)$ (b) $(\frac{2}{3}, 0)$
(c) 1,3 (d) (2,3)
- Find the area of quadrilateral formed by joining points (4,2),(8,2),(4,10) and (8,14).
(a) 5 Sq. units (b) 10 Sq. units
(c) 25 Sq. units (d) 40 sq. units
- If two opposite vertex of a Rectangle are (4,2) and (10,6) and the

equation of other diagonal is $x - 3y + k = 0$. Find the value of k?

- (a) $\frac{1}{2}$ (b) 3 (c) 5 (d) 4
- Find the Intercepts made by the line $7x + 8y - 56 = 0$. On the axis
(a) (5,6) (b) (8,7)
(c) (7,8) (d) None of these
- Find the slope of line passing through the points (2,8) and (6,9).
(a) 5 (b) -5
(c) .25 (d) -.25
- Find the area of triangle formed by lines $8x - 3y = 24$, x axis and y - axis
(a) 12 sq. units (b) 6 sq. units
(c) 18 sq. units (d) 9 sq. units
- Find the area of triangle formed by lines $-6x + 9y = 36$, x -axis and y -axis
(a) 6 sq. units (b) 12 sq. units
(c) 3 sq. units (d) 8 sq. units
- Find the area of triangle formed by lines $2x - 2y = 10$, $2x + 3y = 10$ and x - axis

(a) $\frac{75}{6}$ sq. units

(b) $\frac{125}{6}$ sq. units

(c) $-\frac{125}{6}$ sq. units

(d) None of these

- Find the area of $4x + 3y = 12$; $5x + 7y = 35$. On x -axis

(a) $\frac{160}{13}$ sq. units

(b) $\frac{80}{13}$ sq. units

(c) $\frac{40}{13}$ sq. units

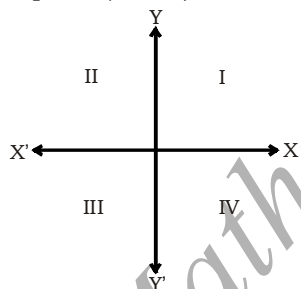
(d) $\frac{150}{13}$ sq. units

12. Find the area of quadrilateral intercepted by straight lines $4x + 2y = 12$, $8x + 4y = 48$ between the axis.
(a) 36 sq. units (b) 27 sq. units
(c) 18 sq. units (d) 9 sq. units
13. Find the area of quadrilateral intercepted by straight lines $x + y = 2$, $3x + 4y = 24$ between the axis
(a) 22 sq. units (b) 20 sq. units
(c) 18 sq. unit (d) 16 sq. units
14. Find the area of quadrilateral formed by straight lines $x = 1$, $x = 3$, $y = 2$ and $x = y + 3$.
(a) 6 sq. units (b) 12 sq. units
(c) 3 sq. units (d) None of these
15. For what value of k given system of equations $5x + 20y = 11$ and $2x + ky = 17$ are intersecting lines?
(a) $k \neq 8$ (b) $k \neq 4$
(c) $k \neq 14$ (d) $k \neq 2$
16. For what value of k the lines $4x + ky = 3$ and $3x + 2y = 7$ are perpendicular to each other?
(a) 6 (b) ± 6
(c) -6 (d) 4
17. For what value of k the lines $(k+1)x + ky = 3$ and $5x - 2y = 7$ are perpendicular to each other?
(a) $\frac{1}{3}$ (b) $-\frac{5}{3}$ (c) $-\frac{1}{3}$ (d) 5
18. If the points A (4,3) and B (x, 5) are on the circle with centre O (2,3), find the value of x.
(a) 1 (b) 2 (c) 2 (d) -2
19. Find a point on the y-axis which is equidistant from the points A (6, 5) and B (-4, 3)
(a) (0,9) (b) (9,0)
(c) (3,0) (d) (4,0)
20. Find the equation of the line which cuts off intercepts 2 and 3 on the axis.
(a) $9x - 7y = 6$
(b) $3x - 2y = 5$
(c) $4x - 3y = 7$
(d) $3x + 2y = 6$
21. Find the co-ordinates of the centroid of a triangle whose vertices are (0, 6) (8, 12) and (8, 0).
(a) $\left(\frac{16}{3}, 6\right)$ (b) $\left(6, \frac{16}{3}\right)$
(c) (6,3) (d) (6,3)
22. The vertices of a triangle are (3, -5) and (-7,4). If its centroid is (2, -12), find the third vertex.
(a) (10, -35) (b) (-2, 10)
(c) (10, 35) (d) (-3, 10)
23. Rhombus ABCD has A at (2,0) and B at (4,4). If one of the sides of the rhombus lies on the x-axis, what is the area of this rhombus?
(a) 40 sq. units
(b) $8\sqrt{6}$ sq. units
(c) $4\sqrt{5}$ sq. units
(d) $8\sqrt{5}$ sq. units
24. What are the co-ordinates of the incentre of triangle given by (3,4), (3,7) and (7,4)?
(a) (5,5) (b) (5,4)
(c) (4,5) (d) (5,5)
25. The equation of a line passing through the point (4,4) and cutting off intercepts on the axis whose sum is 18?
(a) $x + 2y - 12 = 0$
but not $2x + y - 12 = 0$
(b) neither $x + 2y - 12 = 0$ nor $2x + y - 12 = 0$
(c) $2x + y - 12 = 0$
but not $x + 2y - 12 = 0$
(d) $x + 2y - 12 = 0$ or $2x + y - 12 = 0$
26. The co-ordinates of the point P which divides the join of A (5, -2) and B (9,6) in the ratio 3: 1 will be -
(a) (4, -7) (b) $\left(\frac{7}{2}, 4\right)$
(c) (8,4) (d) (12,8)
27. If line $x \cos \theta + y \sin \theta = 2$ is perpendicular to the line $x - y = 3$, then what is one of the value of θ ?
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$
28. The foot of the perpendicular drawn from the point (2,-1) to a straight line L is (1,3). The equation of straight line L is -
(a) $x - 4y + 11 = 0$
(b) $x + 4y + 13 = 0$
(c) $4x - y - 1 = 0$
(d) $4x + y - 7 = 0$
29. Find the area of triangle of lines $\frac{x}{6} + \frac{y}{7} = 1$, $-\frac{x}{4} + \frac{y}{7} = 1$ and x-axis.
(a) 30 sq. units (b) 35 sq. units
(c) 70 sq. units (d) 60 sq. units
30. Let the vertices of a triangle ABC be (4,3), (7,-1), (9,3) then the triangle is :
(a) Scalene (b) Isosceles
(c) Equilateral (d) None of these
31. Let the vertices of a triangle ABC be (4,4), (3,5), (-1,-1), then the triangle is :
(a) scalene (b) equilateral
(c) right angled (d) None of these
32. A(-3,2) and B(5,4) are the end points of a line segment, find the coordinates of the mid points of the line segment:
(a) (1,3) (b) (2,3)
(c) (3,2) (d) (4,3)
33. Find the co-ordinates of the point which divides the join of the points (2,4) and (6,8) externally in the ratio 5 : 3.
(a) (5,6) (b) (12,14)
(c) (3,8) (d) (2,7)
34. Find the co-ordinates of the incentre of the triangle whose vertices are the points (4,-2), (5,5) and (-2,4).
(a) $\left(\frac{7}{3}, \frac{2}{3}\right)$ (b) $\left(\frac{5}{2}, \frac{5}{2}\right)$
(c) $\left(\frac{6}{5}, 5\right)$ (d) None of these
35. A(-2,-1), B(1,0), C(4,3) and D(1,2) are the four points of a quadrilateral. The quadrilateral is a:
(a) Square
(b) Rhombus
(c) Parallelogram
(d) None of these
36. If A(-2,1), B(2,3) and C(-2,-4) are three points, find the angle between BA and BC.
(a) $\tan^{-1}\left(\frac{3}{5}\right)$ (b) $\tan^{-1}\left(\frac{2}{3}\right)$
(c) $\tan^{-1}\left(\frac{3}{2}\right)$ (d) None of these
37. Find the intercepts made by the line $3x + 4y - 12 = 0$ on the axes:
(a) 2 and 3 (b) 4 and 3
(c) 3 and 5 (d) none of these
38. A firm produces 50 units of a good of ₹ 320 and 80 units for ₹ 380. Supposing that the cost curve is a straight line, estimate the cost of producing 110 units.
(a) ₹ 330 (b) ₹ 1665
(c) ₹ 440 (d) ₹ 365
39. Find the length of the perpendicular from the point (3, -2) to the straight line $12x - 5y + 6 = 0$.

- (a) 5 (b) 4 (c) 6 (d) 8
40. Find the equation of the line through the intersection of the lines $3x + 4y = 7$ and $x - y + 2 = 0$ having slope 3.
 (a) $4x - 3x + 4y = 7$
 (b) $21x - 7y + 16 = 0$
 (c) $8x + y + 8 = 0$
 (d) none of these
41. A straight line intersects the x-axis at A and the y-axis at B. AB is divided internally at C(8,10) in the ratio 5 : 4. Find the equation of AB.
 (a) $x + y = 18$ (b) $x + y + 2 = 0$
 (c) $x + y - 2 = 0$ (d) None of these
42. Find the distance between the points (-5,3) and (3,1).
 (a) $2\sqrt{7}$ (b) $3\sqrt{14}$
 (c) $5\sqrt{17}$ (d) $2\sqrt{17}$
43. The point (6, -3) lies in the quadrant:
 (a) First (b) Second
 (c) Third (d) Fourth
44. If $x < 0$ and $y > 0$, then the point (x, y) lies in :
 (a) quadrant I (b) quadrant II
 (c) quadrant III (d) quadrant IV
45. Which of the following points lies on the line $y = 3x + 5$?
 (a) (2, 11) (b) (3, 15)
 (c) (4, 19) (d) (5, 15)
46. The co-ordinates of a point situated on x-axis at a distance of 7-units from y-axis is :
 (a) (0, 7) (b) (7, 0)
 (c) (7, 7) (d) (-7, 7)
47. The co-ordinates of a point below x-axis at a distance of 8 units from x-axis but lying on y-axis is :
 (a) (0, 8) (b) (-8, 0)
 (c) (0, -8) (d) (8, -8)
48. The point of intersection of the lines $2x + 7y = 1$ and $4x + 5y = 11$ is :
 (a) (4, -1) (b) (2, 3)
 (c) (-1, 4) (d) (4, -2)
49. Which of the following points does not lie on the line $y = 2x + 3$?
 (a) (2, 7) (b) (3, 9)
 (c) (4, 10) (d) (5, 13)
50. The area of ΔOAB with O(0, 0), A (8, 0) and B(0, 6) is :
 (a) 12 sq. units (b) 48 sq. units
 (c) 36 sq. units (d) 24 sq. units
51. The line $2x - 3y = 6$ meets y-axis at the point :
 (a) (-2, 0) (b) (0, -2)
 (c) (0, 3) (d) (3, 0)
52. The line $4x + 7y = 12$ meets x-axis at the point :
 (a) (3, 1) (b) (0, 3)
 (c) (3, 0) (d) (4, 0)
53. Graphical equation of the line $x = 0$ represents :
 (a) y-axis (b) x-axis
 (c) origin (d) both x-axis and y-axis
54. The graph of the line $x = 5$ is parallel to :
 (a) x-axis (b) y-axis
 (c) both x-axis and y-axis
 (d) none of these
55. The graph of the line $x = -3$ is parallel to :
 (a) x-axis (b) y-axis
 (c) both x-axis and y-axis
 (d) none of these
56. when we draw the graph of $y = 2$, the line is parallel to :
 (a) x-axis (b) y-axis
 (c) both x-axis and y-axis
 (d) none of these
57. If point $\left(3, \frac{11}{2}\right)$ lies on the graph of the equation $2y = ax + 5$, then a is equal to :
 (a) -2 (b) 4
 (c) 2 (d) 3
58. If point $\left(3, \frac{5}{2}\right)$ lies on the graph of the equation $2y = ax - 5$, then a is equal to :
 (a) $\frac{10}{3}$ (b) $-\frac{10}{3}$
 (c) $\frac{7}{3}$ (d) 0
59. Find the distance between pair of points (8, 0) and (0, 6) :
 (a) 9 (b) 8
 (c) -10 (d) 10
60. Distance of the point (7, 24) from the origin is :
 (a) 24 (b) 25
 (c) -25 (d) 22
61. When we plot the points (4, 3), (0, 0) and (-4, -3) on the graph paper, then the graph shows :
 (a) straight line (b) curve
 (c) zig-zag line (d) None of these
62. The equation of the line which passes through the origin is :
 (a) $3x + 4y = 5$ (b) $3x + 4y = 7$
 (c) $3y + 4x = 0$ (d) None of these
63. If we plot the points (4, 3), (-3, 3), (-3, -2), (-4, -2) on the graph paper, the shape formed is:
 (a) parallelogram
 (b) rectangle
 (c) square (d) rhombus
64. If the side of a figure are represented by $x = 2$, $x = -2$, $y = 2$, $y = -2$ then the graph shows :
 (a) Rhombus
 (b) Rectangle
 (c) Parallelogram
 (d) Square
65. If a straight line $ax + by = c$ meets x-axis at P and y-axis at Q. Then area of the triangle OPQ where O is the point of intersection of co-ordinate axes is :
 (a) $\frac{c^2}{ab}$ (b) $\frac{c^2}{2ab}$
 (c) $\frac{c^2}{a^2b}$ (d) $\frac{c^2}{ab^2}$
66. The line passing through the points (-3, 7) and (4, 6) :
 (a) cut x-axis only
 (b) cuts y-axis only
 (c) cuts both the axes
 (d) does not cut any axes.
67. For what value of k will the equations $x + 2y + 6 = 0$ and $3x + ky + 18 = 0$ represent coincident lines ?
 (a) $k = 8$ (b) $k = 4$
 (c) $k = 5$ (d) $k = 6$
68. The value of k for which the system of equations $x + 3y + 7 = 0$, $4x + ky + 19 = 0$ has no solution is :
 (a) $k = 6$ (b) $k = -12$
 (c) $k = 12$ (d) $k = 8$
69. The pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ in two variables represents pair straight lines which are intersecting, if :
 (a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

- (b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
 (c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
 (d) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$

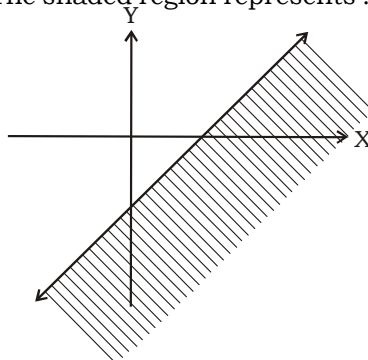
70. The point of the form (b, b) always lies on the straight lines :
 (a) $y = b$ (b) $x - y = 0$
 (c) $x = b$ (d) $x + y = 0$
71. The straight lines $2x + y = 4$ and $x - 3y = 9$ intersect at a point, the point of intersection is :
 (a) $(3, -2)$ (b) $(-3, 2)$
 (c) $(2, -3)$ (d) $(-2, 3)$
72. The value of k for which the system of equations $kx + y = 7$, $9x + 3y = 17$ has a unique solution :
 (a) $k = 3$ (b) $k = 0$
 (c) $k \neq 3$ (d) $k \neq 0$
73. When we plot the points A $(-1, 2)$, B $(-1, -2)$ and C $(2, -2)$ on a graph paper, the figure formed by these points is :
 (a) right angled triangle
 (b) scalene triangle
 (c) equilateral triangle
 (d) isosceles triangle
74. If $ab > 0$ and the point (a, b) lies in the third quadrant in which the point $(-b, -a)$ lies is :



- (a) I (b) II
 (c) III (d) IV
75. The area of the triangle formed by the lines $5x + 7y = 35$, $4x + 3y = 12$ and x -axis is :
 (a) $\frac{160}{13}$ sq. unit
 (b) $\frac{150}{13}$ sq. unit
 (c) $\frac{140}{13}$ sq. unit
 (d) 10 sq. unit

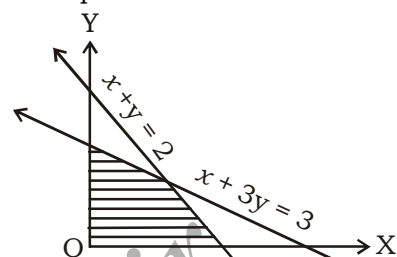
76. The area of triangle formed by the lines $3x - 6y = 12$, $3x - y = 3$ and x -axis is :
 (a) 5.4 sq. units
 (b) 2.7 sq. units
 (c) 4.5 sq. units
 (d) 3.6 sq. units

77. The shaded region represents :



- (a) $y \geq x$ (b) $y \leq -x$
 (c) $y \leq x$ (d) $y \geq -x$
78. The three vertices of $\triangle ABC$ are A $(2, 4)$, B $(-3, 2)$, C $(4, 2)$. The area is :
 (a) 7 sq. units (b) 9 sq. units
 (c) 14 sq. units (d) 10 sq. units
79. If we plot the point A $(4, 11)$ and B $(-2, 3)$ on graph, then the distance between them is :
 (a) 15 (b) 9
 (c) 12 (d) 10
80. Find the area of the region bounded by lines $3x + 4y = 24$, $x + y = 2$ and the coordinate axes :
 (a) 25 sq. units (b) 24 sq. units
 (c) 22 sq. units (d) 20 sq. units
81. Find the area of the region bounded by lines $3x + 4y = 12$, $6x + 8y = 60$, $x = 0$ and $y = 0$:
 (a) 37.5 sq. units
 (b) 31.5 sq. units
 (c) 25 sq. units
 (d) 32 sq. units
82. The area of the region bounded by the lines $x - y = 0$, $x + 2y = 0$ and $y = 3$ is:
 (a) 17 sq. units
 (b) 6.75 sq. units
 (c) 27 sq. units
 (d) 13.5 sq. units
83. The region specified by $x \geq 0$, $x + y \geq 0$ includes :
 (a) 1st quadrant
 (b) 2nd quadrant
 (c) 3rd quadrant
 (d) 4th quadrant

84. The shaded region in the given figure is the solution set of the inequalities :



- (a) $x + y \leq 2$, $x + 3y \geq 3$, $x \geq 0$, $y \geq 0$
 (b) $x + y \geq 2$, $x + 3y \geq 3$, $x \geq 0$, $y \geq 0$
 (c) $x + y \geq 2$, $x + 3y \leq 3$, $x \geq 0$, $y \geq 0$
 (d) $x + y \leq 2$, $x + 3y \leq 3$, $x \geq 0$, $y \geq 0$
85. Area of the rectangular region $2 \leq x \leq 5$, $-1 \leq y \leq 3$ is :
 (a) 9 sq. units (b) 12 sq. units
 (c) 15 sq. unit (d) 20 sq. units
86. Graph of the inequation $2x - 5y \geq 5$ in cartesian plane is :
 (a) above the line $2x - 5y = 5$
 (b) below the line $2x - 5y = 5$
 (c) on & below the line $2x - 5y = 5$
 (d) on & above the line $2x - 5y = 5$
87. Find the area bounded by $|x| + |y| = 6$
 (a) 72 sq. units (b) 48 sq. units
 (c) 54 sq. units (d) 84 sq. units
88. The area of the region bounded by $y = |x| - 1$ and $y = 1 - |x|$
 (a) 3 sq. units (b) 4 sq. units
 (c) 2 sq. units (d) 1 sq. unit
89. The point $(-5, 7)$ lies in the quadrant:
 (a) First (b) Second
 (c) Third (d) Fourth
90. The point $(7, -5)$ lies in the quadrant:
 (a) First (b) Second
 (c) Third (d) Fourth
91. Find the distance between the points $(-6, 2)$ and $(2, 4)$:
 (a) $2\sqrt{17}$ (b) $4\sqrt{17}$
 (c) $2\sqrt{5}$ (d) 10
92. The distance between the points A $(b, 0)$ and B $(0, a)$ is :
 (a) $\sqrt{a^2 - b^2}$ (b) $\sqrt{a^2 + b^2}$
 (c) $\sqrt{a + b}$ (d) $a + b$

93. The distance between the points A (7, 4) and B(3, 1) is :
 (a) 6 units (b) 3 units
 (c) 4 units (d) 5 units
94. The co-ordinates of point situated on x -axis at a distance of 5 units from y -axis is :
 (a) (0, 5) (b) (5, 0)
 (c) (5, 5) (d) (-5, 5)
95. The co-ordinates of a point situated on y -axis at a distance of 7 units from x -axis is :
 (a) (0, 7) (b) (7, 0)
 (c) (7, 7) (d) (-7, 7)
96. The co-ordinates of a point below x -axis at a distance of 6 units from x -axis but lying on y -axis is :
 (a) (0, 6) (b) (-6, 0)
 (c) (0, -6) (d) (6, -6)
97. The distance of the point (6, -8) from the origin is :
 (a) 2 units (b) 14 units
 (c) 7 units (d) 10 units
98. The point of intersection of the lines $2x + 7y = 1$ and $4x + 5y = 11$ is :
 (a) (4, -1) (b) (2, 3)
 (c) (-1, 4) (d) (4, -2)
99. The line $4x + 7y = 12$ meets x -axis at the point :
 (a) (3, 1) (b) (0, 3)
 (c) (3, 0) (d) (4, 0)
100. The line $4x - 9y = 11$ meets y -axis at the point :
 (a) $\left(-\frac{11}{9}, 0\right)$ (b) $\left(0, -\frac{11}{9}\right)$
 (c) $\left(0, \frac{11}{4}\right)$ (d) $\left(0, -\frac{11}{4}\right)$
101. The slope of the line $3x + 7y + 8 = 0$ is :
 (a) 3 (b) 7
 (c) $-\frac{3}{7}$ (d) $\frac{3}{7}$
102. The slope of the line joining P(-4, 7) and Q(2, 3) is :
 (a) $-\frac{2}{3}$ (b) $\frac{2}{3}$
 (c) $-\frac{3}{2}$ (d) $\frac{3}{2}$
103. The equation of a line parallel to x -axis at a distance of 6 units and above x -axis is :
 (a) $x = 6$ (b) $y = 6x$
 (c) $x = 6y$ (d) $y = 6$
104. The equation of a line parallel to y -axis at a distance of 5 units to the left of y -axis, is :
 (a) $y = -5$ (b) $x = -5$
 (c) $x + 5y = 0$ (d) $y + 5x = 0$
105. The equation of a line parallel to x -axis and at a distance of 7 units below x -axis is :
 (a) $y = -7$ (b) $x = 7$
 (c) $x = -7$ (d) $y = -7x$
106. The area of the triangle whose vertices are P (4, 5), Q(-3, 8) and R (3, -4), (in square units) is :
 (a) 66 (b) $16\frac{1}{2}$
 (c) 33 (d) 35
107. The points A(0, 0), B(0, 3) and C(4, 0) are the vertices of a triangle which is :
 (a) Isosceles
 (b) Right angled
 (c) Equilateral
 (d) None of these
108. The co-ordinates of the centroid of $\triangle PQR$ with vertices P(-2, 0), Q(9, -3) and R (8, 3) is :
 (a) (1, 0) (b) $\left(\frac{19}{3}, 0\right)$
 (c) (0, 5) (d) (5, 0)
109. The equation of a line passing through the points A (0, -3) and B (-5, 2) is :
 (a) $x + y + 3 = 0$ (b) $x + y - 3 = 0$
 (c) $x - y + 3 = 0$ (d) $x - y - 3 = 0$
110. The length of perpendicular from the origin to the line $12x + 5y + 7 = 0$ is :
 (a) 2 units (b) 1 unit
 (c) $\frac{7}{13}$ units (d) $\frac{7}{11}$ units
111. The angle which the line joining the points $(\sqrt{3}, 1)$ and $(\sqrt{15}, \sqrt{5})$ makes with x -axis is :
 (a) 30° (b) 45°
 (c) 60° (d) 90°
112. The lines whose equations are $2x - 5y + 7 = 0$ and $8x - 20y + 28 = 0$ are:
 (a) parallel
 (b) perpendicular
 (c) coincident
 (d) intersecting
113. If the distance of the point P(x, y) from A(a, 0) is $a + x$, then $y^2 = ?$
 (a) $2ax$ (b) $4ax$
 (c) $6ax$ (d) $8ax$
114. If the point (x, y) is equidistant from the points (a + b, b - a) and (a - b, a + b) then $bx = ?$
 (a) a^2y (b) ay^2
 (c) ay (d) a^2y^2
115. If the sum of the square of the distance of the point (x, y) from the point (a, 0) and (-a, 0) is $2b^2$, then :
 (a) $x^2 + a^2 = b^2 + y^2$
 (b) $x^2 + a^2 = 2b^2 - y^2$
 (c) $x^2 - a^2 = b^2 + y^2$
 (d) $x^2 + a^2 = b^2 - y^2$
116. P (-4, a) and Q(2, a + 4) are two points and the co-ordinates of the middle point of PQ are (-1, 4). The value of a is :
 (a) 0 (b) 2
 (c) -2 (d) 3
117. If the points P(2, 3), Q(5, a) and R (6, 7) are collinear, the value of a is :
 (a) $5/2$ (b) $-4/3$
 (c) 6 (d) 5
118. The equation of a line parallel to x -axis and passing through (-6, -5) is :
 (a) $y = -5$ (b) $x = -6$
 (c) $y = -5x$ (d) $y = -6x - 5$
119. The equation of a line parallel to y -axis and passing through (2, -5) is :
 (a) $x = 2$ (b) $y = -5$
 (c) $y = 2x$ (d) $x = -5y$
120. Two vertices of a triangle PQR are P(-1, 0) and Q(5, -2) and its centroid is (4, 0). The co-ordinates of R are :
 (a) (8, -2) (b) (8, 2)
 (c) (-8, 2) (d) (-8, -2)
121. The co-ordinates of the point of intersection of the medians of a triangle with vertices P(0, 6), Q(5, 3) and R(7, 3) are :
 (a) (4, 5) (b) (3, 4)
 (c) (4, 4) (d) (5, 4)
122. The ratio in which the line segment joining A(3, -5) and B(5, 4) is divided by x -axis is :
 (a) 4 : 5 (b) 5 : 4
 (c) 5 : 7 (d) 6 : 5
123. The ratio in which the line segment joining P(-3, 7) and Q (7, 5) is divided by y -axis is :
 (a) 3 : 7 (b) 4 : 7
 (c) 3 : 5 (d) 4 : 5

124. The ratio in which the point $P\left(1, \frac{10}{3}\right)$ divides the join of the point $A(-3, 2)$ and $B(3, 4)$ is :
 (a) $2 : 3$ (b) $1 : 2$
 (c) $2 : 1$ (d) $3 : 1$
125. The equation of a line with slope 5 and passing through the point $(-4, 1)$ is :
 (a) $y = 5x + 21$ (b) $y = 5x - 21$
 (c) $5y = x + 21$ (d) $5y = x - 21$
126. The value of 'a' so that the lines $x + 3y - 8 = 0$ and $ax + 12y + 5 = 0$ are parallel is :
 (a) 0 (b) 1
 (c) 4 (d) -4
127. The value of P for which the lines $3x + 8y + 9 = 0$ and $24x + py + 19 = 0$ are perpendicular is :
 (a) -12 (b) -9
 (c) -11 (d) 9
128. The value of a so that line joining $P(-2, 5)$ and $Q(0, -7)$ and the line joining $A(-4, -2)$ and $B(8, a)$ are perpendicular to each other is :
 (a) -1 (b) 5
 (c) 1 (d) 0
129. The angle between the lines represented by the equations $2y - \sqrt{12}x - 9 = 0$ and $\sqrt{3}y - x + 7 = 0$, is:
 (a) 30° (b) 45°
 (c) 60° (d) $22\frac{1}{2}^\circ$
130. If $P(3, 5)$, $Q(4, 5)$ and $R(4, 6)$ be any three points, the angle between PQ and PR is :
 (a) 30° (b) 45°
 (c) 60° (d) 90°
131. Given a ΔPQR with vertices $P(2, 3)$, $Q(-3, 7)$ and $R(-1, -3)$. The equation of median PM is :
 (a) $x - y + 10 = 0$
 (b) $x - 4y - 10 = 0$
 (c) $x - 4y + 10 = 0$
 (d) None of these
132. The co-ordinates of the point P which divides the join of $A(3, -2)$ and $B\left(\frac{11}{2}, \frac{21}{2}\right)$ in the ratio $2 : 3$ are :
 (a) $(4, 3)$ (b) $(4, 5)$
 (c) $\left(4, \frac{5}{2}\right)$ (d) $\left(\frac{3}{2}, \frac{7}{2}\right)$
133. The length of the portion of the straight line $8x + 15y = 120$ intercepted between the axes is :
 (a) 14 units (b) 15 units
 (c) 16 units (d) 17 units
134. The equation of the line passing through the point $(1, 1)$ and perpendicular to the line $3x + 4y - 5 = 0$, is :
 (a) $3x + 4y - 7 = 0$
 (b) $3x + 4y + k = 0$
 (c) $3x - 4y - 1 = 0$
 (d) $4x - 3y - 1 = 0$
135. The equation of a line passing through the point $(5, 3)$ and parallel to the line $2x - 5y + 3 = 0$, is :
 (a) $2x - 5y - 7 = 0$
 (b) $2x - 5y + 5 = 0$
 (c) $2x - 2y + 5 = 0$
 (d) $2x - 5y = 0$
136. The sides PQ , QR , RS and SP of a quadrilateral have the equations $x + 2y = 3$, $x = 1$, $x - 3y = 4$, $5x + y + 12 = 0$ respectively, then the angle between the diagonals PR and QS is:
 (a) 30° (b) 45°
 (c) 60° (d) 90°
137. The equations of two equal sides of an isosceles triangle are $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point $(1, -10)$. The equation of the third side is :
 (a) $x - 3y - 31 = 0$ but not $x - 3y - 31 = 0$
 (b) neither $3x + y + 7 = 0$ nor $x - 3y - 31 = 0$
 (c) $3x + y + 7 = 0$ or $x - 3y - 31 = 0$
 (d) $3x + y + 7 = 0$ but not $x - 3y - 31 = 0$
138. If P_1 and P_2 be perpendicular from the origin upon the straight lines $x \sec \theta + y \operatorname{cosec} \theta = a$ and $x \cos \theta - y \sin \theta = a \cos 2\theta$ respectively, then the value of $4P_1^2 + P_2^2$ is :
 (a) a^2 (b) $2a^2$
 (c) $\sqrt{2}a^2$ (d) $3a^2$
139. Find the equation of the line passing through the point $(2, 2)$ and cutting off intercepts on the axes whose sum is 9 ?
 (a) $x + 2y - 6 = 0$ but not $2x + y - 6 = 0$
 (b) neither $x + 2y - 6 = 0$ nor $2x + y - 6 = 0$
 (c) $2x + y - 6 = 0$ but not $x + 2y - 6 = 0$
 (d) $x + 2y - 6 = 0$ or $2x + y - 6 = 0$
140. Romila went to a stationery shop and purchased 2 pencils and 3 erasers for ₹ 9. Her friend Sonali saw the new variety of pencils and erasers with and she also bought 4 pencils and 6 erasers of the same kind for ₹ 18. Represent this situation algebraically and graphically.
141. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were. Also, three years from now, I shall be three times as old as you will be." interesting? Represent this situation algebraically and graphically.
142. 5 Pens and 7 Pencil together cost ₹ 50. Whereas 7 pens and 5 pencils together Cost ₹ 46. find the cost of one pencil and that of one pen.

ANSWER KEY

1. (c)	15. (a)	29. (b)	43. (d)	57. (c)	71. (a)	85. (b)	99. (c)	113.(b)	127.(b)
2. (c)	16. (c)	30. (b)	44. (b)	58. (a)	72. (c)	86. (c)	100.(b)	114.(c)	128.(d)
3. (c)	17. (b)	31. (c)	45. (a)	59. (d)	73. (a)	87. (a)	101.(c)	115.(d)	129.(a)
4. (d)	18. (c)	32. (a)	46. (b)	60. (b)	74. (c)	88. (c)	102.(c)	116.(b)	130.(b)
5. (c)	19. (a)	33. (b)	47. (c)	61. (a)	75. (a)	89. (b)	103.(d)	117.(c)	131.(c)
6. (b)	20. (b)	34. (b)	48. (a)	62. (c)	76. (b)	90. (d)	104.(b)	118.(a)	132.(a)
7. (c)	21. (a)	35. (c)	49. (c)	63. (b)	77. (c)	91. (a)	105.(a)	119.(a)	133.(d)
8. (a)	22. (a)	36. (b)	50. (d)	64. (d)	78. (a)	92. (b)	106.(c)	120.(b)	134.(d)
9. (b)	23. (d)	37. (b)	51. (b)	65. (b)	79. (d)	93. (d)	107.(b)	121.(c)	135.(b)
10. (b)	24. (c)	38. (c)	52. (c)	66. (b)	80. (c)	94. (b)	108.(d)	122.(b)	136.(d)
11. (a)	25. (d)	39. (b)	53. (a)	67. (d)	81. (b)	95. (a)	109.(a)	123.(a)	137.(c)
12. (b)	26. (c)	40. (b)	54. (b)	68. (c)	82. (d)	96. (c)	110.(c)	124.(c)	138.(a)
13. (a)	27. (b)	41. (a)	55. (b)	69. (a)	83. (a)	97. (d)	111.(a)	125.(a)	139.(d)
14. (a)	28. (a)	42. (d)	56. (a)	70. (b)	84. (d)	98. (a)	112.(c)	126.(c)	

EXERCISE

1. (c) Distance between the points

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$5 = \sqrt{(8-4)^2 + (A-3)^2}$$

$$25 = (4)^2 + (A-3)^2$$

$$25 = 16 + A^2 + 9 - 6A$$

$$9 - 9 = A^2 - 6A$$

$$\Rightarrow A^2 - 6A = 0$$

$$\Rightarrow A(A-6) = 0$$

$$A = (6, 0)$$

2. (c) $(5)^2 = (C-0)^2 + (4-0)^2$

$$25 = C^2 + 16$$

$$C^2 = 9$$

$$C = \pm 3$$

3. (c) Co-ordinates of Mid-points

$$\text{of AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-3+5}{2} \right) \left(\frac{2+4}{2} \right) = 1, 3$$

$$= 16 \text{ sq. units}$$

Area of $\triangle ABD$

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [4(2-14) + 8(14-2) + 8(2-2)]$$

$$= \frac{1}{2} [4 \times (-12) + 8 \times 12 + 8 \times 0]$$

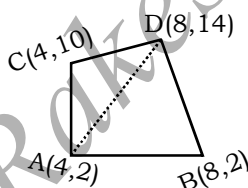
$$= \frac{1}{2} [-48 + 96] = 24 \text{ sq. units}$$

Hence Area of Quadrilateral ABCD

$$= \text{Area of } [\triangle ABD + \triangle ADC]$$

$$= 16 + 24 = 40 \text{ Sq. units}$$

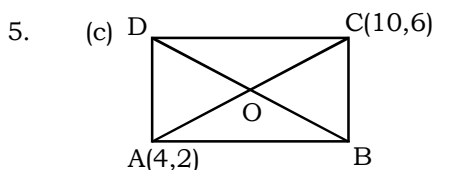
4. (d) Area of Triangle ADC



$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2} [4(14-10) + 8(10-2) + 4(2-14)]$$

$$= \frac{1}{2} [4 \times 4 + 8 \times 8 - 4 \times 12]$$



O is the mid point of diagonal AC and BD Co-ordinate

$$\text{of } O = \left(\frac{4+10}{2}, \frac{2+6}{2} \right)$$

$$= (7,4)$$

Point O is also situated on line $x - 3y + k = 0$

$$\Rightarrow 7 - 3 \times 4 + k = 0$$

$$\Rightarrow 7 - 12 + k = 0$$

$$\Rightarrow k = 5$$

6. (b) $7x + 8y - 56 = 0$

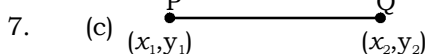
$$7x + 8y = 56$$

$$\frac{x}{8} + \frac{y}{7} = 1$$

Length of Intercepts made at

x axis at $x = 8$

y axis at $y = 7$



$$\text{Slope of line } m = \frac{y_2 - y_1}{x_2 - x_1}$$

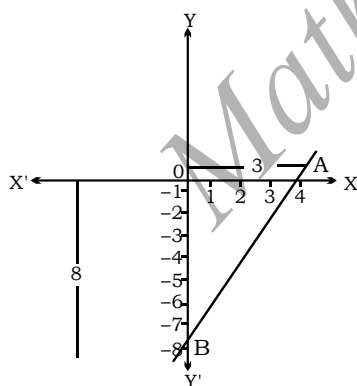
P(2,8) and Q(6,9)

$$\text{Slope } m = \frac{9-8}{6-2} = \frac{1}{4} = .25$$

8. (a) $8x - 3y = 24$

$$\frac{x}{3} - \frac{y}{8} = 1$$

$$\frac{x}{3} + \frac{y}{-8} = 1$$



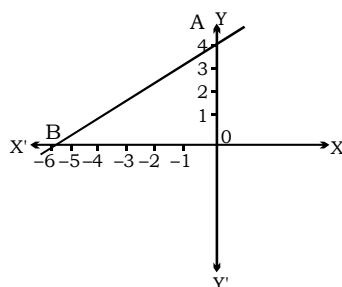
Area of ΔOAB

$$= \frac{1}{2} \times 8 \times 3$$

$$= 12 \text{ Sq. units}$$

9. (b) $-6x + 9y = 36$

$$\frac{x}{-6} + \frac{y}{4} = 1$$



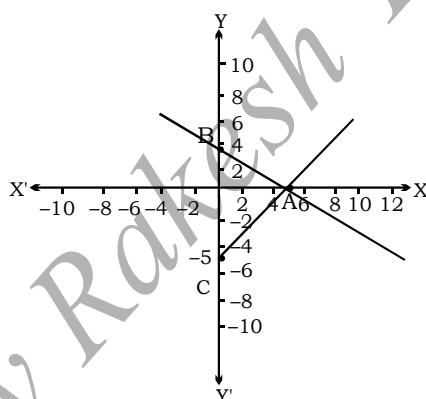
Area of ΔOAB

$$= \frac{1}{2} \times 6 \times 4 = 12 \text{ sq. units}$$

10. (b) $2x - 2y = 10$

$$\Rightarrow \frac{x}{5} - \frac{y}{5} = 1$$

$$\Rightarrow 2x + 3y = 10 = \frac{x}{5} + \frac{3y}{10} = 1$$



$$OA = 5$$

$$OB = 10/3$$

OC = -5 (But distance is always positive)

Area of ΔABC = area of ΔOAB + area of ΔOAC

$$= \frac{1}{2} \times 5 \times \frac{10}{3} + \frac{1}{2} \times 5 \times 5$$

$$= \frac{50}{6} + \left(\frac{.25}{2} \right) = \frac{25}{3} + \frac{25}{2}$$

$$= \frac{50+75}{6} = \frac{125}{6} \text{ sq. units.}$$

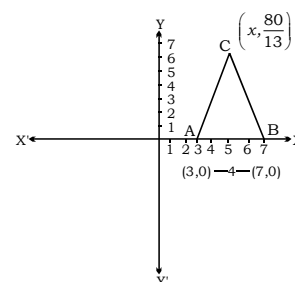
Sol.11(a) On x-axis; $y = 0$

$$4x + 3y = 12$$

$$= x = (3,0) \dots\dots(i)$$

$$5x + 7y = 35$$

$$\Rightarrow x = (7,0) \dots\dots(ii)$$



$$5 \times 4x + 3y = 12 \times 5$$

$$4 \times 5x + 7y = 35 \times 4$$

$$20x + 15y = 60$$

$$20x + 28y = 140$$

$$-13y = -80$$

$$\Rightarrow y = \frac{80}{13}$$

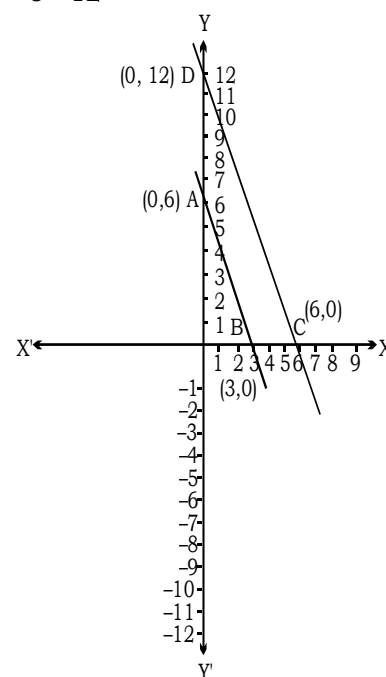
Area of ΔABC

$$= \frac{1}{2} \times 4 \times \frac{80}{13} = \frac{160}{13} \text{ sq. units}$$

12. (b) $4x + 2y = 12$
 $8x + 4y = 48$

$$\Rightarrow \frac{x}{3} + \frac{y}{6} = 1$$

$$\Rightarrow \frac{x}{6} + \frac{y}{12} = 1$$

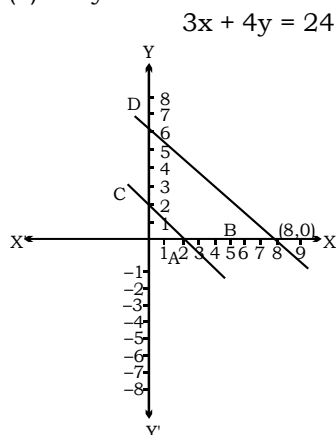


Area of quadrilateral = Area of ΔOCD - Area of ΔOAB

$$= \frac{1}{2} \times 6 \times 12 - \frac{1}{2} \times 3 \times 6$$

$$= 27 \text{ sq. units}$$

13. (a) $x + y = 2$



$$\Rightarrow \frac{x}{2} + \frac{y}{2} = 1 \dots\dots(i)$$

$$\Rightarrow \frac{x}{8} + \frac{y}{6} = 1 \dots\dots(ii)$$

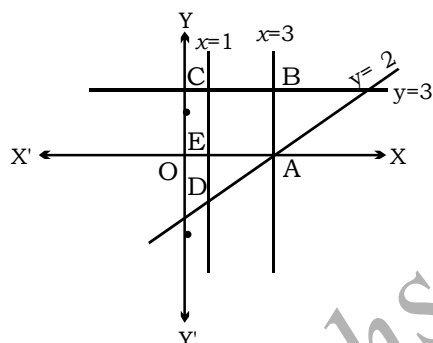
Area of quadrilateral = Area of $\triangle OBD$
- Area of $\triangle OAC$

$$= \frac{1}{2} \times 8 \times 6 - \frac{1}{2} \times 2 \times 2$$

$$= 22 \text{ Sq. units}$$

14. (a) $x = 1, x = 3, y = 2$
and $x = y + 3$

$$\Rightarrow \frac{x}{3} + \frac{y}{-3} = 1$$



Point D will be (1, -2)

ABCD is a trapezium.

$$AB = 2, CD = CE + DE = 2 + 2 = 4$$

$$\text{height} = AE = 3 - 1 = 2$$

Area of trapezium ABCD

$$= \frac{1}{2} AE(AB + CD)$$

$$= \frac{1}{2} \times 2 (2 + 4) = 6 \text{ sq. unit.}$$

15. (a) $5x + 20y = 11$ and $2x + ky = 17$ will be intersecting lines or having unique solution

$$\Rightarrow \frac{5}{2} \neq \frac{20}{k}$$

$$\Rightarrow k \neq 8$$

16. (c) $4x + ky = 3$ and $3x + 2y = 7$ are perpendicular to each other.

Condition for perpendicular :

$$a_1a_2 + b_1b_2 = 0$$

$$\Rightarrow 4 \times 3 + k \times 2 = 0$$

$$\Rightarrow k = -6$$

17. (b) $(k+1)x + ky = 3$ and $5x - 2y = 7$ are perpendicular for perpendicular :

$$a_1a_2 + b_1b_2 = 0$$

$$\Rightarrow (k+1) \times 5 + k \times (-2) = 0$$

$$\Rightarrow 5k + 5 - 2k = 0$$

$$\Rightarrow k = \frac{-5}{3}$$

18. (c) $OA = OB = \text{radius}$

$$= \sqrt{(4-2)^2 + (3-3)^2}$$

$$= \sqrt{(x-2)^2 + (5-3)^2}$$

$$\Rightarrow (x-2)^2 = 0$$

$$\Rightarrow x = 2$$

19. (a) Let the required point be P (0,y).

$$\therefore PA = PB$$

$$= \sqrt{(0-6)^2 + (y-5)^2}$$

$$= \sqrt{(0-4)^2 + (y-3)^2}$$

$$\Rightarrow 36 + (y-5)^2 = 16 + (y-3)^2$$

$$\Rightarrow 36 + y^2 - 10y + 25$$

$$\Rightarrow 16 + y^2 - 6y + 9$$

$$\Rightarrow 4y = 36$$

$$\Rightarrow y = 9$$

$\therefore$ Required point is (0,9).

20. (d) $a = 2, b = 3.$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\Rightarrow \frac{x}{2} + \frac{y}{3} = 1$$

$$\Rightarrow 3x + 2y = 6$$

21. (a) Centroid a triangle =

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$$= \left(\frac{0+8+8}{3}, \frac{6+12+0}{3} \right)$$

$$= \left(\frac{16}{3}, 6 \right)$$

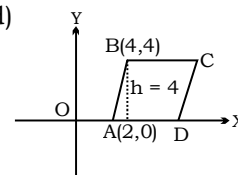
22. (a) Vertices of triangle are (3, -5), (-7, 4) and (h, k) and centroid is (2, -12)

$$(2, -12) = \left(\frac{3-7+h}{3}, \frac{-5+4+k}{3} \right)$$

On solving.

$$(h, k) = (10, -35)$$

23. (d)



$$AB = \sqrt{(4-2)^2 + (4-0)^2}$$

$$\sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

Area of Rhombus =

Base $\times$ height

$$2\sqrt{5} \times 4 = 8\sqrt{5} \text{ sq. units}$$

24. (c)

$$a = \sqrt{(7-3)^2 + (4-7)^2} = 5$$

$$b = \sqrt{(7-3)^2 + (4-4)^2} = 4$$

$$c = \sqrt{(3-3)^2 + (7-4)^2} = 3$$

Incentre I =

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

$$= \left(\frac{5 \times 3 + 4 \times 3 + 3 \times 7}{5+4+3}, \frac{5 \times 4 + 4 \times 7 + 3 \times 4}{5+4+3} \right)$$

$$= (4, 5)$$

25. (d) Let a line $\frac{x}{a} + \frac{y}{b} = 1$

Given, $a + b = 18 \dots\dots(i)$

Passing through (4,4)

$$\text{It means } \frac{4}{a} + \frac{4}{b} = 1$$

$$\Rightarrow ab = 4(a+b) = 72$$

$$(a-b)^2 = (a+b)^2 - 4ab$$

$$= (18)^2 - 4 \times 72$$

$$= 324 - 288 = 36$$

$$a - b = \pm 6 \dots\dots(ii)$$

On solving equation (i) and (ii)

$$(a, b) = (12, 6) \text{ or } (6, 12)$$

$$\text{Line will be } \frac{x}{12} + \frac{y}{6} = 1$$

$$\Rightarrow x + 2y = 12$$

Or,

$$\frac{x}{6} + \frac{y}{12} = 1$$

$$\Rightarrow 2x + y = 12$$

26. (c) $\frac{A \dots B}{(5, -2) \dots (9, 6)}$

$$(x, y) = \left(\frac{9 \times 3 + 1 \times 5}{3+1}, \frac{6 \times 3 + 1 \times -2}{3+1} \right)$$

$$= (8, 4)$$

27. (b) $x \cos \theta + y \sin \theta = 2$
 $\Rightarrow y \sin \theta = -x \cos \theta + 2$
 $\Rightarrow y = -x \cot \theta + 2 \operatorname{cosec} \theta$
 $m_1 = -\cot \theta$
 $x - y = 3 \Rightarrow y = x - 3$
 $m_2 = 1$
Both lines are perpendiculars to each other so $m_1 \cdot m_2 = -1$
 $(-\cot \theta) \cdot (1) = -1$
 $\Rightarrow \cot \theta = 1$
 $\Rightarrow \theta = \frac{\pi}{4}$

28. (a) $m_1 = \tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$
 $= \frac{3+1}{1-2} = -4$

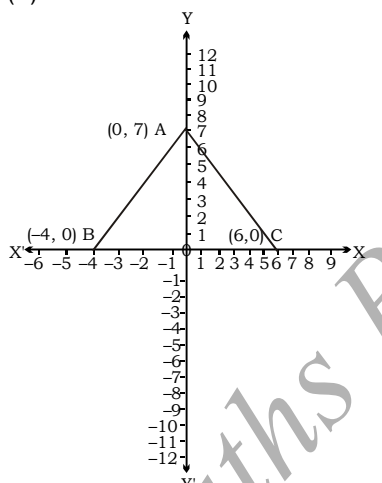
Slope of line L = $\frac{-1}{m_1} = \frac{-1}{-4} = \frac{1}{4}$
Equation of line L will be

$$(y - 3) = \frac{1}{4}(x - 1)$$

$$4(y - 3) = x - 1$$

$$x - 4y + 11 = 0$$

29. (b)



$$\frac{x}{6} + \frac{y}{7} = 1, \quad \frac{x}{-4} + \frac{y}{7} = 1 \quad \text{and}$$

x-axis
 $BC = 4 + 6 = 10$ unit
 $AO = 7$ units

$$\text{Area of triangle} = \frac{1}{2} \times BC \times OA$$

$$= \frac{1}{2} \times 10 \times 7 = 35 \text{ units}$$

30. (b) Let $A = (4, 3)$, $B = (7, -1)$, $C = (9, 3)$
 $AB = \sqrt{(7-4)^2 + (-1-3)^2}$

$$= \sqrt{25} = 5$$

$$BC = \sqrt{(9-7)^2 + (3+1)^2}$$

$$= \sqrt{20} = 2\sqrt{5}$$

$$CA = \sqrt{(4-9)^2 + (3+3)^2}$$

$$= \sqrt{25} = 5$$

$$\therefore AB = CA = 5$$

Hence ABC is an isosceles triangle

31. (c) Let $A = (4, 4)$, $B = (3, 5)$, $C = (-1, -1)$
Then

$$AB = \sqrt{(3-4)^2 + (5-4)^2}$$

$$= \sqrt{2}$$

$$BC = \sqrt{(-1-3)^2 + (-1-5)^2}$$

$$= \sqrt{52}$$

$$AC = \sqrt{(-1-4)^2 + (-1-4)^2}$$

$$= \sqrt{50}$$

$\therefore AB^2 + AC^2 = BC^2$
Hence by the Pythagoras Theorem, ABC is a right angled triangle.

32. (a) The coordinates of the mid points of AB

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{-3+5}{2}, \frac{2+4}{2} \right) = (1, 3)$$

33. (b) The required coordinates of the point which divides AB in the ratio 5 : 3

$$\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}$$

$$= \frac{5 \times 6 - 3 \times 2}{5-3}, \frac{5 \times 8 - 3 \times 4}{5-3}$$

$$= (12, 14)$$

34. (b) Let $A = (4, -2)$, $B = (5, 5)$ and $C = (-2, 4)$
Then

$$a = BC = \sqrt{(-2-5)^2 + (4-5)^2}$$

$$= 5\sqrt{2}$$

$$b = AC = \sqrt{(4+2)^2 + (-2-4)^2}$$

$$= 6\sqrt{2}$$

$$c = AB = \sqrt{(5-4)^2 + (5+2)^2}$$

$$= 5\sqrt{2}$$

and
 $(x_1, y_1) = (4, -2)$, $(x_2, y_2) = (5, 5)$.
 $(x_3, y_3) = (-2, 4)$

$\therefore$ The coordinates of the incentre of the ΔABC are

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

$$= \left(\frac{5}{2}, \frac{5}{2} \right)$$

{substitute the value of (a,b,c) $\{x_1, x_2, x_3, y_1, y_2$ and $y_3\}$.

35. (c) Let A, B, C and D be the vertices of the quadrilateral whose coordinates are $(-2, 1)$, $(1, 0)$, $(4, 3)$ and $(1, 2)$ respectively.

Now, $AB = \sqrt{10}$, $BC = \sqrt{18}$, $DC = \sqrt{10}$, $AD = \sqrt{18}$

$\therefore AB = CD$ and $BC = AD$ i.e., the opposite sides are equal.

Hence ABCD is a parallelogram.

36. (b) Let m_1 and m_2 be the slope of BA and BC respectively. Then

$$m_1 = \frac{3-1}{2-(-2)} = \frac{1}{2}$$

and

$$m_2 = \frac{-4-3}{-2-2} = \frac{7}{4}$$

Let θ be the angle between BA and BC. Then

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

$$= \left| \frac{\frac{7}{4} - \frac{1}{2}}{1 + \frac{7}{4} \times \frac{1}{2}} \right| = \pm \frac{2}{3}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{2}{3} \right)$$

37. (b) We have $3x + 4y - 12 = 0$
 $\Rightarrow 3x + 4y = 12$

$$\Rightarrow \frac{3x}{12} + \frac{4y}{12} = 1$$

$$\Rightarrow \frac{x}{4} + \frac{y}{3} = 1$$

Which is of the form $\frac{x}{a} + \frac{y}{b} = 1$

Then the required intercepts on the axes are 4 and 3.

38. (c) Let the equation of the cost curve as a straight line be $y = mx + c$ (i)

Where x = number of units of a good produced and y = cost of x units in rupees.

Given, when $x = 50$, $y = 320$
and when $x = 80$, $y = 380$

$$\text{from (1) } 320 = 50m + c \dots(2)$$

$$380 = 80m + c \dots(3)$$

Subtracting (2) from (3), we get
 $m = 2$

Substituting $m = 2$ in equation (2), we get $c = 220$

$$\therefore \text{From (1) } y = 2x + 220$$

$$\text{When } x = 110, y = 2 \times 110 + 220 = 440$$

Hence, the required cost of producing 110 units is ₹ 440.

39. (b) Length of the perpendicular from the point $(3, -2)$ to the straight line $12x - 5y + 6 = 0$ is

$$\frac{12 \times 3 - 5 \times (-2) + 6}{\sqrt{(12)^2 + (-5)^2}}$$

$$= \frac{36 + 10 + 6}{\sqrt{169}} = 4 \text{ units.}$$

40. (b) The intersection point of the lines $3x + 4y - 7 = 0$ and $x - y + 2 = 0$ is,

$$3x + 4y - 7 = 0 \dots\dots (i)$$

$$x - y + 2 = 0 \dots\dots (ii)$$

$$x = y - 2 \text{ putting in equation (i)}$$

$$3y - 6 + 4y - 7 = 0$$

$$y = \frac{13}{7}, x = y - 2 = \frac{13}{7} - 2 = \frac{1}{7}$$

$$\text{equation of line}$$

$$y = mx + c$$

$$m = \text{slope of line}$$

$$y = 3x + c \dots\dots (iii)$$

$$\text{equation (iii) passing the point } \left(-\frac{1}{7}, \frac{13}{7}\right)$$

$$\frac{13}{7} = \frac{-3}{7} + c$$

$$c = \frac{16}{7} \text{ putting in equation (iii)}$$

$$y = 3x + \frac{16}{7}$$

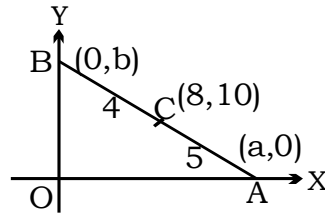
$$21x - 7y + 16 = 0$$

41. (a) Let the equation of the line

$$AB \text{ be } \frac{x}{a} + \frac{y}{b} = 1 \dots\dots(i)$$

Then the coordinates of A and B are respectively $(a, 0)$ and $(0, b)$.

since C $(8, 10)$ divides AB in the ratio $5 : 4$, we have



$$\frac{5 \times 0 + 4 \times a}{5 + 4} = 8$$

$$\text{and } \frac{5 \times b + 4 \times 0}{5 + 4} = 10$$

$$\text{or } a = 18 \text{ and } b = 18$$

Hence from (1), the required equation of the line AB is

$$\frac{x}{18} + \frac{y}{18} = 1$$

$$\text{or } x + y = 18$$

42. (d) Distance between two points

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Let } (x_1, y_1) = (-5, 3) \text{ and } (x_2, y_2) = (3, 1)$$

$$\therefore \text{Required distance}$$

$$= \sqrt{(3 + 5)^2 + (1 - 3)^2}$$

$$= \sqrt{64 + 4} = \sqrt{68} = 2\sqrt{17} \text{ units}$$

43. (d) The point $(6, -3)$ lies in the fourth quadrant.

44. (b) If $x < 0$ & $y > 0$, (x, y) lies in quadrant II

45. (a)????? When $x = 2$, $y = 3 \times 2 + 5 = 6 + 5 = 11$

So, $(2, 11)$ lies on $y = 3x + 5$

46. (b) Clearly, the point of x -axis has ordinate 7 and abscissa 0

So, the point is $(7, 0)$

47. (c) Clearly, the point is $(0, -8)$.

48. (a) $2x + 7y = 1$ (i)

$$4x + 5y = 11 \dots\dots(ii)$$

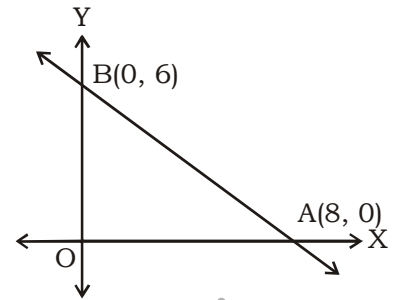
On solving (i) and (ii), we get $x = 4$ and $y = -1$

$\therefore$ Required point of intersection = $(4, -1)$

49. (c) When $x = 4 \Rightarrow y = 2 \times 4 + 3 = 11$

So, $(4, 11)$ lies on $y = 2x + 3$ but $(4, 10)$ does not lie on it.

50. (d)



Clearly, $OA = 8$ units and $OB = 6$ units

$$\therefore \text{ar}(\Delta OAB) = \frac{1}{2} \times OA \times OB$$

$$= \frac{1}{2} \times 8 \times 6 = 24 \text{ sq. units}$$

51. (b) at y -axis, $x = 0$

$$\therefore 2 \times 0 - 3y = 6 \Rightarrow y = -2$$

$$\therefore \text{Required point } (0, -2)$$

52. (c) at x -axis, $y = 0$

$$\therefore 4x + 7 \times 0 = 12 \Rightarrow x = 3$$

$$\therefore \text{Required point} = (3, 0)$$

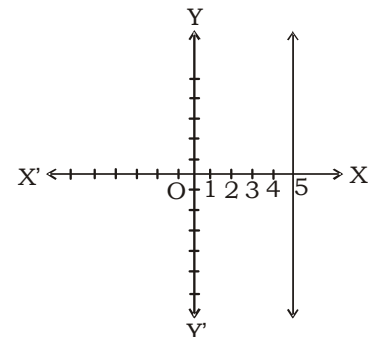
53. (a) The equation of y -axis is $x = 0$

54. (b) The given equation is $x = 5$.

Here, y -coordinate is 0.

So,

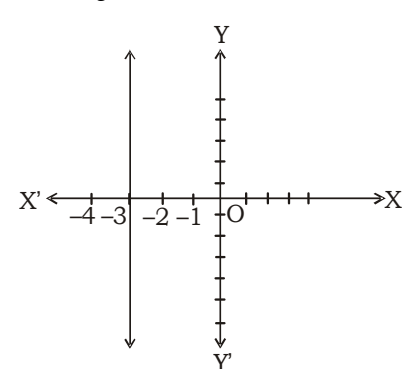
the given line is parallel to y -axis



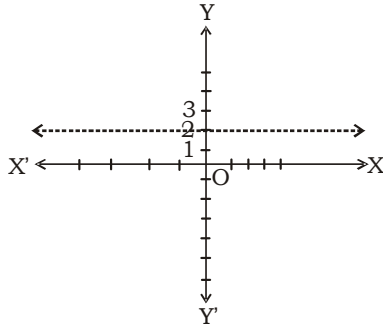
55. (b) The given equation is $x = -3$

Here y -coordinate is 0.

So, the given line is parallel to y -axis



56. (a) The given equation is $y = 2$, here x -coordinate is 0.
So,
the given line is parallel to x -axis.



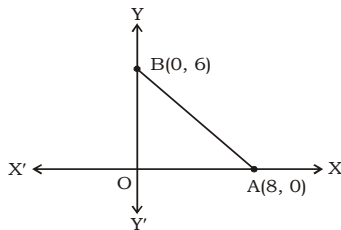
57. (c) Point $\left(3, \frac{11}{2}\right)$ will satisfy the given equation.

$$\therefore 2 \times \frac{11}{2} = a \times 3 + 5 \Rightarrow 11 = 3a + 5 \Rightarrow 6 = 3a \text{ or } a = 2$$

58. (a) Point $\left(3, \frac{5}{2}\right)$ will satisfy the given equation.

$$\therefore 2 \times \frac{5}{2} = a \times 3 - 5 \Rightarrow 5 = 3a - 5 \Rightarrow 3a = 10 \text{ or } a = \frac{10}{3}$$

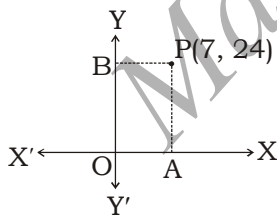
59. (d)



Required distance = AB

$$\begin{aligned} & \sqrt{(OA)^2 + (OB)^2} \\ &= \sqrt{8^2 + 6^2} = 10 \text{ units} \end{aligned}$$

60. (b)

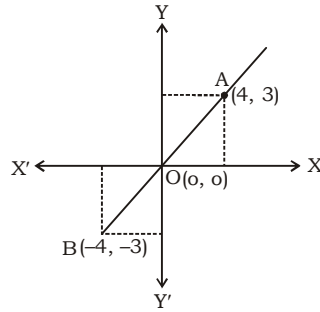


Clearly, $OA = 7$, $AP = 24$

$$\therefore OP^2 = OA^2 + AP^2 = 7^2 + 24^2 = 625$$

$$\therefore OP = \sqrt{625} = 25$$

61. (a) This graph shows AB is a straight line which passes through the origin.



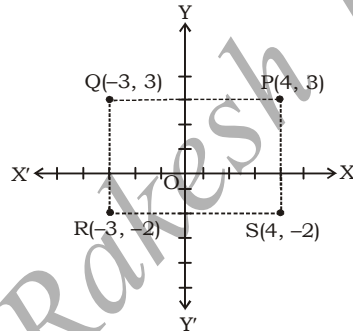
62. (c) $3y + 4x = 0$ or $y = -\frac{4}{3}x$

which passes through origin as $(0, 0)$ satisfy it.

63. (b) Taking the points in the order given, it is easily seen that they are represented P, Q, R, S. Then the shape shows rectangle (PQRS) having sides.

$$PS = QR = 3 - (-2) = 5$$

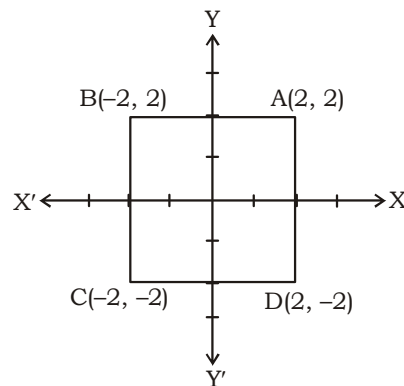
$$PQ = SR = 4 - (-3) = 7$$



64. (d) The graph of $x = 2$ will be a line parallel to y -axis at a distance of 2 units to its right.

Similarly, the graph of $x = -2$ will be a line parallel to y -axis at a distance of 2 units to its left.
and $y = 2$ and $y = -2$ will be the lines parallel to x -axis at a distance of 2 units to its right and 2 units to its left respectively.

Thus ABCD is a square having each side of 4 units.



65. (b) The equation is $-ax + by = c$ (given)

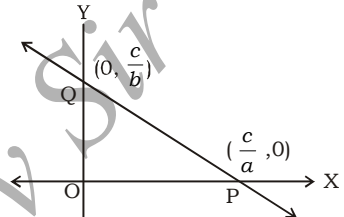
$$\text{When } x = 0 \Rightarrow by = c$$

or

$$y = \frac{c}{b}$$

$$\text{When } y = 0 \Rightarrow -ax = c$$

$$\text{or } x = -\frac{c}{a}$$

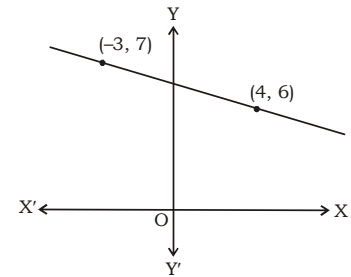


$$\text{Clearly, } OP = \frac{c}{a}, \quad OQ = \frac{c}{b}$$

$$\therefore \text{ar}(\triangle OPQ) = \frac{1}{2} \times OP \times OQ$$

$$= \frac{1}{2} \times \frac{c}{a} \times \frac{c}{b} = \frac{c^2}{2ab}$$

66. (b) $(-3, 7)$ lies in second quadrant and $(4, 6)$ lies in first quadrant.



67. (d) The given equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. These will represent coincident lines if they have infinitely many solutions. The condition for which is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{1}{3} = \frac{2}{k} = \frac{6}{18}$$

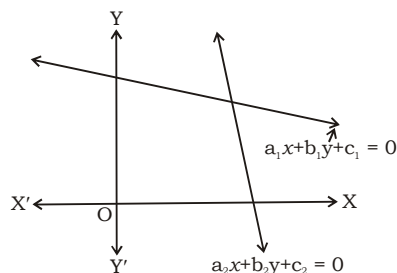
$$\Rightarrow k = 6$$

68. (c) $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ will have no solution if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{1}{4} = \frac{3}{k} \Rightarrow k = 12$$

69. (a)



Given that lines are intersecting i.e. they have unique solution.

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

70. (b) The point (b, b) always lies on the line $x - y = 0$

71. (a) $2x + y = 4$ (i)

$x - 3y = 9$ (ii)

$3 \times (i) + (ii)$ we get

$$\Rightarrow 7x = 21 \Rightarrow x = 21/7$$

$$\Rightarrow x = 3$$

From (i),

$$y = 4 - 6 \Rightarrow y = -2$$

The straight lines intersect at (3, -2)

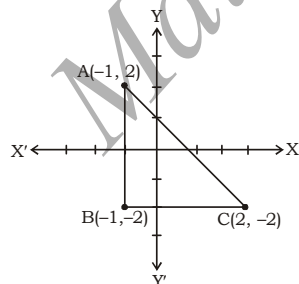
Alternatively:-

(a) Since the point (3, -2) satisfies both the equations. Put the values of x, y in the given equations. You get LHS = RHS

72. (c) $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has a unique solution if

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{k}{9} \neq \frac{1}{3} \Rightarrow k \neq 3$$

73. (a)



Here, $AB = 4$, $BC = 3$

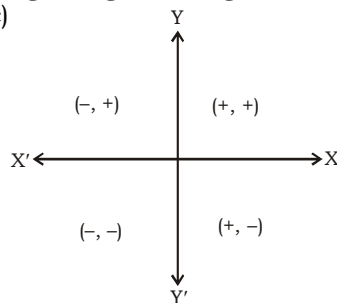
$$\text{and } AC = \sqrt{(2+1)^2 + (-2+2)^2}$$

$$= \sqrt{9+16} = 5$$

$$\text{i.e. } AC^2 = AB^2 + BC^2$$

hence, the figure represents right angled triangle.

74. (c)



$$ab > 0$$

Hence, a and b are both positive or negative.

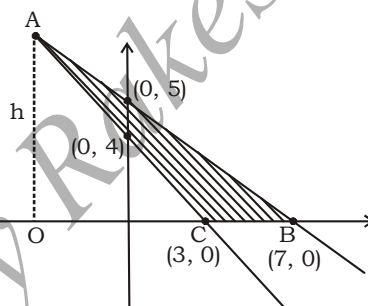
$\therefore$ If (a, b) lies in first quadrant, $(-b, -a)$ will lie in third quadrant.

75. (a) $5x + 7y = 35$

$$\text{or } \frac{x}{7} + \frac{y}{5} = 1 \text{(i)}$$

$$\text{and } 4x + 3y = 12$$

$$\text{or } \frac{x}{3} + \frac{y}{4} = 1 \text{(ii)}$$



$$\therefore \text{Base of } \Delta ABC = BC = 7 - 3 = 4 \text{ unit}$$

let height (OA) = h

$$\text{from (i) and (ii) we get } y = \frac{80}{13}$$

$$\therefore \text{ar}(\Delta ABC)$$

$$= \frac{1}{2} \times 4 \times \frac{80}{13} = \frac{160}{13} \text{ sq.unit}$$

76. (b) $3x - 6y = 12$ (i)

$$3x - y = 3 \text{(ii)}$$

$$(i) - (ii) \text{ we get, } -5y = 9$$

$$\text{or } y = -\frac{9}{5}$$

$$\Rightarrow \text{Height of triangle} = \left| \frac{-9}{5} \right| = \frac{9}{5}$$

put $y = 0$ in (i), we get $x = 4$

i.e. point of intersection of (i) on x -axis = (4, 0)

put $y = 0$ in (ii), we get $x = 1$

i.e. point of intersection of (ii) on x -axis = (1, 0)

$$\therefore \text{Base} = 4 - 1 = 3$$

$$\text{area of} = \frac{1}{2} \times 3 \times \frac{9}{5}$$

$$= \frac{27}{10} = 2.7 \text{ sq.unit}$$

77. (c) Consider any point in the shaded portion

let (3, -3)

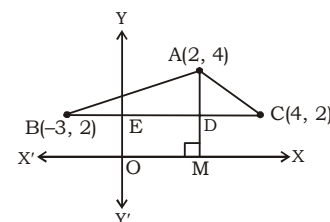
$$\text{i.e. } x = 3, y = -3 \Rightarrow -3 < 3 \text{ i.e. } y < x$$

$$\& \text{ when } x = 3, y = 3 \Rightarrow 3 = 3 \text{ i.e.}$$

$$y = x \therefore y \leq x$$

78. (a) Draw $AM \perp x$ -axis meeting BC at D .

Now, $BC = BE + EC = 3 + 4 = 7$ units

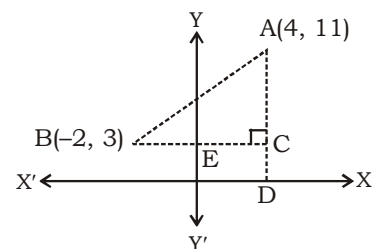


$$\text{and } AD = AM - DM = 4 - 2 = 2 \text{ units}$$

$$\therefore \text{ar}(\Delta ABC)$$

$$= \frac{1}{2} \times 7 \times 2 = 7 \text{ sq.unit}$$

79.(d)



$$AC = AD - CD = 11 - 3 = 8 \& BC = BE + EC = 2 + 4 = 6$$

$$\therefore AB = \sqrt{8^2 + 6^2} = 10$$

Alternatively:-

Required distance =

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

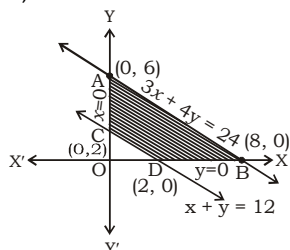
$$\therefore AB = \sqrt{(4+2)^2 + (11-3)^2}$$

$$= \sqrt{6^2 + 8^2}$$

$$\text{or } AB = \sqrt{100} = 10$$

$$80.(c) \quad 3x + 4y = 24 \text{ or } \frac{x}{8} + \frac{y}{6} = 1$$

i.e. it passes through (8, 0) and (0, 6)



Now, $x + y = 2$

put $x = 0$, $0 + y = 2$ or $y = 2$

again put $y = 0$, we get $x = 2$

i.e. it passes through (2, 0) & (0, 2)

$\therefore$ the required region is ABDC

Now, $\text{ar}(\triangle AOB)$

$$= \frac{1}{2} \times OB \times OA$$

$$= \frac{1}{2} \times 8 \times 6 = 24 \text{ sq. units}$$

and $\text{ar}(\triangle OCD)$

$$= \frac{1}{2} \times OD \times OC$$

$$= \frac{1}{2} \times 2 \times 2 = 2 \text{ sq. units}$$

$$\therefore \text{ar}(\square ABDC) = 24 - 2$$

$$= 22 \text{ sq. units}$$

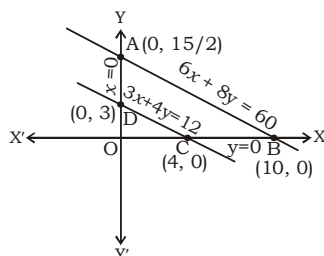
$$81.(b) \quad 3x + 4y = 12 \text{ or } \frac{x}{4} + \frac{y}{3} = 1$$

i.e. it passes through (4, 0) and (0, 3)

Similarly, $6x + 8y = 60$ or

$$\frac{x}{10} + \frac{y}{15/2} = 1$$

i.e. it passes through (10, 0) and (0, 15/2)



$\therefore$ the required region is ABCD
Now,

$$\text{ar}(\triangle OAB) = \frac{1}{2} \times 10 \times \frac{15}{2}$$

$$= 37.5 \text{ sq. units}$$

$$\text{and } \text{ar}(\triangle ODC) = \frac{1}{2} \times 4 \times 3$$

$$= 6 \text{ sq. units}$$

$$\therefore \text{ar}(\square ABCD) = 37.5 - 6$$

$$= 31.5 \text{ units}$$

$$82.(d) \quad x - y = 0 \text{ or } x = y$$

i.e. when $y = 3 \Rightarrow x = 3$

and $x + 2y = 0$

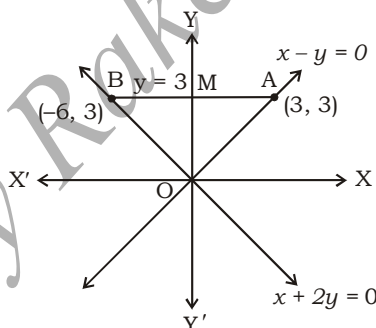
when $y = 3 \Rightarrow x = -6$

$\therefore$ the required region is a $\triangle OAB$

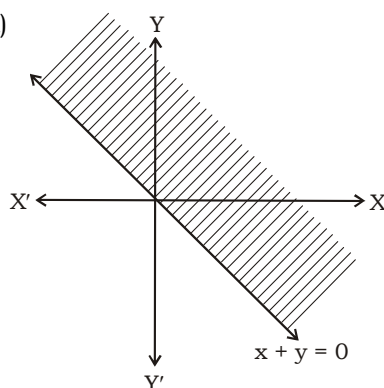
$$\therefore \text{ar}(\triangle OAB) = \frac{1}{2} \times AB \times OM$$

$$= \frac{1}{2} \times 9 \times 3$$

$$= \frac{27}{2} = 13.5 \text{ sq. units}$$



83.(a)



Consider the line $x + y = 0$
when, $x = 0$, $y = 0$ and
when, $x = 1$, $y = -1$

Now join the points (0, 0) & (1, -1) to get a line $x + y = 0$

Now, consider any point (1, 1)

Clearly (1, 1) satisfies the

inequality, $x + y \geq 0$

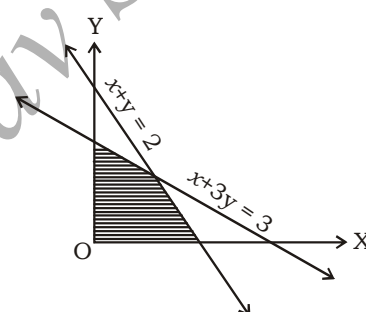
Shade the part of the plane containing (1, 1)

when $x \geq 0$, clearly, the first quadrant will be include as a whole.

84.(d) Clearly (0, 0) satisfies

$x + 3y \leq 3$

and (0, 0) satisfies $x + y \leq 2$



Clearly, shaded region is the portion common to the line $x + 3y = 3$ & below it and that of the line $x + y = 2$ and below it. So shaded region is the solution set of (d)

85.(b) Clearly, the region is a rectangle bounded by the lines

$x = 2$, $x = 5$, $y = -1$ & $y = 3$

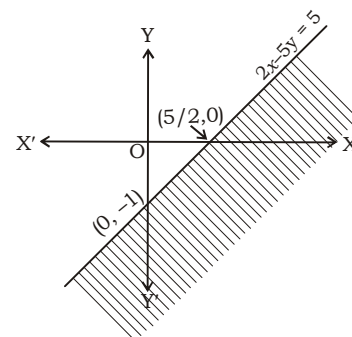
Clearly, the length of rectangle = $5 - 2 = 3$ units

and the breadth of rectangle = $3 - (-1) = 4$ units

$$\therefore \text{Area of the rectangle} = 3 \times 4$$

$$= 12 \text{ sq. unit.}$$

$$86.(c) \quad 2x - 5y = 5, \text{ or } \frac{x}{5/2} + \frac{y}{-1} = 1$$



i.e. it passes through (5/2, 0) & (0, -1). Plot these points and join them with a thick line.

Clearly, (0, 0) does not satisfy $2x - 5y \geq 5$.

- ∴ Shade the portion of the plane not containing (0, 0)
So, required graph is on & below the line $2x - 5y = 5$

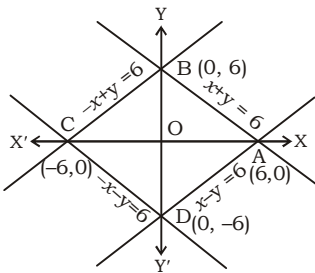
87.(a) As we know, area bounded by $|x| + |y| = k$ is $2k^2$

- ∴ area bounded by $|x| + |y| = 6$ is $2 \times 6^2 = 72$ sq. units

Alternatively :

$|x| + |y| = 6$, this represents four lines -

$x + y = 6$, $x - y = 6$, $-x + y = 6$ and $-x - y = 6$



Hence, the required region is ABCD.

$$\text{Now ar}(\Delta OAB) = \frac{1}{2} \times OA \times OB$$

$$= \frac{1}{2} \times 6 \times 6$$

$$= 18$$

$$\therefore \text{ar}(\square ABCD) = 4 \times \text{ar}(\Delta OAB)$$

$$= 4 \times 18$$

$$= 72 \text{ sq. units}$$

88.(c) $y = |x| - 1$ represents two lines.
 $y = x - 1$ (i)

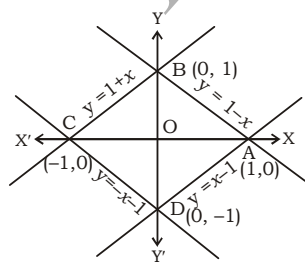
and $y = -x - 1$ (ii)

Similarly, $y = 1 - |x|$ represents two lines

$y = 1 - x$ (iii)

and $y = 1 + x$ (iv)

Now, draw the graph of (i), (ii), (iii) & (iv)



∴ Required region is ABCD

$$\text{Now ar}(\Delta OAB) = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$$

$$\therefore \text{ar}(\square ABCD) = 4 \times \text{ar}(\Delta OAB)$$

$$= 4 \times \frac{1}{2} = 2 \text{ sq. units}$$

89.(b) The point (-5, 7) lies in the second quadrant.

90.(d) The point (7, -5) lies in the fourth quadrant.

91.(a) Distance between two points

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

here $(x_1, y_1) = (-6, 2)$ and $(x_2, y_2) = (2, 4)$

∴ Required distance

$$= \sqrt{(-6 - 2)^2 + (2 - 4)^2}$$

$$= \sqrt{64 + 4} = \sqrt{68} = 2\sqrt{17} \text{ unit}$$

$$92.(b) AB = \sqrt{(b - 0)^2 + (0 - a)^2}$$

$$= \sqrt{b^2 + a^2} = \sqrt{a^2 + b^2}$$

$$93.(d) AB^2 = (7 - 3)^2 + (4 - 1)^2$$

$$= 4^2 + 3^2$$

$$= 16 + 9 = 25$$

$$\Rightarrow AB = \sqrt{25} = 5 \text{ units}$$

94.(b) Clearly, the point of x -axis has ordinate 5 and abscissa 0.

So, the point is (5, 0)

95.(a) Clearly, the point on y -axis has abscissa 0.

So, the point is (0, 7)

96.(c) Clearly, the point is (0, -6)

97.(d) Required distance

$$= \sqrt{(6 - 0)^2 + (-8 - 0)^2}$$

$$= \sqrt{36 + 64} = \sqrt{100} = 10 \text{ units}$$

$$98.(a) 2x + 7y = 1 \text{ (i)}$$

$$4x + 5y = 11 \text{(ii)}$$

on solving (i) and (ii), we get $x = 4$ and $y = -1$

∴ Required point of intersection = (4, -1)

99.(c) Equation of x -axis is $y = 0$

put $y = 0$ in $4x + 7y$

$$= 12 \text{ we get } x = 3$$

∴ Required point = (3, 0)

100.(b) Equation of y -axis is $x = 0$

put $x = 0$ in $4x - 9y$

$$= 11 \text{ we get } y = -\frac{11}{9}$$

$$\therefore \text{ Required point} = \left(0, -\frac{11}{9}\right)$$

$$101.(c) 3x + 7y + 8 = 0 \Rightarrow 7y = -3x - 8$$

$$\Rightarrow y = \left(-\frac{3}{7}\right)x - \left(\frac{8}{7}\right)$$

$$\therefore \text{ Slope of the line is } = -\frac{3}{7}$$

$$102.(c) \text{ Slope of PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3 - 7}{2 - (-4)} = \frac{-4}{6} = -\frac{2}{3}$$

103.(d) Clearly; the equation of the line is, $y = 6$

104.(b) Clearly, the equation of the line is, $x = -5$

105.(a) Clearly, the equation of the line is $y = -7$

106.(c)

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3$$

$$(y_1 - y_2)]$$

$$\Delta = \frac{1}{2} |4(4 + 8) - 3(-4 - 5) + 3(5 - 8)|$$

$$= \frac{1}{2} |66| = 33 \text{ sq. units}$$

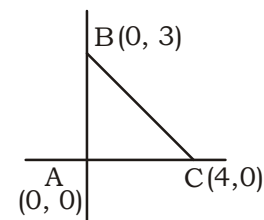
$$107.(b) AB = \sqrt{(0 + 0)^2 + (0 - 3)^2} = 3$$

$$AC = \sqrt{(4 - 0)^2 + (0 + 0)^2} = 4$$

$$\text{and } BC = \sqrt{(4 - 0)^2 + (0 - 3)^2} = 5$$

$$\therefore AB^2 + AC^2 = BC^2$$

∴ ΔABC is a right angled triangle.



108.(d) The co-ordinates of the centroid of ΔPQR are -

$$\left(\frac{-2 + 9 + 8}{3}, \frac{0 - 3 + 3}{3}\right) = (5, 0)$$

109.(a) The required equation is

$$(y+3) = \frac{2+3}{-5-0}(x-0)$$

$$\Rightarrow y+3 = -x \Rightarrow x+y+3 = 0$$

110.(c) Length of perpendicular =

$$= \frac{12 \times 0 + 5 \times 0 + 7}{\sqrt{12^2 + 5^2}} = \frac{7}{13} \text{ units}$$

111.(a) The slope of the line is

$$\frac{\sqrt{5}-1}{\sqrt{15}-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \tan \theta = \frac{1}{\sqrt{3}} \text{ or } \theta = 30^\circ$$

112.(c) Here,

$$\frac{a_1}{a_2} = \frac{2}{8} = \frac{1}{4}, \frac{b_1}{b_2} = \frac{-5}{-20} = \frac{1}{4}$$

$$\text{and } \frac{c_1}{c_2} = \frac{7}{28} = \frac{1}{4}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2},$$

So the given lines are coincident.

$$113.(b) \sqrt{(x-a)^2 + (y-0)^2} = a+x$$

$$\Rightarrow (x-a)^2 + y^2 = (a+x)^2$$

$$\Rightarrow y^2 = (x+a)^2 - (x-a)^2$$

$$\Rightarrow y^2 = 4ax$$

114.(c) Let P (x, y), Q(a + b, b - a) and R(a - b, a + b) are given points.

$$\therefore PQ = PR.$$

$$\Rightarrow \sqrt{\{x-(a+b)\}^2 + \{y-(b-a)\}^2}$$

$$= \sqrt{\{x-(a-b)\}^2 + \{y-(a+b)\}^2}$$

$$\Rightarrow x^2 - 2x(a+b) + (a+b)^2 + y^2 - 2y(b-a) + (b-a)^2 = x^2 + (a-b)^2 - 2x(a-b) + y^2 + (a+b)^2 - 2y(a+b)$$

$$\Rightarrow ax + bx + by - ay = ax - bx + ay + by$$

$$\Rightarrow 2bx = 2ay$$

$$\Rightarrow bx = ay$$

115.(d) Let A (x, y), P(a, 0) and Q(-a, 0), Then,

$$\Rightarrow AP^2 + AQ^2 = 2b^2$$

$$\Rightarrow [(x-a)^2 + (y-0)^2] + [(x+a)^2 + (y-0)^2] = 2b^2$$

$$\Rightarrow x^2 + a^2 - 2ax + y^2 + x^2 + a^2 + 2ax + y^2 = 2b^2$$

$$\Rightarrow 2(x^2 + a^2 + y^2) = 2b^2$$

$$\Rightarrow x^2 + a^2 + y^2 = b^2$$

$$\Rightarrow x^2 + a^2 = b^2 - y^2$$

116.(b) co-ordinates of middle point = (-1, 4)

$$\therefore \frac{a+a+4}{2} = 4$$

$$\Rightarrow 2a+4 = 8$$

$$\Rightarrow 2a = 4$$

$$a = 2$$

117(c) Since, P, Q and R collinear

$$\therefore \text{slope of PQ} = \text{slope of PR}$$

$$\Rightarrow \frac{a-3}{5-2} = \frac{7-3}{6-2} = \frac{a-3}{3} = \frac{4}{4}$$

$$\Rightarrow a-3 = 3 \Rightarrow a = 6$$

118.(a) The equation of a line parallel to x-axis is y = b.

Since, it passes through (-6, -5), so b = -5

$$\therefore \text{The required equation is, } y = -5$$

119.(a) The equation of a line parallel to y-axis is, x = a.

Since, it passes through (2, -5), so a = 2

$$\therefore \text{The required equation is, } x = 2$$

120.(b) Let the co-ordinates of R be (x, y). Then,

$$\frac{-1+5+x}{3} = 4 \text{ and } \frac{0-2+y}{3} = 0$$

$$\text{or } 4+x = 12 \text{ and } -2+y = 0$$

$$\text{or } x = 8 \text{ and } y = 2$$

$$\therefore R = (x, y) = (8, 2)$$

121.(c) Since, point of intersection of median is "centroid".

$$\therefore \text{co-ordinates of centroid}$$

$$= \left(\frac{0+5+7}{3}, \frac{6+3+3}{3} \right)$$

$$= \left(\frac{12}{3}, \frac{12}{3} \right) = (4, 4)$$

122.(b) Let the ratio be k : 1

The ordinate of a point lying on x-axis must be zero

$$\therefore \frac{4k-5 \times 1}{k+1} = 0 \Rightarrow 4k = 5 \Rightarrow k = \frac{5}{4}$$

$$\therefore \text{Required ratio is } \frac{5}{4} : 1 = 5 : 4$$

123.(a) Let the ratio be k : 1

The abscissa of a point lying on y-axis must be zero

$$\therefore \frac{7k-3 \times 1}{k+1} = 0$$

$$\Rightarrow 7k-3 = 0$$

$$\Rightarrow k = \frac{3}{7}$$

$$\therefore \text{Required ratio is } \frac{3}{7} : 1 = 3 : 7$$

124.(c) Let the ratio be k : 1

$$\therefore \frac{3k-3 \times 1}{k+1} = 1$$

$$\Rightarrow 3k-3 = k+1$$

$$\Rightarrow 2k = 4 \Rightarrow k = 2$$

$$\therefore \text{Required ratio is } 2 : 1$$

125.(a) Let the equation be y = 5x + c

Since it passes through (-4, 1), we have 1 = 5(-4) + c

$$\therefore c = 21 \text{ so, its equation is,}$$

$$y = 5x + 21$$

126.(c) Condition of parallelism

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\therefore \frac{1}{a} = \frac{3}{12} \Rightarrow a = 4$$

Alternatively,

$$x+3y-8+0$$

$$\Rightarrow y = \left(-\frac{1}{3} \right) x + \left(\frac{8}{3} \right)$$

$$\therefore m_1 = -\frac{1}{3}$$

$$ax+12y+5+0 = 0$$

$$\Rightarrow y = \left(-\frac{a}{12} \right) x - \frac{5}{12}$$

$$\therefore m_2 = \frac{a}{12}$$

$$\text{for parallelism, } m_1 = m_2$$

$$\therefore -\frac{1}{3} = -\frac{a}{12} \Rightarrow a = 4$$

127.(b) Condition of perpendicularism,

$$a_1 a_2 + b_1 b_2 = 0$$

$$\therefore 3 \times 24 + 8 \times p = 0$$

$$\Rightarrow 8p = -3 \times 24 \Rightarrow p = -9$$

Alternatively

$$3x+8y+9=0 \Rightarrow$$

$$y = \left(-\frac{3}{8} \right) x - \frac{9}{8} \therefore m_1 = -\frac{3}{8}$$

$$24x + py + 19 = 0$$

$$\Rightarrow y = \left(-\frac{24}{p}\right)x - \frac{19}{p}$$

$$\therefore m_2 = \frac{24}{p}$$

for perpendicularism, $m_1 \cdot m_2 = -1$

$$\therefore \left(-\frac{3}{8}\right) \left(-\frac{24}{p}\right) = -1$$

$$\Rightarrow p = -9$$

128.(d) $m_1 = \text{Slope of PQ}$

$$= \frac{-7-5}{0+2} = \frac{-12}{2} = -6$$

$$m_2 = \text{Slope of AB} = \frac{a+2}{8+4} = \frac{a+2}{12}$$

$$\therefore m_1 m_2 = -1 \Rightarrow -6 \times \frac{a+2}{12} = -1$$

$$\Rightarrow a + 2 = 2 \Rightarrow a = 0$$

$$129.(a) \quad 2y - \sqrt{12}x - 9 = 0$$

$$\Rightarrow y = \frac{\sqrt{12}}{2}x + \frac{9}{2}$$

$$\Rightarrow y = \frac{\sqrt{12}}{2}x + \frac{9}{2}$$

$$\Rightarrow m_1 = \frac{\sqrt{12}}{2} = \sqrt{3}$$

$$\sqrt{3}y - x + 7 = 0$$

$$\Rightarrow m_2 = \frac{1}{\sqrt{3}}$$

$$\therefore \tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}} = \frac{1}{\sqrt{3}}$$

$$\text{So, } \theta = 30^\circ$$

130. (b) Slope of PQ,

$$m_1 = \frac{5-5}{4-3} = 0$$

$$\text{Slope of PR, } m_2 = \frac{6-5}{4-3} = 1$$

$$\therefore \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{0-1}{1+0} \right| = 1$$

$$\text{So, } \theta = 45^\circ$$

131. (c) Clearly, M is the mid-point of QR.

$\therefore$ Co-ordinates of M are

$$\left(\frac{-3-1}{2}, \frac{7-3}{2} \right)$$

i.e. M (-2, 2)

Now, find the equation of the line joining P(2, 3) and M(-2, 2)

Required equation is, $(y-3)$

$$= \frac{2-3}{-2-2} (x-2)$$

$$\Rightarrow y-3 = \frac{1}{4} (x-2)$$

$$\Rightarrow 4y-12 = x-2$$

$$\Rightarrow x-4y+10=0$$

132. (a) Required point is :

$$\left(\frac{3 \times 3 + 2 \times \frac{11}{2}}{3+2}, \frac{3(-2) + 2 \times \frac{21}{2}}{3+2} \right)$$

$$= \left(\frac{20}{5}, \frac{15}{5} \right) = (4, 3)$$

133. (d) Point of intersection at x-axis

$$= (x, 0)$$

$$\therefore 8x + 15y = 120$$

$$\Rightarrow 8x + 15 \times 0 = 120 \Rightarrow x = 15$$

$$\Rightarrow \text{point of intersection} = (15, 0)$$

Point of intersection at y-axis

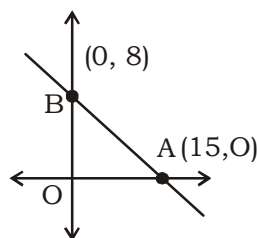
$$= (0, y)$$

$$\therefore 8x + 15y = 120$$

$$\Rightarrow 0 + 15y = 120 \Rightarrow y = 8$$

$$\Rightarrow \text{Point of intersection} = (0, 8)$$

$$\therefore \text{Required length} = AB$$



$$= \sqrt{(15-0)^2 + (0-8)^2}$$

$$= \sqrt{225 + 64} = \sqrt{289}$$

$$= 17 \text{ units}$$

134.(d) Given line - $3x + 4y - 5 = 0$

$$\Rightarrow y = \left(-\frac{3}{4}\right)x + \frac{5}{4}$$

$$\therefore \text{its slope, } m_1 = -\frac{3}{4}$$

Let m_2 be the slope of required line.

$$\text{Then, } m_1 m_2 = -1 \text{ or } \left(-\frac{3}{4}\right) m_2 = -1$$

$$\Rightarrow m_2 = \frac{4}{3}$$

Let the required equation be, $y = m_2 x + c$

$$\Rightarrow y = \frac{4}{3}x + c$$

Since, it passes through (1, 1)

$$\therefore 1 = \frac{4}{3} \times 1 + c \Rightarrow c = 1 - \frac{4}{3} = -\frac{1}{3}$$

$\therefore$ The required equation is ,

$$y = \frac{4}{3}x - \frac{1}{3}$$

$$\text{or } 4x - 3y - 1 = 0$$

135.(b) $2x - 5y + 3 = 0$

$$\Rightarrow y = \left(\frac{2}{5}\right)x + \left(\frac{3}{5}\right)$$

$$\therefore \text{its slope } m_1 = \frac{2}{5}$$

Let the slope of line which is parallel to the given line is m_2

$$\therefore m_2 = m_1 = \frac{2}{5}$$

Let the required equation be,

$$y = \frac{2}{5}x + c$$

Since, it passes through (5, 3)

$$\therefore 3 = \frac{2}{5} \times 5 + c \Rightarrow c = 1$$

Required equation is,

$$y = \frac{2}{5}x + 1$$

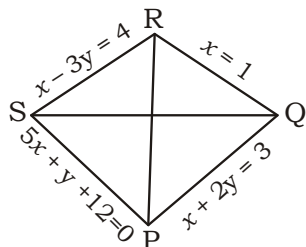
$$\text{or } 2x - 5y + 5 = 0$$

136.(d) $x + 2y = 3$ (i)

$$5x + y = -12$$
 (ii)

On solving (i) and (ii), we get $x = -3, y = 3$

∴ co-ordinates of P(-3, 3)



Similarly, Q(1, 1), R(1, -1) and S(-2, 2)

Now, m_1 = slope of PR

$$= \frac{-1-3}{1+3} = -1$$

m_2 = slope of QS

$$= \frac{-2-1}{-2-1} = 1$$

$$\therefore m_1 m_2 = -1$$

∴ the required angle is 90°

137.(c) ∴ Third side passes through (1, -10)

so its equation $y + 10 = m(x - 1)$ (i)

This side makes equal angle with the given two sides.

let this angle be θ .

Now, slope of line $7x - y + 3 = 0$ is m_1 ,

$$\therefore m_1 = 7$$

and slope of line $x + y - 3 = 0$ is m_2 ,

$$\therefore m_2 = -1$$

angle between (i) and $7x - y + 3 = 0$ = angle between (i) and $x + y - 3 = 0$

$$\therefore \tan \theta = \frac{m-7}{1+7m} = \frac{m-(-1)}{1+m(-1)}$$

$$\Rightarrow m = -3 \text{ or } 1/3$$

Hence possible equations of third side are $y + 10 = -3(x - 1)$

$$\text{and } y + 10 = \frac{1}{3}(x - 1)$$

$$\text{or } 3x + y + 7 = 0$$

$$\text{and } x - 3y - 31 = 0$$

138.(a) P_1 = length of perpendicular from (0, 0) on $x \sec \theta + y \csc \theta = a$

$$\therefore P_1 = \frac{a}{\sqrt{\sec^2 \theta + \csc^2 \theta}}$$

$$= \frac{a}{\sqrt{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}}}$$

$$= a \sin \theta \cdot \cos \theta$$

$$\text{or } 2P_1 = a(2 \sin \theta \cdot \cos \theta)$$

$$\Rightarrow 2P_1 = a \sin 2\theta$$

Similarly,

$$P_2 = \frac{a \cos 2\theta}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = a \cos 2\theta$$

$$\therefore 4P_1^2 + P_2^2 = a^2$$

$$(\sin^2 2\theta + \cos^2 2\theta) = a^2$$

139.(d) Let a and b are the intercepts on x and y-axes respectively.

$$\therefore a + b = 9 \Rightarrow b = 9 - a \text{(i)}$$

and the equation of the line is

$$\frac{x}{a} + \frac{y}{b} = 1 \text{(ii)}$$

From (i) and (ii)

$$\frac{x}{a} + \frac{y}{9-a} = 1 \text{(iii)}$$

this line also passes through the point (2, 2)

$$\therefore \text{from (iii)} \frac{2}{a} + \frac{2}{9-a} = 1$$

On solving we get $a = 6$

$$\text{or } a = 3$$

$$\text{If } a = 6 \text{ then } b = 9 - 6 = 3$$

$$\therefore \text{equation of the line is } \frac{x}{6} + \frac{y}{3} = 1$$

$$\text{or } x + 2y - 6 = 0$$

$$\text{If } a = 3 \text{ then } b = 9 - 3 = 6$$

$$\therefore \text{equation of the line is } \frac{x}{3} + \frac{y}{6} = 1$$

$$\text{or } 2x + y - 6 = 0$$

Hence, required equation is

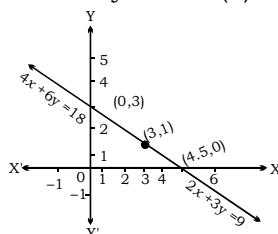
$$x + 2y - 6 = 0 \text{ or } 2x + y - 6 = 0$$

Note : Solve this type of question with the help of given options.

140. Let the C.P. of Pencil and Eraser be x and y

$$2x + 3y = 9 \text{(i)}$$

$$4x + 6y = 18 \text{(ii)}$$



After solving equation (i)

x	0	4.5
y	3	0

$$2x + 3y = 9$$

After solving equation(ii)

x	0	3
y	3	1

$$4x + 6y = 18$$

141. let the present age of Aftab and his daughter will be x and y respectively.

	Aftab	Daughter
Present age	x	y

Before 7 yrs,

$$x - 7 = 7(y - 7) \text{(i)}$$

Present age

After 3 yrs,

$$x + 3 = 3(y + 3) \text{(ii)}$$

After solving eq. (i)

$$= x - 7 = 7y - 49$$

$$x - 7y = -42$$

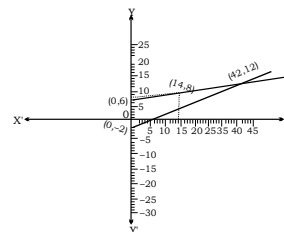
x	0	14
y	6	8

After Eq. (ii)

$$= x + 3 = 3y + 9$$

$$x - 3y = 6$$

x	0	6
y	-2	0



The two lines intersect each other at (42, 12) Present age of after and his daughter will be 42 yrs and 12 yrs respectively

142. Cost of 1 Pen be x and 1 Pencil be y

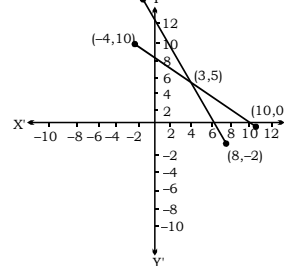
$$5x + 7y = 50 \text{(i)}$$

$$7x + 5y = 46 \text{(ii)}$$

From eq. (i) value of (x, y) after solving = P(x, y) = (10, 0), (-4, 10)

eq.(ii)

$$P(x, y) = (8, -2), (-2, 12)$$

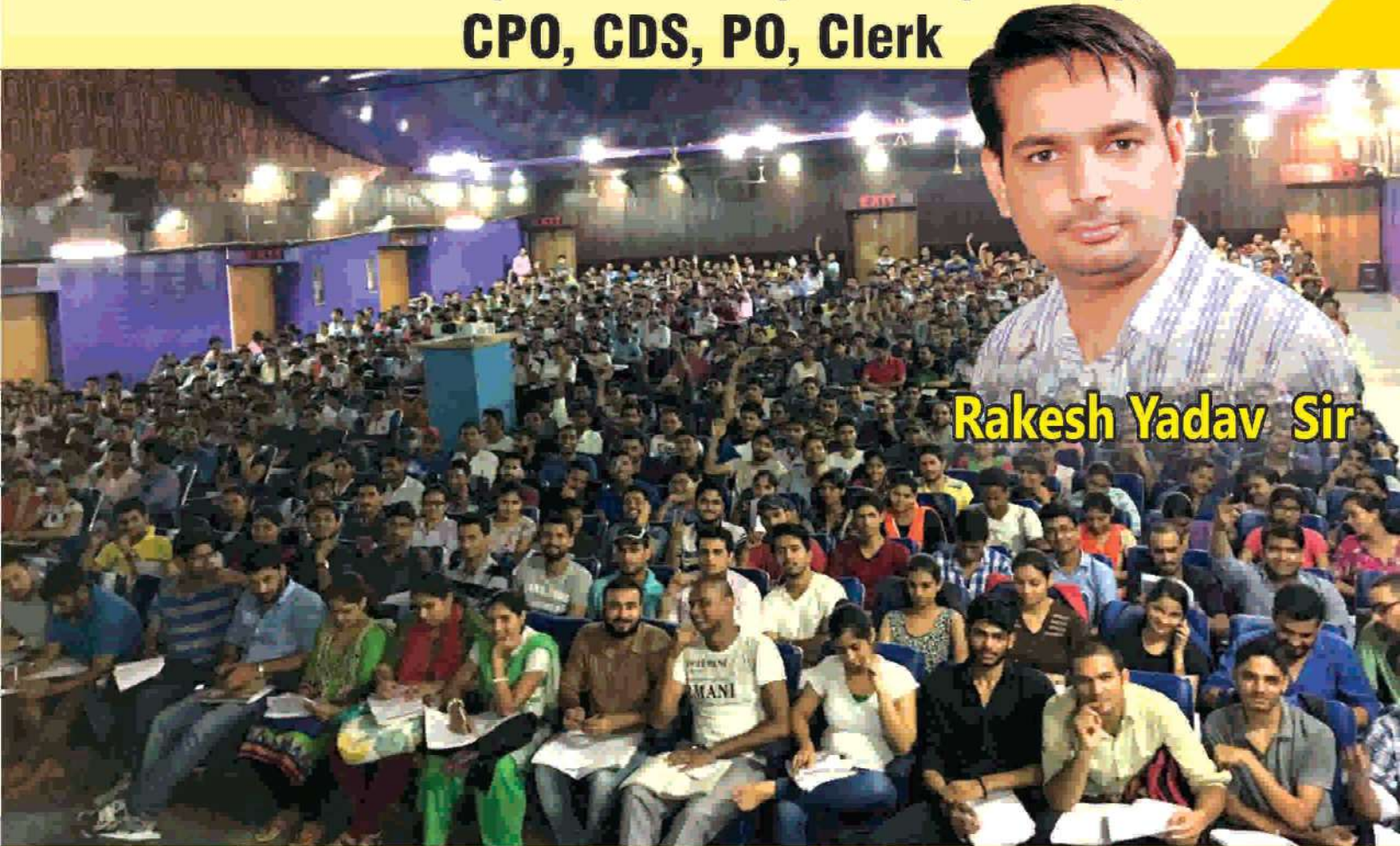


Cost of Pen = ₹ 3
Pencil = ₹ 5

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